“본 강의 동영상 및 자료는 대한민국 저작권법을 준수합니다. 본 강의 동영상 및 자료는 상명대학교 재학생들의 수업목적으로 제작・배포되는 것이므로, 수업목적으로 내려받은 강의 동영상 및 자료는 수업목적 이외에 다른 용도로 사용할 수 없으며, 다른 장소 및 타인에게 복제, 전송하여 공유할 수 없습니다. 이를 위반해서 발생하는 모든 법적 책임은 행위 주체인 본인에게 있습니다.”

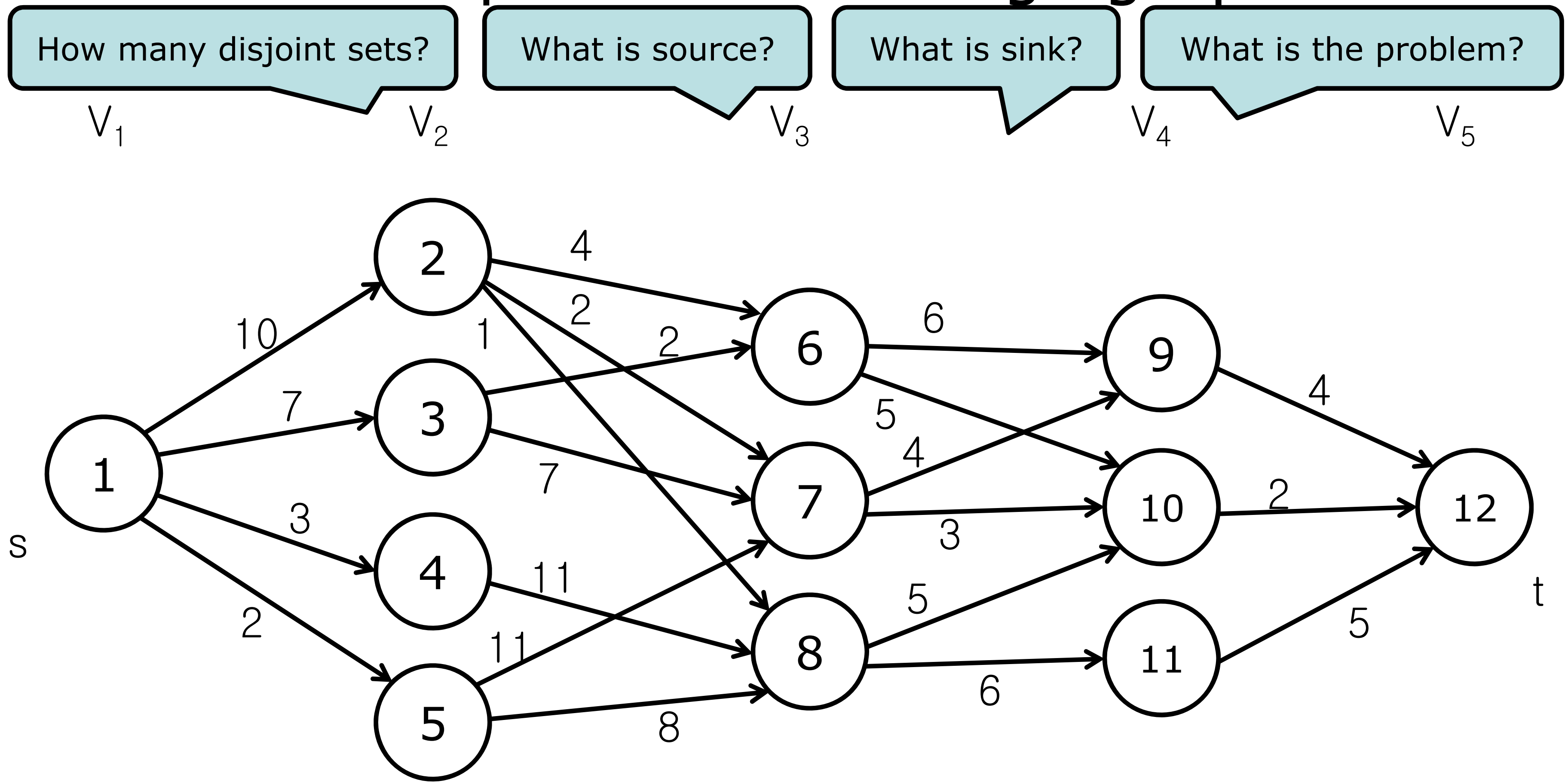
Contents

6.3 Multistage graph

- Definition of multistage graph
  - A directed graph
    - The vertices are partitioned into $k \geq 2$ disjoint sets V_i, $1 \leq i \leq k$.
    - If $<u, v>$ is an edge in E, then $u \in V_i$ and $v \in V_{i+1}$, for some i.
    - $|V_1| = |V_k| = 1$.
    - If $s \in V_1$, then s is the source
    - If $t \in V_k$, then t is the sink
  - Let $c(i, j)$ be the cost of edge $<i, j>$.
    - The cost of a path from s to t is the sum of the costs of the edges on the path.
    - The multistage graph problem is to find a minimum cost path from s to t.

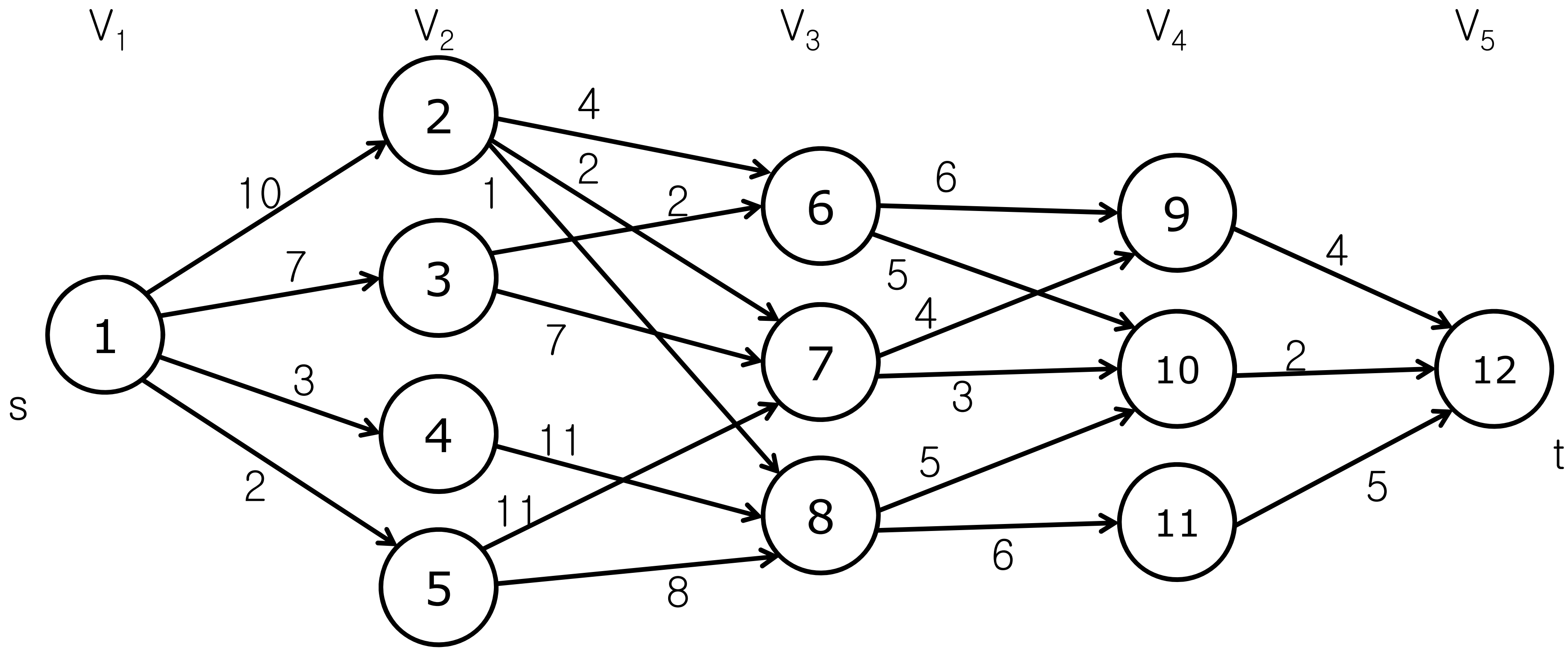
6.3 Multistage graph

- Definition of multistage graph
  - An example of a multistage graph



6.3 Multistage graph

- Problem
  - Find an optimal path from source to sink



6.3 Multistage graph

- Strategy
  - P(i, j)
    - minimum cost path from vertex j in V_i to sink
  - COST(i, j)
    - the cost of P(i, j)

Algorithm

$$\boxed{COST(i, j) = \min_{l \in V_{i+1}} \{ c(j, l) + COST(i+1, l) \}}$$

  - What is the problem to solve?
    - P(1, 1)
    - COST(1, 1)

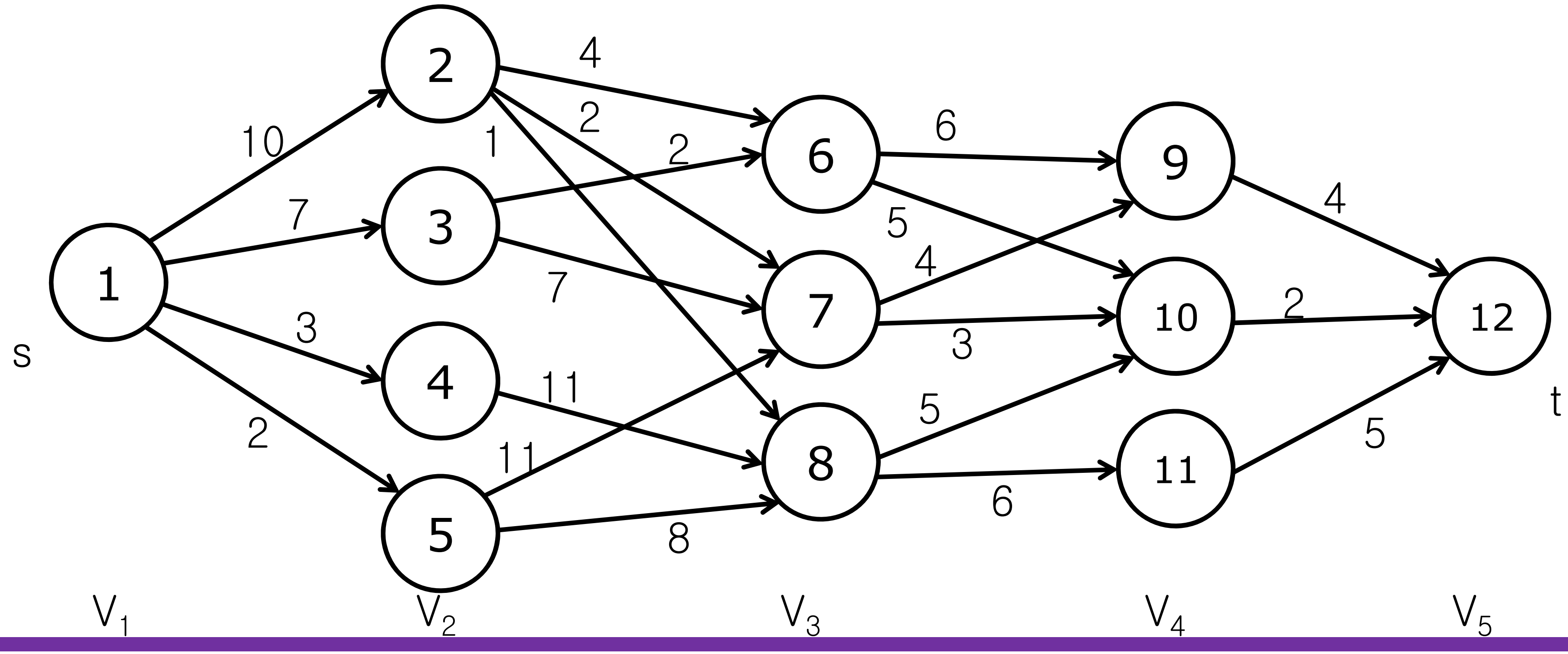
6.3 Multistage graph

- Algorithm

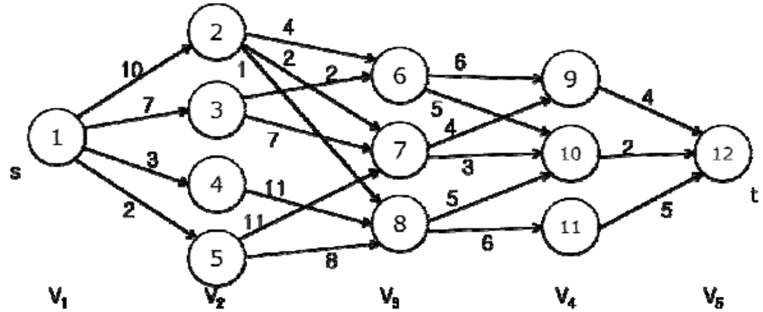
```
void MULTIGRAPH ( Graph G)
{
    int **COST, **D;

    estimate COST(k - 1);
    for ( i = k – 2; i >= 1; i-- ) {
        for all vertices j at stage i
            COST(i, j) ← min(cost(j, l) + COST(i+1, l));
            D(i, j) ← l;
    }
}
```

6.3 Multistage graph

- Example (At V_1)
  - COST(1, 1) =



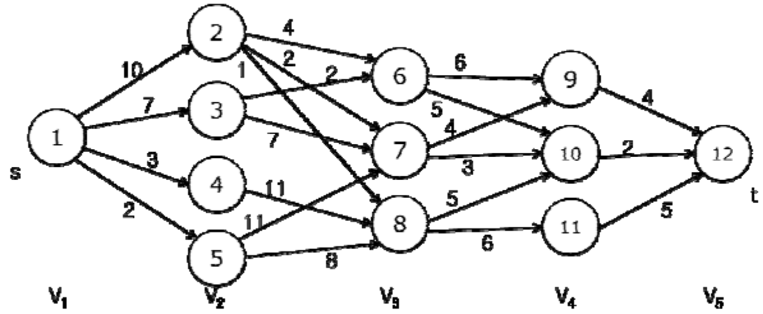
6.3 Multistage graph



- Example (At V_4)
  - COST(4, 9) =
  - COST(4, 10) =
  - COST(4, 11) =

6.3 Multistage graph

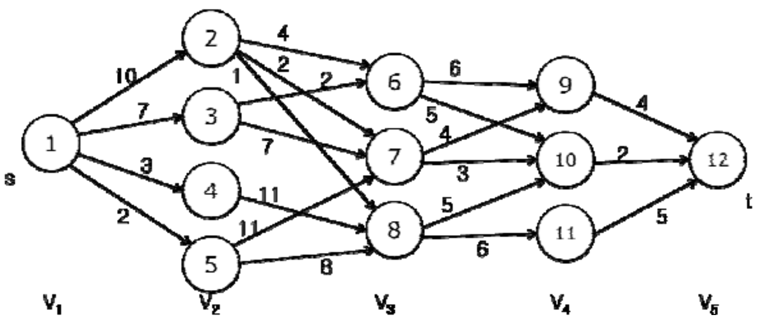
- Example (At V_4)
  - COST(4, 9) = 4
  - COST(4, 10) = 2
  - COST(4, 11) = 5



6.3 Multistage graph

- Example (At V_3)
  - COST(3, 6) =
  - COST(3, 7) =
  - COST(3, 8) =

$$COST(i,j) = \min_{l \in V_{i+1}} \{ c(j,l) + COST(i+1,l) \}$$



6.3 Multistage graph

- Example (At V_3)
  - $COST(3, 6) = \min\{6 + COST(4, 9),$
    $5 + COST(4, 10)\}$
  - $COST(3, 7) = \min\{4 + COST(4, 9),$
    $3 + COST(4, 10)\}$
  - $COST(3, 8) = \min\{5 + COST(4, 10),$
    $6 + COST(4, 11)\}$

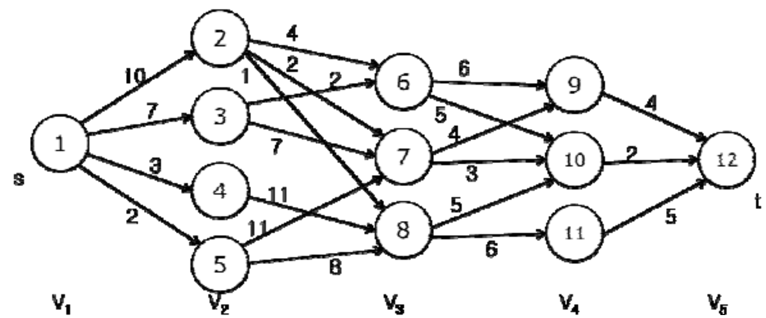
6.3 Multistage graph

- Example (At V_3)
  - $COST(3, 6) = \min\{6 + COST(4, 9),$
    $5 + COST(4, 10)\}$
    $= \min\{6 + 4, 5 + 2\} = 7$
  - $COST(3, 7) = \min\{4 + COST(4, 9),$
    $3 + COST(4, 10)\}$
    $= \min\{4 + 4, 3 + 2\} = 5$
  - $COST(3, 8) = \min\{5 + COST(4, 10),$
    $6 + COST(4, 11)\}$
    $= \min\{5 + 2, 6 + 5\} = 7$

6.3 Multistage graph

- Example (At V_2)
  - COST(2, 2) =
  - COST(2, 3) =
  - COST(2, 4) =
  - COST(2, 5) =

$$COST(i,j) = \min_{l \in V_{i+1}} \{c(j,l) + COST(i+1,l)\}$$



6.3 Multistage graph

- Example (At V_2)
  - COST(2, 2) = min{4 + COST(3, 6),
    2 + COST(3, 7),
    1 + COST(3, 8)}
  - COST(2, 3) = min{2 + COST(3, 6),
    7 + COST(3, 7)}
  - COST(2, 4) = min{11 + COST(3, 8)}
  - COST(2, 5) = min{11 + COST(3, 7),
    8 + COST(3, 8)}

$$COST(i, j) = \min_{l \in V_{i+1}} \{c(j, l) + COST(i+1, l)\}$$

6.3 Multistage graph

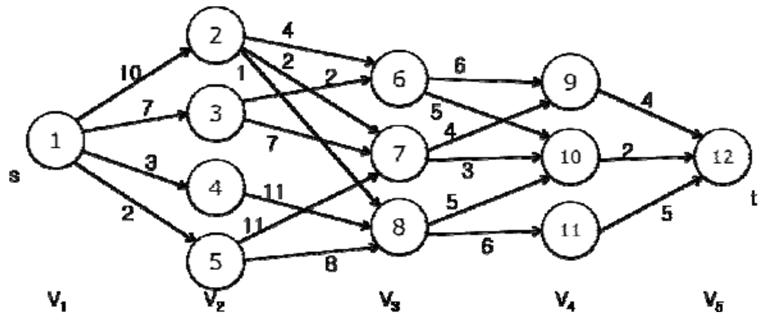
- Example (At V_2)
  - $$\begin{aligned}\mathrm{COST}(2, 2) &= \min\{4 + \mathrm{COST}(3, 6),\\ &\qquad 2 + \mathrm{COST}(3, 7),\\ &\qquad 1 + \mathrm{COST}(3, 8)\}\\ &= \min\{4 + 7, 2 + 5, 1 + 7\} = 7\end{aligned}$$
  - $$\begin{aligned}\mathrm{COST}(2, 3) &= \min\{2 + \mathrm{COST}(3, 6),\\ &\qquad 7 + \mathrm{COST}(3, 7)\}\\ &= \min\{2 + 7, 7 + 5\} = 9\end{aligned}$$
  - $\mathrm{COST}(2, 4) = \min\{11 + \mathrm{COST}(3, 8)\} = 11 + 7 = 18$
  - $$\begin{aligned}\mathrm{COST}(2, 5) &= \min\{11 + \mathrm{COST}(3, 7),\\ &\qquad 8 + \mathrm{COST}(3, 8)\}\\ &= \min\{11 + 5, 8 + 7\} = 15\end{aligned}$$

6.3 Multistage graph

- Example (At V_1)
  - COST(1, 1) =

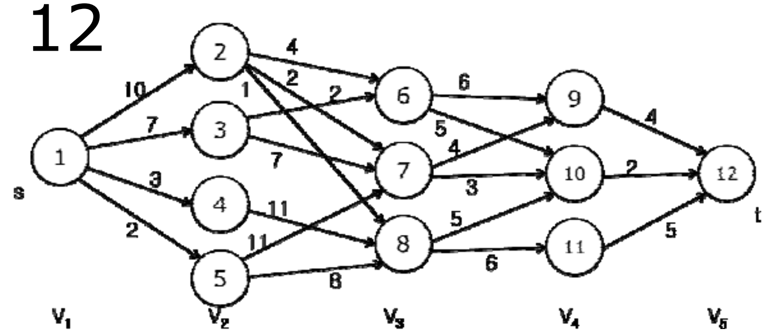
  - P(1, 1) =

$$COST(i,j) = \min_{l \in V_{i+1}} \{ c(j,l) + COST(i+1,l) \}$$



6.3 Multistage graph

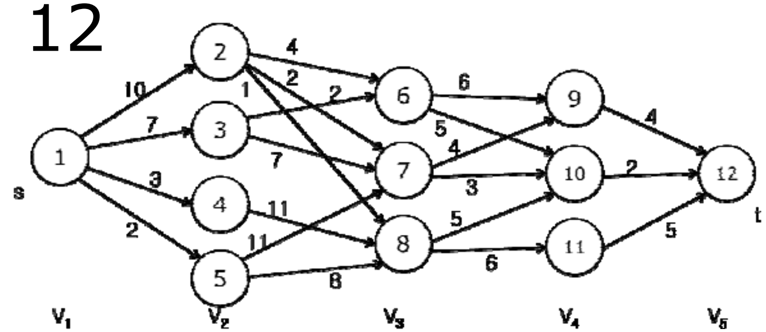
- Example (At V_1)
  - $\text{COST}(1, 1) = \min\{10 + \text{COST}(2, 2),$
    $7 + \text{COST}(2, 3),$
    $3 + \text{COST}(2, 4),$
    $2 + \text{COST}(2, 5)\}$
  - $P(1, 1) = 1 \rightarrow 3 \rightarrow 6 \rightarrow 10 \rightarrow 12$



6.3 Multistage graph

- Example (At V_1)
  - $\mathrm{COST}(1, 1) = \min\{10 + \mathrm{COST}(2, 2),$
    $7 + \mathrm{COST}(2, 3),$
    $3 + \mathrm{COST}(2, 4),$
    $2 + \mathrm{COST}(2, 5)\}$
    $= \min\{10 + 7, 7 + 9, 3 + 18, 2 + 15\}$
    $= 16$

  - $P(1, 1) = 1 \rightarrow 3 \rightarrow 6 \rightarrow 10 \rightarrow 12$



6.3 Multistage graph

- Application: Resource allocation problem
  - Allocating n units of resources to r projects
  - N(i, j)
    - The net profit of allocating j units of resources to project i.
  - How to maximize total net profit?
  - Formulate this problem using multistage graph

6.3 Multistage graph

- Application: Resource allocation problem
  - Multistage graph
    - (r+1) stage graph problem
    - Stage i represents project i.
    - (n+1) vertices V(i, j) are associated with stage i.
    - Stage 1 and r+1 has one vertex: V(1, 0) = s and V(r+1, n) = t.
    - Vertex V(i, j) represents the stage in which a total of j units of resources have been allocated to projects 1, 2, …, i-1.
    - The edges are of the form <V(i, j), V(i+1, l)>.
      - The cost of the edges is N(i, l-j).

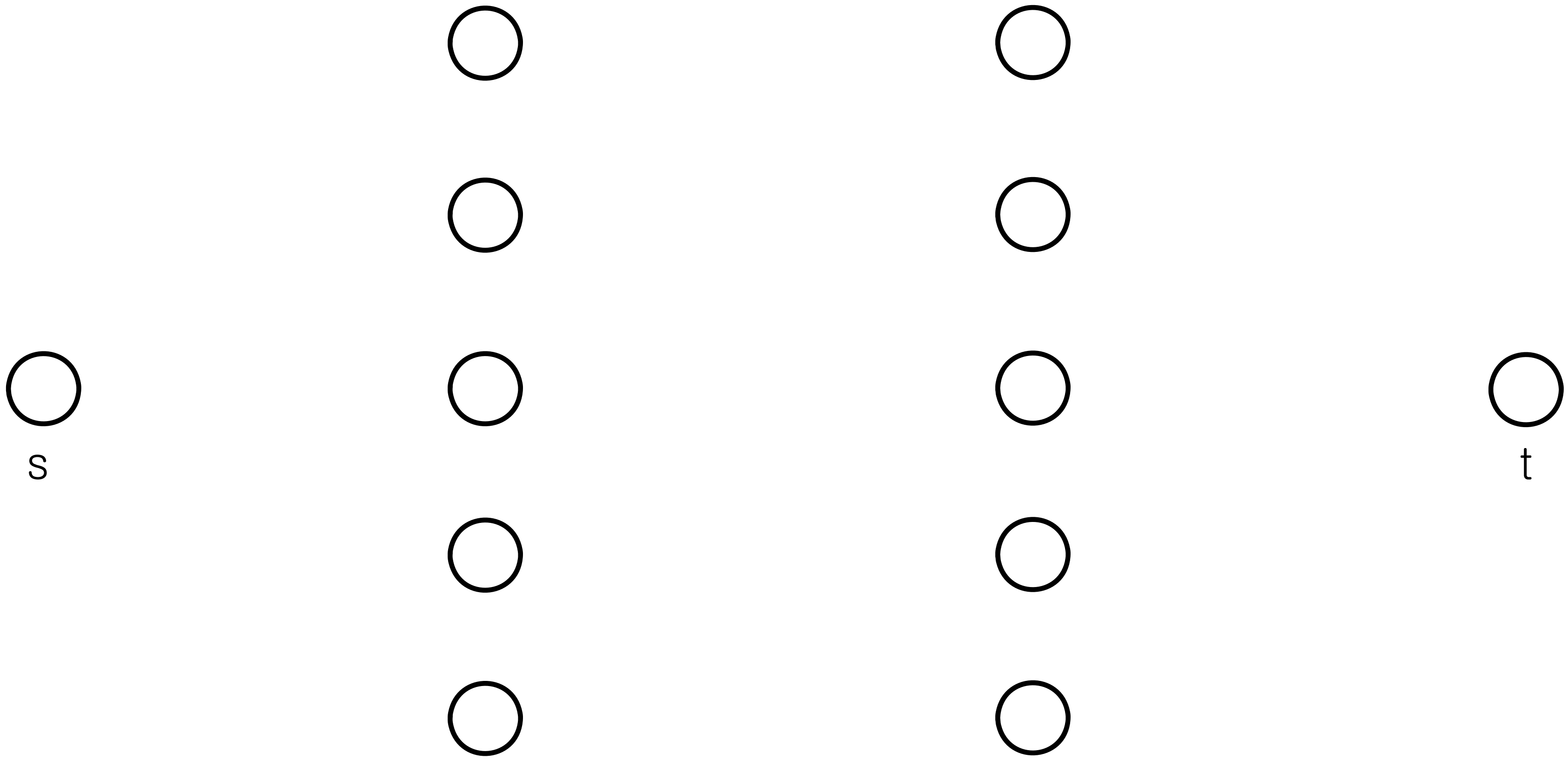
6.3 Multistage graph

- Example
  - 4 units to 3 teams

	1st team	2nd team	3rd team
N=0	0	5	3
N=1	10	15	5
N=2	20	20	10
N=3	25	22	15
N=4	25	25	25

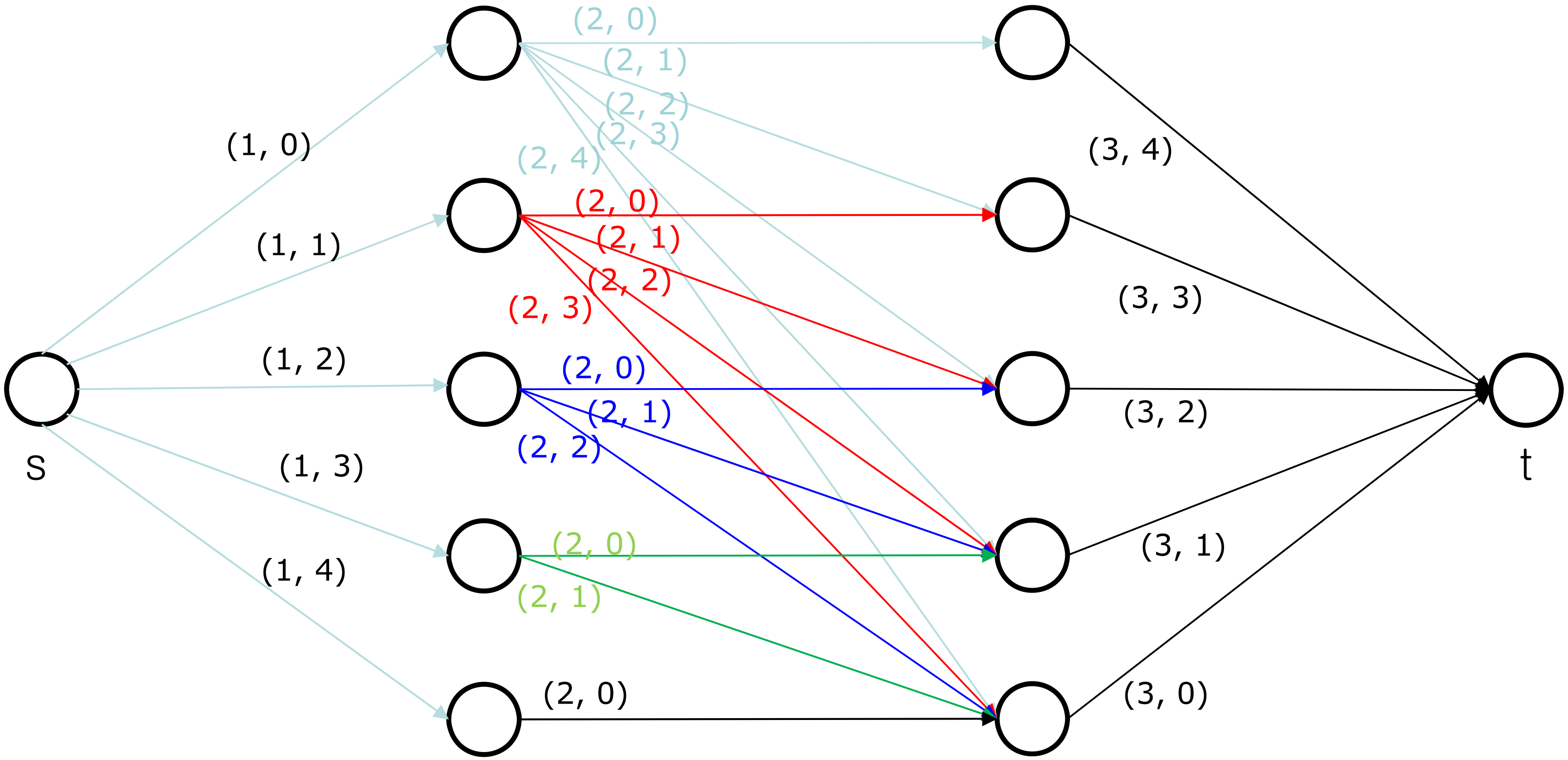
6.3 Multistage graph

- Application: Resource allocation problem
  - Multistage graph of 4 units to 3 teams



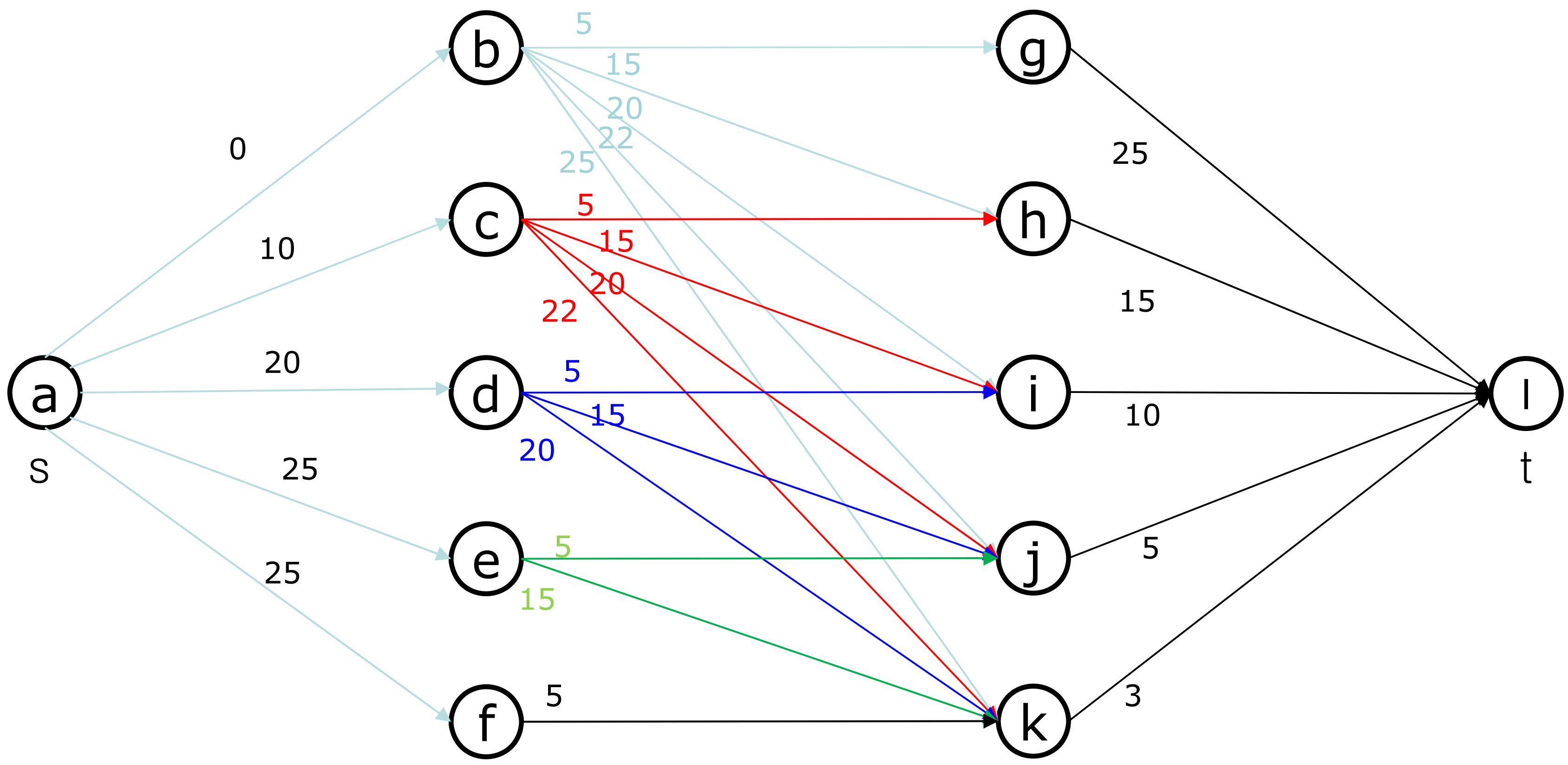
6.3 Multistage graph

- Application: Resource allocation problem
  - Multistage graph of 4 units to 3 teams



6.3 Multistage graph

- Application: Resource allocation problem
  - Multistage graph of 4 units to 3 teams



6.3 Multistage graph

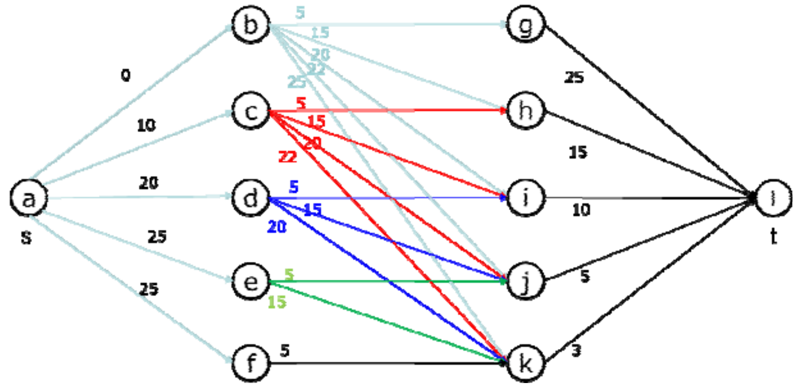
- Application: Resource allocation problem
  - Multistage graph of 4 units to 3 teams

  - Cost(1, a) = ?

6.3 Multistage graph

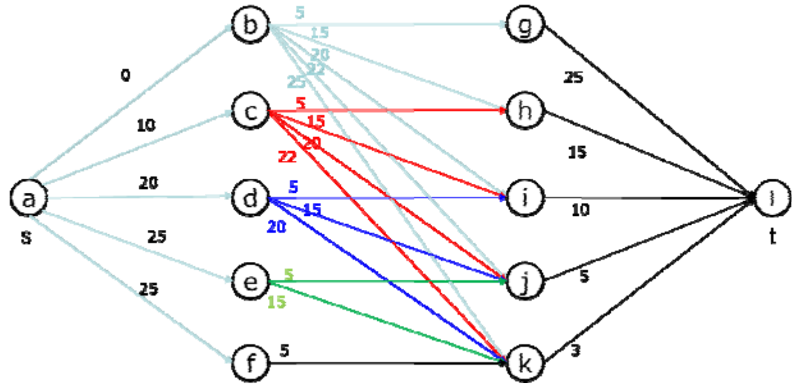
- Application: Resource allocation problem
  - Multistage graph of 4 units to 3 teams

  - Cost(3, g) =
  - Cost(3, h) =
  - Cost(3, i) =
  - Cost(3, j) =
  - Cost(3, k) =



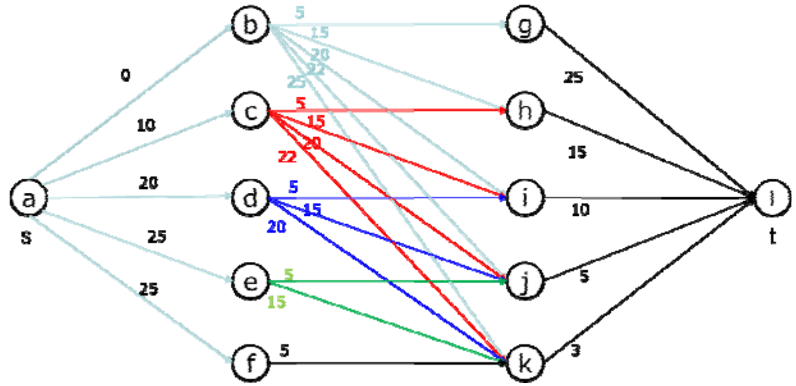
6.3 Multistage graph

- Application: Resource allocation problem
  - Multistage graph of 4 units to 3 teams
  - Cost(2, b) =
  - Cost(2, c) =
  - Cost(2, d) =
  - Cost(2, e) =
  - Cost(2, f) =



6.3 Multistage graph

- Application: Resource allocation problem
  - Multistage graph of 4 units to 3 teams
  - Cost (1, a) =



All about Dynamic Programming

Type	Stepwise approach	Recursive structure	Approach	Time	Specialty
0/1 KNAP	Choosing objects	KNAP(1, n, M) = max {KNAP(2, n, M), KNAP(2, n, M-w_1)+p_1}	Forward	$O(2^n)$	Smart Degenerate Case
Weighted Interval Scheduling	Selecting intervals	OPT(j) = max {OPT(p(j)) + v_j, OPT(j-1)}	Backward	O(n)	Auxiliary memory
Multistage Graph	Determining vertex in the next stage	COST(i, j) = $min_{l \in V_{i+1}}$ {c(j,l) + COST(i+1, l)}	Backward	O(m)	Resource allocation problem
All Pairs Shortest Path					

6.3 Multistage graph

- 다음 중 옳지 않은 문장을 모두 고르시오.
- (a) multistage graph는 acyclic graph이다.
- (b) multistage graph에서 계층의 수는 vertex의 수에 비례한다.
- (c) multistage graph 문제는 Dijkstra 알고리즘으로 해결할 수 있다.
- (d) multistage graph 문제는 backward approach이다.