F4 *Solving Quadratic Equations* Introduction

Learning objectives

This unit is focused on methods of solving quadratic equations. After completing Unit F4 you should

- be able to solve quadratic equations that factorise into two linear terms
- * be able to use the formula for solving quadratic equations
- * understand how to complete the square for a quadratic and hence solve the equation.

Introduction

We can tell from Old Babylonian clay tablets dating from around 2000 BC that the Babylonians knew how to solve a pair of simultaneous equations of the form

$$x + y = p, \ xy = q$$

which is equivalent to the equation

$$x^2 + q = px$$

In the 8th Century BC, quadratic equations of the form $ax^2 = c$ and $ax^2 + bx = c$ were explored in ancient India, using geometric methods. Babylonian mathematicians from circa 400 BC and Chinese mathematicians from circa 200 BC used the method of completing the square (see *Section F4.3*) to solve quadratic equations with positive roots, but did not have a general formula.

In 628 AD, Brahmagupta, an Indian mathematician, gave the first explicit solution of the quadratic equation but it was not until 1896 that the first reference to the general solution

$$ax^2 + bx + c = 0 \qquad (a, b, c \text{ are constants}, \ a \neq 0)$$

as

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

was published by Henry Heaton in The American Mathematical Monthly (see information at

http://en.wikipedia.org/wiki/Quadratic_equation

and http://www.jstor.org/stable/2971099?seq=1)

Key points and principles

- There are three methods of solving quadratic equations:

(i) ***quadratic factorisation***, where we can write

$ax^2 + bx + c = a(x - p)(x - q)$ when p and q are rational numbers

which has solution $x = p$ or $x = q$

(ii) ***formula*** $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

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(iii) ***completing the square***, where we write

$$ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$$

- Note that if $(x - a)(x - b) = 0$, then either $x - a = 0$ or $x - b = 0$, giving $x = a$ or $x = b$.

- Quadratic equations have 2, 1 or 0 real roots according to the value of "$b^2 - 4ac$":

(i) If $b^2 > 4ac$, there are 2 real distinct roots

(ii) If $b^2 = 4ac$, there is 1 (repeated) real root

(iii) If $b^2 < 4ac$, there are no real roots.

Facts to remember

- If $(x - a)(x - b) = 0$ then $x = a$ or $x = b$

- The formula for solving the quadratic equation

$$ax^2 + bx + c = 0$$

is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- The number of real roots of a quadratic is

$$2 \text{ if } b^2 > 4ac$$

$$1 \text{ if } b^2 = 4ac$$

$$0 \text{ if } b^2 < 4ac$$

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Glossary of Terms

Quadratic factorisation This is when you can write a quadratic formula in the form

$$ax^2 + bx + c = a(x - p)(x - q)$$

Formula for solving quadratic equation The formula for solving the quadratic equation

$$ax^2 + bx + c = 0$$

is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Completing the square This is when we write the quadratic in the form

$$ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$$