"본 강의 동영상 및 자료는 대한민국 저작권법을 준수합니다. 본 강의 동영상 및 자료는 상명대학교 재학생들의 수업목적으로 제작·배포되는 것이므로, 수업목적으로 내려받은 강의 동영상 및 자료는 수업목적 이외에 다른 용도로 사용할 수 없으며, 다른 장소 및 타인에게 복제, 전송하여 공유할 수 없습니다. 이를 위반해서 발생하는 모든 법적 책임은 행위 주체인 본인에게 있습니다."

Problem:

- Arrange the elements of a set in an ascending (or descending) order
- $-[2, 1, 4, 3, 7, 5, 6] \rightarrow [1, 2, 3, 4, 5, 6, 7]$
- Sorting algorithms
 - O (n²) algorithms for pairwise comparison
 - Bubble sort
 - Insertion sort
 - Selection sort
 - •
 - Can it be faster? → Use divide & conquer
 - Merge sort
 - Quick sort

Merge sort

- Given a sequent of n elements $\{a_1, a_2, ..., a_n\}$, split them into two set.
- Sort each set individually and merge them to produce a single sorted sequence of n elements.
- Three steps
 - Divide
 - Split the list into two halves
 - Conquer
 - Recursively sort each half
 - Combine
 - Merge the two sorted sublists

Merge sort (Divide & Conquer)

```
void msort( int s, int e, int A[] )
{
   if ( s == e )
      return;
   m ← (s+e)/2;
   msort ( s, m, A );
   msort ( m+1, e, A );
   merge ( s, m, m+1, e, A );
}
```

Merge sort (Divide & Conquer)

```
void merge( int ls, int le, int rs, int re, int A[] )
    int lptr = ls, rptr = rs, bptr = 0;
    int *B = (int *) calloc ((le - ls)+(re - rs)+2, sizeof(int));
    while ( lptr <= le && rptr <= re ) {
        if ( A[lptr] < A[rptr] )</pre>
            B[bptr++] = A[lptr++];
        else
            B[bptr++] = A[rptr++];
    if (lptr > le)
        for ( int i = rptr; i <= re; i++ )
            B[bptr++] = A[i];
    if (rptr > re)
        for ( int i = lptr; i <= le; i++ )
            B[bptr++] = A[i];
    A \leftarrow B;
```

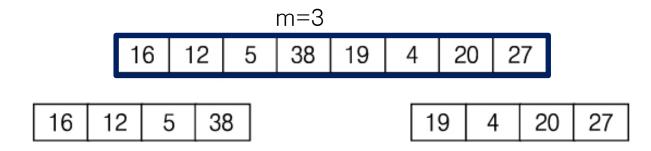
Merge sort (Check three points)

```
void msort( int s, int e, int A[] )
{
   if ( s == e )
      return;
   m ← (s+e)/2;
   msort ( s, m, A );
   msort ( m+1, e, A );
   merge ( s, m, m+1, e, A );
}
```

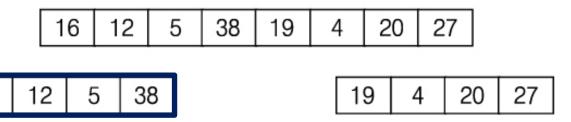
Merge sort (Example)



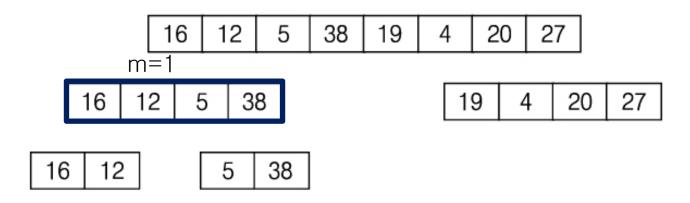
```
merge_sort ( 0, 7, a );
    m = 3;
```



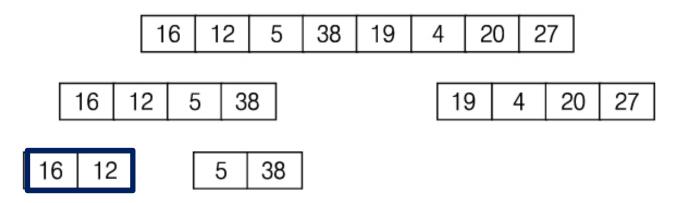
```
merge_sort ( 0, 7, a );
    m = 3;
    merge_sort ( 0, 3, a );
```



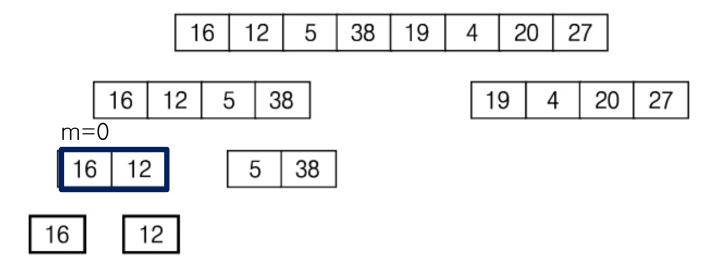
```
merge_sort ( 0, 7, a );
    m = 3;
    merge_sort ( 0, 3, a );
    m = 1;
```



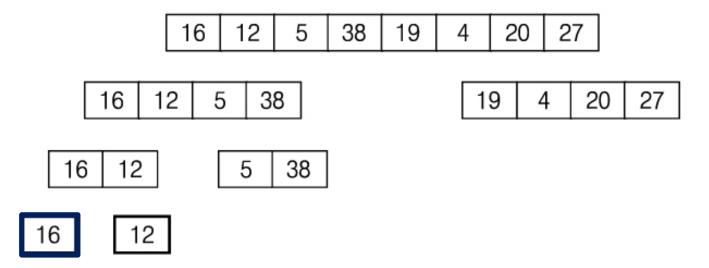
```
merge_sort ( 0, 7, a );
    m = 3;
    merge_sort ( 0, 3, a );
    m = 1;
    merge_sort ( 0, 1, a );
```

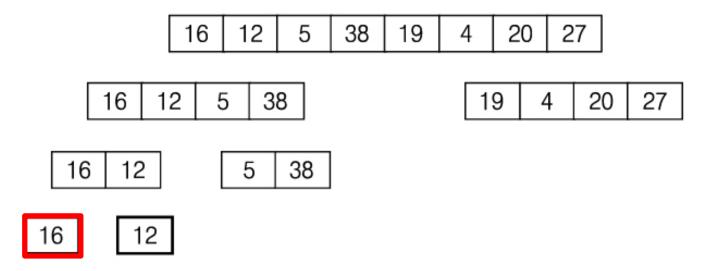


```
merge_sort ( 0, 7, a );
    m = 3;
    merge_sort ( 0, 3, a );
    m = 1;
    merge_sort ( 0, 1, a );
    m = 0;
```



```
merge_sort ( 0, 7, a );
    m = 3;
    merge_sort ( 0, 3, a );
    m = 1;
    merge_sort ( 0, 1, a );
    m = 0;
    merge_sort ( 0, 0, a );
```

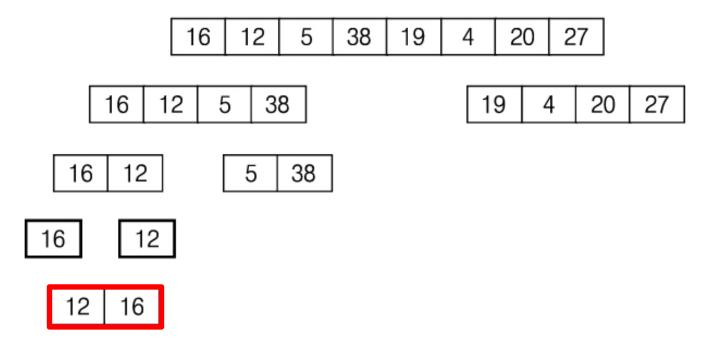


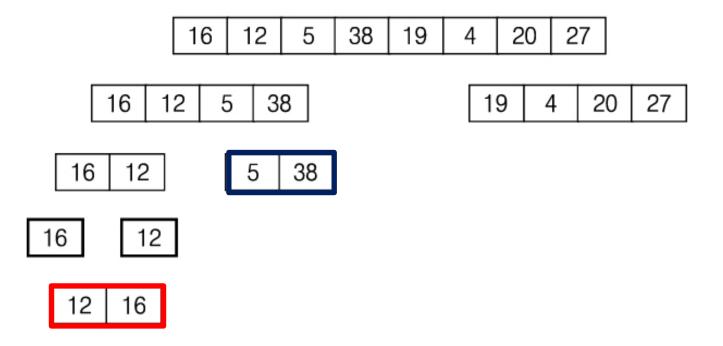


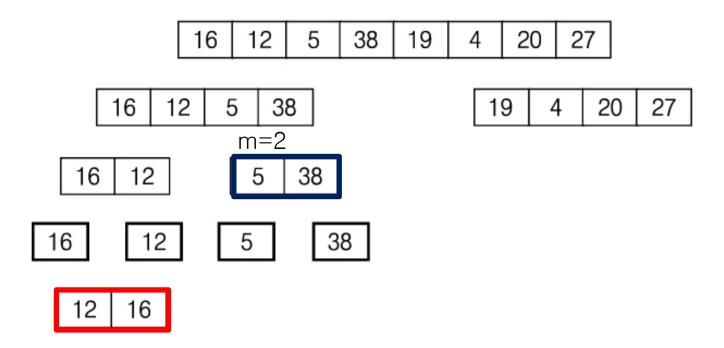
```
merge_sort ( 0, 7, a );
   m = 3;
   merge_sort ( 0, 3, a );
       m = 1;
       merge sort ( 0, 1, a );
           m = 0;
           merge_sort ( 0, 0, a ); \rightarrow sorted
           merge_sort ( 1, 1, a );
                                16
                                     12
                                           5
                                                38
                                                     19
                                                           4
                                                                20
                             12
                                   5
                       16
                                       38
                                                             19
                                                                        20
                                                                             27
                    16
                         12
                                      5
                                          38
```

```
merge_sort ( 0, 7, a );
   m = 3;
   merge_sort ( 0, 3, a );
       m = 1;
       merge sort (0,1,a
           m = 0;
           merge_sort ( 0, 0, a ); \rightarrow sorted
           merge_sort ( 1, 1, a ); → sorted
                               16
                                           5
                                               38
                                                     19
                                                           4
                                                               20
                            12
                                  5
                                                                            27
                       16
                                       38
                                                             19
                                                                       20
                    16
                         12
                                          38
                                     5
```

```
merge_sort ( 0, 7, a );
    m = 3;
    merge_sort ( 0, 3, a );
       m = 1;
       merge sort ( 0, 1, a );
            m = 0;
            merge_sort ( 0, 0, a ); \rightarrow sorted
            merge_sort ( 1, 1, a ); \rightarrow sorted
            merge ( 0, 0, 1, 1, a );
                                 16
                                       12
                                             5
                                                  38
                                                        19
                                                                   20
                                                              4
                              12
                                         38
                                                                            20
                                                                                 27
                        16
                                     5
                                                                19
                     16
                           12
                                       5
                                            38
                  16
```







```
merge_sort ( 0, 3, a );
   m = 1;
   merge sort (0, 1, a); \rightarrow sorted
   merge_sort ( 2, 3, a );
       m = 2;
       merge_sort ( 2, 2, a );
                            16
                                       5
                                            38
                                                 19
                                                            20
                         12
                               5
                                   38
                                                                    20
                                                                         27
                   16
                                                         19
                16
                     12
                                      38
             16
```

merge_sort (0, 7, a);

```
merge_sort ( 2, 2, a ); → sorted
                  16
                           5
                                38
                                    19
                                              20
               12
                        38
                                                         27
          16
                    5
                                           19
                                                     20
       16
            12
                           38
     16
```

merge_sort (0, 7, a);

m = 1;

merge_sort (0, 3, a);

m = 2;

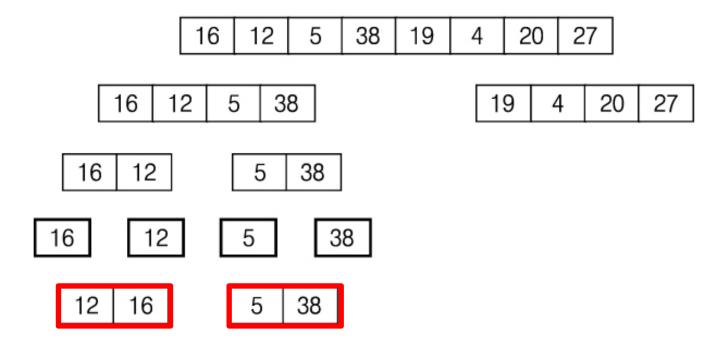
merge sort (0, 1, a); \rightarrow sorted

merge_sort (2, 3, a);

```
merge_sort ( 0, 7, a );
   m = 3;
   merge_sort ( 0, 3, a );
       m = 1;
       merge sort (0, 1, a); \rightarrow sorted
       merge_sort ( 2, 3, a );
           m = 2;
           merge_sort ( 2, 2, a ); → sorted
           merge_sort ( 3, 3, a );
                                16
                                           5
                                                38
                                                     19
                                                                20
                                                           4
                             12
                                   5
                                       38
                                                                             27
                       16
                                                             19
                                                                        20
                    16
                         12
                                          38
                 16
```

```
merge_sort ( 0, 7, a );
   m = 3;
   merge_sort ( 0, 3, a );
       m = 1;
       merge sort (0, 1, a); \rightarrow sorted
       merge_sort ( 2, 3, a );
           m = 2;
           merge_sort ( 2, 2, a ); → sorted
           merge_sort (3, 3, a); → sorted
                               16
                                           5
                                               38
                                                     19
                                                                20
                                                                     27
                                                           4
                             12
                                  5
                                       38
                                                                             27
                       16
                                                             19
                                                                       20
                    16
                         12
                                          38
                 16
```

```
merge_sort ( 0, 7, a );
   m = 3;
   merge_sort ( 0, 3, a );
       m = 1;
       merge sort (0, 1, a); \rightarrow sorted
       merge_sort ( 2, 3, a );
           m = 2;
           merge_sort ( 2, 2, a ); → sorted
           merge_sort ( 3, 3, a ); > sorted
           merge ( 2, 2, 3, 3, a );
                                16
                                                38
                                                                 20
                                           5
                                                      19
                             12
                                   5
                                        38
                       16
                                                              19
                                                                              27
                                                                         20
                    16
                          12
                                          38
                                             38
                 16
```



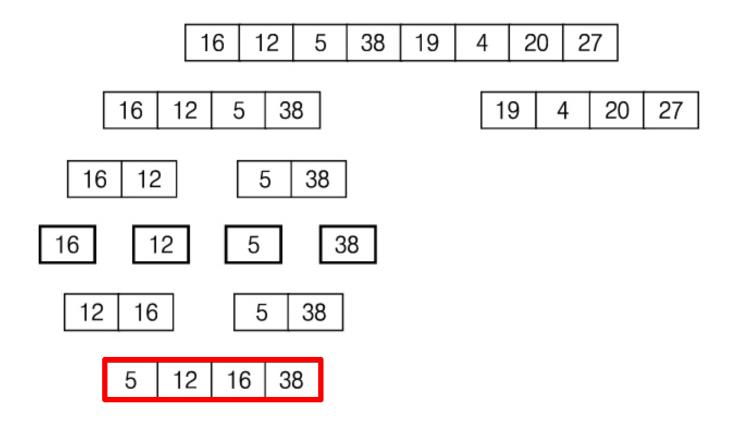
```
merge sort (0, 1, a); \rightarrow sorted
merge_sort ( 2, 3, a ); \rightarrow sorted
merge ( 0, 1, 2, 3, a );
                         16
                                     5
                                          38
                                               19
                                                          20
                                                                27
                                                      4
                      12
                            5
                                 38
                                                                  20
                                                                        27
                16
                                                        19
             16
                   12
                               5
                                    38
          16
                                       38
                  16
                                    38
```

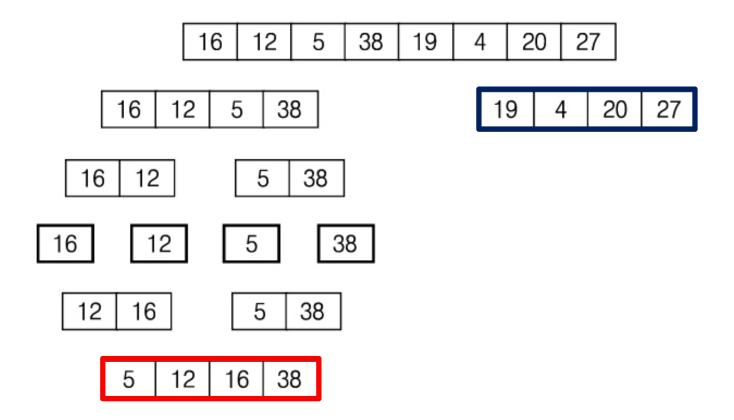
merge_sort (0, 7, a);

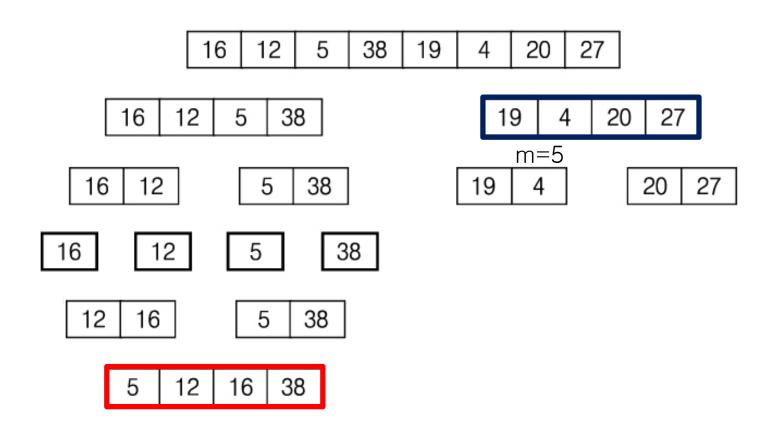
m = 1;

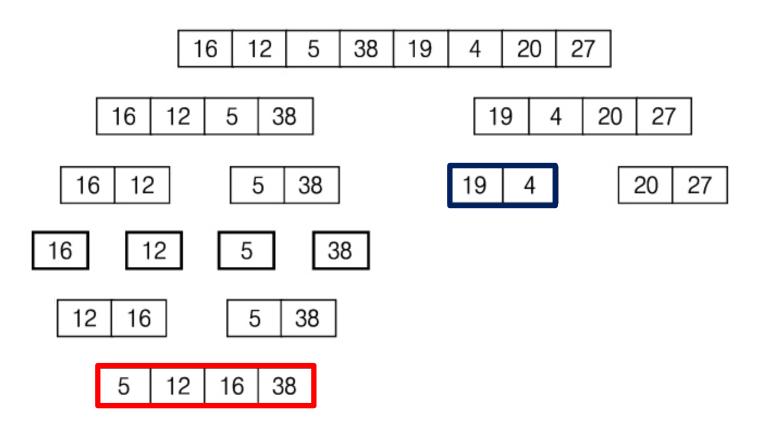
merge_sort (0, 3, a);

```
merge_sort ( 0, 7, a );
    m = 3;
    merge_sort ( 0, 3, a ); > sorted
```

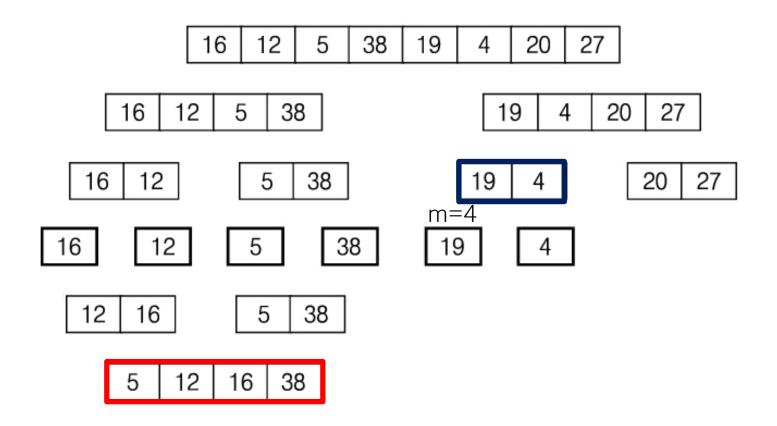








```
merge_sort ( 0, 7, a );
    m = 3;
    merge_sort ( 0, 3, a ); > sorted
    merge_sort ( 4, 7, a );
    m = 5;
    merge_sort ( 4, 5, a );
    m = 4;
```



```
merge_sort ( 0, 7, a );
   m = 3;
   merge_sort ( 0, 3, a ); \rightarrow sorted
   merge_sort ( 4, 7, a );
       merge_sort ( 4, 5, a );
           m = 4;
           merge_sort ( 4, 4, a );
                               16
                                          5
                                               38
                                                    19
                                                               20
                                                                    27
                            12
                                  5
                                       38
                                                                       20
                                                                            27
                       16
                                                            19
                   16
                         12
                                     5
                                         38
                                                          19
                                                                          20
                                                                                27
                                                                4
                 16
                                            38
                   12
                         16
                                         38
                                 16
```

```
merge_sort ( 0, 3, a ); \rightarrow sorted
merge_sort ( 4, 7, a );
   merge_sort ( 4, 5, a );
        m = 4;
       merge_sort ( 4, 4, a ); \rightarrow sorted
                            16
                                        5
                                             38
                                                              20
                                                                   27
                                                   19
                          12
                                5
                                    38
                                                                      20
                                                                           27
                    16
                                                           19
                16
                      12
                                   5
                                       38
                                                        19
                                                                          20
                                                                               27
                                                               4
              16
                                          38
                      16
                                       38
                               16
```

merge_sort (0, 7, a);

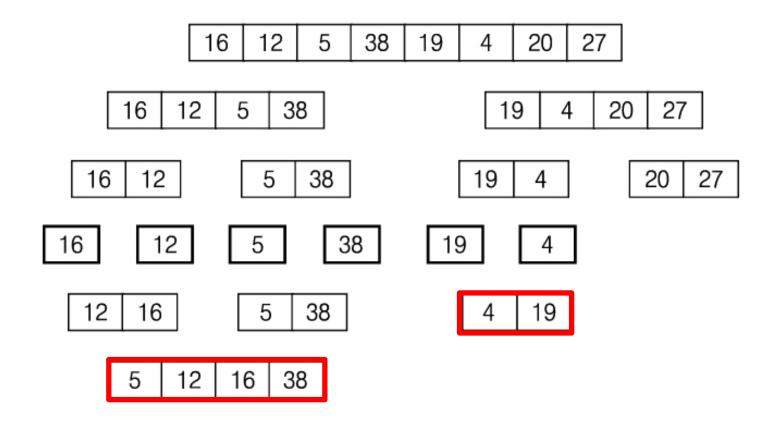
```
merge_sort ( 0, 7, a );
   m = 3;
   merge_sort ( 0, 3, a ); \rightarrow sorted
   merge_sort ( 4, 7, a );
       merge_sort ( 4, 5, a );
           m = 4;
           merge_sort ( 4, 4, a ); \rightarrow sorted
           merge_sort ( 5, 5, a );
                                16
                                      12
                                            5
                                                 38
                                                                  20
                                                                       27
                                                       19
                              12
                                        38
                                                                          20
                                                                               27
                                    5
                                                               19
                        16
                    16
                          12
                                      5
                                           38
                                                            19
                                                                              20
                                                                                   27
                                                                  4
                  16
                                              38
                    12
                          16
                                           38
                                   16
```

```
merge_sort ( 0, 7, a );
   m = 3;
   merge_sort ( 0, 3, a ); \rightarrow sorted
   merge_sort ( 4, 7, a );
       merge_sort ( 4, 5, a );
            m = 4;
            merge_sort ( 4, 4, a ); \rightarrow sorted
            merge_sort (5, 5, a); \rightarrow sorted
                                 16
                                       12
                                             5
                                                  38
                                                                   20
                                                                         27
                                                        19
                              12
                                         38
                                                                                 27
                                    5
                        16
                                                                19
                                                                            20
                     16
                           12
                                       5
                                            38
                                                              19
                                                                                20
                                                                                     27
                                                                    4
                  16
                                               38
                    12
                          16
                                            38
                                    16
```

```
merge_sort ( 0, 7, a );
   m = 3;
   merge_sort (0,3,a); \rightarrow sorted
   merge_sort ( 4, 7, a );
       merge_sort ( 4, 5, a );
            m = 4;
           merge_sort (4, 4, a); \rightarrow sorted
           merge_sort ( 5, 5, a ); \rightarrow sorted
           merge (4, 4, 5, 5, a);
                                 16
                                      12
                                                 38
                                                                   20
                                                                        27
                                             5
                                                       19
                              12
                                         38
                                    5
                        16
                                                                19
                                                                           20
                                                                                27
                     16
                          12
                                       5
                                            38
                                                             19
                                                                               20
                                                                                    27
                                                                   4
                                               38
                  16
                    12
                          16
                                           38
                                   16
```

```
merge_sort ( 0, 7, a );
    m = 3;
    merge_sort ( 0, 3, a ); \rightarrow sorted
    merge_sort ( 4, 7, a );
```

merge_sort (4, 5, a); \rightarrow sorted



```
merge_sort ( 0, 7, a );
   m = 3;
   merge_sort ( 0, 3, a ); \rightarrow sorted
   merge_sort ( 4, 7, a );
       merge_sort ( 4, 5, a ); \rightarrow sorted
       merge_sort ( 6, 7, a );
                                 16
                                                 38
                                                                  20
                                                                        27
                                            5
                                                       19
                              12
                                    5
                                         38
                                                                                27
                                                               19
                        16
                                                                          20
                    16
                          12
                                       5
                                           38
                                                             19
                  16
                                              38
                                                         19
                    12
                          16
                                           38
                                   16
                                         38
```

```
merge_sort ( 0, 7, a );
   m = 3;
   merge_sort ( 0, 3, a ); \rightarrow sorted
    merge_sort ( 4, 7, a );
       merge_sort ( 4, 5, a ); \rightarrow sorted
       merge_sort ( 6, 7, a );
            m = 6;
                                 16
                                       12
                                                  38
                                                                    20
                                                                          27
                                              5
                                                        19
                               12
                                          38
                                     5
                                                                 19
                                                                                  27
                         16
                                                                            20
                     16
                           12
                                        5
                                             38
                                                              19
                                                                             <u>m=6</u>
                  16
                                                38
                                                                              20
                                                           19
                     12
                           16
                                            38
                                    16
                                          38
```

```
merge_sort ( 0, 7, a );
   m = 3;
   merge_sort ( 0, 3, a ); \rightarrow sorted
   merge_sort ( 4, 7, a );
       merge_sort ( 4, 5, a ); \rightarrow sorted
       merge_sort ( 6, 7, a );
           m = 6;
           merge_sort ( 6, 6, a );
                                 16
                                                 38
                                                                  20
                                                                        27
                                            5
                                                       19
                              12
                                         38
                                    5
                                                               19
                                                                                27
                        16
                                                                          20
                    16
                          12
                                       5
                                           38
                                                             19
                                                                              20
                                                                                    27
                                                                   4
                                              38
                                                                                      27
                  16
                    12
                          16
                                           38
                                   16
                                         38
```

```
merge_sort ( 0, 7, a );
   m = 3;
   merge_sort ( 0, 3, a ); \rightarrow sorted
   merge_sort ( 4, 7, a );
       merge_sort (4,5,a); \rightarrow sorted
       merge_sort ( 6, 7, a );
           m = 6;
           merge_sort ( 6, 6, a ); > sorted
                                16
                                      12
                                                38
                                                                 20
                                                                      27
                                            5
                                                      19
                             12
                                   5
                                                                               27
                       16
                                        38
                                                              19
                                                                         20
                    16
                          12
                                      5
                                           38
                                                           19
                                                                             20
                                                                                  27
                                                                  4
                                              38
                 16
                    12
                         16
                                           38
                                   16
                                        38
```

```
merge_sort ( 0, 7, a );
   m = 3;
   merge_sort ( 0, 3, a ); \rightarrow sorted
   merge_sort ( 4, 7, a );
       merge_sort (4,5,a); \rightarrow sorted
       merge_sort ( 6, 7, a );
           m = 6;
           merge_sort ( 6, 6, a ); > sorted
           merge_sort ( 7, 7, a );
                                16
                                      12
                                                 38
                                                                       27
                                            5
                                                      19
                                                                 20
                              12
                                   5
                        16
                                        38
                                                               19
                                                                          20
                                                                               27
                    16
                          12
                                      5
                                           38
                                                            19
                                                                             20
                                                                                   27
                                                                  4
                                              38
                 16
                    12
                          16
                                           38
                                   16
```

```
merge_sort ( 0, 7, a );
   m = 3;
   merge_sort (0,3,a); \rightarrow sorted
   merge_sort ( 4, 7, a );
       merge_sort (4,5,a); \rightarrow sorted
       merge_sort ( 6, 7, a );
            m = 6;
           merge_sort ( 6, 6, a ); > sorted
           merge_sort (7, 7, a); \rightarrow sorted
                                 16
                                                 38
                                      12
                                            5
                                                       19
                                                                  20
                                                                        27
                        16
                              12
                                    5
                                         38
                                                               19
                                                                          20
                                                                                27
                    16
                          12
                                       5
                                           38
                                                            19
                                                                              20
                                                                                    27
                                                                   4
                                              38
                  16
                    12
                          16
                                           38
                                   16
```

```
merge_sort ( 0, 7, a );
   m = 3;
   merge_sort (0,3,a); \rightarrow sorted
   merge_sort ( 4, 7, a );
       merge_sort (4,5,a); \rightarrow sorted
       merge_sort ( 6, 7, a );
           m = 6;
           merge_sort (6,6,a); → sorted
           merge_sort (7, 7, a); \rightarrow sorted
           merge ( 6, 6, 7, 7, a );
                                            5
                                                 38
                                                       19
                                                                  20
                                                                       27
                                 16
                                      12
                        16
                              12
                                    5
                                        38
                                                               19
                                                                          20
                                                                               27
                    16
                          12
                                      5
                                           38
                                                            19
                                                                                   27
                                                                              20
                                                                  4
                                              38
                 16
                                                                            20
                    12
                          16
                                           38
                                   16
```

```
merge_sort ( 4, 5, a ); \rightarrow sorted
merge_sort ( 6, 7, a ); \rightarrow sorted
                         16
                                         38
                                                          20
                                                                27
                                    5
                                               19
                      12
                                 38
                            5
                                                       19
                                                                        27
                16
                                                                  20
             16
                  12
                               5
                                   38
                                                    19
                                                                      20
                                                                           27
          16
                                      38
                                                                              27
                                                                    20
            12
                  16
                                   38
                           16
                                 38
```

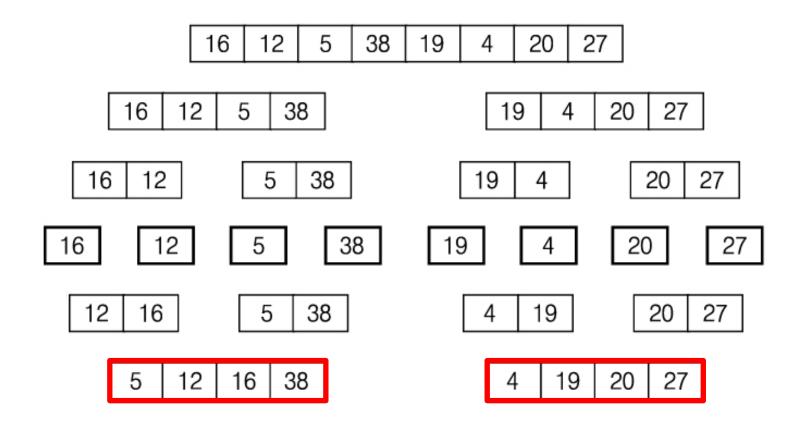
merge_sort (0, 7, a);

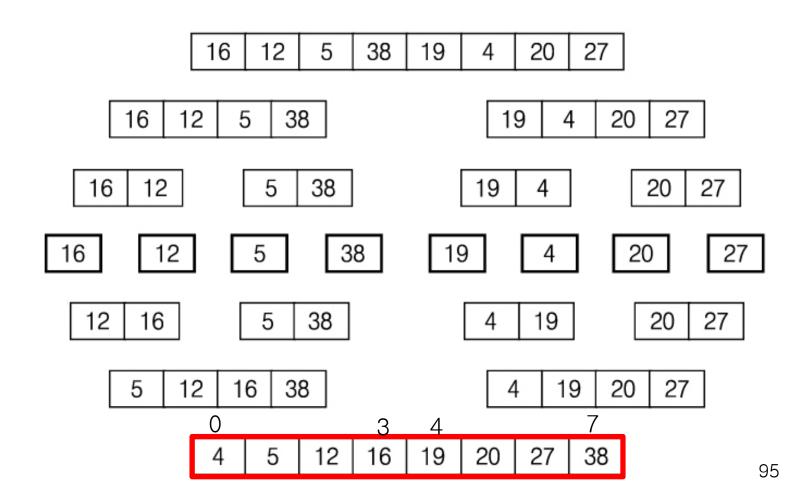
merge_sort (4, 7, a);

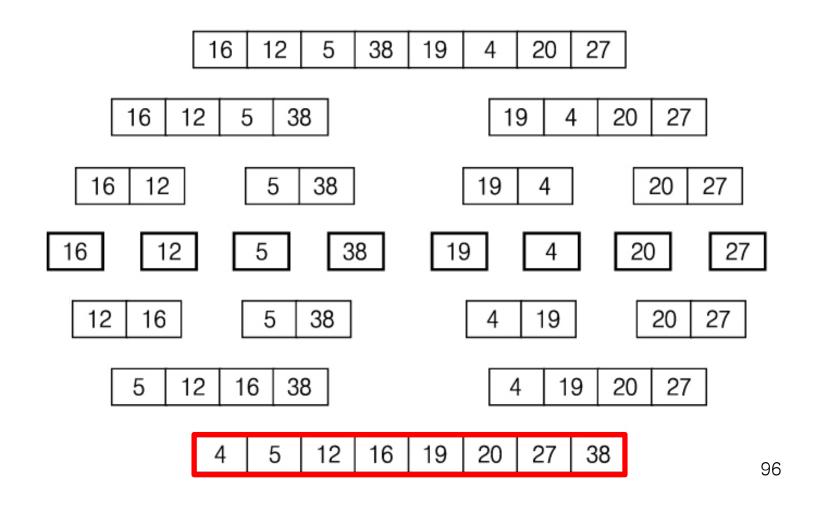
merge_sort (0, 3, a); \rightarrow sorted

m = 3;

```
merge_sort ( 0, 7, a );
   m = 3;
   merge_sort ( 0, 3, a ); \rightarrow sorted
   merge_sort ( 4, 7, a );
       merge_sort ( 4, 5, a ); \rightarrow sorted
       merge_sort ( 6, 7, a ); → sorted
       merge ( 4, 5, 6, 7, a );
                                16
                                                 38
                                                                 20
                                      12
                                            5
                                                      19
                                                                       27
                             12
                                        38
                                   5
                        16
                                                               19
                                                                         20
                                                                               27
                    16
                          12
                                      5
                                           38
                                                            19
                                                                             20
                                                                                   27
                                                                  4
                                              38
                                                                           20
                                                                                     27
                                                         19
                 16
                                                                  5
                    12
                          16
                                           38
                                                                  19
                                                                              20
                                   16
                                        38
                                                                    19
```







3.3 Sorting

- Merge sort (Performance analysis)
 - Recurrence relation

$$T(n) = \begin{cases} 2T(n/2) + cn, & n > 1 \\ a, & n = 1 \end{cases}$$

$$T(n) = an + cn \log n$$

3.3 Sorting

- Iterative merge sort
 - A merge sort without recursive call

```
int *iterative_mergesort ( int n, int *a )
{
   Q <- empty Queue;

   for ( int i = 0; i < n; i++ )
        Q.push ( a[i] )

   while ( Q.count ( ) > 1 ) {
        Q.push ( merge ( Q.pop ( ), Q.pop ( ) ) );

   return Q.pop ( );
}
```

3.3 Sorting

Comparison

	degenerate case	divide	conquer	combine	performance
tournament	n = 1 (s = e)	m = (s+e)/2	champ (s,m); champ (m+1,e);	win (LW, RW);	2T(n/2) + O(1) = O(n)
binary search	n = 1 (s = e)	m = (s+e)/2	bs (s, m-1); or bs (m+1, e);	-	T(n/2) + O(1) = $O(\log n)$
integer multiplication	n = 1	s = n/2 w = u div 2 ^s ;	mult (w+x, y+z); mult (w, y); mult (x, z);	p 2 ⁿ + (r – p – q) 2 ^s + q;	3T(n/2) + O(n) = $O(n^{\log_2 2})$
merge sort	n = 1 (s = e)	m = (s+e)/2	ms (s, m); ms (m+1, e);	merge (s, m, e);	2T(n/2) + O(n) = O(n log n)
quick sort					
median					
matrix multiplication					

퀴즈 3

- 다음은 합병 정렬에 대한 설명이다. 올바른 문장을 모두 고르시오.
 - (a) 합병 정렬은 최선의 경우와 최악의 경우에 모두 O(n log n)이 걸리는 알고리즘이다.
 - (b) 합병 정렬은 분할-정복-결합의 3단계를 모두 요구한다.
 - (c) 합병 정렬의 분할은 O(1)이, 결합은 O(n)의 시간 복잡 도가 요구된다.
 - (d) merge_sort (0, 7, a)를 완료하기 위해서는 8번의 merge 연산을 수행한다.