Processing and Ellipsoid adjusting of r-tables

M1 CSMI Internship

August 23rd, 2023

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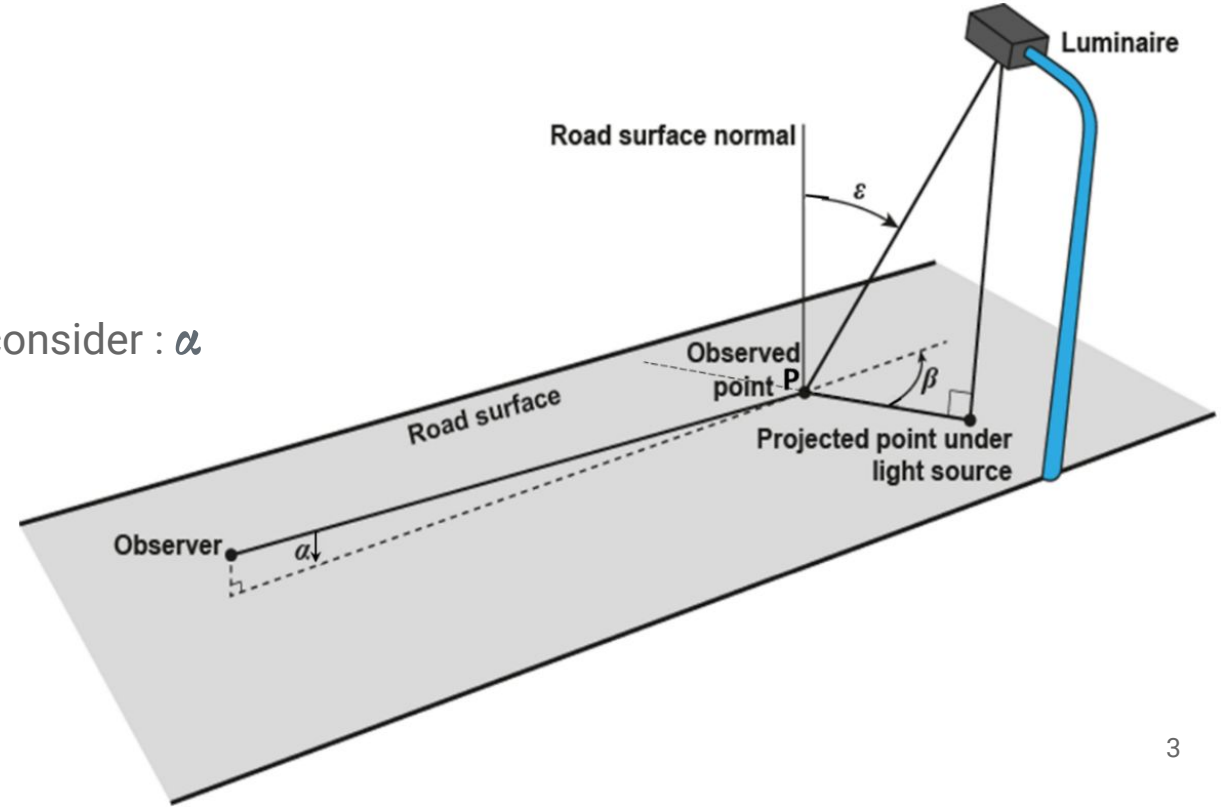
R-tables

Angles :

- 20 values of β
- 29 values of tan(ε)
- For all measurements, we consider : α = 1°



R-table measurements

Standard r-table:

beta / tan gamma	0	2	5	10	15	20	25	30	35	40	45	60	75	90	105	120	135	150	165	180
0	655	655	655	655	655	655	655	655	655	655	655	655	655	655	655	655	655	655	655	655
0,25	619	619	619	619	610	610	610	610	610	610	610	610	610	601	601	601	601	601	601	601
0,5	539	539	539	539	539	539	521	521	521	521	521	503	503	503	503	503	503	503	503	503
0,75	431	431	431	431	431	431	431	431	431	431	395	386	371	371	371	371	371	386	395	395
1	341	341	341	341	323	323	305	296	287	287	278	269	269	269	269	269	269	278	278	278
1,25	269	269	269	260	251	242	224	207	198	189	189	180	180	180	180	180	189	198	207	224
1,5	224	224	224	215	198	180	171	162	153	148	144	144	139	139	139	144	148	153	162	180
1,75	189	189	189	171	153	139	130	121	117	112	108	103	99	99	103	108	112	121	130	139
2	162	162	157	135	117	108	99	94	90	85	85	83	84	84	86	90	94	99	103	111
2,5	121	121	117	95	79	66	60	57	54	52	51	50	51	52	54	58	61	65	69	75
3	94	94	86	66	49	41	38	36	34	33	32	31	31	33	35	38	40	43	47	51
3,5	81	80	66	46	33	28	25	23	22	22	21	21	22	22	24	27	29	31	34	38
4	71	69	55	32	23	20	18	16	15	14	14	14	15	17	19	20	22	23	25	27
4,5	63	59	43	24	17	14	13	12	12	11	11	11	12	13	14	14	16	17	19	21
5	57	52	36	19	14	12	10	9,0	9,0	8,8	8,7	8,7	9,0	10	11	13	14	15	16	16
5,5	51	47	31	15	11	9,0	8,1	7,8	7,7	7,7										
6	47	42	25	12	8,5	7,2	6,5	6,3	6,2											
6,5	43	38	22	10	6,7	5,8	5,2	5,0												
7	40	34	18	8,1	5,6	4,8	4,4	4,2												
7,5	37	31	15	6,9	4,7	4,0	3,8													
8	35	28	14	5,7	4,0	3,6	3,2													
8,5	33	25	12	4,8	3,6	3,1	2,9													
9	31	23	10	4,1	3,2	2,8														
9,5	30	22	9,0	3,7	2,8	2,5														
10	29	20	8,2	3,2	2,4	2,2														
10,5	28	18	7,3	3,0	2,2	1,9														
11	27	16	6,6	2,7	1,9	1,7														
11,5	26	15	6,1	2,4	1,7															
12	25	14	5,6	2,2	1,6															

How R-values are computed :

- L : luminance
- E : illuminance

$$q = \frac{L}{E}$$

$$r = q \cdot cos^3(\epsilon) \cdot 10^4$$

The reflection indicatrix

R-table standard:

beta \ gamma	0	2	5	10	15	20	25	30	35	40	45	60	75	90	105	120	135	150	165	180
0	655	655	655	655	655	655	655	655	655	655	655	655	655	655	655	655	655	655	655	655
0,25	619	619	619	619	610	610	610	610	610	610	610	610	610	601	601	601	601	601	601	601
0,5	539	539	539	539	539	539	521	521	521	521	521	503	503	503	503	503	503	503	503	503
0,75	431	431	431	431	431	431	431	431	431	431	395	386	371	371	371	371	371	386	395	395
1	341	341	341	341	323	323	305	296	287	287	278	269	269	269	269	269	269	278	278	278
1,25	269	269	269	260	251	242	224	207	198	189	189	180	180	180	180	180	189	198	207	224
1,5	224	224	224	215	198	180	171	162	153	148	144	144	139	139	139	144	148	153	162	180
1,75	189	189	189	171	153	139	130	121	117	112	108	103	99	99	103	108	112	121	130	139
2	169	162	157	135	117	108	99	94	90	85	85	83	84	84	86	90	94	99	103	111
2,5	121	121	117	95	79	66	60	57	54	52	51	50	51	52	56	58	61	65	69	75
3	94	94	86	66	49	41	38	36	34	33	32	31	31	33	35	38	40	43	47	51
3,5	81	80	66	46	33	28	25	23	22	22	21	21	22	22	24	27	29	31	34	38
4	71	69	55	32	23	20	18	16	15	14	14	14	15	17	19	20	22	23	25	27
4,5	63	59	43	24	17	14	13	12	12	11	11	11	12	13	14	14	16	17	19	21
5	57	52	36	19	14	12	10	9,0	9,0	8,8	8,7	8,7	9,0	10	11	13	14	15	16	16
5,5	51	47	31	15	11	9,0	8,1	7,8	7,7	7,7										
6	47	42	25	12	8,5	7,2	6,5	6,3	6,2											
6,5	43	38	22	10	6,7	5,8	5,2	5,0												
7	40	34	18	8,1	5,6	4,8	4,4	4,2												
7,5	37	31	15	6,9	4,7	4,0	3,8													
8	35	28	14	5,7	4,0	3,5	3,2													
8,5	33	25	12	4,8	3,6	3,1	2,9													
9	31	23	10	4,1	3,2	2,8														
9,5	30	22	9,0	3,7	2,8	2,5														
10	29	20	8,2	3,2	2,4	2,2														
10,5	28	18	7,3	3,0	2,2	1,9														
11	27	16	6,6	2,7	1,9	1,7														
11,5	26	15	6,1	2,4	1,7															
12	25	14	5,6	2,2	1,6															

$$q = \frac{L}{E}$$

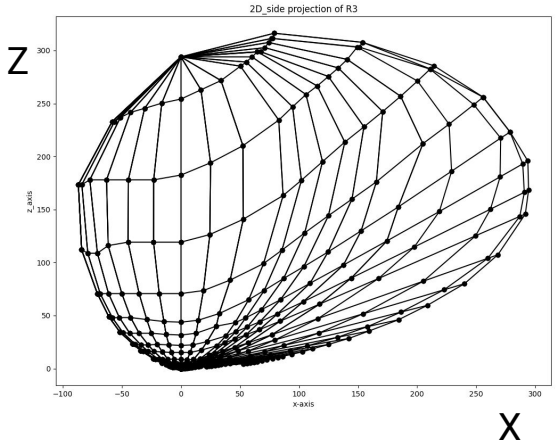
$$r = q \cdot cos^3(\epsilon) \cdot 10^4$$

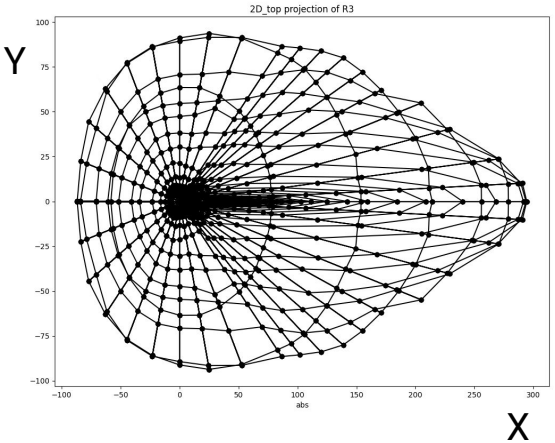
Projection :

$$x = r \cdot sin(\epsilon) \cdot cos(\beta)$$

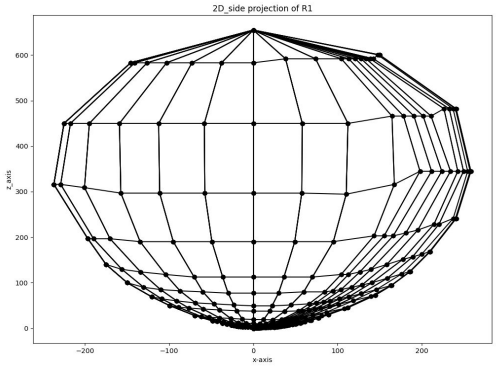
$$y = r \cdot sin(\epsilon) \cdot sin(\beta)$$

$$z = r \cdot cos(\epsilon)$$





How to sum up r-tables in fewer values while retaining the best possible information?



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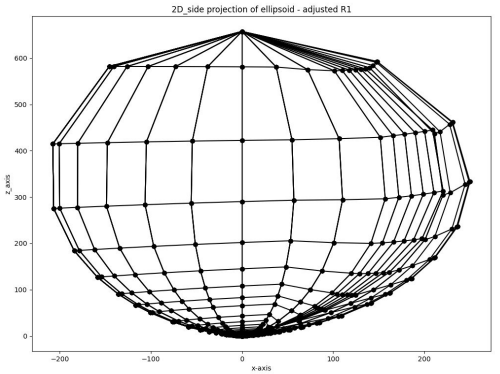

392 data points

Encoding

small data vector (size 15 ~ 30)

Decoding

Original Publication:

Ellipsoid-based approximation method for the estimation of the actual reduced luminance coefficients of road surfaces for accurate lighting simulations, by Ana Ogando-Martínez, Francisco Troncoso-Pastoriza, Enrique Granada-Álvarez, Pablo Eguía-Oller.

2020

Journal : Sustainable Cities and Society
www.elsevier.com/locate/scs

Dividing r-tables in 3 groups

beta / tan gamma	0	2	5	10	15	20	25	30	35	40	45	60	75	90	105	120	135	150	165	180
0	655	655	655	655	655	655	655	655	655	655	655	655	655	655	655	655	655	655	655	655
0,25	619	619	619	619	610	610	610	610	610	610	610	610	610	601	601	601	601	601	601	601
0,5	539	539	539	539	539	539	521	521	521	521	521	503	503	503	503	503	503	503	503	503
0,75	431	431	431	431	431	431	431	431	431	431	395	386	371	371	371	371	371	386	395	395
1	341	341	341	341	323	323	305	296	287	287	278	269	269	269	269	269	269	278	278	278
1,25	269	269	269	260	251	242	224	207	198	189	189	180	180	180	180	180	189	198	207	224
1,5	224	224	224	215	198	180	171	162	153	148	144	144	139	139	139	144	148	153	162	180
1,75	189	189	189	171	153	139	130	121	117	112	108	103	99	99	103	108	112	121	130	139
2	162	162	157	135	117	108	99	94	90	85	85	83	84	84	86	90	94	99	103	111
2,5	121	121	117	95	79	66	60	57	54	52	51	50	51	52	54	58	61	65	69	75
3	94	94	86	66	49	41	38	36	34	33	32	31	31	33	35	38	40	43	47	51
3,5	81	80	66	46	33	28	25	23	22	22	21	21	22	22	24	27	29	31	34	38
4	71	69	55	32	23	20	18	16	15	14	14	14	15	17	19	20	22	23	25	27
4,5	63	59	43	24	17	14	13	12	12	11	11	11	12	13	14	14	16	17	19	21
5	57	52	36	19	14	12	10	9,0	9,0	8,8	8,7	8,7	9,0	10	11	13	14	15	16	16
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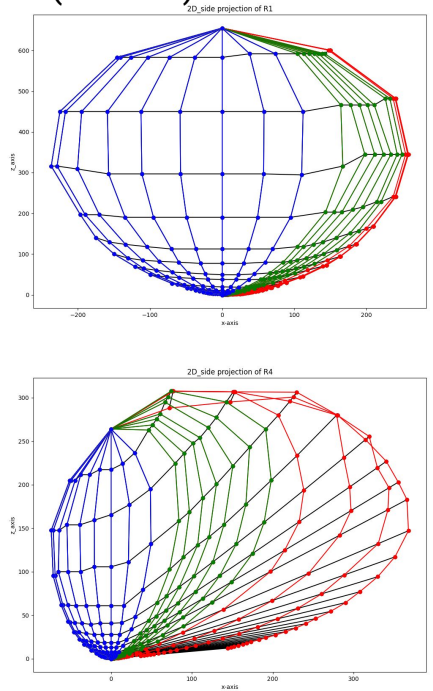
11,5	26	15	6,1	2,4	1,7															
12	25	14	5,6	2,2	1,6															

R1 (top) and R4 (bottom)

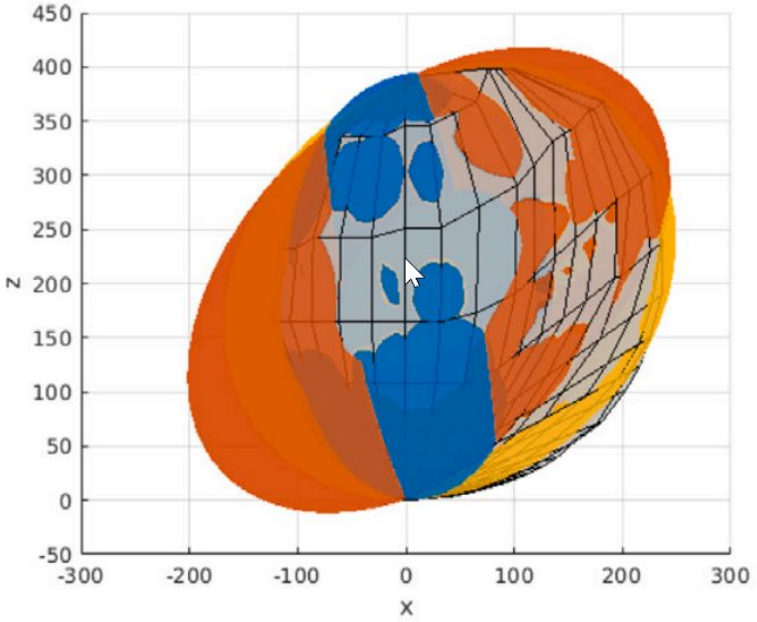


3 ellipsoid adjusting

Using a method of least squares to adjust one ellipsoid to each group of points:

Hypotheses made:

- symmetric about the y=0 plane,
- no constant term,
- no rotation around the X or Z axes



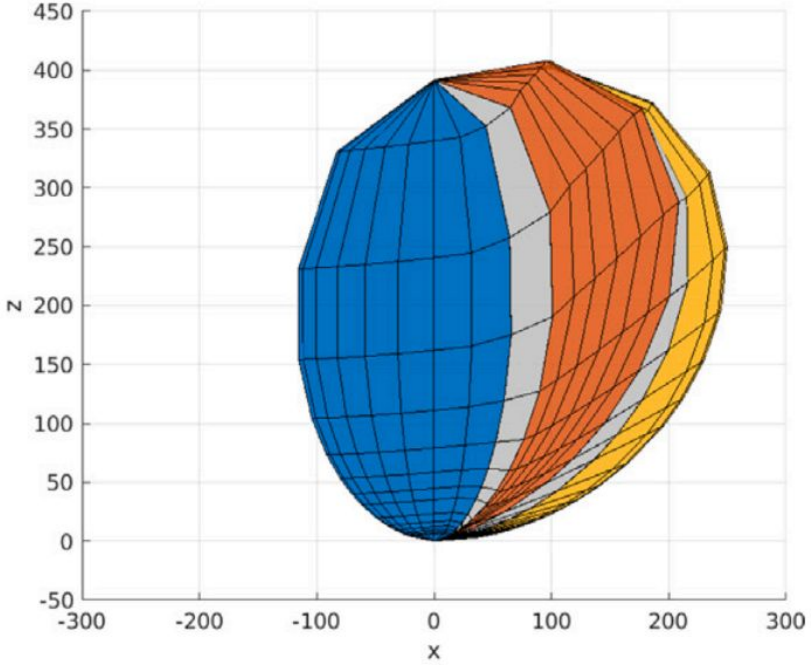
(b) Ellipsoid Fitting.

Adjusting 3 ellipsoids

Each ellipsoid is restricted to it's original group

- At the bounding zones between ellipsoids a rolling average is applied
- The top line is set to its own average everywhere, to ensure the isotropy hypothesis is respected

Compression rate : 580 / 15 = 38,66
Data saving rate : 97,4%



(c) R-table Reconstruction.

Implementation

Application to the databases:

- Standard CIE tables
- Kai Sørensen's measurements
- LCPC
- METAS

Total : 833 r-tables

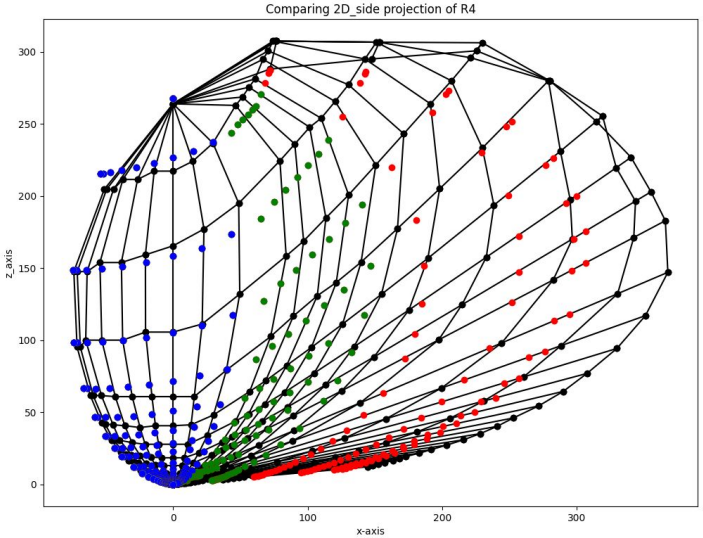
- Computing various indicators, such as Q0, S1
- Adjusting ellipsoids to r-tables using the Ogando-Martinez method
- Computing the error over a given r-table

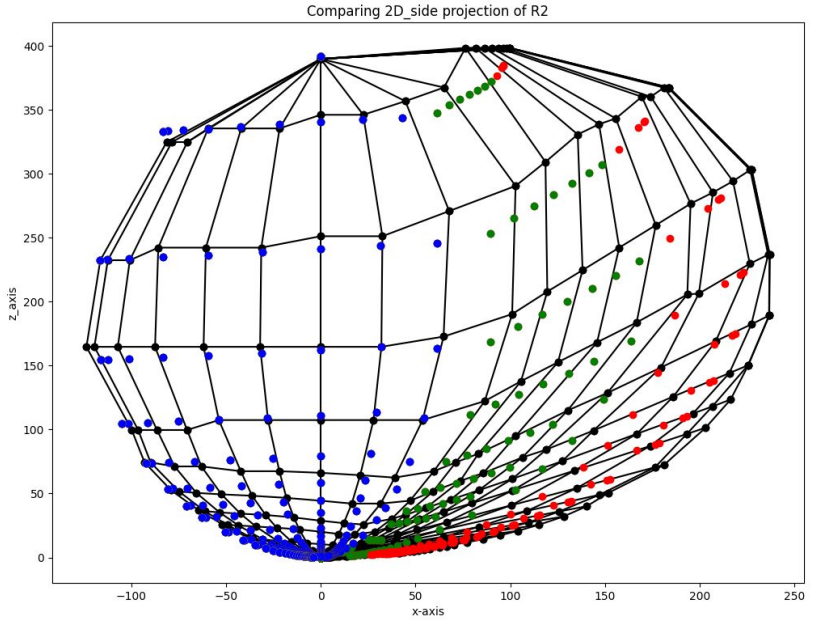
$$Q_0 = \int_0^{\Omega_0} q(\beta, \epsilon) d\Omega.$$

$$S_1 = \frac{r(0,2)}{r(0,0)}.$$

$$\Delta_{r-table} = \frac{\sqrt{\frac{\sum_k \sum_l \left(r^i_{k,l} - r^j_{k,l}\right)^2}{580}}}{\frac{\frac{\sum_k \sum_l r^i_{k,l}}{580} + \frac{\sum_k \sum_l r^j_{k,l}}{580}}{2}}$$

Results of the Ogando-Martinez method

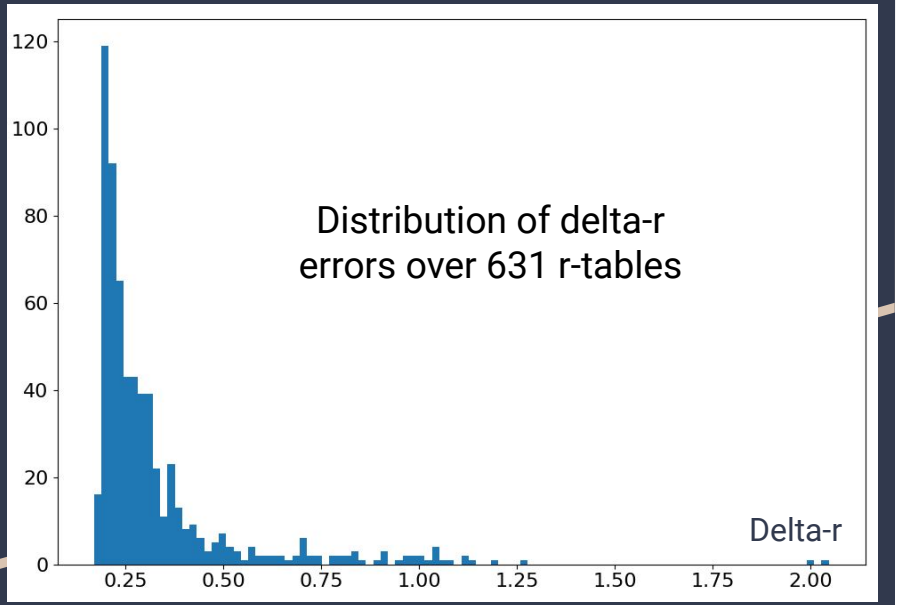




Comparison graphs

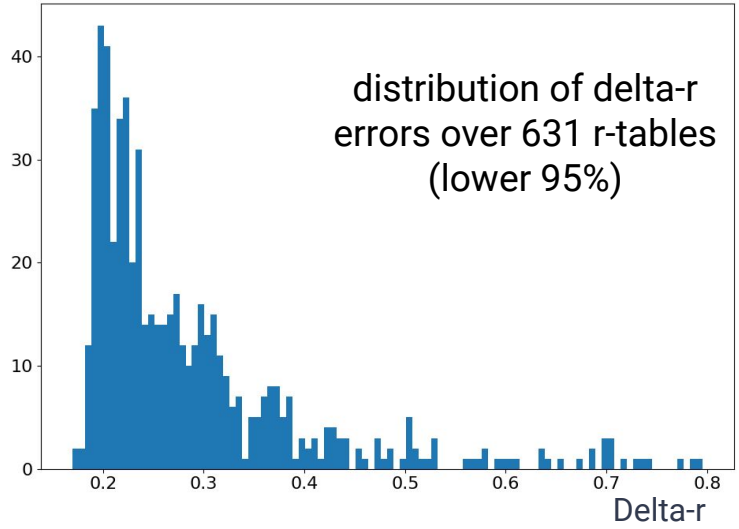
- Original tables in black
- Adjusted tables in colors

Error comparison, measured in delta-r error



r-table	Delta-r
R1	0,20
R2	0,24
R3	0,25
R4	0,37

min 0.17
max 2.04
mean 0.32
median 0.26

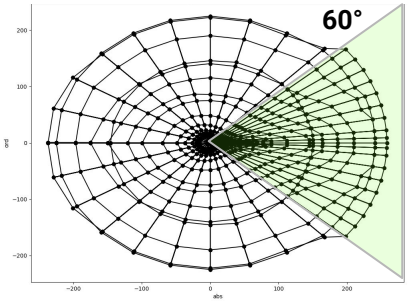


Where does the arbitrary choice for setting the partitioning spots come from ?

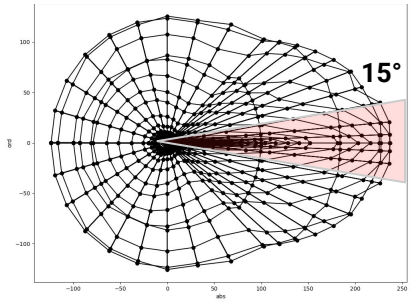
- The authors choose to split the tables at :
  - beta = 15°
  - beta = 60°

Top-down representations justify this choice :

R1



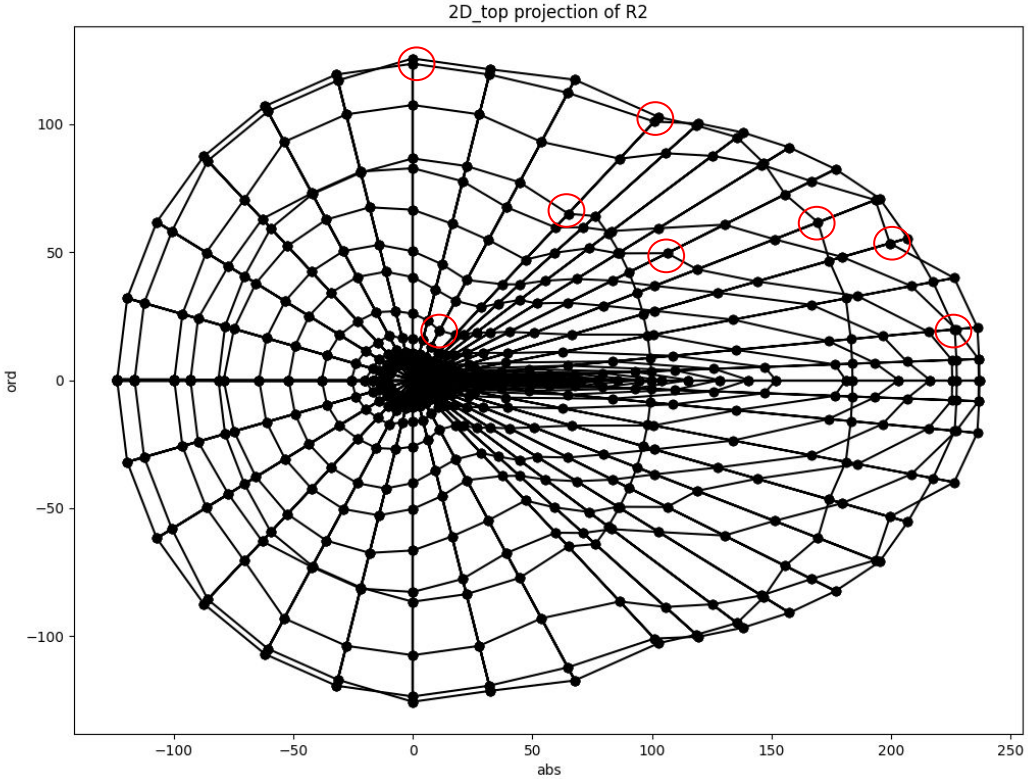
R3



Inflection points, but not the only ones

Many potential inflection points:

Any top-down view always represents the data twice, to respect the symmetry around y=0 hypothesis.

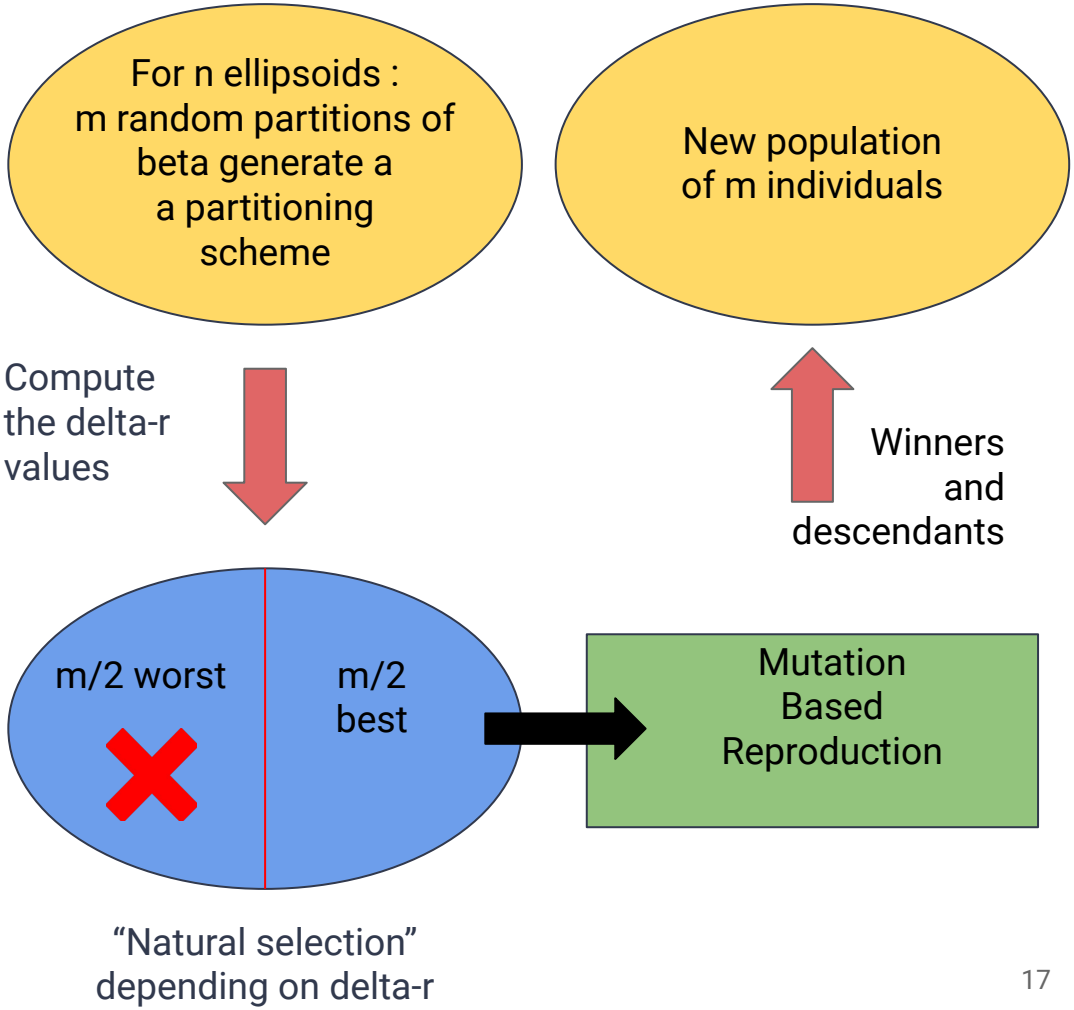


How to partition the r-table ?

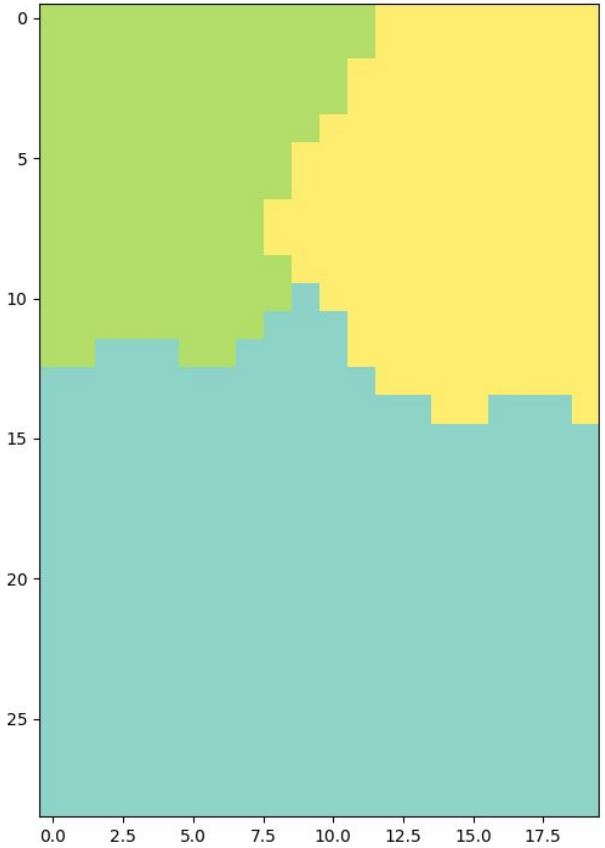
Table: R2, CIE standard

Genetic Algorithms

For n ellipsoids, m individuals and k generations



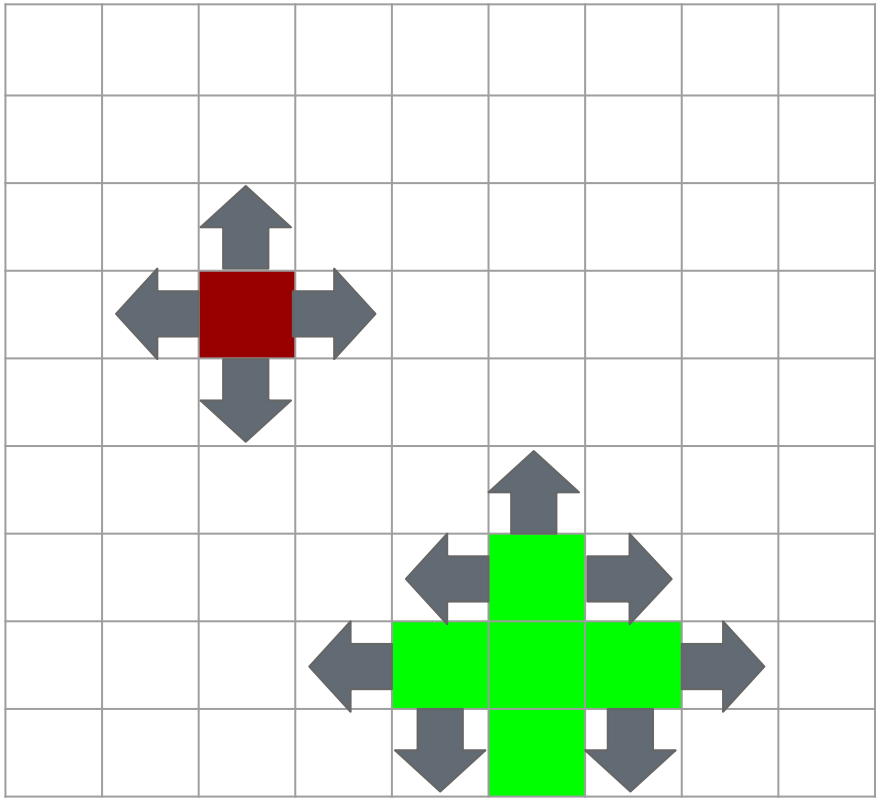
Partitioning r-tables randomly and adapting a genetic algorithm to the task



A random r-table partition

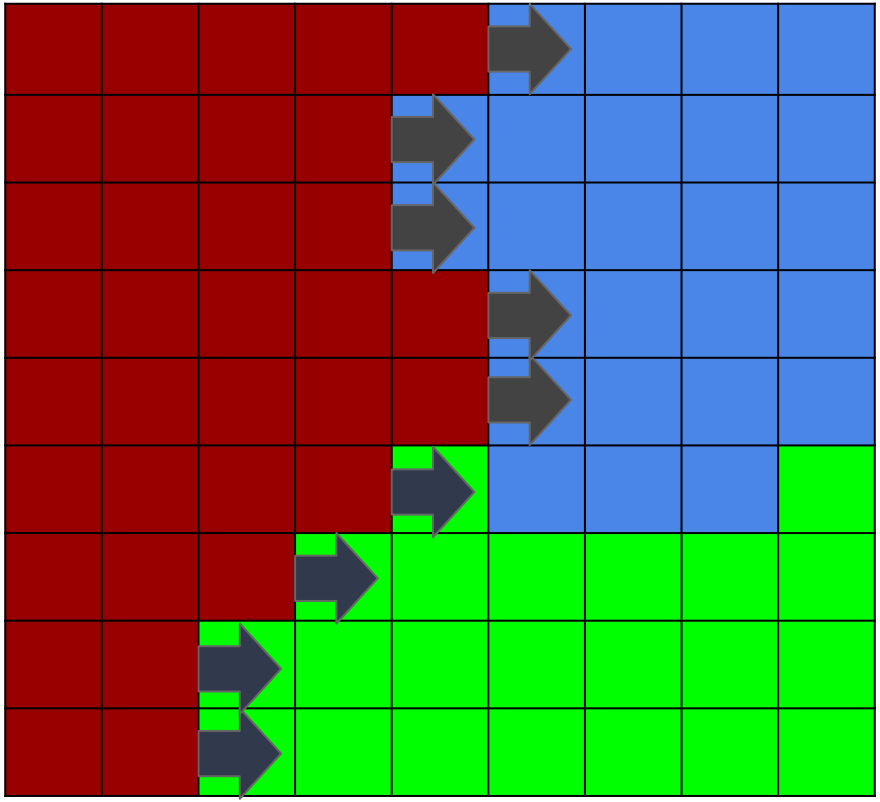
Genetic algorithm

- In order to implement a GA, a number of issues have to be fixed first
  - Making the fitting fast
  - Reducing the search space
  - Finding good partitioning generation and mutation algorithms



- Number of starting points = number of regions to be partitioned
- Simultaneous 4-way Floodfill to fill a 20 x 29 grid (r-table shaped)
- Updating the grid in random order, to make sure no bias toward any propagation directions

Examples of a 4-way flood fill (https://en.wikipedia.org/wiki/Flood_fill)



For a given mutation rate m, m repetitions of the following steps are performed :

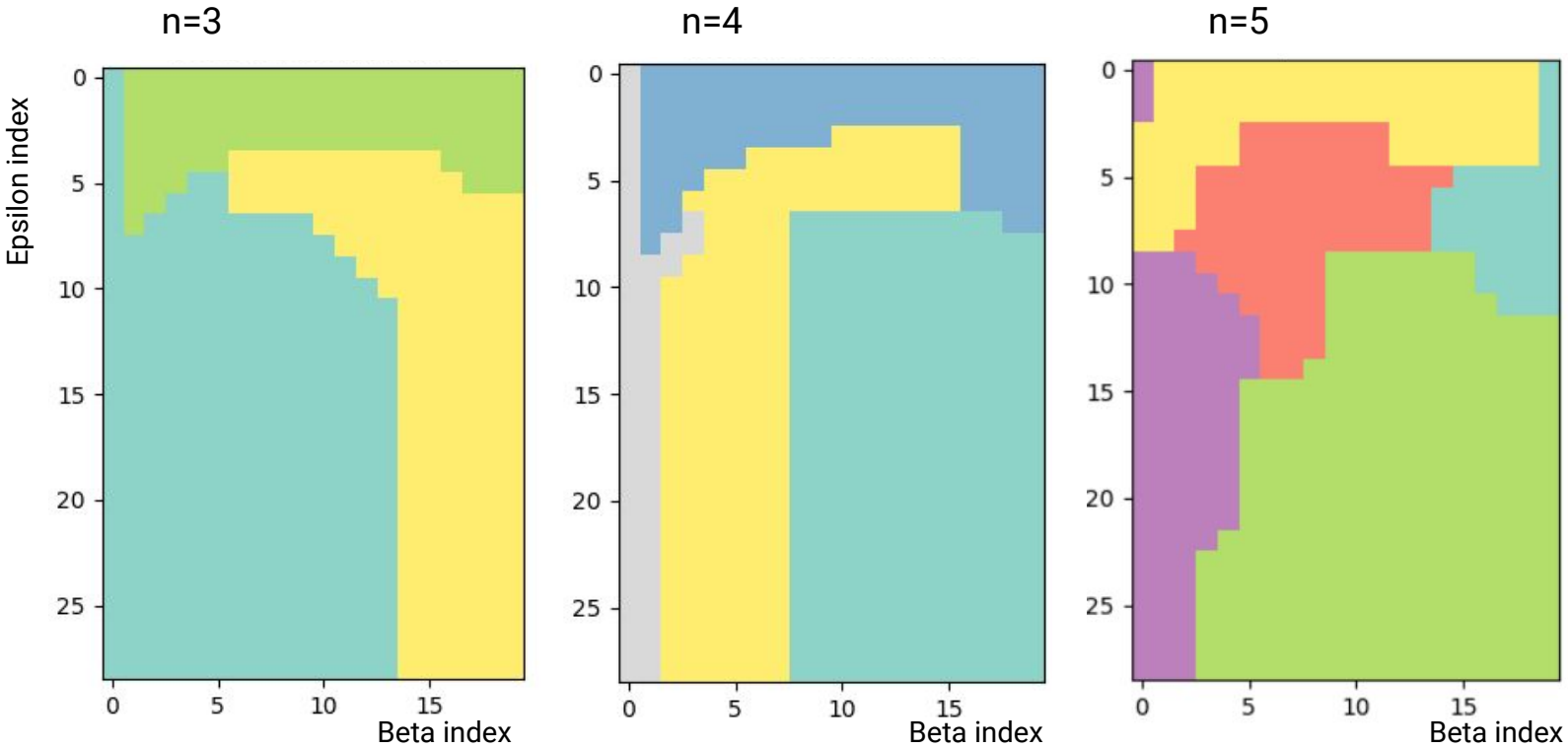
- Random selection of an area (red)
- Choose a direction from (right)
- The zone is moved one square in the chosen direction
- To prevent shrinking:
  - Zones with less then 30 elements are automatically selected

Recap of the results of the final product

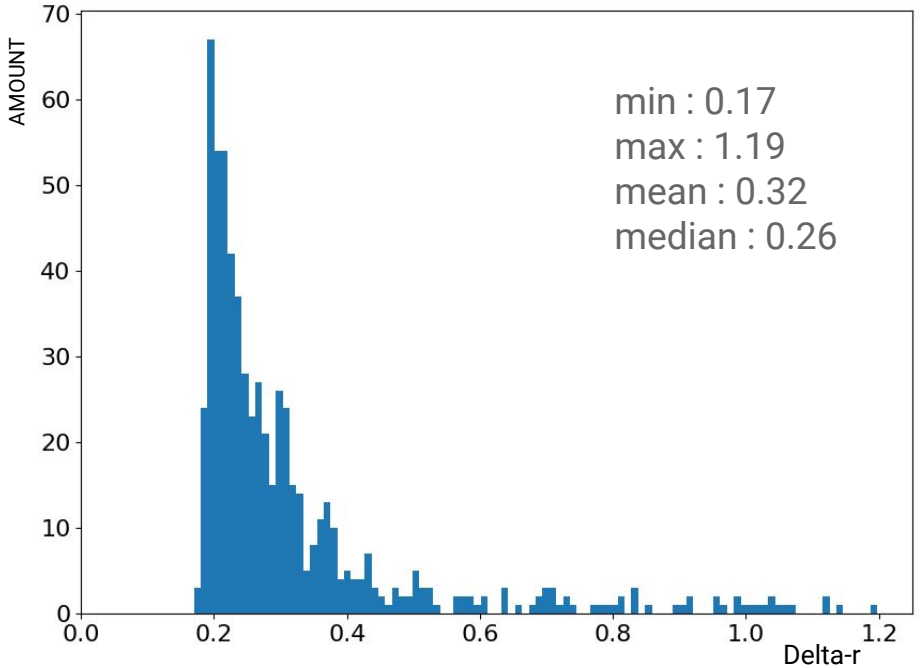
Database: 631 dry r-tables	Ogando-M method	best partitions	Gain on the delta -r
delta-r	0,37	0,24	**35%**

- Average gain of 35% on the error measured by delta-r between the Ogando-Martinez method and the optimal partitions for dry r-tables

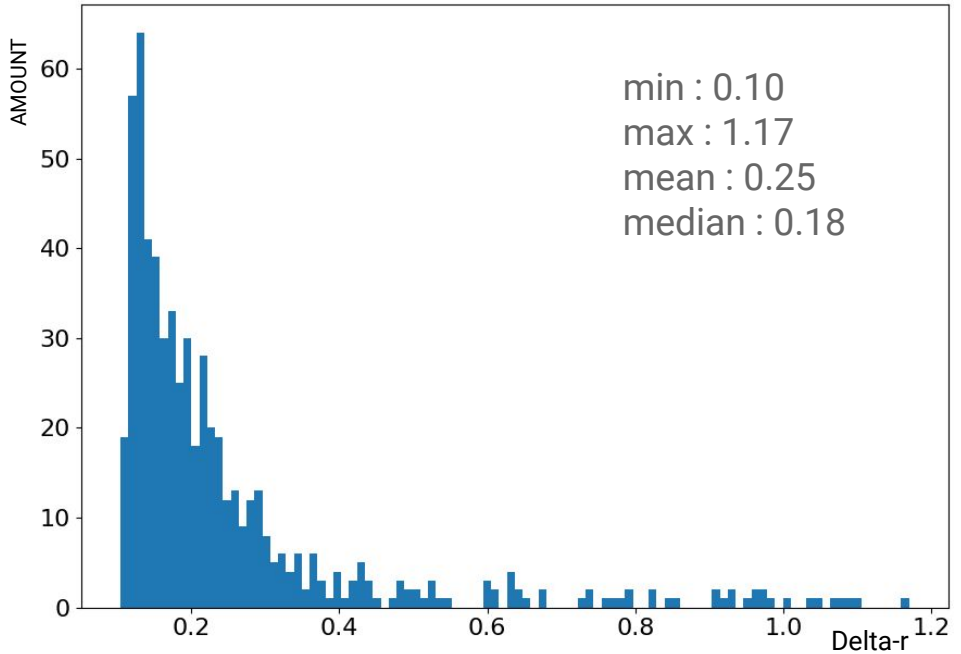
Best partitions found for n ellipsoids (dry)



Compare histograms of the 95% best r-tables



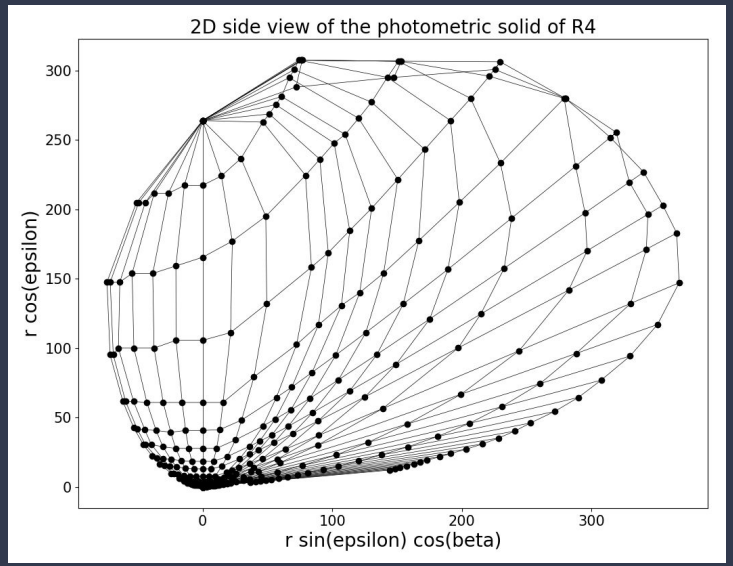
Ogando-Martinez method

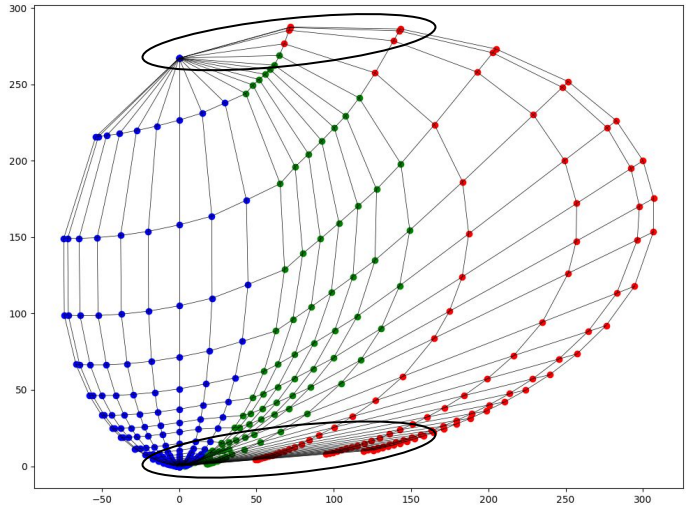


Best 3-ellipsoid partitioning

Example on R4 for 3 ellipsoids

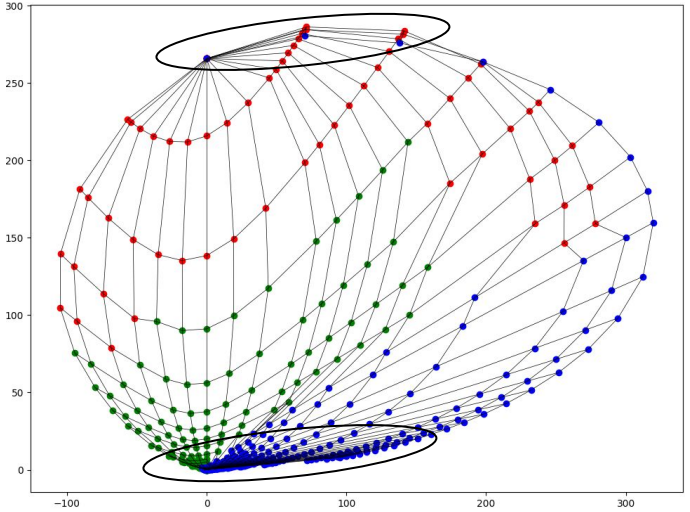
R4 as measured :





The result of the Ogando-Martinez model

The result of the optimal partitioning scheme



Conclusion

- Test of the Ogando-Martinez Model on larger databases
- Contributing models allowing for a much lower error overall
  - ➢ Lower delta-r error
  - ➢ Lower error on all parameters, Q0, S1, and other indicators

Outlook

- Tests on complete r-tables (580 values instead of 392)
- Usefulness of switching to more ellipsoids
- Evolution of ellipses as a function of coating age:
  - Can we predict ageing by studying ellipse parameters?