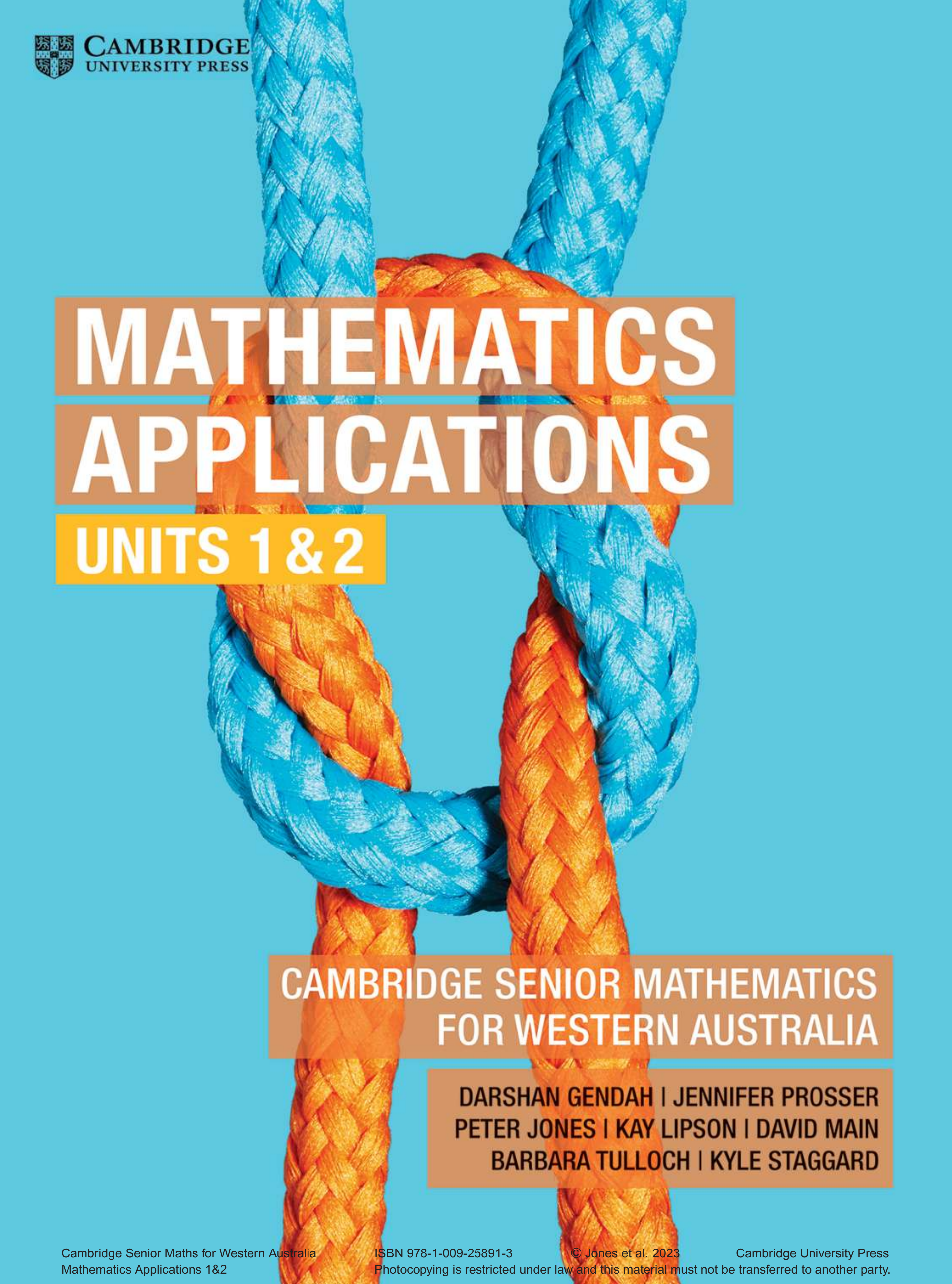


CAMBRIDGE
UNIVERSITY PRESS

MATHEMATICS APPLICATIONS
UNITS 1 & 2

CAMBRIDGE SENIOR MATHEMATICS
FOR WESTERN AUSTRALIA

DARSHAN GENDAH | JENNIFER PROSSER
PETER JONES | KAY LIPSON | DAVID MAIN
BARBARA TULLOCH | KYLE STAGGARD

ISBN 978-1-009-25891-3

CAMBRIDGE
UNIVERSITY PRESS

Shaftesbury Road, Cambridge CB2 8EA, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of the Cambridge University Press & Assessment, a department of the University of Cambridge.

We share the University's mission to contribute to society through the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org

First published 2023
20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1

Cover design by Denise Lane (Sardine Designs)
Text design by Jane Pitkethly
Typeset by diacriTech
Printed in Australia by Ligare Pty Ltd

A catalogue record for this book is available from the National Library of Australia at www.nla.gov.au

ISBN 978-1-009-25891-3 Paperback

Additional resources for this publication at www.cambridge.edu.au/GO

Cambridge University Press & Assessment acknowledges the Australian Aboriginal and Torres Strait Islander peoples of this nation. We acknowledge the traditional custodians of the lands on which our company is located and where we conduct our business. We pay our respects to ancestors and Elders, past and present. Cambridge University Press & Assessment is committed to honouring Australian Aboriginal and Torres Strait Islander peoples' unique cultural and spiritual relationships to the land, waters and seas and their rich contribution to society.

Contents

ISBN 978-1-009-25891-3

ISBN 978-1-009-25891-3

ISBN 978-1-009-25891-3

ISBN 978-1-009-25891-3

Online appendices accessed through the Interactive Textbook or PDF Textbook

ISBN 978-1-009-25891-3

Introduction

Cambridge Mathematics Applications for Western Australia Units 1 & 2 is a new edition aligned specifically to the Western Australian Mathematics Applications Year 11 syllabus. Covering both Units 1 and 2 in one resource, it has been written with practical contexts and worded questions as its priority alongside ample practice offered through worked examples and exercises. This course follows on from the K–10 Australian Curriculum and expands upon the fundamental skills and knowledge developed in earlier levels.

Compared to the previous Australian Curriculum edition, this WA edition has undergone a number of revisions. Careful adjustments to notation and language have been made throughout to match that used in the WA syllabus, and in WA classrooms more generally. Multiple-choice questions that were formerly located in the chapter reviews and revision chapters have been removed for this edition.

This edition begins by introducing computation and practical arithmetic and then consumer arithmetic, building upon the non-calculator skills necessary for the Year 11 and 12 Applications course, before leading to shape and measurement, linear and non-linear expressions, and matrices. The study of the normal distribution and investigating and comparing data distributions covers the statistical strand of the syllabus. Right-angled trigonometry and non-right-angled trigonometry are now covered in separate chapters to better facilitate teaching and learning. Chapters on linear graphs and models and simultaneous linear equations focus largely on new applications of previous knowledge and skills, and getting confident with using a CAS calculator in appropriate contexts.

Worked examples utilising CAS calculators are provided throughout, with screenshots and detailed user instructions for both ClassPad and TI-Nspire included for each CAS example. The CAS 'ribbons' used within each exercise direct students to questions which should be completed with the aid of technology, and are intended to help prepare students for examinations and other assessments.

The TI-Nspire calculator examples and instructions have been completed by Russell Brown and those for the Casio ClassPad have been completed by Maria Schaffner.

The integration of the features of the textbook and the new digital components of the package, powered by Cambridge HOTmaths, are illustrated on pages ix to xii.

About Cambridge HOTmaths

Cambridge HOTmaths is a comprehensive, award-winning mathematics learning system – an interactive online maths learning, teaching and assessment resource for students and teachers, for individuals or whole classes, for school and at home. Its digital engine or platform is used to host and power the Interactive Textbook and the Online Teaching Suite, and selected topics from HOTmaths' own Years 9 and 10 courses area are available for revision of prior knowledge. All this is included in the price of the textbook.

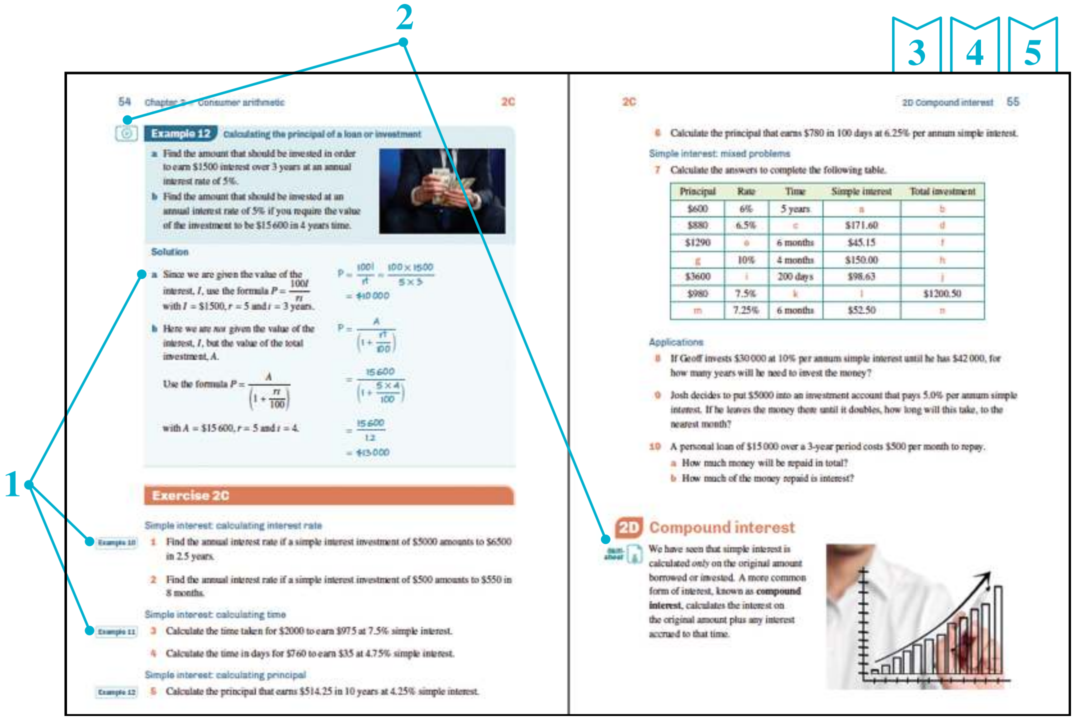
ISBN 978-1-009-25891-3

Overview

Overview of the print book

1. Graded step-by-step worked examples with precise explanations (and video versions) encourage independent learning, and are linked to exercise questions.
2. Additional linked resources in the Interactive Textbook are indicated by icons, such as skillsheets and video versions of examples.
3. Questions that suit the use of a CAS calculator to solve them are identified within exercises.
4. Chapter reviews contain a chapter summary and short-answer and extended-response questions.
5. The glossary includes page numbers of the main explanation of each term.

Numbers refer to descriptions above.

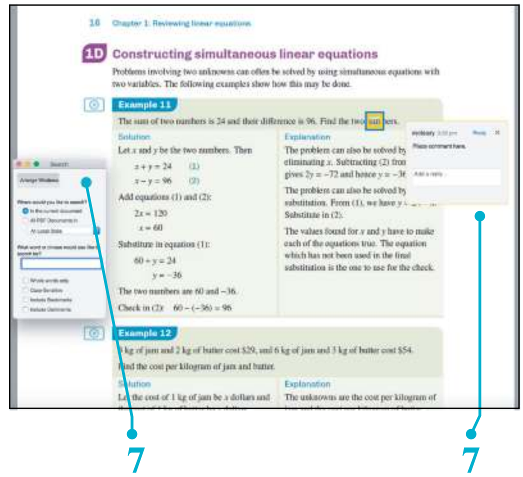


ISBN 978-1-009-25891-3

Overview of the downloadable PDF textbook

6 The convenience of a downloadable PDF textbook has been retained for times when users cannot go online.

7 PDF annotation and search features are enabled.



Overview of the Interactive Textbook

The **Interactive Textbook (ITB)** is an online HTML version of the print textbook powered by the HOTmaths platform, included with the print book or available as a separate purchase.

8 The material is formatted for on screen use with a convenient and easy-to-use navigation system and links to all resources.

9 **Workspaces** for all questions, which can be enabled or disabled by the teacher, allow students to enter working and answers online and to save them. Input is by typing, with the help of a symbol palette, handwriting and drawing on tablets, or by uploading images of writing or drawing done on paper.

10 **Self-assessment tools** enable students to check answers, mark their own work, and rate their confidence level in their work. This helps develop responsibility for learning and communicates progress and performance to the teacher. Student accounts can be linked to the learning management system used by the teacher in the Online Teaching Suite, so that teachers can review student self-assessment and provide feedback or adjust marks.

11 All worked examples have **video versions** to encourage independent learning.

12 **Worked solutions** are included and can be enabled or disabled in the student ITB accounts by the teacher.

13 An expanded and revised set of **Desmos interactives** and activities based on embedded graphics calculator and geometry tool windows demonstrate key concepts and enable students to visualise the mathematics.

14 The **Desmos graphics calculator, scientific calculator**, and **geometry tool** are also embedded for students to use for their own calculations and exploration.

15 **Revision of prior knowledge** is provided with links to diagnostic tests and Year 10 HOTmaths lessons.

16 **Quick quizzes** containing automarked multiple-choice questions have been thoroughly expanded and revised, enabling students to check their understanding.

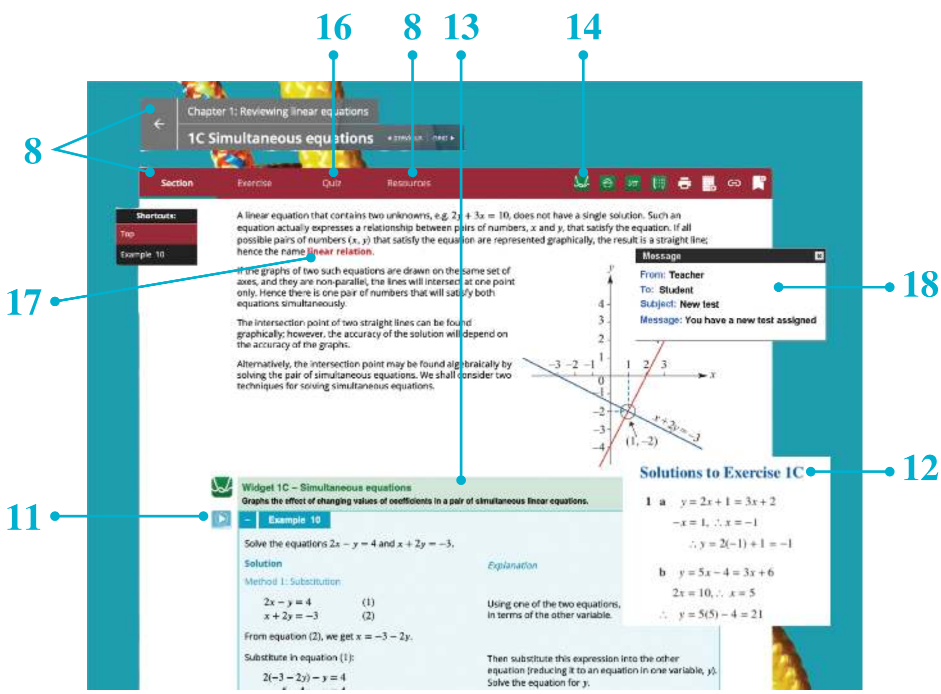
17 **Definitions** pop up for key terms in the text, and are also provided in a dictionary.

18 Messages from the teacher assign tasks and tests.

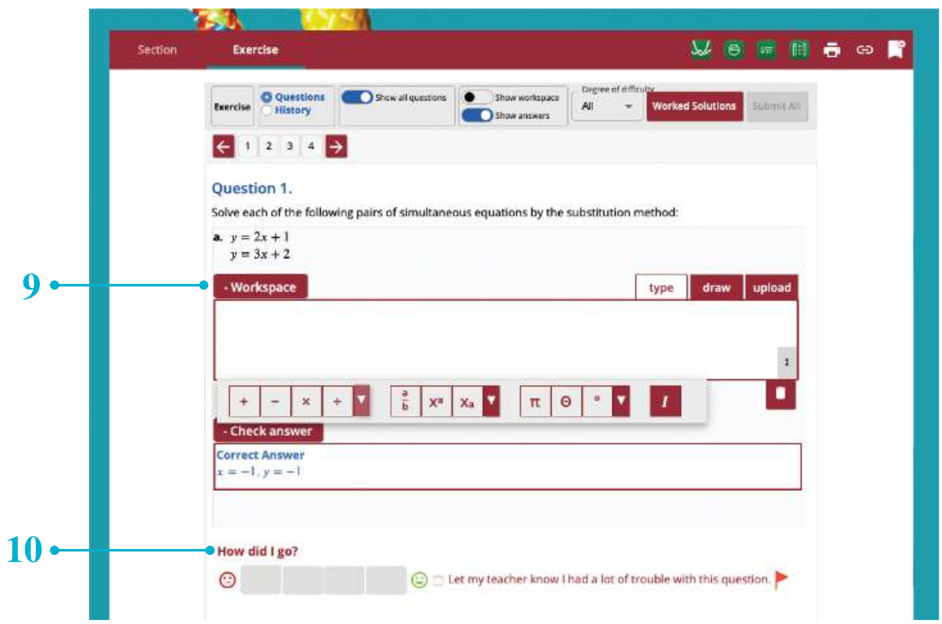
ISBN 978-1-009-25891-3

INTERACTIVE TEXTBOOK POWERED BY THE HOTmaths PLATFORM

A selection of features is shown. Numbers refer to the descriptions on pages xi–xii. HOTmaths platform features are updated regularly



WORKSPACES AND SELF-ASSESSMENT



 ISBN 978-1-009-25891-3

Overview of the Online Teaching Suite powered by the HOTmaths platform

The Online Teaching Suite is automatically enabled with a teacher account and is integrated with the teacher's copy of the Interactive Textbook. All the teacher resources are in one place for easy access. The features include:

19 The HOTmaths learning management system with class and student analytics and reports, and communication tools.

20 Teacher's view of a student's working and self-assessment which enables them to modify the student's self-assessed marks, and respond where students flag that they had difficulty.

21 A HOTmaths-style test generator.

22 A suite of chapter tests and assignments.

23 Editable curriculum grids and teaching programs.

24 A brand-new **Exam Generator**, allowing the creation of customised printable and online trial exams (see below for more).

More about the Exam Generator

The Online Teaching Suite includes a comprehensive bank of SCSA exam questions, augmented by exam-style questions written by experts, to allow teachers to create custom trial exams.

Custom exams can model end-of-year exams, or target specific topics or types of questions that students may be having difficulty with.

Features include:

- Filtering by question-type, topic and degree of difficulty
- Searchable by key words
- Answers provided to teachers
- Worked solutions for all questions
- SCSA marking scheme
- All custom exams can be printed and completed under exam-like conditions or used as revision.

ISBN 978-1-009-25891-3

Acknowledgements

The author and publisher wish to thank the following sources for permission to reproduce material:

Cover: © Getty Images / Richard Drury

Images: © Getty Images / JamieB, Chapter 1 Opener / Philip Wallick, Chapter 2 Opener / dashu83, Chapter 3 Opener / Matt Henry Gunther, Chapter 4 Opener / Jasmin Merdan, Chapter 5 Opener / Andriy Onufriyenko, Chapter 6 Opener / The normal distribution, Chapter 7 opener / Francesco Riccardo Iacomino, Chapter 8 Opener / edchin, Chapter 9 Opener / Karl Hendon, Chapter 10 Opener / mbbirdy, Chapter 11 Opener. Chad Baker, Exercise 1D (1) / Calvin Chan Wai Meng, Exercise 1E (1) / Upengyu, Exercise 1E (2) / t_kimura, Exercise 1G (1) / Subin Pumsom, Exercise 1G (2) / Monty Rakusen, Exercise 1G (3) / Nastasic, Exercise 1G (4) / MarkGillow, Exercise 1H (1) / Westend61, Exercise 1H (2) / Kyri_n Geisser, Exercise 1H (3) / Nick Brundle Photography, Exercise 1H (4) / Sean Gladwell, Exercise 1H (5) / Robynmac, Exercise 1I (1) / Scott Barber, Exercise 1J (1) / MicrovOne, Exercise 1J (3 unltra clean) / MariaAverburg, Exercise 1J (4 ultra wash) / Jacobs Stock Photography Ltd, Exercise 1J (5 store A) / Image Source, Exercise 1J (6 store B) / Edwin Tan, Exercise 1J (7 store C) / Tom Werner, Exercise 1J (8 store D) / Kinga Krzeminska, Exercise 1J (9) / Pierre-Yves Babelon, Chapter 1 review / Oleksandr Hruts, 2A (1) / zhudifeng, 2A (2) / Tina Siegmann, 2A (3) / 3DMAVR, 2B (1) / Rinelle, 2B (2) / KLH49, 2B (3) / Burak Karademir, 2B (4) / hocus-focus, 2D (1) / Yuichiro Chino, 2D (2) / Mgstudyo, Exercise 2E (1) / Lauryn Ishak, Exercise 2E (2) / hoozone, 2F (1) /c Shaunl, 2G (1) / Erik Isakson, 2G (2) / Chatchai Chaihan, 2I (1) / fstop123, 2I (2) / Newsday LLC, 2I (3) / 97, 2I (4) / nazar_ab, 2J (1) / Pramote Polyamate, 3A (1) / Jackyenjoyphotography, 3D (1) / amoklv, 3D (2) / 101cats, 3E (1) / Kae Cheatham, 3F (1) / Oliver Strewe, 3F (2) / JEAN-SEBASTIEN EVRARD, 3G (1) / Paul Souders, 3G (2) / saulgranda, 3J (1) / Grant Faint, 3K (1) / Jose Luis Pelaez Inc, 3K (2) / Daniel Kaesler, 3K (3) / SafakOguz, 3M (1) / artpartner-images, 3M (2) / Creativ Studio Heinemann, 3L (1) / Ben Welsh, 4A (1) / Jean-Philippe Tournut, 4A (2) / Jelena990, 4A (3) / fotograzia, 4A (4) / Christoph Hetzmannseder, 4A (5) / samarets1984, 4A (6) / Allanswart, 4A (7) / kiboka, 4A (8) / StefaNikolic, 4C (1) / Edward Berthelot, 4C (2) / Nurphoto, 4E (1) / Skodonnell, 4F (1) / Nick Bubb www.imagecaptured.co.uk, 4F (2) / GK Hart/Vikki Hart, 4F (3) / Pamspix, 4F (4) / Louise Beaumont, 4F (5) / Gabriel Martinez / EyeEm, 5A (1) / Atlantide Phototravel, 5A (2) / Godong, 5A (3) / Bim, 5A (4) / Peter Cade, 5A (5) / Jacobs Stock Photography Ltd, 5C (1), 5D (2) / The Good Brigade, 5D (3) / Westend61, 5F (2) / Renate Frost / EyeEm, 5F (3) / Nipaporn Panyacharoen / EyeEm, 5F (4) / Photoevent, 5F (5) / Diane Keough, 5G (1) / Peter

ISBN 978-1-009-25891-3

Dazeley, 6A (1) / MoMo Productions, 6A (2) / monkeybusinessimages, 6C (1) / Xavier Lorenzo, 6C (2) / Joos Mind, 6D (1) / Thomas Barwick, 6D (2-3) / Paul Kane/Stringer, 6F (1) / Mgstudyo, 6F (2) / Solstock, 6G (1) / Dylan Burns/AFL Photos/Stringer, 6G (2) / Maskot, 6G (3) / Peter Muller, 6G (5) / Stephen Welstead, 6G (4) / Getty, 6G (5) / Peter Muller, 6I (1) / Kevin Dodge, 6I (2) / Solskin, 6I (3-4) / gradyreese, 6I (5) / Luis Alvarez, 6I (6) / FreshSplash, Exercise 7C / Nomad Outing, 8D (1) / Evgeni Dinev Photography, 8E (1) / Reinhard Krull/EyeEm, 8E (2) / Luis Puga, 8F (1) / ProPIC, 8F (2) / Artur Debat, 9B (1) / Reinhard Krull, 9C (1) / Christoph Jorda, 10B (1) / Jackyenjoyphotography, 10B (2) / zilli, 10G-1 (1) / SimpleImages, 10G-2 (1) / Yaorusheng, 11A (1) / Malte Mueller, 11B (1) / jayk7, 11B (2) / Rosemary Calvert, 11D (1) / ouchi_iro, 11D (2) / Yevgen Romanenko, 11D (3) / Jonathan Paciullo, 11D (4) / Reinhard Dirscherl, 11D (5) / Photoevent, 11D (6) / fcafotodigital, 11D (7) / Richard Newstead, 11E (1) / Jose Luis Pelaez Inc, 11E (2) / kyoshino, 11E (3).

© GOOGLE MAPS, 9D (1).

Subtopics in chapter openers © School Curriculum and Standards Authority (2017) Mathematics Applications Year 11 Syllabus. The School Curriculum and Standards Authority does not endorse this publication or product.

Every effort has been made to trace and acknowledge copyright. The publisher apologises for any accidental infringement and welcomes information that would redress this situation.

ISBN 978-1-009-25891-3

1 Computation and practical arithmetic

In this chapter

Syllabus references

Topic: Applications of rates and percentages

Subtopics: 1.1.4 – 1.1.6

This chapter revises basic methods of computation used in general mathematics. It will allow you to carry out the necessary numerical calculations for solving problems. We will begin with the fundamentals.

ISBN 978-1-009-25891-3

1A Order of operations

Adding, subtracting, multiplying, dividing and squaring are some examples of operations that are used in mathematics. When carrying out a sequence of arithmetic operations, it is necessary to observe a definite sequence of rules. These rules, defining the **order of operations**, have been devised and standardised to avoid confusion.

Order of operation

The rules are to:

- always complete the operations in brackets first
- then carry out the multiplication and division operations (in order, from left to right)
- then carry out the addition and subtraction operations (in order, from left to right).

These rules can also be remembered by using **BIMDAS**.

B **B**rackets come first

I If **I**ndices are involved (powers, square roots), you complete that next

MD **M**ultiplication and **D**ivision, working left to right across the page

AS **A**ddition and **S**ubtraction, working left to right across the page

A calculator, with *algebraic logic*, will carry out calculations in the correct order of operations. However, particular care must be taken with brackets.

Pronumeral

A number or **pronumeral** (letter) placed in front of a bracket means that you multiply everything in the bracket by that number or pronumeral.

$4(8)$ means $4 \times 8 = 32$

$5(x - 9) = 5x - 45$

$a(3a + 6) = 3a^2 + 6a$

Example 1 Using correct order of operation

Evaluate the following.

a $3 + 6 \times 8$ **b** $(3 + 6) \times 8$ **c** $8 \div 2 - 2$ **d** $23 - (8 - 5)$

e $(4)\,3 - 2$ **f** $3 + 5(x - 1)$ **g** $(3 \times 8.5 - 4) - (4.1 + 5.4 \div 2)$

Solution

a $3 + 6 \times 8 = 3 + 48$
$= 51$

b $(3 + 6) \times 8 = 9 \times 8$
$= 72$

ISBN 978-1-009-25891-3

c $8 \div 2 - 2 = 4 - 2$
$= 2$

d $23 - (8 - 5) = 23 - 3$
$= 20$

e $(4)3 - 2 = 12 - 2$
$= 10$

f $3 + 5(x - 1) = 3 + 5x - 5$
$= 5x - 2$

g $(3 \times 8.5 - 4) - (4.1 + 5.4 \div 2) = (25.5 - 4) - (4.1 + 2.7)$
$= 21.5 - 6.8$
$= 14.7$

Exercise 1A

Example 1a–d

1 Evaluate the following, without using a calculator.

a $5 + 4 \times 8$
b $4 \times 3 - 7$
c $7 \times 6 - 4 + 4 \times 3$
d $15 \div 3 + 2$
e $3 + 12.6 \div 3$
f $4 \times (8 + 4)$
g $15 - 9 \div 2 + 4 \times (10 - 4)$
h $(3.7 + 5.3) \div 2$
i $8.6 - 3 \times 2 - 6 \div 3$
j $(3 \times 4 - 3) \div (2 - 3 \times 4)$

Example 1g

2 Use your calculator to find the answers to the following.

a $(8.23 - 4.5) + (3.6 + 5.2)$
b $(17 - 8.7) - (73 - 37.7)$
c $(6.2 + 33.17) \times (6.9 - 6.1)$
d $(3.2 + 0.5 \div 2.5) \div (8.6 - 1.3 \times 4)$

Example 1f–g

3 Evaluate the following.

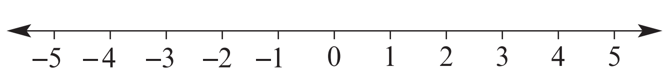
a $9(3)$
b $2(x - 7)$
c $10(5 - y)$
d $w(8 - 2)$
e $k(k + 8)$
f $27(2) - 3(8)$
g $(5 - 3)x + 7(2)$
h $3(5) \times 2 - 8$
i $3(x + 1) - 8$
j $4 - 2(x + 3)$

1B Directed numbers

Positive and negative numbers are **directed numbers** and can be shown on a number line.

Addition and subtraction

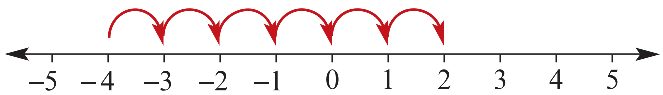
It is often useful to use a number line when adding directed numbers.



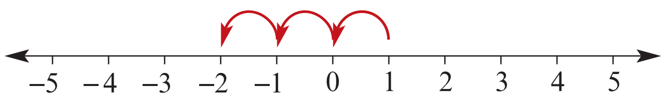
ISBN 978-1-009-25891-3

Adding a positive number means that you move to the right.

Example: $-4 + 6 = 2$



Adding a negative number means that you move to the left.
Example: $1 + (-3) = -2$



When subtracting directed numbers, you add its opposite.
Example: $-2 - 3$ is the same as $-2 + (-3) = -5$
Example: $7 - (-9) = 7 + 9 = 16$

Multiplication and division

Multiplying or dividing two numbers with the *same* sign gives a *positive* value.
Multiplying or dividing two numbers with *different* signs gives a *negative* value.

Multiplication and division with directed numbers

$+ \times + = +$	$+ \times - = -$	$- \times - = +$	$- \times + = -$
$+ \div + = +$	$+ \div - = -$	$- \div - = +$	$- \div + = -$

Example 2 **Using directed numbers**

Evaluate the following.

a $6 - 13$ **b** $(-5) - 11$ **c** $9 - (-7)$ **d** $(-10) - (-9)$
e 5×-3 **f** $(-8) \times (-7)$ **g** $(-16) \div 4$ **h** $(-60) \div (-5)$
i $(-100) \div (-4) \div (-5)$ **j** $(-3)^2$

Solution

a $6 - 13 = 6 + (-13) = -7$

b $(-5) - 11 = (-5) + (-11)$
$= -16$

c $9 - (-7) = 9 + 7$
$= 16$

d $(-10) - (-9) = (-10) + 9$
$= -1$

e $5 \times -3 = -15$

f $(-8) \times (-7) = 56$

g $(-16) \div 4 = -4$

h $(-60) \div (-5) = 12$

i $(-100) \div (-4) \div (-5) = 25 \div (-5)$
$= -5$

j $(-3)^2 = (-3) \times (-3)$
$= 9$

ISBN 978-1-009-25891-3

Exercise 1B

Example 2a–d

1 Without using a calculator, find the answers to the following.

a $6 - 7$ **b** $-10 + 6$ **c** $-13 + (-3)$ **d** $-7 + 10$

e $-7 - 19$ **f** $(-18) - 7$ **g** $(-9) - 3$ **h** $4 - (-18)$

i $18 - (-4)$ **j** $15 - (-17)$ **k** $16 - (-12)$ **l** $(-3) - (-13)$

m $(-12) - (-6)$ **n** $(-21) - (-8)$

Example 2e–j

2 Without using a calculator, find the answers to the following.

a $(-6) \times 2$ **b** $(-6)(-4)$ **c** $(-10) \div (-4)$ **d** $15 \div (-3)$

e $(5 + 2) \times 6 - 6$ **f** $-(-4) \times (-3)$ **g** $-7(-2 + 3)$ **h** $-4(-7 - (2)(4))$

i $-(3 - 2)$ **j** $-6 \times (-5 \times 2)$ **k** $-6(-4 + 3)$ **l** $-(-12 - 9) - 2$

m $-4 - 3$ **n** $-(-4 - 7(-6))$ **o** $(-5)(-5) + (-3)(-3)$

p $8^2 + 4(0.5)(8)(6)$

1C Powers and roots

Squares and square roots

When a number is multiplied by itself, we call this the *square* of the number.

$$4 \times 4 = 4^2 = 16$$

- 16 is called the *square* of 4 (or 4 squared).
- 4 is called the *square root* of 16.
- The square root of 16 can be written as $\sqrt{16} = 4$. ($\sqrt{\ }$ is the square root symbol)

Cubes and cube roots

When a number is squared and then multiplied by itself again, we call this the *cube* of the number.

$$4 \times 4 \times 4 = 4^3 = 64$$

- 64 is called the *cube* of 4 (or 4 cubed).
- 4 is called the *cube root* of 64.
- The cube root of 64 can be written as $\sqrt[3]{64} = 4$. ($\sqrt[3]{\ }$ is the cube root symbol)

Other powers

When a number is multiplied by itself a number of times, the values obtained are called *powers* of the original number.

ISBN 978-1-009-25891-3

For example, $4 \times 4 \times 4 \times 4 \times 4 = 1024 = 4^5$, which is read as '4 to the *power* of 5'.

- 4 is the fifth root of 1024.
- $\sqrt[5]{1024}$ means the fifth root of 1024.
- Another way of writing $\sqrt{16}$ is $16^{\frac{1}{2}}$, which is read as '16 to the half'.
- Likewise, $8^{\frac{1}{3}}$, read as '8 to the third', means $\sqrt[3]{8} = 2$.
- Powers and roots of numbers can be evaluated on the calculator by using the ^ button.

Example 3 Finding the power or root of a number using a calculator

a Find 8^3. **b** Find $8^{\frac{1}{3}}$.

Solution

a

```
8^3
                512
```

b

```
8^(1/3)
                  2
```

Exercise 1C

Example 3

1 Find the value of the following.

a 10^4 **b** 7^3 **c** $\sqrt{25}$ **d** $\sqrt[3]{8}$

e 2^6 **f** 12^4 **g** $9^{\frac{1}{2}}$ **h** $169^{\frac{1}{2}}$

i $1\,000\,000^{\frac{1}{2}}$ **j** $64^{\frac{1}{3}}$ **k** $32^{\frac{1}{5}}$

2 Find the value of the following.

a $\sqrt{10^2 + 24^2}$ **b** $\sqrt{39^2 - 36^2}$ **c** $\sqrt{12^2 + 35^2}$

d $\sqrt{(4 + 2)^2 - 11}$ **e** $10(3 + 5) - (\sqrt{9} - 2)$ **f** $\sqrt{(3 + 2)^2 - (5 - 2)^2}$

1D Approximations, decimal places and significant figures

Approximations occur when we are not able to give exact numerical values in mathematics. Some numbers are too long (e.g. 0.573 128 9 or 107 000 000 000) to work with and they are rounded to make calculations easier. Calculators are powerful tools and have made many tasks easier that previously took a considerable amount of time. Nevertheless, it is still important to understand the processes of **rounding** and estimation.

Some questions do not require an exact answer and a stated degree of accuracy is often sufficient. Some questions may only need an answer rounded to the nearest tenth, hundredth etc. Other questions may ask for an answer correct to two decimal places or to three significant figures.

ISBN 978-1-009-25891-3

Rules for rounding

Rules for rounding

1. Look at the value of the digit to the right of the specified digit.
2. If the value is 5, 6, 7, 8 or 9, *round the digit up*.
3. If the value is 0, 1, 2, 3 or 4, *leave the digit unchanged*.

Example 4 Rounding to the nearest thousand

Round 34 867 to the nearest thousand.

Solution

1. Look at the first digit after the thousands. It is an 8. — 34 867
2. As it is 5 or more, increase the digit to its left by one. So the 4 becomes a 5. The digits to the right all become zero. Write your answer. — 35 000

Note: 34 867 is closer to 35 000 than 34 000

Scientific notation (standard form)

When we work with very large or very small numbers, we often use **scientific notation**, also called *standard form*.

To write a number in scientific notation we express it as a number between 1 and 10 multiplied by a power of 10.

Scientific notation

Large numbers

$$249000000000 = 2.49 \times 100\,000\,000\,000 = 2.49 \times 10^{11}$$

The decimal point needs to be moved 11 places to the right to obtain the basic numeral.

Multiplying by $10^{\text{positive power}}$ gives the effect of moving the decimal point to the right to make the number larger.

Small numbers

$$0.000000002 = 2.0 \div 1\,000\,000\,000 = 2.0 \times 10^{-9}$$

The decimal point needs to be moved 9 places to the left to obtain the basic numeral.

Multiplying by $10^{\text{negative power}}$ gives the effect of moving the decimal point to the left to make the number smaller.

ISBN 978-1-009-25891-3

Example 5 Writing a number in scientific notation

Write the following numbers in scientific notation.

a 7 800 000 **b** 0.000 000 5

Solution

a **1** Write 7 800 000 as a number between 1 and 10 (7.8) and decide what to multiply it by to make 7 800 000.

$7\,800\,000 = 7.8 \times 1\,000\,000$

6 places

7 8 0 0 0 0 0

2 Count the number of places the decimal point needs to move and whether it is to the left or right.

Decimal point needs to move 6 places to the right from 7.8 to make 7 800 000.

3 Write your answer.

$7\,800\,000 = 7.8 \times 10^6$

b **1** Write 0.000 000 5 as a number between 1 and 10 (5.0) and decide what to divide it by to make 0.000 000 5.

$0.000\,000\,5 = 5.0 \div 10\,000\,000$

7 places

0. 0 0 0 0 0 0 5

2 Count the number of places the decimal point needs to move and whether it is to the left or right.

Decimal point needs to move 7 places to the left from 5.0 to make 0.000 000 5.

3 Write your answer.

$0.000\,000\,5 = 5.0 \times 10^{-7}$

Example 6 Writing a scientific notation number as a basic numeral

Write the following scientific notation numbers as basic numerals.

a 3.576×10^7 **b** 7.9×10^{-5}

Solution

a **1** Multiplying 3.576 by 10^7 means that the decimal point needs to be moved 7 places to the right.

3.576×10^7

7 places

3. 5 6 7 0 0 0 0 $\times 10^7$

2 Move the decimal place 7 places to the right and write your answer. Zeros will need to be added as placeholders.

$= 35\,760\,000$

b **1** Multiplying 7.9 by 10^{-5} means that the decimal point needs to be moved 5 places to the left.

7.9×10^{-5}

5 places

0. 0 0 0 0 7 9 $\times 10^{-5}$

2 Move the decimal place 5 places to the left and write your answer.

$= 0.000\,079$

ISBN 978-1-009-25891-3

Significant figures

The first non-zero digit, reading from left to right in a number, is the first **significant figure**. It is easy to think of significant figures as all non-zero figures, except where the zero is between non-zero figures. The number of significant figures is shown in red below.

For example:

Number	Significant figures	Explanation
596.36	5	All numbers provide useful information.
5000	1	We do not know anything for certain about the hundreds, tens or units places. The zeros may be just placeholders or they may have been rounded off to give this value.
0.0057	2	Only the 5 and 7 tell us something. The other zeros are placeholders.
0.00570	3	The last zero tells us that the measurement was made accurate to the last digit.
8.508	4	Any zeros between significant digits are significant.
0.00906	3	Any zeros between significant digits are significant.
560.0	4	The zero in the tenths place means that the measurement was made accurate to the tenths place. The first zero is between significant digits and is therefore significant.

Rules for significant figures

1. All non-zero digits are significant.
2. All zeros between significant digits are significant.
3. After a decimal point, all zeros to the right of non-zero digits are significant.

Example 7 Rounding to a certain number of significant figures

Round 93.738 095 to:

a two significant figures **b** one significant figure **c** five significant figures

Solution

a	**1** Count the significant figures in 93.738 095.	There are eight significant figures.
	2 For two significant figures, start counting two non-zero numbers from the left.	93.738 095
	3 The next number (7) is 5 or more so we increase the previous number (3) by one (making it 4). Write your answer.	= 94 (two significant figures)

ISBN 978-1-009-25891-3

b **1** For one significant figure, count one non-zero number from the left.	93.738 095
2 The next number (3) is less than 5 so we leave the previous number (9) as it is and replace the 3 with 0 to make only one significant figure. Write your answer.	= 90 (one significant figure)
c **1** For five significant figures, start counting five non-zero numbers from the left.	93.738 095
2 The next number (0) is less than 5 so do not change the previous number (8). Write your answer.	= 93.738 (five significant figures)

Example 8 Rounding to a certain number of significant figures

Round 0.006 473 5 to:

a four significant figures **b** three significant figures **c** one significant figure

Solution

a **1** Count the significant figures.	There are five significant figures.
2 Count four non-zero numbers starting from the left.	0.006 473 5
3 The next number (5) is 5 or more. Increase the previous number (3) by one (4). Write your answer.	= 0.006 474 (four significant figures)
b **1** For three significant figures, count three non-zero numbers from the left.	0.006 473 5
2 The next number (3) is less than 5 so we leave the previous number (7) as it is. Write your answer.	= 0.006 47 (three significant figures)
c **1** For one significant figure, count one non-zero number from the left.	0.006 473 5
2 The next number (4) is less than 5 so do not change the previous number (6). Write your answer.	= 0.006 (one significant figure)

ISBN 978-1-009-25891-3

Decimal places

23.798 is a decimal number with three digits after the decimal point. The first digit (7) after the decimal point is the first (or one) decimal place. Depending on the required accuracy we round to one decimal place, two decimal places, etc.

Example 9 Rounding correct to a number of decimal places

Round 94.738 295 to:

a two decimal places **b** one decimal place **c** five decimal places

Solution

a **1** For two decimal places, count two places after the decimal point and look at the next digit (8).

94.738 295

2 As 8 is 5 or more, increase the digit to the left of 8 by one. (3 becomes 4) Write your answer.

= 94.74 (to two decimal places)

b **1** For one decimal place, count one place after the decimal point and look at the next digit (3).

94.738 295

2 As 3 is less than 5, the digit to the left of 3 remains unchanged. Write your answer.

= 94.7 (to one decimal place)

c **1** For five decimal places, count five places after the decimal point and look at the next digit (5).

94.738 295

2 As the next digit (5) is 5 or more, the digit to the left of 5 needs to be increased by one. As this is a 9, the next higher number is 10, so the previous digit also needs to change to the next higher number. Write your answer.

= 94.738 30 (to five decimal places)

Exercise 1D

Example 4

1 Round to the nearest whole number.

a 87.15 **b** 605.99 **c** 2.5 **d** 33.63

Example 4

2 Round to the nearest hundred.

a 6827 **b** 46 770 **c** 79 999 **d** 313.4

ISBN 978-1-009-25891-3

Example 6

3 Write these scientific notation numbers as basic numerals.

a 5.3467×10^4 **b** 3.8×10^6 **c** 7.89×10^5 **d** 9.21×10^{-3}
e 1.03×10^{-7} **f** 2.907×10^6 **g** 3.8×10^{-12} **h** 2.1×10^{10}

Example 5

4 Write these numbers in scientific notation.

a 792 000 **b** 14 600 000 **c** 500 000 000 000 **d** 0.000 009 8
e 0.145 697 **f** 0.000 000 000 06 **g** 2 679 886 **h** 0.0087

5 Express the following approximate numbers, using scientific notation.

a The mass of the Earth is 6 000 000 000 000 000 000 000 000 kg.
b The circumference of the Earth is 40 000 000 m.
c The diameter of an atom is 0.000 000 000 1 m.
d The radius of the Earth's orbit around the Sun is 150 000 000 km.

6 For each of the following numbers, state the number of significant figures.

a 89 156 **b** 608 765 **c** 900 000 000 000 **d** 0.709
e 0.10 **f** 0.006 **g** 450 000 **h** 0.008 007

Example 7
Example 8

7 Round the following to the number of significant figures indicated in each of the brackets.

a 4.8976 (2) **b** 0.078 74 (3)
c 1506.892 (5) **d** 5.523 (1)

8 Calculate the following and give your answer correct to the number of significant figures indicated in each of the brackets.

a $4.3968 \times 0.000\,743\,8$ (2) **b** $0.611\,35 \div 4.1119$ (5)
c $3.4572 \div 0.0109$ (3) **d** $50\,042 \times 0.0067$ (3)

Example 9

9 Use a calculator to find answers to the following. Give each answer correct to the number of decimal places indicated in the brackets.

a 3.185×0.49 (2) **b** $0.064 \div 2.536$ (3)
c 0.474×0.0693 (2) **d** $12.943 \div 6.876$ (4)
e $0.006\,749 \div 0.000\,382$ (3) **f** $38.374\,306 \times 0.007\,493$ (4)

10 Calculate the following, correct to two decimal places.

a $\sqrt{7^2 + 14^2}$ **b** $\sqrt{3.9^2 + 2.6^2}$ **c** $\sqrt{48.71^2 - 29^2}$

ISBN 978-1-009-25891-3

1E Currency exchange rates

The money that you use to pay for goods and services in one country cannot usually be used in any other country. If you take Australian dollars to New Zealand, for example, they must be exchanged with, or converted to, New Zealand dollars. Even though the Australian dollar (AUD) and the New Zealand dollar (NZD) have the same name, they have different values.

The table below shows the rate of exchange between Australian dollars and other currencies on a particular day, rounded to five decimal places.

Currency exchange: Australian Dollar (AUD)				
Country	Currency name	Symbol	Code	units per AUD
United States of America	Dollar	$	USD	0.743 99
European Union	Euro	€	EUR	0.675 97
Great Britain	Pound	£	GBP	0.522 82
Japan	Yen	¥	JPY	84.648 03
South Africa	Rand	R	ZAR	11.417 48
Brazil	Real	R$	BRL	2.780 55
United Arab Emirates	Dirham	إ.د	AED	2.732 69

Source: *http://www.xe.com*

The numbers in the column 'units per AUD' are the **exchange rates** for each currency and are used to convert between Australian dollars and other currencies, in a similar way to converting between units of measurement.

The units per AUD for the Japanese yen is 84.648 03, which means that one AUD will be exchanged for 84.648 03 yen in Japan.

$1 AUD = 84.648 03 JPY

Ten Australian dollars would be exchanged for ten times this amount.

Converting between currencies

An *exchange rate* between Australian dollars and other currencies is given as units per AUD.
Convert Australian dollars to other currencies by *multiplying* the amount by the exchange rate.
Convert other currencies to Australian dollars by *dividing* the amount by the exchange rate.

In Australia, the dollar consists of 100 cents. Most countries divide their main currency unit into 100 smaller units and so it is usual to round currency amounts to two decimal places, even though the conversion rates are usually expressed with many more decimal places than this.

ISBN 978-1-009-25891-3

Example 10 Converting between Australian Dollars and other currencies

Use the table of currency exchange for the Australian dollar (on page 13) to convert.

a 300 AUD into British pounds

b 2500 ZAR into Australian dollars

Solution

a **1** Write the exchange rate for AUD to GBP.

$1 \text{ AUD} = 0.522\,82 \text{ GBP}$

2 Multiply 300 AUD by the exchange rate to convert to GBP.

$$300 \text{ AUD} = 300 \times 0.522\,82 \text{ GBP}$$
$$= 156.846 \text{ GBP}$$

3 Round your answer to two decimal places.

\$300 AUD is converted to £156.85

b **1** Write the exchange rate for AUD to ZAR.

$1 \text{ AUD} = 11.417\,48 \text{ ZAR}$

2 Divide 2500 ZAR by the exchange rate to convert to AUD.

$$2500 \text{ ZAR} = \frac{2500}{11.417\,48} \text{AUD}$$
$$= 218.96\,2503\,109$$

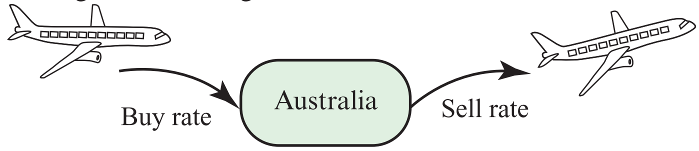
3 Round your answer to two decimal places.

2500 ZAR is converted to \$218.96 AUD.

Banks and currency exchange services use different rates depending on whether a customer is wanting to exchange their local currency for foreign currency or vice versa. These rates are based on what the bank is doing in the exchange, rather than the customer.

- **Sell rate** – The rate at which the bank **sells** you foreign currency in exchange for your local currency. For example, when leaving Australia for New Zealand the bank exchanges your Australian dollars for New Zealand dollars at the sell rate. That is, the bank *sells you* the foreign currency.
- **Buy rate** – The rate at which the bank **buys** your foreign currency in exchange for local currency. For example, when returning to Australia from Japan the bank exchanges your remaining Japanese yen for Australian dollars at the buy rate. That is, the bank *buys back* the foreign currency.

Generally, this exchange can be thought of as:



The following table displays the current buying and selling rates of one Australian dollar (AUD) in relation to four different currencies.

ISBN 978-1-009-25891-3

Currency	Code	Buy	Sell
Japanese yen	JPY	109.00	102.00
Euro	EUR	0.6644	0.6100
Singapore dollar	SGD	1.3712	1.2630
New Zealand dollar	NZD	1.1646	1.0675

Multiplying by the lower figure, the selling rate, will provide us with foreign currency in exchange for Australian dollars.

If John wishes to convert 500 Australian dollars (AUD) to Japanese yen (JPY), he will get $500 \times 102.00 = 51\,000$ yen.

Likewise, if Jenny brings 800 AUD with her to New Zealand, she will get $800 \times 1.0675 = 854$ NZD in return.

On the other hand, to convert foreign currencies to Australian dollars, we need to divide by the larger number, which is the buying rate.

For example, if John had 11 000 JPY left over on his way back to Australia from Japan, he could trade them for $\dfrac{11\,000}{109.00} = 100.92$ AUD.

Jenny also had 254 NZD left over after her trip to Auckland. How many AUD can she get if she trades 254 NZD? It comes down to the same maths. We divide by the bigger number, and Jenny can keep $\dfrac{254}{1.1646} = 218.10$ AUD.

Exercise 1E

Example 10

1 Use the table of currency exchange values on page 13 to convert the following currency amounts into the currency in brackets. Round your answer to two decimal places.

a \$750 AUD (EUR) **b** \$4800 AUD (USD) **c** \$184 AUD (BRL)

d €1500 (AUD) **e** R\$8500 BRL (AUD) **f** د.إ16 000 AED (AUD)

2 Find the total sum of these measurements. Express your answer in the units given in brackets.

a 14 cm, 18 mm (mm) **b** 589 km, 169 m (km)

c 3.4 m, 17 cm, 76 mm (cm) **d** 300 mm^2, 10.5 cm^2 (cm^2)

3 A wall in a house is 7860 mm long. How many metres is this?

4 A truck weighs 3 tonne. How heavy is this in kilograms?

5 An Olympic swimming pool holds approximately 2.25 megalitres of water. How many litres is this?

ISBN 978-1-009-25891-3

6 Baking paper is sold on a roll 30 cm wide and 10 m long. How many baking trays of width 30 cm and length 32 cm could be covered with one roll of baking paper?

7 On a particular day, one Australian dollar was worth 8.6226 Botswana pula (BWP). How many pula would Tapiwa need to exchange if she wanted to receive \$2000 AUD?

8 On a particular day, \$850 AUD could be exchanged to €581.40.
How many euros would be exchanged for \$480 AUD?

9 The exchange rate between pounds and Australian dollar is £1 = \$1.76. Adele converts \$280 into pounds. Calculate the number of pounds Adele receives.

10 The exchange rate between Australian dollars and euros is \$1 = €0.85. Ben exchanges \$260 into euros. Calculate the number of euros Ben receives.

11 Abe exchanges New Zealand dollars into Australian dollars. Ren exchanges British pounds sterling into Australian dollars as well. Given that both received \$250 (AUD) in return for their exchanges, determine how many NZD Abe exchanges and the number of GBP Ren exchanges.

12 The exchange rate between Australian dollar and Hong Kong dollar is 1 AUD = 7.26 HKD. Nicola travels from Australia to Hong Kong. She changes \$450 into Hong Kong dollars.

a How many Hong Kong dollars does she receive?

b When Nicola returns to Australia she has 81 HKD. How many \$ AUD does she receive in return?

13 On one day the rate of exchange between pounds (£) and Australian dollars (\$) was £1 = \$1.85. Calculate

a the number of dollars received in exchange for £150

b the number of pounds received in exchange for \$264.

c Yasin buys 24 postcards for £1.30 each. Calculate the total cost, in dollars, of the postcards.

14 Five items bought at Freddo's Supermarket are shown on the receipt.
The part showing the cost of the apples is missing.

Apples	
Roll	1.35
Mineral water	1.20
Cheese	1.64
Tomatoes	1.20
Total \$	5.90

a How much did the apples cost?

b The total cost of \$5.90 when converted to euros is €4.80. Determine the exchange rate that will enable you to convert dollars (\$) to euros (€).

c Use your answer to part **b** to estimate the cost of cheese in euros.

ISBN 978-1-009-25891-3

Use the following currency exchange table below for questions 15 – 18.

The following table displays the current buying and selling rates of one Australian dollar (AUD) in relation to four different currencies.

Currency	Code	Buy	Sell
Japanese yen	JPY	109.00	102.00
Euro	EUR	0.6644	0.6100
Singapore dollar	SGD	1.3712	1.2630
New Zealand dollar	NZD	1.1646	1.0675

15 Alex is making plans to go to Singapore for business. How many Singapore dollars will he get in exchange for 2500 AUD?

16 Alex flew back to Perth with 520 SGD left over from his 4-day trip to Singapore. How many Australian dollars can he receive back for this amount?

17 A few months later, Alex plans to go to Paris again for business. At the Perth International Airport, he changes 4800 AUD into Euro. Unfortunately, due to a last-minute change of plans Alex was unable to go, so had to exchange his money back to Australian dollars from Euros. Determine how much Alex lost on this deal, rounding your answer to the nearest AUD.

18 Nishioka embarked on a journey to Broome from Japan. When he departed Osaka, he converted 100 000 JPY to AUD. He had a 10-day holiday and returned to Osaka with 6000 JPY still in his wallet. What was his total Australian dollar expenditure, in AUD?

ISBN 978-1-009-25891-3

1F Percentages

Per cent is an abbreviation of the Latin words *per centum*, which mean 'by the hundred'. A **percentage** is a rate or a proportion expressed as part of one hundred. The symbol used to indicate percentage is %. Percentages can be expressed as common fractions or as decimals.

For example: 17% (17 per cent) means 17 parts out of every 100.

$$17\% = \frac{17}{100} = 0.17$$

Conversions

1. To convert a fraction or a decimal to a percentage, multiply by 100.
2. To convert a percentage to a decimal or a fraction, divide by 100.

Example 11 Converting fractions to percentages

Express $\frac{36}{90}$ as a percentage.

Solution

Method 1 (by hand)

1. Multiply the fraction $\frac{36}{90}$ by 100. — $\frac{36}{90} \times 100$
2. Simplify the fraction by dividing both the numerator and denominator by a common factor of 9. — $= \frac{36 \div 9}{90 \div 9} \times 100$ $= \frac{4}{10} \times 100$
3. Cancel out one zero from both 10 and 100 since both values have a common factor of 10. — $= \frac{4}{1} \times 10$
4. Evaluate and write your answer. — $= 40\%$

   Note: The above calculation can be performed on the ClassPad calculator.

Method 2 (using CAS)

1. Enter 36 ÷ 90 on calculator.
2. Press % sign and EXE (Casio) or ENTER (Ti-Nspire).
3. Write your answer.

```
36/90%
                40
```

Expressed as a percentage, $\frac{36}{90}$ is 40%.

Example 12 Converting a decimal to a percentage

Express 0.75 as a percentage.

Solution

1. Multiply 0.75 by 100. — 0.75×100
2. Evaluate and write your answer. — $= 75\%$

ISBN 978-1-009-25891-3

Example 13 Converting a percentage to a fraction

Express 62% as a common fraction.

Solution

1 As 62% means 62 out of 100, this can be written as a fraction $\frac{62}{100}$.

$62\% = \frac{62}{100}$

2 Simplify the fraction by dividing both the numerator and the denominator by 2.

$= \frac{62 \div 2}{100 \div 2}$

$= \frac{31}{50}$

Example 14 Converting a percentage to a decimal

Express 72% as a decimal.

Solution

Write 72% as a fraction over 100 and express this as a decimal.

$\frac{72}{100} = 0.72$

Finding a percentage of a quantity

To find a percentage *of* a number or a quantity, remember that in mathematics 'of' means 'multiply'.

Example 15 Finding a percentage of a quantity

Find 15% of \$140.

Solution

Method 1

1 Write out the problem and rewrite 15% as a fraction out of 100.

15% of 140

$= \frac{15}{100}$ of 140

2 Change 'of' to 'multiply'.

$= \frac{15}{100} \times 140$

3 Perform the calculation and write your answer.

$= 21$

Note: The above calculation can be performed on the CAS calculator.

Method 2 (using CAS)

1 Enter 15%140 on a calculator.

2 Press EXE (Casio) or ENTER (Ti-Nspire).

15%140	
	21

3 Write your answer.

21

ISBN 978-1-009-25891-3

Comparing two quantities

One quantity or number may be expressed as a percentage of another quantity or number (both quantities must always be in the same units). Divide the quantity by what you are comparing it with and then multiply by 100 to convert it to a percentage.

Example 16 Expressing a quantity as a percentage of another quantity

There are 18 girls in a class of 25 students. What percentage of the class are girls?

Solution

1 Work out the fraction of girls in the class. Girls $= \frac{18}{25}$

2 Convert the fraction to a percentage by multiplying by 100. $\frac{18}{25} \times 100$

3 Evaluate and write your answer. $= 72$

72% of the class are girls.

Example 17 Expressing a quantity as a percentage of another quantity with different units

Express 76 mm as a percentage of 40 cm.

Solution

1 First convert 40 centimetres to millimetres by multiplying by 10, as there are 10 millimetres in 1 centimetre. $40\text{ cm} = 40 \times 10$
$= 400\text{ mm}$

2 Write 76 millimetres as a fraction of 400 millimetres. $\frac{76}{400}$

3 Multiply by 100 to convert to a percentage. $\frac{76}{400} \times 100$

4 Evaluate and write your answer. $= 19\%$

Exercise 1F

Example 11
Example 12

1 Express the following as percentages.

a $\frac{1}{4}$ **b** $\frac{2}{5}$ **c** $\frac{3}{20}$ **d** $\frac{7}{10}$

e 0.19 **f** 0.79 **g** 2.15 **h** 39.57

i 0.073 **j** 1

ISBN 978-1-009-25891-3

Example 13
Example 14

2 Express the following as:

i common fractions, in their lowest terms **ii** decimals.

a 25% **b** 50% **c** 75% **d** 68% **e** 5.75%
f 27.2% **g** 0.45% **h** 0.03% **i** 0.0065% **j** 100%

Example 15

3 Find the following, correct to three significant figures.

a 15% of \$760 **b** 22% of \$500 **c** 17% of 150 m
d $13\frac{1}{2}$% of \$10 000 **e** 2% of 79.34 cm **f** 19.6% of 13.46
g 0.46% of €35 **h** 15.9% of \$28 740 **i** 22.4% of \$346 900
j 1.98% of \$1 000 000

Example 16

4 From a class, 28 out of 35 students wanted to take part in a project. What percentage of the class wanted to take part?

5 A farmer lost 450 sheep out of a flock of 1200 during a drought. What percentage of the flock were lost?

6 In a laboratory test on 360 light globes, 16 globes were found to be defective. What percentage were satisfactory, correct to one decimal place?

7 After three rounds of a competition, a basketball team had scored 300 points and 360 points had been scored against them. Express the points scored by the team as a percentage of the points scored against them. Give your answer correct to two decimal places.

8 In a school of 624 students, 125 are in year 10. What percentage of the students are in year 10? Give your answer to the nearest whole number.

Example 17

9 Express 75 cm as a percentage of 2 m.

10 In a population of $3\frac{1}{4}$ million people, 2 115 000 are under the age of 16. Calculate the percentage, to two decimal places, of the population who are under the age of 16.

11 Andrew bought a rare model train for \$450. He later sold the train for \$600. Calculate:

a the profit Andrew made on the sale of the train
b the profit Andrew made as a percentage of the purchase price of the train correct to one decimal place.

12 A bookseller bought 8 copies of a book for \$12.50 each. They were eventually sold for \$10.00 each. Calculate:

a the loss that the bookseller made on the sale of the books
b the loss that the bookseller made as a percentage of the purchase price of the books.

13 The cost of producing a chocolate bar that sells for \$1.50 is 60c. Calculate the profit made on a bar of chocolate as a percentage of the production cost of a bar of chocolate.

ISBN 978-1-009-25891-3

1G Percentage increase and decrease

When increasing or decreasing a quantity by a given percentage, the percentage increase or decrease is always calculated as a percentage of the *original* quantity.

Example 18 Calculating the new price following a percentage increase

Sally's daily wage of \$175 is increased by 15%. Calculate her new daily wage.

Solution

Method 1

1 First find 15% of \$175 by rewriting 15% as a fraction out of 100 and changing 'of' to multiply (or use a calculator).	15% of 175 $= \frac{15}{100} \times 175$
2 Perform the calculation and write your answer.	$= 26.25$
3 As \$175 is to be increased by 15%, add \$26.25 to the original amount of \$175.	$175 + 26.25$ $= 201.25$
4 Write your answer in a sentence.	Sally's new daily wage is \$201.25.

Method 2

1 An increase of 15% means that the new amount will be the original amount (in other words, 100%) plus an extra 15%. Find 115% of 175.	115% of 175 $= \frac{115}{100} \times 175$ $= 201.25$
2 Perform the calculation.	
3 Write your answer in a sentence.	Sally's new daily wage is \$201.25.

Example 19 Calculating the new amount following a percentage decrease

A primary school's fun run distance of 2.75 km is decreased by 20% for students in years 2 to 4. Find the new distance.

Solution

Method 1

1 First find 20% of 2.75 by writing 20% as a fraction out of 100 and changing 'of' to 'multiply' (or use a calculator).	20% of 2.75 $= \frac{20}{100} \times 2.75$
2 Evaluate and write your answer.	$= 0.55$
3 As 2.75 km is to be decreased by 20%, subtract 0.55 km from the original 2.75 km.	$2.75 - 0.55$ $= 2.2$
4 Write your answer in a sentence.	The new distance is 2.2 km.

ISBN 978-1-009-25891-3

Method 2

1 A decrease of 20% means that the new amount will be the original amount (100%) minus 20%. Find 80% of 2.75.

80% of 2.75

$= \frac{80}{100} \times 2.75$

2 Perform the calculation.

$= 2.2$

3 Write your answer in a sentence.

The new distance is 2.2 km.

Example 20 **Calculating a new price with a percentage discount**

If a shop offers a discount of 15% on items in a sale, what would be the sale price of a pair of jeans originally priced at $95?

Solution

Method 1

1 Find 15% of 95.

$15\% \text{ of } 95 = \frac{15}{100} \times 95$

$= 14.25$

2 As jeans are discounted by 15%, this is a decrease, so we need to subtract the discounted price of $14.25 from the original price of $95.

$95 - 14.25$

$= 80.75$

3 Write your answer in a sentence.

The sale price would be $80.75.

Method 2

1 A discount of 15% means that the new amount is 85% of 95.

85% of 95

$= \frac{85}{100} \times 95$

2 Perform the calculation.

$= 80.75$

3 Write your answer in a sentence.

The sale price would be $80.75.

Finding a percentage change

If we are given the original price and the new price of an item, we can find the percentage change. To find a percentage change, we compare the change (increase or decrease) with the original number.

Percentage change

$$\text{Percentage change} = \frac{\text{change}}{\text{original}} \times 100$$

Thus:

$$\text{Percentage discount} = \frac{\text{discount}}{\text{original}} \times 100$$

$$\text{Percentage increase} = \frac{\text{increase}}{\text{original}} \times 100$$

ISBN 978-1-009-25891-3

Example 21 Calculating a percentage increase

A university increased its total size at the beginning of an academic year by 3000 students. If the previous number of students was 35 000, by what percentage, correct to two decimal places, did the student population increase?

Solution

1 To find the percentage increase, use the formula:

$$\text{Percentage increase} = \frac{\text{increase}}{\text{original}} \times 100$$

$$\text{Percentage increase} = \frac{\text{increase}}{\text{original}} \times 100$$

Substitute increase as 3000 and original as 35 000.

$$= \frac{3000}{35\,000} \times 100$$

2 Evaluate.

$$= 8.5714\ldots$$

3 Write your answer correct to two decimal places.

Student population increased by 8.57%.

Example 22 Calculating the percentage discount

Calculate the percentage discount obtained when a calculator with a normal price of \$38 is sold for \$32 to the nearest whole per cent.

Solution

1 Find the amount of discount given by subtracting the new price, \$32, from the original price \$38.

$$\text{Discount} = \$38 - \$32$$
$$= \$6$$

2 To find the percentage discount, use formula:

$$\text{Percentage discount} = \frac{\text{discount}}{\text{original}} \times 100$$

$$\text{Percentage discount} = \frac{\text{discount}}{\text{original}} \times 100$$

Substitute discount as 6 and original as 38 and evaluate.

$$= \frac{6}{38} \times 100$$
$$= 15.7895\ldots$$

3 Write your answer to the nearest whole per cent.

The percentage discount is 16%.

ISBN 978-1-009-25891-3

Exercise 1G

Example 20

1 A jewellery store has a promotion of 20% discount on all watches.

a How much discount will you get on a watch marked \$185?

b What is the sale price of the watch?

2 A store gave different savings discounts on a range of items in a sale. Copy and complete the following table.

	Normal price	% Discount	Saving	Sale price
a	\$89.99	5		
b	\$189.00	10		
c	\$499.00	15		
d	\$249.00	20		
e	\$79.95	22.5		
f	\$22.95	25		
g	\$599.00	27.5		
h	\$63.50	30		
i	\$1000.00	33		

3 In a particular shop the employees are given a $12\frac{1}{2}\%$ discount on any items they purchase. Calculate the actual price an employee would pay for each of the following:

a \$486 laptop

b \$799 HD LED television

c \$260 iPod

d \$750 digital camera

e \$246 digital video recorder

4 A clothing store offers 6% discount for cash sales. A customer who paid cash purchased the following items:
One pair of jeans \$95.95
A leather belt at \$29.95
Two jumpers at \$45 each
Calculate:

a the total saving

b the actual amount paid for the goods.

5 Which results in the larger sum of money, increasing \$50 by 10% or decreasing \$60 by 8%?

ISBN 978-1-009-25891-3

Example 18

6 The production of a particular model of car is increased from 14 000 by 6% over a 12-month period. What is the new production figure?

7 If a new car is sold for $23 960 and three years later it is valued at $18 700, calculate the percentage depreciation, correct to two decimal places.

8 A leading tyre manufacturer claims that a new tyre will average 12% more life than a previous tyre. The owner of a taxi fleet finds that the previous tyre averaged 24 000 km before replacement. How many kilometres should the new tyre average?

Example 22

9 Calculate the percentage discount for each of the following, to the nearest whole number.

	Normal price	Selling price	% Discount
a	$60.00	$52.00	
b	$250.00	$185.00	
c	$5000.00	$4700.00	
d	$3.80	$2.90	
e	$29.75	$24.50	
f	$12.95	$10.00	

10 A second-hand car advertised for sale at $13 990 was sold for $13 000. Calculate, correct to two decimal places, the percentage discount obtained by the purchaser.

ISBN 978-1-009-25891-3

11 A sport shop advertised the following items in their end-of-year sale. Calculate the percentage discount for each of the items to the nearest whole number.

		Normal price	Selling price	% Discount
a	Shoes	\$79.99	\$65.00	
b	12 pack of golf balls	\$29.99	\$19.99	
c	Exercise bike	\$1099.00	\$599.00	
d	Basket ball	\$49.99	\$39.99	
e	Sports socks	\$14.95	\$10.00	
f	Hockey stick	\$299.00	\$250.00	

12 Find the percentage increase that has been applied in each of the following:

a a book that is increased from \$20 to \$25

b an airfare that is increased from \$300 to \$420

c accommodation costs that are increased from \$540 to \$580.50.

1H Ratio

Ratios are used to numerically compare the values of two or more quantities.

A *ratio* can be written as $a : b$ (read as 'a to b'). It can also be written as a fraction $\frac{a}{b}$.

The order of the numbers or numerals in a ratio is important. a : b is *not* the same as b : a.

Example 23 Expressing quantities as a ratio

In a year 10 class of 26 students there are 14 girls and 12 boys. Express the number of girls to boys as a ratio.

Solution

As there are 14 girls and 12 boys, the ratio of girls to boys is 14 : 12.

Note: This could also be written as a fraction $\frac{14}{12}$.

Example 24 Expressing more than two quantities as a ratio

A survey of the same group of 26 students showed that 10 students walked to school, 11 came by public transport, and 5 were driven by their parents. Express as a ratio the number of students who walked to school to the number of students who came by public transport to the number of students who were driven to school.

Solution

The order of the numbers in a ratio is important.

10 students walked, 11 used public transport and 5 were driven so the ratio is 10 : 11 : 5.

ISBN 978-1-009-25891-3

Example 25 Expressing quantities as a ratio

A cordial bottle has instructions to mix 1 part cordial with 4 parts water. Express this as a ratio.

Solution

The ratio of cordial to water is 1 : 4. This could also be written as $\frac{1}{4}$.

Note: The reverse ratio of water *to* cordial is 4 : 1, which could also be written as $\frac{4}{1}$.

Skill-sheet

Exercise 1H

Example 23

1 A survey of a group of 50 year 11 students in a school showed that 35 of them have a part-time job and 15 do not. Express the number of students having a part-time job to those who do not as a ratio.

Example 23
Example 24

2 The table below shows the average life expectancy of some animals.

Animal	Life expectancy
Chimpanzee	40 years
Elephant	70 years
Horse	40 years
Kangaroo	9 years
Tortoise	120 years
Mouse	4 years
Whale	80 years

Find the ratios between the life expectancies of the following animals.

a Whale to horse
b Elephant to kangaroo
c Whale to tortoise
d Chimpanzee to mouse
e Horse to mouse to whale

ISBN 978-1-009-25891-3

1I Expressing ratios in their simplest form

Ratios can be simplified by dividing through by a common factor or by multiplying each term as required.

Example 26 Simplifying ratios

Simplify the follow ratios.

a 15 : 20 **b** 0.4 : 1.7 **c** $\frac{3}{4} : \frac{5}{3}$

Solution

a **1** Divide both 15 and 20 by 5.

2 Evaluate and write your answer.

$$15 : 20$$
$$= \frac{15}{5} : \frac{20}{5}$$
$$= 3 : 4$$

b **1** Multiply both 0.4 and 1.7 by 10 to give whole numbers.

2 Evaluate and write your answer.

$$0.4 : 1.7$$
$$= 0.4 \times 10 : 1.7 \times 10$$
$$= 4 : 17$$

c Method 1

1 Multiply both fractions by 4.

$$\frac{3}{4} \times 4 : \frac{5}{3} \times 4$$
$$= 3 : \frac{20}{3}$$

2 Multiply both sides of the ratio by 3.

3 Write your answer.

$$= 3 \times 3 : \frac{20}{3} \times 3$$
$$= 9 : 20$$

Method 2

1 Multiply both $\frac{3}{4}$ and $\frac{5}{3}$ by the lowest common multiple (LCM) of 3 and 4, which is 12, to eliminate fractions.

2 Evaluate and write your answer.

$$\frac{3}{4} : \frac{5}{3}$$
$$= \frac{3}{4} \times 12 : \frac{5}{3} \times 12$$
$$= 9 : 20$$

In each of the above examples, the ratios are equivalent and the information is unchanged. For example, the ratio:

12 : 8 is equivalent to the ratio 24 : 16 (multiply both 12 and 8 by 2)

and

12 : 8 is also equivalent to the ratio 3 : 2 (divide both 12 and 8 by 4).

Ratios

1. When ratios are written in terms of the smallest possible whole numbers, they are expressed in their *simplest form*.
2. The order of the figures in a ratio is important. 3 : 5 is *not* the same as 5 : 3.
3. Both parts of a ratio must be expressed in the same unit of measurement.

ISBN 978-1-009-25891-3

Example 27 Simplifying ratios with different units

Express 15 cm to 3 m as a ratio in its simplest form.

Solution

1 Write the ratio.

$15\text{ cm} : 3\text{ m}$

2 Convert 3 m to cm, by multiplying 3 m by 100, so that both parts of the ratio will be in the same units.

$15\text{ cm} : 3 \times 100\text{ cm}$

$= 15\text{ cm} : 300\text{ cm}$

3 Simplify the ratio by dividing both 15 and 300 by 15.

4 Write your answer.

$= 15 : 300$

$= \frac{15}{15} : \frac{300}{15}$

$= 1 : 20$

Example 28 Finding missing values in a ratio

Find the missing value for the equivalent ratios 3 : 7 = $\boxed{}$: 28.

Solution

1 Let the unknown value be x and write the ratios as fractions.

2 Solve for x.

$3 : 7 = x : 28$

$\frac{3}{7} = \frac{x}{28}$

Method 1 (by hand)

1 Multiply both sides of equation by 28.

$\frac{3}{7} \times 28 = \frac{x}{28} \times 28$

2 Evaluate and write your answer.

$x = 12$

$3 : 7 = 12 : 28$

Method 2 (using CAS)

Use the solve function.

$\text{solve}\left(\frac{3}{7} = \frac{x}{28}, x\right)$

$x = 12$

ISBN 978-1-009-25891-3

Exercise 1I

Example 26

1 Express the following ratios in their simplest forms.

a 12 : 15 **b** 10 : 45 **c** 22 : 55 : 33 **d** 1.3 : 3.9

e 2.7 : 0.9 **f** $\frac{5}{3} : \frac{1}{4}$ **g** 18 : 8

Example 27

2 Express the following ratios in their simplest form after making sure that each quantity is expressed in the same units.

a 60 L to 25 L **b** \$2.50 to \$50 **c** 75 cm to 2 m

d 5 kg to 600 g **e** 15 mm to 50 cm to 3 m **f** 1 km to 1 m to 1 cm

g 5.6 g to 91 g **h** \$30 to \$6 to \$1.20 to 60c

Example 28

3 For each of the following equivalent ratios find the missing value.

a 1 : 4 = ☐ : 20 **b** 15 : 8 = 135 : ☐ **c** 600 : 5 = ☐ : 1

d 2 : 5 = 2000 : ☐ **e** 3 : 7 = ☐ : 56

4 Which of the following statements are true and which are false? For those that are false, suggest a correct replacement statement, if possible.

a The ratio 4 : 3 is the same as 3 : 4.

b The ratio 3 : 4 is equivalent to 20 : 15.

c 9 : 45 is equivalent to 1 : 5.

d The ratio 60 to 12 is equivalent to 15 to 3, which is the same as 4 to 1.

e If the ratio of a father's age to his daughter's age is 7 : 1, then the girl is 7 years old when her father is 56.

f If my weekly allowance is $\frac{5}{8}$ of that of my friend, then the ratio of my monthly allowance to the allowance of my friend is 20 : 32.

5 The following recipe is for Anzac biscuits.

Anzac biscuits (makes 25)

100 grams rolled oats	60 grams desiccated coconut
175 grams plain all-purpose flour, sifted	125 grams soft brown sugar
125 grams butter	3 tablespoons boiling water
2 tablespoons golden syrup	1 teaspoon bicarbonate of soda

a What is the unsimplified ratio of rolled oats : coconut : flour : brown sugar : butter?

b Simplify the ratio from part **a**.

c You want to adapt the recipe to make 75 biscuits. What quantity of each ingredient do you need?

ISBN 978-1-009-25891-3

1J Unit cost method

Many products in a supermarket are sold in packets containing multiple individual items. For example, chocolate frogs might be sold individually, but also in packets of 12 frogs.

A single chocolate frog in a particular supermarket is sold for 85 cents.

A bag of 12 chocolate frogs is sold in the same supermarket for $5.28.

Would you buy 12 individual chocolate frogs or a bag of 12 chocolate frogs?

We can answer this question by calculating the **unit cost**, or the cost of one single chocolate frog from the bag.

$$\text{one frog from bag} = \frac{cost\ for\ whole\ bag}{12} = \frac{\$5.28}{12} = \$0.44$$

It is obviously better value to buy a bag of frogs because the *unit cost* of a frog from the bag is less than the individual price.

Example 29 Using the unit cost method

If 24 golf balls cost $86.40, how much do 7 golf balls cost?

Solution

1. Find the cost of 1 golf ball by dividing $86.40 (the total cost) by 24 (the number of golf balls).
   $\$86.40 \div 24 = \$3.60$
2. Multiply the cost of one golf ball ($3.60) by 7. Write your answer.
   $\$3.60 \times 7 = \$25.20$
   7 golf balls cost $25.20

Using the unit cost method to compare items

The unit cost method is used to compare the cost of items using the unit cost of the contents. This enables us to calculate which item is the best buy.

Example 30 Using the unit cost method to compare items

Two different brands of kitchen plastic wrap are sold in a shop.
Brand A contains 50 metres of plastic wrap and costs $4.48.
Brand B contains 90 metres of plastic wrap and costs $5.94.
Which brand is the better value?

Solution

The 'unit' in each pack is a metre of plastic wrap. The prices of both brands can be compared based on this unit.

1. Calculate the unit cost, per metre, for each brand by dividing the package cost by the number of units inside.
   $$\text{Unit cost brand A} = \frac{\$4.48}{50\text{m}} = \$0.0896 \text{ per metre}$$
   $$\text{Unit cost brand B} = \frac{\$5.94}{90\text{m}} = \$0.066 \text{ per metre}$$
2. Choose the brand that has the lowest unit cost.
   Brand B has the lowest unit cost per metre of plastic wrap so it is the better value brand.

ISBN 978-1-009-25891-3

Exercise 1J

Example 29

1 Use the unit cost method to answer the following questions.

a If 12 cakes cost \$14.40, how much do 13 cakes cost?

b If a clock gains 20 seconds in 5 days, how much does the clock gain in three weeks?

c If 17 textbooks cost \$501.50, how much would 30 textbooks cost?

d If an athlete can run 4.5 kilometres in 18 minutes, how far could she run in 40 minutes at the same pace?

2 If one tin of red paint is mixed with four tins of yellow paint, it produces five tins of orange paint. How many tins of the red and yellow paint would be needed to make 35 tins of the same shade of orange paint?

3 If a train travels 165 kilometres in 1 hour 50 minutes at a constant speed, calculate how far it could travel in:

a 3 hours
b $2\frac{1}{2}$ hours
c 20 minutes
d 70 minutes
e 3 hours and 40 minutes
f $\frac{3}{4}$ hour

Example 30

4 Ice creams are sold in two different sizes. A 35 g cone costs \$1.25 and a 73 g cone costs \$2.00. Which is the better buy?

5 A shop sells 2 L containers of Brand A milk for \$2.99, 1 L of Brand B milk for \$1.95 and 600 mL of Brand C milk for \$1.42. Calculate the best buy.

6 You need 6 large eggs to bake 2 chocolate cakes. How many eggs will you need to bake 17 chocolate cakes?

7 A car uses 45 litres of petrol to travel 495 kilometres. Under the same driving conditions calculate:

a how far the car could travel on 50 litres of petrol

b how much petrol the car would use to travel 187 kilometres.

ISBN 978-1-009-25891-3

Key ideas and chapter summary

Assign-ment

Order of operation

The order of operations is important. Remember BIMDAS
Brackets come first
Indices (powers, square roots)
Multiplication and **D**ivision come next, working from left to right then
Addition and **S**ubtraction, working from left to right

Directed numbers

Multiplying or dividing two numbers with the **same** sign gives a **positive** value.
Multiplying or dividing two numbers with **different** signs gives a **negative** value.

Scientific notation

To write a number in scientific notation express it as a number between 1 and 10 multiplied by a power of 10.

Rounding

5.417 rounded to two decimal places is 5.42 (number after the 1 is 7 so round up).

Significant figures

All non-zero digits are significant.
All zeros between significant digits are significant.
After a decimal point, all zeros to the right of non-zero digits are significant.

Percentages

To convert a fraction or a decimal to a percentage, multiply by 100.
To convert a percentage to a decimal or a fraction, divide by 100.

$$\text{Percentage change} = \frac{\text{change}}{\text{original quantity or price}} \times 100$$

Ratios

The order of the figures in a ratio is important.
4 : 3 is not the same as 3 : 4.
Ratios can be simplified. For example, 6 : 2 = 3 : 1

Unit cost

The cost of a single item, which is used to compare which item is the best buy.

ISBN 978-1-009-25891-3

Review

Skills check

Having completed this chapter you should be able to:

- use a variety of mathematical operations in the correct order
- add, subtract, multiply and divide directed numbers
- find powers and roots of numbers
- round numbers to specific place values
- write numbers in scientific notation (standard form)
- understand and use significant figures
- express ratios in their simplest form
- solve practical problems involving ratios, percentages and the unit cost method
- compare the price (cost) of items to calculate the best buy
- use currency exchange rates to convert between Australian dollars and other foreign currencies.

Short-answer questions

1 Evaluate the following.

a $3 + 2 \times 4$ **b** $25 \div (10 - 5) + 5$ **c** $14 - 21 \div 3$

d $(12 + 12) \div 12 + 12$ **e** $27 \div 3 \times 5 + 4$ **f** $4 \times (-2) + 3$

g $\dfrac{10 - 8}{2}$ **h** $\dfrac{4(3 + 12)}{2}$ **i** $\dfrac{-5 + 9}{2}$

2 Calculate the following and give your answer correct to two decimal places where appropriate.

a 5^3 **b** $\sqrt{64} - 5$ **c** $9^{\frac{1}{2}} + 9^{\frac{1}{2}}$ **d** $\sqrt{8}$

e $\sqrt{25 - 9}$ **f** $\sqrt{25} - 9$ **g** $\dfrac{6^3}{(10 \div 2)^2}$ **h** $\sqrt{6^2 + 10^2}$

3 Write each of the following in scientific notation.

a 2945 **b** 0.057 **c** 369 000 **d** 850.9

4 Write the basic numeral for each of the following.

a 7.5×10^3 **b** 1.07×10^{-3} **c** 4.56×10^{-1}

5 Write the following correct to the number of significant figures indicated in the brackets.

a 8.916 (2) **b** 0.0589 (2) **c** 809 (1)

ISBN 978-1-009-25891-3

Review

6 Write the following correct to the number of decimal places indicated in the brackets.

a 7.145 (2)
b 598.241 (1)
c 4.0789 (3)

7 On a particular day, one Australian dollar (AUD) can be exchanged for 0.7562 United States dollars (USD).

a What is the equivalent amount of USD for \$350.00 AUD?
b A tourist from the US is visiting Australia. A tour to Phillip Island will cost \$140.00 USD per person. What is the cost in AUD ?

8 An internet shop sells computer equipment and lists the prices of items in Australian dollars, US dollars and British pounds (GBP).
One Australian dollar exchanges for \$0.842 USD and £0.53 GBP on a particular day.
If a hard drive is listed with a price of \$125.60 AUD, what is the price for a customer in:

a USA?
b Great Britain?

9 Express the following percentages as decimals.

a 75%
b 40%
c 27.5%

10 Express the following percentages as fractions, in their lowest terms.

a 10%
b 20%
c 22%

11 Evaluate the following.

a 30% of 80
b 15% of \$70
c $12\frac{1}{2}$% of \$106

12 A new LED smart television was valued at \$1038. During a sale it was discounted by 5%.

a What was the amount of discount?
b What was the sale price?

13 Tom's weekly wage of \$750 is increased by 15%. What is his new weekly wage?

14 A 15-year-old girl working at a local bakery is paid \$12.50 per hour. Her pay will increase to \$15 per hour when she turns 16. What will be the percentage increase to her pay?

15 A leather jacket is reduced from \$516 to \$278. Calculate the percentage discount (to the nearest per cent).

16 After dieting for three months, Melissa who weighed 78 kg lost 4 kg and Jody's weight dropped from 68 kg to 65 kg. Calculate the percentage weight loss, correct to two decimal places, for each girl.

ISBN 978-1-009-25891-3

17 True or false?

a The ratio 3 : 2 is the same as 2 : 3

b 1 : 5 = 3 : 12

c 20 cm : 1 m is written as 20 : 1 in simplest form

d 3 : 4 = 9 : 12

18 A recipe for pizza dough requires 3 parts wholemeal flour for each 4 parts of plain flour. How many cups of wholemeal flour are needed if 24 cups of plain flour are used?

19 A supermarket sells Brand A tomatoes in a 580 g tin for $2.50 and Brand B tomatoes in a 220 g tin for $1.20. Use the unit cost method to determine which of the two brands is the better value.

20 Georgie needs to buy 3 light-globes to replace broken ones in her house. A pack containing one light-globe will cost $2.40 but there is a box of three available for $5.99. How much will Georgie save by buying the box instead of three individual globes?

21 If 5 kilograms of mincemeat costs $50, how much does 2 kilograms of mincemeat cost?

22 A truck uses 12 litres of petrol to travel 86 kilometres. How far will it travel on 42 litres of petrol?

Extended-response questions

1 The following are the pricing for three different brands of washing powder.

Brand A

400 g ($4.90)

Brand B

750 g ($8.80)

Brand C

1.2 kg ($15.00)

a Calculate the price per 100 gram of *Brand A*.

b Calculate the price per 100 gram of *Brand B*.

c Calculate the price per 100 gram of *Brand C*.

d In light of this, arrange the brands in increasing order of price.

ISBN 978-1-009-25891-3

Review

2 Powdered milk prices from four different stores are shown below.

Store A

600 g ($3.84)

Store B

250 g ($1.60)

Store C

2 kg ($12.80)

Store D

5 kg ($32.00)

Harry wanted to list the four shops' pricing in order from least expensive to most expensive.

a Explain how Harry arrived at the solution of B, A, C, D without resorting to any computations.

b Calculate the price per 100 g to see how the various stores compare.

3 The table below displays the current buy and sell exchange rates for the Australian dollar compared to three other hard currencies.

Currency	Buy	Sell
USD	0.8030	0.7282
EUR	0.7396	0.6507
GBP	0.6054	0.5448

a Peter's business travels from London to Australia. What is the total amount of Australian dollars that he will receive for £900?

b Thomas has a Working Visa and is moving from Sydney to London. His meal cost him £6.40 when he arrived at Heathrow. To make his purchase, he produced an Australian twenty-dollar note. How much change would Thomas receive in British pounds?

c Samantha is departing from Perth International Airport to continue her travels in Europe. She converts 2000 AUD into Euros. She is then required to convert her euros into British pounds upon her arrival at London Gatwick Airport due to unforeseen circumstances.
Given that 1 EUR = 0.83 GBP, determine the overall change in value of Samantha's money in British pounds, and state if she has made a profit or loss from these transactions.

ISBN 978-1-009-25891-3

4 Visitor numbers to one island resort during the last four years are shown in the table below.

Year	Number of visitors
2018	12 980
2019	13 600
2020	2 800
2021	7 900

a Calculate the percentage increase in visitors in 2019 compared to 2018. Give your answer to two decimal places.

b There was a massive decrease in the number of visitors in 2020 compared to 2019. State a plausible reason for the change and then calculate the percentage decrease in the number of visitors from 2019 to 2020. Give your answer to one decimal place.

c In 2018 the percentage of visitors increased by 10% in comparison to 2017. Calculate the number of visitors in 2017.

ISBN 978-1-009-25891-3

2 Consumer arithmetic

In this chapter

Syllabus references

Topics: Applications of rates and percentage; Use of spread sheets; Linear and non-linear expressions

Subtopics: 1.1.1 – 1.1.3, 1.1.5, 1.1.7 – 1.1.8, 1.2.1

There is no doubt that an understanding of financial arithmetic will be the most useful life skill that you will develop in mathematics. Without this knowledge you could end up spending a lot of money unnecessarily.

ISBN 978-1-009-25891-3

2A Percentages and applications

Note: If you need help with percentages, the skills are covered in Chapter 1, page 18.

Discounts and mark-ups

Suppose an item is discounted, or **marked down**, by 10%. The amount of the **discount** and the new price are:

$$\begin{aligned}\text{discount} &= 10\% \text{ of original price} \\ &= 0.10 \times \text{original price}\end{aligned} \quad \textit{and} \quad \begin{aligned}\text{new price} &= 100\% \text{ of old price} - 10\% \text{ of old price} \\ &= 90\% \text{ of old price} \\ &= 0.90 \times \text{old price}\end{aligned}$$

Applying discounts

In general, if $r\%$ discount is applied:

$$\text{discount} = \frac{r}{100} \times \text{original price} \qquad \begin{aligned}\text{new price} &= \text{original price} - \text{discount} \\ &= \frac{(100 - r)}{100} \times \text{original price}\end{aligned}$$

Example 1 Calculating the discount and the new price

a How much is saved if a 10% discount is offered on an item marked $50.00?

b What is the new discounted price of this item?

Solution

a Evaluate the discount.

$$\text{Discount} = \frac{10}{100} \times \$50 = \$5.00$$

b Evaluate the new price by either:

- subtracting the discount from the original price

$$\begin{aligned}\text{New price} &= \text{original price} - \text{discount} \\ &= \$50.00 - \$5.00 = \$45.00\end{aligned}$$

or

- calculating 90% of the original price.

$$\text{New price} = \frac{90}{100} \times \$50 = \$45.00$$

ISBN 978-1-009-25891-3

Sometimes, prices are increased or marked *up*.

If a price is increased by 10%:

$$\begin{aligned}\text{increase} &= 10\% \text{ of original price} \\ &= 0.10 \times \text{original price}\end{aligned} \quad \textit{and} \quad \begin{aligned}\text{new price} &= 100\% \text{ of old price} + 10\% \text{ of old price} \\ &= 110\% \text{ of old price} \\ &= 1.10 \times \text{old price}\end{aligned}$$

Applying mark-ups

In general, if $r\%$ increase is applied:

$$\text{increase} = \frac{r}{100} \times \text{original price}$$

$$\begin{aligned}\text{new price} &= \text{original price} + \text{increase} \\ &= \frac{(100 + r)}{100} \times \text{original price}\end{aligned}$$

Example 2 Calculating the increase and the new price

a How much is added if a 10% increase is applied to an item marked \$50?

b What is the new increased price of this item?

Solution

a Evaluate the increase.

$$\text{Increase} = \frac{10}{100} \times 50 = \$5.00$$

b Evaluate the new price by either:

- adding the increase to the original price, or
- calculating 110% of the original price.

$$\begin{aligned}\text{New price} &= \text{original price} + \text{increase} \\ &= \$50.00 + 5.00 = \$55.00\end{aligned}$$

or

$$\text{New price} = \frac{110}{100} \times 50 = \$55.00$$

Calculating the percentage change

Given the original and new price of an item, we can work out the **percentage change**.

Calculating percentage discount or increase

$$\text{Percentage discount} = \frac{\text{discount}}{\text{original price}} \times \frac{100}{1}\%$$

$$\text{Percentage increase} = \frac{\text{increase}}{\text{original price}} \times \frac{100}{1}\%$$

ISBN 978-1-009-25891-3

Example 3 **Calculating the percentage discount or increase**

a The price of an item was reduced from \$50 to \$45. What percentage discount was applied?

b The price of an item was increased from \$50 to \$55. What percentage increase was applied?

Solution

a **1** Determine the amount of the discount.

discount = original price − new price
= 50.00 − 45.00 = \$5.00

2 Express this amount as a percentage of the original price.

$$\text{percentage discount} = \frac{5.00}{50.00} \times \frac{100}{1}$$
$$= 10\%$$

b **1** Determine the amount of the increase.

increase = new price − original price
= 55.00 − 50.00 = \$5.00

2 Express this amount as a percentage of the original price.

$$\text{percentage increase} = \frac{5.00}{50.00} \times \frac{100}{1}$$
$$= 10\%$$

Calculating the original price

Sometimes we are given the new price and the percentage increase or decrease ($r\%$), and asked to determine the original price. Since we know that:

- for a discount: $\text{new price} = \dfrac{(100 - r)}{100} \times \text{original price}$
- for an increase: $\text{new price} = \dfrac{(100 + r)}{100} \times \text{original price}$

we can rearrange these formulas to give rules for determining the original price as follows.

Calculating the original price

When $r\%$ discount has been applied: $\text{original price} = \text{new price} \times \dfrac{100}{(100 - r)}$

When $r\%$ increase has been applied: $\text{original price} = \text{new price} \times \dfrac{100}{(100 + r)}$

ISBN 978-1-009-25891-3

Example 4 Calculating the original price

Suppose that Cate has a $50 gift voucher from her favourite shop.

a If the store has a '10% off' sale, what is the original value of the goods she can now purchase? Give answer correct to the nearest cent.

b If the store raises its prices by 10%, what is the original value of the goods she can now purchase? Give answer correct to the nearest cent.

Solution

a Substitute new price = 50 and $r = 10$ into the formula for a $r\%$ discount.

$$\text{Original price} = 50 \times \frac{100}{90} = \$55.555\ldots$$
$$= \$55.56 \text{ to nearest cent}$$

b Substitute new price = 50 and $r = 10$ into the formula for a $r\%$ increase.

$$\text{Original price} = 50 \times \frac{100}{110} = \$45.454\ldots$$
$$= \$45.45 \text{ to nearest cent}$$

Goods and services tax (GST)

The **goods and services tax (GST)** is a tax of 10% that is added to the price of most goods (such as cars) and services (such as insurance). We can consider this a special case of the previous rules, where $r = 10$.

Consider the cost of an item after GST is added – this is the same as finding the new price when there has been a 10% increase in the cost of the item. Thus:

$$\text{cost with GST} = \text{cost without GST} \times \frac{110}{100} = \text{cost without GST} \times 1.1$$

Similarly, finding the cost of an item before GST was added is the same as finding the original cost when a 10% increase has been applied. Thus:

$$\text{cost without GST} = \text{cost with GST} \times \frac{100}{110} = \frac{\text{cost with GST}}{1.1}$$

Finding the cost with and without GST

- Cost with GST = cost without GST × 1.1

$$\begin{tikzcd} \boxed{\text{without GST}} \arrow[r, bend left, "\times 1.1"] & \boxed{\text{with GST}} \arrow[l, bend left, "\div 1.1"] \end{tikzcd}$$

- Cost without GST $= \dfrac{\text{cost with GST}}{1.1}$

We can also directly calculate the actual amount of GST from either the cost without GST or the cost with GST.

ISBN 978-1-009-25891-3

Finding the amount of GST

- Amount of GST $= \dfrac{\text{cost without GST}}{10}$

$$\begin{tikzcd} \boxed{\text{without GST}} \arrow[r, bend left, "\div 10"] & \boxed{\text{with GST}} \arrow[l, bend left, "\div 11"] \end{tikzcd}$$

- Amount of GST $= \dfrac{\text{cost with GST}}{11}$

Example 5 Calculating GST

a If the cost of electricity supplied in one quarter is \$288.50, how much GST will be added to the bill?

b If the selling price of a washing machine is \$990:

i what is the price without GST?

ii how much of this is GST?

Solution

a Substitute \$288.50 into the rule for GST from cost without GST.

GST $= 288.50 \div 10 = \$28.85$

b **i** Substitute \$990 into the rule for cost with GST.

Cost without GST $= 990 \div 1.1$
$= \$900$

ii We can either determine the amount of the GST by subtraction or by direct substitution into the formula.

GST $= 990 - 900 = 90$
or
GST $= 990 \div 11 = \$90$

Exercise 2A

Review of percentages

1 Calculate the following as percentages. Give answers correct to one decimal place.

a \$200 of \$410 **b** \$6 of \$24.60 **c** \$1.50 of \$13.50
d \$24 of \$260 **e** 30c of 90c **f** 50c of \$2

2 Calculate the amount of the following percentage increases and decreases. Give answers to the nearest cent.

a 10% increase on \$26 000 **b** 5% increase on \$4000
c 12.5% increase on \$1600 **d** 15% increase on \$12
e 10% decrease on \$18 650 **f** 2% decrease on \$1 000 000

ISBN 978-1-009-25891-3

Discounts, mark-ups and mark-downs

Example 1

3 Calculate the amount of the discount for the following, to the nearest cent.

a 24% discount on $360 **b** 72% discount on $250

c 6% discount on $9.60 **d** 9% discount on $812

Example 2

4 Calculate the new increased price for each of the following.

a $260 marked up by 12% **b** $580 marked up by 8%

c $42.50 marked up by 60% **d** $5400 marked up by 17%

5 Calculate the new discounted price for each of the following.

a $2050 discounted by 9% **b** $11.60 discounted by 4%

c $154 discounted by 82% **d** $10 600 discounted by 3%

e $980 discounted by 13.5% **f** $2860 discounted by 8%

Example 3

6 **a** The price of an item was reduced from $25 to $19. What percentage discount was applied?

b The price of an item was increased from $25 to $30. What percentage increase was applied?

Example 4

7 Find the original prices of the items that have been marked down as follows.

a Marked down by 10%, now priced $54.00

b Marked down by 25%, now priced $37.50

c Marked down by 30%, now priced $50.00

d Marked down by 12.5%, now priced $77.00

8 Find the original prices of the items that have been marked up as follows.

a Marked up by 20%, now priced $15.96

b Marked up by 12.5%, now priced $70.00

c Marked up by 5%, now priced $109.73

d Marked up by 2.5%, now priced $5118.75

9 Mikki has a card that entitles her to a 7.5% discount at the store where she works. How much will she pay for boots marked at $230?

10 The price per litre of petrol is $1.80 on Friday. When Rafik goes to fill up his car, he finds that the price has increased by 2.3%. If his car holds 50 L of petrol, how much will he pay to fill the tank?

GST calculations

Example 5

11 Find the GST payable on each of the following (give your answer correct to the nearest cent).

a A gas bill of $121.30 **b** A telephone bill of $67.55

c A television set costing $985.50 **d** Gardening services of $395

ISBN 978-1-009-25891-3

12 The following prices are without GST. Find the price after GST has been added to the following.

a A dress worth \$139

b A bedroom suite worth \$2678

c A home video system worth \$9850

d Painting services of \$1395

13 If a computer is advertised for \$2399 including GST, how much would the computer have cost without GST?

14 What is the amount of the GST that has been added if the price of a car is advertised as \$39 990 including GST?

15 The telephone bill is \$318.97 after GST is added.

a What was the price before GST was added?

b How much GST must be paid?

2B Simple interest

When you borrow money, you have to pay for the use of that money. When you invest money, someone else will pay you for the use of your money. The amount you pay when you borrow or the amount you are paid when you invest is called **interest**. There are many different ways of calculating interest. The simplest of all is called, rather obviously, **simple interest**. Simple interest is a fixed percentage of the amount invested or borrowed and is *calculated on the original amount*.

Suppose we invest \$1000 in a bank account that pays simple interest at the rate of 5% per annum. This means that, for each year we leave the money in the account, interest of 5% of the original amount will be paid to us.

In this instance, the amount of interest paid to us is 5% of \$1000 or $\$1000 \times \dfrac{5}{100} = \$50$

If the money is left in the account for several years, the interest will be paid yearly.

To calculate simple interest we need to know:

- the initial investment, called the **principal**
- the **interest rate**, usually as % per annum (p.a.)
- the length of time the money is invested.

ISBN 978-1-009-25891-3

Example 6 **Calculating simple interest from first principles**

How much interest will be earned if investing \$1000 at 5% p.a. simple interest for 3 years?

Solution

1 Calculate the interest for the first year.
$\text{Interest} = 1000 \times \frac{5}{100} = \$50$

2 Calculate the interest for the second year.
$\text{Interest} = 1000 \times \frac{5}{100} = \$50$

3 Calculate the interest for the third year.
$\text{Interest} = 1000 \times \frac{5}{100} = \$50$

4 Calculate the total interest.
$$\begin{aligned}\text{Interest for 3 years} &= 50 + 50 + 50 \\ &= \$150\end{aligned}$$

The same rules apply when simple interest is applied to a loan rather than an investment.

The simple interest formula

Since the amount of interest in a simple interest investment is the same each year, we can apply a general rule.

$$\text{interest} = \frac{\text{amount invested or borrowed} \times \text{interest rate (per annum)} \times \text{length of time (in years)}}{100}$$

This rule gives rise to the following formula.

Simple interest formula

To calculate the simple interest earned or owed:

$$I = \frac{P \times r \times t}{100} = \frac{Prt}{100}$$

where I = the total interest earned or paid in dollars

P = is the principal (the initial amount borrowed or invested) in dollars

r = is the percentage interest rate per annum

t = the time in years of the loan or investment.

Example 7 **Calculating simple interest for periods other than one year**

Calculate the amount of simple interest that will be paid on an investment of \$5000 at 10% simple interest per annum for 3 years and 6 months.

ISBN 978-1-009-25891-3

Solution

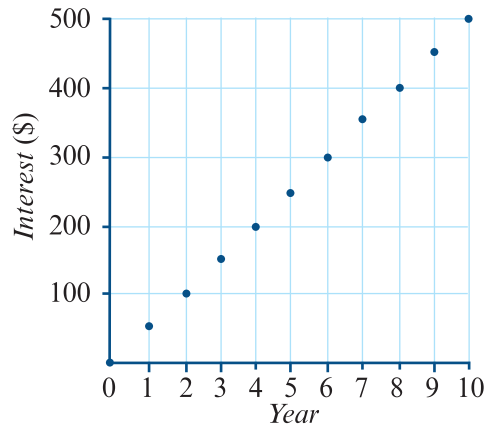
Apply the formula with $P = \$5000$, $r = 10\%$ and $t = 3.5$ (since 3 years and 6 months is equal to 3.5 years).

$$I = \frac{Prt}{100}$$
$$= 5000 \times \frac{10}{100} \times 3.5$$
$$= \$1750$$

The graph below shows the total amount of interest earned after $1, 2, 3, 4, \ldots$ years, when \$1000 is invested at 5% per annum simple interest for a period of years.

As we would expect from the simple interest rule, the graph is linear.

The slope of a line which could be drawn through these points is equal to the amount of interest added each year, in this case \$50.



A CAS calculator enables us to investigate the growth in simple interest with time using both the tables and graphing facilities of the calculator.

Using the TI-Nspire CAS to explore a simple interest investment

\$10 000 is invested at a simple interest rate of 8.25% per annum for a ten-year period. Plot the growth in interest earned over this period.

Steps

1 Find a rule for the interest earned after t years for a simple interest investment when $P = \$10\,000$ and $r = 8.25\%$.

$$I = \frac{Prt}{100} = \frac{10\,000 \times 8.25 \times t}{100}$$
$$= 825t$$

2 Start a new document (ctrl + N) and select **Add Lists & Spreadsheet**.

Name the lists **time** (to represent time in years) and **interest**.

Enter the data 1–10 into the list named **time** as shown.

Note: You can also use the sequence command to do this.

ISBN 978-1-009-25891-3

3 Place the cursor in the grey formula cell in the list named **interest** and type **=825 ×time**.

Note: You can also use the h key and paste time from the variable list.

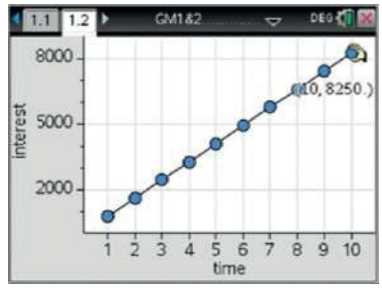
Press [enter] to display the values.

By scrolling down the table (use ▼) we can see interest of \$8250 will be earned after 10 years.

4 Press [ctrl]+[I] and select **Data & Statistics** and plot the graph as shown.

a To connect the data points. Move the cursor to the graphing area and press [ctrl]+[menu]. Select **Connect Data Points**.

b To display a value:
Move the cursor over the data points or use [menu]>**Analyze>Graph Trace** and the horizontal arrow keys to move from point to point.



From the plot we can see that the graph of the amount of simple interest earned is linear. The slope of the graph is equal to the interest added each year.

Note: You can also graph this example in the **Graphs** application.

Using the ClassPad to explore the growth of the interest earned in a simple interest investment

\$10 000 is invested at a simple interest rate of 8.25% per annum for a ten year period. Plot the growth in interest earned over this period.

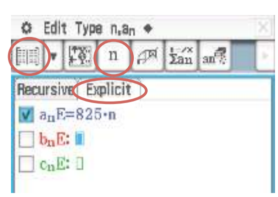
Steps

1 Find a rule for the interest earned after t years for a simple interest investment when $P = \$10\,000$ and $r = 8.25\%$.

$$I = \frac{Prt}{100} = \frac{10\,000 \times 8.25 \times t}{100}$$
$$= 825t$$

2 Enter the rule in the sequence.

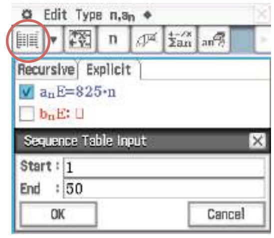
- Open the **Sequence** application .
- Select the **Explicit** tab.
- Move the cursor to the box opposite $\boldsymbol{a_n}$**E:**
- Type **825*n*** using the [n] in the toolbar for n years.
- Press [EXE] to confirm your entry indicated by a tick in the square to the left of $\boldsymbol{a_n}$**E:**

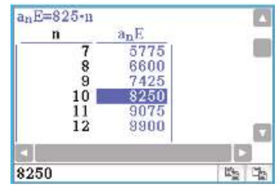


ISBN 978-1-009-25891-3

3 To display the terms of the sequence in a table.

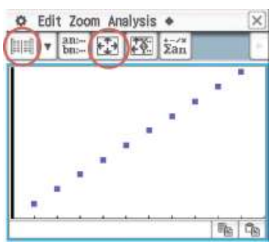
- Tap the icon.
- Tap the **Sequence TableInput** icon in the toolbar.
- Adjust the **Start** and **End** values if required.
- Scroll down the table to find the interest amount $8250 earned after 10 years.

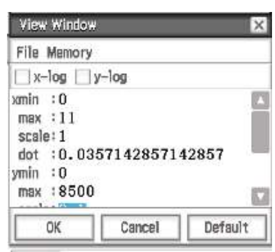


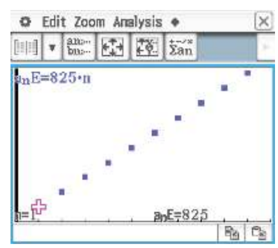


4 To graph the sequence of simple interest values.

- Select the **Sequence Grapher** icon from the toolbar.
- Select the **View Window** icon from the toolbar.
- Set the values as shown and Tap **OK** to confirm your settings (leave dot settings as they are).
- Select **Analysis** and then **Trace** to place a cursor on the values.
- Use the cursor key to view other points.







From the plot we can see that the graph of the amount of simple interest earned is linear. The slope of the graph is equal to the interest paid each year.

Calculating the amount of a simple interest loan or investment

To determine the total value or amount of a **simple interest** loan or investment, the total interest used is added to the initial amount borrowed or invested (the principal).

Total value of a simple interest loan

Total amount after t years (A) = principal (P) + interest (I)

$$\text{or } A = P + I$$

ISBN 978-1-009-25891-3

Example 8 **Calculating the total amount owed on a simple interest loan**

Find the total amount owed on a simple interest loan of $16 000 at 8% per annum after 2 years.

Solution

1 Apply the formula $\frac{Prt}{100}$ with $P = \$16\,000$, $r = 8\%$ and $t = 2$ to find the total interest accrued.

$$I = \frac{Prt}{100}$$
$$= 16\,000 \times \frac{8}{100} \times 2 = \$2560$$

2 Find the total amount owed by adding the interest to the principal.

$$A = P + I$$
$$= 16\,000 + 2560 = \$18\,560$$

Interest paid to bank accounts

One very useful application of simple interest is in the calculation of the interest earned on a bank account. When we keep money in the bank, interest is paid. The amount of interest paid depends on:

- the rate of interest the bank is paying
- the amount on which the interest is calculated.

Generally, banks will pay interest on the **minimum monthly balance**, which is the lowest amount the account contains in each calendar month. When this principle is used, we will assume that all months are of equal length, as illustrated in the next example.

Example 9 **Calculating interest paid to a bank account**

The table shows the entries in Tom's bank account.

Date	Transaction	Debit	Credit	Total
30 June	Pay		400.00	400.00
3 July	Cash	50.00		350.00
15 July	Cash		100.00	450.00
1 August				450.00

If the bank pays interest at a rate of 3% per annum on the minimum monthly balance, find the interest payable for the month of July correct to the nearest cent.

Solution

1 Determine the minimum monthly balance for July.

The minimum balance in the account for July was $350.00.

2 Determine the interest payable on $350.00.

$$I = \frac{Prt}{100}$$
$$= 350 \times \frac{3}{100} \times \frac{1}{12} = 0.875$$
$$= \$0.88 \text{ or } 88 \text{ cents}$$

ISBN 978-1-009-25891-3

Daily balance

Banks may also pay interest on the **daily balance** of a customer's savings account. This is calculated by multiplying the daily interest rate by the balance by the number of days that the balance is for. This means that the interest is adjusted each time the balance in the account changes and is usually totalled at the end of the month. This type of interest is usually found in short-term loans or transactional savings accounts where there is money frequently flowing into and out of the account.

The table below shows Robin's savings account for the month of June. The monthly interest rate on this savings account is 3.2%.

Date	Transaction	Balance ($)
1 June	Opening balance	900
8 June	Withdrawal $200	700
20 June	Deposit $500	1200

Suppose we wish to calculate the amount of interest he will be paid for the month. To do so, we must calculate his average daily balance and multiply it by the monthly interest rate. The table below shows how this can be done.

Date	Number of days	Balance ($)	Total
1–7 June	7 days	900	$7 \times 900 = 6300$
8–19 June	12 days	700	$12 \times 700 = 8400$
20–30 June	11 days	1200	$11 \times 1200 = 13\,200$
Total			27 900

The average daily balance is $\dfrac{27\,900}{30} = \$930$.

Therefore, the interest receivable will be $930 \times 0.032 = \$29.76$.

An alternative way to work out the amount of interest is to use the simple interest formula which we have previously seen is, $I = \dfrac{P \times r \times t}{100}$.

To use this formula, we need to work out the annual interest rate, with time as the number of days in the year.

Given the monthly interest rate is 3.2%

- Annual interest rate = $3.2 \times 12 = 38.4\%$

We will keep all values to at least four decimal places for the purposes of accuracy, and then round our final answer to two decimal places.

ISBN 978-1-009-25891-3

Date	Number of days	Balance ($)	Amount of interest
1–7 June	7 days	900	$900 \times \frac{38.4}{100} \times \frac{7}{365} = 6.6279$
8–19 June	12 days	700	$700 \times \frac{38.4}{100} \times \frac{12}{365} = 8.8373$
20–30 June	11 days	1200	$1200 \times \frac{38.4}{100} \times \frac{11}{365} = 13.8871$
Total interest			29.3523

Therefore, the interest receivable will be 6.63 + 8.84 + 13.89 = $29.36

Exercise 2B

Calculating simple interest

Example 7

1 Calculate the amount of interest earned from each of the following simple interest investments. Give answers correct to the nearest cent.

	Principal	Interest rate	Time
a	$400	5%	4 years
b	$750	8%	5 years
c	$1000	7.5%	8 years
d	$1250	10.25%	3 years
e	$2400	12.75%	15 years
f	$865	15%	2.5 years
g	$599	10%	6 months
h	$85.50	22.5%	9 months
i	$15 000	33.3%	1.25 years

Exploring the growth of interest in a simple interest loan or investment

CAS

2 **a** A loan of $900 is taken out at a simple interest rate of 16.5% per annum.
Use your CAS calculator to construct a graph of the simple interest owed against time (in years) for the next 10 years.

b Use the table of values to determine the amount of interest owed after 5 years.

3 **a** Ben decides to invest his savings of $1850 from his holiday job for five years at 13.25% per annum simple interest.
Use your graphics calculator to construct a graph of the simple interest earned against time (in years) for the next 10 years.

b Use the table of values to determine the amount of interest owed after 4 years.

ISBN 978-1-009-25891-3

Simple interest loans and investments

Example 8

4 Calculate the total amount to be repaid for each following simple interest loans. Give answers correct to the nearest cent.

	Principal	Interest rate	Time
a	\$500	5%	4 years
b	\$780	6.5%	3 years
c	\$1200	7.25%	6 months
d	\$2250	10.75%	8 months
e	\$2400	12%	18 months

5 A simple interest loan of \$20 000 is taken out for 5 years. Calculate:

a the simple interest owed after 5 years if the rate of interest is 12% per annum

b the total amount to be repaid after 5 years.

6 A sum of \$10 000 was invested in a fixed term account for 3 years paying a simple interest rate of 6.5% per annum.
Calculate:

a the total amount of interest earned after 3 years

b the total amount of the investment at the end of 3 years.

7 A loan of \$1200 is taken out at a simple interest rate of 14.5% per annum.
How much is owed, in total, after 3 months?

8 A company invests \$1 000 000 in the short-term money market at 11% per annum simple interest. How much interest is earned by this investment in 30 days? Give your answer to the nearest cent.

9 A building society offers the following interest rates for its cash management accounts.

	Interest rate (per annum) on term (months)				
Balance	1–<3	3–<6	6–<12	12–<24	24–<36
\$20 000–\$49 999	2.85%	3.35%	3.85%	4.35%	4.85%
\$50 000–\$99 999	3.00%	3.50%	4.00%	4.50%	5.00%
\$100 000–\$199 999	3.40%	3.90%	4.40%	4.90%	5.40%
\$200 000 and over	4.00%	4.50%	5.00%	5.50%	6.00%

Using this table, find the simple interest earned by each of the following investments. Give your answers to the nearest cent.

a \$25 000 for 2 months

b \$125 000 for 6 months

c \$37 750 for 18 months

d \$200 000 for 2 years

e \$74 386 for 8 months

f \$145 000 for 23 months

ISBN 978-1-009-25891-3

Interest paid into bank accounts

Example 9

10 An account at a bank is paid interest of 4% per annum on the minimum monthly balance, credited to the account at the beginning of the next month.

Date	Transaction	Debit	Credit	Balance
1 October				5000.00
7 October	Cash	1000.00		4000.00
31 October	Cash		500.00	

a What was the balance of the account at the end of October?

b How much interest was paid for the month?

11 The minimum monthly balances for three consecutive months are:

$240.00 $350.50 $478.95

How much interest is earned over the three-month period if it is calculated on the minimum monthly balance at a rate of 3.5% per annum?

12 The bank statement below shows transactions for a savings account that earns simple interest at a rate of 4.5% per annum on the minimum monthly balance.

Date	Transaction	Debit	Credit	Balance
1 March				500.00
15 March	Cash		250.00	750.00
31 March	Cash		250.00	1000.00
1 April				1000.00

How much interest was earned in March?

13 The bank statement below shows transactions over a three-month period for a savings account that earns simple interest at a rate of 3.75% per annum on the minimum monthly balance.

Date	Transaction	Debit	Credit	Balance
1 March				650.72
8 April	Cash		250.00	900.72
21 May	Cash		250.00	1150.72
1 June				1150.72

a What were the minimum monthly balances in March, April and May?

b How much was earned over this three-month period?

ISBN 978-1-009-25891-3

14 John's savings for April is given in the table below. The savings account interest rate is 2.85% per month.

Days	Transaction	Balance ($)	Number of days
1–5	Opening balance	1500	5
6–15	Withdrawal $300	1200	10
16–30	Deposit $150	1350	15

Calculate his average daily balance and hence the interest earned for the month of April.

15 Rita receives 8.6% per annum interest on her savings account at Alpha Bank. Her July financial activity and account balance are shown in the table below.

Days	Debit	Credit	Balance ($)	Number of days
1–8	Opening balance		6200	
9–15	1200		5000	
16–20		500	5500	
21–31	1500		4000	

Complete the last column of the table and then calculate how much interest she will earn for the month of July.

16 At the present time, the rate of interest on Jaxon's savings account at Delta Bank is 3.4% per month. The transactions he made and his balance for the month of August is shown in the table below.

Date	Transaction	Balance ($)
1 August	Opening balance	12 500
8 August	Deposit $1100	
14 August	Deposit $900	
22 August	Withdrawal $5000	

a Determine the balance in the account after each transaction and hence determine the average daily balance.

b Calculate the amount of interest Jaxon will be paid for the month.

17 Rebecca has a savings account at Gamma Bank that gives her 7.8% interest per annum. The table on the next page shows her transactions and her balance during the month of April 2022.

ISBN 978-1-009-25891-3

Date	Debit	Credit	Balance ($)
1 April	Opening balance		24 000
10 April	5000		
17 April		3000	
23 April	10 000		

a Determine the balance in the account after each transaction.

b Calculate the amount of interest Rebecca will be paid for the month of April.

2C Rearranging the simple interest formula

The simple interest formula can be rearranged to find any one of the variables as long as the other three are known.

Calculating the interest rate

Interest rate

To find the annual interest rate, $r\%$, given the values of P, I and t:

$$r = \frac{100I}{Pt}$$

where P is the principal, I is the amount of interest accrued in t years.

Example 10 Calculating the interest rate

Find the rate of simple interest if:

a a principal of \$8000 increases to \$11 040 in 4 years

b a principal of \$5000 increases to \$5500 in 9 months.

Solution

a **1** Find the amount of interest earned on the investment.

Interest:

$I = 11\,040 - 8000$

$= \$3040$

2 Apply the formula $r = \dfrac{100I}{Pt}$ with $P = \$8000$, $I = \$3040$ and $t = 4$.

$r = \dfrac{100I}{Pt} = \dfrac{100 \times 3040}{8000 \times 4}$

$= 9.5\%$

Interest rate is 9.5% per annum

ISBN 978-1-009-25891-3

b **1** Find the amount of interest earned on the investment.

Interest:

$I = 5500 - 5000$

$= \$500$

2 Apply the same formula with $P = \$5500$, $I = \$500$ and $t = \dfrac{9}{12} = 0.75$ years

$r = \dfrac{100I}{Pt} = \dfrac{100 \times 500}{5000 \times 0.75}$

$= 13.3\%$ to one decimal place

Interest rate is 13.3% per annum

Note: You need to convert the time in months to years by substituting in the formula.

Calculating the time period

Time period

To find the number of years or term of an investment, t years, given P, I and r:

$$t = \frac{100I}{Pr}$$

where P is the principal, I is the amount of interest and $r\%$ is the annual interest rate.

Example 11 Calculating the time period of a loan or investment

Find the length of time it would take for \$5000 invested at an interest rate of 12% per annum to:

a earn \$1800 interest

b earn \$404 interest.

Give answer in days to the nearest day.

Solution

a Apply the formula $t = \dfrac{100I}{Pr}$ with $P = \$5000$, $I = \$1800$ and $r = 12$.

$t = \dfrac{100I}{Pr} = \dfrac{100 \times 1800}{5000 \times 12}$

$= 3$ years

b Apply the same formula with $P = \$5000$, $I = \$404$ and $r = 12$ assuming that there are 365 days in a year.

$t = \dfrac{100I}{Pr} = \dfrac{100 \times 404}{5000 \times 12}$

$= 0.673\ldots$ years

$= 365 \times 0.673$

$= 245.766\ldots$ days

$= 246$ days (to the nearest day)

ISBN 978-1-009-25891-3

Calculating the principal

Calculating the principal

- To find the value of the principal, P, given the values of I, r and t use the formula:

  $$P = \frac{100I}{rt}$$

  where I is the amount of interest accrued, $r\%$ is the annual interest rate and t is the time in years.

- To find the value of the principal, P, given the values of A, r and t:

  $$P = \frac{A}{\left(1 + \frac{rt}{100}\right)}$$

  where A is the amount of the investment or loan, $r\%$ is the annual interest rate and t is the time in years.

Example 12 Calculating the principal of a loan or investment

a Find the amount that should be invested in order to earn \$1500 interest over 3 years at an annual interest rate of 5%.

b Find the amount that should be invested at an annual interest rate of 5% if you require the value of the investment to be \$15 600 in 4 years time.

Solution

a Since we are given the value of the interest, I, use the formula $P = \frac{100I}{rt}$ with $I = \$1500$, $r = 5$ and $t = 3$ years.

$$P = \frac{100I}{rt} = \frac{100 \times 1500}{5 \times 3}$$
$$= \$10\,000$$

b Here we are *not* given the value of the interest, I, but the value of the total investment, A.

Use the formula $P = \frac{A}{\left(1 + \frac{rt}{100}\right)}$

with $A = \$15\,600$, $r = 5$ and $t = 4$.

$$P = \frac{A}{\left(1 + \frac{rt}{100}\right)}$$
$$= \frac{15\,600}{\left(1 + \frac{5 \times 4}{100}\right)}$$
$$= \frac{15\,600}{1.2}$$
$$= \$13\,000$$

ISBN 978-1-009-25891-3

Skill-sheet

Exercise 2C

Simple interest: calculating interest rate

Example 10

1 Find the annual interest rate if a simple interest investment of $5000 amounts to $6500 in 2.5 years.

2 Find the annual interest rate if a simple interest investment of $500 amounts to $550 in 8 months.

Simple interest: calculating time

Example 11

3 Calculate the time taken for $2000 to earn $975 at 7.5% simple interest.

4 Calculate the time in days for $760 to earn $35 at 4.75% simple interest.

Simple interest: calculating principal

Example 12

5 Calculate the principal that earns $514.25 in 10 years at 4.25% simple interest.

6 Calculate the principal that earns $780 in 100 days at 6.25% per annum simple interest.

Simple interest: mixed problems

7 Calculate the answers to complete the following table.

Principal	Rate	Time	Simple interest	Total investment
$600	6%	5 years	a	b
$880	6.5%	c	$171.60	d
$1290	e	6 months	$45.15	f
g	10%	4 months	$150.00	h
$3600	i	200 days	$98.63	j
$980	7.5%	k	l	$1200.50
m	7.25%	6 months	$52.50	n

Applications

8 If Geoff invests $30 000 at 10% per annum simple interest until he has $42 000, for how many years will he need to invest the money?

9 Josh decides to put $5000 into an investment account that pays 5.0% per annum simple interest. If he leaves the money there until it doubles, how long will this take?

10 A personal loan of $15 000 over a 3-year period costs $500 per month to repay.

a How much money will be repaid in total?

b How much of the money repaid is interest?

ISBN 978-1-009-25891-3

2D Compound interest

We have seen that simple interest is calculated *only* on the original amount borrowed or invested. A more common form of interest, known as **compound interest**, calculates the interest on the original amount plus any interest accrued to that time.

Calculating compound interest

Consider, for example, \$250 invested at 10% per annum, where the interest is added to the account each year.

After 1 year:

$$\begin{aligned}\text{interest} &= \text{amount invested} \times \text{interest rate} \times \text{time}\\ &= \$250 \times 10\% \times 1\\ &= 250 \times \frac{10}{100} \times 1\\ &= \$25\end{aligned}$$

so that after one year, the amount of money in the account is:

$$\begin{aligned}\text{amount} &= \text{amount at the start of year} + \text{interest earned}\\ &= \$250 + \$25\\ &= \$275\end{aligned}$$

After 2 years:

$$\text{interest} = \$275 \times 10\% \times 1 = \$27.50$$

so that after two years, the amount of money in the account is:

$$\$275 + \$27.50 = \$302.50$$

After 3 years:

$$\text{interest} = \$302.50 \times 10\% \times 1 = \$30.25$$

so that after three years, the amount of money in the account is:

$$\$302.50 + \$30.25 = \$332.75$$

and so on.

ISBN 978-1-009-25891-3

If we tabulate this information, we will see that using compound interest, the amount of interest owed or paid increases each year.

After	Amount invested	Interest earned	Total amount of investment
1 year	\$250	\$25	\$(250 + 25) = \$275
2 years	\$275	\$27.50	\$(275 + 27.50) = \$302.50
3 years and so on	\$302.50	\$30.25	\$(302.50 + 30.25) = \$332.75

Calculating compound interest in this way can be very tedious. However, there is a pattern to the calculations that enables us to develop a formula.

Start by recalling that the multiplying factor to increase a quantity by 10% is $\left(1 + \frac{10}{100}\right) = 1.1$

Using this factor we have the value of the investment, A, is:

$$A = \$250 \times 1.1 = \$275 \qquad \text{(after 1 year)}$$

$$A = \$250 \times 1.1 \times 1.1$$

$$= \$250 \times (1.1)^2 = \$302.50 \qquad \text{(after 2 years)}$$

$$A = \$250 \times 1.1 \times 1.1 \times 1.1$$

$$= \$250 \times (1.1)^3 = \$332.75 \qquad \text{(after 3 years)}$$

and so on until, the value of the investment:

$$A = \$250 \times (1.1)^n \qquad \text{(after } n \text{ years)}$$

Thus the value of the investment after 10 years would be: $A = \$250 \times 1.1^{10} = \$648.44$

Following this pattern, we can write down a general formula for calculating the amount of a compound investment after a given amount of time.

The compound interest formula

In general, the amount, A, of a compound interest investment is given by:

$$A = P\left(1 + \frac{r}{100}\right)$$

where P is the initial amount invested (the principal) $r\%$ the the annual interest rate t is the time in years

Note: This formula can also be used to determine the amount of debt accrued by a compound interest loan.

To find the *total amount of interest* earned subtract the initial investment from the final amount.

Determining the interest earned

Interest earned (I) = value of the investment (A) – initial amount invested (P)

$$I = A - P$$

ISBN 978-1-009-25891-3

Example 13 **Calculating the amount of the investment and interest**

a Determine, to the nearest dollar, the amount of money accumulated after 3 years if $2000 is invested at an interest rate of 8% per annum, compounded annually.

b Determine the total amount of interest earned.

Solution

a Substitute $P = \$2000, t = 3, r = 8$ into the formula giving the amount of the investment.

$$A = P \times \left(1 + \frac{r}{100}\right)^t = 2000 \times \left(1 + \frac{8}{100}\right)^3$$
$$= \$2519 \text{ to the nearest dollar}$$

b Subtract the principal from this amount to determine the interest earned.

$$I = A - P = 2519 - 2000$$
$$= \$519$$

The formulas for compound interest can also be applied when money is borrowed, as shown in the following example.

Example 14 **Calculating the amount of the debt and interest owed**

a Determine, to the nearest dollar, the amount of money owed after 2 years if $10 000 is borrowed at an interest rate of 10% per annum, compounded annually.

b Determine the amount of interest owed.

Solution

a Substitute $P = \$10\,000, t = 2, r = 10$ into the formula giving the amount of the debt.

$$A = P \times \left(1 + \frac{r}{100}\right)^t = 10\,000 \times \left(1 + \frac{10}{100}\right)^2$$
$$= \$12\,100$$

b Subtract the principal from this amount to determine the interest owed.

$$I = A - P$$
$$= 12\,100 - 10\,000$$
$$= \$2100$$

Another way to determine compound interest is to enter the appropriate formula into a CAS calculator, and examine the interest earned using the calculator's tables and graphing facilities.

ISBN 978-1-009-25891-3

Using the TI-Nspire CAS to investigate compound interest problems

a Set up a table to enable the amount of money accumulated after t years if \$2000 is invested at an interest rate of 8% per annum compounding.

b Use the table to determine the value of the loan after three years and the amount of interest earned.

c Plot the growth in the amount of money in the investment for ten years and note the shape of the graph.

Steps

1 Substitute P = \$2000 and r = 8 into the formula for compound interest.

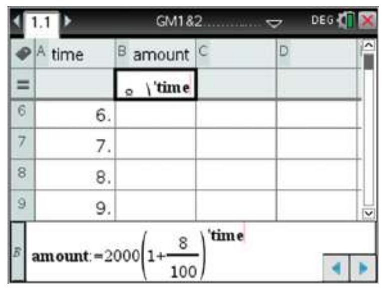
$$A = 2000 \times \left(1 + \frac{8}{100}\right)^t$$

2 Start a new document (ctrl + N) and select **Add Lists & Spreadsheet**.

Name the lists **time** (to represent time in years) and **amount**.

Enter the data 1–10 into the list named **time** as shown.

Note: You can also use the sequence command to do this.



3 Place the cursor in the grey formula cell in the list named **amount** and type in:

= **2000** ×(**1** + **8** ÷ **100**)^ **time**

Note: You can also use the var key and paste **time** from the variable list.

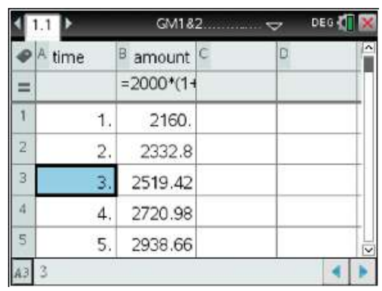
Press enter to display the values as shown.

By scrolling down the table we can see that:

a the amount of money accumulated after 3 years is \$2519.42

b interest earned = \$2519.42 – \$2000

= \$519.42



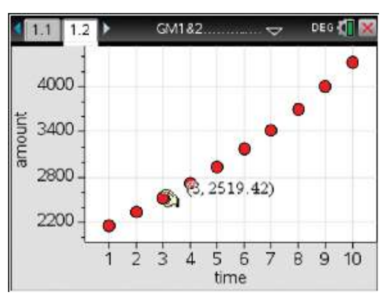
4 Press ctrl + I and select **Add Data & Statistics** and plot the graph as shown.

Note:

a To connect the data points: Move the cursor to the graphing area and press ctrl + menu. Select **Connect Data Points**.

b To display a value: Move the cursor over the data points or use b **>Analyze>Graph Trace**.

c You can use ctrl + menu and select **Zoom>Window Settings** and set the Ymin to 0 if you prefer.



From the plot we see that, for compound interest, the graph of the amount of money accumulated curves upwards with time.

ISBN 978-1-009-25891-3

Using the ClassPad to investigate compound interest problems

a Set up a table to enable the amount of money accumulated after t years if \$2000 is invested at an interest rate of 8% per annum compounding.

b Use the table to determine the value of the loan after three years and the amount of interest earned.

c Plot the growth in the amount of money in the investment for ten years and note the shape of the graph.

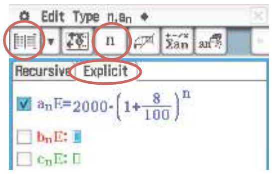
Steps

1 Substitute P = \$2000 and r = 8 into the formula for compound interest.

$$A = 2000 \times \left(1 + \frac{8}{100}\right)^t$$

2 To form a table of values.

- Open the **Sequence** application.
- Select the **Explicit** tab.
- Move the cursor to the box opposite $\mathbf{a_nE}$::
- Type **2000** × (**1** + **8**/**100**)^n. Use the **n** is found in the toolbar $\boxed{\text{n}}$.
- Press $\boxed{\text{EXE}}$ to confirm your entry, indicated by a tick in the square to the left of $\mathbf{a_nE}$:



3 To view a table of values.

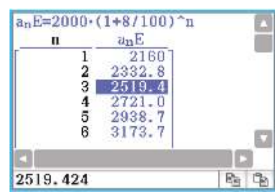
- Tap from the toolbar.
- Scroll down the table to see that the:

a amount of money accumulated after 3 years is \$2519.42

b interest earned

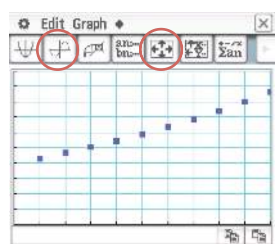
= \$2519.42 – \$2000

= \$519.42



4 To graph the sequence of compound interest values.

- Select the **Sequence Grapher** icon.
- Select the **View Window** icon.
- Set the values as shown. Use a y scale of 500.
- Tap **OK** to confirm your settings.



From the plot we see that, for compound interest, the graph of amount of money accumulated curves upwards with time.

ISBN 978-1-009-25891-3

Comparing simple and compound interest

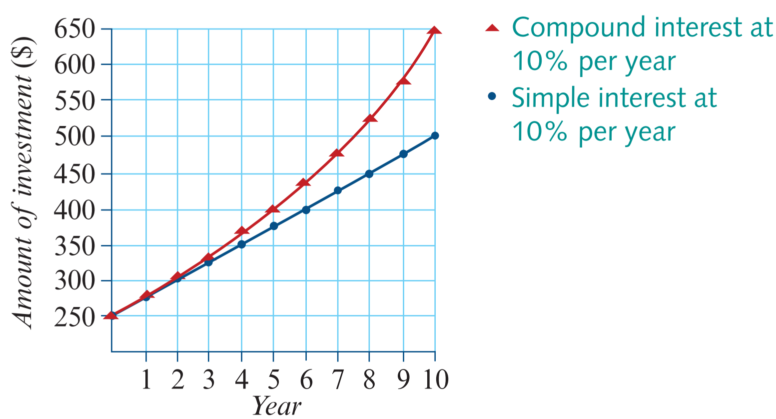
Earlier we saw that the growth in the value of simple interest investments and loans was linear. By contrast, as we have just seen, the growth in compound investments or loans was non-linear following a curve that became increasingly steep.

The difference in the growth pattern becomes clear if we compare a simple interest investment with a compound interest investment with the same principal ($250) and rate of interest (10% per annum). The results are displayed in the table below.

Amount of investment ($)		
Year (n)	10% simple interest	10% compound interest
0	250	250.00
1	275	275.00
2	300	302.50
3	325	332.75
4	350	366.03
5	375	402.63
6	400	442.89
7	425	487.18
8	450	535.90
9	475	589.49
10	500	648.44

From the table, we can see that after the first month the value of the compound interest investment is higher and this advantage increases over time.

The difference between the two investment strategies is even clearer when graphed.



ISBN 978-1-009-25891-3

Compounding periods other than one year

All of the compound interest questions completed so far in this chapter have involved interest that is calculated and added every year. It is very common for interest to be calculated and added to an account, either loan or investment, more often than this. The interest rate is still given as an annual one, but this is adjusted to take into account the more frequent interest calculations.

A particular bank loan might charge interest at an annual rate of 4.8%. However, if the bank calculates the interest owing and adds this to the loan after every month, the rate of interest will have to be a monthly one. Since there are 12 months in a year, the yearly interest rate is converted to a monthly rate by dividing by 12.

Annual interest rate = 4.8% per year

Monthly interest rate $= \dfrac{4.8}{12} = 0.4\%$ per month

Converting interest rates

Assume that, in one year, there are:

- 365 days (ignore the possibility of leap years)
- 52 weeks (even though there are slightly more than this)
- 26 fortnights (even though there are slightly more than this)
- 12 months
- 4 quarters

Convert an annual interest rate to another time period interest rate by dividing by these numbers.

The compounding interest formula you have been using will need to be adjusted so that it can be applied to loans and investments with **compounding time periods** other than one year.

The compound interest formula

In general, the amount, A, of a compound interest investment is given by:

$$A = P\left(1 + \frac{r}{100 \times n}\right)^{n \times t}$$

where:

- A is the amount of a compound interest investment
- P is the initial amount invested or borrowed (the principal)
- $r\%$ is the annual interest rate
- n is the number of compounding periods in a year
- t is the time in years

Note: Notice that the formula is raised to a power of $n \times t$ where the number of compounding periods per year are multiplied by the length of the investment, in years.

For example, for a loan of three years with interest compounding monthly, $n \times t = 3 \times 12 = 36$

ISBN 978-1-009-25891-3

Example 15 Compound interest with compounding periods other than one year

Courtney invests \$50 000 in an account that earns interest at the rate of 4.5% per annum, compounding monthly. How much is in her account after three and a half years?

Solution

1 Write the value of P, r, n and t.

\$50 000 is invested, so $P = 50\,000$
The annual interest rate is 4.5%, so $r = 4.5$
The interest compounds monthly, so $n = 12$
The investment lasts 3.5 years, so $t = 3.5$

2 Use the compound interest formula to determine the amount in the account.

$$A = P\left(1 + \frac{r}{100 \times n}\right)^{n \times t}$$

$$A = 50\,000\left(1 + \frac{4.5}{100 \times 12}\right)^{12 \times 3.5}$$

$$A = 58\,511.799\,171\,5$$

3 Write your answer.

After three and a half years, the amount in Courtney's investment account is \$58 511.80.

Skill-sheet

Exercise 2D

Note: In the following exercises, give all answers correct to the nearest cent.

Compound interest investments

Example 13

1 An amount of \$3500 is invested at 5% compound interest per annum for 5 years. Determine:

a the final value of this investment **b** the total amount of interest earned.

2 An amount of \$7000 is invested at 8% compound interest per annum for 4 years. Determine:

a the final value of this investment **b** the total amount of interest earned.

3 Calculate the difference between the simple interest and the compound interest on an investment of \$3000 at 7.9% per annum over 5 years.

ISBN 978-1-009-25891-3

Compound interest loans

Example 14

4 A person borrows $1250 at 7.5% compound interest per annum for 3 years. Determine:

a the total amount of money owed after 3 years

b the amount of interest owed.

5 A person borrows $1000 at 6.0% compound interest per annum for 5 years. Determine:

a the total amount of money owed after 5 years

b the amount of interest owed.

6 Calculate the difference between the simple interest and the compound interest on a loan of $2000 at 7% per annum over 5 years.

Exploring compound interest loans and investments with a CAS calculator

CAS

7 $850 is borrowed at 13.25% per annum compound interest for 8 years.

a Construct a table to display the total amount owed after t years for up to 8 years.

b How much is owed in total after 5 years, and how much of that is interest?

c Plot the growth in the amount of money in the investment for 8 years and note the shape of the graph.

8 Peter invests $3000 at 5.65% per annum compound interest for 10 years.

a Construct a table to display the total amount owed after t years for up to 5 years.

b How much is owed in total after 4 years, and how much of that is interest?

c Plot the growth in the amount of money in the investment for 10 years and note the shape of the graph.

Compound interest with compounding time periods other than one year

Example 15

9 A bank offers a loan with a compound interest rate of 3.6% per annum, compounding monthly.

a What is the monthly interest rate for this loan?

b If $10 000 is borrowed for a period of 6 months, how much must be paid back to the bank?

c If $25 000 is borrowed for a period of 3 years, how much must be paid back to the bank?

10 A bank will pay compound of interest at the rate of 5.2% per annum, compounding fortnightly.

a What is the fortnightly interest rate for this investment?

b If $5000 is invested for a period of 5 years, what is the final value of the investment?

c How much interest is earned on this investment after 5 years?

ISBN 978-1-009-25891-3

11 Millicent invests \$18 000 in an account that pays compound interest at the rate of 3.8% per annum.

a Calculate the amount in the account after 1 year if the interest compounds:

i monthly ii fortnightly iii daily

b How much extra will Millicent earn in interest over the first year if she chooses fortnightly compounds instead of monthly compounds?

2E Time payment agreements

When you go shopping there are generally two options for payment. The first option is to pay at the time using *cash* or a *debit card*, a card that directly debits (i.e. deducts) money from your bank account. Of course, this option is only possible if you have enough money with you, or in the bank, at the time of purchase.

The second option is to purchase on **credit**. In a financial context, credit means taking delivery of a good or service without paying for it at the time but with a commitment to pay for it later. These are sometimes called *time payment agreements*. Common forms of time payment agreements include **hire-purchase** agreements, **personal loans** and *credit cards*.

The cost of credit includes interest and possibly other fees and charges. It is really important to calculate what this total cost is before purchasing on credit so we can understand the real cost of the purchase.

Hire purchase

In a hire-purchase payment, the buyer will take the item they have purchased and then pay it off by making regular payments of an agreed amount. When all the payments have been made, the buyer owns the item. In some agreements, a deposit is paid at the beginning of the agreement and this reduces the amount of the payments. We are interested in calculating the interest rate being charged in these contracts because it is not always stated explicitly.

For a hire-purchase agreement we do can this in two ways. Firstly, by calculating the flat rate of interest charged and, secondly, by calculating the effective rate of interest.

Flat rate of interest

If we calculate the total interest paid as a proportion of the original debt and express this as an annual rate this is called the **flat rate of interest**.

The flat rate of interest is exactly the same as the simple rate of interest but is often called by this name in time payment agreements.

ISBN 978-1-009-25891-3

Flat rate of interest r_f

The flat rate of interest is calculated as a percentage of the *original amount* owed.

The annual flat rate of interest rate, r_f, is given by:

$$r_f = \frac{100I}{Pt}$$

where I = total interest paid

P = principal owing after the deposit has been deducted

t = the time in years

The following example illustrates the calculation of the flat rate interest for a typical hire-purchase agreement.

Example 16 Calculating the flat rate of interest for a hire-purchase agreement

Josh buys a sound system costing \$1400. He pays a deposit of \$500. The remaining \$900 he owes must be repaid by making six monthly payments of \$160. Calculate the flat rate of interest for this hire-purchase agreement, as a percentage per annum.

Solution

1 To calculate the flat rate of interest we must first work out how much interest Josh will pay.

Interest = total paid − purchase price

Total paid = deposit + repayments
= 500 + 6 × 160
= \$1460

Interest paid = 1460 − 1400
= \$60

2 To calculate the flat rate of interest charged, apply the formula $r_f = \frac{100I}{Pt}$.

- P = \$900 (since only \$900 is owed after the deposit is paid)
- I = \$60 (calculated in **1**)
- t = 0.5 (since the \$900 owed is repaid in 6 months).

$$r_f = \frac{100I}{Pt} = \frac{100 \times 60}{900 \times 0.5}$$

= 13.3% to one decimal place

Thus, the flat rate of interest for this hire-purchase agreement is 13.3% p.a.

In the example above, we see that under this hire-purchase agreement, Josh is paying a flat rate of interest of 13.3%. However, in reality, Josh is actually paying a much higher effective rate of interest. This is because he is making regular repayments throughout the course of the agreement. As a result, for a lot of the time he actually owes quite a lot less than the original \$900 he borrowed.

ISBN 978-1-009-25891-3

Exercise 2E-1

Calculating the real cost of an item purchased under hire purchase

Example 16

1 The cash price of a tennis racquet is \$330. To buy it through a hire-purchase agreement requires a deposit of \$30 and 12 equal monthly instalments of \$28. Calculate:

a the total cost of buying the racquet by hire purchase

b the extra cost of buying by hire purchase.

2 A bicycle is on sale price for \$300. It can be bought through hire purchase with a deposit of \$60 and 10% interest on the outstanding balance, to be repaid in 10 monthly instalments. Calculate:

a the amount of each monthly instalment

b the total cost of buying the bicycle by hire purchase.

3 A hire-purchase agreement offers gym equipment, with a marked price of \$897, for \$87 deposit and \$46.80 a month payable over 2 years.
Calculate:

a the total hire-purchase price

b the amount of interest charged.

Credit cards

Calculating credit card debt is an application of **compound interest** where interest is calculated daily.

Calculating credit card debt

If a credit card debt of $\$P$ accumulates at the rate of $r\%$ per annum, compounding daily, then the amount of debt accumulated after n days is given by:

$$A = P\left(1 + \frac{r \div 365}{100}\right)^n = P\left(1 + \frac{r}{36\,500}\right)^n$$

and the amount of interest payable after n days is given by:

$$I = A - P$$

Note: To determine the daily interest rate the annual interest rate r is divided by 365.

ISBN 978-1-009-25891-3

Example 17 Calculating credit card interest

Determine how much interest is payable on a credit card debt of \$5630 at an interest rate of 17.8% per annum for 27 days.

Solution

1 Calculate the value of debt after 27 days using the rule:
$$A = P\left(1 + \frac{r}{36\,500}\right)^n$$

$P = \$5630$, $r = 17.8\%$ and $n = 27$

$$A = P\left(1 + \frac{r}{36\,500}\right)^n$$
$$= 5630\left(1 + \frac{17.8}{36\,500}\right)^{27}$$
$= \$5704.60$ to the nearest cent

2 The amount of interest payable is obtained by subtracting the original debt P from the value of the debt after 27 days.

$I = A - P$
$= \$5704.60 - \$5630.00$
$= \$74.60$

Taking into account interest-free periods

Most credit cards offer a maximum interest-free period, which means that if you pay for your purchase within that time you won't pay any interest.

Usually the credit card has a statement period, which runs for about 30 days. After the statement closes, there are an additional number of days, usually 15 to 25, to pay the full balance before an interest rate applies. The actual number of interest-free days varies depending on when you make your purchase and the number of days remaining in your statement period. For example, suppose your statement begins on 1 June and ends on 30 June. Your bank gives you another 25 days after 30 June to pay your bill in full before you are charged any interest on the items you bought in June.

Thus:

- if you make a purchase on 10 June, you will have 20 days remaining until your statement period closes plus 25 days to make your full payment. This means that you have 45 interest-free days before you will be charged interest.
- if you make a purchase on 28 June, you only have 2 days remaining until your statement period closes. Adding that to the 25 days available to make the payment and you only have 27 days before you will be charged interest.

Not all statement periods will start at the beginning of the month, this will vary from person to person. However, the statement periods will be the same for all individuals with a particular credit card.

These principles are illustrated in the next example.

ISBN 978-1-009-25891-3

Example 18 Calculating credit card interest with an interest-free period

Janelle pays for her holiday in Bali using her credit card. Her bank offers a 30-day statement period plus a further 25 days interest free. After that time, the bank charges interest at a rate of 20% per annum compounding daily.

The cost of the holiday is \$1500 and Janelle makes the purchase on 16 August, which is day 10 of her statement period. She intends to pay off the credit card on 1 November. At this date, how much will she need to pay back? (Assume no interest is payable on the last day).

Solution

1 Determine the number of interest free days.

Since Janelle is in day 10 of her statement period she has 20 + 25 days = 45 interest free days.

2 Determine the number of days for which interest is payable. Since the purchase was made on 16 August, start counting from 17 August.

Number of days:

17 August–30 August = 14 days

1 September–30 September = 30 days

1 October–31 October = 31 days

Total days = 14 + 30 + 31 = 75

$$\text{Total interest payable days} = 75 - 45$$
$$= 30$$

3 Calculate the amount payable using the rule $A = P\left(1 + \frac{r}{36\,500}\right)^n$.

$P = \$1500$, $r = 20\%$, $n = 30$

$$A = 1500\left(1 + \frac{20}{36\,500}\right)^{30}$$

$= \$1524.85$ to the nearest cent

Janelle will need to pay back \$1524.85.

ISBN 978-1-009-25891-3

Exercise 2E-2

Calculating credit card interest

Example 17

1 Determine the amount of interest payable on the following credit card debts.

a $2000 at an interest rate of 18.9% per annum for 52 days

b $785 at an interest rate of 24% per annum for 200 days

c $12 000 at an interest rate of 22.5% per annum for 60 days

d $837 at an interest rate of 21.7% per annum for 90 days

Example 18

2 Matt has two credit cards, each with different borrowing terms.

- Credit card A charges 22% p.a. interest and offers up to 60 days interest free.
- Credit card B charges 19% p.a. but only offers 40 days interest free.

He wishes to buy an item costing $2000 on his credit card, which he will purchase at the beginning of the statement period whichever card he uses, so as have the maximum interest free days. Which credit card should he use:

a if he is going to pay off the card 30 days after purchase?

b if he is going to pay off the card 60 days after purchase?

c if he is going to pay off the card 90 days after purchase?

d if he is going to pay off the card 240 days after purchase?

3 Joe buys a skateboard costing $830 on his credit card. He buys it on the first day of his statement period so he has the maximum number of interest-free days, which is 55. After that time, the bank charges interest at a rate of 24% per annum compounding daily. How much will he owe on his credit card:

a at the end of 20 weeks?

b at the end of 40 weeks?

4 Brett spends $3000 on some home theatre equipment on his credit card. His bank offers a 30-day statement period and then a further 30 days interest free. After that time, the bank charges interest at a rate of 23.5% per annum compounding daily.
Brett makes the purchase on 2 April, which is day 2 of his statement period. He intends to pay off the credit card on 1 July. At this date, how much will he need to pay back? (Assume no interest is payable on the last day).

5 Sacha buys a computer costing $1470 on her credit card. Her bank offers a 30-day statement period and then a further 10 days interest free. After that time, the bank charges interest at a rate of 18.5% per annum compounding daily.
Sacha makes the purchase on 10 January, which is day 12 of her statement period. She intends to pay off the credit card on 1 March. At this date, how much will she need to pay back? (Assume no interest is payable on the last day).

ISBN 978-1-009-25891-3

2F Inflation

Effect of inflation on prices

Inflation is a term that describes the continuous upward movement in the general level of prices. This has the effect of steadily *reducing* the **purchasing power** of your money; that is, what you can actually buy with your money.

In the early 1970s, inflation rates were very high, up to around 16% and 17%. Inflation in Australia has been relatively low in recent years.

- Since 1970, inflation has averaged 6.8% per year.
- Since 1990, it has averaged 2.1% per year.

Example 19 Determining the effect of inflation on prices over a short period of time

Suppose that inflation is recorded as 2.7% in 2012 and 3.5% in 2013 and that a loaf of bread costs \$2.20 at the end of 2011. If the price of bread increases with inflation, what will be the price of the loaf at the end of 2013?

Solution

1 Determine the price of the loaf of bread at the end of 2012 after a 2.7% increase.

$$\text{Increase on the 2012 price} = 2.20 \times \frac{2.7}{100} = 0.06$$

2 Calculate the price at the end of 2012.

$$\text{Price (2012)} = 2.20 + 0.06 = \$2.26$$

3 Determine the price of the loaf of bread at the end of 2013 after a further 3.5% increase.

$$\text{Increase in price (2013)} = 2.26 \times \frac{3.5}{100} = 0.08$$

4 Calculate the price at the end of 2013.

$$\text{Price (2013)} = 2.26 + 0.08 = \$2.34$$

While the difference in price seen in Example 19 does not seem significant, you will be aware from earlier compound interest examples that even if inflation holds steady at a low 2.1% per year for 20 years, prices will still increase significantly, as the following example shows.

Example 20 Determining the effect of inflation on prices over a long period

Suppose that a one-litre carton of milk costs \$1.70 today.

a What will be the price of the one-litre carton of milk in 20 years time if the average annual inflation rate is 2.1%?

b What will be the price of the one-litre carton of milk in 20 years time if the average annual inflation rate is 6.8%?

ISBN 978-1-009-25891-3

Solution

a **1** This is the equivalent of investing \$1.70 at 2.1% interest compounding annually, so we can use the compound interest formula.

$$A = P \times \left(1 + \frac{r}{100}\right)^t$$

2 Substitute $P = \$1.70$, $t = 20$ and $r = 2.1$ in the formula to find the price in 20 years.

$$\text{Price} = 1.70 \times \left(1 + \frac{2.1}{100}\right)^{20}$$
$$= \$2.58 \text{ to the nearest cent}$$

b Substitute $P = 1.70$, $t = 20$ and $r = 6.8$ in the formula and evaluate.

$$\text{Price} = 1.70 \times \left(1 + \frac{6.8}{100}\right)^{20}$$
$$= \$6.34 \text{ to the nearest cent}$$

Effect of inflation on the purchasing power of money

Another way of looking at the effect of inflation on our money is to consider what a sum of money today would buy in the future. That is, to convert projected dollar numbers back into present-day values so you can think in today's money values.

Suppose, for example, that you put \$100 in a box under the bed and leave it there for 10 years. When you go back to the box, there is still \$100, but, what could you buy with this amount in 10 years time? To find out we need to 'deflate' this amount back to current-day purchasing power dollars.

We can do this using the compound interest formula.

Suppose there has been an average inflation rate of 4% over the 10-year period. Substituting $A = 100$, $r = 4$ and $t = 10$ gives:

$$100 = P \times \left(1 + \frac{4}{100}\right)^{10} = P \times (1 + 0.04)^{10}$$

Rearranging this equation, or using your CAS calculator to solve it, gives:

$$P = \frac{100}{(1 + 0.04)^{10}}$$
$$= \$67.56 \text{ to the nearest cent}$$

That is, the money that was worth \$100 when it was put away has a purchasing power of only \$67.58 after 10 years if the inflation rate has averaged 4% per annum.

Example 21 **Investigating purchasing power**

If savings of \$100 000 are hidden in a mattress in 2016, what is the purchasing power of this amount in 8 years time if the average inflation rate over this period is 3.7%? Give your answer to the nearest dollar.

ISBN 978-1-009-25891-3

Solution

1 Write the compound interest formula with P (the purchasing power, which is unknown), $A = 100\,000$ (current value), $r = 3.7$ and $t = 8$.

$$A = P \times \left(1 + \frac{r}{100}\right)^t$$

$$100\,000 = P \times \left(1 + \frac{3.7}{100}\right)^8$$

2 Use your CAS calculator to solve this equation for P and write your answer.

The purchasing power of \$100 000 in 8 years is \$74 777, to the nearest dollar.

Exercise 2F

Effect of inflation on prices

Example 19

1 Suppose that inflation is recorded as 2.7% in 2017 and 3.5% in 2018, and that a magazine costs \$3.50 at the end of 2016. Assume that the price increases with inflation.

a What will be the price of the magazine at the end of 2017?

b What will be the price of the magazine at the end of 2018?

2 Suppose that Henry receives a salary increase at the end of each year equal to the rate of inflation for that year. Inflation is recorded as 3.2% in 2017 and 5.3% in 2018, and Henry's weekly salary is \$825 at the end of 2016.

a What will Henry's salary be at the end of 2017?

b What will Henry's salary be at the end of 2018?

Example 20

3 Suppose that the cost of petrol per litre is \$1.80 today.

a What will be the price of petrol per litre in 20 years time if the average annual inflation rate is 1.9%?

b What will be the price of petrol per litre in 20 years time if the average annual inflation rate is 7.1%?

4 A house is sold at auction for \$500 000. If the price of the house increases with the inflation rate, what will be the price of the house in 12 years time:

a if the average inflation rate over the 12-year period is 2.6%?

b if the average inflation rate over the 12-year period is 6.9%?

Effect of inflation on purchasing power

Example 21

5 If savings of \$200 000 are hidden in a mattress today, what is the purchasing power of that money in 10 years time:

CAS

a if the average inflation rate over the 10-year period is 3%?

b if the average inflation rate over the 10-year period is 13%?

ISBN 978-1-009-25891-3

CAS

6 If Jo puts \$1000 cash in her safe, what is its purchasing power in 20 years time:

a if the average inflation rate over the 20-year period is 2.6%?

b if the average inflation rate over the 20-year period is 6.9%?

c if the average inflation rate over the 20-year period is 14.3%?

2G Calculating income and preparing personal budgets

Financial calculations are important for businesses and individuals alike. Businesses employ people to do work and they must keep track of hours worked and the rate at which their employees are paid. It is important for individuals to plan how they save and spend their money. In this section, we will use tables of simple calculations to do this, before using technology in the form of spreadsheets to set up and explore situations of income and budgets.

Calculating income

John is a shop owner who employs three people to help run the shop. Each of these employees has a different salary rate as shown in Table 1.

Table 1

Employee	Position	Hourly pay rate
Melissa	Store Manager	\$28.90
Tristan	Senior Sales Staff	\$23.50
Sheila	Casual Sales Staff	\$15.60

John pays his staff at the end of each week and must keep track of the number of hours that each staff member works. Table 2 below shows the hours worked by each staff member in a particular week.

Table 2

Employee	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Melissa	7.5	6.5	7.5	7	3.5	0	0
Tristan	5.0	4.5	5.0	5.0	5.0	5.0	4.0
Sheila	0	0	0	2.5	3.0	3.0	5.0

We can easily calculate the salary that each staff member will receive for this week, by adding the hours worked each day and multiplying by their particular hourly pay rate.

Melissa's salary $= (7.5 + 6.5 + 7.5 + 7 + 3.5 + 0 + 0) \times \$28.90 = \$924.80$

Tristan's salary $= (5.0 + 4.5 + 5.0 + 5.0 + 5.0 + 5.0 + 4.0) \times \$23.50 = \$787.25$

Sheila's salary $= (0 + 0 + 0 + 2.5 + 3.0 + 3.0 + 5.0) \times \$15.60 = \$210.60$

ISBN 978-1-009-25891-3

These values are the **gross salary** for each employee; that is, the salary before tax has been withheld. The tax withheld can be estimated as a percentage of their weekly salary[1].

Melissa's tax rate is 16.9%, while Tristan and Sheila have 13.8% of their salary withheld as tax.

The amount of tax for each employee can easily be calculated using simple percentages.

$$\text{Melissa's tax} = \frac{16.9}{100} \times \$924.80 = \$156.29$$

$$\text{Tristan's tax} = \frac{13.8}{100} \times \$787.25 = \$108.64$$

$$\text{Sheila's tax} = \frac{13.8}{100} \times \$210.60 = \$29.06$$

The **net salary** for each employee is the salary after tax has been withheld. This is commonly referred to as the '*take home pay*' and can be calculated with a simple subtraction.

Melissa's net salary = \$924.80 − \$156.29 = \$768.51

Tristan's net salary = \$787.25 − \$108.64 = \$678.61

Sheila's net salary = \$210.60 − \$29.06 = \$181.54

All of these calculations and values can be summarised in one table as shown below.

	M	T	W	T	F	S	S	Total	Rate/hr	Gross salary	Tax rate	Tax	Net salary
Melissa	7.5	6.5	7.5	7	3.5	0	0	32.	\$28.90	\$924.80	16.9%	\$156.29	\$768.51
Tristan	5.0	4.5	5.0	5.0	5.0	5.0	4.0	33.5	\$23.50	\$787.25	13.8%	\$108.64	\$678.61
Sheila	0	0	0	2.5	3.0	3.0	5.0	13.5	\$15.60	\$210.60	13.8%	\$29.06	\$181.54

A spreadsheet can greatly simplify the calculations above and can allow changes to be made to the information, resulting in automatic recalculation of all connected values. To see how to use a spreadsheet to calculate salaries like this, visit the online resources for this textbook.

[1] The calculation of tax is simplified as a percentage of weekly salary in this example. In reality, the determination of the amount of tax withheld is more complicated than shown here and is beyond the scope of this course.

ISBN 978-1-009-25891-3

Calculating wages from an hourly rate

A **wage** is how much a person is paid to do a job. It can be measured weekly, monthly, or annually.

Hourly rate, on the other hand, is simply how much a person gets paid for an hour of work.

Example 22 Calculating wages from an hourly rate

a Luke works at Maccas and his hourly rate of pay is \$11.50. Last weekend he worked for 12 hours. Calculate his total pay for the weekend.

b Peter, a trainee, also works at Maccas starting his shift at 7:30 a.m. and finishes at 4 p.m. He has an unpaid lunch break between 12 p.m. and 1 p.m. Calculate his total pay for this shift if he earns \$10.75 per hour of work.

Solution

a 1 Write his hourly wage	Hourly wage = \$11.50
2 Write hours worked	Number of hours worked = 12
3 Calculate his wage	Wage = 11.50×12 = \$138
4 Write down the answer	Luke's wage is \$138
b 1 Write his hourly wage	Hourly wage = \$10.75
2 Calculate hours worked	7:30 a.m. $\xrightarrow{\text{30 mins}}$ 8:00 a.m. 8:00 a.m. $\xrightarrow{\text{4 hours}}$ 12:00 p.m. 1 p.m. $\xrightarrow{\text{3 hours}}$ 4 p.m. Number of hours worked = 7.5
c 1 Calculate his wage	Wage = 7.5×10.75 = \$80.63
2 Write down the answer	Peter's wage is \$80.63

Overtime and Other Allowances

Overtime refers to doing extra work over and above your normal hours. Overtime allows people to earn a better hourly rate and thus more income. In this part of the chapter, emphasis will be laid on the two most common rates of overtime: double time ($\times 2$) and time-and-a-half ($\times 1.5$).

Example 23 Calculating overtime

Paul, the junior plumber, works for \$22 per hour. His overtime rate is 'double time'. How much does he get paid for 3 hours overtime?

Solution

a Write his hourly wage	Hourly wage = \$22
b Calculate double time rate	Double time rate = \$22 $\times 2$ = \$44
c Calculate his wage	Wage = 3×44 = \$132
d Write down the answer	Paul's wage is \$132

ISBN 978-1-009-25891-3

Example 24

Jacob is a painter and works a basic 40-hour week from Monday to Friday. On Saturday he does 2 hours overtime and 4 hours on Sunday. His hourly rate is $18 per hour, with time-and-a-half for any hours worked on Saturday and double time for Sunday. Calculate Jacob's total pay for the week.

Solution

1 Calculate his basic pay	$40 \times 18 = \$720$
2 Calculate his Saturday pay	$2 \times 18 \times 1.5 = \$54$
3 Calculate his Sunday pay	$4 \times 18 \times 2 = \$144$
4 Work out his total pay	$720 + 54 + 144 = \$918$
5 Write down the answer	Jacob's pay for the week is $918

Preparing personal budgets

Personal budgets allow people to manage their personal finances, plan for their future and ensure that their salary is used to cover all essential items such as rent, food or bills.

Melissa, from the start of this section, has created a simple personal budget using a table, as shown below.

Melissa's weekly budget			
Income		Expenses	
Salary	768.51	Rent	120.00
		Food	80.00
		Bills	140.00
		Car loan	25.00
		Savings	150.00
		Clothes	30.00
		Gifts	40.00
		Entertainment	50.00
Total income	$768.51	Total Expenses	$635.00
		Surplus	$133.51

We can see from Melissa's budget that she has some regular, constant payments that she is planning for. There is a weekly rent payment of $120.00 and Melissa is estimating that she will spend $80.00 per week on food. Melissa is saving $140.00 each week to cover bills when they arrive and there is a regular repayment on a car loan of $25.00 each week.

ISBN 978-1-009-25891-3

Melissa's budget also includes a constant amount of \$150.00 saved from her salary every week. Perhaps Melissa is depositing this into an account that earns interest? Other items are covered in Melissa's budget as well, such as clothes, entertainment and birthday gifts for her friends.

Melissa's surplus can be calculated by subtracting her total expenses from her total income. In the simplest of terms, the goal of most budgets is to ensure that spending is less than the overall income. It is always a good idea to have some money left over (surplus) at the end of a week or month that can be put towards further savings or unexpected costs.

According to Melissa's budget, she can expect to have approximately \$133.51 left over at the end of her salary. Of course, all of these amounts are just estimates that help Melissa plan her finances. Other payments and bills might need to be paid from time to time.

Spread-sheet

A spreadsheet can help Melissa create her budget. For an example of how to use a spreadsheet to prepare personal budgets like this, visit the Interactive Textbook for this section.

Discretionary vs fixed spending

Part of a personal budget includes considering fixed and discretionary spending.

Fixed spending is money that is spent paying for goods or services that are ongoing costs and do not change in amount or frequency across the year. These costs are predictable within the budget and often are difficult to reduce in the short term. Examples of fixed spending are mortgage or other loan repayments, car insurance, mobile phone plans, transport costs (such as bus or train fares) or household expenses such as water, gas and electricity.

Comparatively, **discretionary spending** is money that is spent on goods or services that are desirable, but there is an element of individual choice (discretion) over how much of your budget goes towards these costs. These are often the first things that can be reduced or cut from a budget when looking for money-saving opportunities. Examples of discretionary spending might be entertainment, eating out or ordering takeaway food, streaming subscription services, gifts, or holidays.

In the example of Melissa's personal budget her rent, food, bills and car loan are all **fixed costs**, whereas clothes, gifts and entertainment are **discretionary costs**. If Melissa was looking to save more money or had an unexpected cost to pay (such as a medical bill or car maintenance) she could reduce the amount spent on these within her budget and put the money towards her savings or use it to pay her additional costs.

ISBN 978-1-009-25891-3

Exercise 2G

Calculating income

1 Complete the following table of income calculations for five employees in a shop, by writing values in the shaded cells of the table.

	Hours worked												
	M	T	W	T	F	S	S	Total	Rate/hr	Gross salary	Tax rate	Tax	Net salary
Penny	0.0	5.0	0.0	6.0	0.0	2.0	2.0		\$12.50		13.8%		
Wilson	7.5	7.5	7.5	7.5	7.5	0.0	0.0		\$26.75		16.9%		
Vimbai	5.5	8.0	6.5	0.0	3.5	2.0	2.0		\$31.85		16.9%		
Brendon	6.5	0.0	7.5	7.5	6.5	5.5	0.0		\$25.80		16.9%		
Keith	7.5	0.0	7.5	7.5	7.5	0.0	5.5		\$29.65		16.9%		

2 **a** Convert the table above to a spreadsheet to calculate the incomes for the employees. Use a formula in each of the shaded cells.

b Vimbai has incorrectly been recorded as working 2 hours on Saturday and Sunday. Remove these values from the spreadsheet and write Vimbai's corrected salary for this week.

c If Brendon should have been paid at the rate of \$26.90 for this week, write his corrected salary.

Personal budgets

3 **a** Use Wilson's salary from the table above to complete a weekly budget in table form. Wilson has rent of \$150.00 per week and estimates he will spend about \$100.00 each week on food and \$120.00 on other bills. He gives \$10 pocket money to his son and pays \$140 into an investment account.

b How much money is left over from Wilson's salary in this particular week?

c Wilson's car registration is \$560 every year. Add an amount to his weekly budget to cover this expense.

d Wilson will add a pay-tv subscription to his budget, for a total of \$55 per month. He would also like to take out a loan to buy a new car. If Wilson would like to have a surplus of at least \$200 from his salary each week, what is the maximum loan payment he could afford?

4 Confirm your answers to question **3** above by creating Wilson's personal budget using a spreadsheet.

ISBN 978-1-009-25891-3

5 Create a personal budget for yourself.

- **a** Choose a career and use internet research to find out an estimate of the weekly salary.
- **b** Choose a suburb to live in and use internet research to find out an estimate of weekly rent.
- **c** Estimate weekly costs for electricity, gas and water.
- **d** Use a supermarket catalogue or online shopping sites to estimate a food bill for a week.
- **e** Estimate how much you would spend on clothes, entertainment, transport and any other expenses that you can think of.
- **f** Use a spreadsheet to compile your budget. Do your estimates allow any salary left over?

Discretionary vs fixed spending

6 The following table shows the income and expenditure for the Williams family for one year.

Income	(\$)	Expenditure	(\$)
Salaries & wages	28 240.00	Food	5 400.00
Centre Link	6 350.64	Rent	9 600.00
		Power	3 453.60
		Insurance	950.55
		Telephone	1 456.14
		Other	6 523.41
		Entertainment	4 523.69
Total		**Total**	

- **a** Complete the table by calculating the total income and expenditure.
- **b** What percentage of their income did the Williams family save?
- **c** The Williams family is planning for a holiday costing \$12 000. If they plan to set aside 75% of their savings for the holiday, how long will it take for the family to attain their goal?
- **d** State one example of fixed spending and one discretionary spending seen in the Williams family budget.

ISBN 978-1-009-25891-3

7 Jack and Kelly are both 14 years of age attending the same school. Their parents have agreed they can purchase a car in three years when they both get their licenses at 17. Their aim is to save enough over the next three years to buy the car themselves. They also must pay for costs like petrol, repairs, and insurance. Jack and Kelly both find reasonably priced cars for $5000, an amount that they think they can afford. Bearing inflation in mind, the cars might cost them each $5500 in 3 years' time. They decide to make a budget estimate of their expected income and expenditures. Their budget is tabulated below for each month.

Monthly income & Expenditure	Jack	Kelly
Allowance	$75	$75
Games rental	$8	$0
Part-time job	$128	$116
Snacks	$21	$13
School supplies	$11	$19
Phone rental	$15	$15
Entertainment	$22	$15

a List the income and expenditure for both Jack and Kelly and calculate how much they each save per month.

b After three years, will they have saved enough to afford their car?

c How much more money does each one need to save per month to afford to buy their car?

d Jack and Kelly are budgeting to make sure they save enough to buy the car. They also must consider the expenses they will face to operate the car after they buy it. List a few operating expenses they might include.

ISBN 978-1-009-25891-3

8 The table below shows the charges associated with water usage in Royal City.

Rates for reading the water meter	
Usage (KL) per year	Meters read January–December
First 150 kL	85.2 c/kL
Next 200 kL	120.5 c/kL
Next 150 kL	132.6 c/kL
Over 500 kL	169.4 c/kL

a The account shows that the water usage for the Smith family was 230 kL. Calculate the amount they are required to pay for their water usage.

b The Bligh's water usage was 450 kL. Calculate the amount they are required to pay for their water usage.

c The Simpson's have a budget of $500 for their water usage. Calculate by how much they exceeded their budget if their water consumption was exactly 540 kL?

Calculating wages from an hourly rate

9 Peter works at Starving John's and his hourly rate of pay is $10.80. Last week he worked for 14 hours. Calculate his basic pay.

10 Ebba starts work at 8:30 a.m. and finishes at 3 p.m. She had an unpaid lunch break between 12 p.m. and 12.30 p.m. Calculate her basic pay if she earns $9.80 per hour of work.

11 Paloma works as a waitress at the Dessert Yard and makes $10.40/hour plus tips. Last week she worked 32 hours and made $463 in tips. What was her gross pay for the week?

12 Ren is a data entry operator. He makes $14.50 an hour and works 32 hours a week. What would be his fortnightly gross pay?

13 Jeremy works as a waiter in a Japanese restaurant. In addition to his regular pay of $11.20/hour, Jeremy keeps 85% of all the tips he receives. Calculate his gross weekly pay for a week in which he works 36 hours and receives $260 in tips.

ISBN 978-1-009-25891-3

14 Ryan works full time four days a week at Scarborough Beach Fish N' Chips for $12.40 per hour. The table below shows his Week 13 work schedule.

Name : Ryan Biggs			Employee No. 1831		Week 13
	In	Out	In	Out	Total hours worked
Mon	0900	1200	1300	1600	
Tue	0800	1200	1400	1700	
Wed	1030	1230	1300	1530	
Fri	0800	1200	1400	1930	
				Total	

a Complete the table, stating the total hours worked each day.

b Hence, calculate his income for the week.

Overtime and other allowances

15 Paul, the trainee electrician, works for $24.50 per hour. His overtime rate is time-and-a-half. How much does he get paid for 4 hours overtime?

16 A machinist is paid $21.60 per hour during normal working hours (9 a.m. – 4 p.m.). For each hour after 4 p.m., he is paid time-and-a-half. Calculate the amount received by the machinist on a day when he worked 9 a.m. until 6 p.m.

17 Santino is employed at a hairdressing salon. He receives $16.50 per hour for a standard 35-hour Monday to Friday week. Santino also receives $45 per week as travel allowance and $200 per year as laundry allowance. Calculate Santino's earnings for a standard week, assuming he receives his laundry allowance on a weekly basis.

18 Cai is paid time-and-a-half for each hour she works over 32 hours in a week. Last week she worked 40 hours for a total of $726. What is Cai's normal hourly rate?

19 A receptionist works a 35-hour week for which he is paid $444.50. In a particular week he works 4 hours overtime on Saturdays which is paid for at time-and-a-half, and 2.5 hours overtime on Sunday which is paid for at double-time. Calculate his gross wage for that week.

ISBN 978-1-009-25891-3

2H Government allowances and pensions

Family tax benefit

Family Tax Benefit (FTB) is an allowance that helps eligible families with the cost of raising their children. It is made up of two parts FTB Part A and FTB Part B. FTB Part A is paid depending on the number of children in a family and their financial status. FTB Part B, on the other hand, is paid per-family and gives extra help to single parents and families with one main income.

The maximum amounts of Family Tax Benefit Part A received per child are updated on 1 July each year. The table below shows the current rates per child within different age brackets and circumstances. For the scope of this book, only FTB Part A has been included as examples.

Current rates FTB Part A	
For each child	Per fortnight
0 to 12 years	\$169.68
13–15 years	\$220.64
16–19 years	\$220.64
16–17 years (completed secondary study)	\$54.32
18–21 years (completed secondary study)	\$54.32

To be eligible for the above rates, a particular family must undergo one of the following tests depending on their yearly income.

	FTB Part A tests
Test 1	Maximum rate for Family Tax Benefit Part A 20 cents less for each dollar above \$50 151.
Test 2	Maximum rate for Family Tax Benefit Part A 30 cents less for each dollar above \$94 316, plus \$3796 for each Family Tax Benefit child after the first.

Example 25

The Smith family has two children aged 5 and 8 years. The combined family income is \$63 500. Calculate the amount of FTB Part A received by the Smith family.

Solution

Yearly maximum amount for both children	$169.68 \times 2 \times 26 = \$8823.36$
Applying the first test	$(63\,500 - 50\,151) \times 0.20 = \$2669.80$
FTB Part A received	$8823.36 - 2669.80 = \$6153.56$

ISBN 978-1-009-25891-3

Exercise 2H-1

Referring to Tables 1 and 2 on the previous page, answer the following questions.

1. The Gordon family has two children aged 10 and 17 years. The 17-year-old son has completed his secondary study and works part-time in a restaurant. The family net income is \$59 860. Determine the amount of FTB Part A received by the Gordon family.
2. A family's income is \$55 765 a year. The family has two children of 3 and 9 years respectively. Calculate the amount of FTB Part A received by this family.
3. Mr and Mrs Pavilion have three children: a son aged 11 years and twin daughters of 15 years of age. The Pavilions' run their own business and earn \$75 420 a year. Calculate the amount of FTB Part A received by the family.
4. Calculate the amount of FTB Part A received by the Packard family having only one child aged 14 years and having a net annual income of \$90 000.
5. A family's income is \$1985 a week. The family has two children of 7 and 16 years respectively. Calculate the amount of FTB Part A received by this family.

Old age pensions

A person must meet the residency requirements in order to be eligible for age pensions in Australia. If you are an Australian citizen and have lived in Australia for at least ten years, you may be eligible for the age pension. Permanent residents in Australia may be eligible for the age pension as well.

In addition to residency requirements, the government will conduct an assets and income test to determine whether a person is eligible for the age pension. The tests consider factors such as age pension, wife pension, caregiver payment, and so on.

Table 1 below shows the pension rates for age pensions.

Table 1

Pension rates for age pensions		
Pension rates per fortnight	Single	Couple each
Maximum basic rate	\$776.70	\$585.50

ISBN 978-1-009-25891-3

The **assets test** has two thresholds and is split into two categories as shown in Table 2 below.

Table 2

	The assets test			
	Home owners		Non-home owners	
	Lower threshold	Upper threshold	Lower threshold	Upper threshold
Single	192 500	707 750	332 000	847 250
Couples	273 000	1 050 000	412 500	1 189 500

The full age pension is received when a lower assets test threshold is not exceeded. Once the lower thresholds are exceeded a person or couple's entitlement to the age pension is reduced by \$1.50 a fortnight for every \$1000 their assets exceed that threshold. Unfortunately, no age pension is received once an upper threshold is exceeded.

Table 3 below shows the **income test** for singles and couples.

Table 3

	The income test	
	Payment per fortnight	Reductions
Single	Up to \$160	None – full payment
	Above \$160	50 cents for each dollar over \$160
Couples combined	Up to \$284	None – full payment
	Above \$284	50 cents for each dollar over \$284

For a person being single and earning income up to \$160 a fortnight, there is no deduction in his or her pension. However, for income exceeding \$160, there is a deduction of 50 cents per dollar.

Example 26

Using the information from the tables above, determine the fortnightly age pension of a single non-homeowner of pension age owning assets worth \$390 000.

Solution

1 Calculate fortnightly age pension from asset test

$$776.70 - \left(\frac{390\,000 - 332\,000}{1000}\right) \times 1.50$$

2 Write the answer

\$689.70

ISBN 978-1-009-25891-3

Example 27

Mr and Mrs Zhang, both in their late sixties, have invested a sum of $200 000 which earns them a yearly simple interest of 5%. Calculate their combined fortnightly age pension using Table 1 and Table 3.

Solution

1 Calculate interest earned	$200\,000 \times 0.05 = \$10\,000$
2 Calculate fortnightly income	$10\,000 \div 26 \approx \$385$
3 Calculate fortnightly age pension	$585.50 \times 2 - (385 - 284) \times 0.50$
4 Write the answer	$1120.50

Exercise 2H-2

1 Using the information from Table 1 and Table 2 on the previous pages, determine the fortnightly age pension of a single homeowner of pension age owning assets worth $405 000.

2 Mr Adan Bligh, a widower in his early seventies, has invested a sum of $150 000 which earns him a flat fixed rate of 6% each year. Calculate his fortnightly age pension using Table 1 and Table 3.

3 Using the information from Table 1 and Table 2 on the previous pages, determine the fortnightly age pension of a home owning couple both of pension age owning assets worth $580 000.

4 Mrs Franco is a 69-year-old widow. She has invested $90 000 as a shareholder in an IT company. Last year she received 2% flat interest on her investment plus $450 as dividend each fortnight. Using Tables 1 and 3, calculate her fortnightly age pension.

5 Agent Marco is a single retired army officer and has reached the pension age. He has assets worth $365 000. He has invested $165 000 of their assets in Delta Bank earning 4% per annum as interest.

a Determine using the assets test his fortnightly pension.

b Determine using the income test his fortnightly age pension.

c It is customary for the government to pay the lower pension out of the income test or assets test carried out. State agent Marco's fortnightly pension rounded up to the nearest dollar.

ISBN 978-1-009-25891-3

2I Shares and dividends

Investors often choose to invest their money in shares. A **share** is a unit of ownership in a company. All shares are equal in value, and each share entitles the person who owns it to an equal claim on the company's **profits**.

For example:

- if there are 100 shares in a company and you own 20, then you own 20% of the shares in a company
- if the company makes a profit of \$100 000 in one year, then you are entitled to 20% of that profit (which would be \$20 000).

Example 28 Calculating profit from shares

There are 500 shares in the Kanz Construction Company. Richard owns 25 shares.

a What percentage of the company does Richard own?

b The company declares an annual profit of \$780 000. How much profit is Richard entitled to?

Solution

a We need to convert 25 out of 500 into a percentage.

Percentage ownership

$$= \frac{25}{500} \times \frac{100}{1}$$

$$= 5\%$$

b Richard is entitled to 5% of the profit.

$$\text{Profit} = 780\,000 \times \frac{5}{100}$$

$$= \$39\,000$$

Investors are not only interested in the amount of profit they are entitled to, they also want to interpret this profit in light of the amount that they have invested in shares. One measure of this is the **price-to-earnings ratio** of the shares.

ISBN 978-1-009-25891-3

Price-to-earnings ratio

$$\text{Price-to-earnings ratio} = \frac{\text{Market price per share}}{\text{Annual earnings per share}}$$

The market price per share is the value that the share currently trades for on the stock market. It is the amount of money paid by investors to buy a single share in a company. The market price is also referred to as just the *share price*.

Earnings per share is a figure describing the company's profit per share.

The lower the price-to-earnings ratio the less you are investing for each dollar of profit, which is better for the investor.

Example 29 Price-to-earnings ratio

Suppose shares in Company A have a market value of \$20, and an annual earnings per share of \$1.85 while shares in Company B have a market value of \$50, an annual earnings per share of \$3.30.

a What is the price-to-earnings ratio for each company? Give answers correct to one decimal place.

b Which shares are a better investment?

Solution

a Substitute in the formula above.

A: price-to-earnings ratio

$$= \frac{20}{1.85} = 10.8$$

B: price-to-earnings ratio

$$= \frac{50}{3.30}$$

$$= 15.15\ldots = 15.2 \text{ to one d.p.}$$

b Compare the ratios.

Company A: price-to-earnings ratio lower.

In practice, companies do not share all of their profits (or earnings) with shareholders, but they do pay **dividends**. Dividends can be specified in one of two ways:

- as the number of dollars each share receives
- as a percentage of the current price of the shares, called the dividend yield.

Percentage dividend yield

$$\text{Dividend yield} = \frac{\text{dividend per share}}{\text{current share price}} \times \frac{100}{1}\%$$

ISBN 978-1-009-25891-3

Example 30 Dividends

Miller has 3000 shares in Alphabet Childcare Centres. The current market price of the shares is \$3.50 each and the company has recently paid a dividend of 40 cents per share to each shareholder.

a How much does Miller receive in dividends in total?

b What is the percentage dividend yield for this share? Give answer correct to one decimal place.

Solution

a Total dividend = number of shares × dividend per share

$$\text{Total dividend} = 3000 \times 0.40 = \$1200$$

b Use the percentage dividend rule by substituting \$0.40 for the share dividend and \$3.50 for the share price.

$$\text{Dividend yield} = \frac{0.40}{3.50} \times \frac{100}{1}\% = 11.4\% \text{ to one d.p.}$$

Exercise 2I

Shares and dividends

Example 28

1 Nick owns 500 of the 100 000 shares available in the Lucky Insurance Company.

a What percentage of the company does Nick own?

b If the company declares an annual profit of \$2 500 000, how much profit is Nick entitled to?

Example 29

2 Suppose shares in Company A have a market value of \$42.50, and the company makes an annual earnings of \$4.85 per share, while shares in Company B have a market value of \$8, and they make an annual earnings per share of \$0.80.

a What is the price-to-earnings ratio for each company, correct to one decimal place?

b Which shares are a better investment?

3 Michael has \$5000 to invest in shares. He has decided to invest in either the Alpha Oil Company or Omega Mining.

a The price-to-earnings ratio for the Alpha Oil Company is 10. If the share price is \$5.00, what is the annual earnings per share?

b The price-to-earnings ratio for Omega Mining is also 10. If the share price is \$10.00, what is the annual earnings per share?

ISBN 978-1-009-25891-3

c Michael decides to spend his money equally between the two share investments. How many shares in each company does he buy?

d Suppose that, in the next 12 months, the share price of:

- Alpha Oil is expected to increase by 10% while the price-to-earnings ratio is expected to remain at 10
- Omega Mining is expected to increase by 8% while the price-to-earnings ratio is expected to reduce to 8.

What is the expected gain to Michael from these changes?

Example 30

4 Suppose Taj has 500 shares in Bunyip Plumbing Supplies. If the current market price of the shares is $4.60 each, and the company declares a dividend of 50 cents per share.

a How much does Taj receive in dividends in total?

b What is the percentage dividend for this share correct to one decimal place?

5 Calculate the total dividend paid on the portfolio of shares given the dividend paid for each share.

Company	Number of shares	Price per share	Dividend per share
AB Co Ltd	2000	$1.20	$0.15
DNDS Inc	450	$4.60	$0.60
Golden Miners	6000	$36.55	$2.40

6 You have been assigned to evaluate the following three companies' stocks within the same industry:

Company	Price per share	Earnings per share
Cybertrons Corporation	$52.40	$12.50
Decepticons Ltd	$8.50	$2.25
Optimus Prime Inc	$9.88	$5.25

Which stock has the best value? Show workings.

ISBN 978-1-009-25891-3

2J Financial investigation: buying a car

Exercise 2J

One of the first big purchases you are likely to make is buying your first car. How much can you afford to pay for that car, and how will you go about financing the purchase?

In this investigation you are going to use available resources to determine the best strategy. You will need to investigate each of the following themes.

1 What can you afford?
Assuming that you will need to finance the car, what can you afford to repay each week or fortnight? You will need to consider your likely salary, as well as your other living costs. Some of the major banks will give advice regarding this amount and include 'affordability' calculators on their websites.

2 How should you finance the car?
Compare some different forms of finance (such as variable interest personal loans, fixed interest personal loans, credit cards), some different financial institutions, as well as some of the financing options offered directly by the car dealerships to determine your best finance option.

3 What car should you buy?
Cars often depreciate in value very quickly, especially if they are purchased new. In the worst-case scenario, you can end up owing more money on a car than its current market value! Compare the depreciation of two or three different brands of car when purchased new, and at various stages over the period of time for which you have decided to finance it. How much will the car be worth when it is finally paid for?

2K Using spreadsheets

The objective of using spreadsheets is to learn how to use formulae to perform calculations and understand how we use cell addresses within a formula.

There are some important rules governing spreadsheets which we must apply when solving problems.

- All formulae start with an = sign to identify them as a formula.
- We use the asterix (*) for multiplication
- For division make use of the forward slash (/)

ISBN 978-1-009-25891-3

Calculating values of formula using a spreadsheet

Use the formula $y = mx + c$ and a spreadsheet to calculate values of y for different sets of values of m, x and c.

Steps

1. Enter values of m, x and c for calculating y into a spreadsheet.
2. To calculate y in the second row enter **= A2*B2 + C2** in cell D2 as shown and the answer will appear as 14.
3. Similarly in cell D3 enter = A3*B3 + C3 and the value of 22 will automatically come up in that cell.
4. To fill the rest of the spreadsheet we can just drag cell D3 down and the other cells D4, D5, etc. will be worked out automatically.

	A	B	C	D
1	m	x	c	y
2	2	5	4	= A2*B2 + C2 **14**
3	3	4	10	= A3*B3 + C3 **22**
4	5	3	−7	**8**
5	10	−2	−5	**−25**
6	8	4	6	**38**

Tracking sales with a spreadsheet

The spreadsheet below shows the T-shirt sales of Top Clothing Ltd over a week for different size shirts. Use a spreadsheet to find the total number of shirts sold of each size over the week.

	A	B	C	D	E	F
1		Monday	Tuesday	Wednesday	Thursday	Friday
2	X Small	12	15	10	21	18
3	Small	10	11	8	13	12
4	Medium	25	21	33	41	38
5	Large	8	7	5	10	6

Steps

1. Insert a total column in column G of the spreadsheet
2. Since we want to find the total shirts sold for each size use the sum function on each row

ISBN 978-1-009-25891-3

3 To calculate the total for the second row enter **= (sum B2 : F2)** in cell G2 as shown and the answer 76 will be displayed.

4 The rest of column G can then be filled by dragging down on cell G2, giving the answers shown.

	A	B	C	D	E	F	G
1		Monday	Tuesday	Wednesday	Thursday	Friday	Total
2	X Small	12	15	10	21	18	=(sum B2 : F2)76
3	Small	10	11	8	13	12	54
4	Medium	25	21	33	41	38	158
5	Large	8	7	5	10	6	36

Example 31

A cleaning firm uses the table below to deliver quotations to consumers. Jobs ranging from one to five hours in length and requiring one to four cleaners are priced in dollars.

Price ($)	Number of hours (n)				
Cleaners (c)	1	2	3	4	5
1	70	100	130	160	190
2	100	160	220	280	340
3	130	220	P	400	490
4	160	280	400	520	640

a Use examples to demonstrate that only one of the following formulae will accurately create all the job costs in the preceding table.

$A = 30 + 20(n + c)$

$B = 40 + 30nc$

b Hence find the value of P.

Solution

a **1** Choose 1 hour and 1 cleaner

$A = 30 + 20(1 + 1) = \$70$(correct)

$B = 40 + 30(1)(1) = \$70$(correct)

2 Choose 2 hours and 3 cleaners

$A = 30 + 20(2 + 3) = \$130$(incorrect)

$B = 40 + 30(2)(3) = \$220$(correct)

$B = 40 + 30nc$ is the correct formula.

b To find P replace $n = 3$ and $c = 3$

$\therefore P = 40 + 30(3)(3) = \$310$

ISBN 978-1-009-25891-3

Exercise 2K

CAS

1 Use a spreadsheet from your calculator to complete the following.

	A	B	C	D
1	Quantity sold	Price	Sales	GST
2	8	5	= A2*B2 **40**	= 0.1*C2 **4**
3	10	9		
4	15	4		
5	7	20		
6	22	3		

2 The spreadsheet below shows the marks of a group of students studying Application Mathematics in four different tests. The tests were marked out of 10 marks each. Use the appropriate formulae in your spreadsheet to complete columns F and G.

	A	B	C	D	E	F	G
1		Test 1	Test 2	Test 3	Test 4	Total Marks	%
2	Alice	9	7	5	8		
3	Berry	5	4	6	4		
4	Carol	9	9	8	9		
5	Damien	10	8	8	5		
6	Essen	2	4	5	1		

3 The area of a trapezium is $A=\dfrac{a+b}{2}\times h$. Use the spreadsheet below and your calculator to complete the table for different sets of values of a, b and h.

	A	B	C	D
1	a	b	h	Area
2	8	6	10	
3	11	19	5	
4	15	17	8	
5	7	23	20	
6	19	21	8	

ISBN 978-1-009-25891-3

4 The table below gives the payroll of five employees at Yummy Fish and Chips over a period of one fortnight.

	A	B	C	D	E	F	G	H	I
1	Name	Pay/ hour ($)	Total hours worked	Overtime /hour ($)	Total over-time hours	Gross pay ($)	Income tax ($)	Other deduc-tions	Net pay ($)
2	Airi M.	30	60	40	4	1960	294	150	1516
3	Daiki W.	35	50	45	8	2110	316.50	-	1793.50
4	Carlos S.	30	45	40	6	a	238.50	240	1111.50
5	Dan F.	35	48	45	10	2130	319.50	180	c
6	Xia K.	40	54	50	2	2260	b	-	1921

a Calculate the value of *a*.

b Given that the income tax rate is fixed, determine this rate as a percentage. Hence calculate the value of *b*.

c Calculate the net pay for Dan F.

5 Mao owns a BMW X3 and the spreadsheet below shows the Vehicle Maintenance Logbook for the first quarter of 2021.

Maintenance log						
Date	Car issues	Workshop	Estimates	Spare cost	Labour cost	Total cost
03-01-2021	Dents and paints	K-Mart	$400	$125	**A**	**B**
14-02-2021	Filter change	My Car	**C**	$120	$90	$210
02-03-2021	Gear problem	Auto Service	$520	$480	$120	$600
31-03-2021	Free Service	BMW Centre	$0.00	$125	$0.00	$125

a Given that the total cost on 03-01-2021 was 20% higher than the estimated cost, determine the values of A and B.

b If the total cost on 14-02-2021 was 5% higher than the estimated cost, determine the value of C.

c Calculate the percentage that Gear repair accounts for the whole quarter's bill.

ISBN 978-1-009-25891-3

Key ideas and chapter summary

Review

Assign-ment

Percentage increase or decrease	**Percentage increase or decrease** is the amount of the increase or decrease of value or quantity expressed as a percentage of the original value or quantity.
Simple interest	**Simple interest** is paid on an investment or loan on the basis of the original amount invested or borrowed called the principal (P). The amount of simple interest is constant from year to year.
GST	**GST** (goods and services tax) is a 10% tax that is added to most purchases.
Shares	A **share** is a unit of ownership of a company.
Price-to-earnings ratio	**Price-to-earnings ratio** is a measure of the profit of a company, given by the market price per share/annual earnings per share. A lower value of the price-to-earnings ratio may indicate a better investment.
Dividend yield	A **dividend** paid by company expressed as a percentage of the share price.
Minimum monthly balance	The lowest amount an account contains in each calendar month is its **minimum monthly balance**
Compound interest	Under **compound interest**, the interest paid on a loan or investment is credited or debited to the account at the end of each period. The interest earned for the next period is based on the principal plus previous interest earned. The amount of interest earned increases each year.
Hire-purchase agreement	Under a **hire-purchase agreement**, the purchaser hires an item from the vendor and makes periodic payments at an agreed rate of interest. At the end of a hire-purchase agreement, the item belongs to the purchaser.
Flat rate of interest	The **flat rate of interest** (r_f) is given by $r_f = \dfrac{100I}{Pt}$ where P is the principal owing after any deposit has been deducted, I is the interest and t is the duration in years.
Purchasing power	**Purchasing power** describes what you can actually buy with your money.

ISBN 978-1-009-25891-3

Credit — **Credit** is an advance of money from a financial institution, such as a bank, that does not have to be paid back immediately but which attracts interest after an interest-free period.

Personal loan — A **personal loan** is a sum of money borrowed from a financial institution such as a bank for buying an item for personal use. Regular repayments are required to repay the loan.

Inflation — **Inflation** is the continuous upward movement of the economy that increases prices over time or, conversely, decreases the spending power of money over time.

Skills check

Having completed this chapter you should be able to:

- calculate the amount of the discount and the new price when an $r\%$ discount is applied
- calculate the amount of the increase and the new price when an $r\%$ increase is applied
- calculate the percentage discount or increase given the old and new prices
- calculate the original price given the new price and the percentage discount or increase
- determine the new price of a good or service after the GST has been added
- determine the original cost of a good or service given the inclusion of GST
- determine the price-to-earnings ratio of shares
- determine dividend yield of shares
- use the formula for simple interest to find the value of any one of the variables I, P, r or t when the values of the other three are known
- determine the interest payable on a bank account, paid on the minimum monthly balance or daily balance
- calculate the amount of an investment after simple interest has been added
- plot the value of simple interest (I) against time (t) to show a linear relationship
- use the formula for compound interest to solve problems involving investments and loans
- plot the value of compound interest (I) against time (t) to show a non-linear relationship
- determine the annual flat rate of interest for a hire-purchase agreement or a personal loan
- determine the interest payable when an item is purchased using a credit card
- determine the new price of an item after a period of inflation
- determine the effect of inflation on the purchasing power of money
- calculate income and prepare a personal budget.

ISBN 978-1-009-25891-3

Short-answer questions

1 State the amount saved if a 10% discount is offered on an item marked \$120.

2 If a 20% discount is offered on an item marked \$30, find the new discounted price of the item.

3 If a 15% increase is applied to an item marked \$60, determine the new price of the item.

4 If GST is applied to an item marked \$121.50, determine the new price of the item.

5 The telephone bill including GST is \$318.97. Calculate the price before GST.

6 Shares in Company A have a market value of \$22.50. If the company makes an annual earnings of \$2.85 per share, calculate the price-to-earnings ratio.

7 How much interest is earned if \$2000 is invested for 1 year at a simple interest rate of 4% per annum?

8 Determine the total value of an investment of \$1000 after 3 years if simple interest is paid at the rate of 5.5% per annum.

9 What is the interest rate, per annum, if a deposit of \$1500 earns interest of \$50 over a period of 6 months?

10 \$2400 is invested at a rate of 4.25% compound interest, paid annually. Calculate the value of this investment after 6 years.

11 \$3600 is invested at a rate of 5% p.a. compounding annually. Calculate the value of the investment after 4 years.

12 Calculate the value when \$8500 compounding annually for 5 years at 6% p.a.

13 Sue has a credit card debt of \$3000. She has 35 interest-free days left, but she will not be able to pay the amount for 90 days. If the interest rate is 18.9% per annum, calculate, to the nearest dollar, the amount she will need to repay.

14 The price of a newspaper is \$2 today. If the price increases with inflation, what will be the price of the newspaper in 10 years time if the average annual inflation rate is 1.8%?

ISBN 978-1-009-25891-3

The following information relates to Questions 15 to 17

Janet buys a car costing $23 000. She pays a $5000 deposit and then makes payments of $440 per month for the next 5 years.

15 How many payments does Janet make under this arrangement?

16 How much interest does Janet pay under this arrangement?

17 What is the annual flat rate of interest?

The following information relates to Questions 18 to 20

To renovate her kitchen Annie take out a personal loan of $20 000 from the bank, for which she is required to make monthly repayments of $685 for 3 years.

18 Calculate the total cost of paying off the loan.

19 Calculate the total interest paid.

20 Calculate the flat rate of interest for this loan, correct to one decimal place.

21 Rabbit Easter Eggs were reduced from $2.99 to $2.37 because they were not selling quickly. Bilby Easter Eggs were discounted from $4.79 to $3.83.

a Which type of Easter egg had the larger percentage reduction?

b Calculate the difference in the percentage rates.

22 After Christmas, all stock in JDs was discounted by 20%. The sale price of a pair of cross-trainers was $110. Calculate the original marked price.

23 If the selling price of a computer is $1990:

a what is the price without GST? **b** how much of this is GST?

24 How much additional interest is earned if $8000 is invested for 7 years at 6.5% when interest is compounded annually, as compared with simple interest paid at the same rate?

25 Zara buys a leather jacket costing $450 on her credit card. She buys it on the last day of her statement period, so she has the minimum number of interest free days, which is 25. After that time, the bank charges interest at a rate of 22.6% per annum compounding daily. How much will she owe on her credit card:

a in 30 days? **b** in 90 days?

26 A television set, which normally costs $880, can be bought through hire purchase with a $200 deposit and a payment of $30 a month for 30 months. Calculate:

a the amount of interest being charged

b the flat rate of interest.

ISBN 978-1-009-25891-3

Extended-response questions

1 **a** The wholesale price of a digital camera is \$350. The maximum profit that a retailer is allowed to make when selling this particular camera is 75% of the wholesale price. Calculate the maximum retail price of the camera.

b Suppose that the wholesale price of the camera increases at 5% per annum simple interest for the next 5 years.

- **i** What is the new wholesale price of the camera?
- **ii** By how much will the wholesale price have increased at the end of 5 years?
- **iii** What is the new retail price of the camera (with 75% profit)?
- **iv** What percentage increase is this in the retail price determined in part **a**?

2 CAS Suppose that you have \$30 000 to invest, and there are two alternative plans for investment:

Plan A offers 5.3% per annum simple interest.

Plan B offers 5.0% per annum compound interest, compounding annually.

a Use your CAS calculator to construct a graph of the interest earned under Plan A against time.

b On the same axes, use your CAS calculator to construct a graph of the interest earned under Plan B against time.

c Which of the plans would you choose, A or B, if the investment is for:

i 3 years? **ii** 6 years?

3 The Smiths bought a new car priced at \$34 800 and paid a deposit of \$5000 cash. They borrowed the balance of the purchase price. They then agreed to repay the loan plus interest in equal monthly payments of \$850 over 4 years.

a Calculate the total amount of interest to be repaid over the term of this loan.

b Calculate, correct to one decimal place, the annual simple interest rate charged on this loan.

4 The phone that Ebba wants to buy usually costs \$400 but is on sale for \$350.

a What percentage discount does this amount to?

b Ebba considers entering into a hire-purchase agreement to buy the phone where she pays no deposit and 24 monthly payments of \$22.50.

- **i** How much interest would Ebba pay under this agreement on the purchase price of \$350?
- **ii** What is the annual flat rate of interest that this represents? Express your answer as a percentage correct to one decimal place.

 ISBN 978-1-009-25891-3

c Another option available to Ebba is to use her credit card, which attracts an interest rate of 20% per annum compounding daily. How much interest would Ebba pay on the purchase price of $350 if she makes no payments for 2 years, assuming that she has 60 interest free days?

d Would you recommend her to purchase the phone using the hire-purchase agreement or using her credit card?

5 The rate of pay for an actor filming an advertising commercial is shown in the table below:

TV/Digital commercials	Rate of pay per hour
Performers	$42.76
Extras	$35.24

Both performers and extras must have a minimum call time of 4 hours. Actors in this industry are paid overtime at a rate of time-and-a-half for the first two hours after 4 hours and double time thereafter. All work on Sundays is paid at double time and all work on public holidays at double time-and-a-half.

a Simon is an actor cast to perform in a commercial for *Cave Motors*. His call time is 8:00 a.m. on a Monday and he is scheduled to finish at 12:00 p.m. Calculate Simon's total pay for this day's work.

b The commercial filming takes longer than expected and Simon ends his day at 4:30 p.m., after having a half an hour break for lunch between 12:30 and 1 p.m. Determine Simon's new total pay, taking into account his overtime.

c Marek is an extra who has been given the following call sheet for his background appearance in a commercial for a new fast food restaurant. The commercial is due to film across a long-weekend. Calculate Marek's total pay using the times given on the call sheet below.

Bullet feed Commercial film schedule				
Day	Call time	Duration	Description	Cast call
Saturday	0930	4 hours	Interior restaurant	Extras
Sunday	1400	4 hours	Interior restaurant	Extras
Monday (Public holiday)	1100	4 hours	Exterior car park	Extras

ISBN 978-1-009-25891-3

3 Shape and measurement

In this chapter

Syllabus references

Topics: Pythagoras' theorem; Mensuration; Similar figures and scale factors

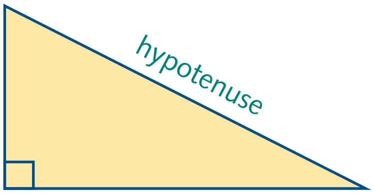
Subtopics: 1.3.1 – 1.3.8

This geometry chapter covers perimeter and area of 2D shapes, and surface area and volume of 3D solids. It also covers similarity within 2D shapes and 3D solids.

ISBN 978-1-009-25891-3

3A Pythagoras' theorem

Pythagoras' theorem is a relationship connecting the side lengths of a right-angled triangle. In a right-angled triangle, the side opposite the **right angle** is called the **hypotenuse**. The hypotenuse is always the longest side of a right-angled triangle.

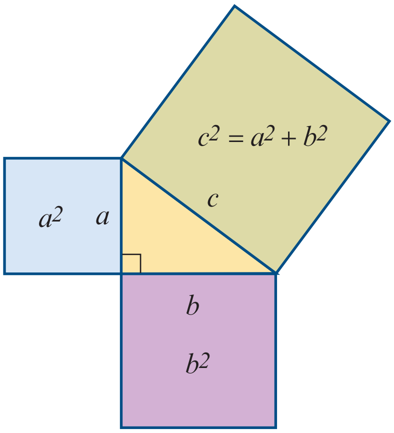


Pythagoras' theorem

Pythagoras' theorem states that, for any right-angled triangle, the sum of the areas of the squares of the two shorter sides (a and b) equals the area of the square of the hypotenuse (c).

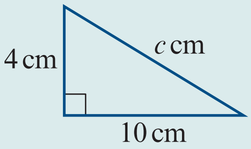
$$c^2 = a^2 + b^2$$

Pythagoras' theorem can be used to find the length of the hypotenuse in a right-angled triangle.



Example 1 Using Pythagoras' theorem to calculate the length of the hypotenuse

Calculate the length of the hypotenuse in the triangle opposite, correct to two decimal places.



Solution

1 Write Pythagoras' theorem. $c^2 = a^2 + b^2$

2 Substitute known values. $c^2 = 10^2 + 4^2$

3 Take the square root of both sides, then evaluate.
$c = \sqrt{10^2 + 4^2}$
$= 10.770\ldots$

4 Write your answer correct to two decimal places, with correct units.
The length of the hypotenuse is 10.77 cm, correct to two decimal places.

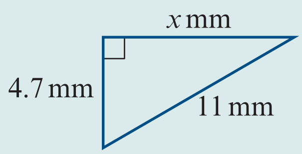
Hint: To ensure that you get a decimal answer, set your calculator to approximate or decimal mode. (See the Appendix.)

ISBN 978-1-009-25891-3

Pythagoras' theorem can also be rearranged to find sides other than the hypotenuse.

Example 2 **Using Pythagoras' theorem to calculate the length of an unknown side in a right-angled triangle**

Calculate the length of the unknown side, x, in the triangle opposite, correct to one decimal place.



Solution

1 Write Pythagoras' theorem. $a^2 + b^2 = c^2$

2 Substitute known values and the given variable. $x^2 + 4.7^2 = 11^2$

3 Rearrange the formula to make x the subject, then evaluate.

$$x = \sqrt{11^2 - 4.7^2}$$
$$= 9.945\ldots$$

4 Write your answer correct to one decimal place, with correct units. The length of x is 9.9 mm, correct to one decimal place.

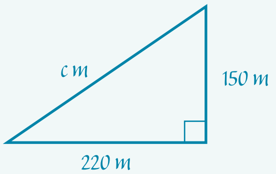
Pythagoras' theorem can be used to solve many practical problems.

Example 3 **Using Pythagoras' theorem to solve a practical problem**

A helicopter hovers at a height of 150 m above the ground and is a horizontal distance of 220 m from a landing pad. Find the direct distance of the helicopter from the landing pad, correct to two decimal places.

Solution

1 Draw a diagram to show which distance is to be found.



2 Write Pythagoras' theorem. $c^2 = a^2 + b^2$

3 Substitute known values. $c^2 = 150^2 + 220^2$

4 Take the square root of both sides, then evaluate.

$$c = \sqrt{150^2 + 220^2}$$
$$= 266.270\ldots$$

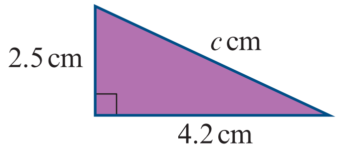
5 Write your answer correct to two decimal places, with correct units. The helicopter is 266.27 m from the landing pad, correct to two decimal places.

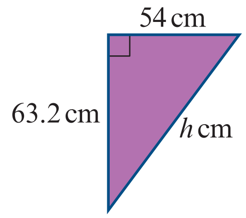
ISBN 978-1-009-25891-3

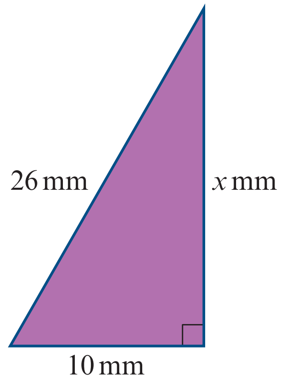
Exercise 3A

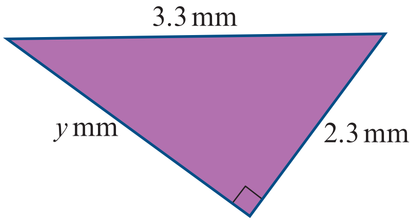
Example 1
Example 2

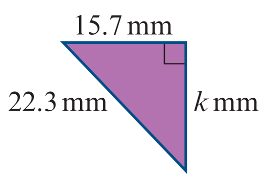
1 Find the length of the unknown side in each of these triangles, correct to one decimal place.

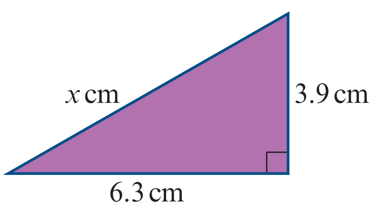
a


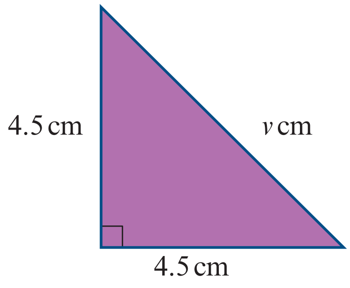
b


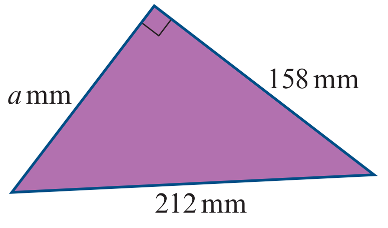
c


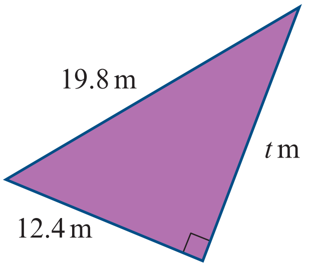
d


e


f


g


h


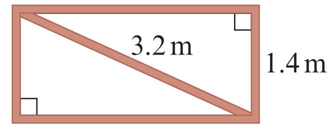
i


ISBN 978-1-009-25891-3

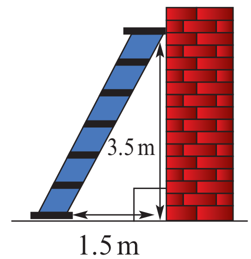
Applications of Pythagoras' theorem

Example 3

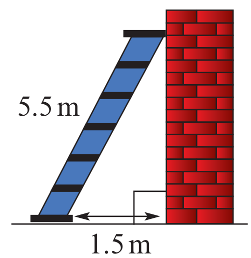
2 A farm gate that is 1.4 m high is supported by a diagonal bar of length 3.2 m. Find the width of the gate, correct to one decimal place.



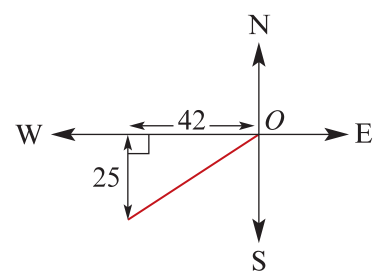
3 A ladder rests against a brick wall as shown in the diagram on the right. The base of the ladder is 1.5 m from the wall, and the top reaches 3.5 m up the wall. Find the length of the ladder, correct to one decimal place.



4 The base of a ladder leaning against a wall is 1.5 m from the base of the wall. If the ladder is 5.5 m long, find how high the top of the ladder is from the ground, correct to one decimal place.



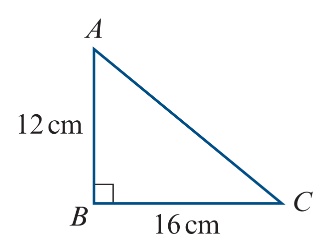
5 A ship sails 42 km due west and then 25 km due south. How far is the ship from its starting point? (Answer correct to two decimal places.)



6 A yacht sails 12 km due east and then 9 km due north. How far is it from its starting point?

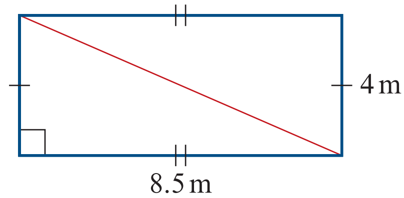
7 A hiker walks 10 km due west and then 8 km due north. How far is she from her starting point? (Answer correct to two decimal places.)

8 In a triangle ABC, there is a right angle at B. AB is 12 cm and BC is 16 cm. Find the length of AC.



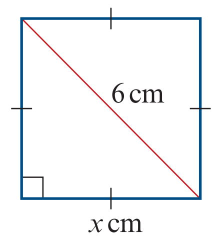
ISBN 978-1-009-25891-3

9 Find, correct to one decimal place, the length of the diagonal of a rectangle with dimensions 8.5 m by 4 m.



10 A rectangular block of land measures 28 m by 55 m. John wants to put a fence along the diagonal. How long will the fence be? (Answer correct to three decimal places.)

11 A square has diagonals of length 6 cm. Find the length of its sides, correct to two decimal places.



12 A flying fox on a school camp starts from a tower 25 m high and lands on the ground 100 metres away. What is the distance a student would travel going down the flying fox?

3B Pythagoras' theorem in three dimensions

When solving three-dimensional problems, it is essential to draw careful diagrams. In general, to find lengths in solid figures, we must first identify the correct right-angled triangle in the plane containing the unknown side. Remember, a plane is a flat surface, such as the cover of a book or a tabletop.

Once it has been identified, the right-angled triangle should be drawn separately from the solid figure, displaying as much information as possible.

ISBN 978-1-009-25891-3

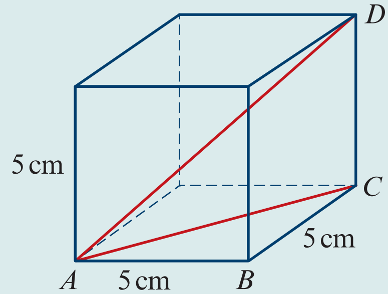
Example 4 Using Pythagoras' theorem in three dimensions

The cube in the diagram on the right has sides of length 5 cm.

Find the length:

a AC, correct to two decimal places

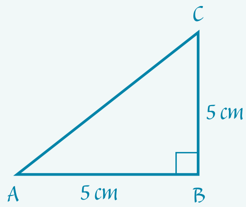
b AD, correct to one decimal place.



Solution

a **1** Locate the relevant right-angled triangle in the diagram.

2 Draw the right-angled triangle ABC that contains AC, and then mark in the known side lengths.



3 Using Pythagoras' theorem, calculate the length AC.

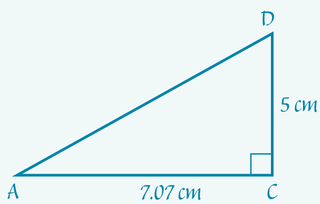
$$AC^2 = AB^2 + BC^2$$
$$\therefore AC = \sqrt{5^2 + 5^2}$$
$$= 7.071\ldots$$

4 Write your answer with correct units and correct to two decimal places.

The length AC is 7.07 cm, correct to two decimal places.

b **1** Locate the relevant right-angled triangle in the diagram.

2 Draw the right-angled triangle ACD that contains AD and mark in the known side lengths. (From part **a**, $AC = 7.07$ cm, correct to two decimal places.)



3 Using Pythagoras' theorem, calculate the length AD.

$$AD^2 = AC^2 + CD^2$$
$$\therefore AD = \sqrt{7.07^2 + 5^2}$$
$$= 8.659\ldots$$

4 Write your answer with correct units and correct to one decimal place.

The length AD is 8.7 cm, correct to one decimal place.

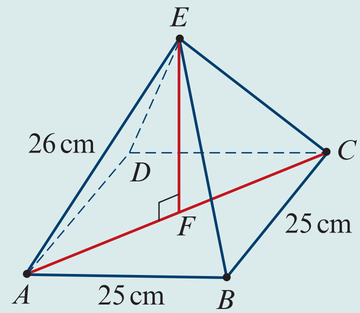
ISBN 978-1-009-25891-3

Example 5 Using Pythagoras' theorem in three-dimensional problems

For the square pyramid shown in the diagram, calculate:

a the length AC, correct to two decimal places

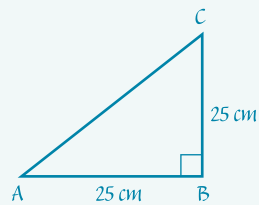
b the height EF, correct to one decimal place.



Solution

a **1** Locate the relevant right-angled triangle in the diagram.

2 Draw the right-angled triangle ABC that contains AC, and mark in known side lengths.



3 Using Pythagoras' theorem, calculate the length AC.

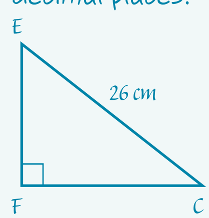
$$AC^2 = AB^2 + BC^2$$
$$\therefore AC = \sqrt{25^2 + 25^2}$$
$$= 35.355\ldots$$

4 Write your answer with correct units and correct to two decimal places.

The length AC is 35.36 cm, correct to two decimal places.

b **1** Locate the relevant right-angled triangle in the diagram.

2 Draw the right-angled triangle EFC that contains EF, and mark in known side lengths.



3 Find FC, which is half of AC. Use the value of AC calculated in part **a**.

$$FC = \frac{AC}{2}$$
$$= \frac{35.36}{2}$$
$$= 17.68 \text{ cm, correct to two decimal places}$$

4 Using Pythagoras' theorem, find EF.

$$EF^2 = EC^2 - FC^2$$
$$\therefore EF = \sqrt{26^2 - 17.68^2}$$
$$= 19.065\ldots$$

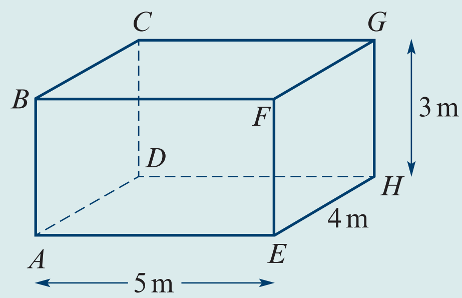
5 Write your answer with correct units and correct to one decimal place.

The height EF is 19.1 cm, correct to one decimal place.

ISBN 978-1-009-25891-3

Example 6 Using Pythagoras' theorem in practical three-dimensional problems

A new home entertainment system needs to be set up in a room with dimensions $4\text{ m} \times 5\text{ m} \times 3\text{ m}$ as shown in the diagram. Expensive cabling is used to wire the room from corner A to corner G.



a What length of cabling is required to go from A to E to H to G?

b What length of cabling is required to go from A to F to G, correct to two decimal places?

c If cabling costs \$9.10 per metre, which is the cheaper option – A to F to G or A to E to G?

Solution

a **1** Add the distances from A to E (5 m), E to H (4 m) and H to G (3 m).

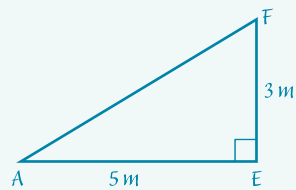
$5 + 4 + 3 = 12$

2 Write your answer with correct units.

The distance is 12 metres.

b **1** First find out the distance from A to F, by locating the relevant right-angled triangle in the diagram.

2 Draw the right-angled triangle AFE that contains AF, and then mark in the known side lengths.



3 Using Pythagoras' theorem, calculate the length AF.

$AF^2 = AE^2 + EF^2$

$AF^2 = \sqrt{5^2 + 3^2}$

$AF^2 = 5.8309\ldots$

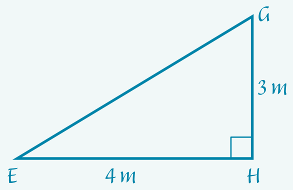
4 Add the lengths AF and FG and give your final answer with correct units and correct to two decimal places.

The total length (A-F-G)

$= 5.83 + 4$

$= 9.83$ m, correct to two decimal places

ISBN 978-1-009-25891-3

c **1**	Work out the cost of cabling from A to F to G by multiplying the length of cabling needed from A to F to G (9.83 m) by the cost of cabling per metre ($9.10).	$9.83 \times 9.10 = 89.453$ Cost of cabling from A to F to G is $89.45.
2	Work out the distance of cable needed to go from A to E to G. The distance from A to E is 5 m. Calculate the distance from E to G by first locating the relevant right-angled triangle in the diagram.	$AE = 5$ m
3	Draw the right-angled triangle EGH that contains EG and mark in known side lengths.	
4	Using Pythagoras' theorem, calculate the length EG.	$EG^2 = EH^2 + HG^2$ $EG = \sqrt{4^2 + 3^2}$ $EG = 5$
5	Add the lengths AE and EG to give total distance from A to E to G.	$A\text{-}E\text{-}G = 5 + 5$ $= 10$ m
6	Work out the cost of cabling from A to E to G by multiplying the length of cabling needed (10 m) by the cost of cabling per metre ($9.10).	$10 \times 9.10 = 91$ Cost of cabling from A to E to G is $91.00.
7	Compare the cost of cabling from A to F to G to the cost of cabling from A to E to G and decide which is the cheaper option.	Cost for A-F-G = $89.45 Cost for A-E-G = $91.00 Thus the cheaper option is to wire cabling from A to F to G.

ISBN 978-1-009-25891-3

Exercise 3B

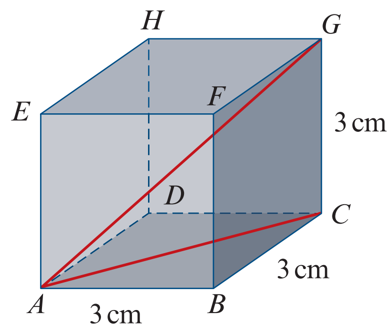
Pythagoras' theorem in three dimensions

Example 4

1 The cube shown in the diagram has sides of 3 cm. Find the length of:

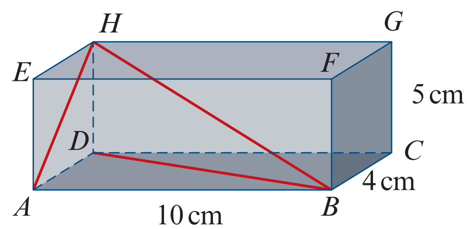
a AC, correct to three decimal places

b AG, correct to two decimal places.



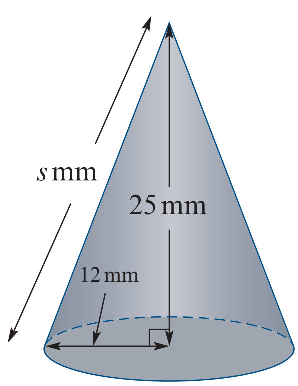
2 For this cuboid, calculate, correct to two decimal places, the length:

a DB **b** BH **c** AH

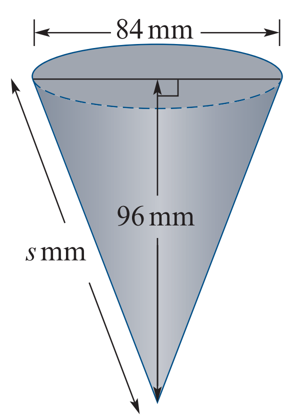


3 Find the sloping height, s, of each of the following cones, correct to two decimal places.

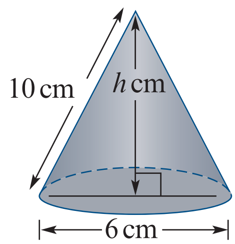
a



b



4 The slant height of this circular cone is 10 cm and the diameter of its base is 6 cm. Calculate the height of the cone, correct to two decimal places.



ISBN 978-1-009-25891-3

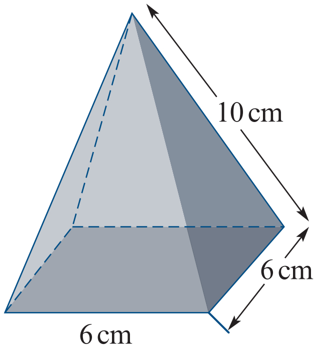
Example 5

5 For each of the following square-based pyramids find, correct to one decimal place:

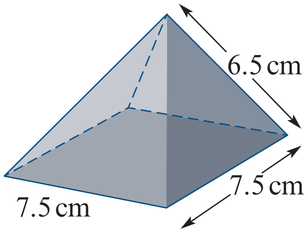
i the length of the diagonal on the base

ii the height of the pyramid.

a



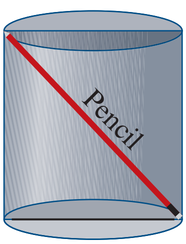
b



Applications of Pythagoras' theorem in three dimensions

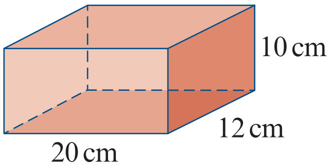
Example 6

6 Find the length of the longest pencil that will fit inside a cylinder with height 15 cm and with circular end surface 8 cm in diameter.



7 Sarah wants to put her pencils in a cylindrical pencil case. What is the length of the longest pencil that would fit inside a cylinder of height 12 cm with a base diameter of 5 cm?

8 Chris wants to use a rectangular pencil box. What is the length of the longest pencil that would fit inside the box shown on the right? (Answer to the nearest centimetre.)



9 A broom is 145 cm long. Would it be able to fit in a cupboard measuring 45 cm by 50 cm and height 140 cm?

10 In order to check the accuracy of the framework and that a room is 'square' a builder often measures the length of the opposing diagonals. What is the distance, correct to two decimal places, from the bottom corner of a room to the top corner diagonally opposite if the room measures 6 m by 4 m by 3.5 m?

11 In the primate enclosure at the zoo, a rope is to be attached from the bottom corner of the enclosure to the opposite top corner for the monkeys to swing and climb on. If the enclosure measures 8 m by 10 m by 12 m, what is the length of the rope? Give your answer correct to two decimal places.

ISBN 978-1-009-25891-3

3C Mensuration: perimeter and area

Mensuration is a part of mathematics that looks at the measurement of length, area and volume. It comes from the Latin word *mensura*, which means ‘measure’.

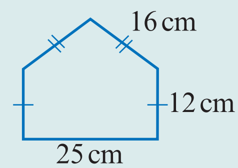
Perimeters of regular shapes

Perimeter

The **perimeter** of a two-dimensional shape is the total distance around its edge.

Example 7 Finding the perimeter of a shape

Find the perimeter of the shape shown.



Solution

To find the perimeter, add up all the side lengths of the shape.

$$\text{Perimeter} = 25 + 12 + 12 + 16 + 16$$
$$= 81 \text{ cm}$$

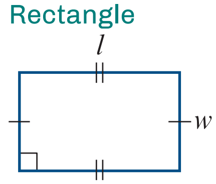
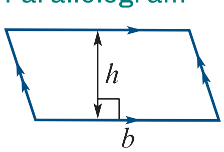
Areas of regular shapes

Area

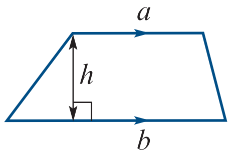
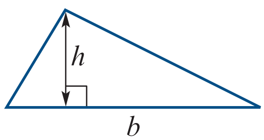
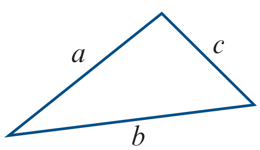
The **area** of a shape is a measure of the region enclosed by its boundaries.

When calculating area, the answer will be in *square units*, i.e. mm^2, cm^2, m^2, km^2.

The **formulas for the areas** of some common shapes are given in the table below, along with the formula for finding the perimeter of a rectangle.

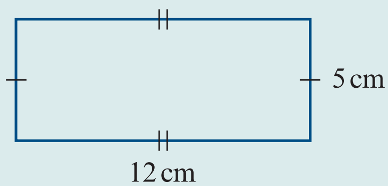
Shape	Area	Perimeter
Rectangle 	$A = lw$	$P = 2l + 2w$ or $P = 2(l + w)$
Parallelogram 	$A = bh$	Sum of four sides

ISBN 978-1-009-25891-3

Shape	Area	Perimeter
Trapezium 	$A = \frac{1}{2}(a+b)h$	Sum of four sides
Triangle 	$A = \frac{1}{2}bh$	Sum of three sides
Heron's formula for finding the area of a triangle with three side lengths known. 	$A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ (s is the semi-perimeter)	$P = a + b + c$

Example 8 **Finding the perimeter of a rectangle**

Find the perimeter of the rectangle shown.



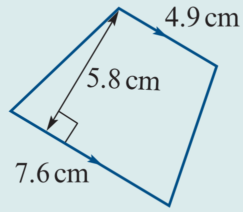
Solution

1. Since the shape is a rectangle, use the formula $P = 2l + 2w$. — $P = 2L + 2W$
2. Substitute length and width values into the formula. — $= 2 \times 12 + 2 \times 5$
3. Evaluate. — $= 34$ cm
4. Give your answer with correct units. — The perimeter of the rectangle is 34 cm.

ISBN 978-1-009-25891-3

Example 9 Finding the area of a shape

Find the area of the given shape.



Solution

1 Since the shape is a trapezium, use the formula $A = \frac{1}{2}(a + b)h$. $A = \frac{1}{2}(a + b)h$

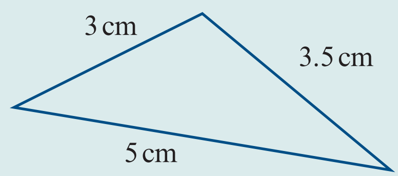
2 Substitute the values for a, b and h. $= \frac{1}{2}(4.9 + 7.6) \times 5.8$

3 Evaluate. $= 36.25\ \text{cm}^2$

4 Give your answer with correct units. The area of the shape is $36.25\ \text{cm}^2$.

Example 10 Finding the area of a triangle using Heron's formula

Find the area of the following triangle. Give your answer correct to two decimal places.



Solution

1 Since the height of the triangle is not given, you need to use Heron's formula as the three side lengths are known.

2 Write down Heron's formula. $A = \sqrt{s(s - a)(s - b)(s - c)}$

3 Find the perimeter of the triangle by adding the three side lengths. $P = 3 + 3.5 + 5$
$= 11.5$

4 Divide the perimeter by 2 to find s. $s = \frac{11.5}{2}$
$= 5.75$

5 Substitute the value for s into Heron's formula to find the area of the triangle. $A = \sqrt{5.75(5.75 - 3)(5.75 - 3.5)(5.75 - 5)}$
$= 5.165\,62\ldots$

6 Give your answer correct to two decimal places and with correct units. The area of the triangle is $5.17\ \text{cm}^2$, correct to two decimal places.

ISBN 978-1-009-25891-3

The formulas for area and perimeter can be applied to many practical situations.

Example 11 **Finding the area and perimeter in a practical problem**

A rectangular display board for a classroom measures 150 cm by 90 cm.

a If ribbon costs \$0.55 per metre, how much will it cost to add a ribbon border around the display board?

b The display board is to be covered with yellow paper. What is the area to be covered? Give you answer in m^2, correct to two decimal places.

Solution

a **1** To find the length of ribbon required, we need to work out the perimeter of the display board. The display board is a rectangle so use the formula $P = 2L + 2w$.	$P = 2L + 2W$
2 Substitute $l = 150$ and $w = 90$ and evaluate.	$= 2(150) + 2(90)$ $= 480$ The length of ribbon required is 480 cm.
3 Convert from centimetres to metres by dividing the length of ribbon by 100.	$= 480 \div 100$ $= 4.8$ m
4 To find the cost of the ribbon, multiply the length of the ribbon by \$0.55.	$4.8 \times 0.55 = 2.64$
5 Evaluate and write your answer.	Cost of ribbon is \$2.64.
b **1** To find the area, use the formula $A = lw$.	$A = LW$
2 Substitute $l = 150$ and $w = 90$ and evaluate.	$= 150 \times 90$ $= 13\,500\ \text{cm}^2$
3 Convert your answer to m^2 by dividing by $(100 \times 100 = 10\,000)$.	$A = 13\,500 \div 10\,000$ $= 1.35$
4 Write your answer with correct units.	Area to be covered with paper is $1.35\ \text{m}^2$.

ISBN 978-1-009-25891-3

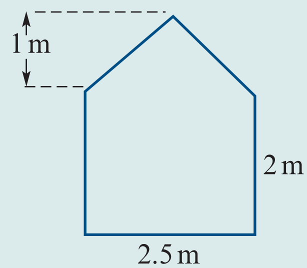
Composite shapes

Composite shape

A **composite shape** is a shape that is made up of two or more basic shapes.

Example 12 Finding the perimeter and area of a composite shape in a practical problem

A gable window at a reception venue is to have LED lights around its perimeter (but not along the bottom of the window). The window is 2.5 m wide and the height of the room is 2 m. The height of the gable is 1 m, as shown in the diagram.

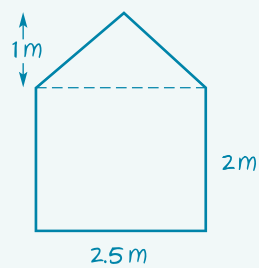


a Calculate the length of LED lights needed, correct to two decimal places.

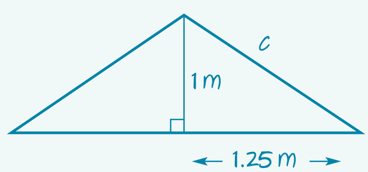
b The glass in the window needs to be replaced. Find the total area of the window, correct to two decimal places.

Solution

a The window is made of two shapes: a rectangle and a triangle.



1 First find the length of the slant edge of the triangle.
Draw a diagram and label the slant edge as c.



Use Pythagoras' theorem to find c.
Note: The length of the base of each triangle is 1.25 m ($\frac{1}{2}$ of 2.5 m).

$c^2 = 1^2 + 1.25^2$

$\therefore c = \sqrt{1^2 + 1.25^2}$

$c = 1.6007\ldots$

$c = 1.60$ m, correct to two decimal places

2 Add all the outside edges of the window but do not include the bottom length.

$2 + 2 + 1.60 + 1.60 = 7.20$

3 Write your answer with correct units.

The length of the LED lights is 7.20 m.

ISBN 978-1-009-25891-3

b **1** To find the total area of the window, first find the area of the rectangle by using the formula $A = bh$.	$A = bh$
2 Substitute the values for b and h.	$= 2.5 \times 2$
3 Evaluate and write your answer with correct units.	$= 5\ \text{m}^2$
4 Find the area of the triangle by using the formula $A = \frac{1}{2}bh$.	$A = \frac{1}{2}bh$
5 Substitute the values for b and h.	$= \frac{1}{2} \times 2.5 \times 1$
6 Evaluate and write your answer with correct units.	$= 1.25\ \text{m}^2$
7 To find the total area of the window, add the area of the rectangle and the area of the triangle.	Total area = area of rectangle + area of triangle $= 5 + 1.25$ $= 6.25\ \text{m}^2$
8 Give your answer with correct units to two decimal places.	Total area of window is $6.25\ \text{m}^2$, correct to two decimal places.

Exercise 3C

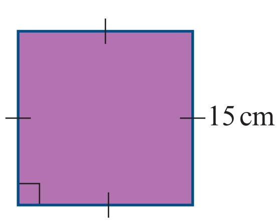
Perimeters and areas

Example 7
Example 8

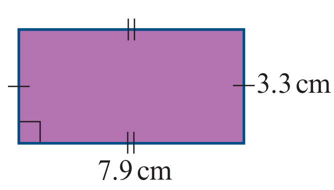
1 For each of the following shapes, find, correct to one decimal place:

i the perimeter **ii** the area

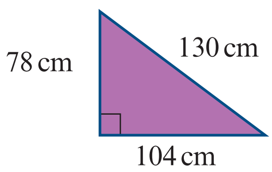
a



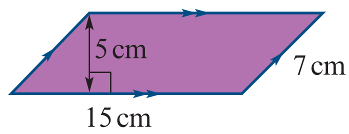
b



c



d

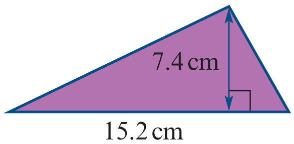


ISBN 978-1-009-25891-3

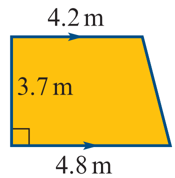
Example 9

2 Find the areas of the given shapes, correct to one decimal place, where appropriate.

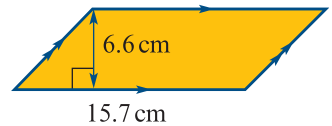
a



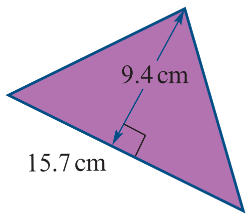
b



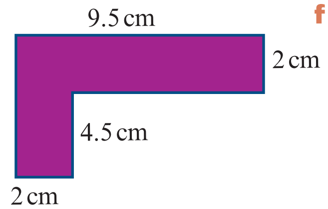
c



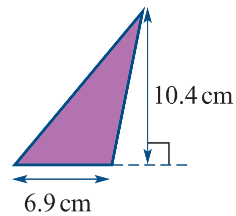
d



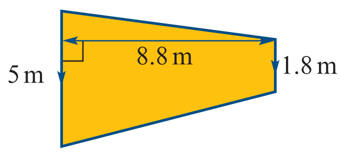
e



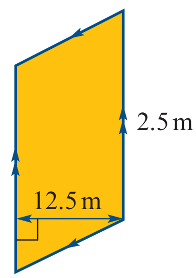
f



g



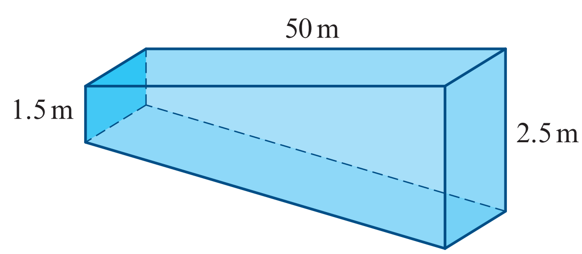
h



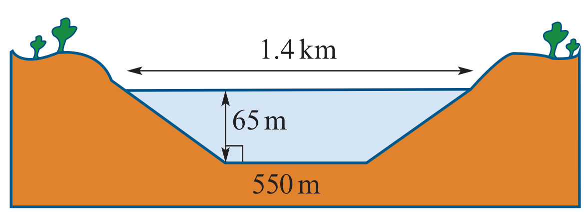
Applications of perimeters and areas

Example 11
Example 12

3 A 50 m swimming pool increases in depth from 1.5 m at the shallow end to 2.5 m at the deep end, as shown in the diagram (*not* to scale). Calculate the area of a side wall of the pool.



4 A dam wall is built across a valley that is 550 m wide at its base and 1.4 km wide at its peak, as shown in the diagram (*not* to scale). The wall is 65 m deep. Calculate the area of the dam wall.



5 Ray wants to tile a rectangular area measuring 1.6 m by 4 m outside his holiday house. The tiles that he wishes to use are 40 cm by 40 cm. How many tiles will he need?

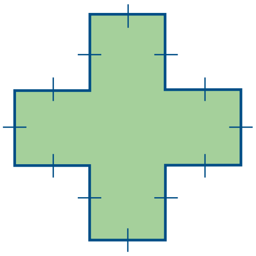
6 One litre of paint covers 9 m^2. How much paint is needed to paint a wall measuring 3 m by 12 m?

ISBN 978-1-009-25891-3

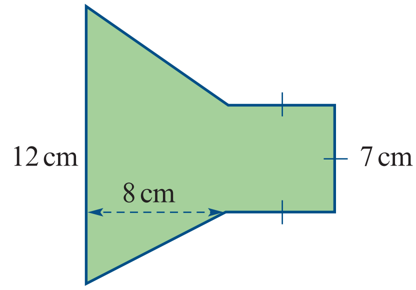
Composite shapes

7 Find the area of the following composite shapes.

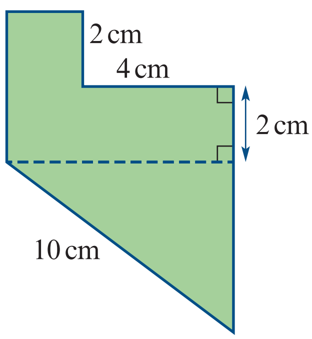
a



b

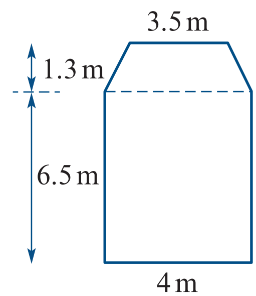


c

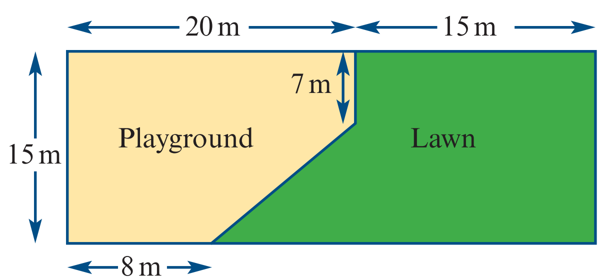


Applications of composite shapes

8 A driveway, as shown in the diagram, is to be paved. What is the area of the driveway, correct to two decimal places?



9 A local council plans to fence a rectangular piece of land to make a children's playground and a lawn as shown. (Not drawn to scale.)



a What is the area of the children's playground?

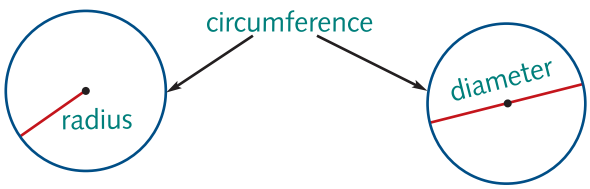
b What is the area of the lawn?

ISBN 978-1-009-25891-3

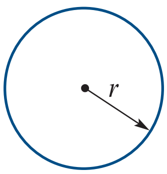
3D Circles

The circumference and area of a circle

The perimeter of a circle is also known as the **circumference** (C) of the circle.



The area and the circumference of a circle are given by the following formulas.

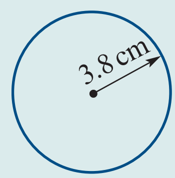
	Area	Circumference
Circle 	$A = \pi r^2$ where r is the **radius**	$C = 2\pi r$ or $C = \pi d$ where d is the **diameter**

Example 13 **Finding the circumference and area of a circle**

For the circle shown, find:

a the circumference, correct to one decimal place

b the area, correct to one decimal place.



Solution

a **1** For the circumference, use the formula $C = 2\pi r$. $C = 2\pi r$

2 Substitute $r = 3.8$ and evaluate. $= 2\pi \times 3.8$

$= 23.876\ldots$

3 Give your answer correct to one decimal place and with correct units. The circumference of the circle is 23.9 cm, correct to one decimal place.

b **1** To find the area of the circle, use the formula $A = \pi r^2$. $A = \pi r^2$

2 Substitute $r = 3.8$ and evaluate. $= \pi \times 3.8^2$

$= 45.364\ldots$

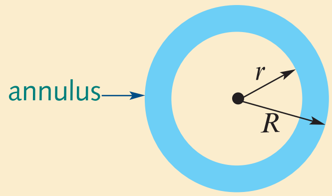
3 Give your answer correct to one decimal place and with correct units. The area of the circle is 45.4 cm^2, correct to one decimal place.

ISBN 978-1-009-25891-3

The annulus

Annulus

An *annulus* is a flat ring shape bounded by two circles that have the same centre.

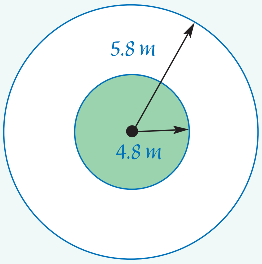


Example 14 Finding the area of an annulus in a practical problem

A path 1 metre wide is to be built around a circular lawn of radius 4.8 m. Find the area of the path, correct to two decimal places.

Solution

1 Draw a diagram to represent the situation. The two circles form an annulus.
The smaller circle has radius of 4.8 m.
With the path of width of 1 m, the larger circle has a radius of 5.8 m



2 Find the area of the larger circle using the formula $A = \pi r^2$.

$A = \pi r^2$

3 Substitute $r = 5.8$ and evaluate.

$A = \pi \times 5.8^2$

$A = 105.68$, correct to two decimal places

4 Find the area of the smaller circle using the formula $A = \pi r^2$.

$A = \pi r^2$

5 Substitute $r = 4.8$ and evaluate.

$A = \pi \times 4.8^2$

$A = 72.38$, correct to two decimal places

6 Subtract the area of the smaller circle from the area of the larger circle to give the required area (the area of the annulus).

Required area = area of large circle − area of small circle
= 105.68 − 72.38
= 33.30

7 Give your answer correct to two decimal places and with correct units.

Area of circular path is 33.30 m^2, correct to two decimal places.

ISBN 978-1-009-25891-3

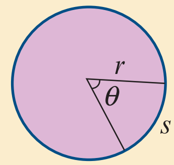
Arc length

Arc of a circle

An **arc** is the length along a circle between two points on the circle. The length of the arc, s, is given by:

$$s = r\left(\frac{\theta}{180}\pi\right)$$

where r is the radius of the circle and $\theta°$ is the angle subtended by the arc at the centre of the circle.

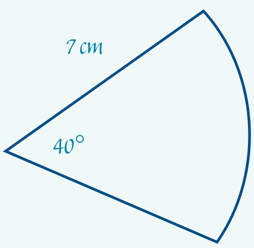


Example 15 Finding the length of an arc

Find, correct to two decimal places, the length of an arc, with an angle of 40° at the centre of the circle and circle radius of 7 cm.

Solution

1 Draw a diagram to represent the situation.



2 Write down the formula for arc length, s.

$s = r\left(\frac{\theta}{180}\pi\right)$

3 Substitute $r = 7$ and $\theta = 40$ and evaluate.

$s = 7\left(\frac{40}{180}\pi\right)$

4 Give your answer correct to two decimal places and with correct units.

$s = 4.8869\ldots$

$s = 4.89$, correct to two decimal places

The length of the arc is 4.89 cm.

ISBN 978-1-009-25891-3

Exercise 3D

Finding the circumference and area of a circle

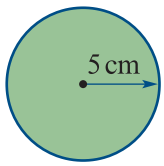
Example 13

1 For each of the following circles, find:

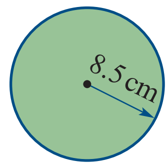
i the circumference, correct to one decimal place

ii the area, correct to one decimal place.

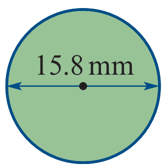
a



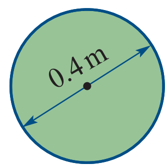
b



c



d



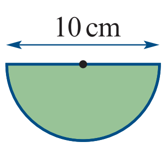
Finding the perimeter and area of shapes involving circles

2 For each of the following shapes, find:

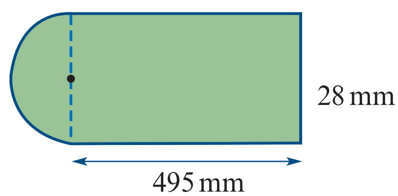
i the perimeter, correct to two decimal places

ii the area, correct to two decimal places.

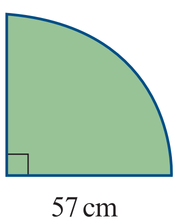
a



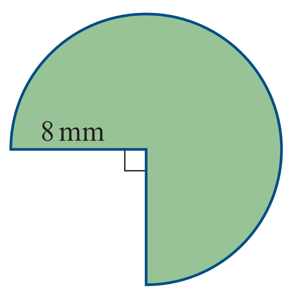
b



c

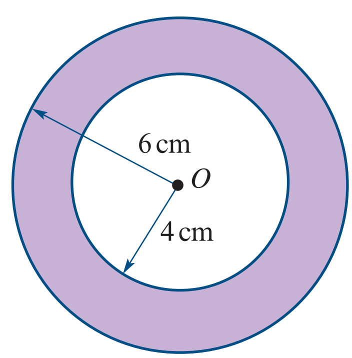


d



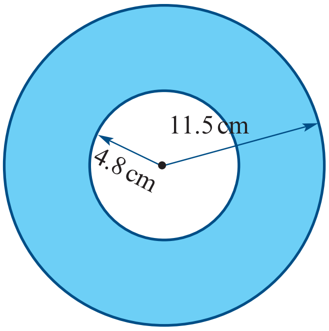
Example 14

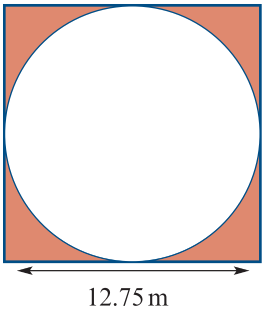
3 The diagram below shows two circles with centre O. The radius of the inner circle is 4 units and the radius of the outer circle is 6 units. What is the area of the annulus (shaded area) correct to two decimal places?

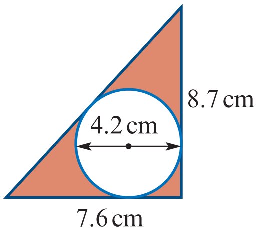


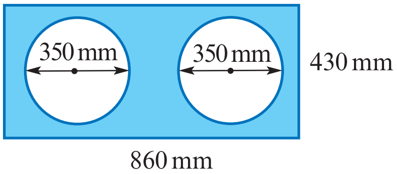
ISBN 978-1-009-25891-3

4 Find the shaded areas in the following diagrams, correct to one decimal place.

a


b


c


d


Applications of perimeters and areas involving circles

5 A fence needs to be built around an athletics track that has straights 400 m long and semicircular ends of diameter 80 m.

a What length of fencing, correct to two decimal places, is required?

b What area will be enclosed by the fencing, correct to two decimal places?

ISBN 978-1-009-25891-3

6 A couple wish to decorate an arch for their wedding. The width of the arch is 1.4 metres and the semicircle at the top begins at a height of 1.9 metres.

a Material is to be attached around the perimeter of the arch. What is this length, to the nearest metre?

b If material is to cover the whole arch so that you cannot see through the arch, what is the minimum amount of square metres of material required, correct to one decimal place?

7 Three juggling rings cut from a thin sheet are to be painted. The diameter of the outer circle of the ring is 25 cm and the diameter of the inside circle is 20 cm. If both sides of the three rings are to be painted, what is the total area to be painted? (Ignore the inside and outside edges.) Round your answer to the nearest cm^2.

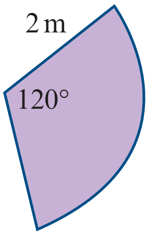
8 A path 1.2 m wide surrounds a circular garden bed whose diameter is 7 metres. What is the area of the path? Give the answer correct to two decimal places.

Arc length and applications

Example 15

9 A circle has a radius of 10 cm. An arc of the circle subtends an angle of 50° at the centre. Calculate the arc length correct to two decimal places.

10 Maria wishes to place edging around the perimeter of her garden bed. The garden bed is in the shape of a sector as shown. What is the perimeter of her garden bed, correct to two decimal places?



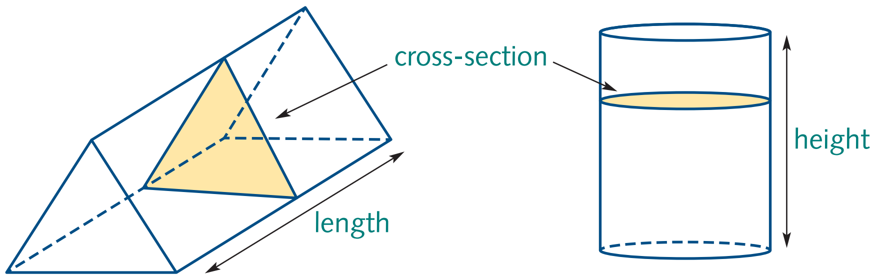
ISBN 978-1-009-25891-3

3E Volume

Volume

Volume is the amount of space occupied by a three-dimensional object.

Prisms and cylinders are three-dimensional objects that have a uniform cross-section along their entire length. The volume of a prism or cylinder is found by using its *cross-sectional area*.

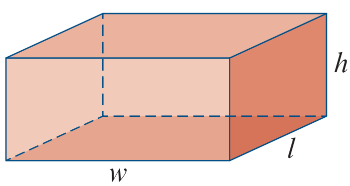
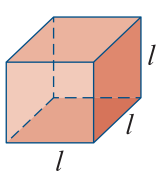


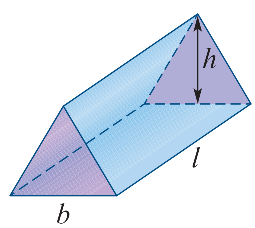
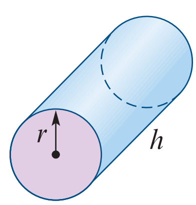
For prisms and cylinders:

volume = area of cross-section × height (or length)

When calculating volume, the answer will be in *cubic units*, i.e. mm^3, cm^3, m^3.

The **formulas for the volumes** of regular prisms and a cylinder are given in the table below.

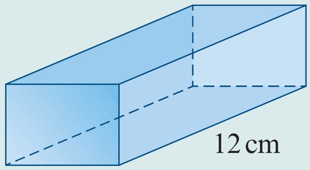
Shape	Volume
Rectangular prism (cuboid) 	$V = lwh$
Square prism (cube) 	$V = l^3$

Shape	Volume
Triangular prism 	$V = \frac{1}{2}bhl$
Cylinder 	$V = \pi r^2 h$

ISBN 978-1-009-25891-3

Example 16 Finding the volume of a cuboid

Find the volume of the following cuboid.



Solution

1 Use the formula $V = lwh$. $V = lwh$

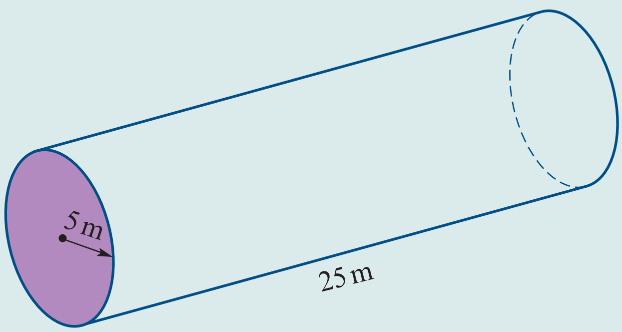
2 Substitute in $l = 12$, $w = 6$ and $h = 4$. $= 12 \times 6 \times 4$

3 Evaluate. $= 288 \text{ cm}^3$

4 Give your answer with correct units. The volume of the cuboid is 288 cm^3.

Example 17 Finding the volume of a cylinder

Find the volume of this cylinder in cubic metres. Give your answer correct to two decimal places.



Solution

1 Use the formula $V = \pi r^2 h$. $V = \pi r^2 h$

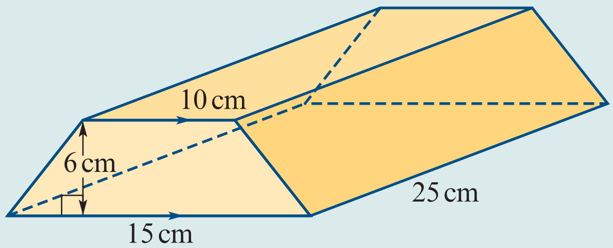
2 Substitute in $r = 5$ and $h = 25$ and evaluate. $= \pi \times 5^2 \times 25$
$= 1963.495\ldots$

3 Write your answer correct to two decimal places and with correct units. The volume of the cylinder is 1963.50 m^3, to two decimal places.

ISBN 978-1-009-25891-3

Example 18 **Finding the volume of a three-dimensional shape**

Find the volume of the three-dimensional shape shown.



Solution

Strategy: To find the volume, find the area of the yellow shaded cross-section and multiply it by the length of the shape.

1 Find the area of the cross-section, which is a trapezium. Use the formula $A = \frac{1}{2}(a + b)h$.
Substitute in $a = 10, b = 15$ and $h = 6$ and evaluate.

$$\begin{aligned}\text{Area of trapezium} &= \frac{1}{2}(a+b)h \\ &= \frac{1}{2}(10+15)\times 6 \\ &= 75\ \text{cm}^2\end{aligned}$$

2 To find the volume, multiply the area of the cross-section by the length of the shape (25 cm).

$$\begin{aligned}V &= \text{area of cross-section} \times \text{length} \\ &= 75 \times 25 \\ &= 1875\ \text{cm}^3\end{aligned}$$

3 Give your answer with correct units.

The volume of the shape is $1875\ \text{cm}^3$.

Capacity

Capacity

Capacity is the amount of substance that an object can hold.

For example, a bucket might have a capacity of 7 litres.

The difference between volume and capacity is that volume is the space available whilst capacity is the amount of substance that fills the volume.

For example:

- a cube that measures 1 metre on each side has a volume of one cubic metre (m^3) and is able to hold 1000 litres (L) (capacity)
- a bucket of volume $7000\ \text{cm}^3$ can hold 7000 mL (or 7 L) of water.

ISBN 978-1-009-25891-3

Capacity conversions

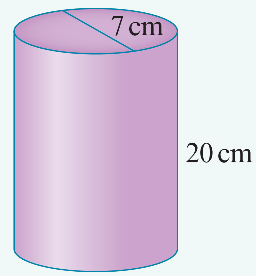
The following conversions are useful to remember.

$$1\ \text{m}^3 = 1000 \text{ litres (L)}$$
$$1\ \text{cm}^3 = 1 \text{ millilitre (mL)}$$
$$1000\ \text{cm}^3 = 1 \text{ litre (L)}$$

Example 19 Finding the capacity of a cylinder

A drink container is in the shape of a cylinder. How many litres of water can it hold if the height of the cylinder is 20 cm and the diameter is 7 cm? Give your answer correct to two decimal places.

Solution



1 Draw a diagram to represent the situation.

2 Use the formula for finding the volume of a cylinder $V = \pi r^2 h$. — $V = \pi r^2 h$

3 Since diameter is 7 cm then the radius is 3.5 cm. Substitute $h = 20$ and $r = 3.5$. — $V = \pi \times 3.5^2 \times 20$

4 Evaluate to find the volume of the cylinder. — $V = 769.6902\ldots$
The volume of the cylinder is 769.69 cm^3.

5 As there are 1000 cm^3 in a litre, divide the volume by 1000 to convert to litres. — $\dfrac{769.69}{1000} = 0.769\,69\ldots$

6 Give your answer correct to two decimal places and with correct units. — Cylinder has capacity of 0.77 litres, correct to two decimal places.

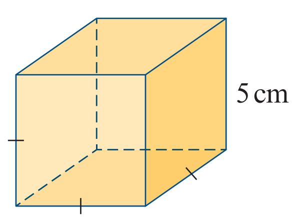
Exercise 3E

Volumes of solids

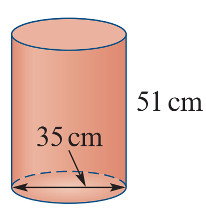
Example 16–18

1 Find the volumes of the following solids. Give your answers correct to one decimal place where appropriate.

a

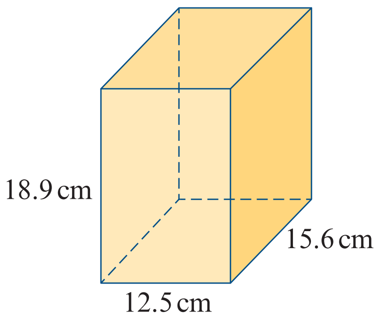


b

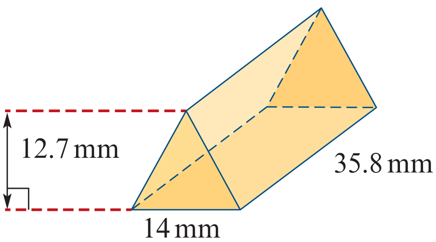


ISBN 978-1-009-25891-3

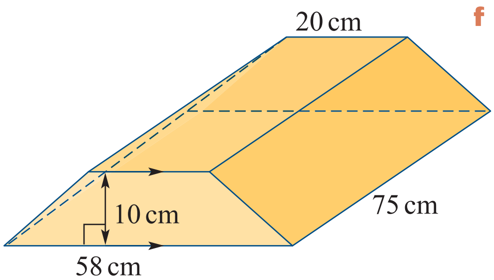
c



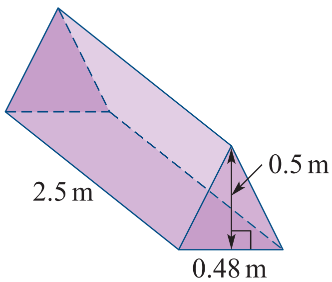
d



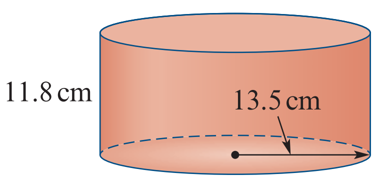
e



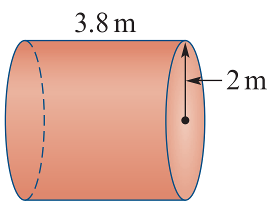
f



g



h



2 A cylindrical plastic container is 15 cm high and its circular end surfaces each have a radius of 3 cm. What is its volume, to the nearest cm^3?

3 What is the volume, to the nearest cm^3, of a rectangular box with dimensions 5.5 cm by 7.5 cm by 12.5 cm?

Applications of volume and capacity

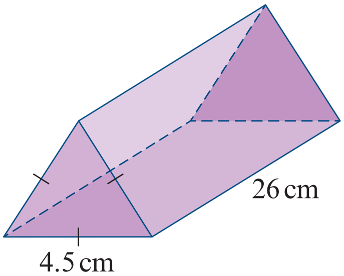
Example 19

4 How many litres of water does a fish tank with dimensions 50 cm by 20 cm by 24 cm hold when full?

5 **a** What is the volume, correct to two decimal places, of a cylindrical paint tin with height 33 cm and diameter 28 cm?

b How many litres of paint would fill this paint tin? Give your answer to the nearest litre.

6 A chocolate bar is made in the shape of an equilateral triangular prism. What is the volume of the box if the length is 26 cm and the side length of the triangle is 4.5 cm? Give your answer to the nearest cm^3.



ISBN 978-1-009-25891-3

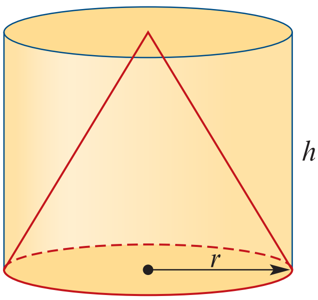
3F Volume of a cone

A cone can fit inside a cylinder, as shown in the diagram. The cone occupies one-third of the volume of the cylinder containing it. Therefore, the formula for finding the volume of a cone is:

$$\text{volume of cone} = \frac{1}{3} \times \text{volume of its cylinder}$$

$$\text{volume of cone} = \frac{1}{3} \times \text{ area of base} \times \text{height}$$

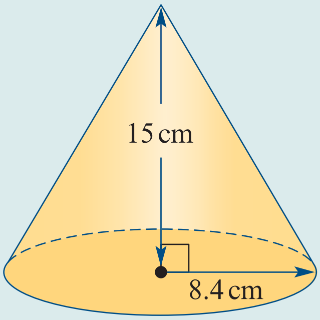
$$V = \frac{1}{3}\pi r^2 h$$



The cone in the above diagram is called a right circular cone because a line drawn from the centre of the circular base to the vertex at the top of the cone is perpendicular to the base.

Example 20 **Finding the volume of a cone**

Find the volume of this right circular cone.
Give your answer to two decimal places.



Solution

1 Use the formula for the volume of a cone, $V = \frac{1}{3}\pi r^2 h$.

$V = \frac{1}{3}\pi r^2 h$

2 Substitute $r = 8.4$ and $h = 15$ and evaluate.

$= \frac{1}{3}\pi(8.4)^2 \times 15$

$= 1108.353\ldots$

3 Give your answer correct to two decimal places and with correct units.

The volume of the cone is 1108.35 cm^3, correct to two decimal places.

ISBN 978-1-009-25891-3

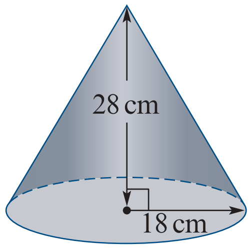
Exercise 3F

Volumes of cones

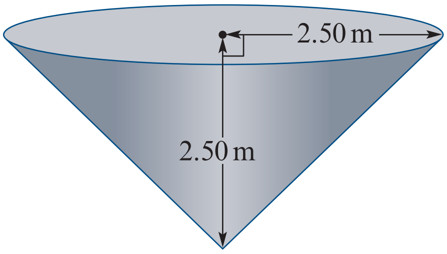
Example 20

1 Find the volume of these cones, correct to two decimal places.

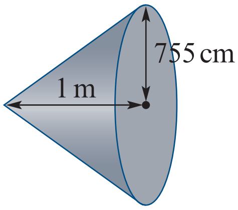
a



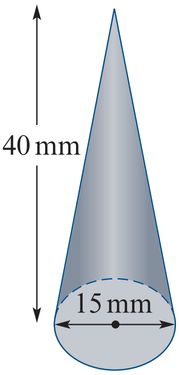
b



c



d



2 Find the volume (to two decimal places) of the cones with the following dimensions.

a Base radius 3.50 cm, height 12 cm

b Base radius 7.90 m, height 10.80 m

c Base diameter 6.60 cm, height 9.03 cm

d Base diameter 13.52 cm, height 30.98 cm

Applications

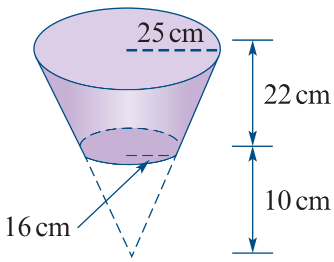
3 What volume of crushed ice will fill a snow cone level to the top if the snow cone has a top radius of 5 cm and a height of 15 cm? Give your answer to the nearest cm^3.

4 A tepee is a conical shaped tent. What is the volume, correct to two decimal places, of a tepee with height 2.6 m and diameter of 3.4 m?

ISBN 978-1-009-25891-3

5 How many litres of water, correct to 2 decimal places, can be poured into a conical flask with a diameter 2.8 cm and a height of 10 cm?

6 A solid figure is *truncated* when a portion of the bottom is cut and removed. Find the volume, correct to two decimal places, of the truncated cone shown in the diagram.



7 A flat-bottomed silo for grain storage is made of a cylinder with a cone on top. The cylinder has a circumference of 53.4 m and a height of 10.8 m. The total height of the silo is 15.3 m. What is the volume of the silo? Give your answer to the nearest m^3.

ISBN 978-1-009-25891-3

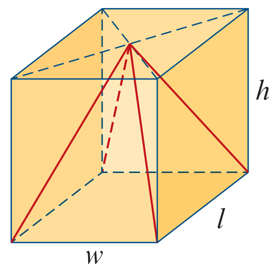
3G Volume of a pyramid

A square pyramid can fit inside a prism, as shown in the diagram. The pyramid occupies one third of the volume of the prism containing it. The formula for finding the volume of a pyramid is therefore:

$$\text{volume of pyramid} = \frac{1}{3} \times \text{volume of its prism}$$

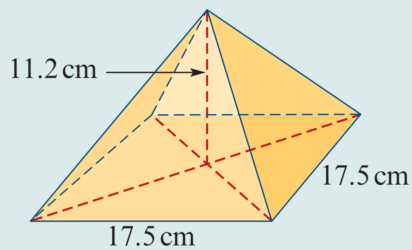
$$\text{volume of pyramid} = \frac{1}{3} \times \text{area of base} \times \text{height}$$

$$V = \frac{1}{3}lwh$$



Example 21 Finding the volume of a square pyramid

Find the volume of a square right pyramid of height 11.2 cm and base 17.5 cm. Give your answer correct to two decimal places.



Solution

1 Use the formula:

$$V = \frac{1}{3} \times \text{area of base} \times \text{height}$$

2 Substitute the values for the area of the base (in this example, the base is a square) and height of the pyramid and evaluate.

$$V = \frac{1}{3} \times \text{area of base} \times \text{height}$$
$$= \frac{1}{3} \times 17.5^2 \times 11.2$$
$$= 1143.333\ldots$$

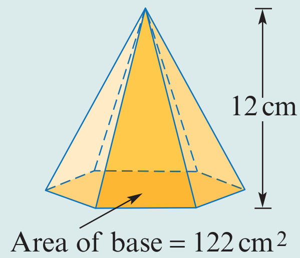
3 Give your answer correct to two decimal places and with correct units.

The volume of the pyramid is 1143.33 cm^3, correct to two decimal places.

ISBN 978-1-009-25891-3

Example 22 Finding the volume of a hexagonal pyramid

Find the volume of this hexagonal pyramid that has a base of area 122 cm^2 and a height of 12 cm.



Solution

1 Use the formula:
$V = \frac{1}{3} \times \text{area of base} \times \text{height}$

$V = \frac{1}{3} \times \text{area of base} \times \text{height}$

2 Substitute the values for area of base (122 cm^2) and height (12 cm) and evaluate.

$= \frac{1}{3} \times 122 \times 12$
$= 488 \text{ cm}^3$

3 Give your answer with correct units.

The volume of the pyramid is 488 cm^3.

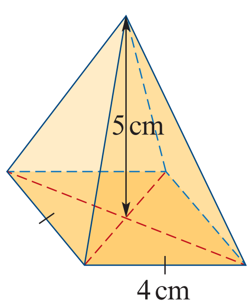
Exercise 3G

Volumes of pyramids

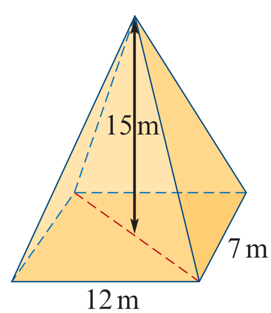
Example 21
Example 22

1 Find the volumes of the following right pyramids, correct to two decimal places where appropriate.

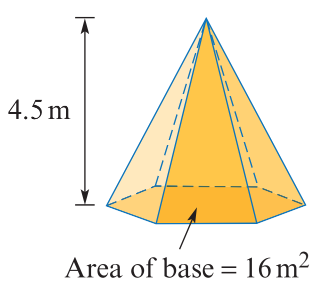
a



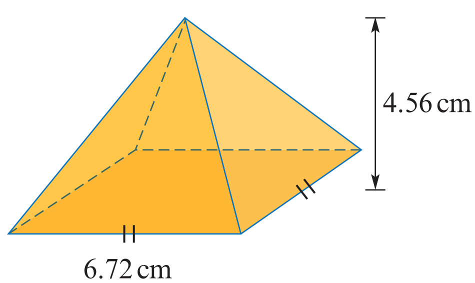
b



c



d



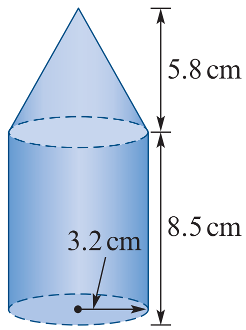
ISBN 978-1-009-25891-3

2 A square-based pyramid has a base side length of 8 cm and a height of 10 cm. What is its volume? Answer correct to three decimal places.

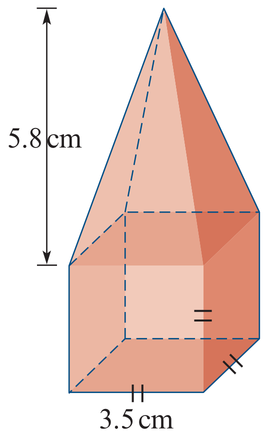
3 The first true pyramid in Egypt is known as the Red Pyramid. It has a square base approximately 220 m long and is about 105 m high. What is its volume?

4 Find the volumes of these composite objects, correct to one decimal place.

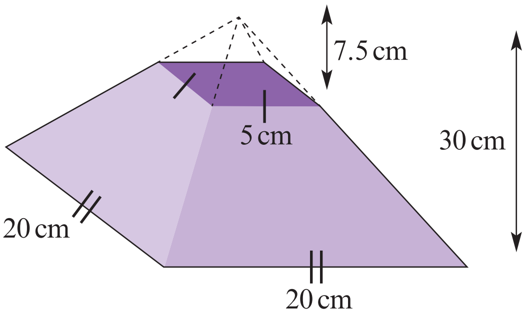
a



b



5 Calculate the volume of the following truncated pyramid, correct to one decimal place.

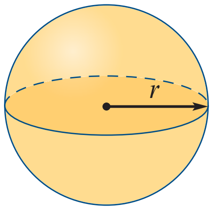


ISBN 978-1-009-25891-3

3H Volume of a sphere

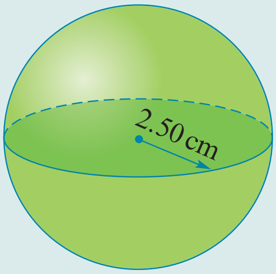
The volume of a sphere of radius r can be found by using the formula:

$V = \frac{4}{3}\pi r^3$



Example 23 Finding the volume of a sphere

Find the volume of this sphere, giving your answer correct to two decimal places.



Solution

1 Use the formula $V = \frac{4}{3}\pi r^3$.

2 Substitute $r = 2.5$ and evaluate.

$$V = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi \times 2.5^3$$
$$= 65.449\ldots$$

3 Give your answer correct to two decimal places and with correct units.

The volume of the sphere is 65.45 cm^3, correct to two decimal places.

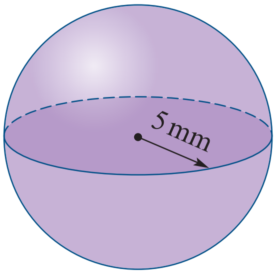
Exercise 3H

Volumes of spheres and hemispheres

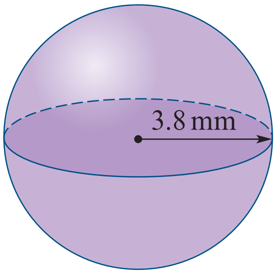
Example 23

1 Find the volumes of these spheres, giving your answers correct to two decimal places.

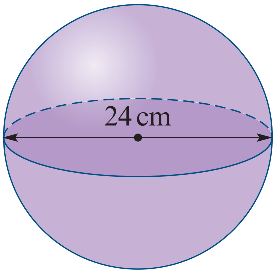
a



b



c



ISBN 978-1-009-25891-3

2 Find the volumes, correct to two decimal places, of the following balls.

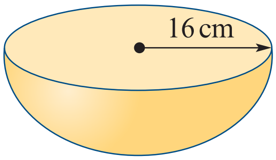
a Tennis ball, radius 3.5 cm

b Basketball, radius 14 cm

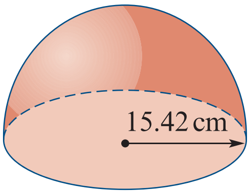
c Golf ball, radius, 2 cm

3 Find the volumes, correct to two decimal places, of the following hemispheres.

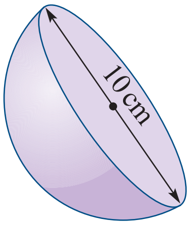
a



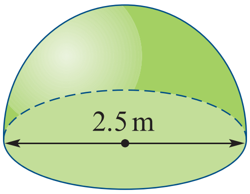
b



c



d



Applications

4 An orange is cut into quarters. If the radius is 35 mm, what is the volume of one quarter to the nearest mm^3?

5 Lois wants to serve punch at Christmas time in her new hemispherical bowl with diameter of 38 cm. How many litres of punch could be served, given that 1 millilitre (mL) is the amount of fluid that fills 1 cm^3? Answer to the nearest litre.

3I Surface area

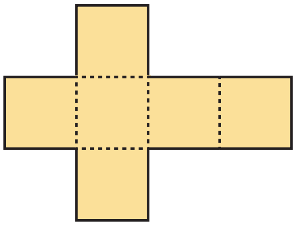
To find the **surface area (*SA*)** of a solid, you need to find the area of each of the faces of the solid and then add these all together.

Solids with plane faces (prisms and pyramids)

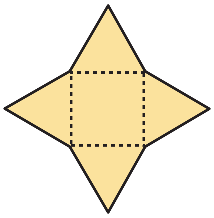
It is often useful to draw the *net* of a solid to ensure that all sides have been added.

A *net* is a flat diagram consisting of the plane faces of a polyhedron, arranged so that the diagram may be folded to form the solid.

For example: The net of a cube and of a square pyramid are shown below.



Net of a cube

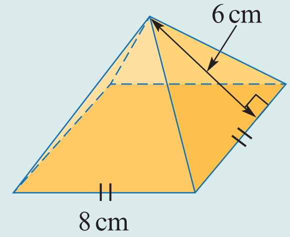


Net of a square pyramid

ISBN 978-1-009-25891-3

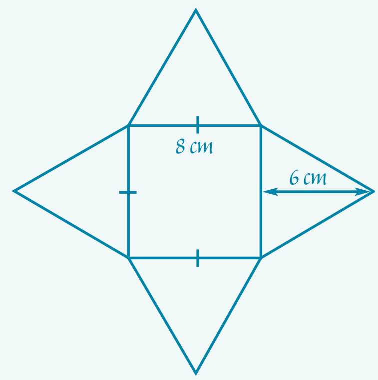
Example 24 Finding the surface area of a pyramid

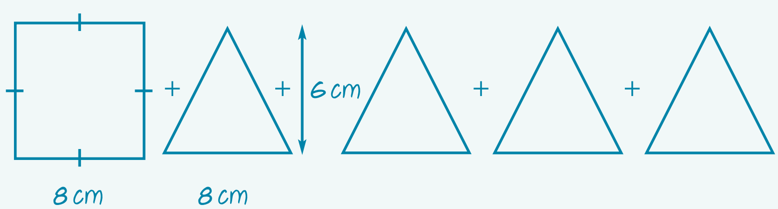
Find the surface area of this square-based pyramid.



Solution

1 Draw a net of the square pyramid. Note that the net is made up of one square and four identical triangles, as shown below.





2 Write down the formula for the total surface area, using the net as a guide, and evaluate.

Total surface area = area of $\square + 4\,\triangle$

$= 8 \times 8 + 4 \times (\frac{1}{2} \times 8 \times 6)$

$= 160$

The surface area of the square pyramid is 160 cm^2.

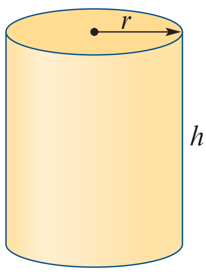
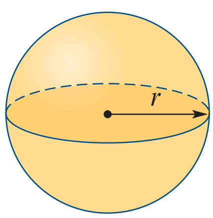
Note: To find the area of the square, multiply the length by the width (8×8).
To find the area of the triangles use $A = \frac{1}{2}bh$, where b is 8 and h is 6.

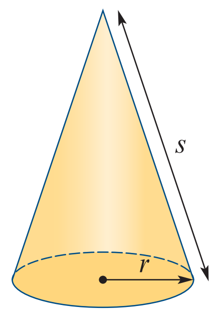
ISBN 978-1-009-25891-3

Solids with curved surfaces (cylinder, cone, sphere)

For some special objects, such as the cylinder, cone and sphere, formulas to calculate the surface area can be developed.

The formulas for the surface area of a cylinder, cone and sphere are given below.

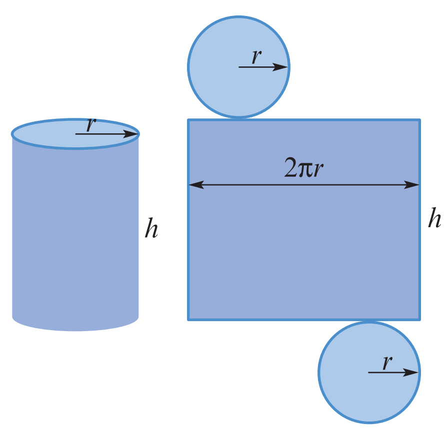
Shape	Surface area
Cylinder 	$S\,A = 2\pi r^2 + 2\pi rh$ $= 2\pi r(r + h)$
Sphere 	$S\,A = 4\pi r^2$

Shape	Surface area
Cone 	$S\,A = \pi r^2 + \pi rs$

To develop the formula for the surface area of a cylinder, we first draw a net, as shown.

The **total surface area** ***(TSA)*** of a cylinder can therefore be found using:

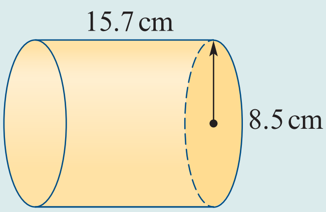
$$\begin{aligned} \text{TSA} &= \text{area of ends} + \text{area of curved surface} \\ &= \text{area of 2 circles} + \text{area of rectangle} \\ &= 2\pi r^2 + 2\pi rh \\ &= 2\pi r(r + h) \end{aligned}$$



ISBN 978-1-009-25891-3

Example 25 Finding the surface area of a cylinder

Find the surface area of this cylinder, correct to one decimal place.



Solution

1 Use the formula for the surface area of a cylinder, $SA = 2\pi r^2 + 2\pi rh$.

$SA = 2\pi r^2 + 2\pi rh$

2 Substitute $r = 8.5$ and $h = 15.7$ and evaluate.

$= 2\pi(8.5)^2 + 2\pi \times 8.5 \times 15.7$

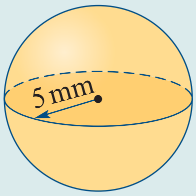
$= 1292.451\ldots$

3 Give your answer correct to one decimal place and with correct units.

The surface area of the cylinder is 1292.5 cm^2, correct to one decimal place.

Example 26 Finding the surface area of a sphere

Find the surface area of a sphere with radius 5 mm, correct to two decimal places.



Solution

1 Use the formula $SA = 4\pi r^2$.

$SA = 4\pi r^2$

2 Substitute $r = 5$ and evaluate.

$= 4\pi \times 5^2$

$= 314.159\ldots$

3 Give your answer correct to two decimal places and with correct units.

The surface area of the sphere is 314.16 mm^2, correct to two decimal places.

ISBN 978-1-009-25891-3

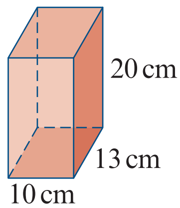
Exercise 3I

Surface areas of prisms and pyramids

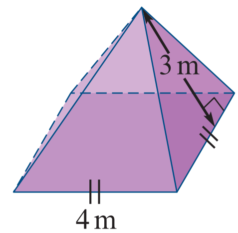
Example 24

1 Find the surface areas of these prisms and pyramids. Where appropriate give your answer correct to one decimal place.

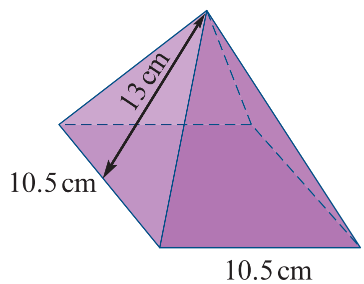
a



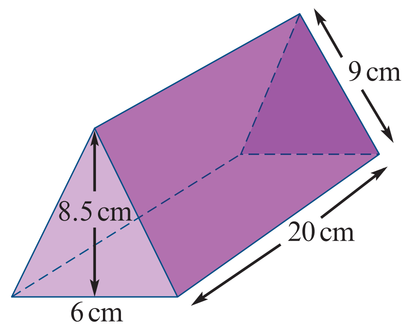
b



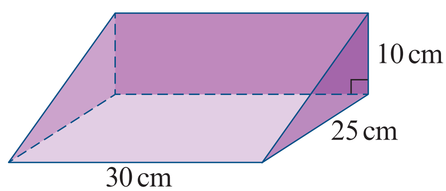
c



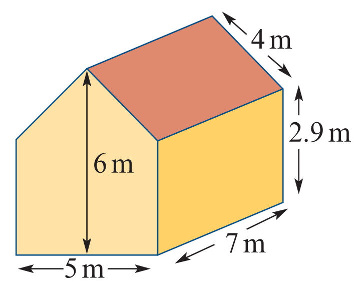
d



e



f

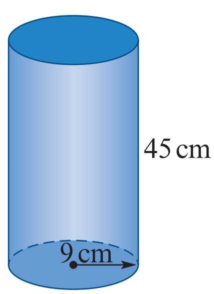


Surface area of curved surfaces

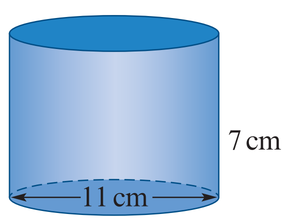
Example 25
Example 26

2 Find the surface area of each of these solids with curved surfaces, correct to two decimal places.

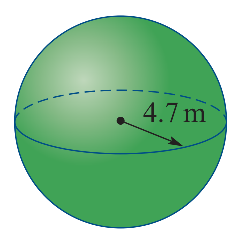
a



b

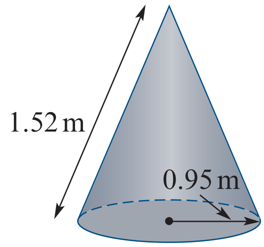


c

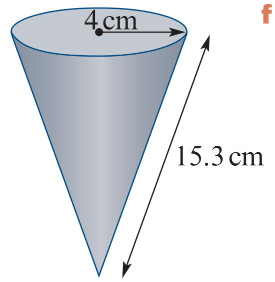


ISBN 978-1-009-25891-3

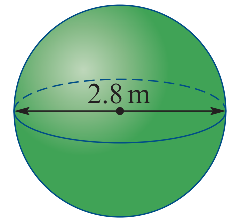
d



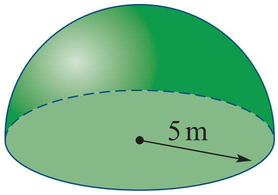
e



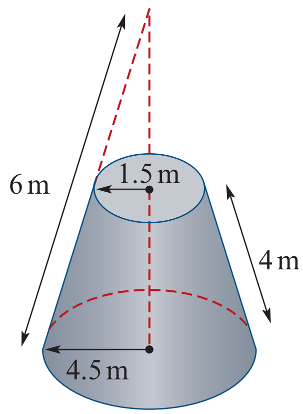
f



g



h



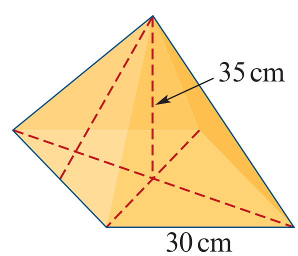
Applications of surface areas

3 A tennis ball has a radius of 3.5 cm. A manufacturer wants to provide sufficient material to cover 100 tennis balls. What area of material is required? Give your answer correct to the nearest cm^2.

4 A set of 10 conical paper hats are covered with material. The height of a hat is 35 cm and the diameter is 19 cm.

- **a** What amount of material, in m^2, will be needed? Give your answer correct to two decimal places.
- **b** Tinsel is to be placed around the base of the hats. How much tinsel, to the nearest metre, is required?

5 For a project, Mark has to cover all sides of a square based pyramid with material (excluding the base). The pyramid has the dimensions as shown in the diagram. How much material will Mark need to cover the sides of the pyramid? Give your answer in square metres, correct to two decimal places.



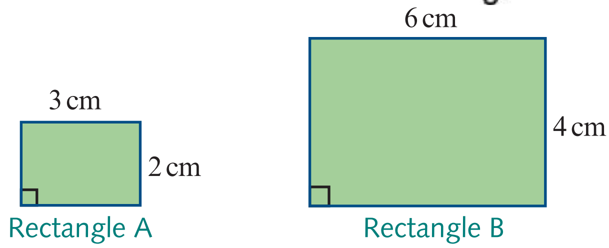
ISBN 978-1-009-25891-3

3J Similar figures

Shapes that are similar have the same shape but are different sizes. The three frogs below are **similar figures**.

Polygons (closed plane figures with straight sides), like the rectangles in the diagram below, are similar if:

- corresponding angles are equal
- corresponding sides are proportional (each pair of corresponding side lengths are in the same ratio).



For example, the two rectangles above are similar as their corresponding angles are equal and their side lengths are in the same ratio.

$$\text{Ratio of side length} = 6 : 3 \text{ or } \frac{6}{3} = \frac{2}{1} = 2$$

$$\text{Ratio of side length} = 4 : 2 \text{ or } \frac{4}{2} = \frac{2}{1} = 2$$

When we enlarge or reduce a shape by a **scale factor**, the *original* and the *image* are similar. In the diagram above, rectangle A has been enlarged by a scale factor, $k = 2$, to give rectangle B.

We can also say that rectangle A has been scaled up to give rectangle B.

We can also compare the ratio of the rectangles' areas.

$$\text{Area of rectangle A} = 6\,\text{cm}^2$$

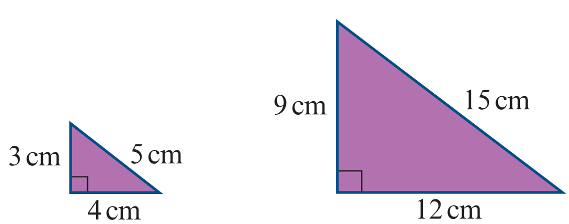
$$\text{Area of rectangle B} = 24\,\text{cm}^2$$

$$\text{Ratio of areas} = 24 : 6 = \frac{24}{6} = \frac{4}{1} = 4$$

The area of rectangle A has been enlarged by a scale factor of $k^2 = 4$ to give rectangle B. We notice that, as the length dimensions are enlarged by a scale factor of 2, the area is enlarged by a scale factor of $2^2 = 4$.

Scaling areas

When all the dimensions are multiplied by a scale factor of k, the area is multiplied by a scale factor of k^2.



ISBN 978-1-009-25891-3

For example, the two triangles on the previous page are similar as their corresponding side lengths are in the same ratio.

$$\text{Scale factor, } k = \text{Ratio of lengths} = \frac{15}{5} = \frac{9}{3} = \frac{12}{4} = \frac{3}{1} = 3$$

We would expect the area scale factor, k^2, or the ratio of the triangles' areas to be $9 (= 3^2)$.

$$\text{Area of small triangle} = 6 \text{ cm}^2$$

$$\text{Area of large triangle} = 54 \text{ cm}^2$$

$$\text{Area scale factor, } k^2 = \text{Ratio of areas} = \frac{54}{6} = \frac{9}{1} = 9.$$

Shapes can be scaled up or scaled down. When a shape is made larger, it is scaled up and when it is made smaller, it is scaled down.

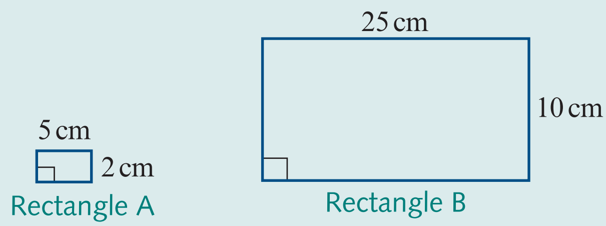
When working out scale factors, the numerator is the length of the second shape and the denominator is the length from the first (or original) shape.

Example 27 Finding the ratio (scale factor) of dimension and area

The rectangles shown are similar.

a Find the ratio of their side lengths.

b Find the ratio of their areas.



Solution

a **1** Since the rectangles are similar, their side lengths are in the same ratio. Compare the corresponding side lengths.

$$\frac{25}{5} = \frac{10}{2} = \frac{5}{1}$$

2 Write your answer.

Note: We can also say that the second rectangle has been scaled up by a scale factor of 5.

The ratio of the side lengths is $\frac{5}{1}$.

b **1** Since the dimensions are multiplied by a scale factor of 5, the area will be multiplied by a scale factor of 5^2. Square the ratio of the side lengths.

$$5^2 = 25$$

2 Write your answer.

Note: We can also say that the area of the second rectangle has been scaled up by a scale factor of 25.

The ratio of the areas is $\frac{25}{1}$.

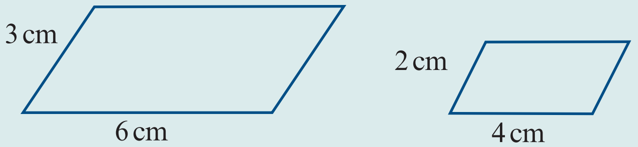
ISBN 978-1-009-25891-3

Example 28 Finding the scale factor

The two shapes shown are similar.

a Determine whether the first shape has been scaled up or down to give the second shape and find the scale factor.

b What is the scale factor for the areas?



Solution

a **1** Since the shape is made smaller it has been scaled down.

Shape has been scaled down.

2 The shapes are similar so their side lengths are in the same ratio. Compare the corresponding side lengths.

$$\frac{4}{6} = \frac{2}{3}$$

3 Write your answer.

The scale factor is $\frac{2}{3}$.

b **1** The scale factor, k, for the shapes is $\frac{2}{3}$ so the scale factor for the area is k^2. Square the value for k and evaluate.

$$k = \frac{2}{3}$$

$$k^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

2 Write your answer.

The scale factor for the area is $\frac{4}{9}$.

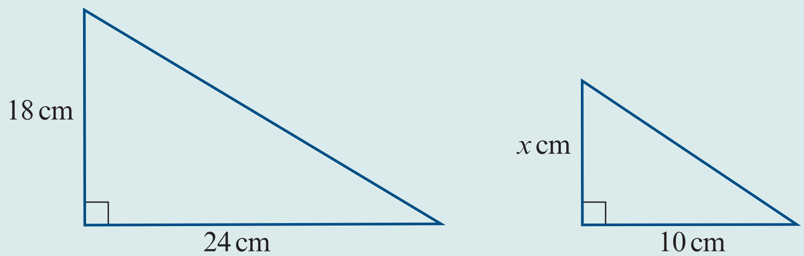
ISBN 978-1-009-25891-3

Example 29 Using a scale factor to find unknown values

The following two triangles are similar.

a Find the value of x.

b What is the scale factor?



Solution

a **1** Since the triangles are similar, their side lengths are in the same ratio. Compare the corresponding side lengths.

Note: Make sure that corresponding sides are compared.

$$\frac{x}{18} = \frac{10}{24}$$

2 Solve for x. Multiply by 18.

$$\frac{x}{18} \times 18 = \frac{10}{24} \times 18$$

3 Evaluate and give your answer with correct units.

$x = 7.5$ cm

b **1** Compare corresponding side lengths and simplify the fraction.

Remember: The numerator of the fraction is the length from the second shape and the denominator is the length from the first (or original) shape.

$$\frac{7.5}{18} = \frac{10}{24} = \frac{5}{12}$$

2 Write your answer.

Note: In this case, the triangle has been scaled down.

Triangle has been scaled down by a scale factor of $\frac{5}{12}$.

ISBN 978-1-009-25891-3

Example 30 Using scaling in maps

A map has a scale of 1 : 20 000. If the measurement on the map between two towns is 5.4 cm, what is the actual distance between these two towns? Give your answer in kilometres, correct to two decimal places.

Solution

A scale of 1 : 20 000 means that 1 cm on the map represents 20 000 cm (or 200 m) on the ground.

Method 1

1 If the map distance between the two towns is 5.4 cm, then multiply this distance by 20 000 to get the actual distance (in cm) between the two towns.

map distance = 5.4 cm
actual distance = 5.4 × 20 000
= 108 000 cm

2 Convert from centimetres to metres by dividing by 100.

108 000 cm ÷ 100 = 1080 m

3 Convert from metres to kilometres by dividing by 1000.

1080 m ÷ 1000 = 1.08 km

Method 2

1 Let x be the actual distance.
A scale of 1 : 20 000 can also be written as a scale factor of $\frac{1}{20\,000}$.
The ratio of the distance on the map to the actual distance will be the same as the scale factor of $\frac{1}{20\,000}$.
Write out the corresponding ratios.

Let x be the actual distance.

$$\frac{5.4}{x} = \frac{1}{20\,000}$$

2 Solve for x. (This can be done by cross-multiplying or by using the solve function on the CAS calculator.)

$$5.4 \times 20\,000 = x$$
$$x = 108\,000 \text{ cm}$$

3 Convert to kilometres by dividing by 100 000.

Note: This is the same as dividing by 100 to convert to metres and then by 1000 to convert to kilometres.

108 000 cm ÷ 100 000 = 1.08 km

4 Write your answer.

The distance between the two towns is 1.08 km.

ISBN 978-1-009-25891-3

Exercise 3J

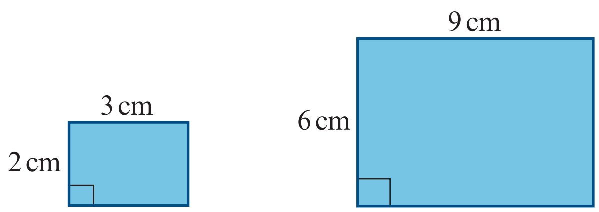
Similarity and ratios

Example 27

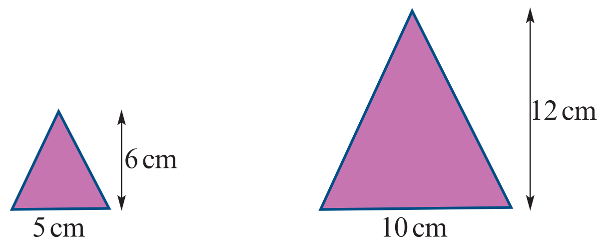
1 The following pairs of figures are similar. For each pair find:

i the ratio of their side lengths **ii** the ratio of their areas.

a

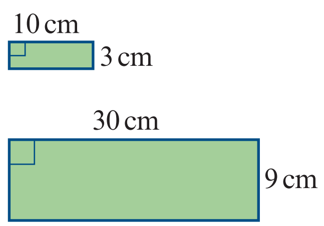


b

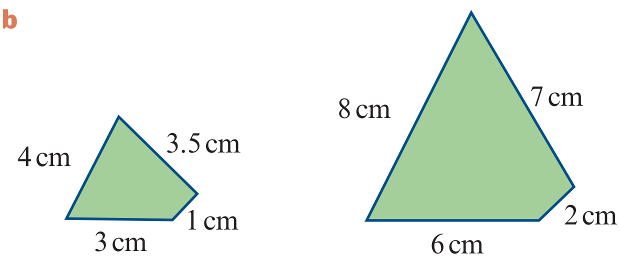


2 Which of the following pairs of figures are similar? For those that are similar, find the ratios of the corresponding sides.

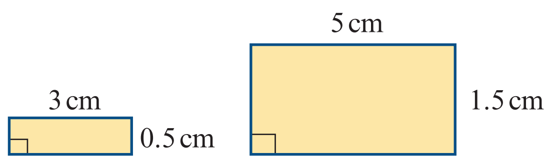
a



b



c



3 Which of the following pairs of figures are similar? State the ratios of the corresponding sides where relevant.

a Two rectangles 8 cm by 3 cm and 16 cm by 4 cm

b Two rectangles 4 cm by 5 cm and 16 cm by 20 cm

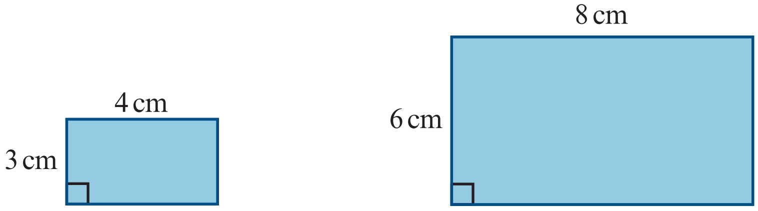
c Two rectangles 4 cm by 6 cm and 2 cm by 4 cm

d Two rectangles 30 cm by 24 cm and 10 cm by 8 cm

e Two triangles, one with sides measuring 3 cm, 4 cm and 5 cm and the other 4.5 cm, 6 cm and 7.5 cm.

ISBN 978-1-009-25891-3

4 The following two rectangles are similar. Find the ratio of their areas.

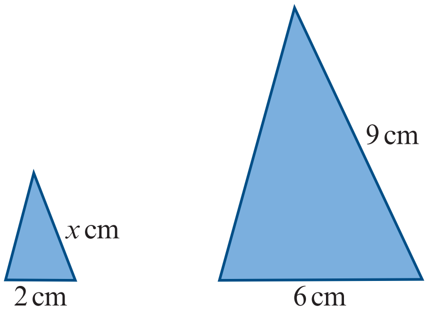


Example 29

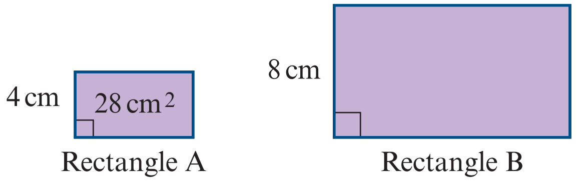
5 The following triangles are similar.

a Find the value of x.

b Find the ratio of their areas.



6 The two rectangles shown to the right are similar. The area of rectangle A is 28 cm^2. Find the area of rectangle B.



Applications of similarity and ratios

7 A photo is 12 cm by 8 cm. It is to be enlarged and then framed. If the dimensions are tripled, what will be the area of the new photo?

8 What is the scale factor if a photo has been enlarged from 15 cm by 9 cm to 25 cm by 15 cm? Give your answer correct to two decimal places.

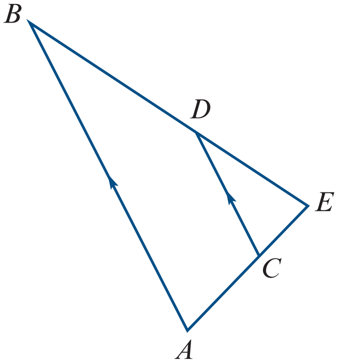
Example 30

9 A scale on a map is 1 : 500 000.

a What is the actual distance between two towns if the distance on the map is 7.2 cm? Give your answer in kilometres.

b If the actual distance between two landmarks is 15 km, what distance would be represented on the map?

10 In triangle ABE, point C lies on side AE and point D lies on side BE. The lines CD and AB are parallel. The length of ED is 5 cm, the length of DB is 7 cm and the length of CD is 6 cm. What is the length of AB?

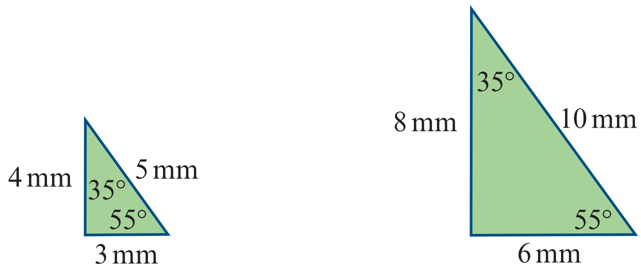


ISBN 978-1-009-25891-3

3K Similar triangles

In mathematics, two **triangles** are said to be **similar** if they have the same shape. As in the previous section, this means that corresponding angles are equal and the lengths of the corresponding sides are in the same ratio.

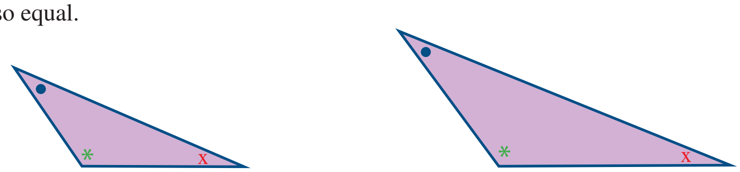
For example, these two triangles are similar.



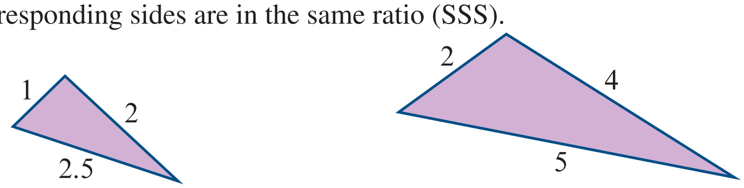
Two triangles can be tested for similarity by considering the following necessary conditions.

- Corresponding angles are equal (AAA or AA).

  Remember: If two pairs of corresponding angles are equal, then the third pair of corresponding angles is also equal.

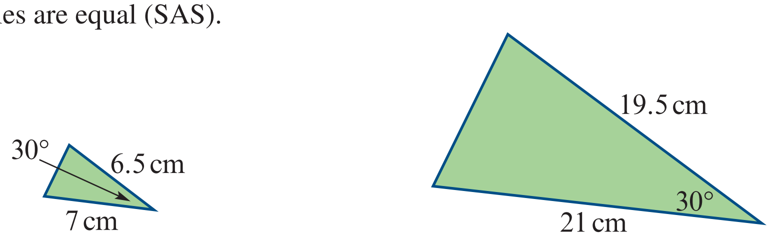


- Corresponding sides are in the same ratio (SSS).



$$\frac{5}{2.5} = \frac{4}{2} = \frac{2}{1} = 2$$

- Two pairs of corresponding sides are in the same ratio and the included corresponding angles are equal (SAS).



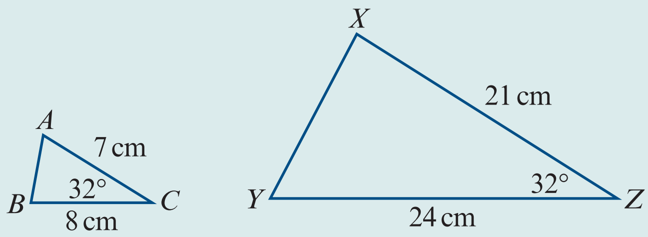
$$\frac{19.5}{6.5} = \frac{21}{7} = 3$$

Both triangles have an included corresponding angle of 30°.

 ISBN 978-1-009-25891-3

Example 31 Checking if triangles are similar

Explain why triangle ABC is similar to triangle XYZ.



Solution

1 Compare corresponding side ratios:
AC and XZ

$$\frac{XZ}{AC} = \frac{21}{7} = \frac{3}{1}$$

BC and YZ.

$$\frac{YZ}{BC} = \frac{24}{8} = \frac{3}{1}$$

2 Triangles ABC and XYZ have an included corresponding angle.

32° is included and corresponding.

3 Write an explanation as to why the two triangles are similar.

Triangles ABC and XYZ are similar as they have two pairs of corresponding sides in the same ratio and the included corresponding angles are equal (SAS).

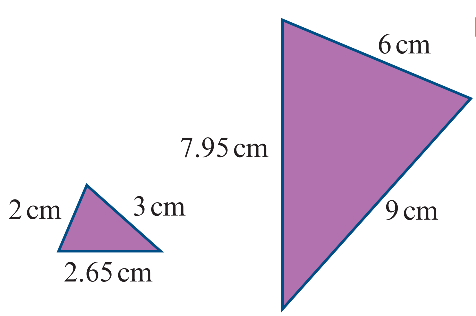
Exercise 3K

Similar triangles

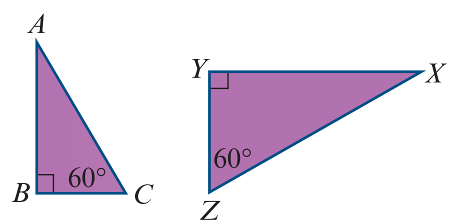
Example 31

1 Three pairs of similar triangles are shown below. Explain why each pair of triangles are similar.

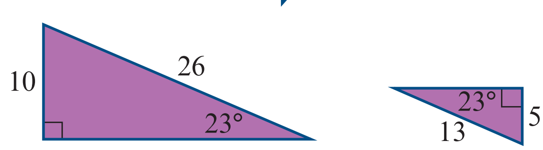
a



b



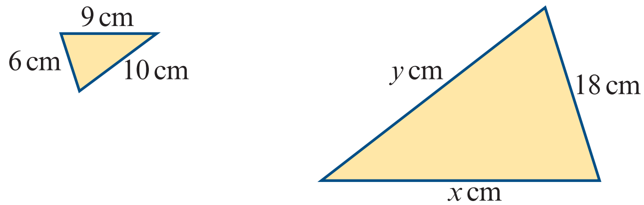
c



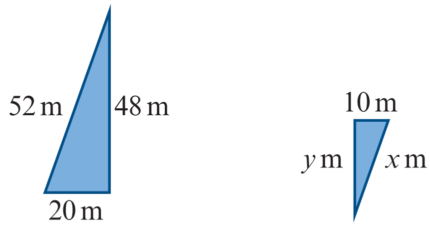
ISBN 978-1-009-25891-3

2 Calculate the missing dimensions, marked x and y, in these pairs of similar triangles.

a



b



3 A triangle with sides 5 cm, 4 cm and 8 cm is similar to a larger triangle with a longest side of 56 cm.

a Find the lengths of the larger triangle's other two sides.

b Find the perimeter of the larger triangle.

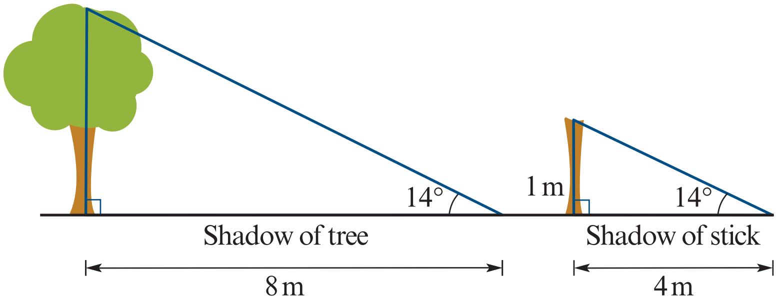
Applications of similar triangles

4 A tree and a 1 m vertical stick cast their shadows at a particular time in the day. The shadow lengths are shown in the diagram below (*not* drawn to scale).

a Give reasons why the two triangles shown are similar.

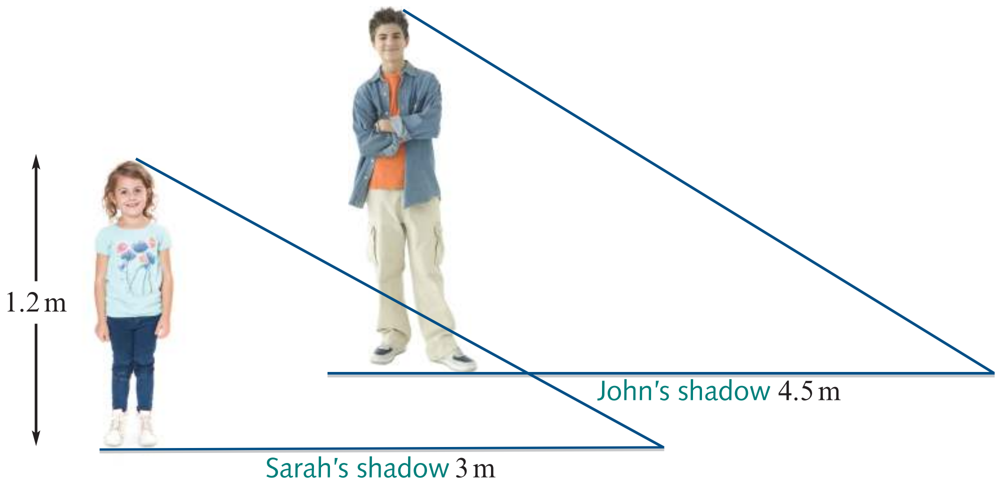
b Find the scale factor for the side lengths of the triangles.

c Find the height of the tree.

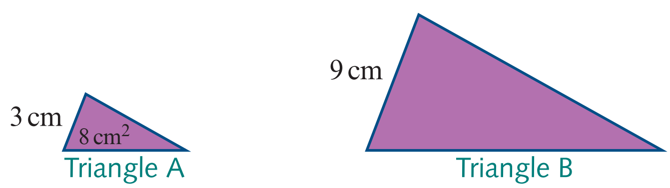


ISBN 978-1-009-25891-3

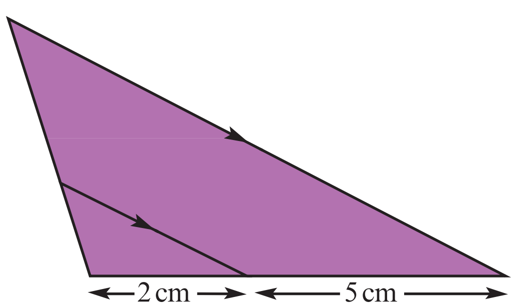
5 John and his younger sister, Sarah, are standing side by side. Sarah is 1.2 m tall and casts a shadow 3 m long. How tall is John if his shadow is 4.5 m long?



6 The area of triangle A is 8 cm^2. Triangle B is similar to triangle A. What is the area of triangle B?



7 Given that the area of the small triangle in the following diagram is 2.4 cm^2, find the area of the larger triangle, correct to two decimal places.

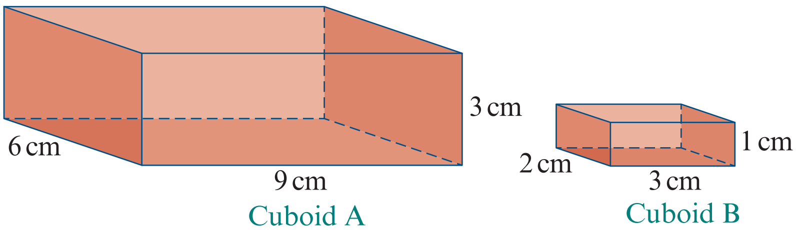


ISBN 978-1-009-25891-3

3L Similar solids

Two solids are similar if they have the same shape and the ratios of their corresponding linear dimensions are equal.

Cuboids



The two cuboids are similar because:

- they are the same shape (both are cuboids)
- the ratios of the corresponding dimensions are the same.

$$\frac{\text{length of cuboid A}}{\text{length of cuboid B}} = \frac{\text{width of cuboid A}}{\text{width of cuboid B}} = \frac{\text{height of cuboid A}}{\text{height of cuboid B}}$$

$$\frac{6}{2} = \frac{9}{3} = \frac{3}{1} = \frac{3}{1}$$

$$\text{Volume scale factor, } k^3 = \text{Ratio of volumes} = \frac{6 \times 9 \times 3}{2 \times 3 \times 1} = \frac{162}{6} = \frac{27}{1} = \frac{3^3}{1}$$

As the length dimensions are enlarged by a scale factor of 3, the volume is enlarged by a scale factor of $3^3 = 27$.

Scaling volumes

When all the dimensions are multiplied by a scale factor of k, the volume is multiplied by a scale factor of k^3.

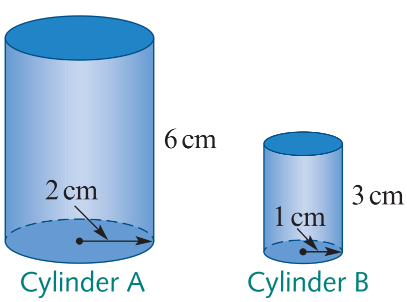
Cylinders

These two cylinders are similar because:

- they are the same shape (both are cylinders)
- the ratios of the corresponding dimensions are the same.

$$\frac{6}{3} = \frac{2}{1}$$

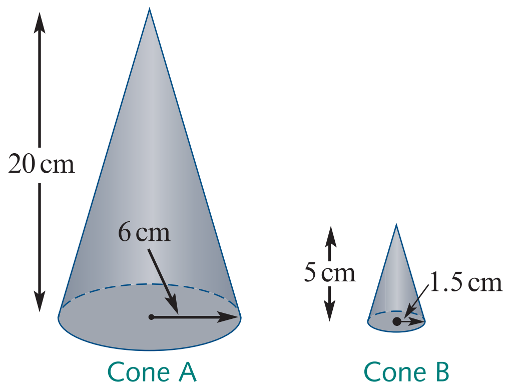
$$\frac{\text{height of cylinder A}}{\text{height of cylinder B}} = \frac{\text{radius of cylinder A}}{\text{radius of cylinder B}}$$



$$\text{Volume scale factor, } k^3 = \text{Ratio of volumes} = \frac{\pi \times 2^2 \times 6}{\pi \times 1^2 \times 3} = \frac{24}{3} = \frac{8}{1} = \frac{2^3}{1}$$

ISBN 978-1-009-25891-3

Cones



These two cones are similar because:

- they are the same shape (both are cones)
- the ratios of the corresponding dimensions are the same.

$$\frac{20}{5} = \frac{6}{1.5} = \frac{4}{1}$$

$$\frac{\text{height of cone A}}{\text{height of cone B}} = \frac{\text{radius of cone A}}{\text{radius of cone B}}$$

$$\text{Volume scale factor, } k^3 = \text{Ratio of volumes} = \frac{\frac{1}{3} \times \pi \times 6^2 \times 20}{\frac{1}{3} \times \pi \times 1.5^2 \times 5} = \frac{720}{11.25}$$

$$= \frac{64}{1} = \frac{4^3}{1}$$

Example 32 **Comparing volumes of similar solids**

Two solids are similar such that the larger one has all of its dimensions three times that of the smaller solid. How many times larger is the larger solid's volume?

Solution

1 Since all of the larger solid's dimensions are 3 times those of the smaller solid, the volume will be 3^3 times larger. Evaluate 3^3.

$3^3 = 27$

2 Write your answer.

The larger solid's volume is 27 times the volume of the smaller solid.

ISBN 978-1-009-25891-3

Exercise 3L

Scaling volumes and surface areas

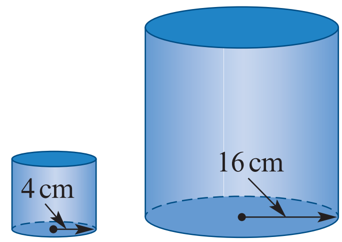
Example 32

1 Two cylindrical water tanks are similar such that the height of the larger tank is 3 times the height of the smaller tank. How many times larger is the volume of the larger tank compared to the volume of the smaller tank?

2 Two cylinders are similar and have radii of 4 cm and 16 cm, respectively.

a What is the ratio of their heights?

b What is the ratio of their volumes?

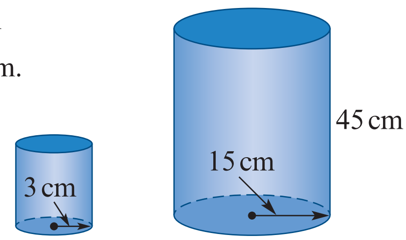


3 Find the ratio of the volumes of two cuboids whose sides are in the ratio $\frac{3}{1}$.

4 The radii of the bases of two similar cylinders are in the ratio $\frac{5}{1}$. The height of the larger cylinder is 45 cm. Calculate:

a the height of the smaller cylinder

b the ratio of the volumes of the two cylinders.

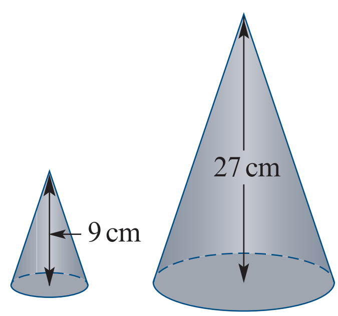


5 Two similar cones are shown at right. The ratio of their heights is $\frac{3}{1}$.

a Determine whether the smaller cone has been scaled up or down to give the larger cone.

b What is the volume scale factor?

c The volume of the smaller cone is 120 cm^3. Find the volume of the larger cone.



6 The radii of the bases of two similar cylinders are in the ratio 3 : 4. The height of the larger cylinder is 8 cm. Calculate:

a the height of the smaller cylinder

b the ratios of the volumes of the two cylinders.

7 A pyramid has a square base of side 4 cm and a volume of 16 cm^3. Calculate:

a the height of the pyramid

b the height and the base length of a similar pyramid with a volume of 1024 cm^3.

8 Two spheres have diameters of 12 cm and 6 cm, respectively. Calculate:

a the ratios of their surface areas

b the ratio of their volumes.

ISBN 978-1-009-25891-3

3M Problem solving and modelling

Exercise 3M

1 Brandon is building a new rectangular deck that will measure 5 m by 3 m. He has to order concrete for the 20 stump holes that he digs which measure 350 mm by 350 mm by 500 mm.

a What amount of concrete (in m^3) will he need to order? Give your answer correct to two decimal places.

b If the decking boards are 90 mm wide, how many 3 m length boards should he order?

c The decking is to be stained. What is the area?

Brandon builds 4 wooden planter boxes to place on the decking. The boxes have a square base of length 40 cm and a height of 60 cm.

d What is the volume of dirt, in cubic metres, required to fill these boxes if he fills them up to 10 cm from the top?

e Brandon also stains the outside surface of the planter boxes, but not the base. What is the total surface area in m^2, correct to one decimal place?

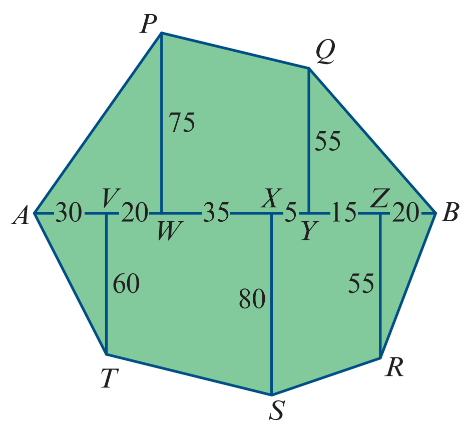
2 Farmer Green owns an irregularly shaped paddock, *APQBRSTA*, as shown in the diagram.

Starting at *A*, he has measured distances in metres along *AB* as well as the distances at right angles from *AB* to each other corner of his paddock.

Use this information to calculate:

a the total area of his paddock in hectares, correct to two decimal places (Note: 1 hectare = 10 000 m^2)

b the length of fencing needed to enclose the paddock, correct to two decimal places.



3 A map is drawn to a scale of 1 : 20 000. A park, drawn on the map, has an area of 6 cm^2. Calculate, in m^2, the actual area of the park. If one hectare = 10 000 m^2, give your answer in hectares.

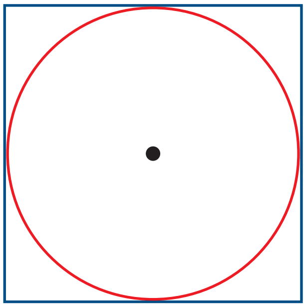
ISBN 978-1-009-25891-3

4 An architect uses a scale of 1 cm : 3 m for the plans of a house she is designing. On the plans, a room has an area of 3.3 cm^2.

a Calculate the actual area of the room in m^2.

b Another room in the same house is to have an actual area of 3.5 m^2. What area, in cm^2, would this be on the plans?

5 A 1 m piece of wire is to be cut into two pieces, one of which is bent into a circle (red). The other piece is bent into a square around the circle (blue).



a What is the length of the side of the square (to the nearest centimetre)?

b What are the lengths of the two pieces of wire?

6 A steel pipe has an outside diameter of 100 mm and an inside diameter of 80 mm.

a What is the surface area of its cross section? Give your answer in square centimetres, correct to two decimal places.

b The pipe is 90 cm long.
What is the inside volume, in cubic centimetres, correct to one decimal place?

c What amount of water, in litres, can pass through the pipe at one time?

d The pipe needs to be coated on the outside with a protective material.
What is its surface area, correct to one decimal place?

7 A cubic box has sides of length 10 cm.

a How many times will you have to enlarge the box by a scale factor of two, before it is too big for a room that is 3 m high with length 4 m and width 3 m.

b What is the volume of the enlarged box?

c What is the surface area, in cm^2, of the enlarged box?

d Using the same cube of side length 10 cm, how many times can you reduce it by a scale factor of $\frac{1}{2}$ before it becomes smaller than 1 mm by 1 mm by 1 mm?

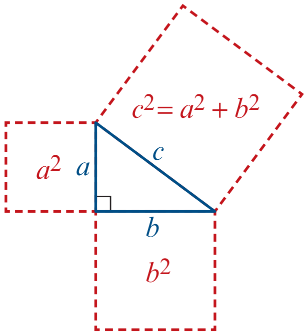
ISBN 978-1-009-25891-3

Key ideas and chapter summary

Assign-
ment

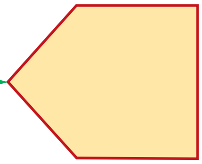
Pythagoras' theorem

Pythagoras' theorem states that:
For any right-angled triangle, the sum of the areas of the squares of the two shorter sides (a and b) equals the area of the square of the hypotenuse (c): $c^2 = a^2 + b^2$



Perimeter (P)

Perimeter is the distance around the edge of a two-dimensional shape.



Perimeter of rectangle

$P = 2l + 2w$

Circumference (C)

Circumference is the perimeter of a circle $C = 2\pi r$

Area (A)

Area is the measure of the region enclosed by the boundaries of a two-dimensional shape.

Area formulas

Area of rectangle $= lw$ Area of parallelogram $= bh$

Area of triangle $= \dfrac{1}{2}bh$ Area of trapezium $= \dfrac{1}{2}(a + b)h$

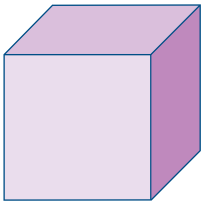
Heron's formula

Area of triangle $= A = \sqrt{s(s - a)(s - b)(s - c)}$ where $s = \dfrac{a + b + c}{2}$ and a, b and c are the sides of the triangle.

Volume (V)

Volume is the amount of space occupied by a three-dimensional object.

- For prisms and cylinders,
  Volume = area of cross-section × height
- For pyramids and cones,
  Volume $= \dfrac{1}{3} \times$ area of base × height



ISBN 978-1-009-25891-3

Review

Volume formulas

Volume of cube = l^3

Volume of cuboid = lwh

Volume of triangular prism = $\frac{1}{2}bhl$

Volume of cylinder = $\pi r^2 h$

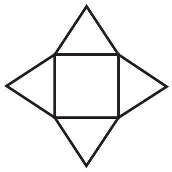
Volume of cone = $\frac{1}{3}\pi r^2 h$

Volume of pyramid = $\frac{1}{3}lwh$

Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area (*SA*)

Surface area is the total of the areas of all the faces of a solid. When finding surface area, it is often useful to draw the net of the shape.



Surface area formulas

Surface area of cylinder $2\pi r^2 + 2\pi rh$

Surface area of cone = $\pi r^2 + \pi rs$

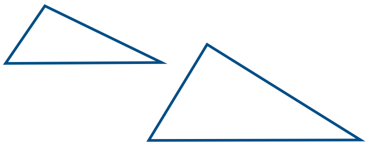
Surface area of sphere = $4\pi r^2$

Similar figures or solids

Similar figures or solids are the same shape but different sizes.

Similar triangles

Triangles are shown to be **similar** if:



- corresponding angles are similar (AAA)
- corresponding sides are in the same ratio (SSS)
- two pairs of corresponding sides are in the same ratio and the included corresponding angles are equal (SAS).

Ratios of area and volume for similar shapes

When all the dimensions of similar shapes are multiplied by a scale factor of k, the areas are multiplied by a scale factor of k^2 and the volumes are multiplied by a scale factor of k^3.

Skills check

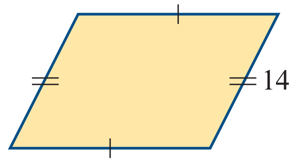
Having completed this chapter you should be able to:

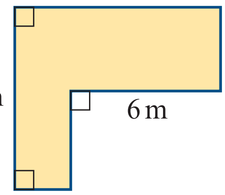
- understand and use Pythagoras' theorem to solve two-dimensional and three-dimensional problems
- find the areas and perimeters of two-dimensional shapes
- find the volumes of common three-dimensional shapes
- find the volumes of pyramids, cones and spheres
- find the surface areas of three-dimensional shapes
- use tests for similarity for two-dimensional and three-dimensional figures.

ISBN 978-1-009-25891-3

Short-answer questions

1 Find the perimeters of these shapes.

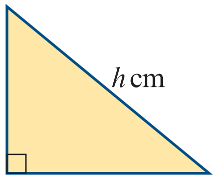
a 

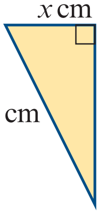
b 

2 Find the perimeter of a square with side length 9 m.

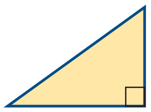
3 Find the perimeter of a rectangle with length 24 cm and width 10 cm.

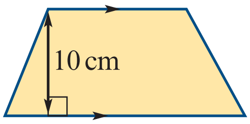
4 Find the lengths of the unknown sides, correct to two decimal places, in the following triangles.

a 

b 

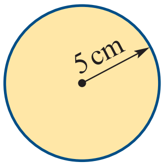
5 Find the areas of the following shapes.

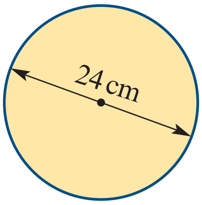
a 

b 

6 Find the surface area of a cube with side length 2.5 m.

7 Find the circumferences of the following circles, correct to two decimal places.

a 

b 

8 Find the areas of the circles in Question **7**, correct to two decimal places.

9 A soup can has a diameter of 7 cm and a height of 13.5 cm.

a How much metal, correct to two decimal places, is needed to make the can?

b A paper label is made for the outside cylindrical shape of the can. How much paper, in m^2, is needed for 100 cans? Give your answer correct to two decimal places.

c What is the capacity of one can, in litres, correct to two decimal places?

 ISBN 978-1-009-25891-3

10 A circular swimming pool has a diameter of 4.5 m and a depth of 2 m. How much water will the pool hold, to the nearest litre?

11 The radius of the Earth is approximately 6400 km. Calculate:

a the surface area, correct to the nearest square kilometre

b the volume, in standard form, correct to four significant figures.

12 The diameter of the base of an oil can in the shape of a cone is 12 cm and its height is 10 cm. Find:

a its volume in cubic centimetres, correct to two decimal places

b its capacity to the nearest millilitre.

13 A right pyramid with a square base of side length 8 m has a height of 3 m. Find the length of a sloping edge, correct to one decimal place.

14 For the solid shown on the right, find correct to two decimal places:

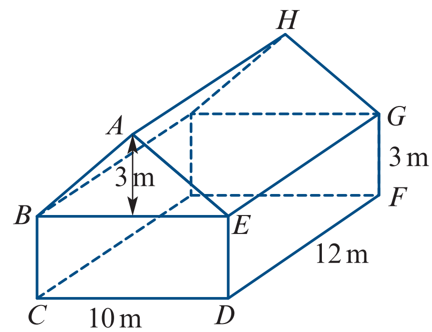
a the area of rectangle $BCDE$

b the area of triangle ABE

c the length AE

d the area of rectangle $AEGH$

e the total surface area.



15 Find the volume of a rectangular prism with length 3.5 m, width 3.4 m and height 2.8 m.

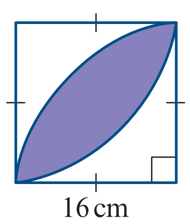
16 You are given a circle of radius r. The radius increases by a scale of factor of 2. By what factor does the area of the circle increase?

17 You are given a circle of diameter d. The diameter decreases by a scale factor of $\frac{1}{2}$. By how much does the area of the circle decrease?

18 For the shaded region, find, correct to two decimal places:

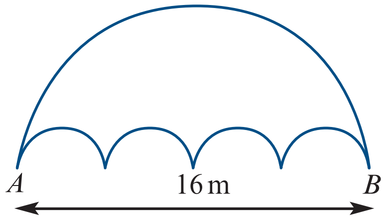
a the perimeter

b the area of the shaded region shown.



 ISBN 978-1-009-25891-3

19 Which is the shorter path from A to B? Is it along the four semi-circles or along the larger semi-circle? Give reasons for your answer.



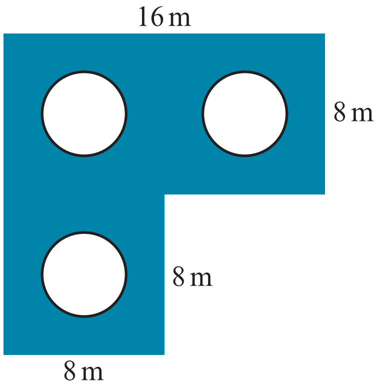
Review

Extended-response questions

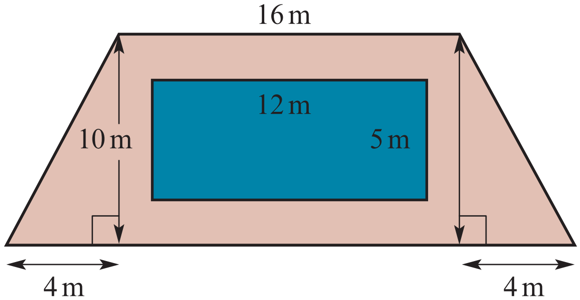
1 A lawn has three circular flowerbeds in it, as shown in the diagram. Each flowerbed has a radius of 2 m. A gardener has to mow the lawn and use a whipper-snipper to trim all the edges. Calculate:

a the area to be mown, correct to two decimal places.

b the length of the edges to be trimmed. Give your answer correct to two decimal places.



2 Chris and Gayle decide to build a swimming pool on their new housing block. The pool will measure 12 m by 5 m and it will be surrounded by timber decking in a trapezium shape. A safety fence will surround the decking. The design layout of the pool and surrounding area is shown in the diagram.



a What length of fencing is required? Give your answer correct to two decimal places.

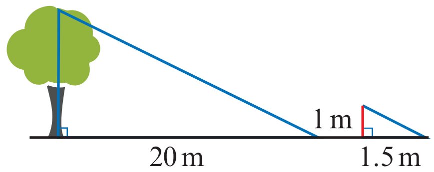
b What area of timber decking is required?

c The pool has a constant depth of 2 m. What is the volume of the pool?

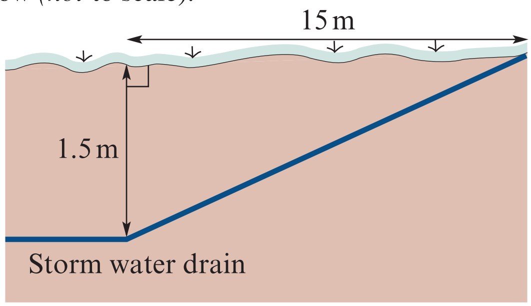
d The interior of the pool is to be painted white. What surface area is to be painted?

ISBN 978-1-009-25891-3

3 A biologist studying gum trees wanted to calculate the height of a particular tree. She placed a wooden stake vertically in the ground, sticking exactly 1 m above the ground, which was level with the base of the tree. The stake cast a shadow on the ground measuring 1.5 m. The gum tree cast a shadow of 20 m, as shown in the diagram below (*not* to scale). Calculate the height of the tree. Give your answer correct to two decimal places.



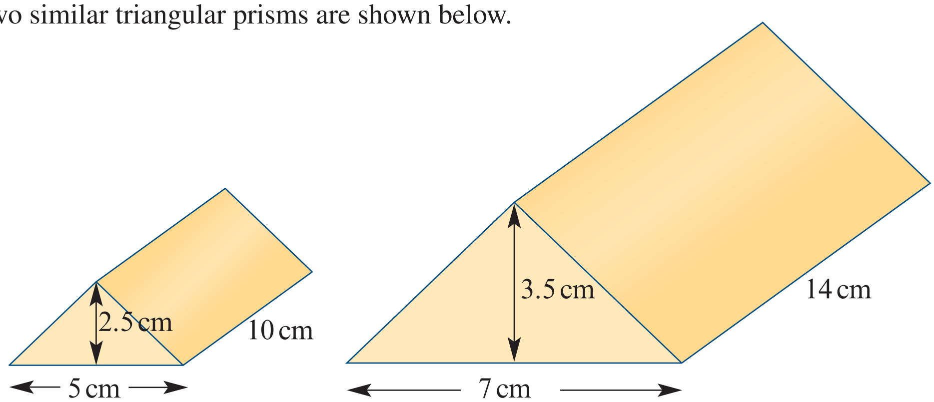
4 A builder is digging a trench for a cylindrical water pipe. From a drain at ground level, the water pipe goes 1.5 m deep, where it joins a storm water drain. The horizontal distance from the surface drain to the storm water drain is 15 m, as indicated in the diagram below (*not* to scale).



a Calculate the length of water pipe required to connect the surface drain to the storm water drain, correct to two decimal places.

b If the radius of the water pipe is 20 cm, what is the volume of the water pipe? Give your answer correct to two decimal places.

5 Two similar triangular prisms are shown below.



a Find the ratio of their surface areas.

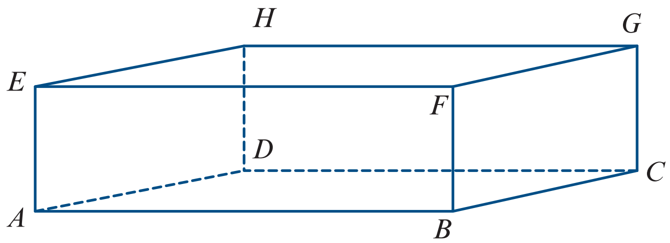
b Find the ratio of their volumes.

c What is the volume of the smaller prism to the nearest cm^3?

ISBN 978-1-009-25891-3

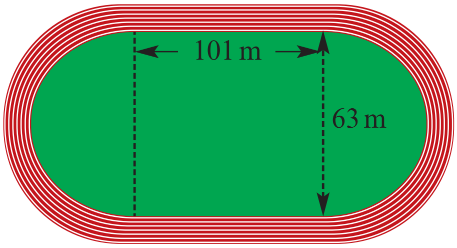
Review

6 The length of a rectangular prism is eight times its height. The width is four times the height. The length of the diagonal between two opposite vertices (AG) is 36 cm. Find the volume of the prism.



7 The volume of a cone of height 28.4 cm is 420 cm^3. Find the height of a similar cone whose volume is 120 cm^3, correct to two decimal places.

8 An athletics track is made up of a straight stretch of 101 m and two semi-circles on the ends as shown in the diagram. There are 6 lanes each 1 metre wide.



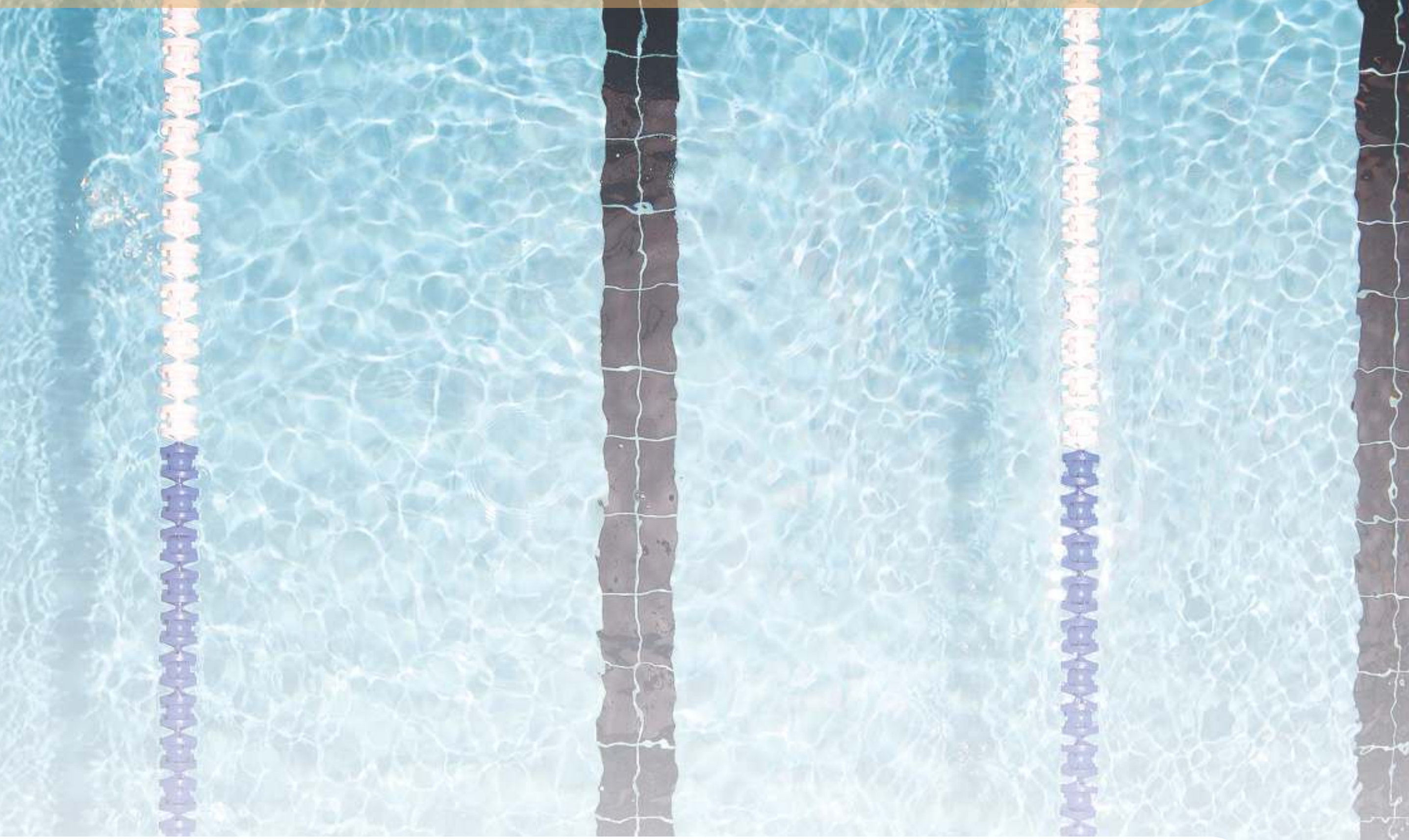
a What is the total distance, to the nearest metre, around the inside lane?

b If 6 athletes run around the track keeping to their own lane, how far, to the nearest metre, would each athlete run?

c Draw a diagram and indicate at which point each runner should start so that they all run the same distance.

ISBN 978-1-009-25891-3

4 Linear and non-linear expressions



In this chapter

Syllabus references

Topics: Linear and non-linear expressions; Linear equations

Subtopics: 1.2.1 – 1.2.3, 2.3.1 – 2.3.2

Linear relations and equations connect two or more variables such that they yield a straight line when graphed. They have many applications in technology, science and business.

ISBN 978-1-009-25891-3

4A Substitution of values into a formula

A **formula** is a mathematical relationship connecting two or more variables.

For example:

- $C = 45t + 150$ is a formula for relating the cost, C dollars, of hiring a plumber for t hours. C and t are the variables.
- $P = 4L$ is a formula for finding the perimeter of a square, where P is the perimeter and L is the side length of the square. P and L are the variables.

By substituting all known variables into a formula, we are able to find the value of an unknown variable.

Example 1 Using a formula

The cost of hiring a windsurfer is given by the rule:

$$C = 40t + 10$$

where C is the cost in dollars and t is the time in hours. How much will it cost to hire a windsurfer for 2 hours?

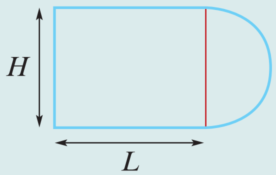
Solution

1 Write the formula.	$C = 40t + 10$
2 To determine the cost of hiring a windsurfer for 2 hours, substitute $t = 2$ into the formula. Note: $40(2)$ means 40×2	$C = 40(2) + 10$
3 Evaluate.	$C = 90$
4 Write your answer.	It will cost \$90 to hire a windsurfer for 2 hours.

Example 2 Using a formula

The perimeter of the shape shown can be given by the formula:

$$P = 2L + H\left(1 + \frac{\pi}{2}\right)$$



In this formula, L is the length of the rectangle and H is the height. Find the perimeter correct to one decimal place, if $L = 16.1$ cm and $H = 3.2$ cm.

ISBN 978-1-009-25891-3

Solution

1 Write the formula. $P = 2L + H\left(1 + \frac{\pi}{2}\right)$

2 Substitute values for L and H into the formula. $P = 2 \times 16.1 + 3.2\left(1 + \frac{\pi}{2}\right)$

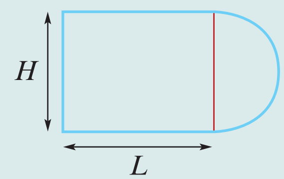
3 Evaluate. $P = 40.4$ (correct to one decimal place)

4 Give your answer with correct units. The perimeter of the shape is 40.4 cm.

Example 3 Using a non-linear formula

The area, A, of the shape shown can be given by the formula:

$$A = HL + \frac{1}{2}\pi\left(\frac{H}{2}\right)^2$$



In this formula, L is the length of the rectangle and H is the height. Find the area correct to one decimal place, if $H = 5.2$ and $L = 18.4$ cm.

Solution

1 Write the formula. $A = HL + \frac{1}{2}\pi\left(\frac{H}{2}\right)^2$

2 Substitute values for H and L into the formula. $A = 5.2 \times 18.4 + \frac{1}{2}\pi\left(\frac{5.2}{2}\right)^2$

3 Evaluate. $A = 106.3$ (correct to one decimal place)

4 Give your answer with correct units. The area of the shape is 106.3 cm^2.

Exercise 4A

Example 1

1 The cost of hiring a dance hall is given by the rule:

$$C = 50t + 1200$$

where C is the total cost in dollars and t is the number of hours for which the hall is hired.

Find the cost of hiring the hall for:

a 4 hours **b** 6 hours **c** 4.5 hours

ISBN 978-1-009-25891-3

2 The distance, d km, travelled by a car in t hours at an average speed of v km/h is given by the formula:

$$d = v \times t$$

Find the distance travelled by a car travelling at a speed of 95 km/h for 4 hours.

3 Taxi fares are calculated using the formula:

$$F = 1.3K + 4$$

where K is the distance travelled in kilometres and F is the cost of the fare in dollars. Find the costs of the following trips.

a 5 km **b** 8 km **c** 20 km

4 The circumference, C, and area, A, of a circle with radius, r, can be calculated using the formulas:

$$C = 2\pi r \qquad A = \pi r^2$$

For the following circles, find,

i the circumference **ii** the area

correct to two decimal places.

a A stained glass window with $r = 25$ cm

b An earring with $r = 3$ mm

c A DVD of $r = 5.4$ cm

d A circular garden bed with $r = 7.2$ m

ISBN 978-1-009-25891-3

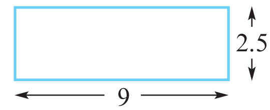
Example 2

5 If the perimeter of a rectangle is given by $P = 2(L + W)$, find the value of P for the following rectangles.

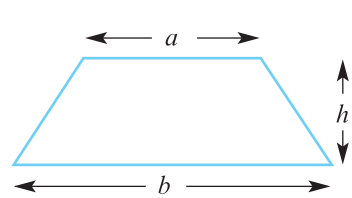
a $L = 3$ and $W = 4$

b $L = 15$ and $W = 8$

c



6 The area of a trapezium as shown is $A = \frac{1}{2}h(a + b)$.
Find A if:



a $h = 1, a = 3, b = 5$

b $h = 5, a = 2.5, b = 3.2$

c $h = 2.7, a = 4.1, b = 8.3$

7 The surface area, A, and volume of a sphere, V, can be calculated using the rules:

$$V = \frac{4}{3}\pi r^3 \qquad A = 4\pi r^2$$

where r is the radius of the sphere.
For the following spheres, find,

i the volume

ii the surface area

correct to two decimal places.

a A basketball with radius $r = 12.1$ cm

b A soap bubble with radius $r = 12.5$ mm

c A plastic sphere with radius $r = 1.35$ m

d An orange with radius $r = 6.3$ cm

8 The formula used to convert temperature from degrees Fahrenheit to degrees Celsius is:

$$C = \frac{5}{9}(F - 32)$$

Use this formula to convert the following temperatures to degrees Celsius. Give your answers correct to one decimal place.

a 50°F

b 0°F

c 212°F

d 92°F

9 The formula for calculating simple interest is:

$$I = \frac{PRT}{100}$$

where P is the principal (amount invested or borrowed), R is the interest rate per annum and T is the time (in years). In the following questions, give your answers to the nearest cent.

a Frank borrows \$5000 at 12% for 4 years. How much interest will he pay?

b Chris borrows \$1500 at 6% for 2 years. How much interest will he pay?

c Jane invests \$2500 at 5% for 3 years. How much interest will she earn?

d Henry invests \$8500 for 3 years with an interest rate of 7.9%. How much interest will he earn?

ISBN 978-1-009-25891-3

10 In Australian Rules football, a goal is worth 6 points and a behind is worth 1 point. The total number of points, is given by:

$$\text{total number of points} = 6G + B$$

where G is the number of goals and B is the number of behinds kicked.

a Find the number of points if:

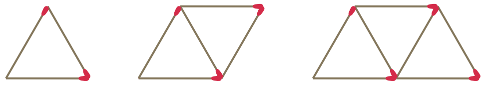
i 2 goals and 3 behinds are kicked

ii 5 goals and 7 behinds are kicked

iii 8 goals and 20 behinds are kicked

b In a match, Redteam scores 4 goals and 2 behinds and Greenteam scores 3 goals and 10 behinds. Which team wins the match?

11 The number of matchsticks used for each shape below follows the pattern $3, 5, 7, \ldots$



The rule for finding the number of matches used in this sequence is:

$$\text{number of matches} = a + (n - 1)d$$

where a is the number of matches in the first shape ($a = 3$), d is the number of extra matches used for each shape ($d = 2$) and n is the shape number.

Find the number of matches in the:

a 6th shape **b** 11th shape **c** 50th shape

ISBN 978-1-009-25891-3

12 Suggested cooking times for roasting x kilograms of meat are given in the following table.

Meat type	Minutes/kilogram
Chicken (well done)	45 min/kg + 20 mins
Lamb (medium)	55 min/kg + 25 mins
Lamb (well done)	65 min/kg + 30 mins
Beef (medium)	55 min/kg + 20 mins
Beef (well done)	65 min/kg + 30 mins

a How long, to the nearest minute, will it take to cook:

i a 2 kg chicken?

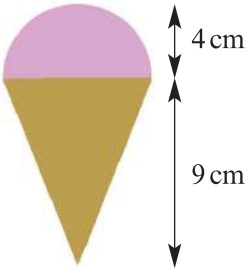
ii 2.25 kg beef (well done)?

iii a piece of lamb weighing 2.4 kg (well done)?

iv 2.5 kg beef (medium)?

b At what time should you put a 2 kg leg of lamb into the oven to have served medium at 7:30 p.m.?

12 An ice cream consists of a wafer in the shape of a cone and a sphere of ice cream on the top, as shown in the diagram on the right. The circular end of the cone has a radius of 4 cm. The formulas for the surface area and volume of cones and spheres are shown below:



surface area of cone $= \pi r\left(r + \sqrt{h^2 + r^2}\right)$

volume of cone $= \dfrac{1}{3}\pi r^2 h$

surface area of sphere $= 4\pi r^2$

volume of sphere $= \dfrac{4}{3}\pi r^3$

where r is the radius of the cone or sphere and h is the height of the cone.

ISBN 978-1-009-25891-3

Use the formulas to find, correct to two decimal places:

a the surface area of ice cream that is above the cone

b the surface area of the wafer

c the volume of ice cream (assume all of the bottom half of the sphere of ice cream is inside the cone)

d the volume of air in the cone

4B Using formulas

At this stage of the chapter, we must determine the value of a variable other than the subject of a formula given the values of the other pronumerals in the formula. This implies that transposition is required to evaluate the unknown variable. If calculators are allowed, we can use the Numerical Solve facility.

Example 4 Using formulas

The formula used to convert degrees Fahrenheit (F) to Celsius (C) is given by $C = \frac{5}{9}(F - 32)$. Use the formula to convert 49℃ into °F.

Solution

Alternative 1: Use Solve facility

1 Write the formula $\quad C = \frac{5}{9}(F - 32)$

2 Substitute the given value of 49 for C $\quad 49 = \frac{5}{9}(F - 32)$

3 Solve using the calculator $\quad \text{Solve}\left(49 = \frac{5}{9}(F - 32),\ F\right)$

4 Write the answer $\quad \therefore F = 120.2$

Alternative 2: Using Transposition of formula

1 Write the formula $\quad C = \frac{5}{9}(F - 32)$

2 Make F the subject of the formula $\quad F = \frac{9C}{5} + 32$

3 Substitute the given value of 49 for C $\quad F = \frac{9(49)}{5} + 32$

4 Write the answer $\quad \therefore F = 120.2$

ISBN 978-1-009-25891-3

Where transposition of formula is complicated, the use of technology is helpful. Use the following steps on the ClassPad calculator.

- Menu
- Num Solve
- Insert your formula
- Input given values
- Solve

Using Numerical Solve facility with formula

Given that $v^2 = u^2 + 2as$, find the value of v ($v > 0$) when $u = 5$, $a = 6$ and $s = 50$.

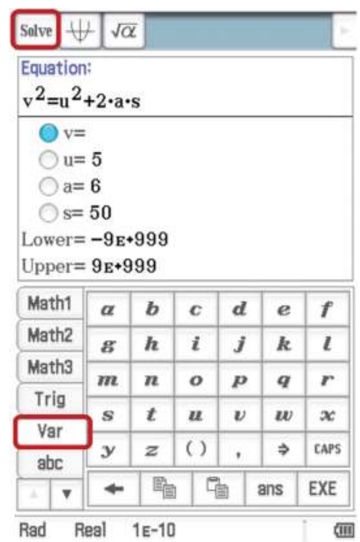
Steps

1 Use the **NumSolve** facility on your calculator:

- Open the **NumSolve** application .
- Open the soft [Keyboard] and select [var] to bring up the variable assignment key.
- Move the cursor to the space under **Equation:** and type the formula using the variables.
  $v^2 = u^2 + 2 \times a \times s$
- Press [EXE] to confirm your entry.

2 Type the values of the variables given. Remember to click the bubble at v as we are solving for v.

3 Select **Solve**.
This gives $v = 25$.



Exercise 4B

1 Given $\frac{1}{x} = \frac{1}{y} + \frac{1}{z}$, evaluate x when $y = 4$ and $z = 10$.

2 Given the formula $v = u + at$, find u when $v = 55$, $a = 10$ and $t = 3$.

3 Given the formula $A = 2\pi r^2 + 2\pi rh$, find h when $r = 4$ and $A = 326.56$.
(Use $\pi = 3.14$)

4 Einstein's famous equation relating energy (E), mass (m) and speed of light (c) is given by $E = mc^2$. Find m when $E = 9 \times 10^{12}$ and $c = 3 \times 10^8$.

5 Given that $s = ut + \frac{1}{2}at^2$, find the value of u when $s = 296.4$, $a = 9.8$ and $t = 6$.

ISBN 978-1-009-25891-3

6 Given that $y = a + bx^2$, find the value of a when $y = 107$, $b = 4$ and $x = -5$.

7 Given that $v^2 = u^2 + 2as$, find the value of s when $u = 5$, $a = 6$ and $v = 25$.

8 Given the formula $y = mx + c$, find m when $y = 3$, $x = -2$ and $c = 11$.

9 The Kinetic Energy (E) is the energy possessed by a body due to its motion and is equal to half the mass (m) of the body times the square of its speed (v). Symbolically, $E = \frac{1}{2}mv^2$.

a Calculate the kinetic energy of a car which has a mass of 1000 kg and is moving at the rate of 25 m/s.

b What is the speed of a horse weighing 345 kg and having a kinetic energy of 1.725×10^4J?

10 Body Mass Index (BMI) is an index of weight-for-height that is commonly used to classify underweight, overweight and obesity in adults. It is defined as the weight (W) in kilograms divided by the square of the height (H) in metres (kg/m^2). Thus,

$$BMI = \frac{W}{H^2}$$

Use the above formula to determine

a the BMI of an adult who weighs 70 kg and whose height is 1.75 m.

b the weight of an adult with BMI of 22.7 and a height of 150 cm.

11 Impulse (I) is defined as the change in the momentum of a body caused over a very short time. If m is the mass and v and u are the final and initial velocities of a body, then $\mathrm{I} = m(v - u)$.

a Calculate the impulse of a body of mass 5 kg whose speed increases from 10 m/s to 15 m/s in a short amount of time.

b Calculate the mass of a body having an impulse of 480 Ns when its speed increases from 18 m/s to 30 m/s.

12 The period of a pendulum (T) can be worked out using the formula

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where T is the time in seconds, l is the length of the pendulum (in metres) and g is the acceleration due to gravity in m/s^2 (use $g = 10$ m/s^2)

a What would be the period of a pendulum if it is 1.5 m long?

b If the pendulum's length in part **a** were to be shortened by one-third its original value, what would be its new period?

ISBN 978-1-009-25891-3

4C Constructing a table of values

We can use a formula to construct a *table of values*. This can be done by substitution (by hand) or using your TI-Nspire or ClassPad.

Example 5 Constructing a table of values

The formula for converting degrees Celsius to degrees Fahrenheit is given by:

$$F = \frac{9}{5}C + 32$$

Use this formula to construct a table of values for F using values of C in intervals of 10 between $C = 0$ and $C = 100$.

Solution

Draw up a table of values for $F = \frac{9}{5}C + 32$, and then substitute values of $C = 0, 10, 20, 30, \ldots$ into the formula to find F.

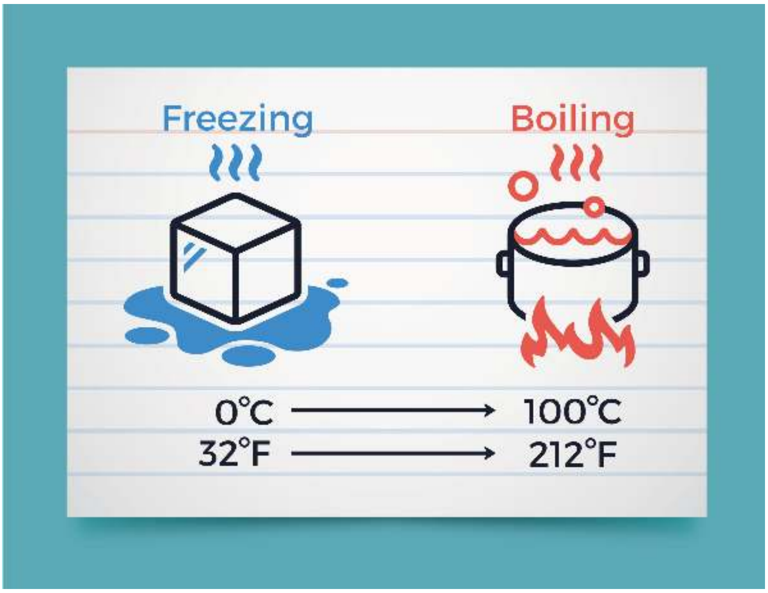
If $C = 0, F = \frac{9}{5}(0) + 32$

If $C = 10, F = \frac{9}{5}(10) + 32 = 50$

and so on.

The table would then look as follows:

C	0	10	20	30	40	50	60	70	80	90	100
F	32	50	68	86	104	122	140	158	176	194	212



ISBN 978-1-009-25891-3

How to construct a table of values using the TI-Nspire CAS

The formula for converting degrees Celsius to degrees Fahrenheit is given by:

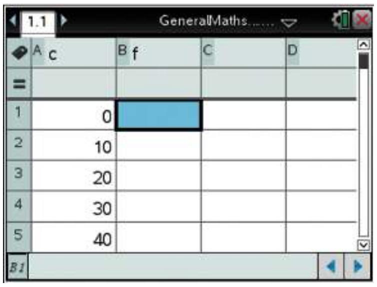
$$F = \frac{9}{5}C + 32$$

Use this formula to construct a table of values for F using values of C in intervals of 10 between $C = 0$ and $C = 100$.

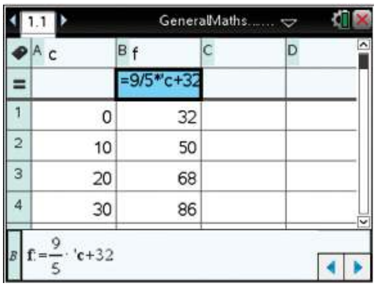
Steps

1 Start a new document: **Press** ctrl + N

2 Select **Add Lists & Spreadsheet**. **Name** the lists c (for Celsius) and f (for Fahrenheit).
Enter the data 0–100 in intervals of 10 into a list named c, as shown.



3 Place cursor in the grey formula cell in column B (i.e. list f) and type in: **= 9 ÷ 5 × c + 32**
Hint: If you typed in c you will need to select **Variable Reference** when prompted. This prompt occurs because c can also be a column name. Alternatively, pressing the var key and selecting **c** from the variable list will avoid this issue.
Press enter to display the values given.
Use the ▼ arrow to move down through the table.



How to construct a table of values using the ClassPad

The formula for converting degrees Celsius to degrees Fahrenheit is given by:

$$F = \frac{9}{5}C + 32$$

Use this formula to construct a table of values for F using values of C in intervals of 10 between $C = 0$ and $C = 100$.

ISBN 978-1-009-25891-3

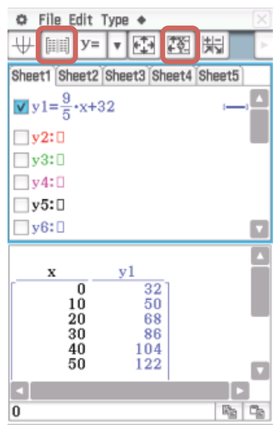
Steps

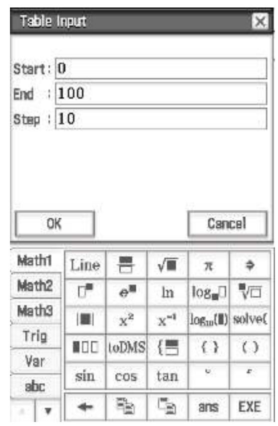
1 Enter the data into your calculator using the **Graph & Table** application. From the application menu screen, locate the built-in **Graph & Table** application, . Tap to open. Tapping Menu from the icon panel (just below the touch screen) will display the application menu if it is not already visible.

2 **a** Adjacent to **y1=** type in the formula $\frac{9}{5}x + 32$. Then press EXE.

b Tap the **Table Input** ( ) icon to set the table entries as shown and tap OK.

c Tap the icon to display the required table of values. Scrolling down will show more values in the table.





Example 6 Constructing a table of values with two variables

The formula for the body mass index, or BMI, of a person is:

$$\text{BMI} = \frac{m}{h^2}$$

where m is the mass of the person in kilograms and h is the height of the person in metres.

a Use the formula to construct a table of values for BMI using values of m in intervals of 10 between $m = 50$ and $m = 100$, and values of h in intervals of 0.2 between $h = 1$ and $h = 2$.

b Use the table to find the BMI for a man with mass 80 kg and height 1.8 m.

Solution

a **1** Determine the values of m in the table.

The values of m will start at 50 and increase by 10, up to 100 .
$m = 50, 60, 70, 80, 90, 100$

ISBN 978-1-009-25891-3

2 Determine the values of h in the table.

The values of h will start at 1 and increase by 0.2, up to 2.
$h = 1, 1.2, 1.4, 1.6, 1.8, 2.0$

3 Draw up a table with one variable in the rows and one variable in the columns.
It won't matter which way it is arranged. Here, the mass is in the rows and height is in the columns.

BMI		Height (m)					
		1	1.2	1.4	1.6	1.8	2.0
Mass (kg)	50						
	60						
	70						
	80						
	90						
	100						

4 Calculate the BMI for each pair of mass and height values and enter them into the table.
For example, the value in the shaded box has been calculated as

$$\text{BMI} = \frac{70}{1.6^2} = 27.34$$

Round answers to two decimal places.

BMI		Height (m)					
		1	1.2	1.4	1.6	1.8	2.0
Mass (kg)	50	50	34.72	25.51	19.53	15.43	12.50
	60	60	41.67	30.61	23.44	18.52	15.00
	70	70	48.61	35.71	27.34	21.60	17.50
	80	80	55.56	40.82	31.25	24.69	20.00
	90	90	62.50	45.92	35.16	27.78	22.50
	100	100	69.44	51.02	39.06	30.86	25.00

b Read the value in the row for 80 kg and the column for 1.8 m, as indicated by the red lines on the table.

The BMI for a man of mass 80 kg and height 1.8 m is 24.69.

Exercise 4C

Example 5

1 A football club wishes to purchase pies at a cost of \$2.15 each. If C is the cost (\$) and x is the number of pies, complete the table showing the amount of money needed to purchase from 40 to 50 pies.

x	40	41	42	43	44	45	46	47	48	49	50
C(\$)	86	88.15	90.3								

ISBN 978-1-009-25891-3

2 The circumference of a circle is given by:

$$C = 2\pi r$$

where r is the radius. Complete the table of values to show the circumferences of circles with radii from 0 to 1 cm in intervals of 0.1 cm. Give your answers correct to three decimal places.

r (cm)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
C (cm)	0	0.628	1.257	1.885							

3 A phone bill is calculated using the formula:

$$C = 40 + 0.18n$$

where C is the total cost and n represents the number of calls made. Complete the table of values to show the cost for 50, 60, 70, . . . 130 calls.

n	50	60	70	80	90	100	110	120	130
C ($)	49	50.80	52.60						

4 The amount of energy (E) in kilojoules expended by an adult male of mass (M) at rest, can be estimated using the formula:

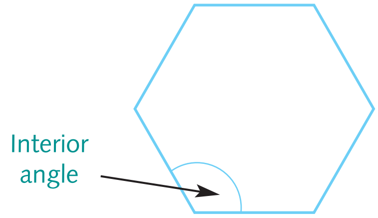
$$E = 110 + 9M$$

Complete the table of values in intervals of 5 kg for males of mass 60–120 kg to show the corresponding values of E.

M (kg)	60	65	70	75	80	85	90	95	100	105	110	115	120
E (kJ)	650	695											

5 The sum, S, of the interior angles of a polygon with n sides is given by the formula:

$$S = 90(2n - 4)$$



Construct a table of values showing the sum of the interior angles of polygons with 3 to 10 sides.

n	3	4	5					
S	180°	360°						

ISBN 978-1-009-25891-3

6 Use the rule $P = M \times T$ to complete the table on the right.

P		T			
		1	2	3	4
M	1				
	2				
	3				
	4				

7 Use the rule $H = \dfrac{R+2}{Z}$ to complete the table on the right. Write the table values as fractions.

H		Z			
		1	2	3	4
R	1				
	2				
	3				
	4				

Example 6

8 A car salesman's weekly wage, E dollars, is given by the formula:

$$E = 60n + 680$$

where n is the number of cars sold.

a Construct a table of values to show how much his weekly wage will be if he sells from 0 to 10 cars.

b Using your table of values, if the salesman earns \$1040 in a week, how many cars did he sell?

9 Anita has \$10 000 that she wishes to invest at a rate of 4.5% per annum. She wants to know how much interest she will earn after $1, 2, 3, \ldots 10$ years. Using the formula:

$$I = \frac{PRT}{100}$$

where P is the principal and R is the interest rate (%), construct a table of values with a calculator to show how much interest, I, she will have after $T = 1, 2, \ldots 10$ years.

10 The formula for finding the amount, A, accumulated at compound interest is given by:

$$A = P \times \left(1 + \frac{r}{100}\right)^t$$

where P is the principal, r is the annual interest rate (%) and t is the time in years. Construct a table of values showing the amount accumulated when \$5000 is invested at a rate of 5.5% over 5, 10, 15, 20 and 25 years. Give your answers to the nearest dollar.

ISBN 978-1-009-25891-3

Example 6 **11** The formula for the cost of sending messages using a particular mobile phone plan, $\$C$, is:

$$C = 0.05t + 0.2p$$

where t is the number of text messages sent and p is the number of photo messages sent.

a Use the formula to construct a table of values for the cost of sending messages using values of t in intervals of 2 between $t = 0$ and $m = 10$ and values of p in intervals of 2 between $p = 0$ and $p = 10$.

b Use the table to find the cost of sending 8 text messages and 4 photo messages.

12 David would like to borrow \$5000 from a bank. The amount of simple interest charged on this loan is given by the formula:

$$I = 50R \times T$$

where I is the interest charged, R is the annual interest rate and T is the number of years before the loan is repaid.

a Use the formula to construct a table of values for the interest charged on the loan using values of R in intervals of 0.2 between $R = 3$ and $R = 4$ and values of T in intervals of 1 between $T = 1$ and $T = 5$.

b Use the table to find the interest charged if the loan has an annual interest rate of 3.8% and is repaid after 4 years.

4D Solving linear equations with one unknown

Practical applications of mathematics often involve the need to be able to solve **linear equations**. An *equation* is a mathematical statement that says that two things are equal. For example, these are all equations:

$$x - 3 = 5 \qquad 2w - 5 = 17 \qquad 3m = 24$$

Linear equations come in many different forms in mathematics but are easy to recognise because the powers on the unknown values are always 1. For example:

- $m - 4 = 8$ is a linear equation, with unknown value m
- $3x = 18$ is a linear equation, with unknown value x
- $4y - 3 = 17$ is a linear equation, with unknown value y
- $a + b = 0$ is a linear equation, with unknown values a and b
- $x^2 + 3 = 9$ is *not* a linear equation (the power of x is 2 not 1), with unknown value x
- $c = 16 - d^2$ is *not* a linear equation (the power of d is 2), with unknowns c and d.

ISBN 978-1-009-25891-3

The process of finding the unknown value is called *solving the equation*. When solving an equation, *opposite* (or *inverse*) operations are used so that the unknown value to be solved is the only **term** remaining on one side of the equation. Opposite operations are indicated in the table below.

Operation	$+$	$-$	$\times$	$\div$	x^2 (power of 2, square)	$\sqrt{x}$ (square root)
Opposite operation	$-$	$+$	$\div$	$\times$	$\sqrt{x}$ (square root)	x^2 (power of 2, square)

Remember: The equation must remain *balanced*. To balance an equation add or subtract the *same* number on *both* sides of the equation or multiply or divide *both* sides of the equation by the *same* number.

Example 7 Solving a linear equation

Solve the equation $x + 6 = 10$.

Solution

Method 1: By inspection

Write the equation. $x + 6 = 10$

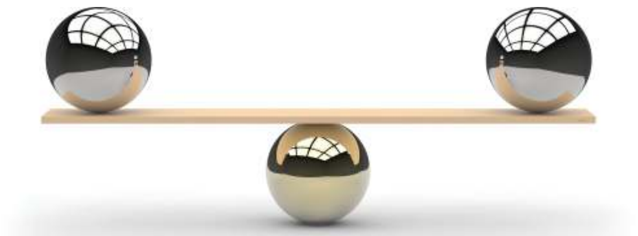
What needs to be added to 6 to make 10?

The answer is 4. $\therefore x = 4$

Method 2: Inverse operations

This method requires the equation to be 'undone', leaving the unknown value by itself on one side of the equation.

1. Write the equation. $x + 6 = 10$
2. Subtract 6 from both sides of the equation. This is the opposite process to adding 6.
$$x + 6 - 6 = 10 - 6$$
$$\therefore x = 4$$
3. Check your answer by substituting the found value for x into the original equation. If each side gives the same value, the solution is correct.
$$\begin{aligned} \text{LHS} &= x + 6 \\ &= 4 + 6 \\ &= 10 \\ &= \text{RHS} \end{aligned}$$
$\therefore$ Solution is correct.



ISBN 978-1-009-25891-3

Example 8 Solving a linear equation by hand

Solve the equation $3y = 18$.

Solution

1 Write the equation.

$$3y = 18$$

2 The opposite process of multiplying by 3 is dividing by 3. Divide both sides of the equation by 3.

$$\frac{3y}{3} = \frac{18}{3}$$

$$\therefore y = 6$$

3 Check that the solution is correct by substituting $y = 6$ into the original equation.

$$\begin{aligned} \text{LHS} &= 3y \\ &= 3 \times 6 \\ &= 18 \\ &= \text{RHS} \end{aligned}$$

$\therefore$ Solution is correct.

Example 9 Solving a linear equation by hand

Solve the equation $4(x - 3) = 24$.

Solution

Method 1

1 Write the equation.

$$4(x - 3) = 24$$

2 Expand the brackets.

$$4x - 12 = 24$$

3 Add 12 to both sides of the equation.

$$4x - 12 + 12 = 24 + 12$$

4 Divide by 4.

$$4x = 36$$

$$\frac{4x}{4} = \frac{36}{4}$$

$$\therefore x = 9$$

5 Check that the solution is correct by substituting $x = 9$ into the original equation (see 4 below).

Method 2

1 Write the equation.

$$4(x - 3) = 24$$

2 Divide by 4.

$$\frac{4(x - 3)}{4} = \frac{24}{4}$$

$$x - 3 = 6$$

3 Add 3.

$$x - 3 + 3 = 6 + 3$$

$$\therefore x = 9$$

4 Check that the solution is correct by substituting $x = 9$ into the orginal equation.

$$\begin{aligned} \text{LHS} &= 4(x - 3) \\ &= 4(9 - 3) \\ &= 4 \times 6 \\ &= 24 \\ &= \text{RHS} \end{aligned}$$

$\therefore$ Solution is correct.

ISBN 978-1-009-25891-3

Example 10 Solving a linear equation using CAS

Solve the equation $-4 - 5b = 8$.

Solution

1 Use the **solve(** command on your CAS calculator to solve for b as shown opposite.

Note: Set the mode of your calculator to Approximate (TI-Nspire) or Decimal (ClassPad) before using **solve(**.

solve $(-4 - 5b = 8, b)$ $\quad b = -2.4$

2 Check that the solution is correct by substituting $x = -2.4$ into the original equation.

$$\begin{aligned} \text{LHS} &= -4 - 5b \\ &= -4 - 5 \times -2.4 \\ &= -4 + 12 \\ &= 8 = \text{RHS} \end{aligned}$$

$\therefore$ Solution is correct.

Exercise 4D

Example 7

1 Solve the following linear equations.

a $x + 6 = 15$	**b** $y + 11 = 26$	**c** $t + 5 = 10$	**d** $m - 5 = 1$
e $g - 3 = 3$	**f** $f - 7 = 12$	**g** $f + 5 = 2$	**h** $v + 7 = 2$
i $x + 11 = 10$	**j** $g - 3 = -2$	**k** $b - 10 = -5$	**l** $m - 5 = -7$
m $2 + y = 8$	**n** $6 + e = 9$	**o** $7 + h = 2$	**p** $3 + a = -1$
q $4 + t = -6$	**r** $8 + s = -3$	**s** $9 - k = 2$	**t** $5 - n = 1$
u $3 - a = -5$	**v** $10 - b = -11$		

Example 8

2 Solve the following linear equations.

a $5x = 15$	**b** $3g = 27$	**c** $9n = 36$	**d** $2x = -16$
e $6j = -24$	**f** $4m = 28$	**g** $2f = 11$	**h** $2x = 7$
i $3y = 15$	**j** $3s = -9$	**k** $-5b = 25$	**l** $4d = -18$
m $\frac{r}{3} = 4$	**n** $\frac{q}{5} = 6$	**o** $\frac{x}{8} = 6$	**p** $\frac{t}{-2} = 6$
q $\frac{h}{-8} = -5$	**r** $\frac{m}{-3} = -7$	**s** $\frac{14}{a} = 7$	**t** $\frac{24}{f} = -12$
u $2a + 15 = 27$	**v** $\frac{y}{4} - 10 = 0$	**w** $13 = 3r - 11$	**x** $\frac{x+1}{3} = 2$
y $\frac{3m}{4} = 6$	**z** $\frac{2x-1}{3} = 4$		

ISBN 978-1-009-25891-3

Example 9

3 Solve the following linear equations.

a $2(y-1)=6$ **b** $8(x-4)=56$ **c** $3(g+2)=12$

d $3(4x-5)=21$ **e** $8(2x+1)=16$ **f** $3(5m-2)=12$

g $\dfrac{2(a-3)}{5}=6$ **h** $\dfrac{4(r+2)}{6}=10$

4 Solve these equations by firstly collecting all like terms.

a $2x=x+5$ **b** $2a+1=a+4$ **c** $4b-10=2b+8$

d $7-5y=3y-17$ **e** $3(x+5)-4=x+11$ **f** $6(c+2)=2(c-2)$

g $2f+3=2-3(f+3)$ **h** $5(1-3y)-2(10-y)=-10y$

Example 10

CAS

5 Solve the following linear equations using CAS. Give answers correct to one decimal place where appropriate.

a $3a+5=11$ **b** $4b+3=27$ **c** $2w+5=9$

d $7c-2=12$ **e** $3y-5=16$ **f** $4f-1=7$

g $3+2h=13$ **h** $2+3k=6$ **i** $-4(g-4)=-18$

j $\dfrac{2(s-6)}{7}=4$ **k** $\dfrac{5(t+1)}{2}=8$ **l** $\dfrac{-4(y-5)}{5}=2.4$

m $2(x-3)+4(x+7)=10$ **n** $5(g+4)-6(g-7)=25$ **o** $5(p+4)=25+(7-p)$

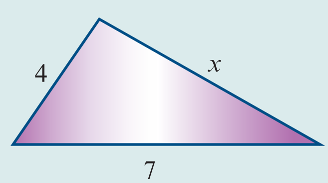
4E Developing a formula: setting up linear equations in one unknown

In many practical problems, we often need to set up a linear equation before finding the solution to a problem. Some practical examples are given below showing how a linear equation is set up and then solved.

Example 11 Setting up a linear equation

Find an equation for the perimeter of the triangle shown.

Note: Perimeter is the distance around the outside of a shape.



Solution

1 Choose a variable to represent the perimeter. Let P be the perimeter.

2 Add up all sides of the triangle and let them equal the perimeter, P.
$P=4+7+x$
$P=11+x$

3 Write your answer. The required equation is $P=11+x$

ISBN 978-1-009-25891-3

Example 12 Setting up and solving a linear equation

If 11 is added to a certain number, the result is 25. Find the number.

Solution

1 Choose a variable to represent the number.	Let n be the number.
2 Using the information, write an equation.	$n + 11 = 25$
3 Solve the equation by subtracting 11 from both sides of the equation.	$n + 11 - 11 = 25 - 11$ $\therefore n = 14$
4 Write your answer.	The required number is 14.

Example 13 Setting up and solving a linear equation

At a recent show, Chris spent \$100 on 8 showbags, each costing the same price.

a Using x as the cost of one showbag, write an equation showing the cost of 8 showbags.

b Use the equation to find the cost of one showbag.

Solution

a **1** Write the cost of one showbag using the variable given.	Let x be the cost of one showbag.
2 Use the information to write an equation. Remember: $8 \times x = 8x$	$8x = 100$
b **1** Write the equation.	$8x = 100$
2 Solve the equation by dividing both sides of the equation by 8.	$\frac{8x}{8} = \frac{100}{8}$ $\therefore x = 12.5$
3 Write your answer.	The cost of one showbag is \$12.50.

ISBN 978-1-009-25891-3

Example 14 Setting up and solving a linear equation

A car rental company has a fixed charge of \$110 plus \$84 per day for the hire of a car. The Brown family have budgeted \$650 for the hire of a car during their family holiday. For how many days can they hire a car?

Solution

1 Choose a variable (d) for the number of days that the car is hired for. Use the information to write an equation.

Let d be the number of days that the car is hired for.

$$110 + 84d = 650$$

2 Solve the equation.
First, subtract 110 from both sides of the equation.
Then divide both sides of the equation by 84.

$$110 + 84d - 110 = 650 - 110$$
$$84d = 540$$
$$\frac{84d}{84} = \frac{540}{84}$$
$$\therefore d = 6.428$$

3 Write your answer in terms of complete days.

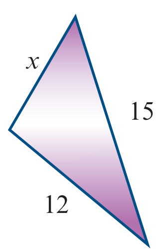
Car hire works on a daily rate so 6.428 days is not an option. We therefore round down to 6 days to ensure that the Brown family stays within their budget of \$650.
The Brown family could hire a car for 6 days.

Exercise 4E

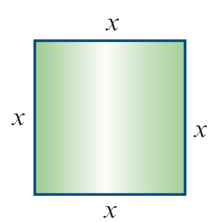
Example 11

1 Find an expression for the perimeter, P, of each of the following shapes.

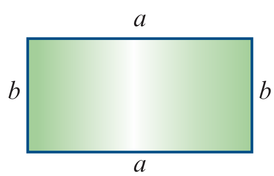
a



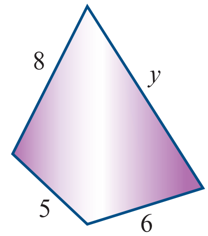
b



c



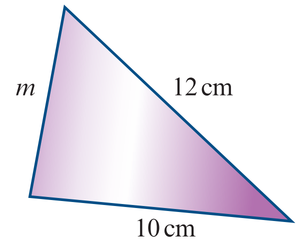
d



ISBN 978-1-009-25891-3

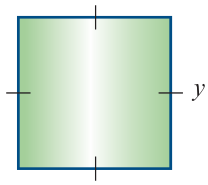
2 **a** Write an expression for the perimeter of the triangle shown.

b If the perimeter, P, of the triangle is 30 cm, solve the equation to find the value of m.



3 **a** Write an equation for the perimeter of the square shown.

b If the perimeter is 52 cm, what is the length of one side?



Example 12

4 Seven is added to a number and the result is 15.

a Write an equation using n to represent the number.

b Solve the equation for n.

5 Five is added to twice a number and the result is 17. What is the number?

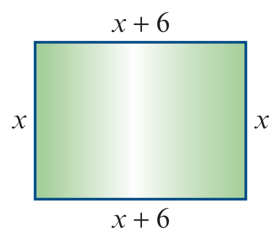
6 When a number is doubled and 15 is subtracted, the result is 103. Find the number.

7 The perimeter of a rectangle is 84 cm. The length of the rectangle is 6 cm longer than the width, as shown in the diagram.

a Write an expression for the perimeter, P, of the rectangle.

b Find the value of x.

c Find the lengths of the sides of the rectangle.



Example 13
Example 14

8 Year 11 students want to run a social. The cost of hiring a band is \$820 and they are selling tickets for \$15 per person. The profit, P, is found by subtracting the band hire cost from the money raised from selling tickets. The students would like to make a profit of \$350. Use the information to write an equation, and then solve the equation to find how many tickets they need to sell.

9 The cost for printing cards at the Stamping Printing Company is \$60 plus \$2.50 per card. Kate paid \$122.50 to print invitations for her party. How many invitations were printed?

10 A raffle prize of \$1000 is divided between Anne and Barry so that Anne receives 3 times as much as Barry. How much does each receive?

ISBN 978-1-009-25891-3

11 Bruce cycles x kilometres, then walks half as far as he cycles. If the total distance covered is 45 km, find the value of x.

12 Amy and Ben live 17.2 km apart. They cycle to meet each other. Ben travels at 12 km/h and Amy travels at 10 km/h.

a How long, to the nearest minute, until they meet each other?

b What distance, correct to one decimal place, have they both travelled?

4F Developing a formula: setting up linear equations in two unknowns

It is often necessary to develop formulas so that problems can be solved. Constructing a formula is similar to developing an equation from a description.

Example 15 Setting up and solving a linear equation in two unknowns

Sausage rolls cost \$1.30 each and party pies cost 75 cents each.

a Construct a formula for finding the cost, C dollars, of buying x sausage rolls and y party pies.

b Find the cost of 12 sausage rolls and 24 party pies.

Solution

a **1** Work out a formula using x.
One sausage roll costs \$1.30.
Two sausage rolls cost $2 \times \$1.30 = \$2.60$.
Three sausage rolls cost $3 \times \$1.30 = \$3.90$ *etc.*
Write a formula using x.

x sausage rolls cost
$x \times 1.30 = 1.3x$

2 Work out a formula using y.
One party pie costs \$0.75.
Two party pies cost $2 \times \$0.75 = \$1.50$.
Three party pies cost $3 \times \$0.75 = \$2.25$ *etc.*
Write a formula using y.

y party pies cost
$y \times 0.75 = 0.75y$

3 Combine to get a formula for total cost, C.

$C = 1.3x + 0.75y$

b **1** Write the formula for C.

$C = 1.3x + 0.75y$

2 Substitute $x = 12$ and $y = 24$ into the formula.

$C = 1.3 \times 12 + 0.75 \times 24$

3 Evaluate.

$C = 33.6$

4 Give your answer in dollars and cents.

The total cost for 12 sausage rolls and 24 party pies is \$33.60.

ISBN 978-1-009-25891-3

Skill-sheet

Exercise 4F

Example 15

1 Balloons cost 50 cents each and streamers costs 20 cents each.

a Construct a formula for the cost, C, of x balloons and y streamers.

b Find the cost of 25 balloons and 20 streamers.

2 Tickets to a concert cost \$40 for adults and \$25 for children.

a Construct a formula for the total amount, C, paid by x adults and y children.

b How much money altogether was paid by 150 adults and 315 children?

3 At the football canteen, chocolate bars cost \$1.60 and muesli bars cost \$1.40.

a Construct a formula to show the total money, C, made by selling x chocolate bars and y muesli bars.

b How much money would be made if 55 chocolate bars and 38 muesli bars were sold?

4 At the bread shop, custard tarts cost \$1.75 and iced doughnuts \$0.70 cents.

a Construct a formula to show the total cost, C, if x custard tarts and y iced doughnuts are purchased.

b On Monday morning, Mary bought 25 custard tarts and 12 iced doughnuts. How much did it cost her?

5 At the beach café, Marion takes orders for coffee and milkshakes. A cup of coffee costs \$3.50 and a milkshake costs \$5.00.

a Let x = number of coffees ordered and y = number of milkshakes ordered. Using x (coffee) and y (milkshakes) write a formula showing the cost, C, of the number of coffee and milkshakes ordered.

b Marion took orders for 52 cups of coffee and 26 milkshakes. How much money did this make?

ISBN 978-1-009-25891-3

6 Joe sells budgerigars for \$30 and parrots for \$60.

a Write a formula showing the money, C, made by selling x budgerigars and y parrots.

b Joe sold 28 parrots and 60 budgerigars. How much money did he make?

7 James has been saving fifty-cent and twenty-cent pieces.

a If James has x fifty-cent pieces and y twenty-cent pieces, write a formula to show the number, N, of coins that James has.

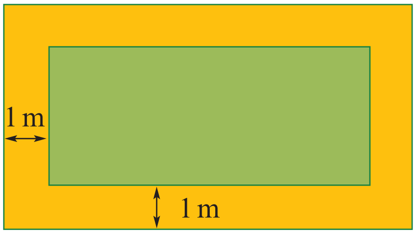
b Write a formula to show the value, V dollars, of James' collection.

c When James counts his coins, he has 45 fifty-cent pieces and 77 twenty-cent pieces. How much money does he have in total?

8 A tennis coach buys four cans of tennis balls and empties them into a large container. The container already has twelve balls in it. Altogether, there are now 32 tennis balls. How many tennis balls were in each can?

9 Maria is five years older than George. The sum of their ages is 37. What are their ages?

10 A rectangular lawn is twice as long as it is wide. The lawn has a path 1 metre wide around it. The length of the perimeter of the outside of the path is 48 metres. What is the width of the lawn? Give your answer correct to the nearest centimetre.



ISBN 978-1-009-25891-3

Review

Key ideas and chapter summary

Assign-ment

Formula A **formula** is a mathematical relationship connecting two or more variables.

Linear equation A **linear equation** is an equation whose unknown values are always to the power of 1.

Non-linear equation A **non-linear equation** is one whose unknown values are *not* all to the power of 1.

Skills check

Having completed the current chapter you should be able to:

- substitute values in linear and non-linear relations and formulas
- construct tables of values from given formulas
- solve linear equations
- use linear equations to solve practical problems
- develop formulas from descriptions

Short-answer questions

1 If $a = 4$, state the value of $3a + 5$.

2 If $b = 1$, state the value of $2b - 9$.

3 If $C = 50t + 14$ and $t = 8$, then the value of C is?

4 If $P = 2L + 2W$, determine the value of P when $L = 6$ and $W = 2$.

5 If $x = -2, y = 3$ and $z = 7$, then state the value of $\frac{z - x}{y}$.

6 If $a = 2, b = 5, c = 6$ and $d = 10$, then evaluate $bd - ac$.

7 The area of a circle is given by $A = \pi r^2$. If $r = 6$ cm, calculate the area of the circle, correct to two decimal places.

8 Solve $4x = 24$.

9 Solve $\frac{x}{3} = -8$.

ISBN 978-1-009-25891-3

10 Solve $2v + 5 = 11$.

11 Solve $3k - 5 = -14$.

12 The cost of hiring a car for a day is \$60 plus 0.25c per kilometre. Michelle travels 750 kilometres. Find their total cost.

13 Given $v = u + at$ and $v = 11.6$ when $u = 6.5$ and $a = 3.7$, calculate the value of t correct to two decimal places.

14 The area of a trapezium is given by $A = \dfrac{(a + b)}{2}h$.
Determine an expression for h in terms of A, a and b.

15 Solve the following equations for x.

a $x + 5 = 15$ **b** $x - 7 = 4$ **c** $16 + x = 24$
d $9 - x = 3$ **e** $2x + 8 = 10$ **f** $3x - 4 = 17$
g $x + 4 = -2$ **h** $3 - x = -8$ **i** $6x + 8 = 26$
j $3x - 4 = 5$ **k** $\dfrac{x}{5} = 3$ **l** $\dfrac{x}{-2} = 12$

16 If $P = 2l + 2b$, find P if:

a $l = 12$ and $b = 8$ **b** $l = 40$ and $b = 25$

17 If $A = \dfrac{1}{2}bh$, find A if:

a $b = 6$ and $h = 10$ **b** $b = 12$ and $h = 9$

18 The formula for finding the circumference of a circle is given by $C = 2\pi r$, where r is the radius. Find the circumference of a circle with radius 15 cm, correct to two decimal places.

19 The formula for the volume of a cylinder is given by $V = \pi r^2 h$, where r is the radius of the circle cross-section of the cylinder and h is the height of the cylinder. Find the volume of a cylinder with a cross-section radius of 3 cm and a height of 15 cm. Round your answer to two decimal places.

20 For the equation $y = 33x - 56$,

a Construct a table of values for values of x in intervals of 5 from -20 to 25.
b For what value of x is $y = 274$?
c When $y = -221$, what value is x?

21 For the equation $k = 2h + g$, construct a table of values for values of h and g in intervals of 1 from -2 to 2.

ISBN 978-1-009-25891-3

22 I think of a number, double it and add 4. If the result is 6, what is the original number?

23 Four less than three times a number is 11. What is the number?

Extended-response questions

1 The cost, C, of hiring a boat is given by $C = 8h + 25$ where h represents hours.

a What is the cost if the boat is hired for 4 hours?

b For how many hours was the boat hired if the cost was \$81?

2 A phone bill is calculated using the formula $C = 25 + 0.50n$, where n is the number of calls made.

a Complete the table of values below for values of n from 60 to 160.

n	60	70	80	90	100	110	120	130	140	150	160
C											

b What is the cost of making 160 phone calls?

3 A ticket to see a play costs \$89.00 per adult and \$42 per child. The total price, \$$P$, for a adult tickets and c child tickets is calculated using the formula $P = 89 \times a + 42 \times c$.

a Construct a table of values that show the price of play tickets for values of a and c in intervals of 1 between 0 and 5.

b What is the total price of 3 adult and 1 child tickets to the play?

4 An electrician charges \$80 up front and \$45 for each hour, h, that he works.

a Write a linear equation for the total charge, C, of any job.

b How much would a 3-hour job cost?

5 Aadya is a full-time electrician and the cost (C \$) for hiring him is made up of a callout fee of \$80 and a fixed rate of \$42 per hour. The equation for hiring Aadya for h hours is given by $C = 0.95(80 + 42h)$.

a Aadya offers a discount to all his clients. State how much discount he allows to his clients.

b Calculate the cost of hiring Aadya for 6 hours.

c Mrs Will paid \$335.35 for some repairs undergone. For how many hours did she hire the electrician?

ISBN 978-1-009-25891-3

Review

6 Appleby Senior High usually hires a school bus to pick and drop its students. The cost (C \$) associated with hiring the bus for d days and travelling k kilometres is given by

$$C = 50 + 25d + 10k$$

a Explain the significance of 50 in the above equation.

b Calculate the cost of hiring the bus for 10 days and travelling a total distance of 165 km.

c In a particular month, the school hired the bus for 21 days for a total cost of \$3005. Determine the number of kilometres the bus travelled.

d The school principal was concerned that the cost of hiring the school bus was way past the available budget. She appointed all the top mathematics teachers and assigned them a task to work out a minimal spanning tree whereby all the students using the school bus could be picked up and dropped by travelling the least possible distance. If the school's budget is \$2500 for 21 days, determine the maximum distance that the bus could travel without exceeding the budget.

ISBN 978-1-009-25891-3

5 Matrices

In this chapter

5A The basics of a matrix
5B Using matrices to model (represent) practical solutions
5C Adding and subtracting matrices
5D Scalar multiplication
5E Matrix multiplication
5F Applications of matrices
5G Communications and connections
5H Identity matrices
5I Extended application and problem-solving tasks
Chapter summary and review

Syllabus references

Topic: Matrices and matrix arithmetic

Subtopics: 1.2.4 – 1.2.7

A **matrix** (plural matrices) is a rectangular group of numbers set out in rows and columns. Matrices can be used to store information, solve sets of simultaneous equations, find optimal solutions in business, analyse networks, transform shapes in geometry, encode information and devise the best strategies in game theory. We will explore some of these applications while learning the basic theory of matrices.

ISBN 978-1-009-25891-3

5A The basics of a matrix

A market stall operates on Friday and Saturday. Sales could be recorded using matrix A.

Matrix A :

$$A = \begin{array}{c} \\ \textit{Friday} \\ \textit{Saturday} \\ \\ \end{array} \begin{array}{ccc} \textit{Shirts} & \textit{Jeans} & \textit{Belts} \\ \left[\begin{matrix} 6 \\ 3 \end{matrix}\right. & \begin{matrix} 8 \\ 7 \end{matrix} & \left.\begin{matrix} 4 \\ 1 \end{matrix}\right] \\ \textit{column } 1 & \textit{column } 2 & \textit{column } 3 \end{array} \begin{array}{c} \\ \textit{row } 1 \\ \textit{row } 2 \\ \\ \end{array}$$

Rows	Columns
Friday sales are listed in row 1.	The number of shirts sold is listed in column 1.
Saturday sales are listed in row 2.	The number of pairs of jeans sold is listed in column 2.
	The number of belts sold is listed in column 3.

We can read the following information from the matrix:

- On Friday 8 pairs of jeans were sold.
- On Saturday 1 belt was sold.
- The total number of items sold on Friday was $6 + 8 + 4 = 18$.
- The total number of belts sold was $4 + 1 = 5$.

Order of a matrix

The **order** (or size) **of a matrix** is written as: number of rows × number of columns

$$\begin{array}{c} \textit{row } 1 \\ \textit{row } 2 \end{array} \begin{bmatrix} 6 & 8 & 4 \\ 3 & 7 & 1 \end{bmatrix} \qquad \begin{array}{ccc} \left[\begin{matrix} 6 \\ 3 \end{matrix}\right. & \begin{matrix} 8 \\ 7 \end{matrix} & \left.\begin{matrix} 4 \\ 1 \end{matrix}\right] \\ \textit{column } 1 & \textit{column } 2 & \textit{column } 3 \end{array}$$

Think: 'rows in a cinema'

Think: 'columns of the Parthenon'

The order of matrix A in the market stall example above is 2×3; that is, 2 rows × 3 columns. It is called a 'two by three' matrix.

In writing down the order of a matrix, the number of rows is always given first, then the number of columns. *Rows first*, then *columns*.

ISBN 978-1-009-25891-3

That is, for any matrix:

- The number of rows is denoted by $\boldsymbol{m}$.
- The number of columns is denoted by $\boldsymbol{n}$.
- The order can therefore be written as $\boldsymbol{m} \times \boldsymbol{n}$.

Remember: When you walk into a cinema, you go to your *row first*.

Matrices are usually named using capital letters such as A, B, O.

Elements of a matrix

The numbers within a matrix are called its **elements**.

Locating an element in a matrix

a_{ij} is the element in *row i*, *column j*.

For example, in the matrix:

$$A = \begin{bmatrix} 6 & 8 & 4 \\ 3 & 7 & 1 \end{bmatrix}$$

- element a_{13} is in row 1, column 3 and its value is 4
- element a_{22} is in row 2, column 2 and its value is 7.

Example 1 Interpreting the elements of a matrix

Matrix B shows the number of boys and girls in years 10 to 12 at a particular school.

$$B = \begin{array}{r c} & \begin{array}{cc} \textit{Boys} & \textit{Girls} \end{array} \\ \begin{array}{r} \textit{Year 10} \\ \textit{Year 11} \\ \textit{Year 12} \end{array} & \begin{bmatrix} 57 & 63 \\ 48 & 54 \\ 39 & 45 \end{bmatrix} \end{array}$$

a Give the order of matrix B.
b What information is given by the element b_{12}?
c Which element gives the number of girls in year 12?
d How many boys in total?
e How many students in year 11?

Solution

a Count the rows, count the columns.
Remember: Order is rows × columns.

The order of matrix B is 3×2.

b The element b_{12} is in row 1 and column 2. This is where the year 10 row meets the girls column.

There are 63 girls in year 10.

ISBN 978-1-009-25891-3

c Year 12 is row 3. Girls are column 2. — The number of year 12 girls is given by b_{32}.

d The sum of the boys column gives the total number of boys. — The total number of boys is 144.

e The sum of the year 11 row gives the total number of students in year 11. — There are 102 students in year 11.

Row matrices

A **row matrix** has a *single row* of elements.

In matrix A, the Friday sales from the market stall can be represented by a 1×3 *row* matrix.

$$A = \begin{array}{c} \\ \textit{Friday} \\ \textit{Saturday} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Shirts} & \textit{Jeans} & \textit{Belts} \end{array} \\ \begin{bmatrix} 6 & 8 & 4 \\ 3 & 7 & 1 \end{bmatrix} \end{array} \qquad \begin{array}{c} \\ \textit{Friday} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Shirts} & \textit{Jeans} & \textit{Belts} \end{array} \\ \begin{bmatrix} 6 & 8 & 4 \end{bmatrix} \end{array}$$

Column matrices

A **column matrix** has a *single column* of elements.

In matrix A, the sales of jeans from the market stall can be represented by a 2×1 *column* matrix.

$$\begin{array}{c} \\ \textit{Friday} \\ \textit{Saturday} \end{array} \begin{array}{c} \textit{Jeans} \\ \begin{bmatrix} 8 \\ 7 \end{bmatrix} \end{array}$$

Although they appear to be very simple, row and column matrices have useful properties that will be explored in this chapter.

Square matrices

In **square matrices** the number of *rows* equals the number of *columns*.

Here are three examples.

$$\underset{1 \times 1}{[9]} \qquad \underset{2 \times 2}{\begin{bmatrix} 5 & 4 \\ 4 & 2 \end{bmatrix}} \qquad \underset{3 \times 3}{\begin{bmatrix} 0 & 4 & 3 \\ 8 & 1 & 6 \\ 2 & 0 & 7 \end{bmatrix}}$$

ISBN 978-1-009-25891-3

How to enter a matrix using the TI-Nspire CAS

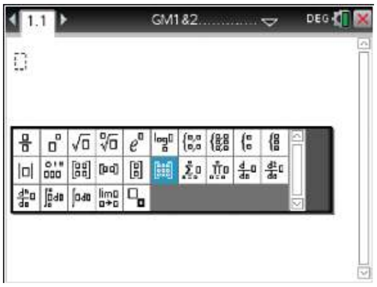
Enter the matrix $B = \begin{bmatrix} 57 & 63 \\ 48 & 51 \\ 39 & 45 \end{bmatrix}$ into the TI-Nspire CAS. Display the element b_{21}.

Steps

1 Press [on] **>New Document>Add Calculator**.

2 Press [templates] and use the cursor ▼ ▶ arrows to highlight the matrix template shown. Press [enter].

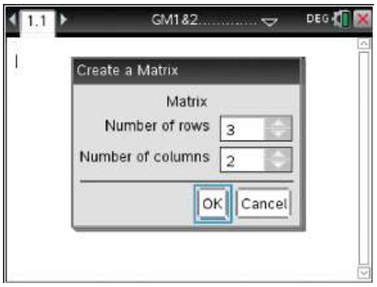
Note: Math Templates can also be accessed by pressing [ctrl] + [menu] **>Math Templates**.



3 Press ◀ then ▲ or ▼ to select the **Number of rows** required (number of rows in this example is 3).

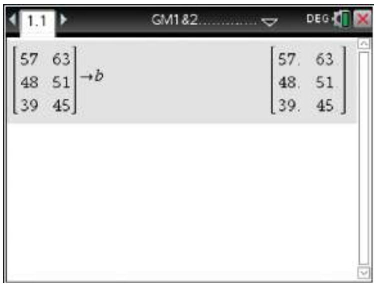
Press [tab] to move to the next entry and repeat for the **Number of columns** (the number of columns in this example is 2).

Press [tab] to highlight **OK** and press [enter].



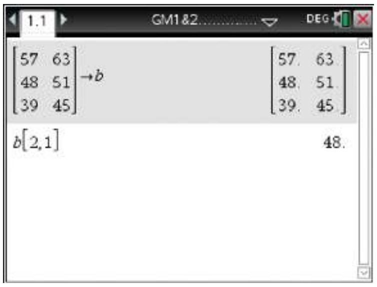
4 Type in the values into the matrix template. Use [tab] or the arrow keys to move to the required position in the matrix to enter each value.

When the matrix has been completed press [tab] or ▶ to move outside the matrix and press [ctrl]+[var] followed by [B]. This will store the matrix as the variable **b**. Press [enter].



5 When you type B (or b) in the graphics calculator, it will paste in the matrix $\begin{bmatrix} 57 & 63 \\ 48 & 51 \\ 39 & 45 \end{bmatrix}$.

6 To display element b_{21} (the element in position Row 2, Column 1), type in **b[2,1]** and press [enter].

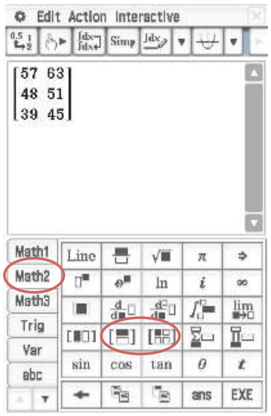


ISBN 978-1-009-25891-3

How to enter a matrix using the ClassPad

Enter the matrix $B = \begin{bmatrix} 57 & 63 \\ 48 & 51 \\ 39 & 45 \end{bmatrix}$ into the ClassPad calculator.

Steps



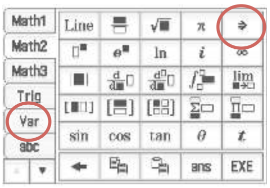
1 Open the soft Keyboard in the **Main** application $\sqrt{\alpha}$.

2 Select the Math2 keyboard.

3 Tap the 2×2 matrix icon followed by the 2×1 matrix icon. This will add a third row and create a 3×2 matrix.

4 Enter the values of $\begin{bmatrix} 57 & 63 \\ 48 & 51 \\ 39 & 45 \end{bmatrix}$.

Note: Tap in each new position to enter the new value or use the cursor key on the hard keyboard to navigate to a new position.

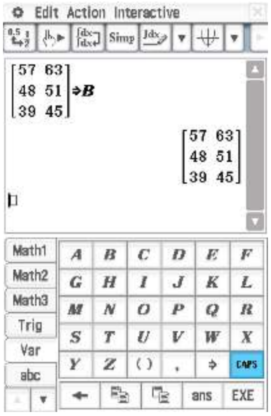


5 To assign the matrix the variable name B.

a Move the cursor to the very right-hand side of the matrix.

b From the keyboard, tap the variable assignment key ⇒, followed by the var, then CAPS (for uppercase letters) and ***B***. Press EXE to confirm your choice.

Note: Until it is reassigned, ***B*** will represent the matrix as defined above.



ISBN 978-1-009-25891-3

Exercise 5A

Example 1

1 Matrix C is shown on the right.

$$C = \begin{bmatrix} 2 & 4 & 16 & 7 \\ 6 & 8 & 9 & 3 \\ 5 & 6 & 10 & 1 \end{bmatrix}$$

a Write down the order of the matrix C.

b State the value of:

i c_{13} **ii** c_{24} **iii** c_{31}

c Find the sum of the elements in row 3.

d Find the sum of the elements in column 2.

2 For each of the following matrices:

i state the order **ii** find the values of the required elements.

a $A = \begin{bmatrix} 5 & 6 & 8 \\ 4 & 7 & 9 \end{bmatrix}$ Find a_{12} and a_{22}

b $B = \begin{bmatrix} 6 & 8 & 2 \end{bmatrix}$ Find b_{13} and b_{11}

c $C = \begin{bmatrix} 4 & 5 \\ 3 & 1 \\ 8 & -4 \end{bmatrix}$ Find c_{32} and c_{12}

d $D = \begin{bmatrix} 8 \\ 6 \\ 9 \end{bmatrix}$ Find d_{31} and d_{11}

e $E = \begin{bmatrix} 10 & 12 \\ 15 & 13 \end{bmatrix}$ Find e_{21} and e_{12}

f $F = \begin{bmatrix} 8 & 11 & 2 & 6 \\ 4 & 1 & 5 & 7 \\ 6 & 14 & 17 & 20 \end{bmatrix}$ Find f_{34} and f_{23}

3 Name which of the matrices in Question **2** are:

a row matrices **b** column matrices **c** square matrices.

4 For matrix D, give the values of the following elements.

$$D = \begin{bmatrix} 3 & 4 & 6 & 11 & 2 \\ 5 & 1 & 9 & 10 & 4 \\ 8 & 7 & 2 & 0 & 1 \\ 6 & 8 & 5 & 8 & 2 \end{bmatrix}$$

a d_{23} **b** d_{45}

c d_{11} **d** d_{24}

e d_{42}

5 From Question **4**, enter the matrix D into your CAS calculator. Use it to check your answers to Question **4**. CAS

ISBN 978-1-009-25891-3

6 Some students were asked which of four sports they preferred to play and the results were entered in the following matrix.

$$S = \begin{array}{c} \\ \textit{Year 10} \\ \textit{Year 11} \\ \textit{Year 12} \end{array} \begin{array}{c} \begin{array}{cccc} \textit{Tennis} & \textit{Basketball} & \textit{Football} & \textit{Hockey} \end{array} \\ \begin{bmatrix} 19 & 18 & 31 & 14 \\ 16 & 32 & 22 & 12 \\ 21 & 25 & 5 & 7 \end{bmatrix} \end{array}$$

a How many year 11 students preferred basketball?

b Write down the order of matrix S.

c What information is given by s_{23}?

7 $A = \begin{bmatrix} 2 & 7 & -1 \\ 0 & 5 & 3 \\ -3 & 4 & 8 \\ 7 & 6 & -6 \end{bmatrix}$ $B = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$ $C = \begin{bmatrix} 8 & -2 \end{bmatrix}$ $D = \begin{bmatrix} 4 & -3 & 0 & 1 & 9 \\ 6 & 11 & 2 & 7 & 5 \end{bmatrix}$

a Write down the order of each matrix A, B, C and D.

b Identify the elements: a_{32}, b_{21}, c_{11} and d_{24} of matrices A, B, C and D respectively.

8 Matrix F shows the number of hectares of land used for different purposes on two farms, X and Y.
Row 1 represents Farm X and row 2 represents Farm Y. Columns 1, 2 and 3 show the amount of land used for wheat, cattle and sheep (W, C, S) respectively, in hectares.

$$F = \begin{array}{c} \begin{array}{ccc} W & C & S \end{array} \\ \begin{bmatrix} 150 & 300 & 75 \\ 200 & 0 & 350 \end{bmatrix} \end{array} \begin{array}{c} \\ X \\ Y \end{array}$$

a How many hectares are used on:

i Farm X for sheep?

ii Farm X for cattle?

iii Farm Y for wheat?

b Calculate the total number of hectares used on both farms for wheat.

c Write down the information that is given by:

i f_{22} **ii** f_{13} **iii** f_{11}

d Which element of matrix F gives the number of hectares used:

i on Farm Y for sheep?

ii on Farm X for cattle?

iii on Farm Y for wheat?

e State the order of matrix F.

ISBN 978-1-009-25891-3

5B Using matrices to model (represent) practical situations

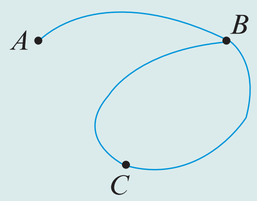
A **network** is a diagram of points (vertices) joined by lines (edges). It can be used to show **connections** or relationships. The information in network diagrams can be recorded in a matrix and used to solve related problems.

Example 2 Using a matrix to represent connections

The network diagram drawn shows the ways to travel between three towns, A, B and C.

a Use a matrix to represent the connections. Each element should describe the number of ways to travel *directly* from one town to another.

b What information is given by the sum of the second column of the matrix?



Solution

a As there are three towns, A, B and C, use a 3×3 matrix to show the direct connections.

There are 0 roads directly connecting any town to itself. So enter 0 where column A crosses row A, and so on.

$$\begin{array}{cc} \begin{array}{ccc} A & B & C \end{array} & \\ \begin{bmatrix} 0 & & \\ & 0 & \\ & & 0 \end{bmatrix} & \begin{array}{c} A \\ B \\ C \end{array} \end{array}$$

If there were a road directly connecting town A to itself, it would be a loop from A back to A.

There is one road directly connecting B to A (or A to B). So enter 1 where column B crosses row A and where column A crosses row B.

$$\begin{array}{cc} \begin{array}{ccc} A & B & C \end{array} & \\ \begin{bmatrix} 0 & 1 & \\ 1 & 0 & \\ & & 0 \end{bmatrix} & \begin{array}{c} A \\ B \\ C \end{array} \end{array}$$

There are no direct roads between C and A. So enter 0 where column C crosses row A and where column A crosses row C.

$$\begin{array}{cc} \begin{array}{ccc} A & B & C \end{array} & \\ \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & \\ 0 & & 0 \end{bmatrix} & \begin{array}{c} A \\ B \\ C \end{array} \end{array}$$

There are 2 roads between C and B. Enter 2 where column C crosses row B and where column B crosses row C.

$$\begin{array}{cc} \begin{array}{ccc} A & B & C \end{array} & \\ \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix} & \begin{array}{c} A \\ B \\ C \end{array} \end{array}$$

b The second column shows the number of roads directly connected to town B.

The sum of the second column is the total number of roads directly connected to town B.

$1 + 0 + 2 = 3$

ISBN 978-1-009-25891-3

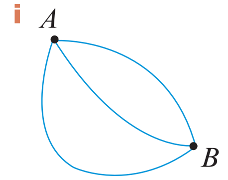
Exercise 5B

Example 2

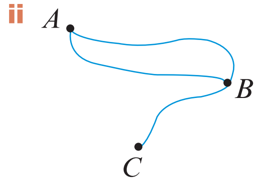
1 The road network show roads connecting towns.

a In each case use a matrix to record the number of ways of travelling *directly* from one town to another.

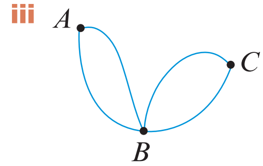
i



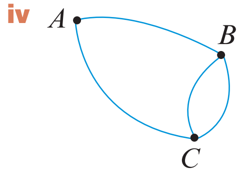
ii



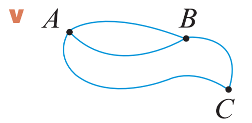
iii



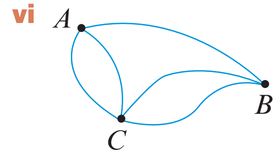
iv



v



vi



b What does the sum of the second column of each matrix represent?

2 The matrices record the number of ways of going directly from one town to another.

a In each case draw graphs to show the direct connections between towns A, B and C.

i $\begin{array}{cc} \begin{array}{ccc} A & B & C \end{array} & \\ \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} & \begin{array}{c} A \\ B \\ C \end{array} \end{array}$

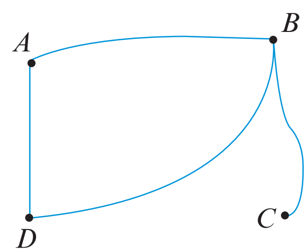
ii $\begin{array}{cc} \begin{array}{ccc} A & B & C \end{array} & \\ \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} & \begin{array}{c} A \\ B \\ C \end{array} \end{array}$

iii $\begin{array}{cc} \begin{array}{ccc} A & B & C \end{array} & \\ \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix} & \begin{array}{c} A \\ B \\ C \end{array} \end{array}$

iv $\begin{array}{cc} \begin{array}{ccc} A & B & C \end{array} & \\ \begin{bmatrix} 0 & 2 & 2 \\ 2 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix} & \begin{array}{c} A \\ B \\ C \end{array} \end{array}$

b State the information that is given by the sum of the first column in the matrices of part **a**.

3 The network diagram opposite has lines showing which people from the four people A, B, C and D have met.



a Represent the graph using a matrix. Use 0 when two people have *not* met and 1 when they have met.

b How can the matrix be used to tell who has met the most people?

c Who has met the most people?

d Who has met the least number of people?

ISBN 978-1-009-25891-3

5C Adding and subtracting matrices

Rules for adding and subtracting matrices

1. Matrices are added by adding the elements that are in the same positions.
2. Matrices are subtracted by subtracting the elements that are in the same positions.
3. **Matrix addition and subtraction** can only be done if the two matrices have the *same order*.

Example 3 Adding and subtracting of matrices

Complete the following addition and subtraction of matrices.

a $\begin{bmatrix} 2 & 4 \\ 5 & 1 \end{bmatrix} + \begin{bmatrix} 9 & 8 \\ 9 & -1 \end{bmatrix}$

b $\begin{bmatrix} 7 & 3 \\ 2 & 8 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ -1 & 9 \\ 3 & 7 \end{bmatrix}$

Solution

a **1** Write the addition.

$$\begin{bmatrix} 2 & 4 \\ 5 & 1 \end{bmatrix} + \begin{bmatrix} 9 & 8 \\ 9 & -1 \end{bmatrix}$$

2 Add the elements that are in the same positions.

$$= \begin{bmatrix} 2+9 & 4+8 \\ 5+9 & 1+(-1) \end{bmatrix}$$

3 Evaluate each element.

$$= \begin{bmatrix} 11 & 12 \\ 14 & 0 \end{bmatrix}$$

b **1** Write the subtraction.

$$\begin{bmatrix} 7 & 3 \\ 2 & 8 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ -1 & 9 \\ 3 & 7 \end{bmatrix}$$

2 Subtract the elements that are in the same positions.

$$= \begin{bmatrix} 7-4 & 3-2 \\ 2-(-1) & 8-9 \\ 1-3 & 0-7 \end{bmatrix}$$

3 Evaluate each element.

$$= \begin{bmatrix} 3 & 1 \\ 3 & -1 \\ -2 & -7 \end{bmatrix}$$

ISBN 978-1-009-25891-3

The zero matrix, 0

In a **zero matrix** every element is zero.

The following are examples of zero matrices.

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Just as in arithmetic with ordinary numbers, adding or subtracting a zero matrix does not make any change to the original matrix. For example:

$$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \qquad \begin{bmatrix} 5 \\ 6 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

Also, subtracting any matrix from itself gives a zero matrix. For example:

$$\begin{bmatrix} 9 & 4 & 8 \end{bmatrix} - \begin{bmatrix} 9 & 4 & 8 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

A zero matrix is denoted by '0'.

Skill-sheet

Exercise 5C

Example 3

1 Complete the following addition and subtraction of matrices.

a $\begin{bmatrix} 4 & 3 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 5 & 7 \\ 6 & 1 \end{bmatrix}$ **b** $\begin{bmatrix} 8 & 6 \\ 9 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 4 & 0 \end{bmatrix}$ **c** $\begin{bmatrix} 3 & 5 \\ 7 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

d $\begin{bmatrix} 9 \\ 8 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ **e** $\begin{bmatrix} 8 & 6 \\ 2 & 9 \end{bmatrix} - \begin{bmatrix} 5 & 2 \\ 1 & 3 \end{bmatrix}$ **f** $\begin{bmatrix} 7 & 4 \\ 5 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 6 \\ 2 & -8 \end{bmatrix}$

g $\begin{bmatrix} 4 & 2 \end{bmatrix} + \begin{bmatrix} 8 & 5 \end{bmatrix}$ **h** $\begin{bmatrix} 7 & -5 \end{bmatrix} - \begin{bmatrix} 7 & -5 \end{bmatrix}$ **i** $\begin{bmatrix} 4 & -3 \end{bmatrix} + \begin{bmatrix} -4 & 3 \end{bmatrix}$

j $\begin{bmatrix} 4 & -3 & 2 & -1 \end{bmatrix} - \begin{bmatrix} 6 & -5 & -1 & 8 \end{bmatrix}$

2 Using the matrices given:

$$A = \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 7 \\ 1 & 3 \end{bmatrix} \quad C = \begin{bmatrix} 6 & 2 \\ 1 & 0 \\ 3 & -8 \end{bmatrix} \quad D = \begin{bmatrix} -3 & 5 \\ 4 & -2 \\ 1 & 7 \end{bmatrix} \quad E = \begin{bmatrix} 8 \\ 3 \end{bmatrix}$$

find, where possible:

a $A + B$ **b** $B + A$ **c** $A - B$ **d** $B - A$

e $B + E$ **f** $C + D$ **g** $B + C$ **h** $D - C$

ISBN 978-1-009-25891-3

3 Two people shared the work of a telephone poll surveying voting intentions. The results for each person's survey are given in matrix form.

Sample 1:

$$\begin{array}{c} \\ Men \\ Women \end{array} \begin{array}{c} \begin{array}{cccc} Liberal & Labor & Democrat & Green \end{array} \\ \begin{bmatrix} 19 & 21 & 7 & 3 \\ 18 & 17 & 11 & 4 \end{bmatrix} \end{array}$$

Sample 2:

$$\begin{array}{c} \\ Men \\ Women \end{array} \begin{array}{c} \begin{array}{cccc} Liberal & Labor & Democrat & Green \end{array} \\ \begin{bmatrix} 24 & 21 & 3 & 2 \\ 19 & 20 & 6 & 5 \end{bmatrix} \end{array}$$

Write a matrix showing the overall result of the survey.

4 The weights and heights of four people were recorded and then checked again one year later.

2004 results:

$$\begin{array}{c} \\ Weight\ (kg) \\ Height\ (cm) \end{array} \begin{array}{c} \begin{array}{cccc} Aida & Bianca & Chloe & Donna \end{array} \\ \begin{bmatrix} 32 & 44 & 59 & 56 \\ 145 & 155 & 160 & 164 \end{bmatrix} \end{array}$$

2005 results:

$$\begin{array}{c} \\ Weight\ (kg) \\ Height\ (cm) \end{array} \begin{array}{c} \begin{array}{cccc} Aida & Bianca & Chloe & Donna \end{array} \\ \begin{bmatrix} 38 & 52 & 57 & 63 \\ 150 & 163 & 167 & 170 \end{bmatrix} \end{array}$$

a Write the matrix that gives the changes in each person's weight and height after one year.

b Who gained the most weight?

c Which person had the greatest height increase?

ISBN 978-1-009-25891-3

5D Scalar multiplication

A *scalar* is just a number. Multiplying a matrix by a number is called **scalar multiplication**.

Multiplying a matrix by a scalar

Scalar multiplication is the process of multiplying a matrix by a number (a scalar).

In scalar multiplication each element is multiplied by that scalar (number).

The following is an example of scalar multiplication of a matrix.

$$5\begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 5\times 1 & 5\times 2 \\ 5\times 2 & 5\times 0 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ 10 & 0 \end{bmatrix}$$

Example 4 Scalar multiplication

If $A = \begin{bmatrix} 5 & 1 \\ -3 & 0 \end{bmatrix}$, find $3A$.

Solution

1 If $A = \begin{bmatrix} 5 & 1 \\ -3 & 0 \end{bmatrix}$, then $3A = 3\begin{bmatrix} 5 & 1 \\ -3 & 0 \end{bmatrix}$.

$$3A = 3\begin{bmatrix} 5 & 1 \\ -3 & 0 \end{bmatrix}$$

2 Multiply each number in the matrix by 3.

$$= \begin{bmatrix} 3\times 5 & 3\times 1 \\ 3\times -3 & 3\times 0 \end{bmatrix}$$

3 Evaluate each element.

$$= \begin{bmatrix} 15 & 3 \\ -9 & 0 \end{bmatrix}$$

Scalar multiplication has many practical applications. It is particularly useful in scaling up the elements of a matrix, for example, add the GST to the cost of the prices of all items in a shop by multiplying a matrix of prices by 1.1.

ISBN 978-1-009-25891-3

Example 5 Application of scalar multiplication

A gymnasium has the enrolments in courses shown in this matrix.

$$\begin{array}{c} \\ \textit{Men} \\ \textit{Women} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Body building} & \textit{Aerobics} & \textit{Fitness} \end{array} \\ \begin{bmatrix} 70 & 20 & 80 \\ 10 & 50 & 60 \end{bmatrix} \end{array}$$

The manager wishes to double the enrolments in each course. Show this in a matrix.

Solution

1 Each element in the matrix is multiplied by 2.

$$2 \times \begin{bmatrix} 70 & 20 & 80 \\ 10 & 50 & 60 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \times 70 & 2 \times 20 & 2 \times 80 \\ 2 \times 10 & 2 \times 50 & 2 \times 60 \end{bmatrix}$$

2 Evaluate each element.

$$= \begin{array}{c} \\ \text{Men} \\ \text{Women} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Body building} & \text{Aerobics} & \text{Fitness} \end{array} \\ \begin{bmatrix} 140 & 40 & 160 \\ 20 & 100 & 120 \end{bmatrix} \end{array}$$

Scalar multiplication can also be used in conjunction with addition and subtraction of matrices.

Example 6 Scalar multiplication and subtraction of matrices

If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, find the matrix equal to $2A - 3B$.

Solution

1 Write $2A - 3B$ in expanded matrix form.

$$2A - 3B = 2 \times \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - 3 \times \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

2 Multiply the elements in A by 2 and the elements in B by 3.

$$= \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix}$$

3 Subtract the elements in corresponding positions.

$$= \begin{bmatrix} 2-0 & 2-3 \\ 0-3 & 2-0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$$

ISBN 978-1-009-25891-3

How to add, subtract and scalar multiply matrices using the TI-Nspire CAS

If $A = \begin{bmatrix} 2 & -3 \\ 5 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 6 \\ 1 & -2 \end{bmatrix}$, find:

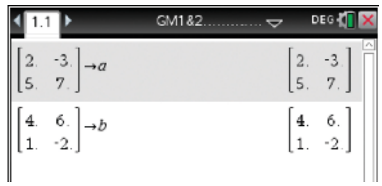
a $A + B$ **b** $A - B$ **c** $9A$ **d** $15A - 11B$

Steps

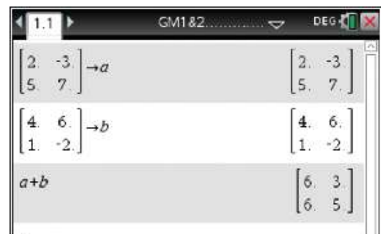
1 Press [on]>**New Document>Add Calculator**.

2 Enter the matrices A and B into your calculator.

Note: Refer to page 211 if you are unsure how to enter a matrix into your calculator.



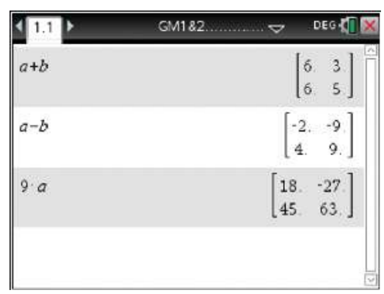
a To calculate $A + B$, type $A + B$ and then press [enter] to evaluate.



$$A + B = \begin{bmatrix} 6 & 3 \\ 6 & 5 \end{bmatrix}$$

b To calculate $A - B$, type A – B and then press [enter] to evaluate.

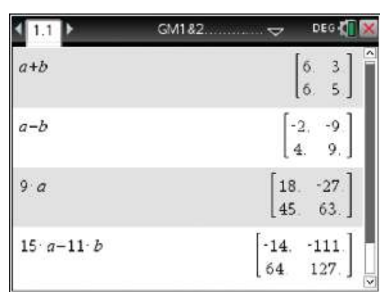
c To calculate $9A$, type $9A$ and then press [enter] to evaluate.



$$A - B = \begin{bmatrix} -2 & -9 \\ 4 & 9 \end{bmatrix}$$

$$9A = \begin{bmatrix} 18 & -27 \\ 45 & 63 \end{bmatrix}$$

d To calculate $15A - 11B$, type $15A - 11B$ and then press [enter] to evaluate.



$$15A - 11B = \begin{bmatrix} -14 & -111 \\ 64 & 127 \end{bmatrix}$$

ISBN 978-1-009-25891-3

How to add, subtract and scalar multiply matrices using the ClassPad

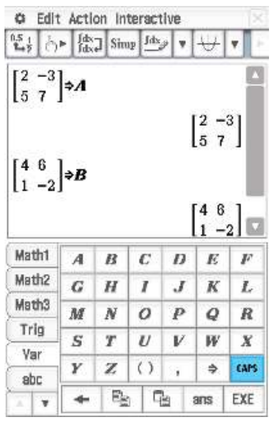
If $A = \begin{bmatrix} 2 & -3 \\ 5 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 6 \\ 1 & -2 \end{bmatrix}$, find:

a $A + B$ **b** $A - B$ **c** $9A$ **d** $15A - 11B$

Steps

1 Enter the matrices A and B into your calculator.

Note: Refer to page 212 if you are unsure how to enter a matrix into your calculator.

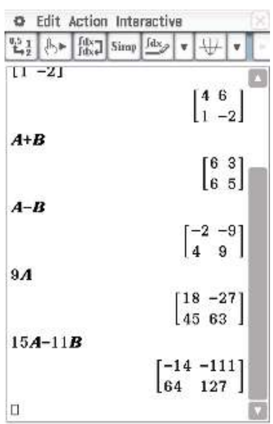


a To calculate $A + B$, type $\boldsymbol{A + B}$ and then press EXE to evaluate.

b To calculate $A - B$, type $\boldsymbol{A - B}$ and then press EXE to evaluate.

c To calculate $9A$, type $\boldsymbol{9A}$ and then press EXE to evaluate.

d To calculate $15A - 11B$, type $\boldsymbol{15A - 11B}$ and then press EXE to evaluate.



$A + B = \begin{bmatrix} 6 & 3 \\ 6 & 5 \end{bmatrix}$

$A - B = \begin{bmatrix} -2 & -9 \\ 4 & 9 \end{bmatrix}$

$9A = \begin{bmatrix} 18 & -27 \\ 45 & 63 \end{bmatrix}$

$15A - 11B = \begin{bmatrix} -14 & -111 \\ 64 & 127 \end{bmatrix}$

ISBN 978-1-009-25891-3

Exercise 5D

Example 4

1 Calculate the values of the following.

a $2\begin{bmatrix} 7 & -1 \\ 4 & 9 \end{bmatrix}$ **b** $5\begin{bmatrix} 0 & -2 \\ 5 & 7 \end{bmatrix}$ **c** $-4\begin{bmatrix} 16 & -3 \\ 1.5 & 3.5 \end{bmatrix}$ **d** $1.5\begin{bmatrix} 1.5 & 0 \\ -2 & 5 \end{bmatrix}$

e $3\begin{bmatrix} 6 & 7 \end{bmatrix}$ **f** $6\begin{bmatrix} -2 \\ 5 \end{bmatrix}$ **g** $\frac{1}{2}\begin{bmatrix} 4 & 6 & 0 \\ 0 & 3 & 1 \end{bmatrix}$ **h** $-1\begin{bmatrix} 3 & 6 & -8 \end{bmatrix}$

Example 6

2 Given the matrices:

$$A = \begin{bmatrix} 3 & -4 \\ 2 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 7 & 6 \\ 1 & -4 \end{bmatrix} \quad C = \begin{bmatrix} -3 & 4 \\ -2 & -5 \end{bmatrix} \quad O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

find the matrix required for:

a $3A$ **b** $2B + 4C$ **c** $5A - 2B$ **d** $2O$ **e** $3B + O$

3 Enter the matrices $A = \begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ 0 & 5 \end{bmatrix}$ into your CAS calculator and use them to evaluate:

CAS

a $17A - 14B$ **b** $29B - 21A$ **c** $9A + 7B$ **d** $3(5A - 4B)$

4 For the matrices:

$$A = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & 1 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 0 & 1 & 1 \end{bmatrix}$$

find the matrix for:

a $3A + 4B$ **b** $5C - 2D$ **c** $2(3A + 4B)$ **d** $3(5C - 2D)$

Example 5

5 The expenses arising from costs and wages for each section of three stores, A, B and C, are shown in the Costs matrix. The Sales matrix shows the money from the sale of goods in each section of the three stores. Figures represent the nearest million dollars.

Costs:

$$\begin{array}{c|ccc} & \textit{Clothing} & \textit{Furniture} & \textit{Electronics} \\ \hline A & 12 & 10 & 15 \\ B & 11 & 8 & 17 \\ C & 15 & 14 & 7 \end{array}$$

Sales:

$$\begin{array}{c|ccc} & \textit{Clothing} & \textit{Furniture} & \textit{Electronics} \\ \hline A & 18 & 12 & 24 \\ B & 16 & 9 & 26 \\ C & 19 & 13 & 12 \end{array}$$

a Write a matrix showing the profits in each section of each store.

b If 30% tax must be paid on profits, show the amount of tax that must be paid by each section of each store. No tax needs to be paid for a section that has made a loss.

ISBN 978-1-009-25891-3

6 Zoe competed in the gymnastics rings and parallel bars events in a three-day gymnastics tournament. A win was recorded as 1 and a loss as 0. The three column matrices show the results for Saturday, Sunday and Monday.

$$\begin{array}{lccc} & \textit{Sat} & \textit{Sun} & \textit{Mon} \\ \textit{Gymnastics rings} & \begin{bmatrix} 1 \\ 0 \end{bmatrix} & \begin{bmatrix} 1 \\ 1 \end{bmatrix} & \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ \textit{Parallel bars} & & & \end{array}$$

a Give a 2×1 column matrix which records her total wins for each of the two types of events.

b Zoe received \$50 for each win. Give a 2×1 matrix which records her total prize money for each of the two types of events.

5E Matrix multiplication

Matrix multiplication is the multiplication of a matrix by another matrix. Not to be confused with the scalar multiplication, which is the multiplication of a matrix by a number.

The matrix multiplication of two matrices A and B can be written as $A \times B$ or just AB. Although it is called multiplication and the symbol $\times$ may be used, matrix multiplication is not the simple multiplication of numbers but a routine involving the sum of pairs of numbers that have been multiplied.

For example, the method of matrix multiplication can be demonstrated by using a practical example. The numbers of CDs and DVDs sold by Fatima and Gaia are recorded in matrix N. The selling prices of the CDs and DVDs are shown in matrix P.

$$N = \begin{array}{l} \\ \textit{Fatima} \\ \textit{Gaia} \end{array} \begin{array}{c} \begin{array}{cc} \textit{CDs} & \textit{DVDs} \end{array} \\ \begin{bmatrix} 7 & 4 \\ 5 & 6 \end{bmatrix} \end{array} \qquad P = \begin{array}{l} \\ \textit{CDs} \\ \textit{DVDs} \end{array} \begin{array}{c} \$ \\ \begin{bmatrix} 20 \\ 30 \end{bmatrix} \end{array}$$

We want to make a matrix, S, that shows the value of the sales made by each person.

Fatima sold: 7 CDs at \$20 + 4 DVDs at \$30.
Gaia sold: 5 CDs at \$20 + 6 DVDs at \$30.

$$S = \begin{array}{l} \\ \textit{Fatima} \\ \textit{Gaia} \end{array} \begin{array}{c} \$ \\ \begin{bmatrix} 7 \times 20 + 4 \times 30 \\ 5 \times 20 + 4 \times 30 \end{bmatrix} \end{array}$$

The steps used in this example follow the routine for the matrix multiplication of $N \times P$.

ISBN 978-1-009-25891-3

As we move across the *first row* of matrix N we move down the *column* of matrix P, adding the products of the pairs of numbers as we go.

Then we move across the *second row* of matrix N and down the *column* of matrix P, adding the products of the pairs of numbers as we go.

$$\begin{array}{cc} N & \times \quad P \end{array}$$

$$\begin{bmatrix} 7 & 4 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} 20 \\ 30 \end{bmatrix} = \begin{bmatrix} 7 \times 20 + 4 \times 30 \\ \square \quad \square \end{bmatrix}$$

$$\begin{bmatrix} 7 & 4 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} 20 \\ 30 \end{bmatrix} = \begin{bmatrix} 7 \times 20 + 4 \times 30 \\ 5 \times 20 + 6 \times 30 \end{bmatrix}$$

$$= \begin{bmatrix} 140 + 120 \\ 100 + 180 \end{bmatrix}$$

$$= \begin{array}{c} \$ \\ \begin{bmatrix} 260 \\ 280 \end{bmatrix} \end{array} \begin{array}{l} \\ \textit{Fatima} \\ \textit{Gaia} \end{array}$$

Rules for matrix multiplication

Because of the way the products are formed, the number of columns in the first matrix must equal the number of rows in the second matrix. Otherwise, we say that matrix multiplication is not defined, meaning it is not possible.

Matrix multiplication

For matrix multiplication to be defined:

Think of the orders as two railway carriages that must be the same where they meet.

order of 1st matrix order of 2nd matrix

$m \times n$ $\quad$ $n \times p$

must be the same

In our example of the CD and DVD sales:

order of 1st matrix order of 2nd matrix

2×2 $\quad$ 2×1

the same

Notice that the outside numbers give the order of the product matrix: the matrix made by multiplying the two matrices. In our case, the answer is a 2×1 matrix.

Order of the product matrix

The order of the product matrix is given by:

Think: when the 'railway carriages' meet, the result has an order given by the end numbers.

order of 1st matrix order of 2nd matrix

$m \times n$ $\quad$ $n \times p$

order of answer

$m \times p$

We will check that these two important rules hold in the examples that follow.

ISBN 978-1-009-25891-3

Methods of matrix multiplication

Some people like to think of the matrix multiplication of $A \times B$ using a *run and dive* description. This procedure can be very tedious and error prone, so we will only do simple cases by hand so that you understand the process. Then a CAS calculator will be used.

Matrix multiplication of $A \times B$

The *run and dive* description of matrix multiplication is to add the products of the pairs made as you:

- *run* along the first row of A and *dive* down the first column of B
- repeat running along the first row of A and diving down the next column of B until all columns of B have been used
- now start running along the next row of A and repeat diving down each column of B, entering your results in a new row
- repeat this routine until all rows of A have been used.

Example 7 Matrix multiplication

For the following matrices: $A = \begin{bmatrix} 5 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 8 \\ 9 \end{bmatrix}$ $C = \begin{bmatrix} 2 & 4 & 7 \end{bmatrix}$ $D = \begin{bmatrix} 8 \\ 6 \\ 5 \end{bmatrix}$

- decide whether the matrix multiplication in each question below is defined
- if matrix multiplication is defined, give the order of the answer matrix and then do the matrix multiplication.

a AB **b** BA **c** CD

Solution

a AB

1 Write the order of each matrix.

$$\begin{array}{cc} A & B \\ 3 \times 2 & 2 \times 1 \end{array}$$

2 The inside numbers are the same.

Matrix multiplication is defined for $A \times B$.

3 The outside numbers give the order of $A \times B$.

The order of the product AB is 3×1.

4 Move across the first row of A and down the column of B, adding the products of the pairs.

$$\begin{bmatrix} 5 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 5 \times 8 + 2 \times 9 \\ \\ \\ \end{bmatrix}$$

5 Move across the second row of A and down the column of B, adding the products of the pairs.

$$\begin{bmatrix} 5 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 5 \times 8 + 2 \times 9 \\ 4 \times 8 + 6 \times 9 \\ \\ \end{bmatrix}$$

ISBN 978-1-009-25891-3

Matrix multiplication AB continued – the matrices in this example are:

$$A = \begin{bmatrix} 5 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 8 \\ 9 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 4 & 7 \end{bmatrix} \quad D = \begin{bmatrix} 8 \\ 6 \\ 5 \end{bmatrix}$$

6 Move across the third row of A and down the column of B, adding the products of the pairs.

$$\begin{bmatrix} 5 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 5 \times 8 + 2 \times 9 \\ 4 \times 8 + 6 \times 9 \\ 1 \times 8 + 3 \times 9 \end{bmatrix}$$

7 Tidy up by doing some arithmetic.

$$= \begin{bmatrix} 40 + 18 \\ 32 + 54 \\ 8 + 27 \end{bmatrix}$$

8 Write your answer.

$$\text{So } A \times B = \begin{bmatrix} 58 \\ 86 \\ 35 \end{bmatrix}$$

b BA

1 Write the order of each matrix.

B 2×1 A 3×2

2 Are the inside numbers the same? No.

Multiplication is not defined for $B \times A$.

c CD

1 Write the order of each matrix.

C 1×3 D 3×1

2 Are the inside numbers the same? Yes.

Multiplication is defined for $C \times D$.

3 The outside numbers give the order of $C \times D$.

The order of the product CD is 1×1.

4 Move across the row of C and down the column of D, adding the products of the pairs.

$$\begin{bmatrix} 2 & 4 & 7 \end{bmatrix} \begin{bmatrix} 8 \\ 6 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \times 8 + 4 \times 6 + 7 \times 5 \end{bmatrix}$$

5 Tidy up by doing some arithmetic.

$$= [16 + 24 + 35]$$

6 Write your answer.

$$\text{So } C \times D = [75]$$

In the previous example, $AB \neq BA$. Usually, when we reverse (*commute*) the order of the matrices in matrix multiplication, we get a different answer. This differs from ordinary arithmetic, where multiplication gives the same answer when the terms are commuted, for example, $3 \times 4 = 4 \times 3$.

Matrix multiplication

In general, matrix multiplication is not commutative. That is: $AB \neq BA$

ISBN 978-1-009-25891-3

How to multiply two matrices using the TI-Nspire CAS

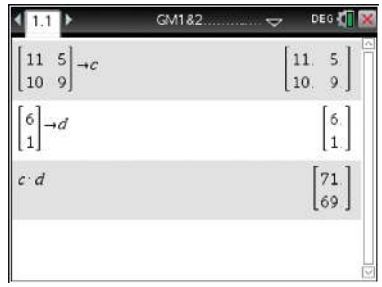
If $C = \begin{bmatrix} 11 & 5 \\ 10 & 9 \end{bmatrix}$ and $D = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$, find the matrix CD.

Steps

1 **[⌂ on]>New Document>Add Calculator**.

2 Enter the matrices C and D into your calculator.

Note: Refer to page 211 if you are unsure how to enter a matrix into your calculator.



3 To calculate matrix CD, type in c × d. Press [enter] to evaluate.

Note: You must put a multiplication sign between the c and d.

Check: C has dimension 2×2 and D has dimension 2×1. So, matrix CD should be a 2×1 matrix, as it is.

4 Write your answer.

$$CD = \begin{bmatrix} 71 \\ 69 \end{bmatrix}$$

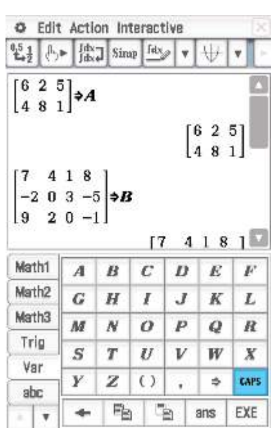
How to multiply two matrices using the ClassPad

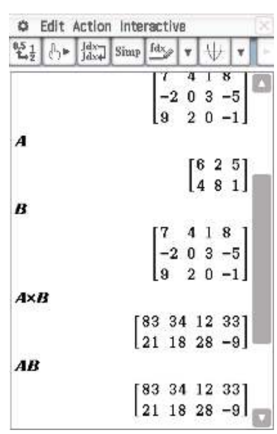
Find $A \times B$: $A = \begin{bmatrix} 6 & 2 & 5 \\ 4 & 8 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 7 & 4 & 1 & 8 \\ -2 & 0 & 3 & -5 \\ 9 & 2 & 0 & -1 \end{bmatrix}$

Steps

1 Enter the matrices A and B into your calculator.

Note: Refer to page 212 if you are unsure how to enter a matrix into your calculator.





2 To calculate $A \times B$, type $\boldsymbol{A} \times \boldsymbol{B}$ or $\boldsymbol{AB}$ and then press [EXE] to evaluate.

3 *Check:* A has dimensions 2×3 and B has dimensions 3×4. So, matrix AB should be a 2×4 matrix, which it is.

4 Write your answer.

$$AB = \begin{bmatrix} 83 & 34 & 12 & 33 \\ 21 & 18 & 28 & -9 \end{bmatrix}$$

ISBN 978-1-009-25891-3

Exercise 5E

Example 7

1 For the following matrices: $A = \begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 3 \\ 0 & 8 \\ 2 & -5 \end{bmatrix}$

- decide whether the matrix multiplication in each question below is defined
- if matrix multiplication is defined, give the order of the answer matrix and then do the matrix multiplication.

a AB **b** BA **c** CB **d** BC
e AA **f** BB **g** AC **h** CA

2 Write the orders of each pair of matrices and decide if matrix multiplication is defined. If matrix multiplication is defined, find the answer.

a $\begin{bmatrix} 3 & 4 \end{bmatrix}\begin{bmatrix} 6 \\ 5 \end{bmatrix}$ **b** $\begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ **c** $\begin{bmatrix} 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

d $\begin{bmatrix} 8 & -2 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ **e** $\begin{bmatrix} 1 & 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$ **f** $\begin{bmatrix} 1 & 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

3 For the matrices: $A = \begin{bmatrix} 2 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$ find:

a **i** $3A$ **ii** $5A$ **iii** $8A$ **iv** $3A + 5A$
b **i** $6B$ **ii** $6B + B$ **iii** $7B$
c **i** $2A$ **ii** $3B$ **iii** AB **iv** $2A \times 3B$ **v** $6AB$

4 Perform these matrix multiplications without using a CAS calculator.

a $\begin{bmatrix} 4 & 2 \\ 1 & 6 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix}$ **b** $\begin{bmatrix} 8 & 4 \\ 5 & 9 \end{bmatrix}\begin{bmatrix} 7 \\ 2 \end{bmatrix}$ **c** $\begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ 6 & 7 \end{bmatrix}$

d $\begin{bmatrix} 3 & 5 \\ 1 & 8 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ -2 & -3 \end{bmatrix}$ **e** $\begin{bmatrix} 8 & 3 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **f** $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 9 & 6 \\ 7 & 8 \end{bmatrix}$

g $\begin{bmatrix} 2 & 5 \\ 4 & 1 \\ 0 & 7 \end{bmatrix}\begin{bmatrix} 8 \\ 3 \end{bmatrix}$ **h** $\begin{bmatrix} 7 & 4 \\ 1 & 0 \\ 6 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ **i** $\begin{bmatrix} 5 & 8 \end{bmatrix}\begin{bmatrix} 7 \\ 6 \end{bmatrix}$

ISBN 978-1-009-25891-3

j $\begin{bmatrix} 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 9 \end{bmatrix}$

k $\begin{bmatrix} 4 & 1 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 6 \\ 1 \end{bmatrix}$

l $\begin{bmatrix} 1 & 0 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \\ 0 \end{bmatrix}$

m $\begin{bmatrix} 6 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \end{bmatrix}$

n $\begin{bmatrix} 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 5 \\ 8 \end{bmatrix}$

o $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$

5 Use your CAS calculator to do the matrix multiplications in Question **4**. CAS

6 $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

a Find AB. **b** Find BA. **c** Does $AB = BA$?

7 Use these matrices to find the required products.

$C = \begin{bmatrix} 9 & 8 \\ 7 & 6 \end{bmatrix}$ $D = \begin{bmatrix} 8 & 6 \\ 4 & 2 \end{bmatrix}$ $E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $F = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

a CD **b** CE **c** CF **d** DE **e** DF

8 Perform the following matrix multiplications using your CAS calculator. CAS

a $\begin{bmatrix} 6 & 8 & 12 \\ 14 & 17 & 11 \end{bmatrix}\begin{bmatrix} 26 & 9 & 21 & 6 \\ 8 & -7 & -4 & 9 \\ 13 & 10 & 5 & 26 \end{bmatrix}$

b $\begin{bmatrix} 15 & 9 & 23 & 72 \end{bmatrix}\begin{bmatrix} -6 \\ 22 \\ -8 \\ 19 \end{bmatrix}$

c $\begin{bmatrix} 16 \\ 10 \\ 24 \\ -18 \end{bmatrix}\begin{bmatrix} -31 & 47 & 61 & -14 \end{bmatrix}$

d $\begin{bmatrix} 8 & -7 & 9 \\ 6 & 11 & 14 \\ 3 & 21 & -5 \end{bmatrix}\begin{bmatrix} 8 & -19 & 24 \\ 33 & 16 & 19 \\ 4 & 0 & 13 \end{bmatrix}$

9 Noting that $A^2 = A \times A, A^3 = A \times A \times A$, etc., calculate:

i A^2 **ii** A^3 **iii** A^4

for each of the following matrices.

a $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ **b** $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ **c** $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

d $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ **e** $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

ISBN 978-1-009-25891-3

5F Applications of matrices

Data represented in matrix form can be multiplied to produce new useful information.

Example 8 Business application of matrices

Fatima and Gaia's store has a special sales promotion. One free cinema ticket is given with each DVD purchased. Two cinema tickets are given with the purchase of each computer game.

The number of DVDs and games sold by Fatima and Gaia are given in matrix S.

The selling price of a DVD and a game, together with the number of free tickets is given by matrix P.

$$S = \begin{array}{c} \\ Fatima \\ Gaia \end{array} \begin{array}{c} \begin{array}{cc} DVDs & Games \end{array} \\ \begin{bmatrix} 7 & 4 \\ 5 & 6 \end{bmatrix} \end{array} \qquad P = \begin{array}{c} \\ DVDs \\ Games \end{array} \begin{array}{c} \begin{array}{cc} \$ & Tickets \end{array} \\ \begin{bmatrix} 20 & 1 \\ 30 & 2 \end{bmatrix} \end{array}$$

From the matrix product $S \times P$ and interpret.

Solution

1 Complete the matrix multiplication, $S \times P$.

$$\begin{array}{c} \\ \text{Fatima} \\ \text{Gaia} \end{array} \begin{array}{c} \begin{array}{cc} \text{DVDs} & \text{Games} \end{array} \\ \begin{bmatrix} 7 & 4 \\ 5 & 6 \end{bmatrix} \end{array} \times \begin{array}{c} \\ \text{DVDs} \\ \text{Games} \end{array} \begin{array}{c} \begin{array}{cc} \$ & \text{Tickets} \end{array} \\ \begin{bmatrix} 20 & 1 \\ 30 & 2 \end{bmatrix} \end{array}$$

$$= \begin{array}{c} \\ \text{Fatima} \\ \text{Gaia} \end{array} \begin{array}{c} \begin{array}{cc} \$ & \text{Tickets} \end{array} \\ \begin{bmatrix} 7 \times 20 + 4 \times 30 & 7 \times 1 + 4 \times 2 \\ 5 \times 20 + 6 \times 30 & 5 \times 1 + 6 \times 2 \end{bmatrix} \end{array}$$

$$= \begin{array}{c} \\ \text{Fatima} \\ \text{Gaia} \end{array} \begin{array}{c} \begin{array}{cc} \$ & \text{Tickets} \end{array} \\ \begin{bmatrix} 260 & 15 \\ 280 & 17 \end{bmatrix} \end{array}$$

2 Interpret the matrix.

Fatima had sales of \$260 and gave out 15 tickets.
Gaia had sales of \$280 and gave out 17 tickets.

ISBN 978-1-009-25891-3

Properties of row and column matrices

Row and column matrices provide efficient ways of extracting information from data stored in large matrices. Matrices of a convenient size will be used to explore some of the surprising and useful properties of row and column matrices.

Example 9 Using row and column matrices to extract information

Three rangers completed their monthly park surveys of feral animal sightings in the matrix S.

$$S = \begin{array}{c} \\ Aaron \\ Barra \\ Chloe \end{array} \begin{array}{c} \begin{array}{cccc} Cats & Dogs & Foxes & Rabbits \end{array} \\ \begin{bmatrix} 27 & 9 & 34 & 59 \\ 18 & 15 & 10 & 89 \\ 35 & 6 & 46 & 29 \end{bmatrix} \end{array} \quad A = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

a Evaluate $S \times B$.

b What information about matrix S is given in the product $S \times B$?

c Evaluate $A \times S$.

d What information about matrix S is given in the product $A \times S$?

Solution

a Matrix multiplication of a 3×4 and a 4×1 matrix produces a 3×1 matrix.

$$S \times B = \begin{bmatrix} 27 & 9 & 34 & 59 \\ 18 & 15 & 10 & 89 \\ 35 & 6 & 46 & 29 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 27 + 9 + 34 + 59 \\ 18 + 15 + 10 + 89 \\ 35 + 6 + 46 + 29 \end{bmatrix} = \begin{bmatrix} 129 \\ 132 \\ 116 \end{bmatrix}$$

b Look at the second last step in the working of $S \times B$.

Each row of SB gives the sum of the rows in S. Namely, the total sightings made by each ranger.

c Matrix multiplication of a 1×3 and a 3×4 matrix produces a 1×4 matrix.

$$A \times S = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 27 & 9 & 34 & 59 \\ 18 & 15 & 10 & 89 \\ 35 & 6 & 46 & 29 \end{bmatrix}$$

d In the second last step of part **c**, we see that each element is the sum of the sightings for each type of animal.

$$A \times S = \begin{bmatrix} 27 + 18 + 35 & 9 + 15 + 6 & 34 + 10 + 46 & 59 + 89 + 29 \end{bmatrix}$$
$$= \begin{bmatrix} 80 & 30 & 90 & 177 \end{bmatrix}$$

Each column of AS gives the sum of the columns in S, which gives the sum of the sightings of each type of animal.

ISBN 978-1-009-25891-3

Exercise 5F

General applications

Example 8

1 One matrix below shows the number of milkshakes and sandwiches that Helen had for lunch. The number of kilojoules (kJ) present in each food is given in the other matrix.

$$\begin{array}{c} \\ Helen \end{array}\begin{array}{c} \begin{array}{cc} Milkshakes & Sandwiches \end{array} \\ \begin{bmatrix} 2 & \quad\quad 3 \end{bmatrix} \end{array} \qquad \begin{array}{r} \\ Milkshakes \\ Sandwiches \end{array}\begin{array}{c} kJ \\ \begin{bmatrix} 1400 \\ 1000 \end{bmatrix} \end{array}$$

Use a matrix product to calculate how many kilojoules Helen had for lunch.

2 The first matrix shows the number of cars and bicycles owned by two families. The second matrix records the wheels and seats for cars and bicycles.

$$\begin{array}{r} \\ Smith \\ Jones \end{array}\begin{array}{c} \begin{array}{cc} Cars & Bicycles \end{array} \\ \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \end{array} \qquad \begin{array}{r} \\ Car \\ Bicycle \end{array}\begin{array}{c} \begin{array}{cc} Wheels & Seats \end{array} \\ \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix} \end{array}$$

Use a matrix product to find a matrix that gives the numbers of wheels and seats owned by each family.

3 Eve played a game of darts. The parts of the dartboard that she hit during one game are recorded in matrix H. The bull's eye is a small area in the centre of the dartboard. The points scored for hitting different regions of the dartboard are shown in matrix P.

$$H = \begin{array}{c} \\ Hits \end{array}\begin{array}{c} \begin{array}{ccc} Bull's\ eye & Inner\ region & Outer\ region \end{array} \\ \begin{bmatrix} 2 & \quad\quad 13 & \quad\quad 5 \end{bmatrix} \end{array}$$

$$P = \begin{array}{c} Points \\ \begin{bmatrix} 20 \\ 5 \\ 1 \end{bmatrix} \end{array}\begin{array}{l} \\ Bull's\ eye \\ Inner\ region \\ Outer\ region \end{array}$$

Use matrix multiplication to find a matrix giving her score for the game.

ISBN 978-1-009-25891-3

Business applications

4 On a Saturday morning Michael's café sold 18 quiches, 12 soups and 64 coffees. A quiche costs \$5, soup costs \$8 and a coffee costs \$3.

a Use a row matrix to record the number of each type of item sold.

b Write the costs of each item in a column matrix.

c Use matrix multiplication of the matrices from parts **a** and **b** to find the total value of the mornings sales.

5 Han's stall at the football made the sales shown in the table.

Tubs of chips	Pasties	Pies	Sausage rolls
90	84	112	73

The selling prices were: chips \$4, pastie \$5, pie \$5 and a sausage roll \$3.

a Record the numbers of each product sold in a row matrix.

b Write the selling prices in a column matrix.

c Find the total value of the sales by using matrix multiplication of the row and column matrices found in parts **a** and **b**.

6 Supermarkets sell eggs in boxes of 12, apples in packets of 8 and yoghurt tubs in sets of 4. This is represented by matrix A.

$$A = \begin{array}{c} \\ \textit{Items per packet} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Eggs} & \textit{Apples} & \textit{Yoghurt} \end{array} \\ \begin{bmatrix} 12 & 8 & 4 \end{bmatrix} \end{array}$$

The cost for each type of packet is given by matrix B.

$$B = \begin{array}{c} \\ \textit{Eggs} \\ \textit{Apples} \\ \textit{Yoghurt} \end{array} \begin{array}{c} \$ \\ \begin{bmatrix} 7 \\ 4 \\ 3 \end{bmatrix} \end{array}$$

The sales of each type of packet are given by matrix C as a column matrix and by matrix D as a row matrix.

$$C = \begin{array}{c} \\ \textit{Eggs} \\ \textit{Apples} \\ \textit{Yoghurt} \end{array} \begin{array}{c} \textit{Packets} \\ \begin{bmatrix} 100 \\ 50 \\ 30 \end{bmatrix} \end{array} \qquad D = \begin{array}{c} \\ \textit{Packets} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Eggs} & \textit{Apples} & \textit{Yoghurt} \end{array} \\ \begin{bmatrix} 100 & 50 & 30 \end{bmatrix} \end{array}$$

Choose the appropriate matrices and use matrix multiplication to find:

a the total number of items sold (counting each egg, apple or yoghurt tub as an item)

b the total value of all sales.

ISBN 978-1-009-25891-3

Using row and column matrices to extract information from a matrix

Example 9

7 The number of study hours completed by three students over four days is shown in matrix H.

$$H = \begin{array}{r} \\ Issie \\ Jack \\ Kaiya \end{array} \begin{array}{c} \begin{array}{cccc} Mon & Tues & Wed & Thur \end{array} \\ \begin{bmatrix} 2 & 3 & 2 & 3 \\ 1 & 4 & 0 & 2 \\ 3 & 4 & 3 & 2 \end{bmatrix} \end{array}$$

Using matrix multiplication with a suitable row or column matrix:

a produce a matrix showing the total study hours for each student

b hence, find a matrix with the average hours of study for each student

c obtain a matrix with the total number of hours studied on each night of the week

d hence, find a matrix with the average number of hours studied each night, correct to 1 decimal place.

8 Matrix R records four students' results in five tests.

$$R = \begin{array}{r} \\ Ellie \\ Felix \\ George \\ Hannah \end{array} \begin{array}{c} \begin{array}{ccccc} T1 & T2 & T3 & T4 & T5 \end{array} \\ \begin{bmatrix} 87 & 91 & 94 & 86 & 88 \\ 93 & 76 & 89 & 62 & 95 \\ 73 & 61 & 58 & 54 & 83 \\ 66 & 79 & 83 & 90 & 91 \end{bmatrix} \end{array}$$

Choose an appropriate row or column matrix and use matrix multiplication to:

a obtain a matrix with the sum of each student's results

b hence, give a matrix with each student's average test score

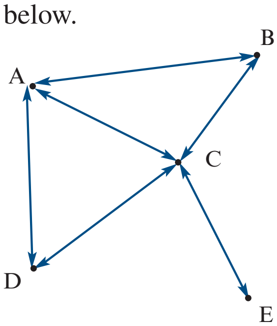
c derive a matrix with the sum of the scores for each test

d hence, give a matrix with the average score on each test.

5G Communications and connections

Social networks, communication pathways and connections can be represented and analysed using matrix techniques.

The connections between five people can be seen in both the network diagram and matrix below.



$$P = \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ A \\ B \\ C \\ D \\ E \end{array}$$

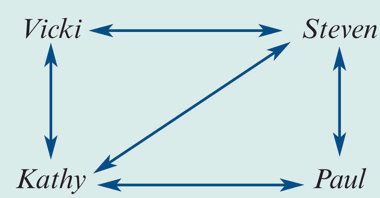
ISBN 978-1-009-25891-3

In a diagram showing the connections between a group of people, the following features can be observed.

- A double-headed arrow connecting two names indicates that those two people communicate with each other.
- If there is no arrow directly connecting two people, they do not communicate.
- These links are called one-step connections because there is just one direct step in contacting the other person.
- These connections can be represented as a single $n \times n$ matrix, where n is the number of people. This will create a square matrix.
- If matrix P shows the number of one-step connections between a group of people, then P^2 gives the number of two-step communications between people. Namely, how many ways one person can communicate with someone via another person.
- Note: In this situation the leading diagonal of the matrix (elements on the diagonal from the top left corner to the bottom right corner of the matrix) is all zeroes as it is not possible for a person to communicate with themselves.

Example 10 **Applying matrices to social networks**

The diagram shows the communications within a group of friends.



a Record the social links in a matrix N, using the first letter of each name to label the columns and rows. Explain how the matrix should be read.

b Explain why there is symmetry about the leading diagonal of the matrix.

c What information is given by the sum of a column or row?

d Find the matrix N^2 and explain what this matrix shows.

e Use the matrix N^2 to find the number of two-step ways Kathy can communicate with Steven and write the connections.

f In the matrix N^2 there is a 3 where S column meets the S row. This indicates that there are three two-step communications Steven can have with himself. Explain how this can be given a sensible interpretation.

Solution

a

$$N = \begin{array}{c} \begin{array}{cccc} V & S & K & P \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ V \\ S \\ K \\ P \end{array}$$

For example, reading from the column S down to the row K, a value of 1 indicates that Steven communicates with Kathy. The number 0 is used where there is no communication.

ISBN 978-1-009-25891-3

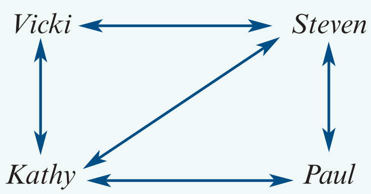
b The symmetry occurs because the communication is two way. For example, Vicki communicates with Steven and Steven communicates with Vicki.

c The sum of a column or row gives the total number of people that a given person can communicate with.
For example, Kathy can communicate with: $1 + 1 + 0 + 1 = 3$ people.

d
$$N^2 = \begin{array}{c} \\ \\ \end{array} \begin{array}{cc} \begin{array}{cccc} V & S & K & P \end{array} & \\ \begin{bmatrix} 2 & 1 & 1 & 2 \\ 1 & 3 & 2 & 1 \\ 1 & 2 & 3 & 1 \\ 2 & 1 & 1 & 2 \end{bmatrix} & \begin{array}{c} V \\ S \\ K \\ P \end{array} \end{array}$$

N^2 gives the number of two-step communications between people. That is, how many ways one person can communicate with someone via another person.

The diagram from the question is repeated here:



e Reading down the K column to the S row the 2 indicates there are 2 two-step communications between Kathy and Steven.
These can be found in the arrows diagram.

Kathy → Vicki → Steven
Kathy → Paul → Steven

f There are three ways Steven can communicate with himself via another person.

Steven → Vicki → Steven
Steven → Kathy → Steven
Steven → Paul → Steven

For example, using the first case above, Steven might ring Vicki and ask her to ring him back later to remind him of an appointment.

The matrix N^3 would give the three-step communications between people. The number of ways of communicating with someone via two people.

The matrix methods of investigating communications can be applied to friendships, travel between towns and other types of two-way connections.

ISBN 978-1-009-25891-3

Exercise 5G

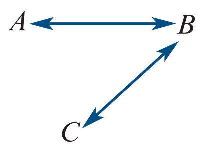
Example 10

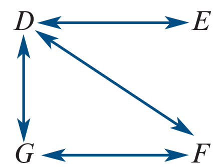
1 Assume that communications are a two-way process. So if A communicates with B then B communicates with A. The letters represent the names of people. Find the error in this communications matrix.

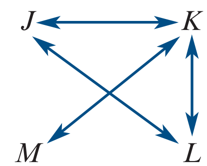
Where column A meets row B the number 1 indicates that A communicates with B. The number 0 is used to show there is no direct communication between two people.

$$\begin{array}{cc} \begin{array}{cccc} A & B & C & D \end{array} & \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} & \begin{array}{c} A \\ B \\ C \\ D \end{array} \end{array}$$

2 Write the matrix for each communications diagram. Use the number 1 when direct communication between two people exists and 0 for no direct communication.

a 

b 

c 

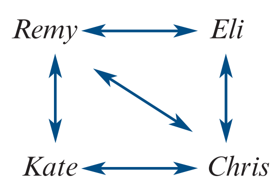
3 Road connections between towns are recorded in the matrices below. The letters represent towns. Where column A meets row C, the number 1 indicates that there is a road directly connecting town A to town C. The number 0 is used to show when there is no road directly connecting two towns.
Draw a diagram corresponding to each matrix showing the roads connecting the towns.

a
$$\begin{array}{cc} \begin{array}{ccc} A & B & C \end{array} & \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} & \begin{array}{c} A \\ B \\ C \end{array} \end{array}$$

b
$$\begin{array}{cc} \begin{array}{cccc} P & Q & R & S \end{array} & \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} & \begin{array}{c} P \\ Q \\ R \\ S \end{array} \end{array}$$

c
$$\begin{array}{cc} \begin{array}{cccc} T & U & V & W \end{array} & \\ \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} & \begin{array}{c} T \\ U \\ V \\ W \end{array} \end{array}$$

4 Communication connections between Chris, Eli, Kate and Remy are shown in the diagram.



a Write a matrix Q to represent the connections. Label the columns and rows in alphabetical order using the first letter of each name. Enter 1 to indicate that two people communicate directly or 0 if they do not.

b What information is given by the sum of column R?

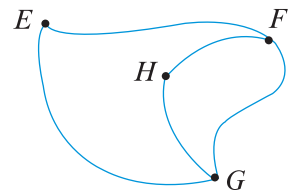
c **i** Find Q^2.

ii Using the matrix Q^2, find the total number of ways that Eli can communicate with a person via another person.

iii Write the chain of connections for each way that Eli can communicate to a person via another person.

ISBN 978-1-009-25891-3

5 Roads connecting the towns Easton, Fields, Hillsville and Gorges are shown in the diagram. The first letter of each town is used.



a Use a matrix R to represent the road connections. Label the columns and rows in alphabetical order using the first letter of town's name. Write 1 when two towns are directly connected by a road and write 0 if they are not connected.

b What does the sum of column F reveal about the town Fields?

c **i** Find R^2.

ii How many ways are there to travel from Fields to a town via another town? Include ways of starting and ending at Fields.

iii List the possible ways of part **ii**.

5H Identity matrices

Identity matrix

In ordinary arithmetic, the number 1 is called the *multiplicative identity element*. When 1 multiplies a number, the answer is always *identical* to the original number. Is there a matrix that can multiply any matrix and give an answer identical to the original matrix? Consider the following example.

Example 11 The identity matrix

$$A = \begin{bmatrix} 5 & 2 \\ 8 & 3 \end{bmatrix} \text{ and } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

a Find AI. **b** Find IA.

Solution

a AI

1 Write the order of each matrix. The inside numbers are the same, so matrix multiplication is defined. The outside numbers tell us that the answer is a 2×2 matrix.

$$\begin{array}{cc} A & I \\ 2 \times 2 & 2 \times 2 \end{array}$$

ISBN 978-1-009-25891-3

2 Do the matrix multiplication by hand or using your calculator.

$$A \times I = \begin{bmatrix} 5 & 2 \\ 8 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 5 \times 1 + 2 \times 0 & 5 \times 0 + 2 \times 1 \\ 8 \times 1 + 3 \times 0 & 8 \times 0 + 3 \times 1 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & 2 \\ 8 & 3 \end{bmatrix}$$

b *IA*

1 Write the order of each matrix. The inside numbers are the same, so matrix multiplication is defined. The outside numbers tell us that the answer is a 2×2 matrix.

$$\begin{array}{cc} I & A \\ 2 \times 2 & 2 \times 2 \end{array}$$

2 Do the matrix multiplication by hand or using your calculator.

$$I \times A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 5 & 2 \\ 8 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 1 \times 5 + 0 \times 8 & 1 \times 2 + 0 \times 3 \\ 0 \times 5 + 1 \times 8 & 0 \times 2 + 1 \times 3 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & 2 \\ 8 & 3 \end{bmatrix}$$

Identity matrix for 2 × 2 matrices

Identity matrix for 2 × 2 matrices

The 2×2 identity matrix is $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. For any 2×2 matrix A, $AI = IA = A$

The **identity matrix**, I, also has the special property that it is *commutative* in matrix multiplication. When I is one of the matrices in the multiplication, the answer is the same when the order of the matrices is commuted (reversed).

In Example 11: $AI = IA = \begin{bmatrix} 5 & 2 \\ 8 & 3 \end{bmatrix}$

Remember that matrix multiplication is not usually commutative.

ISBN 978-1-009-25891-3

Only square matrices have identity matrices. The *identity matrix for any square matrix* is a square matrix of the same order with 1s along the *leading diagonal* (from the top left to the bottom right) and 0s in all the other positions.

$$[1] \qquad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example 12

Given that $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is an identity matrix,

a Express $\begin{bmatrix} 5 & 3 \\ 6 & -1 \end{bmatrix} + 2I$ as a single matrix.

b Find the matrix A such that $2A + 3I = \begin{bmatrix} 9 & 0 \\ -4 & 7 \end{bmatrix}$.

Solution

a Multiply by constants.

$$\begin{bmatrix} 5 & 3 \\ 6 & -1 \end{bmatrix} + 2I = \begin{bmatrix} 5 & 3 \\ 6 & -1 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 3 \\ 6 & -1 \end{bmatrix} + \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$$

Do the matrix addition by hand.

$$= \begin{bmatrix} 7 & 5 \\ 6 & 1 \end{bmatrix}$$

b Write down the equation.

$$2A + 3I = \begin{bmatrix} 9 & 0 \\ -4 & 6 \end{bmatrix}$$

Multiply by constants.

$$2A + 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ -3 & 7 \end{bmatrix}$$

$$2A + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ -4 & 7 \end{bmatrix}$$

ISBN 978-1-009-25891-3

Rearrange equation and complete matrix addition by hand.

$$2A = \begin{bmatrix} 9 & 0 \\ -4 & 7 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$2A = \begin{bmatrix} 6 & 0 \\ -4 & 4 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 0 \\ -2 & 2 \end{bmatrix}$$

Exercise 5H

Given that $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is an identity matrix,

1 Express $\begin{bmatrix} 4 & 5 \\ 5 & -2 \end{bmatrix} + 3I$ as a single matrix.

2 Express $\begin{bmatrix} 8 & -3 \\ 0 & 5 \end{bmatrix} - 5I$ as a single matrix.

3 Find the matrix P such that $P + I = \begin{bmatrix} 7 & -1 \\ 4 & 3 \end{bmatrix}$.

4 Find the matrix Q such that $Q - I = \begin{bmatrix} 2 & 0 \\ 5 & -8 \end{bmatrix}$.

5 Is $AI + BI = (A + B)I$?

6 Is $AI - BI = (A - B)I$?

7 Find the matrix B such that $B + 2I = \begin{bmatrix} 8 & 10 \\ -6 & 0 \end{bmatrix}$.

8 Find the matrix C such that $C - 5I = \begin{bmatrix} 2 & 4 \\ -5 & 3 \end{bmatrix}$.

9 Find the matrix D such that $3D + 4I = \begin{bmatrix} 13 & -3 \\ -6 & 7 \end{bmatrix}$.

10 Find the matrix P such that $5P - 2I = \begin{bmatrix} 8 & -5 \\ 15 & 3 \end{bmatrix}$

ISBN 978-1-009-25891-3

11 $\begin{bmatrix} 4 & -8 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} p & q \\ 0 & r \end{bmatrix} = I$. Work out the values of p, q and r.

12 $\begin{bmatrix} 7 & 5 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 4 & -5 \\ -2 & c \end{bmatrix} = 18I$. Work out the value of c.

5I Extended application and problem-solving tasks

Exercise 5I

1 Scalar multiplication occurs when a number multiplies a matrix.
For a 2×2 matrix it has the general form:

$$k\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$$

Suggest why the matrix $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$ below is called a *scalar matrix*.
Give examples to support your view.

Information can be stored in matrices with hundreds of columns and rows. Using conveniently sized matrices, we will explore methods that can be used for extracting information from huge matrix data banks.

2 The mobile phone bills of Anna, Boyd and Charlie for the four quarters of 2014 are recorded in the matrix P. We will investigate the effect of multiplying by matrix E.

$$P = \begin{array}{r} \\ Anna \\ Boyd \\ Charlie \end{array}\!\!\begin{array}{c} \begin{array}{cccc} Q1 & Q2 & Q3 & Q4 \end{array} \\ \begin{bmatrix} 47 & 43 & 52 & 61 \\ 56 & 50 & 64 & 49 \\ 39 & 41 & 44 & 51 \end{bmatrix} \end{array} \qquad E = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

a Find $P \times E$ and comment on the result of that matrix multiplication.

b State the matrix F needed to extract the second quarter, $Q2$, costs.

A matrix is needed so it can multiply matrix P to extract the four quarterly costs on Charlie's phone bill.

c What will be the order of the matrix that displays Charlie's quarterly costs?

d State the order of a matrix G that when it multiplies a 3×4, gives a 1×4 matrix as the result? Should the matrix G pre-multiply or post-multiply matrix P? Pre-multiply means it multiplies at the front (left) of matrix P. While post-multiply means the matrix multiplies when written after (to the right of) matrix P.

e Suggest a suitable matrix G that will multiply matrix P and produce a matrix of Charlie's quarterly phone bills. Check that it works.

ISBN 978-1-009-25891-3

Key ideas and chapter summary

Assign-ment

Matrix	A **matrix** is a rectangular array of numbers set out in rows and columns within square brackets. The rows are horizontal; the columns are vertical.
Order of a matrix	The **order (size) of a matrix** is: number of rows × number of columns (i.e. $m \times n$). The number of rows is always given first.
Elements of a matrix	The **elements of a matrix** are the numbers within it. The position of an element is given by its row and column in the matrix. Element $a_{i\,j}$ is in row i and column j. Row is always given first.
Connections	A matrix can be used to record various types of connections, such as social communications and roads directly connecting towns.
Equal matrices	Two matrices are equal when they have the same numbers in the same positions. To do this they need to have the same order (shape).
Adding matrices	Matrices of the same order can be *added* by adding numbers in the same positions.
Subtracting matrices	Matrices of the same order can be *subtracted* by subtracting numbers in the same positions.
Zero matrix, *O*	A **zero matrix** is any matrix with zeroes in every position.
Scalar multiplication	**Scalar multiplication** is the multiplication of a matrix by *a number*.
Matrix multiplication	The process of multiplying a matrix by a matrix.
Identity matrix, *I*	An *identity matrix* I behaves like the number 1 in arithmetic. Any matrix multiplied by I remains unchanged. For 2×2 matrices, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ where $AI = A = IA$ Matrix multiplication by the identity matrix is commutative.

ISBN 978-1-009-25891-3

Review

Skills check

Having completed this chapter you should be able to:

- state the order of a given matrix
- describe the location of an element within a matrix
- decide whether two matrices are equal
- add and subtract matrices
- perform scalar multiplication on a matrix
- identify a zero matrix
- decide whether it is possible to do matrix multiplication with two given matrices
- give the order of the matrix resulting from matrix multiplication
- perform matrix multiplication
- state the identity matrix for a $n \times n$ matrix and know its properties
- represent and solve communications

Short-answer questions

Use matrix F for Questions 1 and 2

$$F = \begin{bmatrix} 4 & 8 & 6 \\ 5 & 1 & 7 \end{bmatrix}$$

1 State the order of matrix F.

2 State the value of f_{21}.

3 Three students were asked the number of electronic devices their family owned. The results are shown in the matrix.

$$\begin{array}{l} \\ Caroline \\ Delia \\ Emir \end{array} \begin{array}{c} \begin{array}{ccc} TVs & VCRs & PCs \end{array} \\ \begin{bmatrix} 4 & 3 & 2 \\ 1 & 0 & 5 \\ 2 & 1 & 3 \end{bmatrix} \end{array}$$

State the number of PCs owned by Emir's family.

4 The matrix gives the numbers of roads directly connecting one town to another. State the total number of roads directly connecting town E to other towns.

$$\begin{array}{c} \begin{array}{ccc} D & E & F \end{array} \\ \begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 3 \\ 1 & 3 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ D \\ E \\ F \end{array}$$

ISBN 978-1-009-25891-3

5 If the two matrices are equal, solve for x.

$$\begin{bmatrix} 4 & 3x \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 7 & 9 \end{bmatrix}$$

Use matrices M and N in Questions 6 to 10

$$M = \begin{bmatrix} 7 & 6 \\ 4 & 3 \end{bmatrix} \quad N = \begin{bmatrix} 5 & -2 \\ 1 & 0 \end{bmatrix}$$

6 Find $M + N$.

7 Find $M - N$.

8 Find $N - N$.

9 State the matrix $2N$.

10 Find the matrix $2M + N$.

Use the matrices P, Q, R, S in Questions 11 to 14

$$P \begin{bmatrix} 5 & 4 & 1 \\ 7 & 6 & 8 \end{bmatrix} \quad Q = \begin{bmatrix} 2 \\ 6 \\ 0 \end{bmatrix} \quad R = \begin{bmatrix} 4 & 7 \end{bmatrix} \quad S = \begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix}$$

11 State the order of the matrix SP.

12 State the order of the matrix QR.

13 Which matrix multiplications gives a 1×3 matrix?

14 Find the matrix multiplication PQ.

15 State the 2×2 identity matrix.

Use matrix A in Questions 16 to 19

$$A = \begin{bmatrix} 4 & 2 & 1 & 0 \\ 3 & 4 & 7 & 9 \end{bmatrix}$$

16 State the order of matrix A.

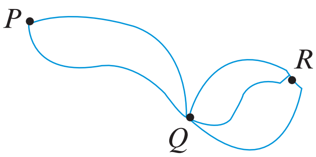
17 Identify the element $a_{1\,3}$.

18 If $C = [5 \quad 6]$, find CA.

19 If the order of a matrix B was 4×1, what would be the order of the matrix resulting from AB?

ISBN 978-1-009-25891-3

20 Roads are shown joining towns P, Q and R. Use a matrix to record the number roads directly connecting one town to another town.



21 Use the matrices below

$$A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 5 \\ 7 & 6 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

to find:

a $3A$ **b** $A + B$ **c** $B - A$ **d** $2A + B$

e $A - A$ **f** AB **g** BA **h** A^2

i AI

Extended-response questions

1 Farms A and B have their livestock numbers recorded in the matrix shown.

$$\begin{array}{r c} & \begin{array}{ccc} \textit{Cattle} & \textit{Pigs} & \textit{Sheep} \end{array} \\ \begin{array}{r} \textit{Farm } A \\ \textit{Farm } B \end{array} & \begin{bmatrix} 420 & 50 & 100 \\ 300 & 40 & 220 \end{bmatrix} \end{array}$$

a How many pigs are on Farm B?

b What is the total number of sheep on both farms?

c Which farm has the largest total number of livestock?

2 A bakery recorded the sales for Shop A and Shop B of cakes, pies and rolls in a Sales matrix, S. The prices were recorded in the Prices matrix, P.

$$S = \begin{array}{r c} & \begin{array}{ccc} \textit{Cakes} & \textit{Pies} & \textit{Rolls} \end{array} \\ \begin{array}{r} A \\ B \end{array} & \begin{bmatrix} 12 & 25 & 18 \\ 15 & 21 & 16 \end{bmatrix} \end{array} \qquad P = \begin{array}{r c} & \$ \\ \begin{array}{r} \textit{Cakes} \\ \textit{Pies} \\ \textit{Rolls} \end{array} & \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \end{array}$$

a How many pies were sold by Shop B?

b What is the selling price of pies?

c Calculate the matrix product SP.

d What information is contained in matrix SP?

e Which shop had the largest income from its sales? How much were its takings?

ISBN 978-1-009-25891-3

3 Patsy and Geoff decided to participate in a charity fun run.

a Patsy plans to walk for 4 hours and jog for 1 hour. Geoff plans to walk for 3 hours and jog for 2 hours. Write out matrix A, filling in the missing information.

$$A = \begin{array}{c} \\ \textit{Patsy} \\ \textit{Geoff} \end{array} \begin{array}{c} \begin{array}{cc} \textit{Hours walking} & \textit{Hours jogging} \end{array} \\ \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix} \end{array}$$

b Walking raises \$2 per hour and consumes 1500 kJ/h (kilojoules per hour).
Jogging raises \$3 per hour and consumes 2500 kJ/h.
Write out matrix B, filling in the missing information.

$$B = \begin{array}{c} \\ \textit{Walking} \\ \textit{Jogging} \end{array} \begin{array}{c} \begin{array}{cc} \$ & kJ \end{array} \\ \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix} \end{array}$$

c Use matrix multiplication to find a matrix that shows the money raised and the kilojoules consumed by each person.

4 A circus show is held over a three-day period – Friday, Saturday and Sunday. The table below shows the entry price per day for an adult and for a child, and the number of adults and children attending on each day.

	Friday	Saturday	Sunday
Price (\$) – Adult	12	10	10
Price (\$) – Child	6	5	5
Number of adults	360	190	400
Number of children	50	60	150

a Write down two matrices such that their product will give the amount of entry money paid on Friday and hence calculate this product.

b Write down two matrices such that their product will give the amount of entry money paid on Sunday and hence calculate this product.

c Calculate the percentage increase in revenue on Sunday compared to Friday.

ISBN 978-1-009-25891-3

6 Investigating and comparing data distributions

In this chapter

6A Types of data
6B Displaying and describing categorical data distributions
6C Interpreting and describing frequency tables
6D Displaying and describing numerical data
6E Characteristics of distributions of numerical data: shape, location and spread
6F Dot plots and stem-and-leaf plots
6G Summarising data
6H Boxplots
6I Comparing the distribution of a numerical variable across two or more groups
6J Statistical investigation
Chapter summary and review

Syllabus references

Topics: The statistical investigation process; Making sense of data relating to a single statistical variable; Comparing data for a numerical variable across two or more groups

Subtopics: 2.1.1 – 2.1.5, 2.1.10 – 2.1.12

In this information age we increasingly have to interpret data. This data may be presented in charts, diagrams or graphs, or it may simply be lists of words or numbers. There may be a lot of relevant information embodied in the data, but the story it has to tell will not always be immediately obvious. Various statistical procedures are available which will help us extract the relevant information from data sets. In this chapter, we will look at some of the techniques that are used when the data are collected from a single variable and that can help us to answer real world questions.

Cambridge Senior Maths for Western Australia Mathematics Applications 1&2 ISBN 978-1-009-25891-3

6A Types of data

Consider the following situation. In completing a survey, students are asked to:

- indicate their sex by circling an ‘F’ for female or an ‘M’ for male on the form
- indicate their preferred coffee cup size when buying takeaway coffee as ‘small’, ‘medium’ or ‘large’
- write down the number of brothers they have
- measure and write their hand span in centimetres.

The information collected from four students is displayed in the table below.

Sex	Coffee size	Number of brothers	Hand span (in cm)
M	Large	0	23.6
F	Small	2	19.6
F	Small	1	20.2
M	Large	1	24.0

Since the answers to each of the questions in the survey will vary from student to student, each question defines a different **variable** namely: *sex*, *coffee size*, *number of brothers* and *hand span*. The values we collect about each of these variables are called **data**.

The data in the table fall into two broad types: *categorical* or *numerical*.

Categorical data

The data arising from the students’ responses to the first and second questions in the survey are called **categorical data** because the data values can be used to place the person into one of several groups or categories. However, the properties of the data generated by these two questions differ slightly.

- The question asking students to use an ‘M’ or ‘F’ to indicate their sex will prompt a response of either M or F. This identifies the respondent as either male or female but tells us no more. We call this **nominal data** because it simply *names* or *nominates*.
- However, the question with responses ‘small’, ‘medium’ and ‘large’ that indicates the students’ preferred coffee size tells us two things. Firstly, it names the coffee size, but secondly it enables us to order the students according to their preferred coffee sizes. We call this **ordinal data** because it enables us to both name and order their responses.

ISBN 978-1-009-25891-3

Numerical data

The data arising from the responses to the third and fourth questions in the survey are called **numerical data** because they have values for which arithmetic operations such as adding and averaging make sense. However, the properties of the data generated by these questions differs slightly.

- The question asking students to write down the number of brothers they have will prompt whole number responses like 0, 1, 2, . . .
  Because the data can only take particular numerical values it is called **discrete data**. Discrete data arises in situations where counting is involved. For this reason, discrete data is sometimes called count data.
- In response to the hand span question, students who wrote 24 cm could have an actual hand span of anywhere between 23.5 and 24.4 cm, depending on the accuracy of the measurement and how the student rounded their answer. This is called **continuous data**, because the variable we are measuring, in this case, *hand span*, can take any numerical value within a specified range.
  Continuous data are often generated when measurement is involved. For this reason, continuous data is sometimes called measurement data.

Types of variables

Categorical variables

Variables that generate categorical data are called **categorical variables** or, if we need a finer distinction, **nominal** or **ordinal** variables. For example, *sex* is a nominal variable, while *coffee size* is an ordinal variable.

Numerical variables

Variables that generate numerical data are called numerical variables or, if we need a finer distinction, discrete or continuous variables. For example, *number of brothers* is a **discrete variable**, while *hand span* is a **continuous variable**.

ISBN 978-1-009-25891-3

Exercise 6A

Classifying data

1 Classify the categorical data arising from people answering the following questions as either nominal or ordinal.

a What is your favourite football team?

b How often do you exercise? Choose one of ‘never’, ‘once a month’, ‘once a week’, ‘every day’.

c Indicate how strongly you agree with ‘alcohol is the major cause of accidents’ by selecting one of ‘strongly agree’, ‘agree’, ‘disagree’, ‘strongly disagree’.

d What language will you study next year, ‘French’, ‘Chinese’, ‘Spanish’ or ‘none’?

2 Classify the data generated in each of the following as categorical or numerical.

a Kindergarten pupils bring along their favourite toys, and they are grouped together under the headings ‘dolls’, ‘soft toys’, ‘games’, ‘cars’ and ‘other’.

b The number of students on each of 20 school buses are counted.

c A group of people each write down their favourite colour.

d Each student in a class is weighed in kilograms.

e Students are weighed and then classified as ‘light’, ‘average’ or ‘heavy’.

f People rate their enthusiasm for a certain rock group as ‘low’, ‘medium’ or ‘high’.

3 Classify the data generated in each of the following situations as nominal, ordinal or numerical (discrete or continuous).

a The different brand names of instant soup sold by a supermarket are recorded.

b A group of people are asked to indicate their attitude to capital punishment by selecting a number from 1 to 5, where 1 = strongly disagree, 2 = disagree, 3 = undecided, 4 = agree and 5 = strongly agree.

c The number of computers per household was recorded during a census.

Classifying variables

4 Classify the numerical variables identified below (in italics) as discrete or continuous.

a The *number of pages* in a book

b The *price* paid to fill the tank of a car with petrol

c The *volume* of petrol (in litres) used to fill the tank of a car

d The *time* between the arrival of successive customers at an ATM

e The *number of people* at a football match

ISBN 978-1-009-25891-3

6B Displaying and describing categorical data distributions

To make sense of data, we first need to organise it into a more manageable form. For categorical data, frequency tables and bar charts are used for this purpose.

The frequency table

Frequency

A **frequency table** is a listing of the values a variable takes in a data set, along with how often (frequently) each value occurs.

Frequency can be recorded as a:

- **frequency**: the number of times a value occurs
- **percentage frequency**: the percentage of times a value occurs, where:

$$\text{percentage frequency} = \frac{\text{count}}{\text{total}} \times 100\%$$

- frequency **distribution**: a listing of the values a variable takes, along with how frequently each of these values occurs.

Example 1 Constructing a frequency table for categorical data

Thirty children chose a sandwich, a salad or a pie for lunch, as follows:

sandwich, salad, salad, pie, sandwich, sandwich, salad, salad, pie, pie, pie, salad, pie, sandwich, salad, pie, salad, pie, sandwich, sandwich, pie, salad, salad, pie, pie, pie, salad, pie, sandwich, pie

Construct a table for the data showing both frequency and percentage frequency.

Solution

	Frequency	
Lunch choice	Number	%
Sandwich	7	23.3
Salad	10	33.3
Pie	13	43.3
Total	30	99.9

1. Set up a table as shown. The variable *lunch choice* has three categories: 'sandwich', 'salad' and 'pie'.
2. Count the number of children choosing a sandwich, a salad or a pie. Record in the 'Number' column.
3. Add the frequencies to find the total number.
4. Convert the frequencies into percentages and record in the '%' column. For example, percentage frequency for pies equals $\frac{13}{30} \times 100\% = 43.3\%$.
5. Total the percentages and record. Note that the percentages add up to 99.9%, not 100%, because of rounding.

ISBN 978-1-009-25891-3

Bar charts

When there is a lot of data, a frequency table can be used to summarise the information, but we generally find that a graphical display is also useful. When the data is categorical, the appropriate display is a **bar chart**.

Bar charts

In a bar chart:

- frequency or percentage frequency is shown on the vertical axis
- the variable being displayed is plotted on the horizontal axis
- the height of the bar (column) gives the frequency (or percentage)
- the bars are drawn with gaps to indicate that each value is a separate category
- there is one bar for each category.

Example 2 Constructing a bar and percentage bar chart from a frequency table

Use the frequency table for lunch choice from Example 1 to construct:

a a bar chart

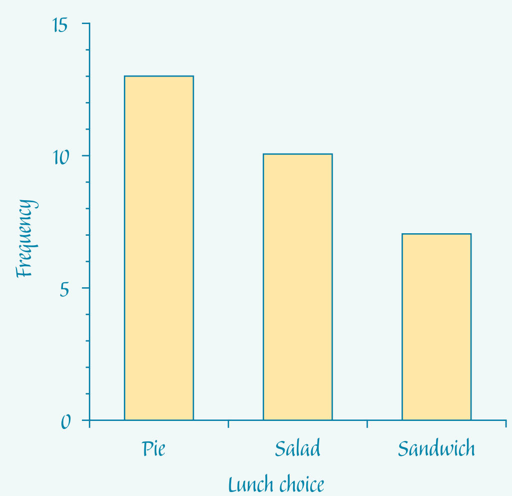
b a percentage bar chart.

Solution

a **1** Label the horizontal axis with the variable name, 'Lunch choice'. Mark the scale off into three equal intervals and label them 'Pie', 'Salad' and 'Sandwich'.

2 Label the vertical axis 'Frequency'. Insert a scale allowing for the maximum frequency of 13. Up to 15 would be appropriate. Mark the scale in 5s.

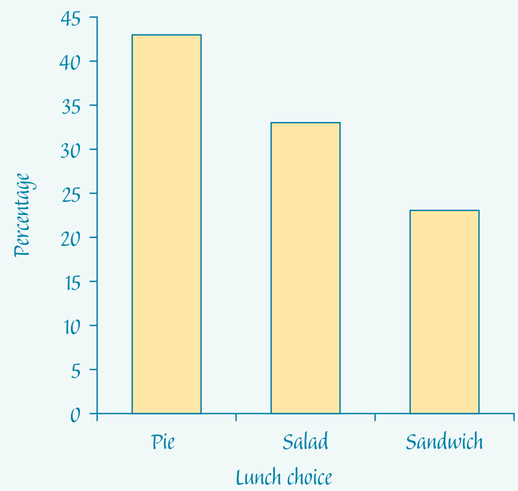
3 For each interval draw in a bar as shown. Make the width of each bar less than the width of the category intervals to show that the categories are quite separate. The height of each bar is equal to the frequency.



Note: For nominal variables it is common, but not necessary, to list categories in decreasing order by frequency. This makes later interpretation easier.

ISBN 978-1-009-25891-3

b To construct a percentage bar chart of the lunch choice data, follow the same procedure as above but label the vertical axis 'Percentage'. Insert a scale allowing for a maximum percentage frequency up to 45%. Mark the vertical scale in intervals of 5%.
The height of each bar is equal to the percentage.



The mode or modal category

One of the features of a data set that is quickly revealed with a bar chart is the **mode** or **modal category**. This is the most frequently occurring category. In a bar chart, this is given by the category with the tallest bar. For the bar chart in Example 2, the modal category is clearly 'pie'. That is, the most frequent or popular lunch choice was a pie.

When is the mode useful?

The mode is most useful when a single value or category in the frequency table occurs more often (frequently) than the others. Modes are of particular importance in popularity polls, answering questions like 'Which is the most frequently watched TV station between the hours of 6 p.m. and 8 p.m.?' or 'When is a supermarket in peak demand?'

Skill-sheet

Exercise 6B

Constructing frequency tables

Example 1

1 The *sex* of 15 people in a bus is as shown (F = female, M = male):

F M M M F M F F M M M F M M M

a Is the variable *sex* nominal or ordinal?

b Construct a frequency table for the data including frequencies and percentages frequency.

2 The UK *shoe size* of 20 eighteen-year-old males are as shown:

8	9	9	10	8	8	7	9	8	9
10	12	8	10	7	8	8	7	11	11

a Is the variable *shoe size* nominal or ordinal?

b Construct a frequency table for the data including frequencies and percentages.

ISBN 978-1-009-25891-3

Analysing frequency tables and constructing bar charts

Example 2

3 The table below shows the frequency distribution of the favourite type of fast food (*food type*) of a group of students.

a Complete the table.

b Is the variable *food type* nominal or ordinal?

c How many students preferred Chinese food?

d What percentage of students chose chicken as their favourite fast food?

e What was the favourite type of fast food for these students?

f Construct a bar chart of the frequencies (number).

	Frequency	
Food type	Number	%
Hamburgers	23	33.3
Chicken	7	10.1
Fish and chips	6	
Chinese	7	10.1
Pizza	18	
Other	8	11.6
Total		99.9

4 The following responses were received to a question regarding the return of capital punishment.

a Complete the table.

b Is the data used to generate this table nominal or ordinal?

c How many people said 'Strongly agree'?

d What percentage of people said 'Strongly disagree'?

e What was the most frequent response?

f Construct a frequencies bar chart.

	Frequency	
Capital punishment	Number	%
Strongly agree	21	8.2
Agree	11	4.3
Don't know	42	
Disagree		
Strongly disagree	129	50.4
Total	256	100.0

5 A bookseller noted the types of books purchased during a particular day, with the following results.

a Complete the table.

b Is the variable *type of book* nominal or ordinal?

c How many books purchased were classified as 'Fiction'?

d What percentage of books were classified as 'Children'?

e How many books were purchased in total?

f Construct a bar chart of the percentage frequencies (%).

	Frequency	
Type of book	Number	%
Children	53	22.8
Fiction	89	
Cooking	42	18.1
Travel	15	
Other	33	14.2
Total	232	

ISBN 978-1-009-25891-3

6 A survey of secondary school students' preferred ways of spending their leisure time at home gave the following results.

a How many students were surveyed?

b Is the variable *leisure activity* nominal or ordinal?

c What percentage of students said that they preferred to spend their leisure time phoning a friend?

d What was the most popular way of spending their leisure time for these students?

e Construct a bar chart of the percentage frequencies (%).

	Frequency	
Leisure activity	Number	%
Watch TV	84	42
Read	26	13
Listen to music	46	23
Watch a movie	24	12
Phone friends	8	4
Other	12	6
Total	200	100

6C Interpreting and describing frequency tables and bar charts

As part of this subject, you will be expected to complete a statistical investigation. Under these circumstances, constructing a frequency table or a bar chart is not an end in itself. It is merely a means to an end. The end is being able to understand something about the variables you are investigating that you didn't know before.

To complete the investigation, you will need to communicate this finding to others. To do this, you will need to know how to describe and interpret any patterns you observe in the context of your data investigation in a written report that is both systematic and concise. The purpose of this section is to help you develop such skills.

Some guidelines for describing the distribution of a categorical variable and communicating your findings

- Briefly summarise the context in which the data were collected including the number of people (or things) involved in the study.
- If there is a clear modal category, make sure that it is mentioned.
- Include relevant counts or percentages in the report.
- If there are a lot of categories, it is not necessary to mention every category.
- Either counts or percentages can be used to describe the distribution.

These guidelines are illustrated in the following examples.

ISBN 978-1-009-25891-3

Example 3 **Using a frequency table to describe the outcome of an investigation involving a categorical variable**

A group of 30 children were offered a choice of a sandwich, a salad or a pie for lunch and their responses collected and summarised in the frequency table opposite.

Lunch choice	Frequency
Sandwich	7
Salad	10
Pie	13
Total	30

Use the frequency table to report on the relative popularity of the three lunch choices quoting appropriate frequencies to support your conclusions.

Solution

Report

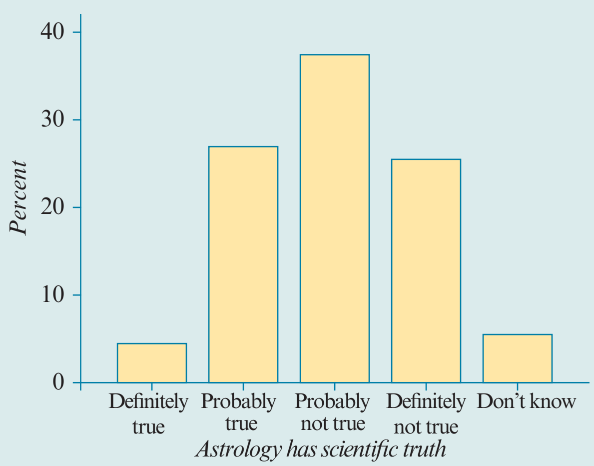
A group of 30 children were offered a choice of a sandwich, a salad or a pie for lunch. The most popular lunch choice was pie, chosen by 13 of the children. Ten of the children chose a salad. The least popular option was sandwich, chosen by only 7 of the children.

Example 4 **Using a frequency table and a percentage bar chart to describe the outcome of an investigation involving a categorical variable**

A sample of 200 people were asked to comment on the statement ‘Astrology has scientific truth’ by selecting one of the options ‘definitely true’, ‘probably true’, ‘probably not true’, definitely not true’ or ‘don’t know’.

The data are summarised in the following frequency table and bar chart. Note that the categories in the frequency table can be ordered in a definite order because the data is ordinal.

Astrology has scientific truth	Frequency	
	Number	%
Definitely true	9	4.5
Probably true	54	27.0
Probably not true	75	37.5
Definitely not true	51	25.5
Don’t know	11	5.5
Total	200	100.0



Write a report summarising the findings of this investigation quoting appropriate percentages to support your conclusion.

ISBN 978-1-009-25891-3

Solution

Report

Two hundred people were asked to respond to the statement 'Astrology has scientific truth'.

The majority of respondents did not agree, with 37.5% responding that they believed that this statement was probably not true, and another 25.5% declaring that the statement was definitely not true. Over one quarter (27%) of the respondents thought that the statement was probably true, while only 4.5% of subjects thought that the statement was definitely true.

Exercise 6C

Interpreting and describing frequency tables and bar charts

Example 3

1 A group of 69 students were asked to nominate their preferred type of fast food. The results are summarised in the percentage frequency table opposite. Use the information in the table to complete the report below by filling in the blanks.

Fast food type	%
Hamburgers	33.3
Chicken	10.1
Fish and chips	8.7
Chinese	10.1
Pizza	26.1
Other	11.6
Total	99.9

Report

A group of ☐ students were asked their favourite type of fast food. The most popular response was ☐ (33.3%), followed by pizza (☐). The rest of the group were almost evenly split between chicken, fish and chips, Chinese and other, all around 10%.

2 Two hundred and fifty-six people were asked whether they agreed that there should be a return to capital punishment in their state. Their responses are summarised in the table opposite. Use the information in the table to complete the report below by filling in the blanks.

Capital punishment	%
Strongly agree	8.2
Agree	4.3
Don't know	16.4
Disagree	20.7
Strongly disagree	50.4
Total	100.0

Report

A group of 256 people were asked whether they agreed that there should be a return to capital punishment in their state. The majority of these people ☐ (50.4%), followed by ☐ who disagreed. Levels of support for return to capital punishment were quite low, with only 4.3% agreeing and 8.2% strongly agreeing. The remaining ☐ said that they didn't know.

ISBN 978-1-009-25891-3

3 A group of 200 students were asked how they prefer to spend their leisure time. The results are summarised in the frequency table below.
Use the information in the table to write a brief report of the results of this investigation.

Leisure activity	%
Internet and digital games	42
Read	13
Listen to music	23
Watch TV or go to movies	12
Phone friends	4
Other	6
Total	100

Example 4

4 A group of 579 employees from a large company were asked to rate the importance of salary in determining how they felt about their job. Their responses are shown in the following frequency table and bar chart.

Importance of salary	%
Very important	33.5
Important	56.8
Somewhat important	7.8
Not at all important	1.9
Total	100.0

Write a report describing how these employees rated the importance of salary in determining how they felt about their job.

ISBN 978-1-009-25891-3

6D Displaying and describing numerical data

Frequency tables can also be used to organise numerical data. For discrete numerical data, the process is the same as that for categorical data, as shown in the following example.

Discrete data

Example 5 Constructing a frequency table for discrete numerical data

The number of brothers and sisters (siblings) reported by each of the 30 students in year 11 are as follows:

2	3	4	0	3	2	3	0	4	1	0	0	1	2	3
0	2	1	1	4	5	3	2	5	6	1	1	1	0	2

Construct a frequency table for these data.

Solution

1. Find the maximum and the minimum values in the data set. Here the minimum is 0 and the maximum is 6.
2. Construct a table as shown, including all the values between the minimum and the maximum.
3. Count the number of 0s, 1s, 2s, etc. in the data set. For example, there are seven 1s. Record these values in the number column.
4. Add the frequencies to find the total.
5. Convert the frequencies to percentages, and record in the per cent (%) column.

   For example, percentage of 1s equals $\frac{7}{30} \times 100 = 23.3\%$.
6. Total the percentages and record.

Number of siblings	Frequency	
	Number	%
0	6	20.0
1	7	23.3
2	6	20.0
3	5	16.7
4	3	10.0
5	2	6.7
6	1	3.3
Total	30	100.0

Grouping data

Some variables can only take on a limited range of values, for example, the variable *number of children in a family*. For these variables, it makes sense to list each of these values individually when forming a frequency distribution.

In other cases, the variable can take on a large range of values, for example, the variable *age* might take values from 0 to 100 or even more. Listing all possible ages would be tedious and would produce a large and unwieldy table. To solve this problem we **group the data** into a small number of convenient intervals.

ISBN 978-1-009-25891-3

These grouping intervals should be chosen according to the following principles:

- Every data value should be in an interval.
- The intervals should not overlap.
- There should be no gaps between the intervals.

The choice of intervals can vary but there are some guidelines.

- A division, which results in about 5 to 15 groups, is preferred.
- Choose an interval width that is easy for the reader to interpret such as 10 units, 100 units or 1000 units (depending on the data).
- By convention, the beginning of the interval is given the appropriate exact value, rather than the end. As a result, intervals of 0–49, 50–99, 100–149 would be preferred over the intervals 1–50, 51–100, 101–150 etc.

Grouped discrete data

Example 6 Constructing a frequency table for a discrete numerical variable

A group of 20 people were asked to record how many cups of coffee they drank in a particular week, with the following results:

2	0	9	10	23	25	0	0	34	32
5	0	17	14	3	6	0	33	23	0

Construct a table of these data showing both frequency (count) and percentage frequency.

Solution

1 The minimum number of cups of coffee drunk is 0 and the maximum is 34. Intervals beginning at 0 and ending at 34 would ensure that all the data are included. Interval width of 5 will mean that there are 7 intervals. Note that the endpoints are within the interval, so that the interval 0 – 4 includes 5 values: 0, 1, 2, 3, 4.

2 Set up the table as shown.

3 Count the number of data values in each interval to complete the number column.

	Frequency	
Cups of coffee	Number	%
0–4	8	40
5–9	3	15
10–14	2	10
15–19	1	5
20–24	2	10
25–29	1	5
30–34	3	15
Total	20	100

4 Convert the frequencies into percentages and record in the per cent (%) column. For example, for the interval 5–9: % frequency $= \frac{3}{20} \times 100 = 15\%$

5 Total the percentages and record.

ISBN 978-1-009-25891-3

Grouped continuous data

Example 7 **Constructing a frequency table for a continuous numerical variable**

The following are the heights of the 41 players in a basketball club, in centimetres.

178.1	185.6	173.3	193.4	183.1
184.6	202.4	170.9	183.3	180.3
185.8	189.1	178.6	194.7	185.3
191.1	189.7	191.1	180.4	180.0
193.8	196.3	189.6	183.9	177.7
178.9	193.0	188.3	189.5	182.0
183.6	184.5	188.7	192.4	203.7
180.1	170.5	179.3	184.1	183.8
174.7				

Construct a frequency table of these data.

Solution

1. Find the minimum and maximum heights, which are 170.5 cm and 203.7 cm. A minimum value of 170 and a maximum of 204.9 will ensure that all the data are included.
2. Interval width of 5 cm will mean that there are 7 intervals from 170 to 204.9, which is within the guidelines of 5–15 intervals.
3. Set up the table as shown. All values of the variable that are from 170 to 174.9 have been included in the first interval. The second interval includes values from 175 to 179.9, and so on for the rest of the table.
4. The number of data values in each interval is then counted to complete the number column of the table.
5. Convert the frequencies into percentages and record in the per cent (%) column. For example, for the interval 175.0–179.9: % frequency $= \frac{5}{41} \times 100 = 12.2\%$
6. Total the percentages and record.

	Frequency	
Heights	Number	%
170–174.9	4	9.8
175–179.9	5	12.2
180–184.9	13	31.7
185–189.9	9	22.0
190–194.9	7	17.1
195–199.9	1	2.4
200–204.9	2	4.9
Total	41	100.1

The interval that has the highest frequency is called the **modal interval**. Here the modal interval is 180.0–184.9, as 13 players (31.7%) have heights that fall into this interval.

ISBN 978-1-009-25891-3

Histograms

As with categorical data, we would like to construct a visual display of a frequency table for numerical data. The graphical display of a frequency table for a numerical variable is called a **histogram**. A histogram looks similar to a bar chart but, because the data is numerical, there is a natural order to the plot and the bar widths depend on the data values.

Histograms

In a histogram:

- frequency (number or percentage) is shown on the vertical axis
- the values of the variable being displayed are plotted on the horizontal axis
- each column corresponds to a data value, or a data interval if the data is grouped; alternatively, for ungrouped discrete data, the actual data value is located at the middle of the column
- the height of the column gives the frequency (number or percentage).

Example 8 Constructing a histogram for ungrouped discrete data

Construct a histogram for the data in the frequency table.

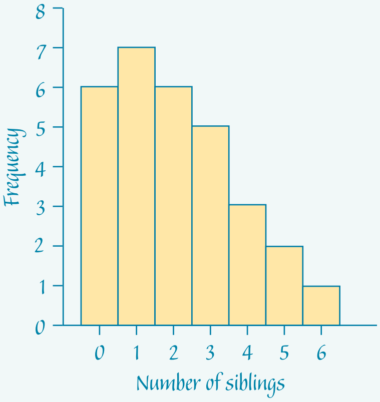
Siblings	Frequency
0	6
1	7
2	6
3	5
4	3
5	2
6	1
Total	30

Solution

1 Label the horizontal axis with the variable name 'Number of siblings'. Mark in the scale in units, so that it includes all possible values.

2 Label the vertical axis 'Frequency'. Insert a scale allowing for the maximum frequency of 7. Up to 8 would be appropriate. Mark the scale in units.

3 For each value for the variable draw in a column. The data is discrete, so make the width of each column 1, starting and ending halfway between data values. For example, the column representing 2 siblings starts at 1.5 and ends at 2.5. The height of each column is equal to the frequency.



ISBN 978-1-009-25891-3

Example 9 Constructing a histogram for continuous data

Construct a histogram for the data in the frequency table.

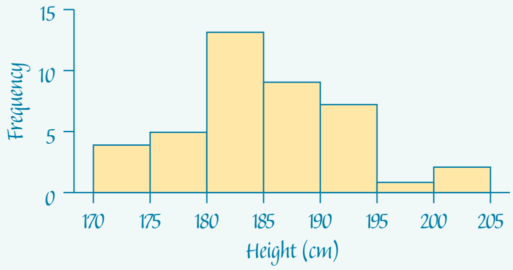
Height (cm)	Frequency
170.0–174.9	4
175.0–179.9	5
180.0–184.9	13
185.0–189.9	9
190.0–194.9	7
195.0–199.9	1
200.0–204.9	2
Total	41

Solution

1 Label the horizontal axis with the variable name 'Height'. Mark in the scale using the beginning of each interval as the scale points; that is, 170, 175, . . .

2 Label the vertical axis 'Frequency'. Insert a scale allowing for the maximum frequency of 13. Up to 15 would be appropriate. Mark the scale in units.

3 For each interval draw in a column. Each column starts at the beginning of the interval and finishes at the beginning of the next interval. Make the height of each column equal to the frequency.



Constructing a histogram using a CAS calculator

It is relatively quick to construct a histogram from a frequency table. However, if you only have the data (as you mostly do), it is a very slow process because you have to construct the frequency table first. Fortunately, a CAS calculator will do this for us.

ISBN 978-1-009-25891-3

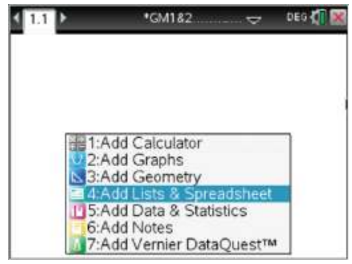
How to construct a histogram using the TI-Nspire CAS

Display the following set of 27 marks in the form of a histogram.

16	11	4	25	15	7	14	13	14	12	15	13	16	14
15	12	18	22	17	18	23	15	13	17	18	22	23	

Steps

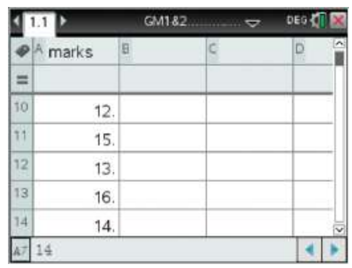
1 Start a new document: Press on and select **New Document** (or use ctrl + N). If prompted to save an existing document, move cursor to **No** and press enter.



2 Select **Add Lists & Spreadsheet**.
Enter the data into a list named *marks*.

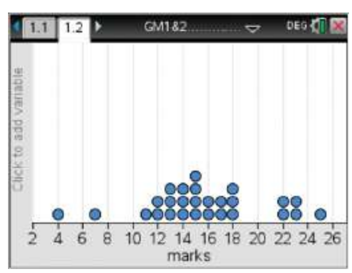
a Move the cursor to the name space of column A (or any other column) and type in *marks* as the list name. Press enter.

b Move the cursor down to row 1, type in the first data value and press enter. Continue until all the data has been entered. Press enter after each entry.

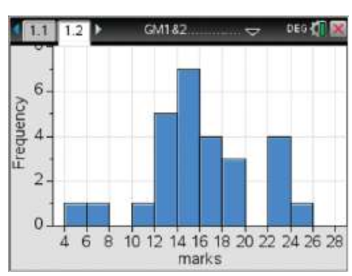


3 Statistical graphing is done through the **Data & Statistics** application.
Press ctrl + I and select **Add Data & Statistics** (or press on, arrow to the Data & Statistics icon, and press enter).
Note: A random display of dots will appear – this is to indicate that data are available for plotting. It is not a statistical plot.

a Press tab to show the list of variables that are available. Select the variable **marks**. Press enter to paste the variable **marks** to that axis.

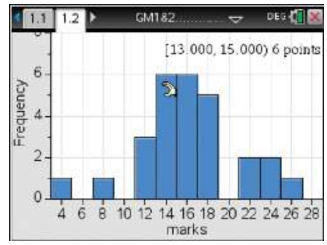


b A dot plot is displayed as the default plot. To change the plot to a histogram, press menu>**Plot Type>Histogram** and then press enter or 'click' (press the click key).
Your screen should now look like that shown opposite. This histogram has a column (or bin) width of 2 and a starting point of 3.



ISBN 978-1-009-25891-3

4 Data analysis



a Move cursor onto any column. A will appear and the column data will be displayed as shown opposite.

b To view other column data values move the cursor to another column.

Note: If you click on a column it will be selected. To deselect any previously selected columns move the cursor to the open area and press .

Hint: If you accidentally move a column or data point, press ctrl + esc to undo the move.

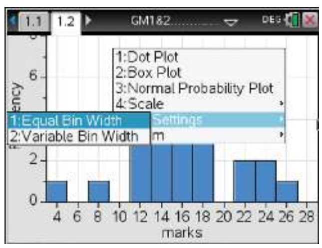
5 Change the histogram column (bin) width to 4 and the starting point to 2.

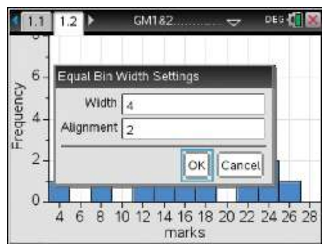
a Press ctrl + menu to get the contextual menu as shown (below left).

Hint: Pressing ctrl + menu with the cursor on the histogram gives you access to a contextual menu that enables you to do things that relate only to histograms.

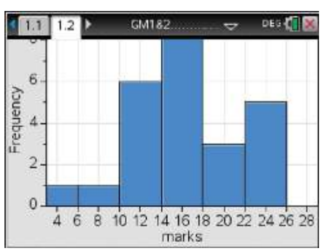
b Select **Bin Settings**.

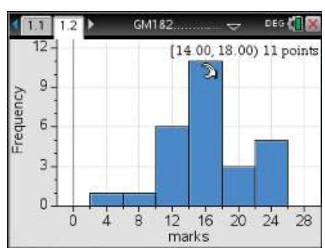
c In the settings menu (below right) change the **Width** to **4** and the **Starting Point (Alignment)** to **2** as shown. Press enter.



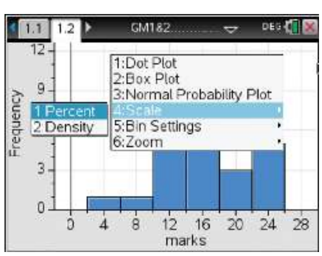


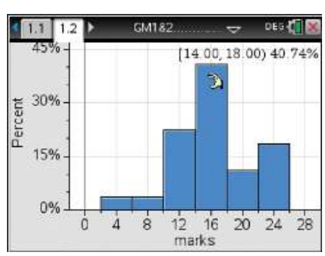
d A new histogram is displayed with column width of 4 and a starting point of 2 but it no longer fits the viewing window (below left). To solve this problem press ctrl + menu **>Zoom>Zoom-Data** and enter to obtain the histogram as shown below right.





6 To change the frequency axis to a percentage axis, press ctrl + menu **>Scale>Percent** and then press enter.





ISBN 978-1-009-25891-3

How to construct a histogram using the ClassPad

Display the following set of 27 marks in the form of a histogram.

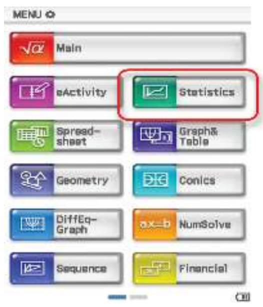
16 11 4 25 15 7 14 13 14 12 15 13 16 14
15 12 18 22 17 18 23 15 13 17 18 22 23

Steps

1 From the application menu screen, locate the **Statistics** application.

Tap Statistics to open.

Note: Tapping Menu from the icon panel (just below the touch screen) will display the application menu if it is not already visible.



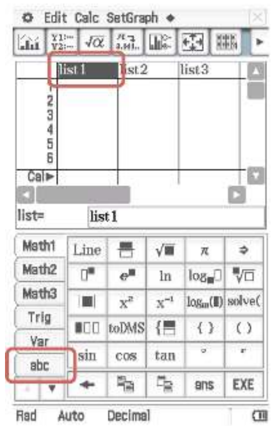
2 Enter the data into a list named **marks**.

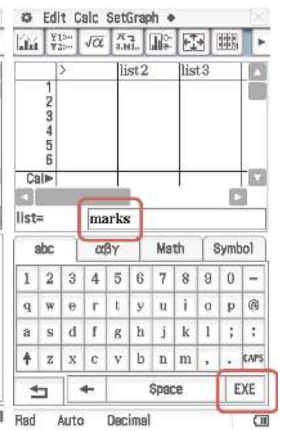
a Highlight the heading of the first list by tapping.

b Press Keyboard and tap abc.

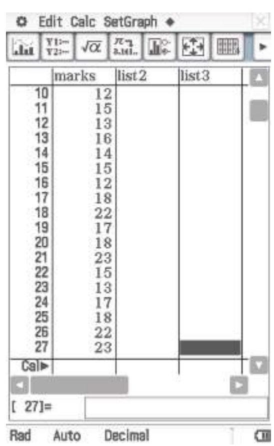
c Type **marks** and press EXE.

d Starting in row 1, type in each data value. Press EXE or ▼ to move down the list.





Your screen should be like the one shown at right.



ISBN 978-1-009-25891-3

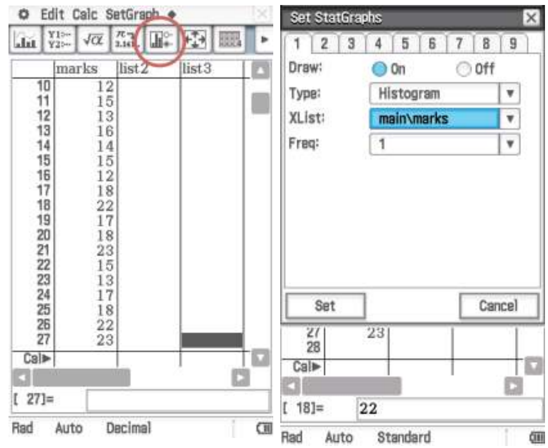
3 To plot a statistical graph:

a Tap at the top of the screen. This opens the **Set StatGraphs** dialog box.

b Complete the dialog box. For:

- **Draw:** select **On**
- **Type:** select **Histogram** (▼)
- **XList:** select **main\marks** (▼)
- **Freq:** leave as **1**.

c Tap Set to confirm your selections.



Note: To make sure only this graph is drawn, select **SetGraph** from the menu bar at the top and confirm there is a tick only beside **StatGraph1** and no other box.

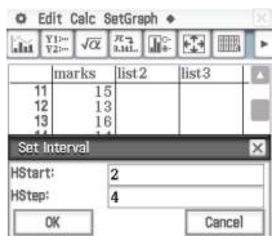
4 To plot the graph:

a Tap in the toolbar.

b Complete the **Set Interval** dialog box as given below. For:

- **HStart:** type in **2**
- **HStep:** type in **4**.

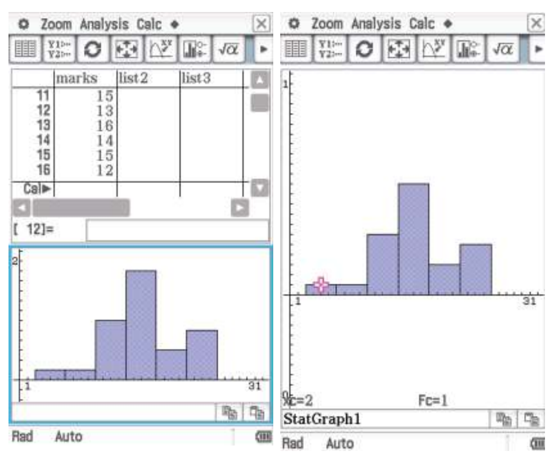
c Tap **OK**.



5 The screen is split in two. Tapping Resize from the icon panel will allow the graph to fill the entire screen. Tap Resize to return to half-screen size.

6 Tapping places a marker on the first column of the histogram and tells us that:

- the first interval begins at 2 ($\mathbf{x_c = 2}$)
- for this interval, the frequency is 1 ($\mathbf{F_c = 1}$).



To find the frequencies and starting points of the other intervals, use the arrow (▶) to move from interval to interval.

ISBN 978-1-009-25891-3

Exercise 6D

Constructing frequency tables for numerical data

Example 5

1 The number of magazines purchased in a month by 15 different people was as follows:

0 5 3 0 1 0 2 4 3 1 0 2 1 2 1

Construct a frequency table for the data, including both the frequency and percentage frequency.

Example 6

2 The amount of money carried by 20 students is as follows:

$4.55 $1.45 $16.70 $0.60 $5.00 $12.30 $3.45 $23.60 $6.90 $4.35
$0.35 $2.90 $1.70 $3.50 $8.30 $3.50 $2.20 $4.30 $0.00 $11.50

Construct a frequency table for the data, including both the number and percentage in each category. Use intervals of $5, starting at $0.

Analysing frequency tables and constructing histograms

Example 7

3 A group of 28 students were asked to draw a line that they estimated to be the same length as a 30 cm ruler. The results are shown in the frequency table below.

a How many students drew a line with a length:

i from 29.0 to 29.9 cm?

ii of less than 30 cm?

iii of 32 cm or more?

b What percentage of students drew a line with a length:

i from 31.0 to 31.9 cm?

ii of less than 31 cm?

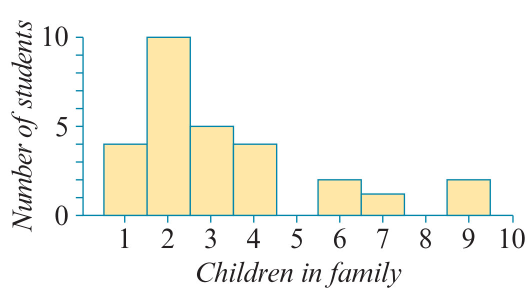
iii of 30 cm or more?

	Frequency	
Length of line (cm)	Number	%
28.0–28.9	1	3.6
29.0–29.9	2	7.1
30.0–30.9	8	28.6
31.0–31.9	9	32.1
32.0–32.9	7	25.0
33.0–33.9	1	3.6
Total	28	100.0

c Use the table to construct a histogram using the counts.

Interpreting histograms

4 The number of children in the family for each student in a class is shown in the histogram.



a How many students are the only child in a family?

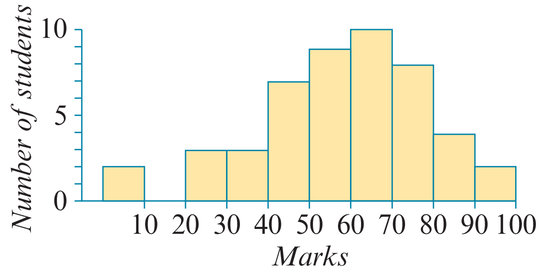
b What is the most common number of children in a family?

c How many students come from families with 6 or more children?

d How many students are there in the class?

ISBN 978-1-009-25891-3

5 The following histogram gives the scores on a general knowledge quiz for a class of year 11 students.



- **a** How many students scored from 10 to 19 marks?
- **b** How many students attempted the quiz?
- **c** What is the modal interval?
- **d** If a mark of 50 or more is designated as a pass, how many students passed the quiz?

Constructing histograms using a CAS calculator and their analysis

6 A student purchased 21 new textbooks from a schoolbook supplier with the following prices (in dollars):

41.65	34.95	32.80	27.95	32.50	53.99	63.99	17.80	13.50	18.99	42.98
38.50	59.95	13.20	18.90	57.15	24.55	21.95	77.60	65.99	14.50	

- **a** Use a CAS calculator to construct a histogram with column width 10 and starting point 10. Name the variable *price*.
- **b** For this histogram:
  - **i** what is the range of the third interval?
  - **ii** what is the 'frequency' for the third interval?
  - **iii** what is the modal interval?

7 The maximum temperatures for several capital cities around the world on a particular day, in degrees Celsius, were:

17	26	36	32	17	12	32	2	16	15	18	25
30	23	33	33	17	23	28	36	45	17	19	37

- **a** Use a CAS calculator to construct a histogram with column width 2 and starting point 0. Name the variable *maxtemp*.
- **b** For this histogram:
  - **i** what is the starting point of the second column?
  - **ii** what is the 'frequency' for this interval?
- **c** Use the window menu to redraw the histogram with a column width of 5 and a starting point of 0.
- **d** For this histogram:
  - **i** how many cities had maximum temperatures from 20°C to 25°C?
  - **ii** what is the modal interval?

ISBN 978-1-009-25891-3

6E Characteristics of distributions of numerical data: shape, location and spread

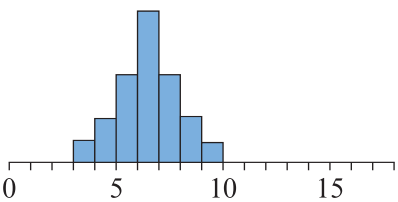
Distributions of numerical data are characterised by their shape and special features such as centre and spread.

Shape of a distribution

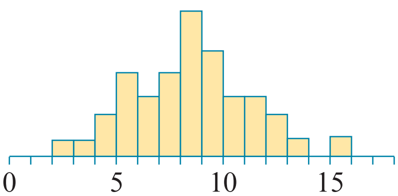
Symmetry and skew

A distribution is said to be **symmetric** if it forms a mirror image of itself when folded in the 'middle' along a vertical axis. Otherwise, the distribution is **skewed**.

Histogram A is symmetric, while Histogram B shows a distribution that is approximately symmetric.



Histogram A



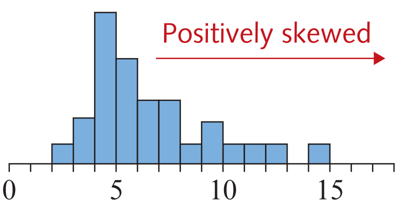
Histogram B

Positive and negative skew

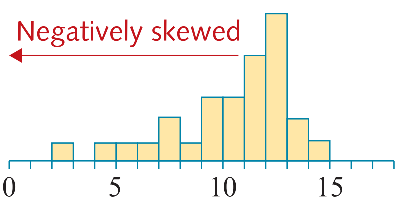
A histogram may be positively or negatively skewed.

- It is **positively skewed** if it has a short tail to the left and a long tail pointing to the right (because of the many values towards the positive end of the distribution).
- It is **negatively skewed** if it has a short tail to the right and a long tail pointing to the left (because of the many values towards the negative end of the distribution).

Histogram C is an example of a positively skewed distribution, and Histogram D is an example of a negatively skewed distribution.



Histogram C



Histogram D

Knowing whether a distribution is skewed or symmetric is important, as this gives considerable information concerning the choice of appropriate summary statistics, as will be seen in the next section.

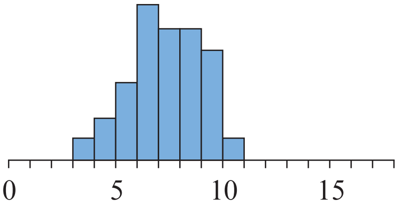
ISBN 978-1-009-25891-3

Location and spread

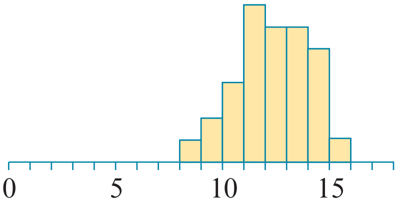
Comparing location

Two distributions are said to differ in **location** if the values of the data in one distribution are generally larger than the values of the data in the other distribution.

Consider, for example, the following histograms, shown on the same scale. Histogram F is identical in shape and width to Histogram E but moved horizontally five units to the right, indicating that these distributions differ in location.



Histogram E

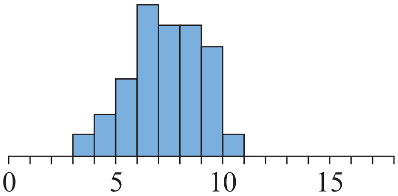


Histogram F

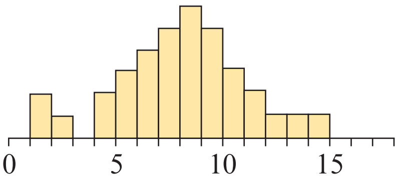
Comparing spread

Two distributions are said to differ in **spread** if the values of the data in one distribution tend to be more variable (spread out) than the values of the data in the other distribution.

Histograms G and H illustrate the difference in spread. While both are centred at about the same place, Histogram H is more spread out.



Histogram G



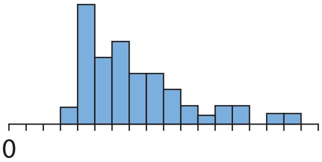
Histogram H

Exercise 6E

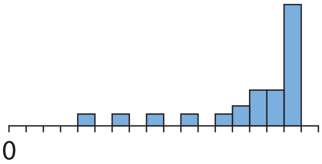
Describing shape using histograms

1 Describe the shape of each of the following histograms.

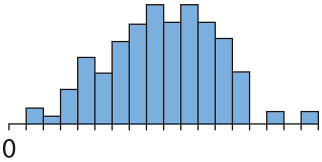
a



b



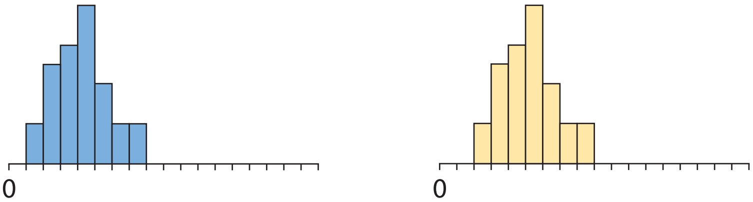
c



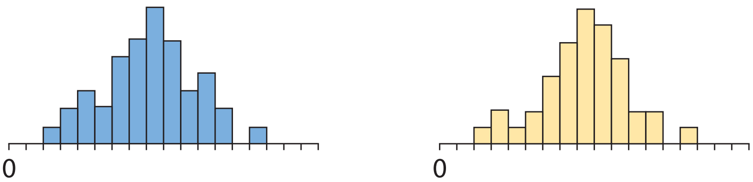
ISBN 978-1-009-25891-3

2 Do the following pairs of distributions differ in spread, location, both or neither? Assume that each pair of histograms is drawn on the same scale.

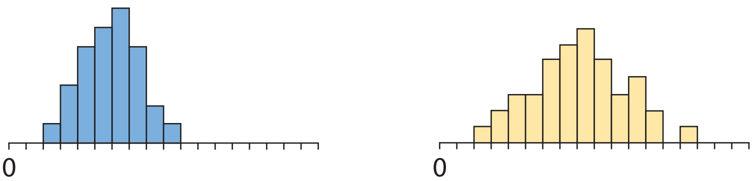
a



b



c



6F Dot plots and stem-and-leaf plots

Dot plots

The simplest display of numerical data is a **dot plot**.

Dot plot

A dot plot consists of a number line with each data point marked by a dot. When several data points have the same value, the points are stacked on top of each other.

Dot plots are a great way to display fairly small data sets where the data takes a limited number of values.

Example 10 Constructing a dot plot

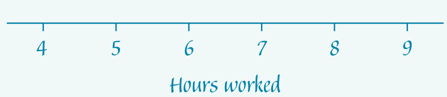
The number of hours worked by each of 10 students in their part-time jobs is as follows:

6 9 5 8 6 4 6 7 6 5

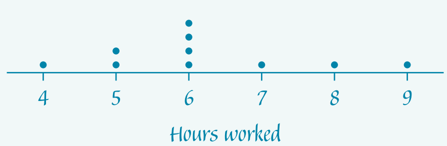
Construct a dot plot of these data.

Solution

1 Draw in a number line, scaled to include all data values. Label the line with the variable being displayed.



2 Plot each data value by marking in a dot above the corresponding value on the number as shown.



ISBN 978-1-009-25891-3

Stem-and-leaf plots

The **stem-and-leaf plot** or **stem plot** is another plot used for small data sets.

Example 11 Constructing a stem plot

The following is a set of marks obtained by a group of students on a test:

15	2	24	30	25	19	24	33	41	60	42	35	35
28	28	19	19	28	25	20	36	38	43	45	39	

Display the data in the form of an ordered stem-and-leaf plot.

Solution

1 The data set has values in the units, tens, twenties, thirties, forties, fifties and sixties. Thus, appropriate stems are 0, 1, 2, 3, 4, 5 and 6. Write these down in ascending order, followed by a vertical line.

0	
1	
2	
3	
4	
5	
6	

2 Now attach the leaves. The first data value is 15. The stem is 1 and the leaf is 5. Opposite the 1 in the stem, write the number 5, as shown.

0	
1	5
2	
3	
4	
5	
6	

The second data value is 2. The stem is 0 and the leaf is 2. Opposite the 0 in the stem, write the number 2, as shown.

0	2
1	5
2	
3	
4	
5	
6	

Continue systematically working through the data, following the same procedure, until all points have been plotted. You will then have the *unordered* stem plot, as shown.

0	2
1	5 9 9 9
2	4 5 4 8 8 8 5 0
3	0 3 5 5 6 8 9
4	1 2 3 5
5	
6	0

ISBN 978-1-009-25891-3

3 Ordering the leaves in increasing value as they move away from the stem gives the *ordered* stem plot, as shown. Write the name of the variable being displayed (*Marks*) at the top of the plot and add a key (1|5 means 15 marks).

Marks 1|5 means 15 marks

0	2
1	5 9 9 9
2	0 4 4 5 5 8 8 8
3	0 3 5 5 6 8 9
4	1 2 3 5
5	
6	0

It can be seen from the preceding plot that the distribution is approximately symmetric, with one test score, 60, which seems to stand out from the rest. When a value sits away from the main body of the data, it is called an **outlier**.

Choosing between plots

We now have three different plots that can be used to display numerical data: the histogram, the dot plot and the stem plot. They all allow us to make judgements concerning the important features of the distribution of the data, so how would we decide which one to use?

While there are no hard and fast rules, the following guidelines are often used.

Plot	Used best when	How usually constructed
Dot plot	small data sets (say $n < 30$) discrete data	by hand or with technology when constructing histograms as well
Stem plot	small data sets (say $n < 50$)	by hand
Histogram	large data sets (say $n > 30$)	with technology

Exercise 6F

Constructing and analysing dot plots

Example 10

1 The number of children in each of 15 families is as follows:

0 7 2 2 2 4 1 3 3 2 2 2 0 0 1

a Construct a dot plot of the number of children.

b What is the mode of this distribution?

2 A group of 20 people were asked how many times in the last week they had shopped at a particular supermarket. Their responses were as follows:

0 1 1 0 0 6 0 1 2 2 3 4 0 0 1 1 2 3 2 0

a Construct a dot plot of this data.

b How many people did not shop at the supermarket in the last week?

ISBN 978-1-009-25891-3

3 The number of goals scored in an AFL game by each player on one team is as follows:

0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 1 1 1 2 2 3 6

a Construct a dot plot of the number of goals scored.
b What is the mode of this distribution?
c What is the shape of the distribution of goals scored?

4 In a study of the service offered at her café, Amanda counted the number of people waiting in the queue every 5 minutes from 12 noon until 1 p.m.:

Time	12:00	12:05	12:10	12:15	12:20	12:25	12:30	12:35	12:40	12:45	12:50	12:55	1:00
Number	0	2	4	4	7	8	6	5	0	1	2	1	1

a Construct a dot plot of the number of people waiting in the queue.
b When does the peak demand at the café seem to be?

Constructing and analysing stem plots

Example 11

5 The marks obtained by a group of students on an English examination are as follows:

92	65	35	89	79	32	38	46	26	43	83	79
50	28	84	97	69	39	93	75	58	49	44	59
78	64	23	17	35	94	83	23	66	46	61	52

a Construct a stem plot of the marks.
b How many students obtained 50 or more marks?
c What was the lowest mark?

6 The stem plot on the right shows the ages, in years, of all the people attending a meeting.

a How many people attended the meeting?
b What is the shape of the distribution of ages?
c How many of these people were less than 43 years old?

Age (years) 1 | 2 represents 12 years

0	2 7
2	1 4 5 5 7 8 9
3	0 3 4 4 5 7 8 9
4	0 1 2 2 3 3 4 5 7 8 8 8
5	2 4 5 6 7 9
6	3 3 3 8
7	0

ISBN 978-1-009-25891-3

7 An investigator recorded the amount of time for which 24 similar batteries lasted in a toy. Her results (in hours) were:

26	40	30	24	27	31	21	27	20	30	33	22
4	26	17	19	46	34	37	28	25	31	41	33

a Make a stem plot of these times.

b How many of the batteries lasted for more than 30 hours?

8 The amount of time (in minutes) that a class of students spent on homework on one particular night were:

10	27	46	63	20	33	15	21	16	14	15
39	70	19	37	56	20	28	23	0	29	10

a Make a stem plot of these times.

b How many students spent more than 60 minutes on homework?

c What is the shape of the distribution?

9 The prices of a selection of shoes at a discount outlet are as follows:

\$49	\$75	\$68	\$79	\$75	\$39	\$35	\$52	\$149	\$84
\$36	\$95	\$28	\$25	\$78	\$45	\$46	\$76	\$82	

a Construct a stem plot of this data.

b What is the shape of the distribution?

ISBN 978-1-009-25891-3

6G Summarising data

A statistic is any number computed from data. Certain special statistics are called **summary statistics**, because they numerically summarise important features of the data set. Of course, whenever any set of data is summarised into just one or two numbers, much information is lost. However, if a summary statistic is well chosen, it may reveal important information hidden in the data set.

For a single data distribution, the most commonly used summary statistics are either measures of centre or measures of spread.

Measures of centre

The mean

The most commonly used measure of the centre of a distribution of a numerical variable is the **mean**. The mean is calculated by summing the data values and then dividing by their number. The mean of a set of data is what many people call the 'average'.

The mean

$$\text{mean} = \frac{\text{sum of data values}}{\text{total number of data values}}$$

For example, consider the set of data: 1, 5, 2, 4

$$\text{Mean} = \frac{1+5+2+4}{4} = \frac{12}{4} = 3$$

Some notation

Because the rule for the mean is relatively simple, it is easy to write in words. However, later you will meet other rules for calculating statistical quantities that are rather complicated and hard to write out in words. To overcome this problem, we use a shorthand notation that enables complex statistical formulas to be written out in a compact form.

In this notation we use:

- the Greek capital letter sigma, $\sum$, as a shorthand way of writing 'sum of'
- a lower case x to represent a data value
- a lower case x with a bar, $\bar{x}$ (pronounced 'x bar'), to represent the mean of the data values
- n to represent the total number of data values.

The rule for calculating the mean then becomes: $\bar{x} = \dfrac{\sum x}{n}$

ISBN 978-1-009-25891-3

Example 12 Calculating the mean

The following data set shows the number of premierships won by each of the current AFL teams, until the end of 2022. Find the mean of the number of premierships won.

Team	Premierships won
Carlton	16
Essendon	16
Collingwood	15
Melbourne	13
Hawthorn	13
Brisbane Lions	3
Richmond	13
Geelong	10
Sydney	5
Kangaroos	4
West Coast	4
Adelaide	2
Port Adelaide	1
Western Bulldogs	2
St Kilda	1
Fremantle	0
Gold Coast	0
GWS	0

Solution

1 Write down the formula and the value of n.

$$\bar{x} = \frac{\sum x}{n} \qquad n = 18$$

2 Substitute into the formula and evaluate.

$$\bar{x} = \frac{16 + 16 + 15 + \cdots + 2 + 1 + 0 + 0 + 0}{18}$$

3 We do not expect the mean to be a whole number, so give your answer to one decimal place.

$$= \frac{120}{18}$$

$$= 6.7$$

ISBN 978-1-009-25891-3

The median

Another useful measure of the centre of a distribution of a numerical variable is the middle value, or **median**. To find the value of the median, all the observations are listed in order and the middle one is the median.

For example, the median of the following data set is 6, as there are five observations on either side of this value when the data are listed in order.

$$\begin{array}{ccccccccccc} & & & & & \text{median} = 6 & & & & & \\ & & & & & \downarrow & & & & & \\ 2 & 3 & 4 & 5 & 5 & \boxed{6} & 7 & 7 & 8 & 8 & 11 \end{array}$$

When there is an even number of data values, the median is defined as the midpoint of the two middle values. For example, the median of the following data set is 6.5, as there are six observations on either side of this value when the data are listed in order.

$$\begin{array}{cccccccccccc} & & & & & & \text{median} = 6.5 & & & & & \\ & & & & & & \downarrow & & & & & \\ 2 & 3 & 4 & 5 & 5 & \boxed{6 \quad 7} & 7 & 8 & 8 & 11 & 11 \end{array}$$

Returning to the premiership data. As the data are already given in order, it only remains to determine the middle observation.

Since there are 18 entries in the table there is no actual middle observation, so the median is chosen as the value halfway between the two middle observations, in this case the ninth and tenth (5 and 4).

$$\text{median} = \frac{1}{2}(5 + 4) = 4.5$$

The interpretation here is that, of the teams in the AFL, half (or 50%) have won the premiership 5 or more times and half (or 50%) have won the premiership 4 or less times.

The following rule is useful for locating the median in a larger data set stem plot.

Determining the median

To compute the median of a distribution:

- arrange all the observations in ascending order according to size
- if n, the number of observations, is odd, then the median is the $\frac{n+1}{2}$th observation from the end of the list
- if n, the number of observations, is even, then the median is found by averaging the two middle observations in the list. That is, to find the median the $\frac{n}{2}$ and the $\left(\frac{n}{2} + 1\right)$th observations are added together and divided by 2.

ISBN 978-1-009-25891-3

Example 13 Determining the median

Find the median age of 23 people whose ages are displayed in the ordered stem plot below.

Age (years) 1 | 2 represents 12 years

0	2 5
2	1 4 5 8
3	0 3 4 6
4	0 1 2 5 7
5	2 4 5 8
6	3 5 9 9

Solution

As the data are already given in order, it only remains to determine the middle observation.

1. Write down the number of observations. $n = 23$
2. The median is located at the $\dfrac{n+1}{2}$th position. median is at the $\dfrac{23+1}{2} = 12$th position
   Thus the median age is 41 years.

Note: We can check to see whether we are correct by counting the number of data values either side of the median. They should be equal.

Comparing the mean and median

In Example 12 we found that the mean number of premierships won by the 18 AFL clubs was $\overline{x} = 6.6$. By contrast, in Example 13, we found that the median number of premierships won was 4.5.

These two values are quite different and the interesting question is: Why are they different, and which is the better measure of centre in this situation?

To help us answer this question, consider a stem plot of these data values.

Premierships won

0	0 0 0 1 1 1 2 3 4
0	5 9
1	0 1 1 2
1	5 6 6

From the stem-and-leaf plot it can be seen that the distribution is positively skewed. This example illustrates a property of the mean. When the distribution is skewed or if there are one or two very extreme values, then the value of the mean may be far from the centre. The median is not so affected by unusual observations and always gives the middle value.

ISBN 978-1-009-25891-3

Measures of spread

A measure of spread is calculated in order to judge the *variability* of a data set. That is, are most of the values clustered together, or are they rather spread out?

The range

The simplest measure of spread can be determined by considering the difference between the smallest and the largest observations. This is called the **range**.

The range

The range (R) is the simplest measure of spread of a distribution.

The range is the difference between the largest and smallest values in the data set.

R = largest data value − smallest data value

Example 14 Finding the range

Consider the marks, for two different tasks, awarded to a group of students:

Task A

2	6	9	10	11	12	13	22	23	24	26	26	27	33	34
35	38	38	39	42	46	47	47	52	52	56	56	59	91	94

Task B

11	16	19	21	23	28	31	31	33	38	41	49	52	53	54
56	59	63	65	68	71	72	73	75	78	78	78	86	88	91

Find the range of each of these distributions.

Solution

For task A the minimum mark is 2 and the maximum mark is 94.

Range for Task A = 94 − 2 = 92

For Task B, the minimum mark is 11 and the maximum mark is 91.

Range for Task B = 91 − 11 = 80

ISBN 978-1-009-25891-3

The range for Task A is greater than the range for Task B. Is the range a useful summary statistic for comparing the spread of the two distributions? To help make this decision, consider the stem plots of the data sets:

Task A

0	2 6 9
1	0 1 2 3
2	2 3 4 6 6 7
3	3 4 5 8 8 9
4	2 6 7 7
5	2 2 6 6 9
6	
7	
8	
9	1 4

Task B

0	
1	1 6 9
2	1 3 8
3	1 1 3 8
4	1 9
5	2 3 4 6 9
6	3 5 8
7	1 2 3 5 8 8 8
8	6 8
9	1

From the stem-and-leaf plots of the data it appears that the spread of marks for the two tasks is not really described by the range. It is clear that the marks for Task A are more concentrated than the marks for Task B, except for the two unusual values for Task A.

Another measure of spread is needed, one which is not so influenced by these extreme values. The statistic we use for this task is the **interquartile range**.

The interquartile range

The interquartile range (IQR) gives the spread of the middle 50% of data values.

Determining the interquartile range

To find the interquartile range of a distribution:

- arrange all observations in order according to size
- divide the observations into two equal-sized groups, and if n is odd, omit the median from both groups
- locate Q_1, the *first quartile*, which is the median of the lower half of the observations, and Q_3, the *third quartile*, which is the median of the upper half of the observations.

The interquartile range IQR is then: IQR = $Q_3 - Q_1$

Definitions of the **quartiles** of a distribution sometimes differ slightly from the one given here. Using different definitions may result in slight differences in the values obtained, but these will be minimal and should not be considered a difficulty.

ISBN 978-1-009-25891-3

Example 15 Finding the interquartile range (IQR)

Find the interquartile ranges for Tasks A and B in Example 14 and compare.

Solution

1 There are 30 values in total. This means that there are fifteen values in the lower 'half', and fifteen in the upper 'half'. The median of the lower half (Q_1) is the 8th value.

Task A

Lower half:

2 6 9 10 11 12 13 22 23 24 26 26 27 33 34

$Q_1 = 22$

2 The median of the upper half (Q_3) is the 8th value.

Upper half:

35 38 38 39 42 46 47 47 52 52 56 56 59 91 94

$Q_3 = 47$

3 Determine the IQR.

$IQR = Q_3 - Q_1 = 47 - 22 = 25$

4 Repeat the process for Task B.

Task B

$Q_1 = 31$

$Q_3 = 73$

$IQR = Q_3 - Q_1 = 73 - 31 = 42$

5 Compare the IQR for Task A to the IQR for Task B.

The IQR shows the variability of Task A marks is smaller than the variability of Task B marks.

The interquartile range describes the range of the middle 50% of the observations. It measures the spread of the data distribution around the median (M). Since the upper 25% and the lower 25% of the observations are discarded, the interquartile range is generally not affected by outliers in the data set, which makes it a reliable measure of spread.

The standard deviation

The **standard deviation (s)**, measures the spread of a data distribution about the mean ($\bar{x}$).

The standard deviation

The standard deviation is defined to be:

$$s = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}}$$

where n is the number of data values (sample size) and $\bar{x}$ is the mean.

Although it is not easy to see from the formula, the standard deviation is the square root of an average of the squared deviations of each data value from the mean. We work with the *squared* deviations because the sum of the deviations around the mean will always be zero. For technical reasons we average by dividing by $n - 1$, not n. In practice this is not a problem, as dividing by $n - 1$ compared to n generally makes very little difference to the final value.

ISBN 978-1-009-25891-3

Normally, you will use your calculator to determine the value of a standard deviation. However, to understand what is involved when your calculator is doing the calculation, you should know how to calculate the standard deviation from the formula.

Example 16 **Calculating the standard deviation**

Use the formula:

$$s = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}}$$

to calculate the standard deviation of the data set: 2, 3, 4.

Solution

1 To calculate s, it is convenient to set up a table with columns for:
x the data values
$(x - \bar{x})$ the deviations from the mean
$(x - \bar{x})^2$ the squared deviations.

	x	$(x - \bar{x})$	$(x - \bar{x})^2$
	2	−1	1
	3	0	0
	4	1	1
Sum	9	0	2

2 First find the mean and then complete the table as shown.

$$\bar{x} = \frac{\sum x}{n} = \frac{2 + 3 + 4}{3} = \frac{9}{3} = 3$$

3 Substitute the required values into the formula and evaluate.

$$s = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}} = \sqrt{\frac{2}{3 - 1}} = 1$$

The standard deviation is a useful measurement when comparing the variability of multiple data sets. It gives an indication of the degree to which the individual data values are spread around their mean. The smaller the standard deviation, the closer the data is to the mean. This indicates that the data shows less variation, with the data being more similar to the mean.

For example, using the data from Example 14, Task A has a standard deviation of 22.45, whilst Task B has a standard deviation of 23.71 (to two decimal places). This indicates that the marks for Task A have less variation than Task B, and therefore the marks from Task A are closer to the mean.

Using a CAS calculator to calculate summary statistics

As you can see, calculating the various summary statistics you have encountered in this section is sometimes rather complicated and generally time consuming. Fortunately, it is no longer necessary to carry out these computations by hand, except in the simplest cases.

ISBN 978-1-009-25891-3

How to calculate measures of centre and spread using the TI-Nspire CAS

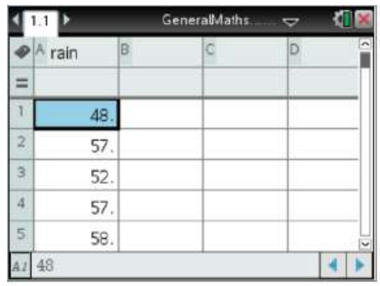
The table shows the monthly rainfall figures for a year in Melbourne.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Rainfall (mm)	48	57	52	57	58	49	49	50	59	67	60	59

Determine the mean and standard deviation, median and interquartile range, and range.

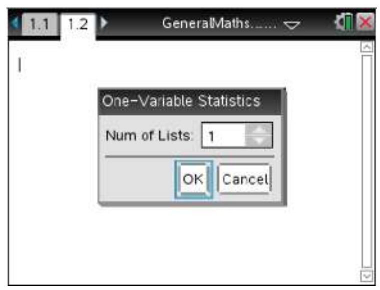
Steps

1 Start a new document: Press [⌂ on] and select **New Document** (or press [ctrl] + [N]).



2 Select **Add Lists & Spreadsheet**.
Enter the data into a list named *rain* as shown.
Statistical calculations can be done in the **Lists & Spreadsheet** application or the **Calculator** application.

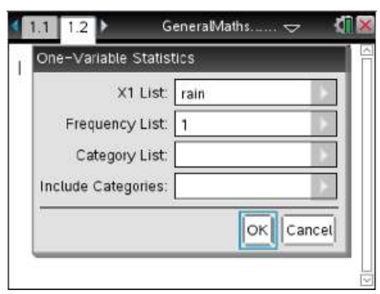
3 Press [ctrl] + [I] and select **Add Calculator** (or press [⌂ on] and arrow to [calculator icon] and press [enter]).



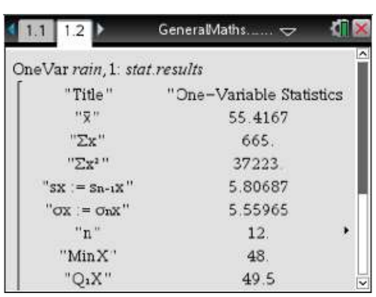
a Press [menu]**>Statistics>Stat Calculations>One-Variable Statistics**, [enter].

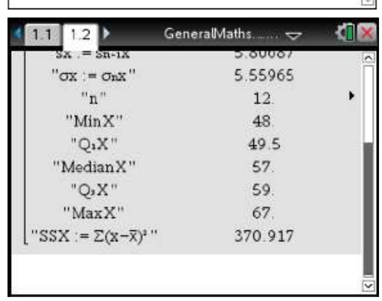
b Press the [tab] key to highlight OK and [enter].

c Use the ▶ arrow and [enter] to paste in the list name **rain**. Press [esc] to exit the popup screen and generate statistical results screen below.



Notes: 1 The sample standard deviation is **sx**.
2 Use the ▲ arrows to scroll through the results screen to see the full range of statistics calculated.





ISBN 978-1-009-25891-3

4 Write the answers correct to one decimal place.

$\bar{x} = 55.4, S = 5.8$

$M = 57$

$IQR = Q_3 - Q_1 = 59 - 49.5 = 9.5$

$R = \max - \min = 67 - 48 = 19$

How to calculate measures of centre and spread using the ClassPad

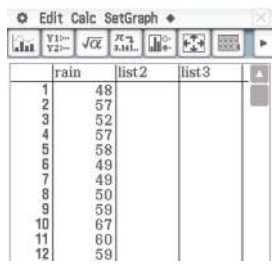
The table shows the monthly rainfall figures for a year in Melbourne.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Rainfall (mm)	48	57	52	57	58	49	49	50	59	67	60	59

Determine the mean and standard deviation, median and interquartile range, and range.

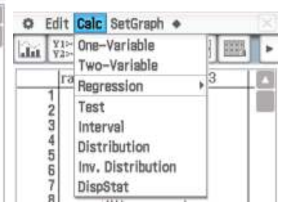
Steps

1 Open the **Statistics** application and enter the data into the column labelled **rain**.



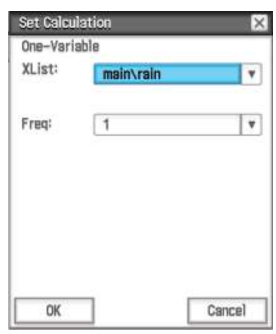
2 To calculate the mean, median, standard deviation, and quartiles:

- Select **Calc** from the menu bar.
- Tap **One-Variable** and open the **Set Calculation** dialog box.



3 Complete the dialog box. For:

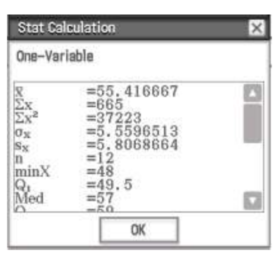
- **XList:** select **main \ rain** (▼)
- **Freq:** leave as **1**.

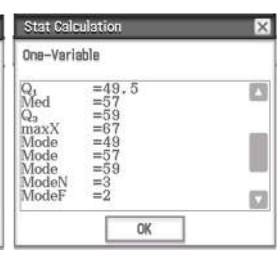


4 Tap **OK** to confirm your selections.

Notes: 1 The sample standard deviation is given by s_x.

2 Use the ▲▼ side-bar arrows to scroll through the results screen to obtain values for additional statistics if required.





ISBN 978-1-009-25891-3

5 Write the answers correct to one decimal place.

$\bar{x} = 55.4, S = 5.8$

$M = 57$

$IQR = Q_3 - Q_1 = 59 - 49.5 = 9.5$

$R = max - min = 67 - 48 = 19$

Exercise 6G

Calculating the mean, median and IQR without a calculator

Example 12

1 Find, without using a calculator, the mean and median for each of these data sets.

a 2, 5, 7, 2, 9

b 4, 11, 3, 5, 6, 1

c 15, 25, 10, 20, 5

d 101, 105, 98, 96, 97, 109

e 1.2, 1.9, 2.3, 3.4, 7.8, 0.2

Example 14
Example 15

2 Find, without using a calculator, the median and IQR and range of each of these ordered data sets.

a 2, 2, 5, 7, 9, 11, 12, 16, 23

b 1, 3, 3, 5, 6, 7, 9, 11, 12, 12

c 21, 23, 24, 25, 27, 27, 29, 31, 32, 33

d 101, 101, 105, 106, 107, 107, 108, 109

e 0.2, 0.9, 1.0, 1.1, 1.2, 1.2, 1.3, 1.9, 2.1, 2.2, 2.9

Example 13

3 Without a calculator, determine the median and the IQR for the data displayed in the following stem plots.

a Monthly rainfall (mm)

4	8 9 9
5	0 2 7 7 8 9 9
6	0 7

b Battery time (hours)

0	4
1	7 9
2	0 1 2 4 5 6 6 7 7 8
3	0 0 1 1 3 3 4 7
4	0 1 6

Using a calculator to determine summary statistics

4 The following table gives the area, in hectares, of each of the suburbs of a city:

3.6	2.1	4.2	2.3	3.4	40.3	11.3	19.4	28.4	27.6	7.4	3.2	9.0

a Find the mean and the median areas, correct to one decimal place.

b Which is a better measure of centre for this data set? Explain your answer.

5 The prices, in dollars, of apartments sold in a particular suburb during one month were:

\$387 500	\$329 500	\$293 400	\$600 000	\$318 000	\$368 000	\$750 000
\$333 500	\$335 500	\$340 000	\$386 000	\$340 000	\$404 000	\$322 000

a Find the mean and the median of the prices.

b Which is a better measure of centre of this data set? Explain your answer.

ISBN 978-1-009-25891-3

CAS

6 A manufacturer advertised that a can of soft drink contains 375 mL of liquid. A sample of 16 cans yielded the following contents:

357	375	366	360	371	363	351	369
358	382	367	372	360	375	356	371

Find the mean and standard deviation, median and IQR, and range for the volume of drink in the cans. Give answers correct to one decimal place.

7 The serum cholesterol levels for a sample of 20 people are:

231	159	203	304	248	238	209	193	225	244
190	192	209	161	206	224	276	196	189	199

Find the mean and standard deviation, median and IQR, and range of the serum cholesterol levels. Give answers correct to one decimal place.

8 Twenty babies were born at a local hospital on one weekend. Their birth weights are given in the stem plot.

Birth weight (kg) 3 | 6 represents 3.6 kg

Stem	Leaf
2	1 5 7 9 9
3	1 3 3 4 4 5 6 7 7 9
4	1 2 2 3 5

Find the mean and standard deviation, median and IQR, and range of the birth weights.

9 The results of a student's chemistry experiment were as follows:

7.3 8.3 5.9 7.4 6.2 7.4 5.8 6.0

Write all answers correct to two decimal places.

a **i** Find the mean and the median of the results.

ii Find the IQR and the standard deviation of the results.

b Unfortunately, when the student was transcribing his results into his chemistry book, he made a small error and wrote:

7.3 8.3 5.9 7.4 6.2 7.4 5.8 60

i Find the mean and the median of these results.

ii Find the interquartile range and the standard deviation of these results.

c Describe the effect the error had on the summary statistics in parts **a** and **b**.

ISBN 978-1-009-25891-3

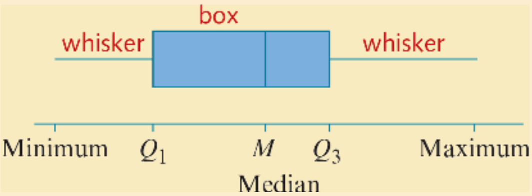
6H Boxplots

Knowing the median and quartiles of a distribution means that quite a lot is known about the central region of the data set. If something is known about the tails of the distribution as well, then a good picture of the whole data set can be obtained. This can be achieved by knowing the **maximum** and **minimum** values of the data.

When we list the *median*, the *quartiles* and the *maximum* and *minimum* values of a data sets, we have what is known as a **five-number summary**. Its pictorial (graphical) representation is called a **boxplot** or a box-and-whisker plot.

Boxplots

- A boxplot is a graphical representation of a five-number summary.
- A box is used to represent the middle 50% of scores.



- The median is shown by a vertical line drawn within the box.
- Lines (whiskers) extend out from the lower and upper ends of the box to the smallest and largest data values of the data set respectively.

Example 17 Constructing a boxplot from a five-number summary

The following are the monthly rainfall figures for a year in Melbourne.

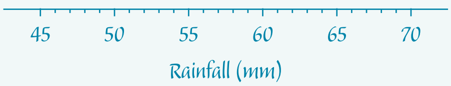
Month	J	F	M	A	M	J	J	A	S	O	N	D
Rainfall (mm)	48	57	52	57	58	49	49	50	59	67	60	59

Construct a boxplot to display this data, given the five-number summary:

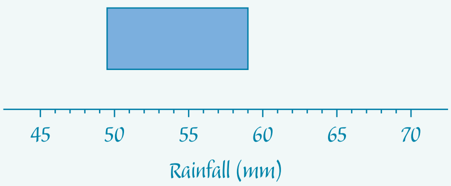
Min = 48, $Q_1 = 49.5$, $M = 57$, $Q_3 = 59$, Max = 67

Solution

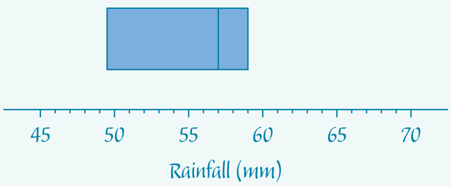
1 Draw in a labelled and scaled number line that covers the full range of values.



2 Draw in a box starting at $Q_1 = 49.5$ and ending at $Q_3 = 59$.

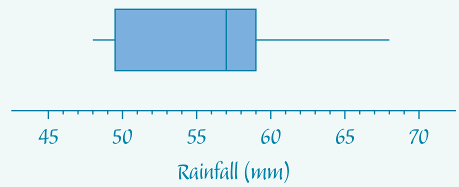


3 Mark in the median value with a vertical line segment at $M = 57$.



ISBN 978-1-009-25891-3

4 Draw in the whiskers, lines joining the midpoint of the ends of the box, to the minimum and maximum values, 48 and 67, respectively.



Boxplots with outliers

An extension of the boxplot can also be used to identify possible outliers in a data set.

Outlier

An *outlier* is a data value that appears to be rather different from other observations.

Sometimes it is difficult to decide whether or not an observation is an outlier. For example, a boxplot might have one extremely long whisker. How might we explain this?

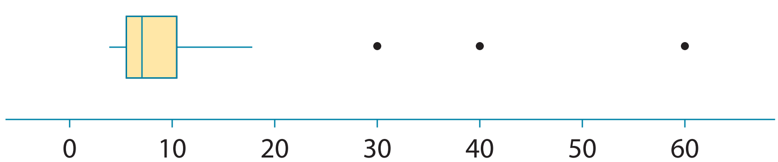
- The data distribution could be extremely skewed with lots of data values in its tail.
- Another explanation is that the long whisker hides one or more outliers.

By modifying the boxplots, we can decide which explanation is most likely.

Designating outliers

Any data point in a distribution that lies more than 1.5 interquartile ranges above the third quartile or more than 1.5 interquartile ranges below the first quartile could be an outlier.

These data values are plotted individually in the boxplot, and the whisker now ends at the largest or smallest data value that is not outside these limits. An example of a boxplot displaying outliers is shown below.



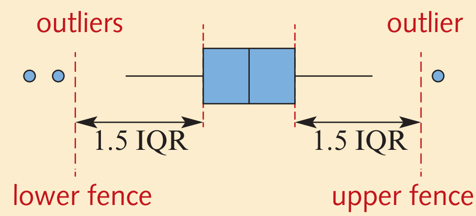
Upper and lower fences

When constructing a boxplot to display outliers, we must first determine the location of what we call the *upper and lower fences*. These are imaginary lines drawn one and a half the interquartile range (or box widths) above and below the ends of the box. Data values outside these fences are classified as possible outliers and plotted separately.

Using a boxplot to display possible outliers

In a boxplot, possible outliers are defined as those values that are:

- greater than $Q_3 + 1.5 \times \text{IQR}$ (upper fence)
- less than $Q_1 - 1.5 \times \text{IQR}$ (lower fence).



ISBN 978-1-009-25891-3

When drawing a boxplot, any observation identified as an outlier is indicated by a dot. The whiskers then end at the smallest and largest values that are not classified as outliers.

Example 18 Constructing a boxplot showing outliers

The number of hours that each of 33 students spent on a school project is shown below.

2	3	4	9	9	13	19	24	27	35	36
37	40	48	56	59	71	76	86	90	92	97
102	102	108	111	146	147	147	166	181	226	264

Construct a boxplot for this data set that can be used to identify possible outliers.

Solution

1 From the ordered list, state the minimum and maximum values. Find the median, the $\frac{1}{2}(33 + 1)$th = 17th value.

min. = 2, max. = 264,
median = 71

2 Determine Q_1 and Q_3. There are 33 values, so Q_1 is halfway between the 8th and 9th values and Q_3 is halfway between the 25th and 26th values.

first quartile, $Q_1 = \dfrac{24 + 27}{2} = 25.5$

third quartile, $Q_3 = \dfrac{108 + 111}{2} = 109.5$

3 Determine the IQR.

$\text{IQR} = Q_3 - Q_1 = 109.5 - 25.5 = 84$

4 Determine the upper and lower fences.

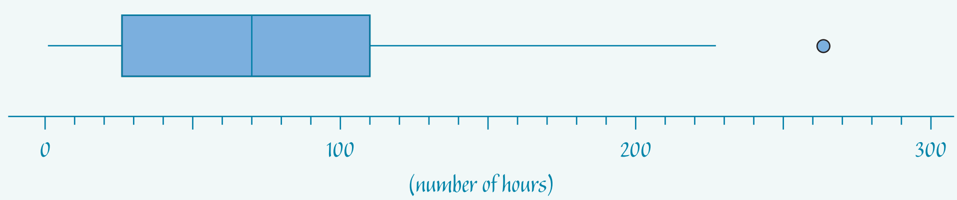
$$\begin{aligned}\text{Lower fence} &= Q_1 - 1.5 \times \text{IQR} \\ &= 25.5 - 1.5 \times 84 \\ &= -100.5\end{aligned}$$

$$\begin{aligned}\text{Upper fence} &= Q_3 + 1.5 \times \text{IQR} \\ &= 109.5 + 1.5 \times 84 \\ &= 235.5\end{aligned}$$

5 Locate any values outside the fences, and the values that lie just inside the limits (the whiskers will extend to these values).

There is one outlier 264.
The largest value that is not an outlier is 226.

6 The boxplot can now be constructed as shown below. The circle denotes the outlier.



There is one possible outlier, the student who spent 264 hours on the project.

ISBN 978-1-009-25891-3

It is clearly very time-consuming to construct boxplots displaying outliers by hand. Fortunately, your CAS calculator will do it for you automatically as we will see below.

How to construct a boxplot using the TI-Nspire CAS

The number of hours that each of 33 students spent on a school project is shown below.

2	3	4	9	9	13	19	24	27	35	36
37	40	48	56	59	71	76	86	90	92	97
102	102	108	111	146	147	147	166	181	226	264

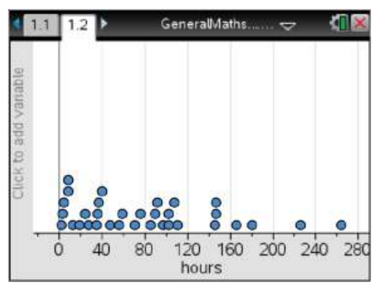
Construct a boxplot for this data set that can be used to identify possible outliers.

Steps

1 Press [on] and select **New Document** (or use [ctrl] + [N]).

2 Select **Add Lists & Spreadsheet**.
Enter the data into a list called **hours** as shown.

3 Statistical graphing is done through the **Data & Statistics** application. Press [ctrl] + [I] and select **Add Data & Statistics** (or press [on], arrow to [icon], and press [enter]).
Note: A random display of dots will appear – this is to indicate list data is available for plotting. It is not a statistical plot.

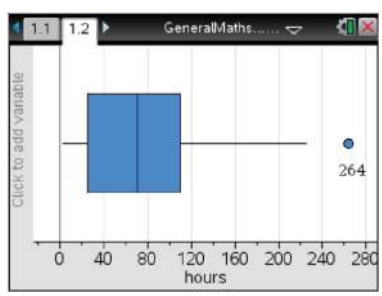


a Press [tab] to show the list of variables. Select the variable **hours**. Press [enter] to paste the variable **hours** to that axis. A dot plot is displayed as the default plot.

b To change the plot to a boxplot press [menu]**>Plot Type>BoxPlot**, then [enter] or click' (press [click]). Outliers are indicated by a dot(s).

4 Data Analysis
Move the cursor over the plot to display the key values (or use [menu]**>Analyze>Graph Trace**).
Starting at the far left of the plot, we see that the:

- minimum value is 2: **minX** = **2**
- first quartile is 25.5: $\mathbf{Q_1}$ = **25.5**
- median is 71: **Median** = **71**
- third quartile is 109.5: $\mathbf{Q_3}$ = **109.5**
- maximum value is 264: **maxX** = **264**. It is also an outlier.



ISBN 978-1-009-25891-3

How to construct a boxplot using the ClassPad

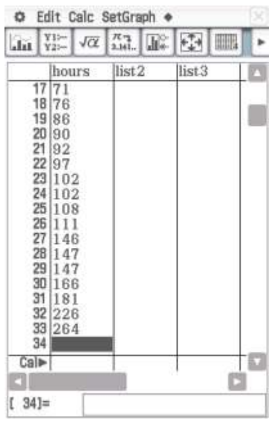
The number of hours that each of 33 students spent on a school project is shown below.

2	3	4	9	9	13	19	24	27	35	36
37	40	48	56	59	71	76	86	90	92	97
102	102	108	111	146	147	147	166	181	226	264

Construct a boxplot for this data set that can be used to identify possible outliers.

Steps

1 Open the **Statistics** application and enter the data into a column labelled **hours**.



2 Open the **Set StatGraphs** dialog box by tapping [icon] in the toolbar. Complete the dialog box as shown, right. For:

- **Draw:** select **On**
- **Type:** select **MedBox** (▼)
- **XList:** select **main\hours** (▼)
- **Freq:** leave as **1**.

Tap the **Show Outliers** box.

Tap [Set] to exit.

3 Tap [icon] to plot the boxplot.

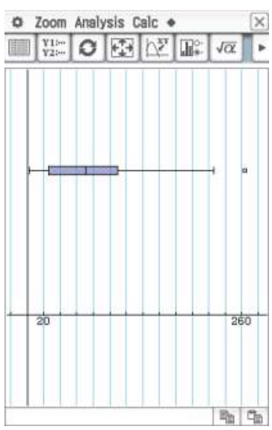
4 Tap [Resize] to obtain a full-screen display.

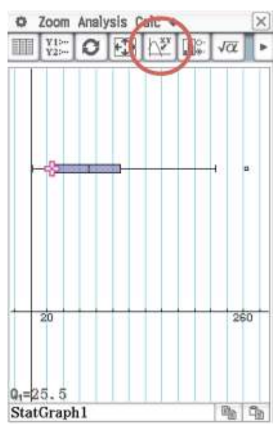
5 Key values can be read from the boxplot by tapping [icon].

Use the arrows (◀ and ▶) to move from point to point on the boxplot.

Starting at the far left of the plot, we see that the:

- minimum value is 2 (**minX = 2**)
- first quartile is 25.5 ($\mathbf{Q_1}$ = **25.5**)
- median is 71 (**Median = 71**)
- third quartile is 109.5 ($\mathbf{Q_3}$ = **109.5**)
- maximum value is 264 (**maxX = 264**). It is also an outlier.





ISBN 978-1-009-25891-3

Exercise 6H

Constructing a boxplot from a five-number summary

Example 17

1 The heights (in centimetres) of a class of girls are:

160	165	123	143	154	180	133	123	157	157
135	140	140	150	154	159	149	167	176	163
154	167	168	132	145	143	157	156		

The five-number summary for this data is:

Min = 123, $Q_1 = 141.5$, $M = 154$, $Q_3 = 161.5$, Max = 180

Use this five-number summary to construct a boxplot (there are no outliers).

2 The data shows how many weeks each of the singles in the Top 41 has been in the charts, in a particular week.

24	11	5	7	4	15	13	4	12	14	3	12	4	4
3	10	17	8	6	2	18	15	5	6	9	14	4	5
14	12	16	11	6	7	12	4	16	2	8	10	1	

The five-number summary for this data is:

Min = 1, $Q_1 = 4$, $M = 8$, $Q_3 = 13.5$, Max = 24

Use this five-number summary to construct a boxplot (there are no outliers).

Example 18

3 The amount of pocket money paid per week to a sample of year 8 students is:

\$5.00	\$10.00	\$12.00	\$8.00	\$7.50	\$12.00	\$15.00
\$10.00	\$10.00	\$0.00	\$5.00	\$10.00	\$20.00	\$15.00
\$26.00	\$13.50	\$15.00	\$5.00	\$15.00	\$25.00	\$16.00

The five-number summary for this data is:

Min = 0, $Q_1 = 7.75$, $M = 12$, $Q_3 = 15$, Max = 26

Use this five-number summary to construct a boxplot (there is one outlier).

Constructing boxplots from raw data

4 The length of time, in years, that employees have been employed by a company is:

CAS

5	1	20	8	6	9	13	15	4	2
15	14	13	4	16	18	26	6	8	2
6	7	20	2	1	1	5	8		

Use a CAS calculator to construct the boxplot.

ISBN 978-1-009-25891-3

CAS

5 The times (in seconds) that 35 children took to tie up a shoelace are:

8	6	18	39	7	10	5	8	6	14	11	10
8	35	6	6	14	15	6	7	6	5	8	11
8	15	8	8	7	8	8	6	29	5	7	

Use a CAS calculator to construct the boxplot.

6 A researcher is interested in the number of books people borrow from a library. She selected a sample of 38 people and recorded the number of books each person had borrowed in the previous year. Here are her results:

7	28	0	2	38	18	0	0	4	0	0	5	13
2	13	1	1	14	1	8	27	0	52	4	11	0
0	12	28	15	10	1	0	2	0	1	11	0	

a Use a CAS calculator to construct a boxplot of the data.

b Use the boxplot to identify any possible outliers and write down their values.

7 The following table gives the prices for units sold in a particular suburb in one month (in thousands of dollars):

356	366	375	389	432
445	450	450	495	510
549	552	579	585	590
595	625	725	760	880
940	950	1017	1180	1625

a Use a CAS calculator to construct a boxplot of the data.

b Use the boxplot to identify any possible outliers and write down their values.

8 The time taken, in seconds, for a group of children to complete a puzzle is:

8	6	18	39	7	10	5	8	6	14	11	5
10	8	60	6	6	14	15	6	7	6	5	7
8	11	8	15	8	8	7	8	8	6	29	

a Use a CAS calculator to construct a boxplot of the data.

b Use the boxplot to identify any possible outliers and write down their values.

ISBN 978-1-009-25891-3

CAS

9 The percentage of people using the internet in 23 countries is given in the table:

Country	Internet users (%)
Afghanistan	5.45
Argentina	55.80
Australia	79.00
Brazil	48.56
Bulgaria	51.90
China	42.30
Colombia	48.98
Greece	55.07
Hong Kong SAR, China	72.90
Iceland	96.21
India	12.58

Country	Internet users (%)
Italy	55.83
Malaysia	65.80
Morocco	55.42
New Zealand	82.00
Saudi Arabia	54.00
Singapore	72.00
Slovenia	68.35
South Africa	41.00
United Kingdom	87.48
United States	79.30
Venezuela	49.05
Vietnam	39.49

a Use a CAS calculator to construct a boxplot of the data.

b Use the boxplot to identify any possible outliers and write down their values.

ISBN 978-1-009-25891-3

6I Comparing the distribution of a numerical variable across two or more groups

It makes sense to compare the distributions of data sets when they are concerned with the *same* numerical variable, say *height* measured for different groups of people, for example, a basketball team and a gymnastic team.

For example, it would be useful to compare the distributions for each of the following:

- the maximum daily temperatures in Melbourne in March and the maximum daily temperatures in Sydney in March
- the test scores for a group of students who had not had a revision class and the test scores for a group of students who had a revision class.

In each of these examples, we can actually identify *two variables*. One is a *numerical variable* and the other is a *categorical variable*.

For example:

- The variable maximum daily *temperature* is numerical while the variable *city*, which takes the values 'Melbourne' or 'Sydney', is categorical.
- The variable *test score* is numerical while the variable *attended a revision class*, which takes the values 'yes' or 'no', is categorical.

Thus, when we compare two data sets in this section, we will be actually investigating the relationship between two variables: a numerical variable and a categorical variable.

The outcome of these investigations will be a brief written report that compares the distribution of the numerical variable across two or more groups defined as categorical variables. The starting point for these investigations will be, as always, a graphical display of the data. To this end you will meet and learn to interpret two new graphical displays: the **back-to-back stem plot** and the **parallel boxplot**.

Comparing distributions using back-to-back stem plots

A back-to-back stem plot differs from the stem plots you have met in the past in that it has a single stem with two sets of leaves, one for each of the two groups being compared.

ISBN 978-1-009-25891-3

Example 19 **Comparing distributions using back-to-back stem plots**

The following back-to-back stem plot displays the distributions of life expectancies for males (in years) in several countries in the years 1970 and 2010.

In this situation, *life expectancy* is the numerical variable. *Year*, which takes the values 1970 and 2010, is the categorical variable.

Male life expectancy (in years)

1970		2010
8	3	
	4	
9 4 2	5	8
9 9 9 8 7 7 1 0	6	9
4	7	1 2 4 4 6 8 9 9
	8	0 0 0

5 | 8 = 58 years

1970	2010
M = 67 years	M = 76 years
IQR = 12.5 years	IQR = 8 years

Use the back-to-back stem plot and the summary statistics provided to compare these distributions in terms of centre and spread and draw an appropriate conclusion.

Solution

1 Centre: Use the medians to compare centres.

The median life expectancy of males in 2010 (M = 76 years) was nine years higher than in 1970 (M = 67 years).

2 Spread: Use the IQRs to compare spreads.

The spread of life expectancies of males in 2010 (IQR = 12.5 years) was different to the spread in 1970 (IQR = 8).

3 Conclusion: Use the above observations to write a general conclusion.

In conclusion, the median life expectancy for these countries has increased over the last 40 years, and the variability in life expectancy between countries has decreased.

Comparing distributions using parallel boxplots

Back-to-back stem plots can be used to compare the distribution of a numerical variable across two groups when the data sets are small. Parallel boxplots can also be used to compare distributions. Unlike back-to-back stem plots, boxplots can also be used when there are more than two groups.

By drawing boxplots on the same axis, both the centre and spread for the distributions are readily identified and can be compared visually.

ISBN 978-1-009-25891-3

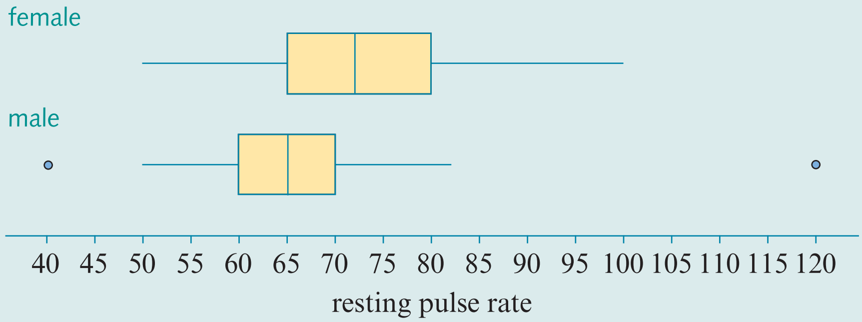
When comparing distributions of a numerical variable across two or more groups using parallel boxplots, the report should address the key features of:

- centre (the median)
- spread (the IQR)
- possible outliers.

Example 20 Comparing distributions across two groups using parallel boxplots

The following parallel boxplots display the distribution of pulse rates (in beats/minute) for a group of female students and a group of male students.

Use the information in the boxplots to write a report comparing these distributions in terms of centre, spread and outliers in the context of the data.



Solution

1 Centre: Compare the medians. Estimate values of these medians from the plot (the vertical lines in the boxes).

The median pulse rate for females ($M = 72$ beats/minute) is higher than that for males ($M = 65$ beats/minute).

2 Spread: Compare the spread of the two distributions using IQRs (the widths of the boxes).

The spread of pulse rates for females ($IQR = 15$) is higher than for males ($IQR = 10$).

3 Outliers: Locate on any outliers and describe.

There are no female outliers. The males with pulse rates of 40 and 120 were outliers.

4 Conclusion: Use the above observations to write a general conclusion.

In conclusion, the median pulse rate for females was higher than for males and female pulse rates were generally more variable than male pulse rates.

ISBN 978-1-009-25891-3

Exercise 6I

Comparing groups using back-to-back stem plots

Example 19

1 The stem plot displays the age distribution of ten females and ten males admitted to a regional hospital on the same day.

a Calculate the median and the IQR for admission ages of the females and males in this sample.

b Write a report comparing these distributions in terms of centre and spread in the context of the data.

Females		Males
9	0	
5 0	1	3 6
7	2	1 4 5 6 7
7 1	3	4
3 0	4	0 7
0	5	
	6	
9	7	

4 | 0 = 40 years

2 The stem plot opposite displays the mark distribution of students from two different mathematics classes (Class A and Class B) who sat the test. The test was marked out of 100.

a How many students in each class scored less than 50%?

b Determine the median and the IQR for the marks obtained by the students in each class.

c Write a report comparing these distributions in terms of centre and spread in the context of the data.

Marks %

Class B		Class A
3 2	1	9
	2	2
	3	9
	4	5 7 8
	5	5 8
9	6	5 8
6 4 3 3 2 2 1 0 0	7	1 6 7 9 9
8 8 4 4 3 2 1 1 0 0	8	0 1 2 2 5 5 9
8 1	9	1 9

7 | 1 = 71

ISBN 978-1-009-25891-3

3 The following table shows the number of nights spent away from home in the past year by a group of 21 Australian tourists and by a group of 21 Japanese tourists:

Australian

3	14	15	3	6	17	2
7	4	8	23	5	7	21
9	11	11	33	4	5	3

Japanese

14	3	14	7	22	5	15
26	28	12	22	29	23	17
32	5	9	23	6	44	19

a Construct a back-to-back stem-and-leaf plot of these data sets.

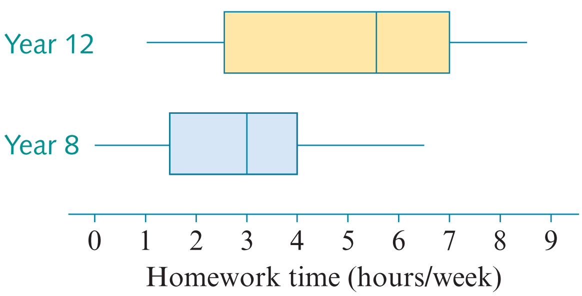
b Determine the median and IQR for the two distributions.

c Write a report comparing the distributions of the number of nights spent away by Australian and Japanese tourists in terms of centre and spread.

Comparing groups using parallel boxplots

Example 20

4 The boxplots below display the distributions of homework time (in hours/week) of a sample of year 8 and a sample of year 12 students.



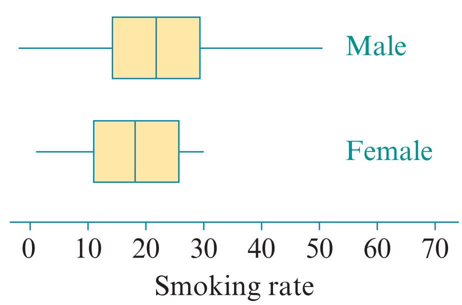
a Estimate the median and IQRs from the boxplots.

b Use these medians and IQRs to write a report comparing these distributions in terms of centre and spread in the context of the data.

5 The boxplots below display the distribution of smoking rates (%) of males and females from several countries.

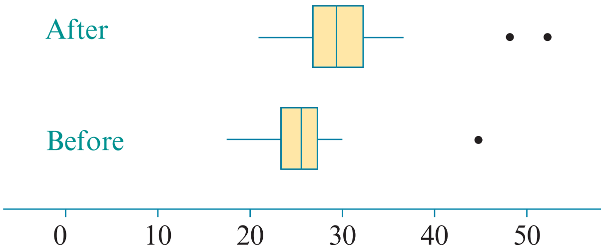
a Estimate the median and IQRs from the boxplots.

b Use the information in the boxplots to write a report comparing these distributions in terms of centre and spread in the context of the data.



ISBN 978-1-009-25891-3

6 The boxplots below display the distributions of the number of sit-ups a person can do in one minute, both before and after a fitness course.



a Estimate the median, IQRs and the values of any outliers from the boxplots.

b Use these medians and IQRs to write a report comparing these distributions in terms of centre and spread in the context of the data.

7 To test the effect of alcohol on coordination, twenty randomly selected participants were timed to complete a task with both 0% blood alcohol and 0.05% blood alcohol. The times taken (in seconds) are shown in the accompanying table.

0% blood alcohol									
38	36	35	35	43	46	42	47	40	48
35	34	40	44	30	25	39	31	29	44

0.05% blood alcohol									
39	32	35	39	36	34	41	64	44	38
43	42	46	46	50	32	32	41	40	50

a Draw boxplots for each of the sets of scores on the same scale.

b Use the information in the boxplots to write a report comparing the distributions of the times taken to complete a task with 0% blood alcohol and 0.05% blood alcohol in terms of centre (medians), spread (IQRs) and outliers.

ISBN 978-1-009-25891-3

6J Statistical investigation

Exercise 6J

1 To investigate the age of parents at the birth of their first child, a hospital recorded the ages of the mothers and fathers for the first 40 babies born in the hospital for each of the years 1970, 1990 and 2010.

The data is given below:

1970 Mother									
23	22	33	19	19	26	20	15	26	17
18	31	24	20	29	28	25	45	28	22
1970 Father									
29	15	39	29	22	35	32	26	37	29
25	31	20	34	28	22	33	25	34	46
1990 Mother									
28	14	38	28	21	34	31	25	36	28
24	30	19	33	27	21	32	24	33	45
1990 Father									
31	27	46	31	26	28	30	27	43	37
39	22	27	35	31	29	32	27	38	35
2010 Mother									
30	26	45	32	25	27	29	26	42	36
38	21	26	34	37	28	28	37	37	34
2010 Father									
37	31	39	36	21	34	34	23	17	37
23	33	31	32	24	39	45	30	35	34

Use appropriate displays and summary statistics to answer the following questions:

a How do the ages of the mothers compare to the ages of fathers in each time period?

b How have the ages of the mothers changed over the three time periods?

c How have the ages of the fathers changed over the three time periods?

d Has the relationship between mothers' ages and fathers' ages changed over time?

In each case write a brief report to summarise your findings.

ISBN 978-1-009-25891-3

Key ideas and chapter summary

Assign-ment

Types of data	Data can be classified as **numerical** or **categorical**.
Frequency table	A **frequency table** is a listing of the values that a variable takes in a data set, along with how often (frequently) each value occurs. **Frequency** can be recorded as the number of times a value occurs or a **percentage**, the percentage of times a value occurs.
Categorical data	**Categorical data** arises when classifying or naming some quality or attribute. When the categories are naming the groups, the data is called **nominal**. When there is an inherent order in the categories, the data is called **ordinal**.
Bar chart	A **bar chart** is used to display the frequency distribution of a categorical variable.
Mode, modal category/class	The **mode** (or modal category) is the value of a variable (or the category) that occurs most frequently. The **modal interval**, for **grouped data**, is the interval that occurs most frequently.
Numerical data	**Numerical data** arises from measuring or counting some quantity. **Discrete** numerical data can only take particular values, usually whole numbers, and often arises from counting. **Continuous** numerical data describes numerical data that can take any value, sometimes in an interval, and often arises from measuring.
Histogram	A **histogram** is used to display the frequency distribution of a numerical variable: suitable for medium to large-sized data sets.
Stem plot	A **stem plot** is a visual display of a numerical data set, an alternative display to the histogram: suitable for small to medium-sized data sets. Leading digits are shown as the stem and the final digit as the leaf.
Dot plot	A **dot plot** consists of a number line with each data point marked by a dot. Suitable for small to medium sized data sets.
Describing the distribution of a numerical variable	The **distribution of a numerical variable** can be described in terms of **shape** (**symmetric** or **skewed**: positive or negative), **centre** (the midpoint of the distribution) and **spread**.
Summary statistics	**Summary statistics** are used to give numerical values to special features of a data distribution such as centre and spread.
Mean	The **mean** ($\bar{x}$) is a summary statistic that can be used to locate the centre of a symmetric distribution.

ISBN 978-1-009-25891-3

Review

Range The **range** (R) is the difference between the smallest and the largest data values. It is the simplest measure of spread.

range = largest value − smallest value

Standard deviation The **standard deviation** (s) is a summary statistic that measures the spread of the data values around the mean.

Median The **median** (M) is a summary statistic that can be used to locate the centre of a distribution. It is the midpoint of a distribution, so that 50% of the data values are less than this value and 50% are more.
If the distribution is clearly skewed or there are outliers, the median is preferred to the mean as a measure of centre.

Quartiles **Quartiles** are summary statistics that divide an ordered data set into four equal groups.

Interquartile range The **interquartile range (IQR)** gives the spread of the middle 50% of data values in an ordered data set. If the distribution is highly skewed or there are outliers, the IQR is preferred to the standard deviation as a measure of spread.

Five-number summary The median, the first quartile, the third quartile, along with the minimum and the maximum values in a data set, are known as a **five-number summary**.

Outliers **Outliers** are data values that appear to stand out from the rest of the data set.

Boxplot A **boxplot** is a visual display of a five-number summary with adjustments made to display outliers separately when they are present.

Skills check

Having completed this chapter you should be able to:

- differentiate between nominal, ordinal, discrete and continuous data
- interpret the information contained in a frequency table
- identify the mode from a frequency table and interpret it
- construct a bar chart or histogram from a frequency table
- construct a histogram from raw data using a graphics calculator
- construct a dot plot and stem-and-leaf plot from raw data

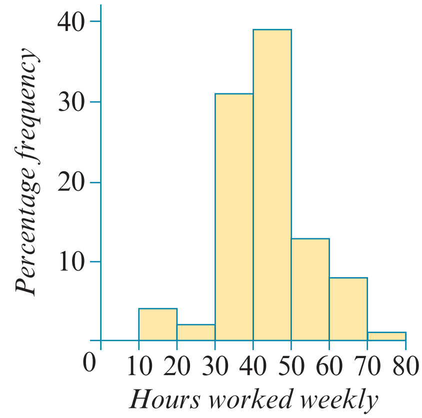
ISBN 978-1-009-25891-3

- recognise symmetric, positively skewed and negatively skewed distributions
- identify potential outliers in a distribution from its histogram or stem plot
- locate the median and quartiles of a data set and hence calculate the IQR
- produce a five-number summary from a set of data
- construct a boxplot from a five-number summary
- construct a boxplot from raw data using a graphics calculator
- use a boxplot to identify key features of a data set such as centre and spread
- use the information in a back-to-back stem plot or a boxplot to describe and compare distributions
- calculate the mean and standard deviation of a data set
- understand the difference between the mean and the median as measures of centre and be able to identify situations where it is more appropriate to use the median
- write a short paragraph comparing distributions in terms of centre, spread and outliers.

Short-answer questions

The following information relates to Questions 1 to 4

The number of hours worked per week by employees in a large company is shown in the following percentage frequency histogram.



1 Find the percentage of employees who work from 20 to less than 30 hours per week.

2 Find the percentage of employees who worked *less* than 30 hours per week.

3 Find the modal interval for hours worked.

4 State the median interval for the number of hours worked.

ISBN 978-1-009-25891-3

Review

The following information relates to Questions 5 to 8

A group of 18 employees of a company were asked to record the number of meetings they had attended in the last month.

1 1 2 3 4 5 5 6 7 9 10 12 14 14 16 22 23 44

5 State the range for the given data.

6 State the median number of meetings.

7 Calculate the mean number of meetings.

8 Determine the interquartile range (IQR) of the number of meetings.

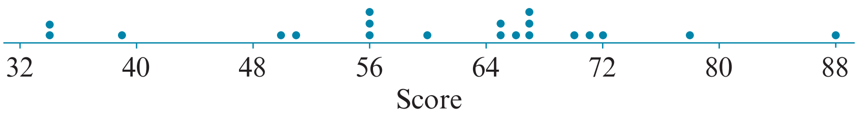
9 The heights of six basketball players (in cm) are:

178.1 185.6 173.3 193.4 183.1 193.0

Determine the mean and standard deviation of the heights of the players.

The following information relates to Questions 10 and 11

The dot plot below gives the examination scores in mathematics for a group of 20 students.

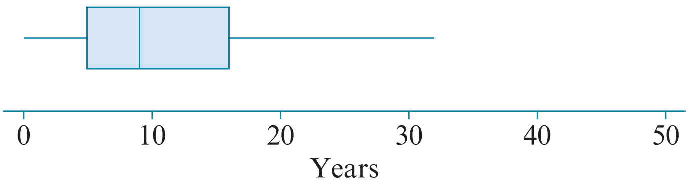


10 State the number of students who scored 56 on the examination.

11 Calculate the percentage of students who scored between 40 and 80 on the exam, giving your answer to the nearest whole number.

The following information relates to Questions 12 to 15

The number of years for which a sample of people have lived at their current address is summarised in the boxplot.



12 State the range of the number of years lived at this address, giving your answer to the nearest 10.

13 State to the nearest whole number the median number of years lived at this address.

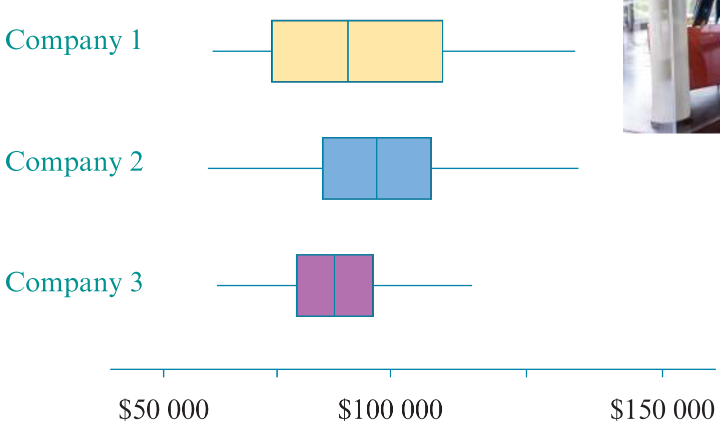
14 Determine the interquartile range of the number of *years lived at this address*.

15 Determine the percentage who have lived at this address for more than 15 years.

ISBN 978-1-009-25891-3

The following information relates to Questions 16 to 17

The amount paid per annum to the employees of each of three large companies is shown in the boxplots.



16 Which company has the lowest median wage?

17 Which company has the largest general spread (IQR) in wage?

18 Classify the data that arises from the following situations as nominal, ordinal, discrete or continuous.

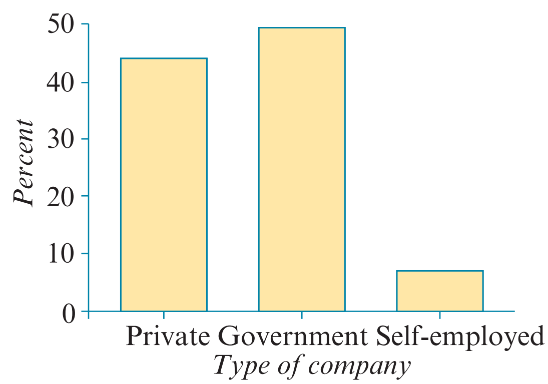
a The number of phone calls a hotel receptionist receives each day.

b Interest in politics on a scale from 1 to 5, where 1 = very interested, 2 = quite interested, 3 = somewhat interested, 4 = not very interested, and 5 = uninterested.

19 The following bar chart shows the percentage of working people in a certain town who are employed in private companies, work for the government or are self-employed.

a Is the data categorical or numerical?

b Approximately what percentage of the people are self-employed?



20 A researcher asked a group of people to record how many cigarettes they had smoked on a particular day. Here are her results:

0	0	9	10	23	25	0	0	34	32	0	0	30	0	4
5	0	17	14	3	6	0	33	23	0	32	13	21	22	6

Using class intervals of width 5, construct a histogram of this data.

ISBN 978-1-009-25891-3

Review

21 A teacher recorded the time taken (in minutes) by each of a class of students to complete a test:

56	57	47	68	52	51	43	22	59	51	39
54	52	69	72	65	45	44	55	56	49	50

a Make a dot plot of this data.

b Make a stem-and-leaf plot of these times.

c Use this stem plot to find the median and quartiles for the time taken.

CAS

22 The weekly rentals, in dollars, for a group of people are given below:

285	185	210	215	320	680	280
265	300	210	270	190	245	315

Find the mean and standard deviation, the median and the IQR, and the range of the weekly rentals. Write your answers correct to two decimal places if they are not exact.

23 Geoff decided to record the time (in minutes) it takes him to complete his mail round each working day for four weeks. His data is recorded below:

170	189	201	183	168	182	161	166	167	173	182	167	188	211
164	176	161	187	180	201	147	188	186	176	174	193	185	183

Find the mean and standard deviation of his mail round times, correct to two decimal places.

24 A group of students was asked to record the number of SMS messages that they sent in one 24-hour period. The following five-number summary was obtained from the data set.

$$\text{Min} = 0, \quad Q_1 = 3, \quad M = 5, \quad Q_3 = 12, \quad \text{Max} = 24$$

Use the summary to construct a boxplot of this data.

25 The following data gives the number of students absent from a large secondary college on each of 36 randomly chosen school days:

7	22	12	15	21	16	23	23	17	23	8	16
7	3	21	30	13	2	7	12	18	14	14	0
15	16	13	21	10	16	11	4	3	0	31	44

a Construct a boxplot of this data.

b What was the median number of students absent each day during this period?

c On what percentage of days, correct to one decimal place, were more than 20 students absent?

ISBN 978-1-009-25891-3

Extended-response questions

1 The divorce rates (in percentages) of 19 countries are:

27	18	14	25	28	6	32	44	53	0
26	8	14	5	15	32	6	19	9	

a Is the data categorical or numerical?

b Construct an ordered stem plot of divorce rates by hand.

c Construct a dot plot of divorce rates by hand.

d What shape is the distribution of divorce rates?

e What percentage, correct to one decimal place, of the 19 countries have divorce rates greater than 30%?

f Calculate the mean and median of the distribution of divorce rates.

g Use your calculator to construct a histogram of the data with class intervals of width 10.

i What is the shape of the histogram?

ii How many of the 19 countries have divorce rates from 10% to less than 20%?

2 Metro has decided to improve its service on the Lilydale train line in Victoria. Trains were timed on the run from Lilydale to Flinders Street Station, Melbourne, and their times recorded over a period of six weeks at the same time each day.
The journey times are shown below (in minutes):

60	61	70	72	68	80	76	65	69	79	82
90	59	86	70	77	64	57	65	60	68	60
63	67	74	78	65	68	82	89	75	62	64
58	64	69	59	62	63	89	74	60		

a Use your CAS calculator to construct a histogram of the times taken for the journey from Lilydale to Flinders Street.

i On how many days did the trip take 65–69 minutes?

ii What shape is the histogram?

iii What percentage of trains, correct to one decimal place, took less than 65 minutes to reach Flinders Street?

b Use your calculator to determine the following summary statistics for the *time* taken (correct to two decimal places):

$$\bar{x},\ s,\ \text{Min},\ Q_1,\ M, Q_3,\ \text{Max}$$

ISBN 978-1-009-25891-3

CAS

c Use the summary statistics to complete the following report.

i The mean time taken from Lilydale to Flinders Street was ☐ minutes.

ii 50% of the trains took more than ☐ minutes to travel from Lilydale to Flinders Street.

iii The range of travelling times was ☐ minutes, while the interquartile range was ☐ minutes.

iv 25% of trains took more than ☐ minutes to travel to Flinders Street.

v The standard deviation of travelling times was ☐ minutes.

d Summary statistics for the year before Metro took over the Lilydale line from Connex are:

$$\text{Min} = 55, \quad Q_1 = 65, \quad M = 70, \quad Q_3 = 89, \quad \text{Max} = 99$$

Construct boxplots for the last year Connex ran the line and for the data from Metro on the same plot.

e Use the information from the boxplots to write a report comparing the distribution of travelling times for the two transport corporations in terms of centre (medians) and spread (IQRs).

ISBN 978-1-009-25891-3

7 The normal distribution

In this chapter

7A The normal distribution and the 68–95–99.7% rule

7B Determination of other percentages using a CAS

7C Quantiles

7D Standard scores

Chapter summary and review

Syllabus references

Topic: Making sense of data relating to a single statistical variable

Subtopics: 2.1.6 – 2.1.9

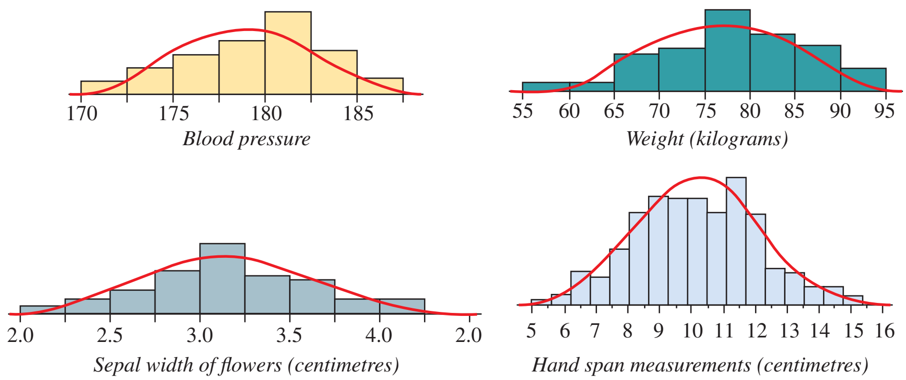
We know that the interquartile range is the spread of the middle 50% of the dataset. Can we find some similar way in which to interpret the standard deviation?

It turns out we can, but we need to restrict ourselves to symmetric distributions that have an approximate *bell shape*. Again, while this may sound very restrictive, many of the data distributions we work with in statistics (but not all) can be well approximated by this type of distribution. In fact, it is so common that it is called the **normal distribution**.

ISBN 978-1-009-25891-3

7A The normal distribution and the 68–95–99.7% rule

Many datasets that arise in practice are roughly symmetrical and have approximate bell shapes, as shown in the four examples below.



Data distributions that are bell-shaped can be modelled by a *normal* distribution.

The 68–95–99.7% rule

In normal distributions, the percentage of observations that lie within a certain number of standard deviations of the mean can always be determined. In particular, we are interested in the percentage of observations that lie within one, two or three standard deviations of the mean. This gives rise to what is known as the **68–95–99.7% rule**.

The 68–95–99.7% rule

For a *normal* distribution, approximately:

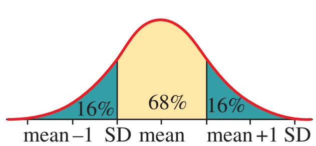
- *68%* of the observations lie within *one* standard deviation of the mean
- *95%* of the observations lie within *two* standard deviations of the mean
- *99.7%* of the observations lie within *three* standard deviations of the mean.

To give you an understanding of what this rule means in practice, it is helpful to view this rule graphically.

The 68–95–99.7% rule in graphical form

If a data distribution is approximately normal, then:

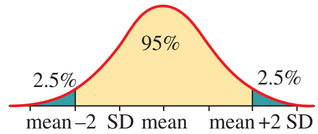
- around 68% of the data values will lie within one *standard deviation (SD)* of the mean.



  This also means that *32%* of values lie outside this region. As the distribution is symmetric, we can also say that around 16% of values lie in each of the tails (shaded blue, above).

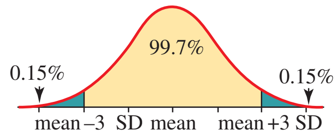
ISBN 978-1-009-25891-3

- around *95%* of the data values will lie within *two standard deviations* of the mean.



This also means that 5% of values lie outside this region. As the distribution is symmetric, we can also say that around 2.5% of values lie in each of the tails (shaded blue, above).

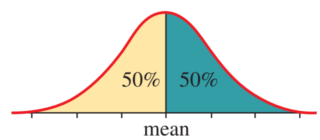
- around *99.7%* of the data values will lie within *three standard deviations* of the mean.



This also means that 0.3% of values lie outside this region. As the distribution is symmetric, we can also say that around 0.15% of values lie in each of the tails (shaded blue, above).

Finally, because the *normal distribution* is *symmetric,* the mean and the median coincide so that:

- *50%* of the data values will lie *above* the mean and *50%* of values will lie *below* the mean.



Example 1 **Applying the 68–95–99.7% rule**

The distribution of delivery times for pizzas made by House of Pizza is approximately normal, with a mean of 25 minutes and a standard deviation of 5 minutes.

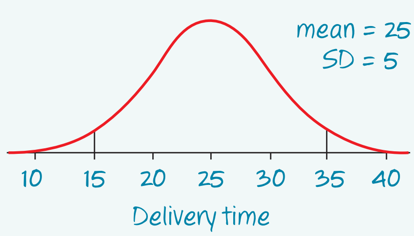
a What percentage of pizzas have delivery times of between 15 and 35 minutes?

b What percentage of pizzas have delivery times of greater than 30 minutes?

c In 1 month, House of Pizza delivers 2000 pizzas. How many of these pizzas are delivered in less than 10 minutes?

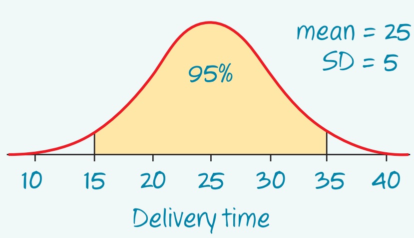
Solution

a **1** Sketch, scale and label a normal distribution curve with a mean of 25 and a standard deviation of 5.



2 Shade the region under the normal curve representing delivery times of between 15 and 35 minutes.

3 Note that delivery times of between 15 and 35 minutes lie within *two* standard deviations of the mean.
($15 = 25 - 2 \times 5$ and $35 = 25 + 2 \times 5$)

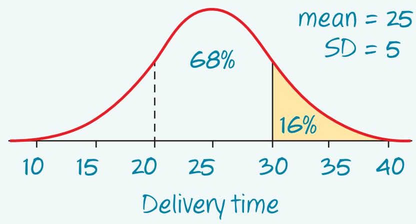


ISBN 978-1-009-25891-3

4 95% of values are within two standard deviations of the mean. Use this information to write your answer.

95% of pizzas will have delivery times of between 15 and 35 minutes.

b **1** As before, draw, scale and label a normal distribution curve with a mean of 25 and a standard deviation of 5. Shade the region under the normal curve representing delivery times of greater than 30 minutes.



2 Delivery times of greater than 30 minutes are more than *one* standard deviation above the mean. $(30 = 25 + 1 \times 5)$

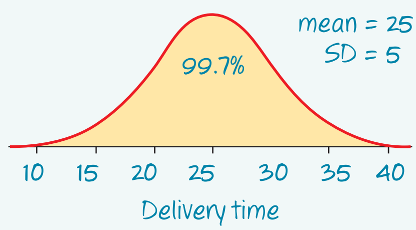
3 16% of values are more than one standard deviation above the mean. Write your answer.

16% of pizzas will have delivery times of greater than 30 minutes.

c **1** Write down the number of pizzas delivered.

Number = 2000

2 Delivery times of less than 10 minutes are more than *three* standard deviations below the mean. $(10 = 25 - 3 \times 5)$.



3 0.15% of values are more than *three* standard deviations below the mean. Record this.

Percentage delivered in less than 10 minutes = 0.15%

4 Therefore, the number of pizzas delivered in less than 10 minutes is 0.15% of 2000.

Number of pizzas delivered in less than 10 minutes $= 0.15\%$ of 2000

$$= \frac{0.15}{100} \times 2000 = 3$$

Exercise 7A

Skill-sheet

Routine applications of the 68–95–99.7% rule

1 The blood pressure readings for executives are approximately normally distributed with a mean systolic blood pressure of 134 and a standard deviation of 20. Given this information it can be concluded that:

a about 68% of the executives have blood pressures between ☐ and ☐

b about 95% of the executives have blood pressures between ☐ and ☐

ISBN 978-1-009-25891-3

c about 99.7% of the executives have blood pressures between ☐ and ☐

d about 16% of the executives have blood pressures above ☐

e about 2.5% of the executives have blood pressures below ☐

f about 0.15% of the executives have blood pressures below ☐

g about 50% of the executives have blood pressures above ☐.

2 The average weight of a bag of 10 blood plums picked at U-Pick Orchard is normally distributed with a mean of 1 .88 kg and a standard deviation of 0.2 kg.
Given this information the percentage of the bags of 10 plums that weigh:

a between 1.68 and 2.08 kg is approximately ☐ %

b between 1.28 and 2.48 kg is approximately ☐ %

c more than 2.08 kg is approximately ☐ %

d more than 2.28 kg is approximately ☐ %

e less than 1.28 kg is approximately ☐ %

f more than 1.88 kg is approximately ☐ %.

Further applications of the 68–95–99.7% rule

3 The distribution of times taken for walkers to complete a circuit in a park is normal, with a mean time of 14 minutes and a standard deviation of 3 minutes.

a What percentage of walkers complete the circuit in:

i more than 11 minutes? **ii** less than 14 minutes?

iii between 14 and 20 minutes?

b In a week, 1000 walkers complete the circuit. How many will take less than 8 minutes?

4 The distribution of heights of 19-year-old women is approximately normal, with a mean of 170 cm and a standard deviation of 5 cm.

a What percentage of these women have heights:

i between 155 and 185 cm? **ii** greater than 180 cm?

iii between 160 and 175 cm?

b In a sample of 5000 of these women, how many have heights greater than 175 cm?

5 The distribution of resting pulse rates of 20-year-old men is approximately normal, with a mean of 66 beats/minute and a standard deviation of 4 beats/minute.

a What percentage of these men have pulse rates of:

i higher than 66? **ii** between 66 and 70? **iii** between 62 and 74?

b In a sample of 2000 of these men, how many have pulse rates between 54 and 78 beats/minute?

ISBN 978-1-009-25891-3

7B Determination of other percentages using a CAS

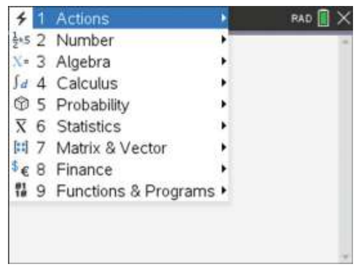
The 68–95–99.7% rule tells us the approximate percentage of the data that lie within 1, 2 and 3 standard deviations either side of the mean. It is possible to calculate the percentage of the data values that lie between two boundaries that are not necessarily multiples of the standard deviation. A CAS calculator is used to find these percentages.

How to find the percentage of data values between two boundaries using a TI-Nspire CAS

Steps

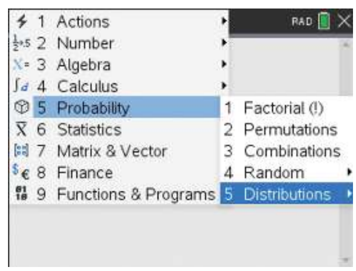
1 Start a new document (ctrl + N) and add a calculator page (1: Add Calculator)

2 Press menu

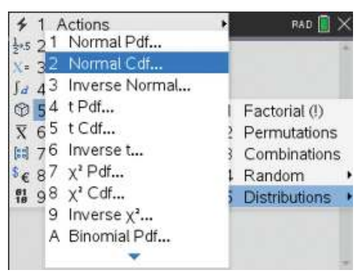


3 Press (▼) repeatedly to 5: Probability.

4 Press (▶) and then (▼) repeatedly to 5: Distributions



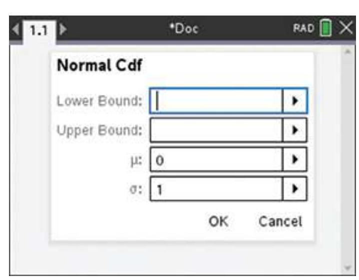
5 Press (▶) and then (▼) repeatedly to 2: Normal Cdf ...



6 Press enter or (centre button).

To use the dialogue box that appears, you need to enter four values. These are:

- Lower Bound: This is the lower boundary for the required percentage.
- Upper Bound: This is the upper boundary for the required percentage.
- μ: This is the mean of the distribution (0 by default).
- σ: This is the standard deviation of the distribution (1 by default).



ISBN 978-1-009-25891-3

7 As an example, consider a normally distributed data set that has mean $\mu = 27$ and standard deviation $\sigma = 3$. To find the percentage of the values in the data set that are expected to lie between 25 and 36, enter the values as shown in the screen opposite, pressing tab after each one.

8 tab to OK and press enter or centre button. The percentage is returned as a decimal. Multiply this decimal by 100 to calculate the required percentage. Approximately 74.6% of the values in this data set lie between 25 and 36.

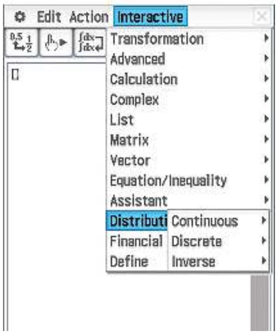
How to find the percentage of data values between two boundaries using a ClassPad CAS

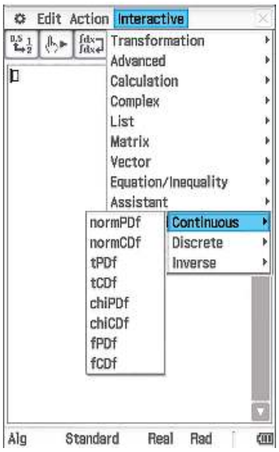
Steps

1 Start a new Main calculation page.

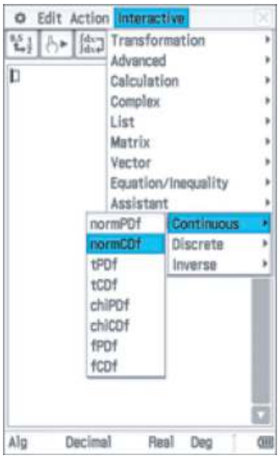
2 Tap 'Interactive' then 'Distribution/Inv. Dist'

3 Tap 'Continuous'





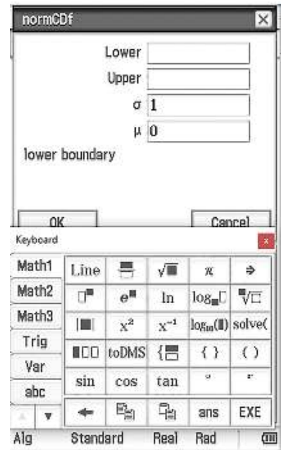
4 Tap 'normCDF'.
An interactive dialogue box will appear.



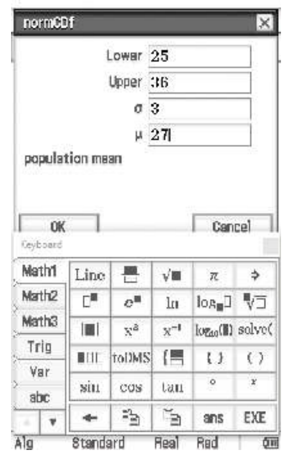
ISBN 978-1-009-25891-3

5 To use this interactive function you need to enter four values. These are:

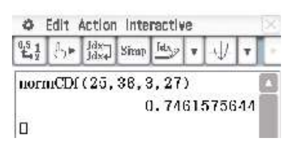
- Lower: This is the lower boundary for the required percentage.
- Upper: This is the upper boundary for the required percentage.
- σ: This is the standard deviation of the distribution (1 by default).
- μ: This is the mean of the distribution (0 by default).



6 As an example, consider a normally distributed data set that has mean $\mu = 27$ and standard deviation $\sigma = 3$. To find the percentage of the values in the data set that are expected to be between 25 and 36, enter the values as shown in the screen opposite, and then tap OK.



7 The percentage is returned as a decimal.
Multiply this decimal by 100 to calculate the required percentage.
Approximately 74.6% of the values in this data set lie between 25 and 36.



Example 2 Determining percentage between two boundaries in a normal distribution

The distribution of the height of a population of women is approximately normally distributed with a mean height of 170 cm and standard deviation of 15 cm.

What percentage of these women are expected to have a height between 153 cm and 158 cm?

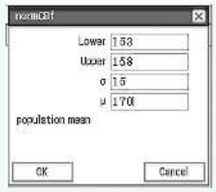
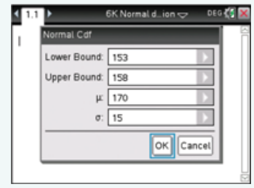
Round your answers to one decimal place.

ISBN 978-1-009-25891-3

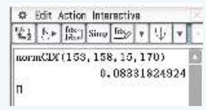
Solution

1 Enter the following values into a CAS calculator:

- Lower Bound / Lower = 153
- Upper Bound / Upper = 158
- $\mu = 170$
- $\sigma = 15$

2 tab to OK and press enter (Nspire) or tap OK (ClassPad).




3 Calculate the percentage.

Required percentage $= 0.0833\ldots \times 100\%$
$\approx 8.3\%$

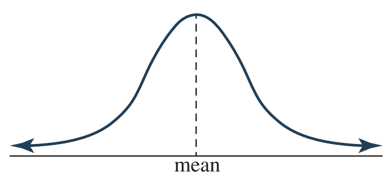
4 Write your answer.

Approximately 8.3% of the women in this population have a height between 153 cm and 158 cm.

Infinite boundaries for normal distributions

If we apply the 68–95–99.7% rule to the example above, we would see that almost all of the women in that population would fall within three standard deviations of the mean. Three standard deviations higher than the mean is a height of 215 cm, and while it is not very likely that a woman from this population would be taller than this, it certainly isn't impossible.

The graph of the normal distribution does not have an end point. The further away from the mean the graph reaches, the closer that graph is to the horizontal axis. The tails of the normal distribution graph get closer and closer to the horizontal axis, but never touch it.



The graphs of normal distributions are not normally drawn with arrows on the tails, but in this diagram they indicate that the tails continue without end.

If we want to calculate the number of data values greater than or less than a particular value in a data set, we must take into account that there is really no upper boundary or lower boundary on the possible data set values. We can never say that a particular value would absolutely never exist, all we can say is that if that value is a long way from the mean, then it isn't very likely to occur. In this case, we use infinity (∞) as the upper boundary, and negative infinity ($-\infty$) as the lower boundary in the normal distribution calculations.

ISBN 978-1-009-25891-3

Example 3 Determining percentage involving infinite boundaries in a normal distribution

The distribution of the height of a population of women is approximately normally distributed with a mean height of 170 cm and standard deviation of 15 cm.

a What percentage of these women are expected to have a height greater than 175 cm?

b What percentage of these women are expected to have a height less than 145 cm?

c In a group of 500 women, how many are expected to have a height less than 145 cm?

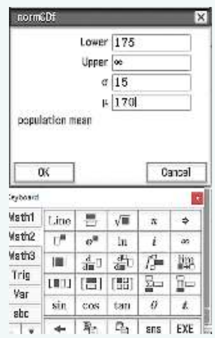
Round your answers to one decimal place.

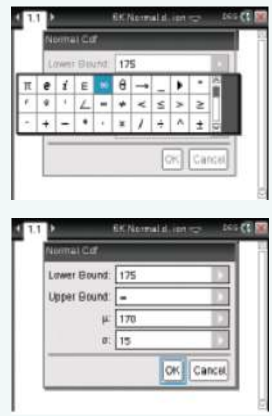
Solution

a **1** Enter the following values into a CAS calculator:

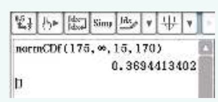
- Lower Bound / Lower = 175
- Upper Bound / Upper = ∞
- μ = 170
- σ = 15

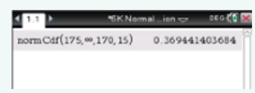
Note: The infinity symbol can be found in the keyboard menu Math 2 (ClassPad) or by pressing Ctrl catalogue (Nspire).





2 tab to OK and press enter (Nspire) or tap OK (ClassPad).





3 Calculate the percentage.

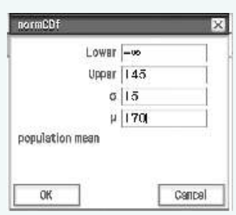
Required percentage $= 0.369\ldots \times 100\%$
$\approx 36.9\%$

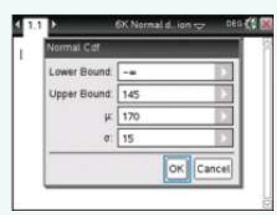
4 Write your answer.

Approximately 36.9% of the women in this population have a height greater than 175 cm.

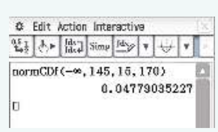
b **1** Enter the following values into a CAS calculator:

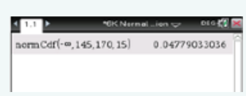
- Lower Bound / Lower = −∞
- Upper Bound / Upper = 145
- μ = 170
- σ = 15





2 tab to OK and press enter (Nspire) or tap OK (ClassPad).





3 Calculate the percentage.

Required percentage $= 0.0477\ldots \times 100\%$
$\approx 4.8\%$

ISBN 978-1-009-25891-3

4 Write your answer.	Approximately 4.8% of the women in this population have a height greater than 145 cm.
c 1 Use the answer to part **b** to find 4.8% of 500.	$4.8\% \text{ of } 500 = \frac{4.8}{100} \times 500$ $= 24$
2 Write your answer.	In a group of 500 women, 24 of them are expected to have a height less than 145 cm.

Exercise 7B

Use a CAS calculator to determine percentages in a normal distribution

CAS

1 The time it takes a population of adults to solve a particular puzzle is normally distributed with a mean time of 5 minutes and a standard deviation of 0.75 minutes. In this question, round your answers to one decimal place.
Find the percentage of adults in this population that are expected to solve the puzzle in a time that is:

a between 4 and 6 minutes.
b between 5.2 and 5.9 minutes,
c greater than 5.3 minutes.
d smaller than 4.4 minutes.

2 The label on a bottle of tomato sauce shows that it contains 500 ml of sauce. In reality though, the volume of sauce in a bottle is normally distributed with a mean of 500 ml and standard deviation 1.4 ml.
Find the percentage of bottles produced by a factory, rounded to one decimal place, that are expected to have a volume of sauce that is:

a between 500 and 503 ml.
b between 495 and 497 ml.
c greater than 500.3 ml.
d smaller than 499 ml.

3 Bottles of fruit juice contain a mean volume of 1000 ml (1 Litre) with a standard deviation of 2 ml. A shop owner received an order of 200 bottles of this fruit juice.

a **i** What percentage of bottles are expected to have a volume of juice between 998 ml and 1004 ml? Write your answer to one decimal place.
ii How many of the 200 bottles are expected to have a volume of juice between 998 ml and 1004 ml? Write your answer to the nearest bottle.

b How many of the 200 bottles are expected to have a volume less than 997 ml? Write your answer to the nearest bottle,

c How many of the 200 bottles are expected to have a volume greater than 999 ml? Write your answer to the nearest bottle.

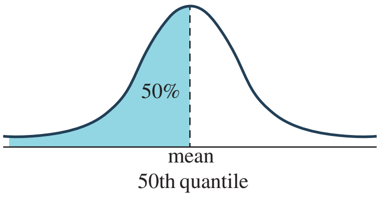
ISBN 978-1-009-25891-3

7C Quantiles

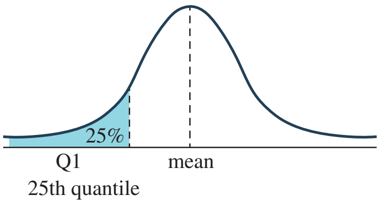
In addition to finding the percentage of data values that are between certain boundaries within a normally distributed data set, we can also determine a boundary for a particular percentage of values.

> A quantile is the value below which a certain percentage of data values are expected to fall.

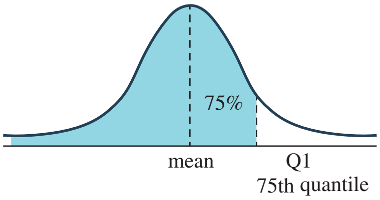
For example, the 50th quantile is the data value below which 50% of the data values are expected to fall. The 50th quantile is also known as the median. It divides the data set in half.



The 25th quantile is the data value below which 25% of the data values are expected to fall. The 25th quantile is also known as Q_1, or the first quartile. It divides the bottom 25% of the data from the rest of the data set.



In a similar way, the 75th quantile is also known as Q_3, or the third quartile. It divides the bottom 75% of the data from the rest of the data set.



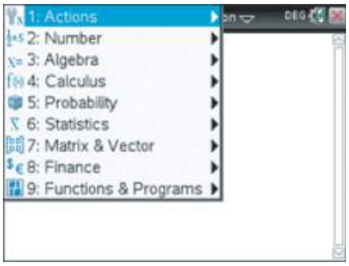
If the mean and standard deviation of a normally distributed data set are known, then any quantile for that data set can be determined using a CAS calculator.

How to find a quantile using a TI-Nspire CAS

Steps

1 Start a new document (ctrl + N) and add a calculator page (1: Add Calculator)

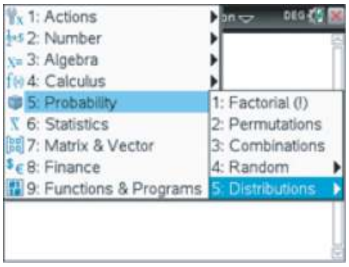
2 Press menu



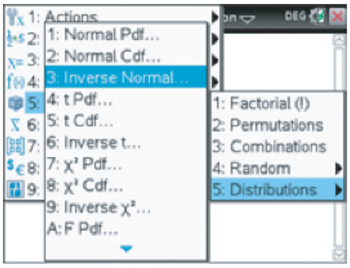
ISBN 978-1-009-25891-3

3 Press (▼) repeatedly to 5: Probability.

4 Press (▶) and then ▼ repeatedly to 5: Distributions

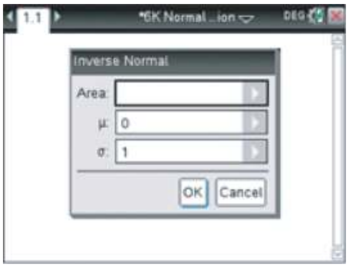


5 Press (▶) and then ▼ repeatedly to 3: Inverse Normal

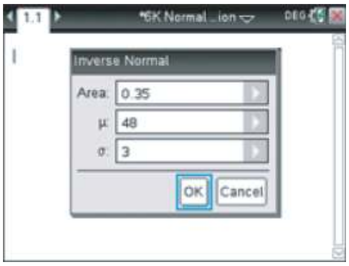


6 Press [enter] or (centre button).
To use the dialogue box that appears, you need to enter three values. These are:

- Area. This is the percentage value for the quantile. It must be entered as a decimal.
- μ is the mean of the distribution (0 by default).
- σ is the standard deviation for the distribution (1 by default).



7 For example, to calculate the 35th quantile for a normally distributed data set with mean 48 and standard deviation 3, enter the values shown in the screen opposite, pressing [tab] after each one.



8 Press [tab] to select [OK] and press [enter] or centre button.
The quantile is returned.
The 35th quantile for this distribution is approximately 46.8.

ISBN 978-1-009-25891-3

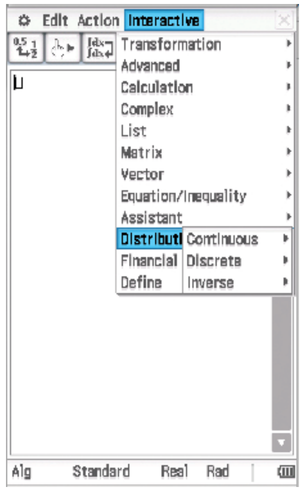
How to find a quantile using a Casio ClassPad

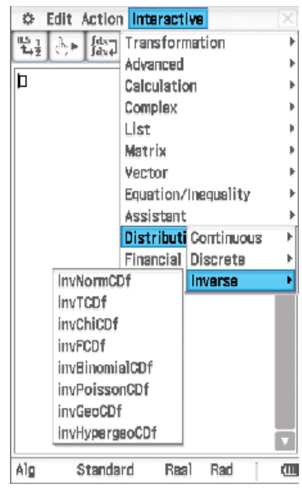
Steps

1 Start a new Main calculation page.

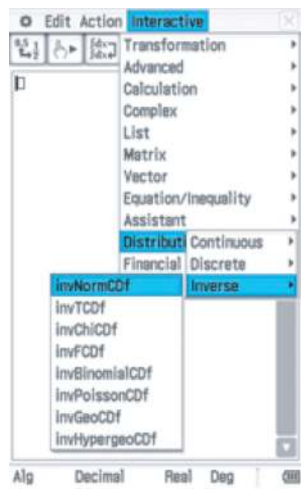
2 Tap ‘Interactive’ then ‘Distribution/Inv. Dist’.

3 Tap ‘Inverse’.



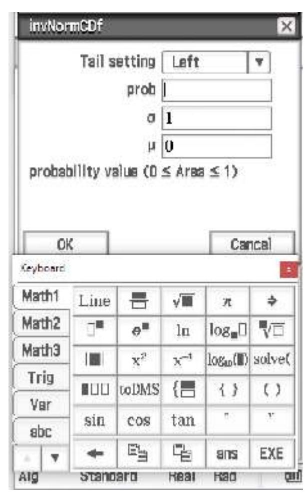


4 Tap ‘invNormCDF’.
An interactive dialogue box will appear.



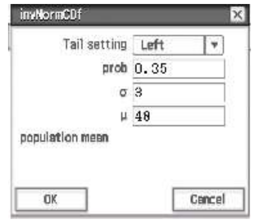
5 To use this interactive function you need to enter four values. These are:

- Tail Setting is the position of the shaded area of the normal distribution graph. For a quantile calculation, this should be set to ‘Left’.
- Prob is the percentage value for the quantile. It must be entered as a decimal.
- σ is the standard deviation for the distribution (1 by default).
- μ is the mean of the distribution (0 by default).

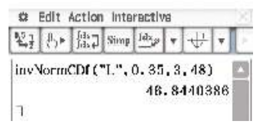


ISBN 978-1-009-25891-3

6 For example, to calculate the 35th quantile for a normally distributed data set with mean 48 and standard deviation 3, enter the values shown in the screen opposite and then tap OK.



7 The quantile is returned.
The 35th quantile for this distribution is approximately 46.8.



Example 4 Finding quantiles using a CAS calculator

A normally distributed data set has mean 28 and standard deviation 2.

a Find the value below which 10% of the data is expected to fall.

b Find the value above which 18% of the data is expected to fall.

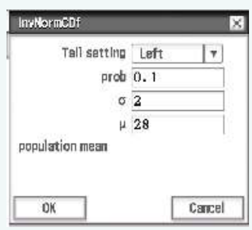
Round your answers to one decimal place.

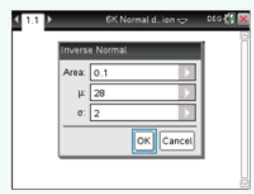
Solution

a **1** The value below which 10% of the data is expected to fall is the 10th quantile.

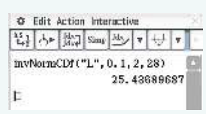
2 Enter the following values into a CAS calculator:

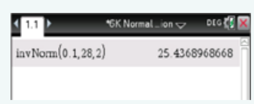
- Area / prob = 0.1
- $\mu = 28$
- $\sigma = 2$





3 tab to OK and press enter (Nspire) or tap OK (ClassPad).

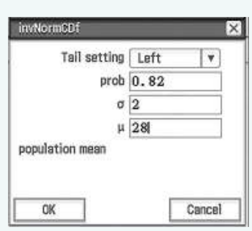


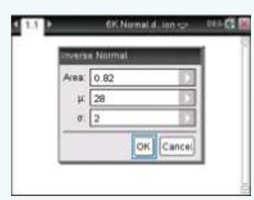


4 Write your answer.

10% of the data in this distribution are expected to fall below the value of 25.4.

b **1** This question requires the value above which 18% of values are expected to fall. This is equivalent to finding the 82nd quantile, since 100% – 18% = 82%.



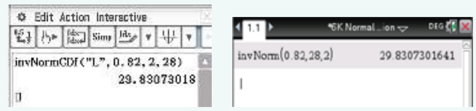


2 Enter the following values into a CAS calculator:

- Area / prob = 0.82
- $\mu = 28$
- $\sigma = 2$

ISBN 978-1-009-25891-3

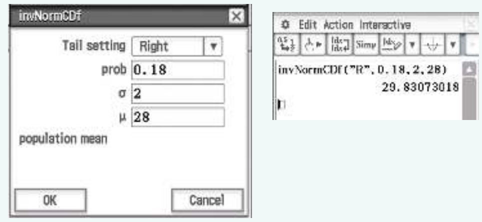
3 tab to OK and press enter (Nspire) or tap OK (ClassPad).



4 Write your answer.

18% of the data in this distribution are expected fall above the value of 29.8.

Note: Casio Classpad users have the option of using a different 'tail' setting in the dialogue box for this calculation.
Select 'Right' to calculate the value ***above*** which 18% of data values will lie, as shown.



Example 5 Practical application of quantiles

The scores on a mathematics examination are normally distributed with a mean of 63 marks and a standard deviation of 4 marks.

The top 5% of students will be given the grade of A+.

What is the minimum number of marks a student must get on the test to receive the grade of A+?

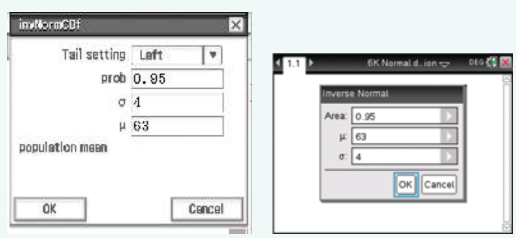
Round your answer to one decimal place.

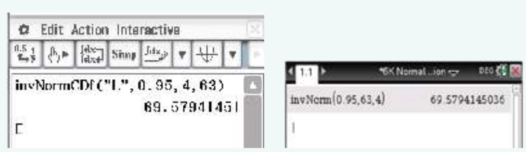
Solution

1 The top 5% of students will be awarded the grade of A+. This implies that 95% will receive a lower grade and so we need to find the 95th quantile.

2 Enter the following values into a CAS calculator:
- Area / prob = 0.95
- $\mu = 63$
- $\sigma = 4$

to calculate the quantile.





3 Write your answer.

The minimum number of marks required to receive a grade of A+ is 69.6.

ISBN 978-1-009-25891-3

Exercise 7C

Determining quantiles using a CAS calculator

CAS

1 In the following questions, round your answers to 1 decimal place.

- **a** Find the 25^{th} quantile for a normal distribution with mean of 30 and standard deviation of 5.
- **b** Find the 75^{th} quantile for a normal distribution with mean of 78 and standard deviation 3.
- **c** Find the 32^{nd} quantile for a normal distribution with mean of 26 and standard deviation 2.3.
- **d** Find the 86^{th} quantile for a normal distribution with mean of 3.6 and standard deviation 0.8.

2 A data set is normally distributed with mean 45 and standard deviation 5. If 36% of the data values in that data set are expected to be lower than p, find the value of p. Round your answer to one decimal place.

3 A data set is normally distributed with mean 26 and standard deviation 4.8. If 22% of the data are higher than q, find the value of q. Round your answer to one decimal place.

Practical applications of quantiles

4 The scores on a history examination are normally distributed with a mean of 78 and a standard deviation of 4.
The top 8% of students will receive an A grade.
What is the minimum number of marks required to receive an A grade?
Round your answer to the nearest whole number.

5 In the first round of an athletics competition, 240 runners entered the 100 m event. The time taken to run 100 m by these competitors is normally distributed with a mean of 12.6 sec and standard deviation 1.8 sec. The fastest 80% of runners will qualify for the second round.

- **a** What is the slowest possible time for a runner that progresses to the second round? Round your answer to one decimal place.
- **b** How many runners progress to the second round? Round your answer to the nearest whole number.

6 The length of fish in a pond is known to be normally distributed with a mean of 25.6 cm and standard deviation 5.3 cm. It is expected that 14% of the fish are too small to be caught and must be thrown back. What is the smallest length of fish that may be caught and kept?
Round your answer to the nearest millimetre.

ISBN 978-1-009-25891-3

CAS

7 The length of carrots harvested on a farm is normally distributed with a mean of 27 cm and standard deviation 3.1 cm. The shortest 20% of carrots were chopped and frozen and the rest were sold at a market. What is the greatest length of carrot that would be chopped and frozen?
Round your answer to one decimal place.

8 Eggs are classified as small, medium or large in size, based on their weight.
The weight of eggs from a chicken farm is normally distributed with a mean of 520 grams and standard deviation 18 grams.
On average, 30% of the eggs from this farm are classified as small, 50% are classified as medium and 20% of the eggs are classified as large.
Round all of your answers to the nearest gram.

- **a** What is the largest weight of an egg that could be classified as small?
- **b** What is the smallest weight of an egg that could be classified as large?
- **c** Between what weights are these eggs classified as medium?

7D Standard scores

The 68–95–99.7% rule makes the standard deviation a natural measuring stick for normally distributed data.

For example, a person who obtained a score of 112 on an IQ test with a mean of 100 and a standard deviation of 15 has an IQ score less than one standard deviation from the mean. Their score is typical of the group as a whole, as it lies well within the middle 68% of scores. In contrast, a person who scores 133 stands out; their score is more than two standard deviations from the mean and this puts them in the top 2.5%.

Because of the additional insight provided by relating the standard deviations to percentages, it is common to transform data into a new set of units that show the number of standard deviations a data value lies from the mean of the distribution. This is called *standardising* and these transformed data values are called **standardised** or ***z*-scores**.

Calculating standardised (z-) scores

To obtain a standard score for an actual score, subtract the mean from the score and then divide the result by the standard deviation. That is:

$$\text{standard score} = \frac{\text{actual score} - \text{mean}}{\text{standard deviation}} \quad \text{or} \quad z = \frac{x - \bar{x}}{s}$$

Let us check to see that the formula works.

ISBN 978-1-009-25891-3

We already know that an IQ score of 115 is one standard deviation above the mean, so it should have a standard or z-score of 1. Substituting into the formula above we find, as we had predicted, that:

$$z = \frac{115 - 100}{15} = \frac{15}{15} = 1$$

Standard scores can be both positive and negative:

- a *positive* z-score indicates that the actual score it represents lies *above* the mean
- a *zero* standardised score indicates that the actual score is *equal* to the mean
- a *negative* z-score indicates that the actual score lies *below* the mean.

Example 6 **Calculating standard scores**

The heights of a group of young women have a mean of $\bar{x} = 160$ cm and a standard deviation of $s = 8$ cm. Determine the standard or z-scores of a woman who is:

a 172 cm tall **b** 150 cm tall **c** 160 cm tall.

Solution

1 Write down the data value (x), the mean ($\bar{x}$) and the standard deviation (s).

2 Substitute the values into the formula $z = \dfrac{x - \bar{x}}{s}$ and evaluate.

a $x = 172,\ \bar{x} = 160,\ s = 8$

$$z = \frac{x - \bar{x}}{s} = \frac{172 - 160}{8} = \frac{12}{8} = 1.5$$

b $x = 150,\ \bar{x} = 160,\ s = 8$

$$z = \frac{x - \bar{x}}{s} = \frac{150 - 160}{8} = -\frac{10}{8} = -1.25$$

c $x = 160,\ \bar{x} = 160,\ s = 8$

$$z = \frac{x - \bar{x}}{s} = \frac{160 - 160}{8} = \frac{0}{8} = 0$$

Using standard scores to compare performance

Standard scores are also useful for comparing groups that have different means and/or standard deviations. For example, consider a student who obtained a mark of 75 in Psychology and a mark of 70 in Statistics. In which subject did she do better?

Calculating standard scores

Subject	*Mark*	*Mean*	*Standard Deviation*
Psychology	75	65	10
Statistics	70	60	5

We could take the marks at face value and say that she did better in Psychology because she got a higher mark in that subject. The assumption that underlies such a comparison is that the marks for both subjects have the same distribution with the same mean and standard deviation. However, in this case the two subjects have very different means and standard deviations, as shown in the table above.

If we assume that the *marks* are *normally distributed*, then *standardisation* and the *68–95–99.7% rule* give us a way of resolving this issue.

ISBN 978-1-009-25891-3

Let us standardise the marks

Psychology: standardised mark $z = \dfrac{75 - 65}{10} = 1$

Statistics: standardised mark $z = \dfrac{70 - 60}{5} = 2$

What do we see? The student obtained a higher score for Psychology than for Statistics. However, relative to her classmates she did better in Statistics.

- Her mark of 70 in Statistics is equivalent to a z-score of 2. This means that her mark was two standard deviations above the mean, placing her in the top 2.5% of students.
- Her mark of 75 for Psychology is equivalent to a z-score of 1. This means that her mark was only one standard deviation above the mean, placing her in the top 16% of students. This is a good performance, but not as good as for Statistics.

Example 7 **Applying standard scores**

Another student studying the same two subjects obtained a mark of 55 for both Psychology and Statistics. Does this mean that she performed equally well in both subjects? Use standardised marks to help you arrive at your conclusion.

Solution

1 Write down her mark (x), the mean ($\bar{x}$) and the standard deviation (s) for each subject and compute a standardised score for both subjects. Evaluate and compare.

Psychology : $x = 55$, $\bar{x} = 65$, $s = 10$

$z = \dfrac{x - \bar{x}}{s} = \dfrac{55 - 65}{10} = \dfrac{-10}{10} = -1$

Statistics : $x = 55$, $\bar{x} = 60$, $s = 5$

$z = \dfrac{x - \bar{x}}{s} = \dfrac{55 - 60}{5} = \dfrac{5}{5} = -1$

2 Write down your conclusion.

Yes, her standardised score, $z = -1$, was the same for both subjects. In both subjects she finished in the bottom 16%.

Converting standard scores into actual scores

Having learnt how to calculate standard scores, you also need to be able to convert a standardised score back into an actual score. The rule for converting a standardised score into an actual score is given below.

Converting standardised scores into actual scores

By making the actual score the subject of the rule for calculating standard scores, we arrive at:

actual score = mean + standard score × standard deviation or $x = \bar{x} + z \times s$

ISBN 978-1-009-25891-3

Example 8 Converting standard scores into actual scores

A class test (out of 50) has a mean mark of $\bar{x} = 34$ and a standard deviation of $s = 4$. Joe's standardised test mark was $z = -1.5$. What was Joe's actual mark?

Solution

1 Write down the mean ($\bar{x}$), the standard deviation (s) and Joe's standardised score (z).

$\bar{x} = 34,\ s = 4,\ z = -1.5$

2 Write down the rule for calculating the actual score and substitute these values into the rule.

$x = \bar{x} + z \times s$

$\quad = 34 + (-1.5) \times 4 = 28$

Joe's actual mark was 28.

Exercise 7D

Calculating standard scores

1 A set of scores has a mean of 100 and a standard deviation of 20. Standardise the following scores.

a 120 **b** 140 **c** 80 **d** 100 **e** 40 **f** 110

Calculating actual scores from standardised scores

2 A set of scores has a mean of 100 and a standard deviation of 20. Calculate the actual score if the standardised score was:

a 1 **b** 0.8 **c** 2.1 **d** 0 **e** −1.4 **f** −2.5

Applications

3 The table below contains the scores a student obtained in a practice test for each of his WACE subjects. Also shown is the mean and standard deviation for each subject.

Subject	*Mark*	*Mean*	*Standard deviation*
English	69	60	4
Biology	75	60	5
Chemistry	55	55	6
Maths Applications	55	44	10
Psychology	73	82	4

a Calculate his standard score for each subject.

b Use the standard score to rate his performance in each subject, assuming a normal distribution of marks and using the 68–95–99.7% rule.

ISBN 978-1-009-25891-3

4 The body weights of a large group of 14-year-old boys have a mean of 54 kg and a standard deviation of 10.0 kg.

a Kareem weighs 56 kg. Determine his standardised weight.

b Leon has a standardised weight of −0.75. Determine his actual weight.

Assuming the boys' weights are approximately normally distributed with a mean of 54 kg and a standard deviation of 10 kg, determine the:

c percentage of these boys who weigh more than 74 kg

d percentage of these boys who weigh between 54 and 64 kg

e percentage of these boys who have standardised weights less than −1.

f percentage of these boys who have standardised weights greater than −2.

ISBN 978-1-009-25891-3

Key ideas and chapter summary

Assign-ment

Normal distribution Data distributions that are bell-shaped can be modelled by a *normal* distribution.

The 68–95–99.7% rule Percentage of observations that lie within one, two or three standard deviations of the mean.

Quantiles A quantile is the value below which a certain percentage of data values are expected to fall. For example, the 25th quantile is the data value below which 25% of the data values are expected to fall.

Standard scores The number of standard deviations a data value lies from the mean of the distribution.
To obtain a standard score for an actual score, subtract the mean from the score and then divide the result by the standard deviation. That is:

$$\textit{standard score} = \frac{\textit{actual score} - \textit{mean}}{\textit{standard deviation}}$$

Skills check

Having completed this chapter you should be able to:

- understand that normal distributions are symmetric distributions that have an approximate *bell shape*
- understand the 68–95–99.7% rule
- calculate probabilities and percentages for normal distributions with known mean and standard deviation in practical situations using CAS
- understand quantiles and find the boundary of a particular percentage of values
- know the meaning of standard scores and calculate z-scores.

Short-answer questions

1 The normal distribution is symmetrical about the mean. True or False?

2 What percentage of data lies within *one standard deviation (SD)* of the mean?

3 95% of data lies within three standard deviation of the mean. True or False?

4 After extensive testing, it was found that the lifetimes of power bulbs had a mean of 2500 hours and a standard deviation of 100 hours. Assuming that the lifetime of a bulb is modelled by a normal distribution, what percentage of power bulbs will have a lifetime between 2400 and 2600 hours?

ISBN 978-1-009-25891-3

Review

5 The height of girls at a particular age follows a normal distribution with a mean of 130 cm and standard deviation 3 cm. Find the probability that a girl picked at random from this age group has a height less than 134 cm. CAS

6 The number of hours of the life of a torch battery is normally distributed with a mean of 120 hours and a standard deviation of 16 hours. Find the probability that a torch battery has a life of more than 140 hours.

7 A normally distributed data set has a mean of 60 and standard deviation 3. Find the value below which 85% of the data is expected to fall.

8 A normally distributed data set has a mean of 20 and standard deviation 2. Find the value below which 55% of the data is expected to fall.

9 Cars currently sold by Cheap Cars Ltd have an average of 145 horsepower with a standard deviation of 25 horsepower. What is the z-score for a car with 180 horsepower?

10 In a normally distributed data set, the mean is 96 and the z-score for a raw value of 104 is 2. Find the value of the standard deviation.

11 The random variable X has a normal distribution with mean 6 and standard deviation 1.5. Calculate the percentage of scores that lie between 4.5 and 9.

12 A certain type of vegetable has a mass which is normally distributed with a mean of 2 kg and standard deviation 0.25 kg. In a lorry load of 600 of these vegetables, estimate how many will have a mass greater than 2.1 kg. CAS

13 The number of marks of 1000 candidates in an examination is normally distributed with a mean of 58 marks and a standard deviation of 10 marks. Given that the pass mark is 50 marks, estimate the number of candidates who pass the examination.

14 A company packing spices knows that the weight of 500 packets form a normal distribution with a mean weight of 16 grams and a standard deviation of 0.2 gram. How many of these 500 packets are expected to weigh less than 15.6 grams?

15 The life span of a species of insects is modelled by a normal distribution with a mean of 360 hours and a standard deviation of 20 hours. Determine the life span exceeded by 7% of the insects. CAS

16 The weight of a consignment of sacks of sugar is normally distributed with a mean of 30 kg and a standard deviation of 2 kg. Determine to the nearest kg, the weight below which 15% of the sacks fall.

ISBN 978-1-009-25891-3

Review

Extended-response questions

1 The weights of oranges in a supermarket shipment are normally distributed with a mean of 160 g and a standard deviation of 20 g. The distribution is such that 68%, 95% and 99.7% of the oranges have weights within one, two and three standard deviations from the mean respectively.
Determine the percentages of oranges from the supermarket that

a weigh between 120 g and 200 g

b weigh more than 160 g

c weigh exactly 100 g

d weigh between 140 g and 160 g.

2 At a hardware store, the lengths of a large number of wooden rods marked as 2 m long, were actually normally distributed with a mean of 202 cm and a standard deviation of 3 cm.

a State the median length of the wooden rods.

b Find the percentage of planks whose length is between 199 cm and 205 cm.

c Find the percentage of planks having length that is less than 1.99 m.

CAS

3 Top Jewellery Ltd purchased freshwater pearls to produce its necklaces. The diameters of the pearls were found to be normally distributed, with a mean of 1.2 cm and a standard deviation of 0.15 cm.

a What proportion of the pearls will have a diameter exceeding 1.05 cm?

b Below what size will the diameter of 5% of the pearls fall?

4 The marks for a mathematics examination at a school are normally distributed with a mean of 59% and a standard deviation of 12%.

a State the median examination score.

b Determine the interquartile range of the examination scores.

c The top 10% students were awarded an A grade. Determine the minimum cut off for an A grade.

5 Suppose a data set is normally distributed with a mean of 120 and a standard deviation of 10.

a What data value is 2 standard deviations above the mean?

b What data value is 1.5 standard deviations below the mean?

ISBN 978-1-009-25891-3

8 Right-angled trigonometry

In this chapter

Syllabus references

Topic: Applications of trigonometry

Subtopics: 2.2.1, 2.2.4

Trigonometry can be used to solve many practical problems. How high is that tree? What is the height of the mountain we can see in the distance? What is the exact location of the fire that has just been seen by fire spotters? How wide is the lake? What is the area of this irregular-shaped paddock?

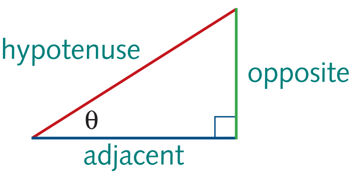
ISBN 978-1-009-25891-3

8A Trigonometry basics

Although you are likely to have studied some trigonometry, it may be helpful to review a few basic ideas.

Naming the sides of a right-angled triangle

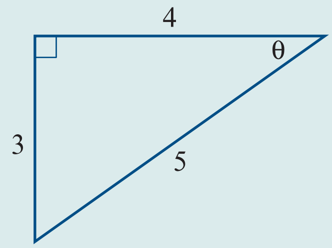
- The *hypotenuse* is the longest side of the right-angled triangle and is always opposite the right angle (90°).
- The *opposite* side is directly opposite the angle θ.
- The *adjacent* side is beside the angle θ, but it is not the hypotenuse. It runs from θ to the right angle.



The opposite and adjacent sides are located in relation to the position of angle θ. If θ was in the other corner, the sides would have to swap their labels. The letter θ is the Greek letter *theta*. It is commonly used to label an angle.

Example 1 **Identifying the sides of a right-angled triangle**

Give the lengths of the hypotenuse, the opposite side and the adjacent side in the triangle shown.



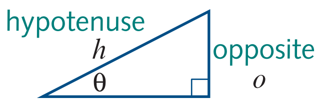
Solution

The hypotenuse is opposite the right angle.	The hypotenuse, $h = 5$
The opposite side is opposite the angle θ.	The opposite side, $o = 3$
The adjacent side is beside θ, but is not the hypotenuse.	The adjacent side, $a = 4$

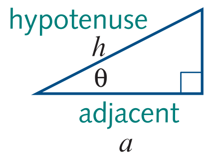
ISBN 978-1-009-25891-3

The trigonometric ratios

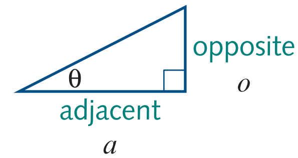
The **trigonometric ratios sin θ**, **cos θ** and **tan θ** can be defined in terms of the sides of a right-angled triangle.



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin\theta = \frac{o}{h}$$



$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos\theta = \frac{a}{h}$$



$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan\theta = \frac{o}{a}$$

"SOH — CAH — TOA"

This mnemonic **SOH-CAH-TOA** is often used by students to help them remember the rule for each trigonometric ratio.

In this mnemonic:

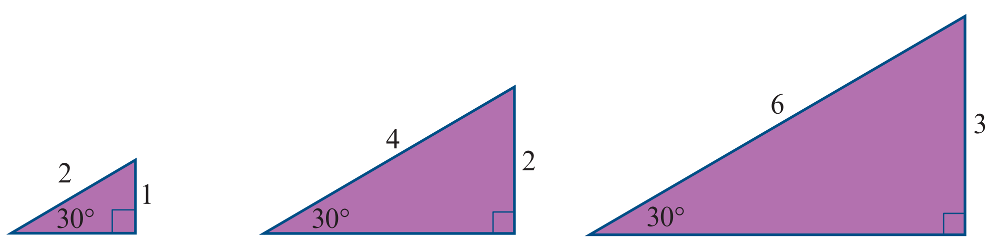
- **SOH** reminds us that sine equals opposite over hypotenuse
- **CAH** reminds us that cosine equals adjacent over hypotenuse
- **TOA** reminds us that tan equals opposite over adjacent

Or you may prefer:

'**S**ir **O**liver's **H**orse **C**ame **A**mbling **H**ome **T**o **O**liver's **A**rms'

The meaning of the trigonometric ratios

Using a calculator, we find, for example, that $\sin 30^\circ = 0.5$. This means that in *all* right-angled triangles with an angle of 30°, the ratio of the side opposite the 30° to the hypotenuse is always 0.5.



$$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{2} = 0.5 \qquad \frac{\text{opposite}}{\text{hypotenuse}} = \frac{2}{4} = 0.5 \qquad \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{6} = 0.5$$

Try drawing any right-angled triangle with an angle of 30° and check that the ratio:

$$\frac{\text{opposite}}{\text{hypotenuse}} = 0.5$$

ISBN 978-1-009-25891-3

Similarly, for *any* right-angled triangle with an angle of 30° the ratios cos 30° and tan 30° always have the same values:

$$\cos 30^\circ = \frac{\text{adjacent}}{\text{hypotenuse}} \text{ is always } \frac{\sqrt{3}}{2} = 0.8660 \text{ (to four decimal places)}$$

$$\tan 30^\circ = \frac{\text{opposite}}{\text{adjacent}} \text{ is always } \frac{1}{\sqrt{3}} = 0.5774 \text{ (to four decimal places).}$$

A calculator gives the value of each trigonometric ratio for any angle entered.

Using you CAS calculator to evaluate trigonometric ratios

Warning!

Make sure that your calculator is set in DEGREE mode before attempting the following example.

See Appendix (available in the digital textbook online).

Example 2 Finding the values of trigonometric ratios

Use your graphics calculator to find, correct to four decimal places, the value of:

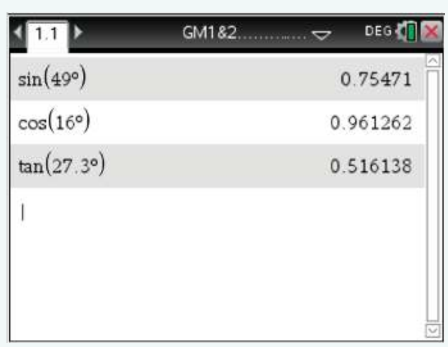
a $\sin 49^\circ$ **b** $\cos 16^\circ$ **c** $\tan 27.3^\circ$

Solution

1 For the **TI-Nspire CAS** ensure that the mode is set in **Degree** and **Approximate (Decimal)**. Refer to Appendix to set mode.

2 In a Calculator page, select **sin** from the palette and type 49.

3 Repeat for **b** and **c** as shown on the calculator screen.
Optional: you can add a degree symbol from the $\boxed{\alpha\beta^\circ}$ palette if desired. This will override any mode settings.



4 Write your answer correct to four decimal places.

a $\sin(49^\circ) = 0.7547$
b $\cos(16^\circ) = 0.9613$
c $\tan(27.3^\circ) = 0.5161$

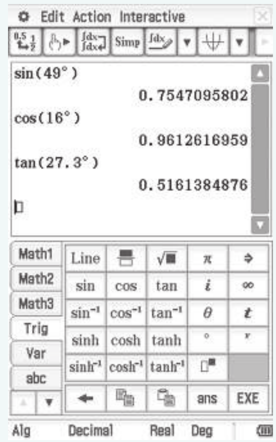
ISBN 978-1-009-25891-3

1 For **ClassPad**, in the Main application ensure that the status bar is set to **Decimal** and **Degree** mode.

2 To enter and evaluate the expression:

- Display the **keyboard**
- In the Trig palette select [sin]
- Type [4][9][°][)]
- Press [EXE].

3 Repeat for **b** and **c** as shown on the calculator screen.



4 Write your answer correct to four decimal places.

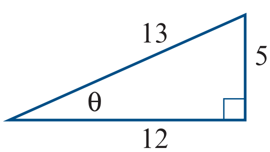
a sin (49°) = 0.7547
b cos (16°) = 0.9613
c tan (27.3°) = 0.5161

Exercise 8A

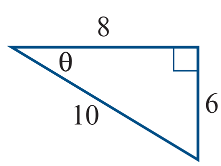
Example 1

1 State the values of the hypotenuse, the opposite side and the adjacent side in each triangle.

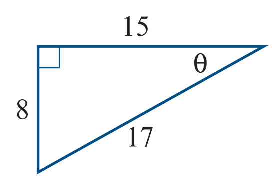
a



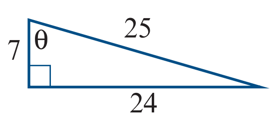
b



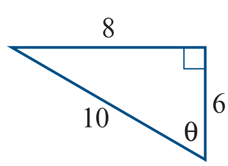
c



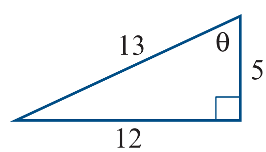
d



e



f



2 Write the ratios for sin θ, cos θ and tan θ for each triangle in Question **1**.

Example 2

3 Find the values of the following trigonometric ratios, correct to four decimal places.

a sin 27° **b** cos 43° **c** tan 62° **d** cos 79°
e tan 14° **f** sin 81° **g** cos 17° **h** tan 48°
i sin 80° **j** sin 49.8° **k** tan 80.2° **l** cos 85.7°

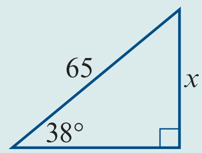
ISBN 978-1-009-25891-3

8B Finding an unknown side in a right-angled triangle

The trigonometric ratios can be used to find unknown sides in a right-angled triangle, given an angle and one side. When the unknown side is in the *numerator* (top) of the trigonometric ratio, proceed as follows.

Example 3 **Finding an unknown side**

Find the length of the unknown side x in the triangle shown, correct to two decimal places.

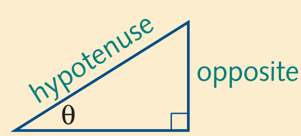


Solution

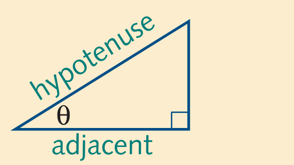
1. The sides involved are the opposite and the hypotenuse, so use sin θ. $\quad \sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$
2. Substitute in the known values. $\quad \sin 38^\circ = \dfrac{x}{65}$
3. Multiply both sides of the equation by 65 to obtain an expression for x. Use a calculator to evaluate.
$$65 \times \sin 38^\circ = x$$
$$x = 65 \times \sin 38^\circ$$
$$= 40.017\ldots$$
4. Write your answer correct to two decimal places. $\quad x = 40.02$

Finding an unknown side in a right-angled triangle

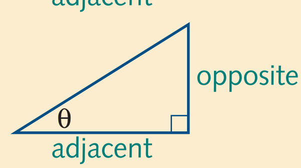
1. Draw the triangle and write in the given angle and side. Label the unknown side as x.
2. Use the trigonometric ratio that includes the given side and the unknown side.
   a. For the opposite and the hypotenuse, use $\quad \sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$



   b. For the adjacent and the hypotenuse, use $\quad \cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$



   c. For the opposite and the adjacent, use $\quad \tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$



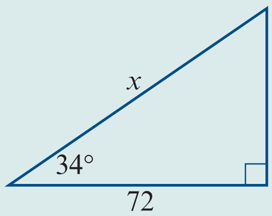
3. Rearrange the equation to make x the subject.
4. Use your calculator to find the value of x to the required number of decimal places.

ISBN 978-1-009-25891-3

An extra step is needed when the unknown side is in the *denominator* (at the bottom) of the trigonometric ratio.

Example 4 **Finding an unknown side which is in the denominator of the trigonometric ratio**

Find the value of x in the triangle shown, correct to two decimal places.



Solution

1 The sides involved are the adjacent and the hypotenuse, so use $\cos\theta$.
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

2 Substitute in the known values.
$$\cos 34^\circ = \frac{72}{x}$$

3 Multiply both sides by x.
$$x\cos 34^\circ = 72$$

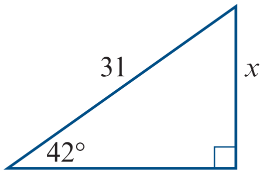
4 Divide both sides by $\cos 34^\circ$ to obtain an expression for x. Use a calculator to evaluate.
$$x = \frac{72}{\cos 34^\circ} = 86.847\ldots$$

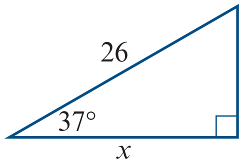
5 Write your answer correct to two decimal places.
$$x = 86.85$$

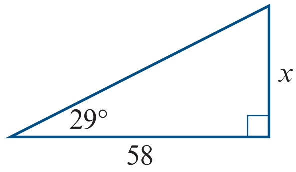
Exercise 8B

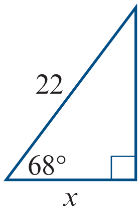
Example 3

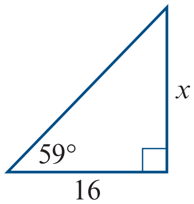
1 In each right-angled triangle below:
- decide whether the sin θ, cos θ or tan θ ratio should be used
- then find the unknown side x, correct to two decimal places.

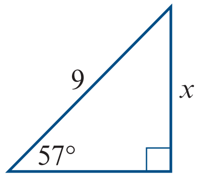
a 

b 

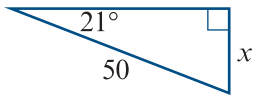
c 

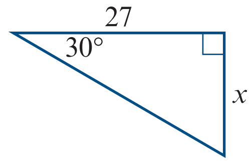
d 

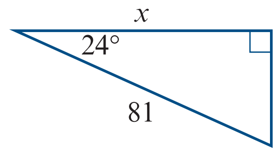
e 

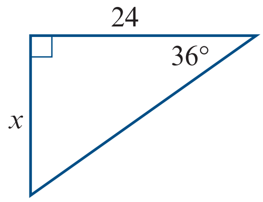
f 

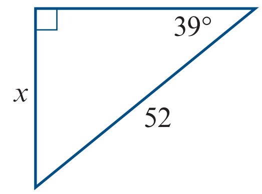
ISBN 978-1-009-25891-3

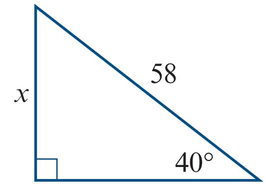
g


h


i


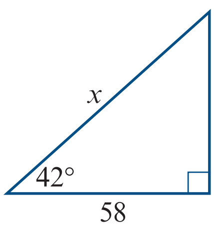
j


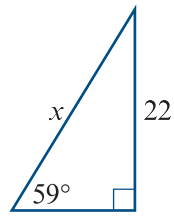
k


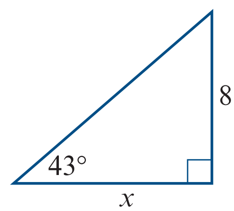
l


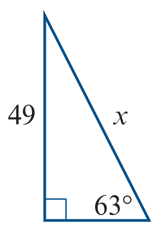
Example 4

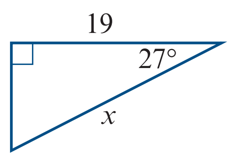
2 Find the unknown side x in each right-angled triangle below, correct to two decimal places.

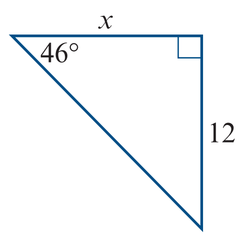
a


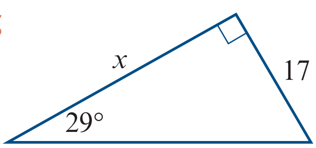
b


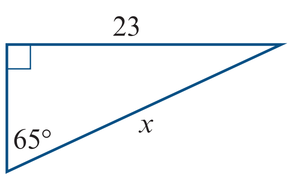
c


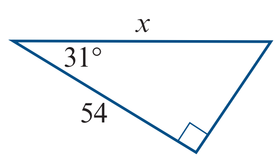
d


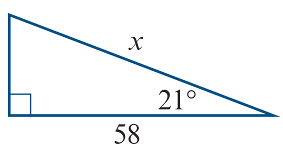
e


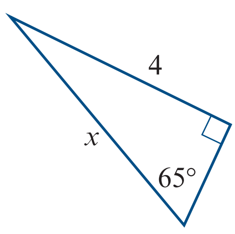
f


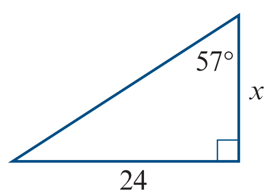
g


h


i


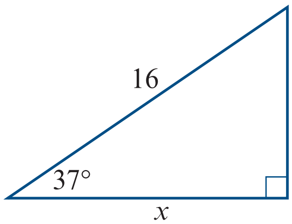
j


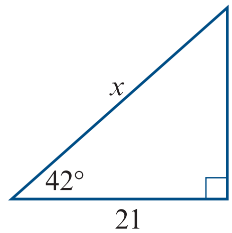
k


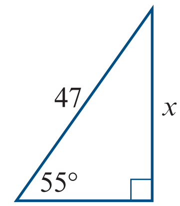
l


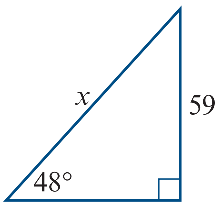
ISBN 978-1-009-25891-3

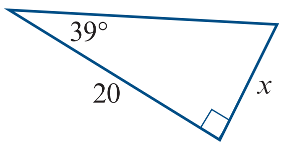
3 Find the length of the unknown side shown in each triangle, correct to one decimal place.

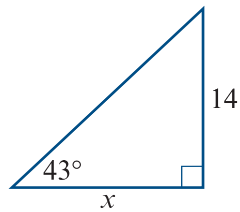
a


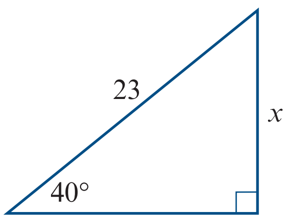
b


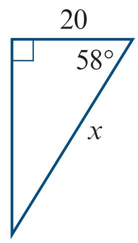
c


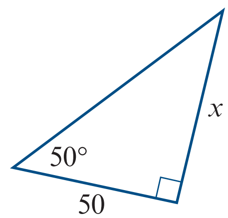
d


e


f


g


h


i


8C Finding an angle in a right-angled triangle

Finding an angle from a trigonometric ratio value

Before we look at how to find an unknown angle in a right-angled triangle, it will be useful to see how to find the angle when we know the value of the trigonometric ratio.

Suppose a friend told you that they found the sine value of a particular angle to be 0.8480 and challenged you to find out the mystery angle that had been used.

This is equivalent to saying:

$\sin\theta = 0.8480$, find the value of angle θ.

To do this, you need to work backwards from 0.8480 by undoing the sine operation to get back to the angle used. It is as if we have to find reverse gear to undo the effect of the sine function.

The reverse gear for sine is called the inverse of sine, written $\sin^{-1}$. The superscript -1 is not a power. It is just saying let us undo, or take one step backwards from using, the sine function.

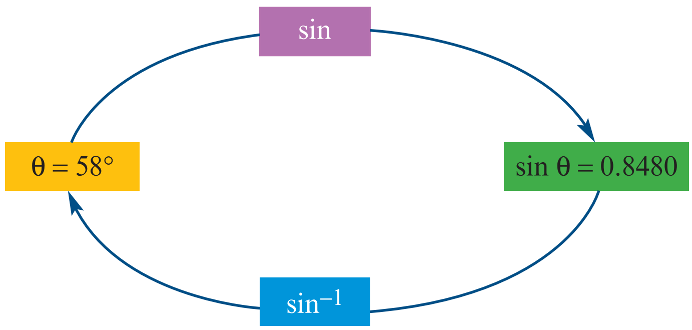
The request to find θ when $\sin\theta = 0.8480$ can be written as:

$$\sin^{-1}(0.8480) = \theta$$

ISBN 978-1-009-25891-3

This process is summarised in the following diagram.

- The *top arrow* in the diagram corresponds to: given θ, find $\sin\theta$. We use the sine function on our calculator to do this by entering $\sin 58°$ into a calculator to obtain the answer 0.8480.
- The *bottom arrow* in the diagram corresponds to: given $\sin\theta = 0.8480$, find θ. We use the $\sin^{-1}$ function on our calculator to do this by entering $\sin^{-1}(0.8480)$ to obtain the answer $58°$.



Similarly:

- The *inverse of cosine*, written as $\cos^{-1}$, is used to find θ when $\cos\theta = 0.5$ (e.g.).
- The *inverse of tangent*, written $\tan^{-1}$, is used to find θ when $\tan\theta = 1.67$ (e.g.).

You will learn how to how to use the $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ function of your calculator in the following example.

Example 5 Finding an angle from a trigonometric ratio

Find the angle θ, correct to one decimal place, given:

a $\sin\theta = 0.8480$ **b** $\cos\theta = 0.5$ **c** $\tan\theta = 1.67$

Solution

a We need to find $\sin^{-1}(0.8480)$.

1 For **TI-Nspire CAS**, press $\boxed{\text{trig}}$, select $\sin^{-1}$, then press $\boxed{0}\,\boxed{.}\,\boxed{8}\,\boxed{4}\,\boxed{8}\,\boxed{0}\,\boxed{\text{enter}}$.

$\sin^{-1}(0.848)$	57.9948

2 For **ClassPad**, tap $\boxed{\sin^{-1}}\,\boxed{0}\,\boxed{.}\,\boxed{8}\,\boxed{4}\,\boxed{8}\,\boxed{0}\,\boxed{)}\,\boxed{\text{EXE}}$.

3 Write your answer correct to one decimal place. $\theta = 58.0°$

ISBN 978-1-009-25891-3

b We need to find $\cos^{-1}(0.5)$.

1 For **TI-Nspire CAS**, press trig, select $\cos^{-1}$, then press 0 . 5 enter.

$\cos^{-1}(0.5) \quad 60$

2 For **ClassPad**, tap $\cos^{-1}$ 0 . 5) EXE.

3 Write your answer correct to one decimal place.

$\theta = 60°$

c We need to find $\tan^{-1}(1.67)$.

1 For **TI-Nspire CAS**, press trig, select $\tan^{-1}$, then press 1 . 6 7 enter.

$\tan^{-1}(1.67) \quad 59.0867$

2 For **ClassPad**, tap $\tan^{-1}$ 1 . 6 7) EXE.

3 Write your answer correct to one decimal place.

$\theta = 59.1°$

Getting the language right

The language we use when finding an angle from a trig ratio is difficult when you first meet it. The samples below are based of the results of Example 5.

- When you see:

  $$\sin(58°) = 0.8480$$

  think 'the sine of the angle 58° equals 0.8480'.

- When you see:

  $$\cos(60°) = 0.5$$

  think 'the cosine of the angle 60° equals 0.5'.

- When you see:

  $$\tan(59.1°) = 1.67$$

  think 'the tan of the angle 59.1° equals 1.67'.

- When you see:

  $$\sin^{-1}(0.8480) = 58°$$

  think 'the angle whose sine is 0.8480 equals 58°'.

- When you see:

  $$\cos^{-1}(0.5) = 60°$$

  think 'the angle whose cosine is 0.5 equals 60°'.

- When you see:

  $$\tan^{-1}(1.67) = 59.1°$$

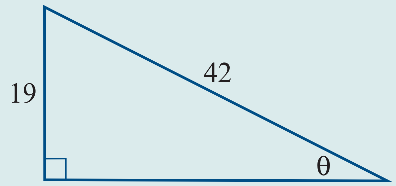
  think 'the angle whose tan is 1.67 equals 59.1°'.

ISBN 978-1-009-25891-3

Finding an angle given two sides

Example 6 Finding an angle given two sides in a right-angled triangle

Find the angle θ, in the right-angled triangle shown, correct to one decimal place.



Solution

1 The sides involved are the opposite and the hypotenuse, so use sin θ.

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$

2 Substitute in the known values.

$\sin\theta = \dfrac{19}{42}$

3 Write the equation to find an expression for θ. Use a calculator to evaluate.

$\theta = \sin^{-1}\left(\dfrac{19}{42}\right)$
$= 26.896\ldots$

4 Write your answer correct to one decimal place.

$\theta = 26.9°$

The three angles in a triangle add to 180°. As the right angle is 90°, the other two angles must add to make up the remaining 90°. When one angle has been found just subtract it from 90° to find the other angle. In Example 6, the other angle must be 90° – 26.9° = 63.1°.

Finding an angle in a right-angled triangle

1. Draw the triangle with the given sides shown. Label the unknown angle as θ.
2. Use the trigonometric ratio that includes the two known sides.
   - If given the opposite and hypotenuse, use $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$
   - If given the adjacent and hypotenuse, use $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
   - If given the opposite and adjacent, use $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$
3. Divide the side lengths to find the value of the trigonometric ratio.
4. Use the appropriate inverse function key to find the angle θ.

ISBN 978-1-009-25891-3

Exercise 8C

Example 5

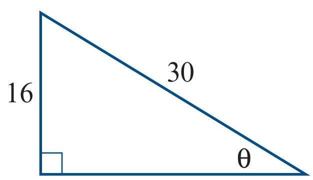
1 Find the angle θ, correct to one decimal place.

- **a** $\sin\theta = 0.4817$
- **b** $\cos\theta = 0.6275$
- **c** $\tan\theta = 0.8666$
- **d** $\sin\theta = 0.5000$
- **e** $\tan\theta = 1.0000$
- **f** $\cos\theta = 0.7071$
- **g** $\sin\theta = 0.8660$
- **h** $\tan\theta = 2.500$
- **i** $\cos\theta = 0.8383$
- **j** $\sin\theta = 0.9564$
- **k** $\cos\theta = 0.9564$
- **l** $\tan\theta = 0.5774$
- **m** $\sin\theta = 0.7071$
- **n** $\tan\theta = 0.5000$
- **o** $\cos\theta = 0.8660$
- **p** $\cos\theta = 0.3414$

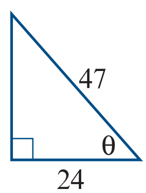
Example 6

2 Find the unknown angle θ in each triangle, correct to one decimal place.

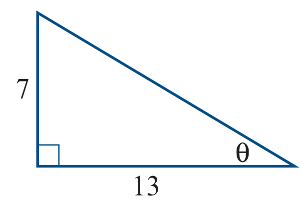
a Use $\sin^{-1}$ for this triangle.



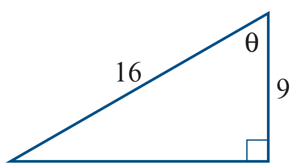
b Use $\cos^{-1}$ for this triangle.



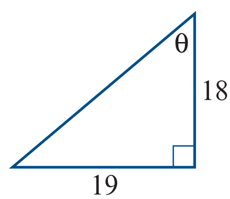
c Use $\tan^{-1}$ for this triangle.



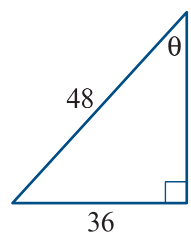
d



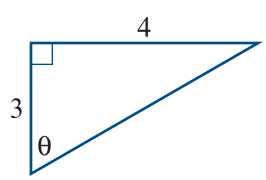
e



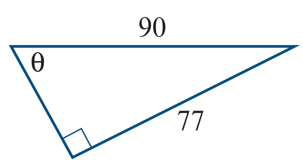
f



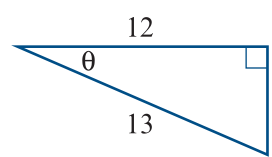
g



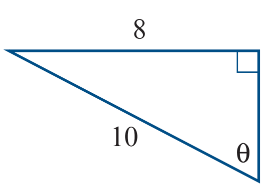
h



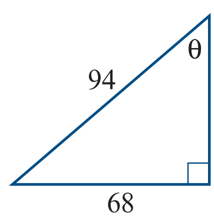
i



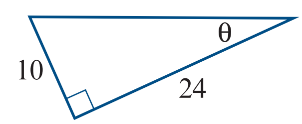
j



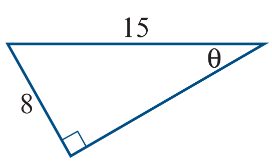
k



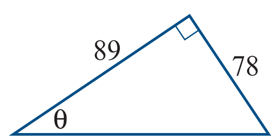
l



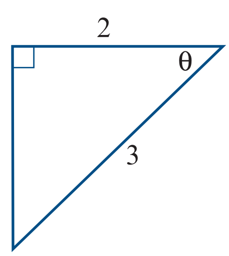
m



n

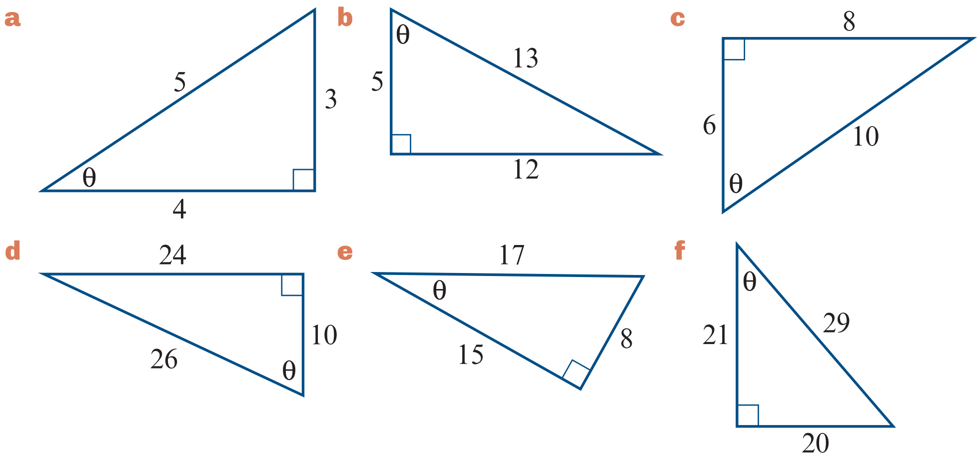


o



ISBN 978-1-009-25891-3

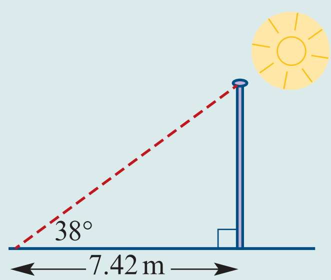
3 Find the value of θ in each triangle, correct to one decimal place.



8D Applications of right-angled triangles

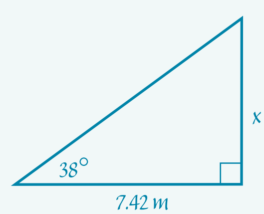
Example 7 Application requiring a length

A flagpole casts a shadow 7.42 m long. The sun's rays make an angle of 38° with the level ground. Find the height of the flagpole, correct to two decimal places.



Solution

1 Draw a diagram showing the right-angled triangle. Include all the known details and label the unknown side.



2 The opposite and adjacent sides are involved so use tan θ.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

3 Substitute in the known values.

$$\tan 38^\circ = \frac{x}{7.42}$$

4 Multiply both sides by 7.42.

$$7.42 \times \tan 38^\circ = x$$

5 Use your calculator to find the value of x.

$$x = 5.797\ldots$$

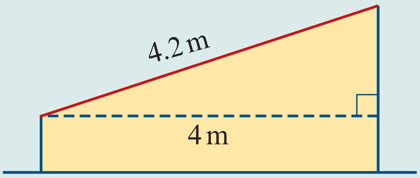
6 Write your answer correct to two decimal places.

The height of the flagpole is 5.80 m.

ISBN 978-1-009-25891-3

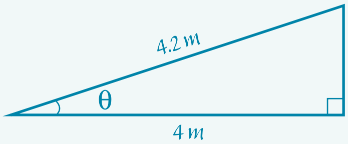
Example 8 Application requiring an angle

A sloping roof uses sheets of corrugated iron 4.2 m long on a shed 4 m wide. There is no overlap of the roof past the sides of the walls. Find the angle the roof makes with the horizontal, correct to one decimal place.



Solution

1 Draw a diagram showing the right-angled triangle. Include all known details and label the required angle.



2 The adjacent and hypotenuse are involved so use cos θ.

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

3 Substitute in the known values.

$$\cos\theta = \frac{4}{4.2}$$

4 Write the equation to find θ.

$$\theta = \cos^{-1}\left(\frac{4}{4.2}\right)$$

$$\theta = 17.752\ldots$$

$\cos^{-1}\left(\frac{4}{4.2}\right)$	17.7528

5 Use your calculator to find the value of θ.

6 Write your answer correct to one decimal place.

The roof makes an angle of 17.8° with the horizontal.

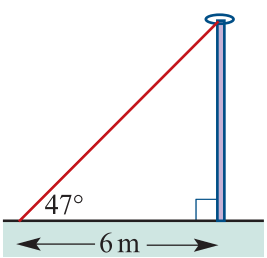
Warning!

Always evaluate a mathematical expression as a whole, rather than breaking it into several smaller calculations. Rounding-off errors accumulate as more approximate answers are fed into the calculations.

Exercise 8D

Example 7

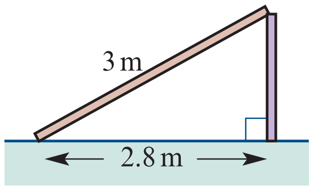
1 A pole is supported by a wire that runs from the top of the pole to a point on the level ground 6 m from the base of the pole. The wire makes an angle of 47° with the ground. Find the height of the pole, correct to two decimal places.



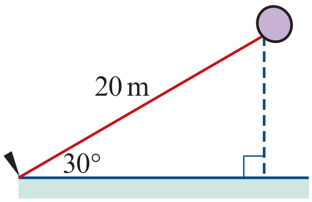
ISBN 978-1-009-25891-3

Example 8

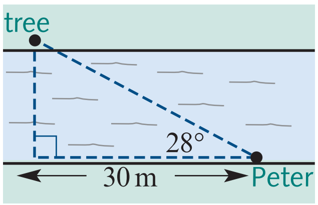
2 A 3 m log rests with one end on the top of a post and the other end on the level ground 2.8 m from the base of the post. Find the angle the log makes with the ground, correct to one decimal place.



3 A balloon is tied to a string 20 m long. The other end of the string is secured by a peg to the surface of a level sports field. The wind blows so that the string forms a straight line making an angle of 30° with the ground. Find the height of the balloon above the ground.



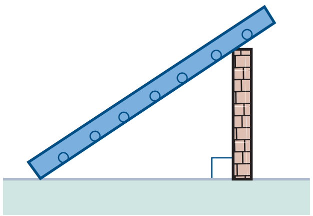
4 Peter noticed that a tree was directly opposite him on the far bank of the river. After he walked 30 m along his side of the river, he found that his line of sight to the tree made an angle of 28° with the riverbank. Find the width of the river, to the nearest metre.



5 A ladder rests on a wall 2 m high. The foot of the ladder is 3 m from the base of the wall on level ground.

a Copy the diagram and include the given information. Label as θ the angle the ladder makes with the ground.

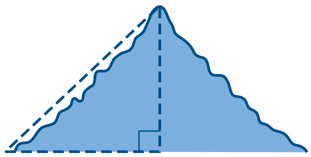
b Find the angle the ladder makes with the ground, correct to one decimal place.



6 The distance measured up the sloping face of a mountain was 3.8 km. The sloping face was at an angle of 52° with the horizontal.

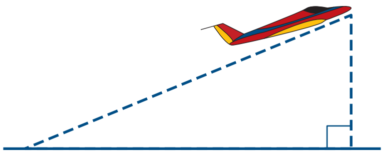
a Make a copy of the diagram and show the known details. Show the height of the mountain as x.

b Find the height of the mountain, correct to one decimal place.



ISBN 978-1-009-25891-3

7 An aeroplane maintains a flight path of 17° with the horizontal after it takes off. It travels for 2 km along that flight path.



a Show the given and required information on a copy of the diagram.

b Find, correct to two decimal places:

i the horizontal distance of the aeroplane from its take-off point

ii the height of the aeroplane above ground level.

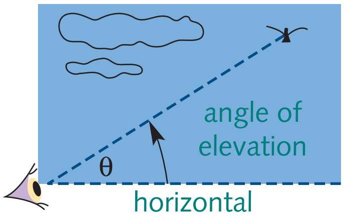
8 A 3 m ladder rests against an internal wall. The foot of the ladder is 1 m from the base of the wall. Find the angle the ladder makes with the floor, correct to one decimal place.

9 The entrance to a horizontal mining tunnel has collapsed, trapping the miners inside. The rescue team decide to drill a vertical escape shaft from a position 200 m further up the hill. If the hill slopes at 23° from the horizontal, how deep does the rescue shaft need to be to meet the horizontal tunnel? Answer correct to one decimal place.

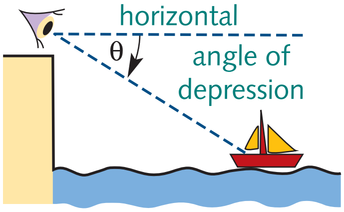
10 A strong rope needs to be fixed with one end attached to the top of a 5 m pole and the other end pegged at an angle of 60° with the level ground. Find the required length of the rope, correct to two decimal places.

8E Angles of elevation and depression

The **angle of elevation** is the angle through which you *raise* your line of sight from the horizontal when you are looking *up* at something.



The **angle of depression** is the angle through which you *lower* your line of sight from the horizontal when you are looking *down* at something.

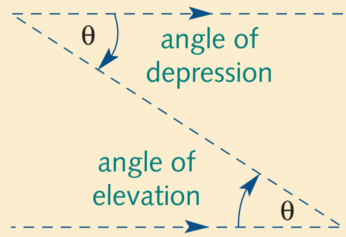


ISBN 978-1-009-25891-3

Angles of elevation and depression

angle of elevation = angle of depression

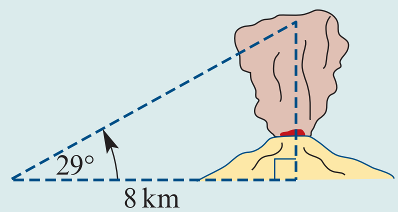
The diagram shows that the angle of elevation and the angle of depression are alternate angles ('Z' angles), so they are equal.



Applications of angles of elevation and depression

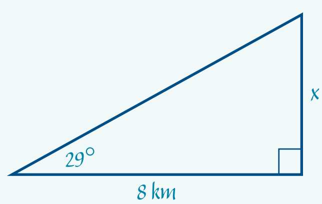
Example 9 Angle of elevation

A park ranger measured the top of a plume of volcanic ash to be at an angle of elevation of 29°. From her map she noted that the volcano was 8 km away. She calculated that the height of the plume to be 4.4 km. Show how she might have done this. Give your answer correct to one decimal place.



Solution

1 Draw a right-angled triangle showing the given information. Label the required height x.



2 The opposite and adjacent sides are involved so use tan θ.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

3 Substitute in the known values.

$$\tan 29^\circ = \frac{x}{8}$$

4 Multiply both sides by 8.

$$8 \times \tan 29^\circ = x$$

5 Use your calculator to find the value of x.

$$x = 4.434\ldots$$

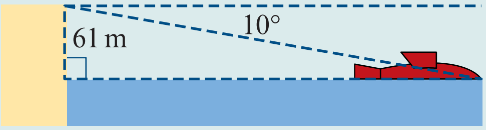
6 Write your answer correct to one decimal place.

The height of the ash plume was 4.4 km.

ISBN 978-1-009-25891-3

Example 10 Angle of depression

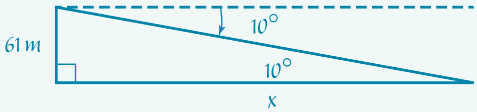
From the top of a cliff 61 m above sea level, Chen saw a capsized yacht. He estimated the angle of depression to be about 10°. How far was the yacht from the base of the cliff, to the nearest metre?



Solution

1. Draw a diagram showing the given information. Label the required distance x.
2. Mark in the angle at the yacht corner of the triangle. This is also 10°, because it and the angle of depression are alternate (or 'Z') angles.

   Warning: The angle between the cliff face and the line of sight is *not* 10°.



3. The opposite and adjacent sides are involved so use tan θ.
   $$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
4. Substitute in the known values.
   $$\tan 10^\circ = \frac{61}{x}$$
5. Multiply both sides by x.
   $$x \times \tan 10^\circ = 61$$
6. Divide both sides by tan 10°.
   $$x = \frac{61}{\tan 10^\circ}$$
7. Do the division using your calculator.
   $$x = 345.948\ldots$$
8. Write your answer to the nearest metre.
   The yacht was 346 m from the base of the cliff.

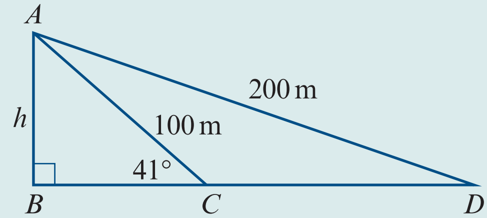
ISBN 978-1-009-25891-3

Example 11 Application with two right-angled triangles

A cable 100 m long makes an angle of elevation of 41° with the top of a tower.

a Find the height, h, of the tower, to the nearest metre.

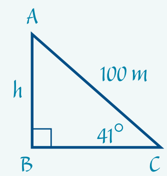
b Find the angle of elevation, α, to the nearest degree, that a cable 200 m long would make with the top of the tower.



Solution

Strategy: Find h in triangle ABC, then use this value to find α in triangle ABD.

a **1** Draw triangle ABC showing the given and required information.



2 The opposite and hypotenuse are involved, so use sin θ.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

3 Substitute in the known values.

$$\sin 41^\circ = \frac{h}{100}$$

4 Multiply both sides by 100.

$$h = 100 \times \sin 41^\circ$$
$$h = 65.605\ldots$$

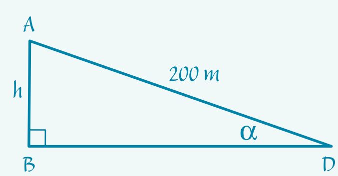
5 Evaluate 100 sin(41°) using your calculator and store the answer as the value of the variable h for later use.

$100 \cdot \sin(41^\circ) \rightarrow h$	65.6059

6 Write your answer to the nearest metre.

The height of the tower is 66 m.

b **1** Draw triangle ABD showing the given and required information



2 The opposite and hypotenuse are involved, so use sin α.

$$\sin\alpha = \frac{\text{opposite}}{\text{hypotenuse}}$$

3 Substitute in the known values. In part **a** we stored the height of the tower as h.

$$\sin\alpha = \frac{h}{200}$$
$$\alpha = \sin^{-1}\left(\frac{h}{200}\right)$$

4 Write the equation to find α.

$$\alpha = 19.149\ldots$$

ISBN 978-1-009-25891-3

5 Use your calculate to evaluate α.

$100 \cdot \sin(41°) \to h$	65.6059
$\sin^{-1}\left(\dfrac{h}{200}\right)$	19.1492

6 Write your answer to the nearest degree.

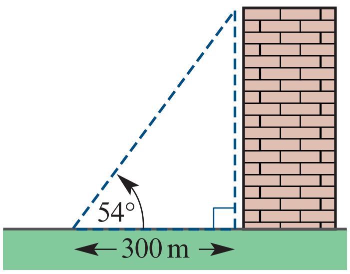
The 200 m cable would have an angle of elevation of 19°.

Skill-sheet

Exercise 8E

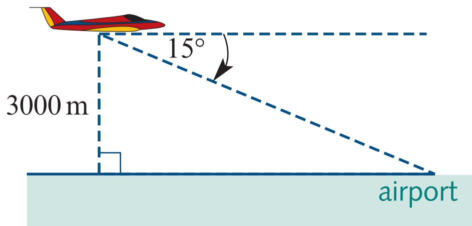
Example 9

1 After walking 300 m away from the base of a tall building, on level ground, Elise measured the angle of elevation to the top of the building to be 54°. Find the height of the building, to the nearest metre.



Example 10

2 The pilot of an aeroplane saw an airport at sea level at an angle of depression of 15°. His altimeter showed that the aeroplane was at a height of 3000 m. Find the horizontal distance of the aeroplane from the airport, to the nearest metre.



3 The angle of elevation measured from ground level to the top of a tall tree was 41°. The distance of the measurer from the base of the tree was 38 m. How tall was the tree? Give your answer to the nearest metre.

4 When Darcy looked from the top of a cliff, 60 m high, he noticed his girlfriend at an angle of depression of 20° on the ground below. How far was she from the cliff? Answer correct to one decimal place.

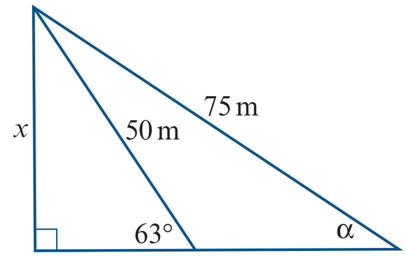
5 From the top of a mountain, I could see a town at an angle of depression of 1.4° across the level plain. Looking at my map I found that the town was 10 km away. Find the height of the mountain above the plain, to the nearest metre.

6 What would be the angle of elevation to the top of a radio transmitting tower 100 m tall and 400 m from the observer? Answer to the nearest degree.

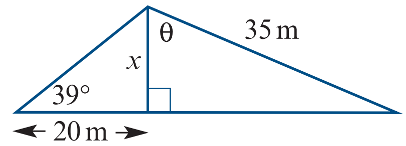
ISBN 978-1-009-25891-3

Example 11

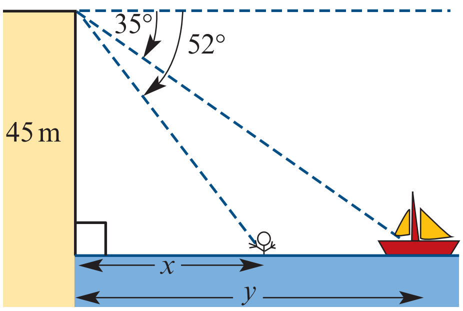
7 **a** Find the length x, correct to one decimal place.
b Find the angle α, to the nearest degree.



8 **a** Find the length x, correct to one decimal place.
b Find the angle θ, to the nearest degree.



9 From the top of a cliff 45 m high, an observer looking along an angle of depression of 52° could see a man swimming in the sea. The observer could also see a boat at an angle of depression of 35°. Calculate, to the nearest metre:



a the distance x of the man from the base of the cliff
b the distance y of the boat from the base of the cliff
c the distance from the man to the boat.

10 A police helicopter hovering in a fixed position at an altitude of 500 m moved its spotlight through an angle of depression of 57° onto a lost child. The pilot sighted the rescue team at an angle of depression of 31°. If the terrain was level, how far, to the nearest metre, was the rescue team from the child?

8F Bearings and navigation

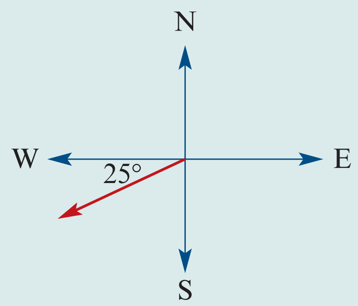
True or three-figure bearings

A **true bearing** is the angle measured clockwise from north around to the required direction. True bearings are also called **three-figure bearings** because they are written using three numbers or figures. For example, 090° is the direction measured 90° clockwise from north, better known as east!
In this section the old style compass bearings such as S 20°E are not used.

ISBN 978-1-009-25891-3

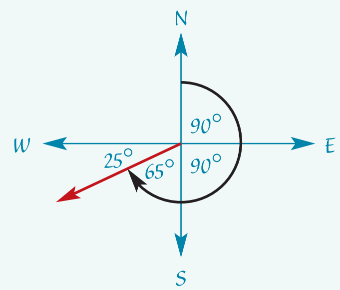
Example 12 Determining three-figure bearings

Give the three-figure bearing for the direction shown.



Solution

1 Calculate the total angles swept out clockwise from north.
There is an angle of 90° between each of the four points of the compass.



2 Write your answer.

The angle from north = 90° + 90° + 65 = 245°
or 270° − 25° = 245°
The three-figure bearing is 245°.

Navigation problems

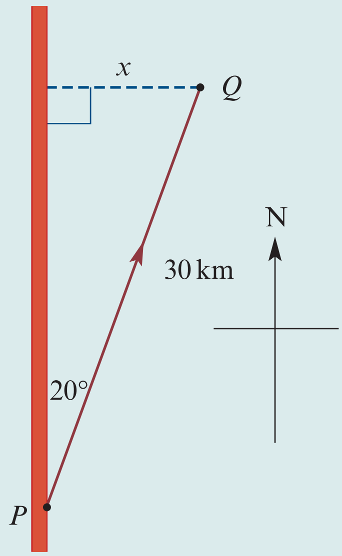
Navigation problems usually involve a consideration of not only the *direction* of travel, given as a bearing, but also the *distance* travelled.

Example 13 Navigating using a three-figure bearing

A group of bushwalkers leave point P, which is on a road that runs north–south, and walk for 30 km on a bearing 020° to reach point Q.

a What is the shortest distance x from Q back to the road, correct to one decimal place?

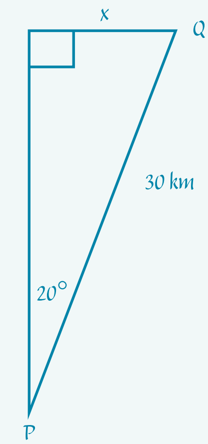
b Looking from point Q, what would be the three-figure bearing of their starting point?



ISBN 978-1-009-25891-3

Solution

a **1** Show the given and required information in a right-angled triangle.
The question asks for the value of x.



2 The opposite and hypotenuse are involved, so use $\sin\theta$.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

3 Substitute in the known values.

$$\sin 20° = \frac{x}{30}$$

4 Multiply both sides by 30.

$$30 \times \sin 20° = x$$

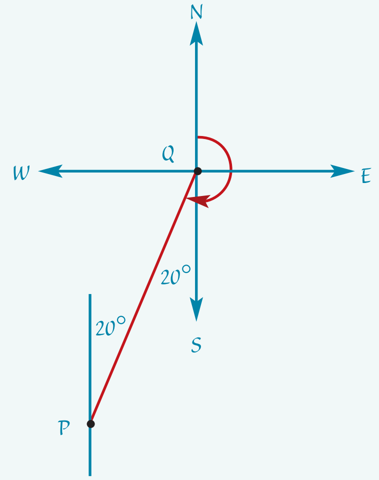
5 Find the value of x using your calculator.

$$x = 10.260\ldots$$

6 Write your answer correct to one decimal place.

The shortest distance to the road is 10.3 km.

b **1** The question asks for the bearing of P from Q.
Draw the compass points at Q.



2 Enter the alternate angle 20°.

3 Standing at Q, add all the angles when facing north and then turning clockwise to look at P. This gives the three-figure bearing of P when looking from Q.

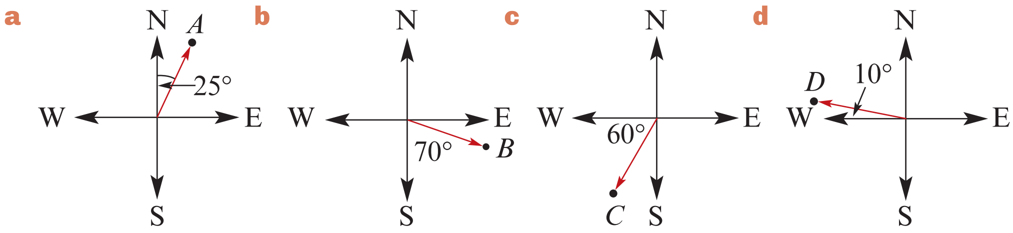
The angle from north is $180° + 20° = 200°$
The three-figure bearing is 200°.

ISBN 978-1-009-25891-3

Exercise 8F

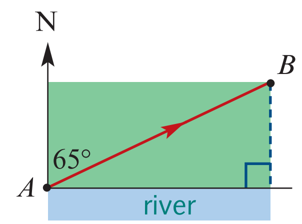
Example 12

1 State the three-figure bearing of each of the points A, B, C and D.



Example 13

2 Eddie camped overnight at point A beside a river that ran east–west. He walked on a bearing of 065° for 18 km to point B.



a What angle did his direction make with the river?

b What is the shortest distance from B to the river, correct to two decimal places?

3 A ship sailed 3 km west, then 2 km south.

a Give its three-figure bearings from an observer who stayed at its starting point, correct to the nearest degree.

b For a person on the ship, what would be the three-figure bearings looking back to the starting point?

4 An aeroplane flew 500 km south, then 600 km east. Give its three-figure bearing from its starting point, to the nearest degree.

5 A ship left port and sailed east for 5 km, then sailed north. After some time an observer at the port could see the ship on a bearing of 050°.

a How far north had the ship travelled? Answer correct to one decimal place.

b Looking from the ship, what would be the three-figure bearing of the port?

ISBN 978-1-009-25891-3

6 A woman walked from point A for 10 km on a bearing of 060° to reach point B. Then she walked for 15 km heading south until she was at point D. Give the following distances correct to one decimal place and directions to the nearest degree.

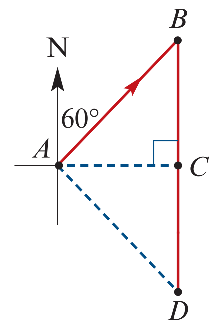
a Find the distances walked from A to B and from B to D.

b How far south did she walk from B to C?

c Find the distance from A to C.

d What is the distance from C to D?

e Find the three-figure bearing and distance she would need to walk to return to her starting point.



7 A ship left port P and sailed 20 km on a bearing of 230°. It then sailed north for 30 km to reach point C. Give the following distances correct to one decimal place and directions to the nearest degree.

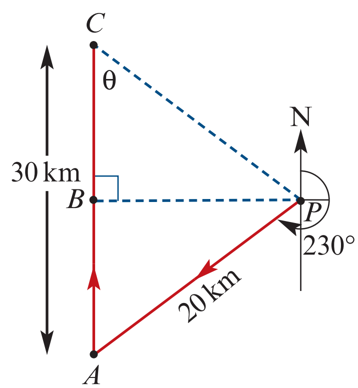
a Find the distance AB.

b Find the distance BP.

c Find the distance BC.

d Find the angle θ at point C.

e State the three-figure bearing and distance of the port P from the ship at C.



ISBN 978-1-009-25891-3

Review

Key ideas and chapter summary

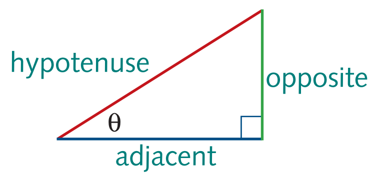
Assignment

Naming the sides of a right-angled triangle

The **hypotenuse** is the longest side and is always opposite the right angle (90°).

The *opposite* side is directly opposite the angle θ (the angle being considered).

The *adjacent* side is beside angle θ and runs from θ to the right angle.



Trigonometric ratios

The **trigonometric ratios** are sin θ, cos θ and tan θ:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

SOH-CAH-TOA

This helps you to remember the trigonometric ratio rules.

Degree mode

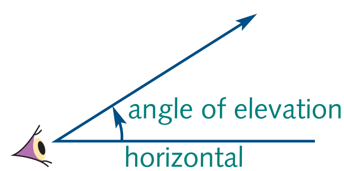
Make sure your calculator is in DEGREE mode when doing calculations with trigonometric ratios.

Applications of right-angled triangles

Always draw well-labelled diagrams showing all known sides and angles. Also label any sides or angles that need to be found.

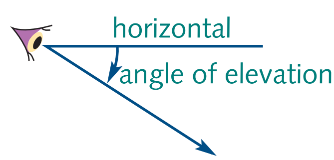
Angle of elevation

The **angle of elevation** is the angle through which you *raise* your line of sight from the horizontal, looking *up* at something.



Angle of depression

The **angle of depression** is the angle through which you *lower* your line of sight from the horizontal, looking *down* at something.

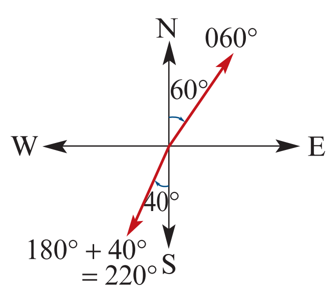


Angle of elevation = angle of depression

The angles of elevation and depression are alternate (‘Z’) angles so they are equal.

Three-figure bearings

Three-figure bearings are measured clockwise from north and always given with three digits, e.g. 060°, 220°.



ISBN 978-1-009-25891-3

Review

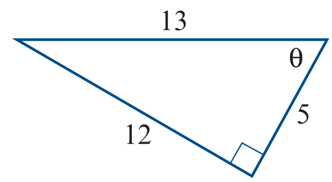
Skills check

Having completed this chapter you should be able to:

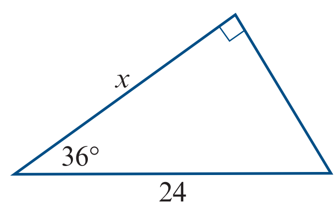
- use trigonometric ratios to find an unknown side or angle in a right-angled triangle
- show the angle of elevation or angle of depression on a well-labelled diagram
- show directions on a diagram by using three-figure bearings
- solve practical problems involving right-angled triangles.

Short-answer questions

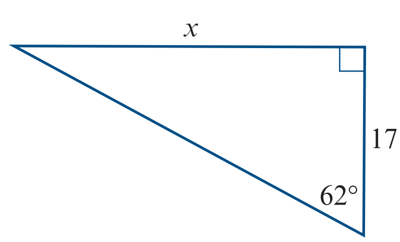
1 In the triangle shown, state the value of $\sin\theta$.



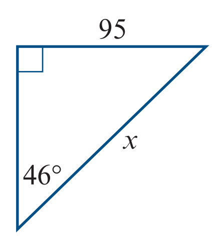
2 Write an expression for the length of side x.



3 Write an expression that can be used to solve for x.

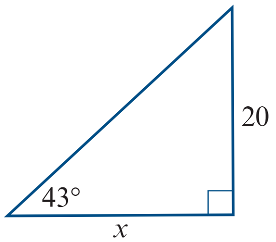


4 Write an expression that can be used to solve for x.

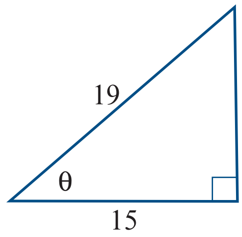


ISBN 978-1-009-25891-3

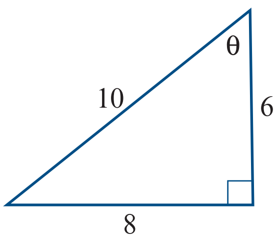
5 Write an expression that can be used to solve for x.



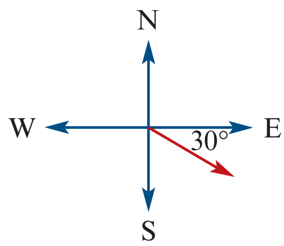
6 Write an expression that can be used to solve for θ.



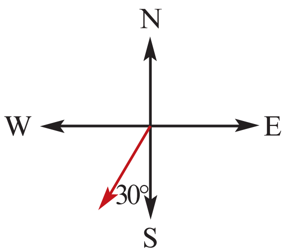
7 Calculate the value of θ, correct to one decimal place.



8 In the given diagram state the true bearing.



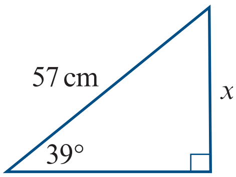
9 In the given diagram state the true bearing.



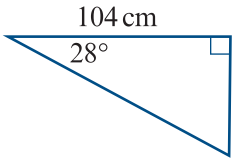
ISBN 978-1-009-25891-3

Review

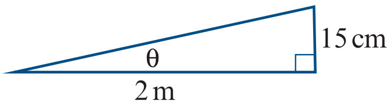
10 Find the length of x, correct to two decimal places.



11 Find the length of the hypotenuse, correct to two decimal places.

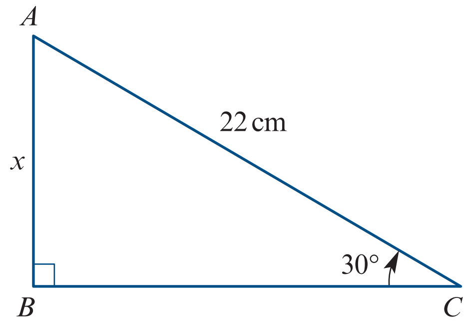


12 A road rises 15 cm for every 2 m travelled horizontally.
Find the angle of slope θ, to the nearest degree.

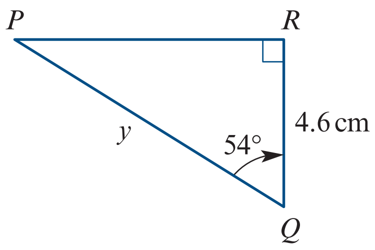


13 **a** Find the sides of a right-angled triangle for which $\cos\theta = \frac{72}{97}$ and $\tan\theta = \frac{65}{72}$.
b Hence find $\sin\theta$.

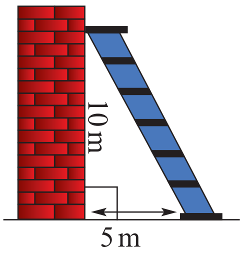
14 Find the value of x.



15 Find the value of y.



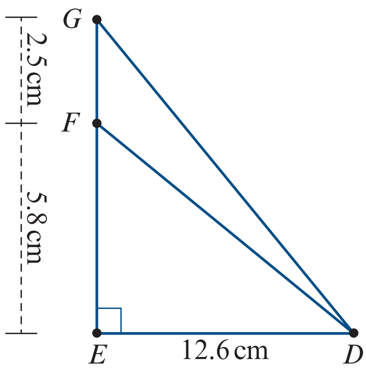
16 Find the angle that the ladder makes with the wall.



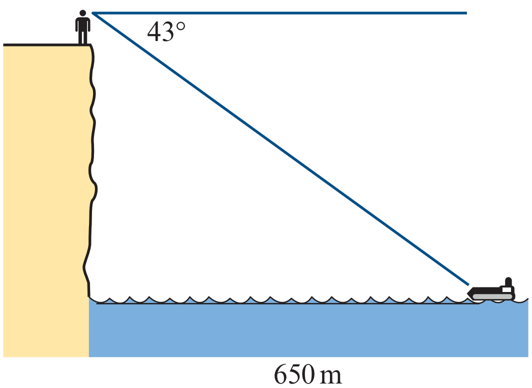
ISBN 978-1-009-25891-3

Review

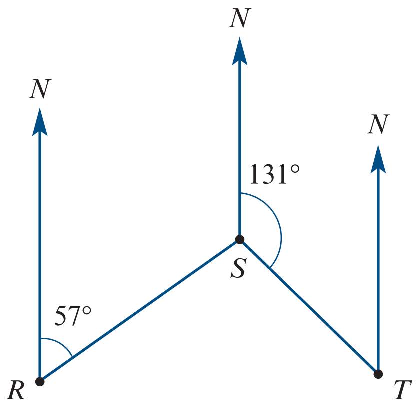
17 Determine the size of angle FDG.



18 Tony standing at the top of a cliff sights a ship 650 m at sea as shown. The angle of depression of the ship from Toni is 43°. If Tony is 1.82 m, find the height of the cliff, to the nearest metre.



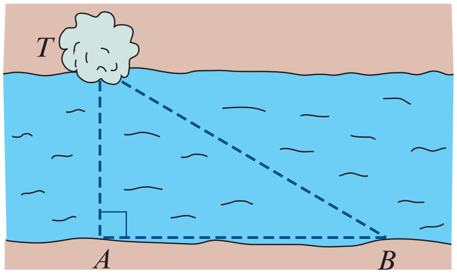
19 The diagram shows the positions of three towns R, S and T. Find the bearing of R from S.



ISBN 978-1-009-25891-3

Extended-response questions

1 Tim was standing at point A when he saw a tree T directly opposite him on the far bank of the river. He walked 100 m along the riverbank to point B and noticed that his line of sight to the tree made an angle of 27° with the riverbank. Answer the following correct to two decimal places.



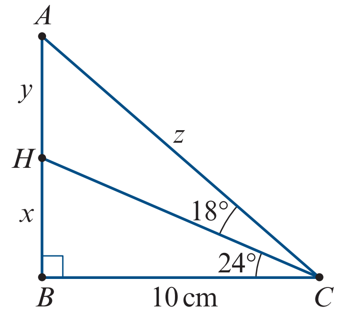
a How wide was the river?

b What is the distance from point B to the tree?

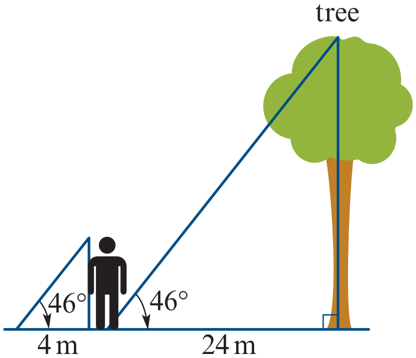
Standing at B, Tim measured the angle of elevation to the top of the tree to be 18°.

c Make a clearly labelled diagram showing distance TB, the height of the tree and the angle of elevation, then find the height of the tree.

2 Find the value of each variable, correct to two decimal places where applicable.



3 The angle of elevation of the top of a tree from a statue's feet is 46° as shown. If the tree casts a shadow of 24 m,



a Determine the height of the tree.

b If a statue casts a shadow of 4 m, find the statue's height, assuming the two triangles are similar.

4 A ship sets sail from a harbour A and travels 30 km to a harbour B on a bearing of 060°. It stops for a few hours, then sails to a harbour C on a bearing of 150°, travelling a distance of 40 km.

a Draw a diagram to illustrate the above situation.

b Hence calculate the distance A to C.

c On what bearing must the ship set sail to go back to harbour A?

ISBN 978-1-009-25891-3

9 Non-right-angled trigonometry

In this chapter

Syllabus references

Topic: Applications of trigonometry

Subtopics: 2.2.2, 2.2.3

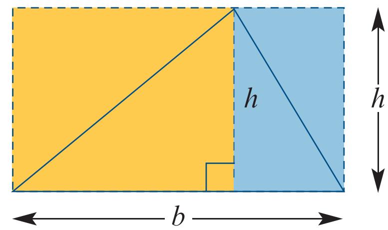
This chapter takes trigonometry to a new level by dealing with non-right-angled triangles. It focuses on three fundamental rules: the area of a triangle, the sine rule, and the cosine rule. All these entail connecting four pieces of information about a particular triangle. Three of these pieces of information will be known, and we will be able to figure out the fourth using certain equations. Trigonometry has direct applications to solving real problems and can be applied in a variety of activities that we love. For example, in music, sound travels in waves, and while this pattern is not as regular as a sine or cosine function, it is nevertheless valuable in generating computer music. Trigonometry is commonly utilised in building and aviation as well.

ISBN 978-1-009-25891-3

9A The area of a triangle

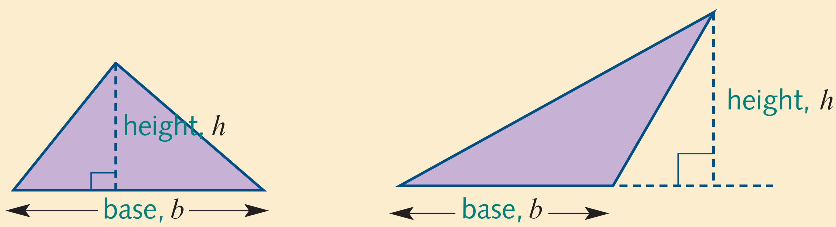
Area of a triangle = $\frac{1}{2}$ × base × height

From the diagram, we see that the area of a triangle with a base b and height h is equal to half the area of the rectangle $b \times h$ that it fits within.



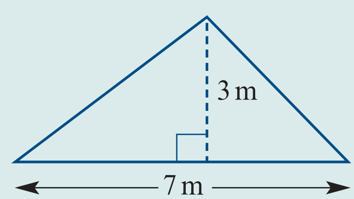
Area of a triangle

$$\text{Area of a triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$
$$= \frac{1}{2} \times b \times h$$



Example 1 Finding the area of a triangle using $\frac{1}{2}$ × base × height

Find the area of the triangle shown, correct to one decimal place.



Solution

1 As we are given values for the base and height of the triangle, use Area = $\frac{1}{2}$ × base × height	Base, $b = 7$ Height, $h = 3$ Area of triangle $= \frac{1}{2} \times b \times h$
2 Substitute the given values.	$= \frac{1}{2} \times 7 \times 3$
3 Evaluate.	$= 10.5\ \text{m}^2$
4 Write your answer.	The area of the triangle is $10.5\ \text{m}^2$.

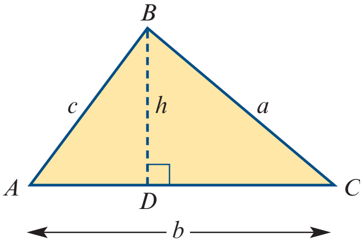
ISBN 978-1-009-25891-3

Area of a triangle = $\frac{1}{2}ab \sin C$

In triangle ABD,

$$\sin C = \frac{h}{a}$$

$$h = a \sin C$$



So we can replace h with $a \sin C$ in the rule:

$$\text{Area of a triangle} = \frac{1}{2} \times b \times h$$

$$\text{Area of a triangle} = \frac{1}{2} \times b \times a \times \sin C$$

Similarly, using side c or a for the base, we can make a complete set of three rules:

Area of a triangle

$$\text{Area of a triangle} = \frac{1}{2} bc \sin A$$

$$\text{Area of a triangle} = \frac{1}{2} ac \sin B$$

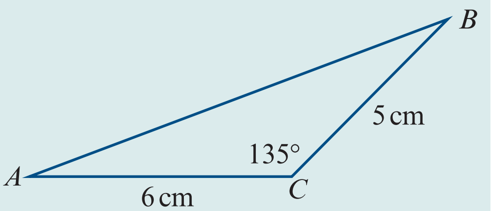
$$\text{Area of a triangle} = \frac{1}{2} ab \sin C$$

Notice that each version of the rule follows the pattern:

$$\text{Area of a triangle} = \frac{1}{2} \times (\text{product of two sides}) \times \sin(\text{angle between those two sides})$$

Example 2 Finding the area of a triangle using $\frac{1}{2}ab \sin C$

Find the area of the triangle shown, correct to one decimal place.



Solution

1 We are given two sides a, b and the angle C between them, so use:

Area of a triangle = $\frac{1}{2} ab \sin C$

$b = 5, c = 6, A = 135°$

Area of triangle $= \frac{1}{2} ab \sin C$

2 Substitute values for a, b and $\sin C$ into the rule.

$= \frac{1}{2} \times 5 \times 6 \times \sin 135°$

3 Use your calculator to find the area.

$= 10.606...$

4 Write your answer correct to one decimal place.

The area of the triangle is 10.6 cm^2.

ISBN 978-1-009-25891-3

Heron's rule for the area of a triangle

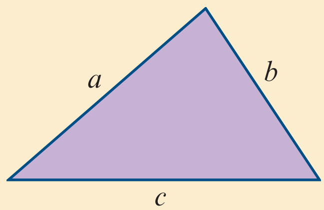
Heron's rule can be used to find the area of any triangle when we know the lengths of the three sides.

Heron's rule for the area of a triangle

Area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

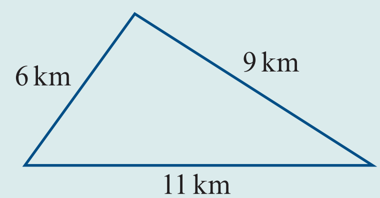
where $s = \frac{1}{2}(a+b+c)$

The variable s is called the *semi-perimeter* because it is equal to half the sum of the sides.



Example 3 **Finding the area of a triangle using Heron's formula**

The boundary fences of a farm are shown in the diagram. Find the area of the farm, to the nearest square kilometre.



Solution

1 As we are given the three sides of the triangle, use Heron's rule. Start by finding s, the semi-perimeter.

Let $a = 6, b = 9, c = 11$

$$s = \frac{1}{2}(a+b+c)$$
$$= \frac{1}{2}(6+9+11) = 13$$

2 Write Heron's rule.

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

3 Substitute the values of s, a, b and c into Heron's rule.

$$= \sqrt{13(13-6)(13-9)(13-11)}$$
$$= \sqrt{13 \times 7 \times 4 \times 2}$$

4 Use your calculator to find the area.

$$= 26.981\ldots$$

5 Write your answer.

The area of the farm, to the nearest square kilometre, is 27 km^2.

ISBN 978-1-009-25891-3

Exercise 9A

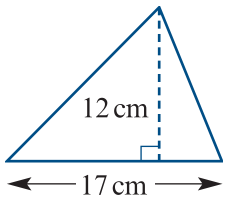
In this exercise, calculate areas correct to one decimal place where necessary.

Finding areas using $\frac{1}{2} \times \text{base} \times \text{height}$

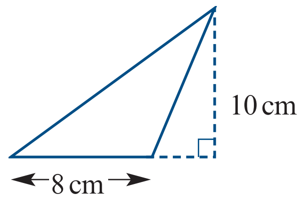
Example 1

1 Find the area of each triangle.

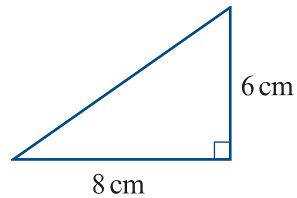
a



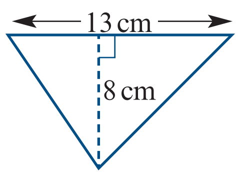
b



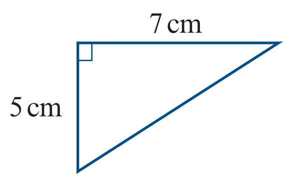
c



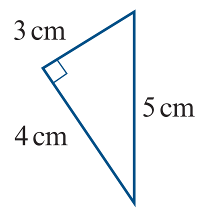
d



e



f

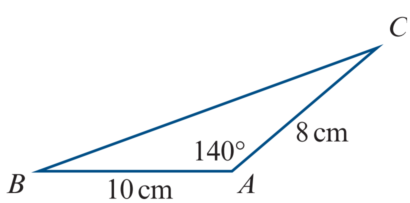


Finding areas using $\frac{1}{2}ab \sin C$

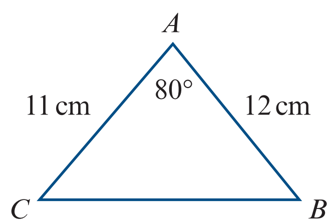
Example 2

2 Find the areas of the triangles shown.

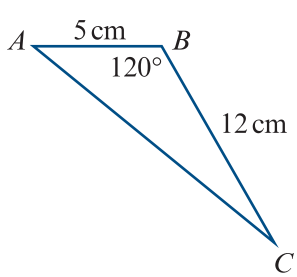
a



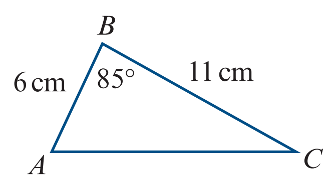
b



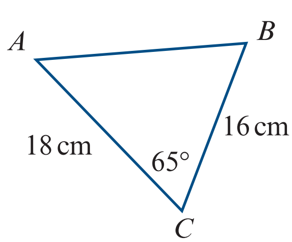
c



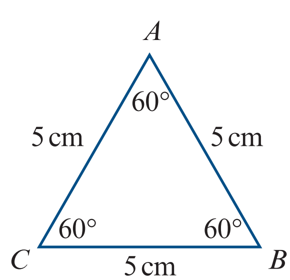
d



e



f



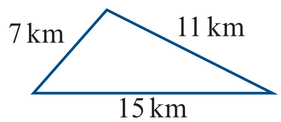
ISBN 978-1-009-25891-3

Finding areas using Heron's formula

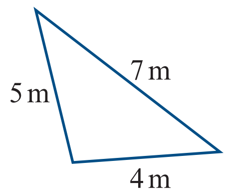
Example 3

3 Find the area of each triangle.

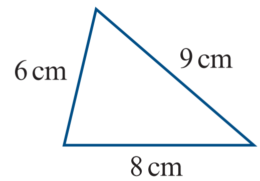
a



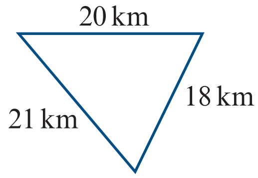
b



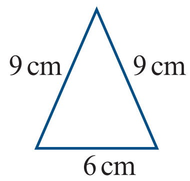
c



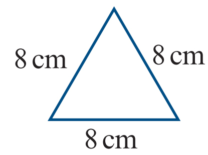
d



e



f



Mixed problems

4 For each triangle below choose the rule for finding its area from:

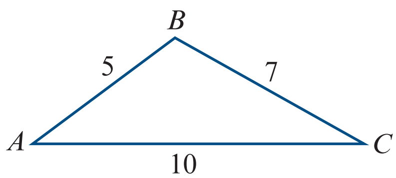
i $\frac{1}{2}$ *base* × *height*

ii $\frac{1}{2}bc\sin A$

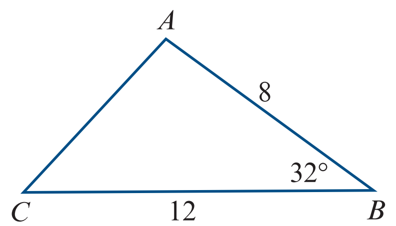
iii $\frac{1}{2}ac\sin B$

iv $\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$

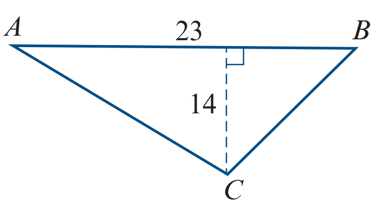
a



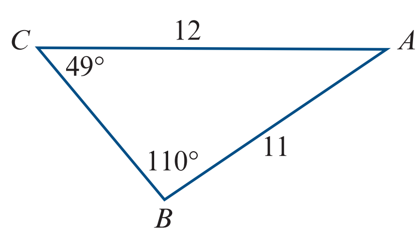
b



c

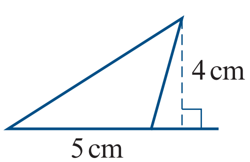


d

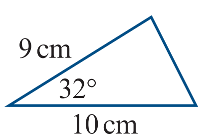


5 Find the area of each triangle shown.

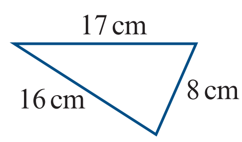
a



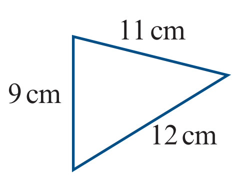
b



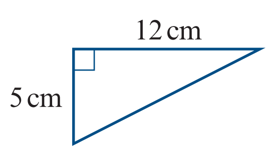
c



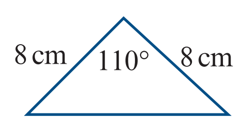
d



e

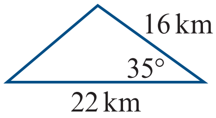


f

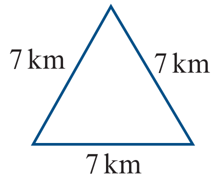


ISBN 978-1-009-25891-3

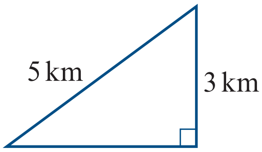
g



h



i



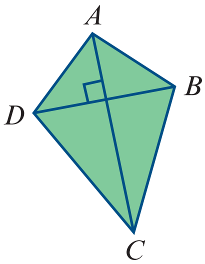
6 Find the area of a triangle with a base of 28 cm and a height of 16 cm.

7 Find the area of triangle ABC with side a 42 cm, side b 57 cm and angle C 70°.

8 Find the area of a triangle with sides of 16 km, 19 km and 23 km.

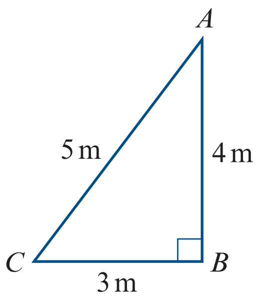
Applications

9 The kite shown is made using two sticks, AC and DB. The length of AC is 100 cm and the length of DB is 70 cm. Find the area of the kite.

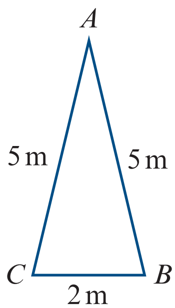


10 Three students A, B and C stretched a rope loop 12 m long into different shapes. Find the area of each shape.

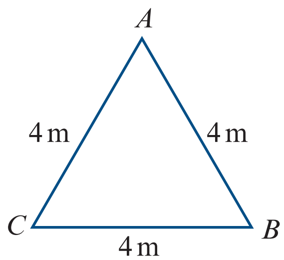
a



b



c



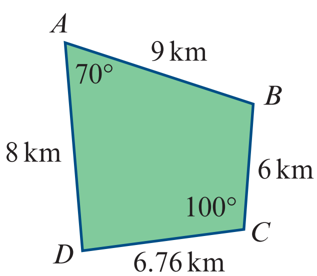
11 A farmer needs to know the area of his property with the boundary fences as shown. The measurements are correct to two decimal places. Write all answers correct to two decimal places.

Hint: Draw a line from B to D to divide the property into two triangles.

a Find the area of triangle ABD.

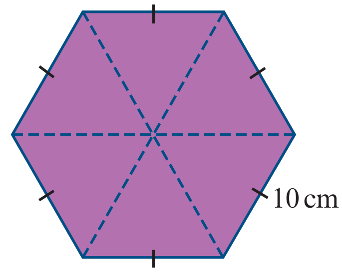
b Find the area of triangle BCD.

c State the total area of the property.



ISBN 978-1-009-25891-3

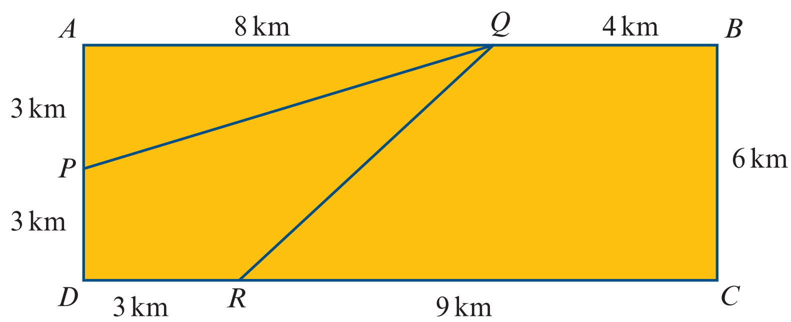
12 A regular hexagon with sides 10 cm long can be divided into six smaller equilateral triangles. (Remember, an equilateral triangle has all sides of equal length.)



a Find the area of each triangle.

b What is the area of the hexagon?

13 A large rectangular area of land, *ABCD* in the diagram, has been subdivided into three regions as shown.



a Find the area of:

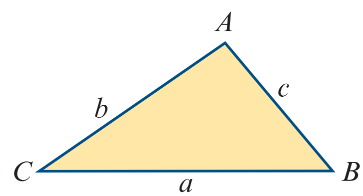
i region *PAQ* **ii** region *QBCR* **iii** region *PQRD*.

b Find the size of angle *PQR*, correct to one decimal place.

9B The sine rule

Standard triangle notation

The convention for labelling a non-right-angled triangle is to use the upper case letters A, B, and C for the angles at each corner. The sides are named using lower case letters so that side a is opposite angle A, and so on.

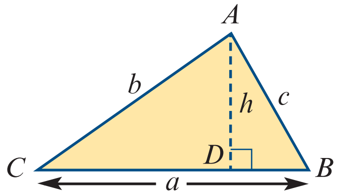


This notation is used for the sine rule and cosine rule (see Section 9C). Both rules can be used to find angles and sides in triangles that do not have a right angle.

ISBN 978-1-009-25891-3

How to derive the sine rule

In triangle ABC, show the height h of the triangle by drawing a perpendicular line from D on the base of the triangle to A.



In triangle ADC, $\sin C = \dfrac{h}{b}$

So $h = b \times \sin C$

In triangle ABD, $\sin B = \dfrac{h}{c}$

So $h = c \times \sin B$

We can make the two rules for h equal to each other. $b \times \sin C = c \times \sin B$

Divide both sides by $\sin C$. $b = \dfrac{c \times \sin B}{\sin C}$

Divide both sides by $\sin B$. $\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

If the triangle was redrawn with side c as the base, then using similar steps we get:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

We can combine the two rules as shown in the following box.

The sine rule

In any triangle ABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The **sine rule** is used to find the sides and angles in a non-right-angled triangle when given:

- two sides and an angle opposite one of the given sides
- two angles and one side.

Note: If neither of the given angles is opposite the given side, find the third angle using $A + B + C = 180°$.

The sine rule can take the form of any of these three possible equations:

$$\frac{a}{\sin A} = \frac{b}{\sin B} \qquad \frac{b}{\sin B} = \frac{c}{\sin C} \qquad \frac{a}{\sin A} = \frac{c}{\sin C}$$

Each equation has two sides and two angles opposite those sides. If we know three of the parts, we can find the fourth.

So if we know two angles and a side opposite one of the angles, we can find the side opposite the other angle. Similarly, if we know two sides and an angle opposite one of those sides, we can find the angle opposite the other side.

ISBN 978-1-009-25891-3

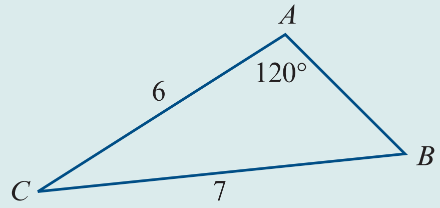
Although we have expressed the sine rule using a triangle ABC, for any triangle, such as PQR, the pattern of fractions consisting of 'side / sine of angle' pairs would appear as:

$$\frac{p}{\sin P} = \frac{q}{\sin Q} \qquad \frac{q}{\sin Q} = \frac{r}{\sin R} \qquad \frac{p}{\sin P} = \frac{r}{\sin R}$$

Using the sine rule

Example 4 **Using the sine rule given two sides and an opposite angle**

Find angle B in the triangle shown, correct to one decimal place.



Solution

1 We have the pairs $a = 7$ and $A = 120°$
$b = 6$ and $B = ?$
with only B unknown.
So use $\frac{a}{\sin A} = \frac{b}{\sin B}$.
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

2 Substitute in the known values.
$$\frac{7}{\sin 120°} = \frac{6}{\sin B}$$

3 Cross-multiply.
$$7 \times \sin B = 6 \times \sin 120°$$

4 Divide both sides by 7.
$$\sin B = \frac{6 \times \sin 120°}{7}$$

5 Write the equation to find angle B.
$$B = \sin^{-1}\left(\frac{6 \times \sin 120°}{7}\right)$$

6 Use your calculator to evaluate the expression for B.
$$B = 47.928\ldots°$$

7 Write your answer correct to one decimal place.
Angle B is $47.9°$.

In Example 4, now that we know that $A = 120°$ and $B = 47.9°$, we can use the fact that the angles in a triangle add to 180° to find C.

$$\begin{aligned} A + B + C &= 180° \\ 120° + 47.9° + C &= 180° \\ 167.9° + C &= 180° \\ C &= 180° - 167.9° = 12.1° \end{aligned}$$

ISBN 978-1-009-25891-3

As we now know that $A = 120°$, $a = 7$ and $C = 12.1°$, we can find side c using

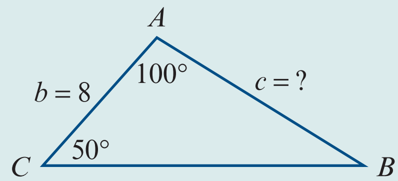
$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

The steps are similar to those in the example.

Finding all the angles and sides of a triangle is called solving the triangle.

Example 5 Using the sine rule given two angles and one side

Find side c in the triangle shown, correct to one decimal place.



Solution

1 Find the angle opposite the given side by using $A + B + C = 180°$.

$$A + B + C = 180°$$
$$100° + B + 50° = 180°$$
$$B + 150° = 180°$$
$$B = 30°$$

2 We have the pairs $b = 8$ and $B = 30°$, $c = ?$, $C = 50°$ with only c unknown. So use $\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

3 Substitute in the known values.

$$\frac{8}{\sin 30°} = \frac{c}{\sin 50°}$$

4 Multiply both sides by $\sin 50°$.

$$c = \frac{8 \times \sin 50°}{\sin 30°}$$

5 Use your calculator to find c.

$$c = 12.256\ldots$$

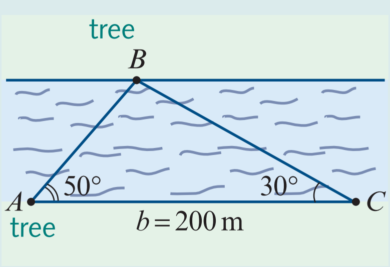
6 Write your answer correct to one decimal place.

Side c is 12.3 units long.

Example 6 Application of the sine rule

Leo wants to tie a rope from a tree at point A to a tree at point B on the other side of the river. He needs to know the length of rope required.

When he stood at A, he saw the tree at B at an angle of 50° with the river-bank. After walking 200 metres east to C, the tree was seen at an angle of 30° with the bank.



Find the length of rope required to reach from A to B, correct to two decimal places.

ISBN 978-1-009-25891-3

Solution

1 To use the sine rule, we need two angle-side pairs with only one item unknown.

So one part of the sine rule equation will be:

$$\frac{c}{\sin C}$$

The unknown is the length of the rope, side c. Angle $C = 30°$ is given.

2 We know side $b = 200$ and need to find angle B to use the sine rule equation:

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

3 Use $A + B + C = 180°$ to find angle B.

$$A + B + C = 180°$$
$$50° + B + 30° = 180°$$
$$B = 100°$$

4 We have the pairs:
$b = 200$ and $B = 100°$
$c = ?$ and $C = 30°$
with only c unknown.

So use $\frac{c}{\sin C} = \frac{b}{\sin B}$.

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

5 Substitute in the known values.

$$\frac{c}{\sin 30°} = \frac{200}{\sin 100°}$$

6 Multiply both sides by sin 30° .

$$c = \frac{200 \times \sin 30°}{\sin 100°}$$

7 Use your calculator to find c.

$$c = 101.542\ldots$$

8 Write your answer correct to two decimal places.

The rope must be 101.54 m long.

The ambiguous case

Note: When finding a missing angle using the sine rule, take care to check that your answer makes sense in the context of the question. That is, should the angle you are finding be an acute or obtuse angle?

This is because for some non-right-angled triangles there is more than one correct angle and depending on your calculator's setting may give an answer that does not fit with the question.

ISBN 978-1-009-25891-3

For example, if you are asked to find the missing angle, x, in the following triangle, then the sine rule can be easily applied.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

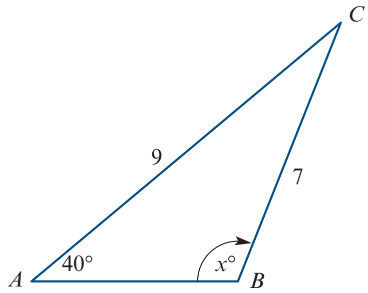
$$\frac{7}{\sin 40^\circ} = \frac{9}{\sin x}$$

$$7 \times \sin x = 9 \times \sin 40^\circ$$

$$\sin x = \frac{9 \times \sin 40^\circ}{7}$$

$$x = \sin^{-1}\left(\frac{9 \times \sin 40^\circ}{7}\right)$$

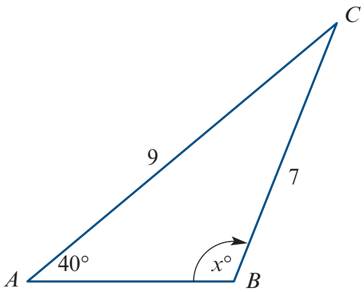
$$x = 55.73^\circ$$



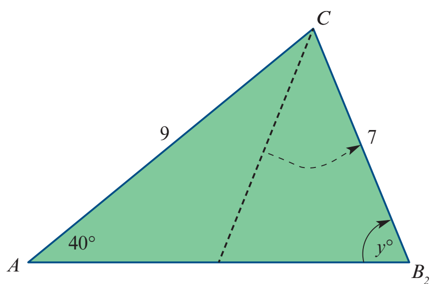
However, notice that the angle found for x is 55.7° which is an acute angle. But it is clear from the diagram that x should be an obtuse angle that is greater than 90°.

The first value of x is not incorrect, and you have not made a mistake in your working. We just need to take into account the *SSA* case for non-right-angled triangles in which two sides and one of their opposite angles are given. In these situations, two possible triangles can be formed using the given information.

The first triangle we were given:



A second possible triangle:

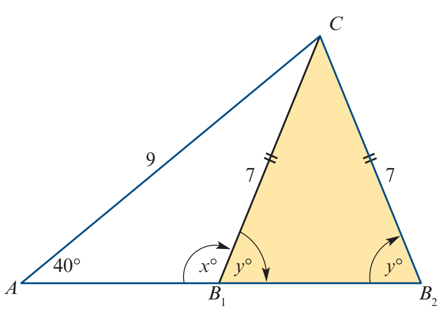


The first triangle has an obtuse angle at B, whereas the second possible triangle – still with side lengths 9 and 7 and a non-included angle of 40° – has an acute angle at B_2 (labelled y).

What we have created is an isosceles triangle with identical angles, $y°$.

It can also be seen that angle x and y are supplementary angles, that add to 180°.

Therefore, when we have found the value of the angle as 55.7°, we have actually determined the value of angle y. To find angle x we simply need to subtract our first calculated value from 180°.



$$\begin{aligned} \therefore x &= 180 - y \\ &= 180 - 55.7 \\ x &= 124.3^\circ \end{aligned}$$

ISBN 978-1-009-25891-3

This is known as *the ambiguous case* and is a concept that is important to be aware of when finding angles use the sine rule. The ambiguous case does not exist for the cosine rule (9C) because it uses two sides and an included angle.

Tips for solving trigonometry problems

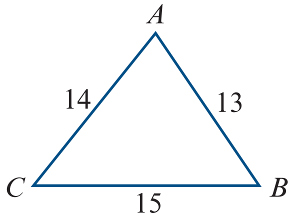
- Always make a rough sketch in pencil as you read the details of a problem. You may need to make changes as you read more, but it is very helpful to have a sketch to guide your understanding.
- In any triangle, the longest side is opposite the largest angle. The shortest side is opposite the smallest angle.
- When you have found a solution, re-read the question and check that your answer fits well with the given information and your diagram.
- Round answers for each part to the required decimal places. Keep more decimal places when the results are used in further calculations. Otherwise, rounding off errors accumulate.

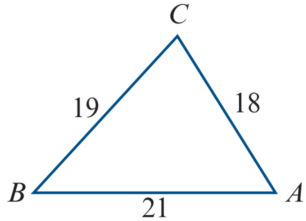
Exercise 9B

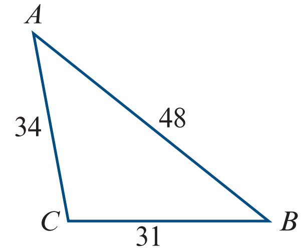
In this exercise, calculate lengths correct to two decimal places and angles correct to one decimal place where necessary.

Basic principles

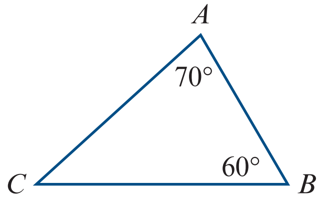
1 In each triangle, state the lengths of sides a, b and c.

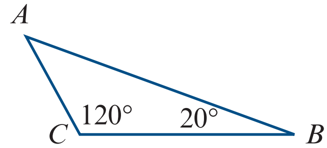
a


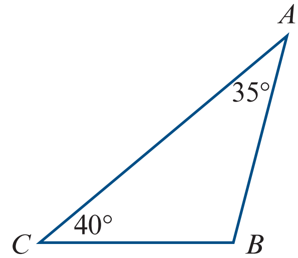
b


c


2 Find the value of the unknown angle in each triangle. Use $A + B + C = 180°$.

a


b


c


ISBN 978-1-009-25891-3

3 In each of the following, a student was using the sine rule to find an unknown part of a triangle, but was unable to complete the final steps of the solution. Find the unknown value by completing each problem.

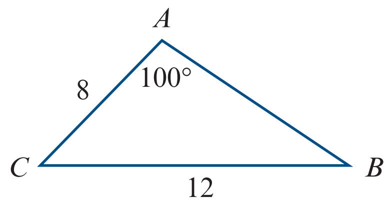
a $\dfrac{a}{\sin 40^\circ} = \dfrac{8}{\sin 60^\circ}$ **b** $\dfrac{b}{\sin 50^\circ} = \dfrac{15}{\sin 72^\circ}$ **c** $\dfrac{c}{\sin 110^\circ} = \dfrac{24}{\sin 30^\circ}$

d $\dfrac{17}{\sin A} = \dfrac{16}{\sin 70^\circ}$ **e** $\dfrac{26}{\sin B} = \dfrac{37}{\sin 95^\circ}$ **f** $\dfrac{21}{\sin C} = \dfrac{47}{\sin 115^\circ}$

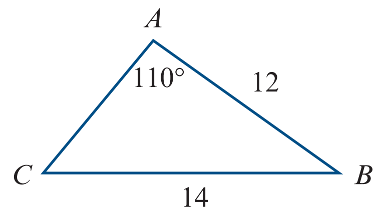
Using the sine rule to find angles

Example 4

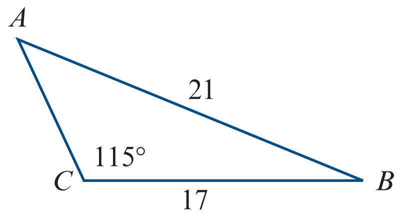
4 **a** Find angle B.



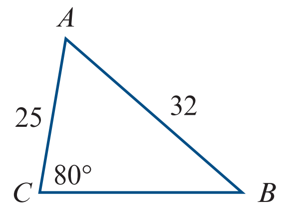
b Find angle C.



c Find angle A.



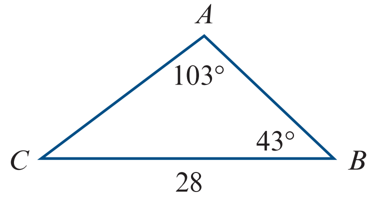
d Find angle B.



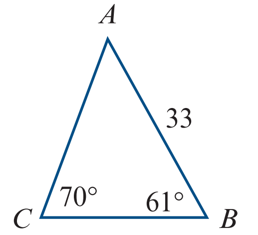
Using the sine rule to find sides

Example 5

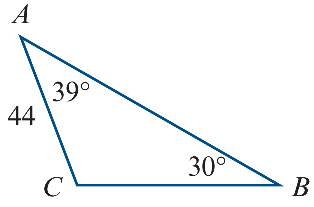
5 **a** Find side b.



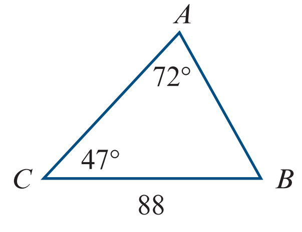
b Find side b.



c Find side a.

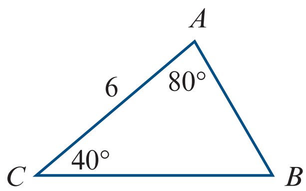


d Find side c.

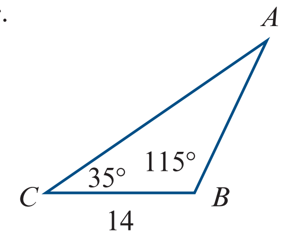


ISBN 978-1-009-25891-3

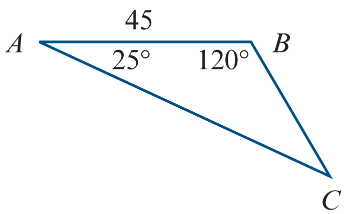
6 **a** Find side c.



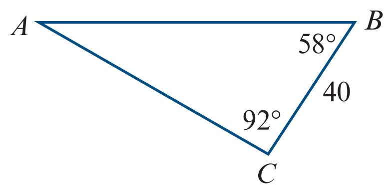
b Find side c.



c Find side b.



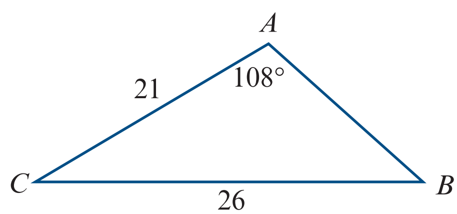
d Find side b.



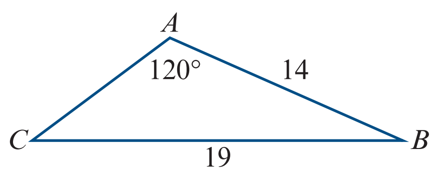
Solving triangles using the sine rule

7 Solve (find all the unknown sides and angles of) the following triangles.

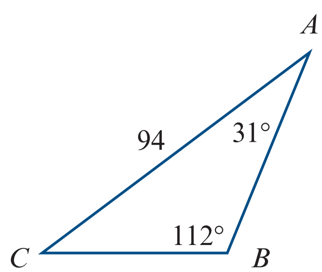
a



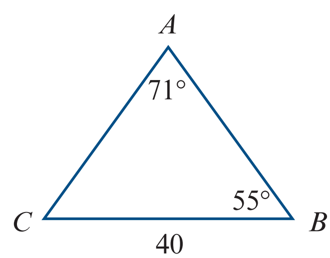
b



c



d



8 In the triangle ABC, $A = 105°$, $B = 39°$ and $a = 60$. Find side b.

9 In the triangle ABC, $A = 112°$, $a = 65$ and $c = 48$. Find angle C.

10 In the triangle ABC, $B = 50°$, $C = 45°$ and $a = 70$. Find side c.

11 In the triangle ABC, $B = 59°$, $C = 74°$ and $c = 41$. Find sides a and b and angle A.

12 In the triangle ABC, $a = 60$, $b = 100$ and $B = 130°$. Find angles A and C and side c.

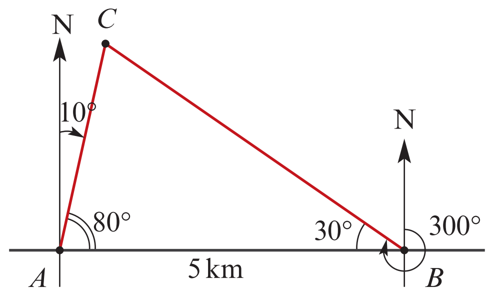
13 In the triangle ABC, $A = 130°$, $B = 30°$ and $c = 69$. Find sides a and b and angle C.

Cambridge Senior Maths for Western Australia
Mathematics Applications 1&2
ISBN 978-1-009-25891-3

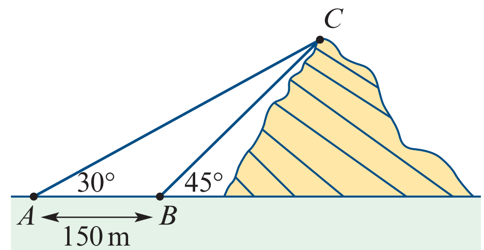
Applications

Example 6

14 A fire-spotter located in a tower at A saw a fire in the direction 010°. Five kilometres to the east of A another fire-spotter at B saw the fire in the direction 300°.
Find the distance of the fire from each tower.



15 A surveyor standing at point A measured the angle of elevation to the top of the mountain as 30°. She moved 150 m closer to the mountain and at point B measured the angle of elevation to the top of the mountain as 45°.
There is a proposal to have a strong cable from point A to the top of the mountain to carry tourists in a cable car. What is the length of the required cable?



16 A naval officer sighted the smoke of a volcanic island on a bearing of 044°. A navigator on another ship 25 km due east of the first ship saw the smoke on a bearing of 342°.

a Find the distance of each ship from the volcano.

b If the ship closest to the volcano can travel at 15 km/h, how long will it take it to reach the volcano?

17 An air-traffic controller at airport A received a distress call from an aeroplane low on fuel. The bearing of the aeroplane from A was 070°. From airport B, 80 km north of airport A, the bearing of the aeroplane was 120°.

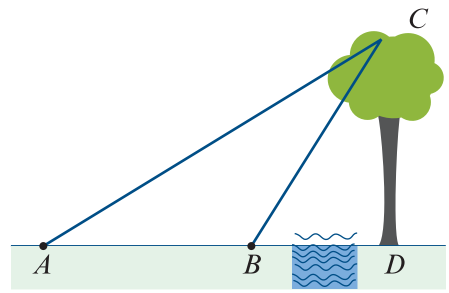
a Which airport was closest for the aeroplane?

b Find the distance to the closest airport.

c The co-pilot estimates fuel consumption to be 1525 litres per 100 km. The fuel gauge reads 1400 litres. Is there enough fuel to reach the destination?

ISBN 978-1-009-25891-3

18 Holly was recording the height of a tall tree. She recorded the angle of elevation of the top of the tree from point A as 25°. Holly walked 80 m towards the tree and recorded the angle of elevation from point B as 50°.



a Copy the diagram shown and add the given information.

b Find the angle at B in triangle ABC.

c Find the angle at C in triangle ABC.

d Find the length b (from A to C).

e Find distance DC, the height of the tree.

9C The cosine rule

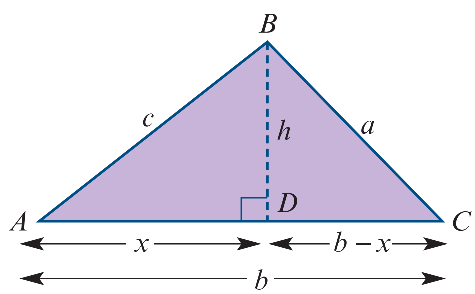
The **cosine rule** can be used to find the length of a side in any non-right-angled triangle when two sides and the angle between them are known. When you know the three sides of a triangle, the cosine rule can be used to find any angle.

How to derive the cosine rule

In the triangle ABC, show the height h of the triangle by drawing a line perpendicular from D on the base of the triangle to B.

Let $AD = x$

As $AC = b$, then $DC = b - x$.



In triangle ABD, $\cos A = \dfrac{x}{c}$

Multiply both sides by c. $x = c\cos A$ (1)

Using Pythagoras' theorem in triangle ABD. $x^2 + h^2 = c^2$ (2)

Using Pythagoras' theorem in triangle CBD. $(b - x)^2 + h^2 = a^2$

Expand (multiply out) the squared bracket. $b^2 - 2bx + x^2 + h^2 = a^2$

Use (1) to replace x with $c\cos A$. $b^2 - 2bc\cos A + x^2 + h^2 = a^2$

Use (2) to replace $x^2 + h^2$ with c^2. $b^2 - 2bc\cos A + c^2 = a^2$

Reverse and rearrange the equation. $a^2 = b^2 + c^2 - 2bc\cos A$

Repeating these steps with side c as the base, we get: $b^2 = a^2 + c^2 - 2ac\cos B$

Repeating these steps with side a as the base, we get: $c^2 = a^2 + b^2 - 2ab\cos C$

ISBN 978-1-009-25891-3

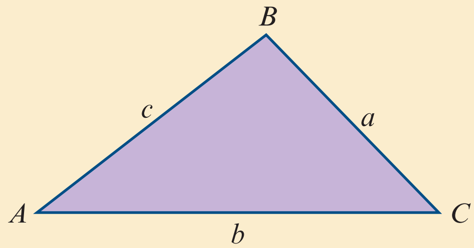
The three versions of the cosine rule can be rearranged to give rules for $\cos A$, $\cos B$, and $\cos C$.

The cosine rule

The cosine rule in any triangle ABC:

- when given two sides and the angle between them, the third side can be found using one of the equations:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$



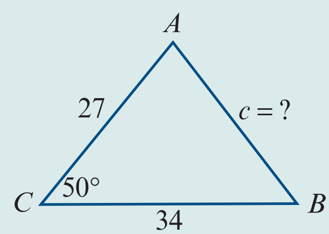
- when given three sides, any angle can be found using one of the following rearrangements of the cosine rule:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \qquad \cos B = \frac{a^2 + c^2 - b^2}{2ac} \qquad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Using the cosine rule

Example 7 **Using the cosine rule given two sides and the angle between them**

Find side c, correct to two decimal places, in the triangle shown.



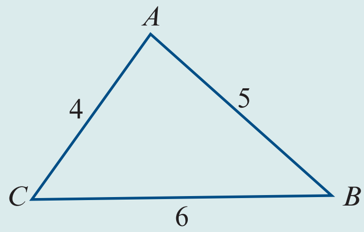
Solution

1. Write down the given values and the required unknown value. $a = 34, b = 27, c = ?, C = 50°$
2. We are given two sides and the angle between them. To find side c use $c^2 = a^2 + b^2 - 2ab\cos C$. $c^2 = a^2 + b^2 - 2ab\cos C$
3. Substitute the given values into the rule. $c^2 = 34^2 + 27^2 - 2 \times 34 \times 27 \times \cos 50°$
4. Take the square root of both sides. $c = \sqrt{(34^2 + 27^2 - 2 \times 34 \times 27 \times \cos 50°)}$
5. Use your calculator to find c. $c = 26.548\ldots$
6. Write your answer correct to two decimal places. The length of side c is 26.55 units.

ISBN 978-1-009-25891-3

Example 8 Using the cosine rule to find an angle given three sides

Find the largest angle, correct to one decimal place, in the triangle shown.



Solution

1 Write down the given values. $a = 6, b = 4, c = 5$

2 The largest angle is always opposite the largest side, so find angle A. $A = ?$

3 We are given three sides. To find angle A use $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

4 Substitute the given values into the rule.

$$\cos A = \frac{4^2 + 5^2 - 6^2}{2 \times 4 \times 5}$$

5 Write the equation to find angle A.

$$A = \cos^{-1}\left(\frac{4^2 + 5^2 - 6^2}{2 \times 4 \times 5}\right)$$

6 Use your calculator to evaluate the expression for A. Make sure that your calculator is in DEGREE mode.

$$A = 82.819\ldots^\circ$$

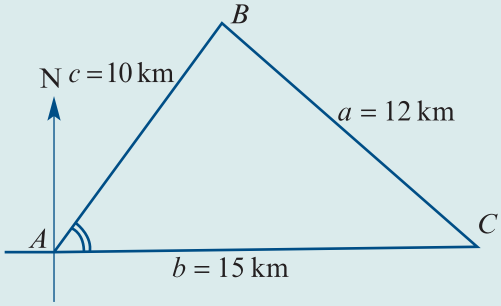
Tip: Wrap all the terms in the numerator (top) within brackets. Also put brackets around all of the terms in the denominator (bottom).

7 Write your answer. The largest angle is 82.8°.

ISBN 978-1-009-25891-3

Example 9 Application of the cosine rule: finding an angle and a bearing

A yacht left point A and sailed 15 km east to point C. Another yacht also started at point A and sailed 10 km to point B, as shown in the diagram. The distance between points B and C is 12 km.



a What was the angle between their directions as they left point A? Give the angle correct to two decimal places.

b Find the bearing of point B from the starting point A to the nearest degree.

Solution

a **1** Write the given values. $a = 12, b = 15, c = 10$

2 Write the form of the cosine rule for the required angle, A.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

3 Substitute the given values into the rule.

$$\cos A = \frac{15^2 + 10^2 - 12^2}{2 \times 15 \times 10}$$

4 Write the equation to find angle A.

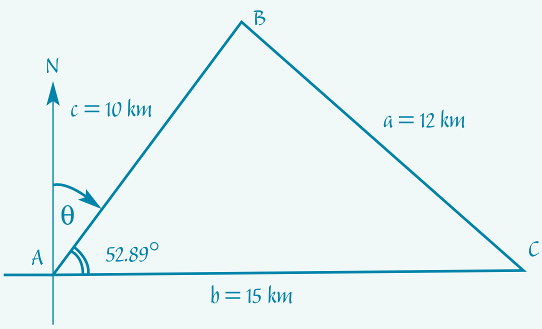
$$A = \cos^{-1}\left(\frac{15^2 + 10^2 - 12^2}{2 \times 15 \times 10}\right)$$

5 Use your calculator to evaluate the expression for A.

$$A = 52.891°$$

6 Give the answer to two decimal places. The angle was 52.89°.

b **1** The bearing θ, of point B from the starting point A, is measured clockwise from north.



$$\theta + 52.89° = 90°$$

2 Consider the angles in the right-angle at point A.

$$\theta = 90° - 52.89°$$
$$= 37.11°$$

3 Find the value of θ.

4 Write your answer. The bearing of point B from point A is 037°.

ISBN 978-1-009-25891-3

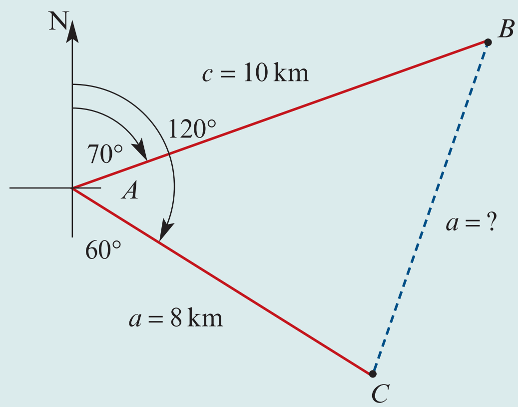
Example 10 Application of the cosine rule involving bearings

A bushwalker left his base camp and walked 10 km in the direction 070° .

His friend also left the base camp but walked 8 km in the direction 120° .

a Find the angle between their paths.

b How far apart were they when they stopped walking? Give your answer correct to two decimal places.



Solution

a **1** Angles lying on a straight line add to 180°.

$$60^\circ + A + 70^\circ = 180^\circ$$
$$A + 130^\circ = 180^\circ$$
$$A = 50^\circ$$

2 Write your answer.

The angle between their paths was 50°.

b **1** Write down the known values and the required unknown value.

$a = ?$, $b = 8$, $c = 10$, $A = 50$

2 We have two sides and the angle between them. To find side a use $a^2 = b^2 + c^2 - 2bc \cos A$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

3 Substitute in the known values.

$$a^2 = 8^2 + 10^2 - 2 \times 8 \times 10 \times \cos 50^\circ$$

4 Take the square root of both sides.

$$a = \sqrt{(8^2 + 10^2 - 2 \times 8 \times 10 \times \cos 50^\circ)}$$

5 Use your calculator to find the value of a.

$$a = 7.820\ldots$$

6 Write your answer correct to two decimal places.

The distance between them was 7.82 km.

ISBN 978-1-009-25891-3

Exercise 9C

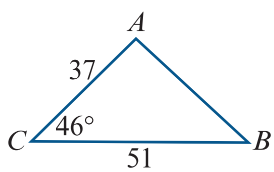
In this exercise, calculate lengths correct to two decimal places and angles correct to one decimal place.

Using the cosine rule to find sides

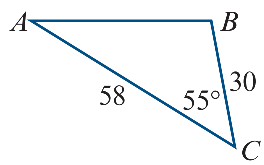
Example 7

1 Find the unknown side in each triangle.

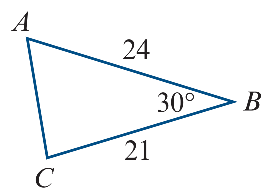
a



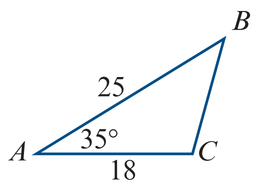
b



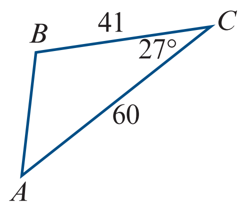
c



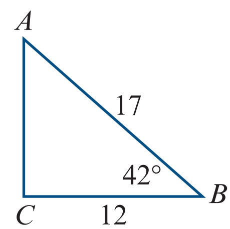
d



e



f



2 In the triangle ABC, $a = 27, b = 22$ and $C = 40°$. Find side c.

3 In the triangle ABC, $a = 18, c = 15$ and $B = 110°$. Find side b.

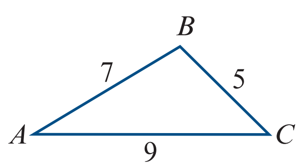
4 In the triangle ABC, $b = 42, c = 38$ and $A = 80°$. Find side a.

Using the cosine rule to find angles

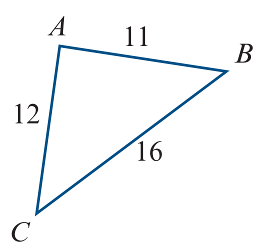
Example 8

5 Find angle A in each triangle.

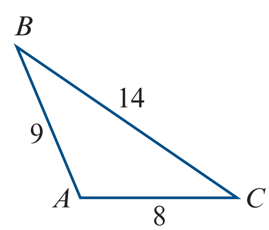
a



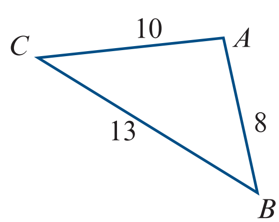
b



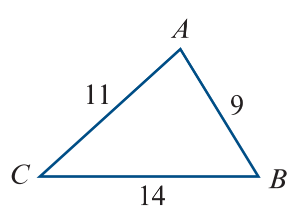
c



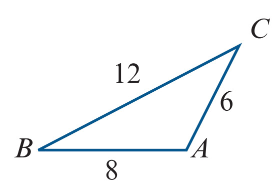
d



e



f



6 In the triangle ABC, $a = 9, b = 10$ and $c = 11$. Find angle A.

7 In the triangle ABC, $a = 31, b = 47$ and $c = 52$. Find angle B.

8 In the triangle ABC, $a = 66, b = 29$ and $c = 48$. Find angle C.

ISBN 978-1-009-25891-3

9 Find the smallest angle in the triangle ABC, with $a = 120$, $b = 90$ and $c = 105$.

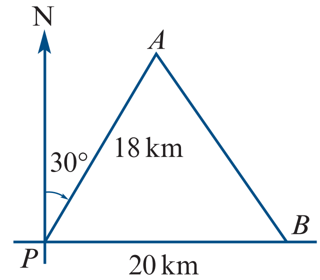
10 In the triangle ABC, $a = 16$, $b = 21$ and $c = 19$. Find the largest angle.

Applications

Example 9
Example 10

11 A farm has a triangular shape with fences of 5 km, 7 km and 9 km in length. Find the size of the smallest angle between the fences. The smallest angle is always opposite the smallest side.

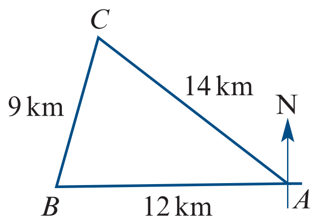
12 A ship left the port P and sailed 18 km on a bearing of 030° to point A. Another ship left port P and sailed 20 km east to point B. Find the distance from A to B, correct to one decimal place.



13 A bushwalker walked 12 km west from point A to point B. Her friend walked 14 km from point A to point C as shown in the diagram. The distance from B to C is 9 km.

a Find the angle at A, between the paths taken by the bushwalkers, correct to one decimal place.

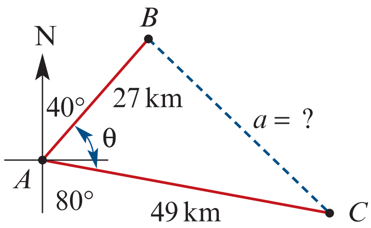
b What is the bearing of point C from A? Give the bearing correct to the nearest degree.



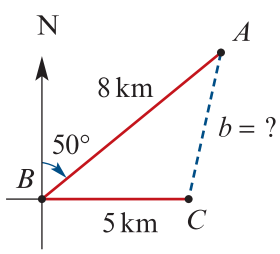
14 A ship left port A and travelled 27 km on a bearing of 40° to reach point B. Another ship left the same port and travelled 49 km on a bearing of 100° to arrive at point C.

a Find the angle θ between the directions of the two ships.

b How far apart were the two ships when they stopped?



15 A battleship B detected a submarine A on a bearing of 050° and at a distance of 8 km. A cargo ship C was 5 km due east of the battleship. How far was the submarine from the cargo ship?



16 From a lookout tower A, a fire-spotter saw a bushfire B at a distance of 15 km in the direction 315°. A township C was located 12 km on a bearing of 265° from the tower. How far was the bushfire from the township?

ISBN 978-1-009-25891-3

17 Passengers, who are travelling in a car west along a road that runs east–west, see a mountain 9 km away on a bearing of 290°. When they have travelled a further 5 km west along the road, what will be the distance to the mountain?

18 At a point A on the ground, the angle of elevation to the top of a radio transmission tower is 60°. From that point a 40 m cable was attached to the top of the tower. At a point B, a further 10 m away from the base of the tower, another cable is to be pegged to the ground and attached to the top of the tower. What length is required for the second cable?

9D Extended application and problem-solving task

Exercise 9D

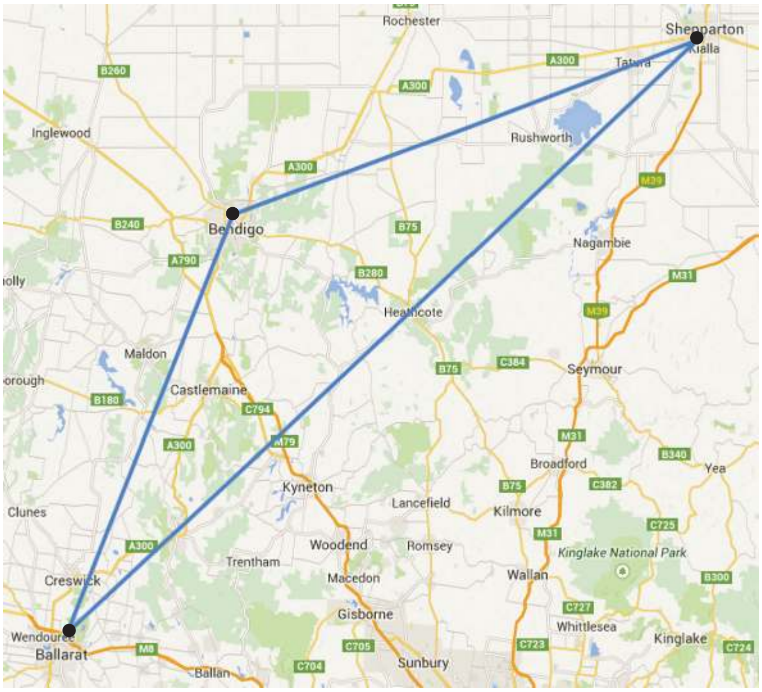
Alice operates a charter flight service for country Victoria. She is planning a round trip from her base at Bendigo to Shepparton, Ballarat and then returning to Bendigo.

a The direct distance from Ballarat to Bendigo is known to be 85 km. By measuring the lengths of the blues lines between the three cities calculate the direct distance from:

  i Bendigo to Shepparton

  ii Shepparton to Ballarat.

b Her Piper Archer TX has a range of 522 nautical miles. One nautical mile equals 1.85 km. The fuel gauge indicates the tank is one third full. Is there enough fuel to complete the round trip?



ISBN 978-1-009-25891-3

c The average cruising speed is 147 miles per hour. One mile equals 1.61 km. Find the total flight time for the round trip.

d Fuel consumption cost is 95 cents per km. What will be her fuel cost for the trip?

e i If a vertical line points North, use the map and a protractor to find the bearing that Alice must fly from Bendigo to Shepparton.
  ii Calculate the bearing that Alice must fly from Shepparton to Ballarat.
  iii Calculate the bearing that Alice must fly from Ballarat to Bendigo.

f Give the direction and distance flight information that Alice will need to use on the Shepparton to Ballarat stage of the circuit.

g Use your answers to part **e** to find the angle between the flight paths to and from Shepparton.

h Show how a trigonometry distance rule can be applied to the Bendigo–Shepparton and Shepparton–Ballarat distances with the angle at Shepparton to find the Ballarat–Bendigo distance. Confirm that the distance is approximately 85 km.

i Find the area enclosed by the flight paths.

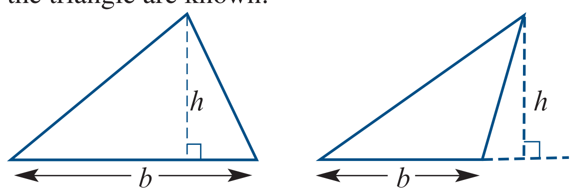
ISBN 978-1-009-25891-3

Key ideas and chapter summary

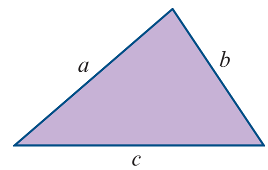
Assign-ment

Area of a triangle

- Use the formula area of triangle $= \frac{1}{2} \times b \times h$ if the base and height of the triangle are known:



- Use the formula area of triangle $= \frac{1}{2} \times ab \sin C$ if two sides and the angle between them are known.
- Use Heron's rule if the lengths of the three sides of the triangle are known.

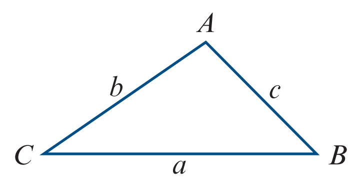


$$\text{Area of a triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where} \qquad s = \frac{1}{2}(a+b+c)$$

Labelling a non-right-angled triangle

Side a is always opposite angle A, and so on.



Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Use the **sine rule** when given:

- two sides and an angle opposite one of those sides
- two angles and one side.

If neither angle is opposite the given side, find the third angle using $A + B + C = 180°$.

Cosine rule

The **cosine rule** has three versions. When given two sides and the angle between them, use the rule that starts with the required side:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

ISBN 978-1-009-25891-3

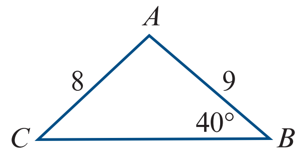
Skills check

Having completed this chapter you should be able to:

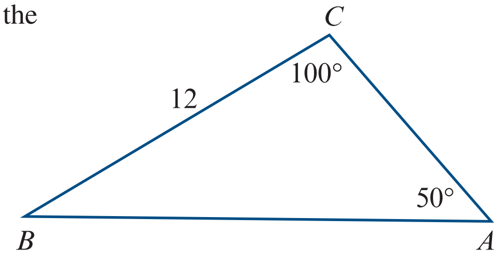
- use the sine rule and cosine rule in non-right-angled triangles to find an unknown side or angle
- use the appropriate rule for finding the area of a triangle
- solve practical problems involving non-right-angled triangles.

Short-answer questions

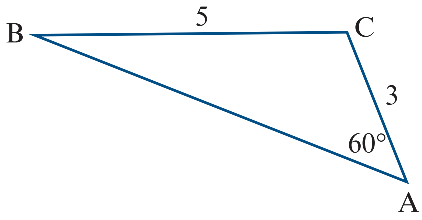
1 Calculate the size of angle C, correct to one decimal place.



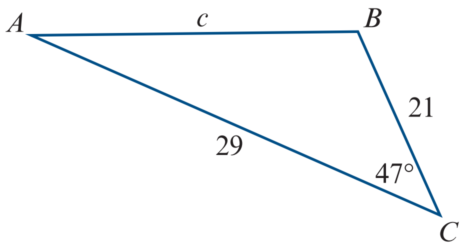
2 Write an expression that can be used to find the length of side AB.



3 Write an expression that can be used to solve for sin B.



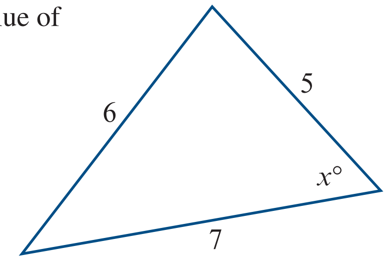
4 Write an expression that should be used to find the length c in triangle ABC.



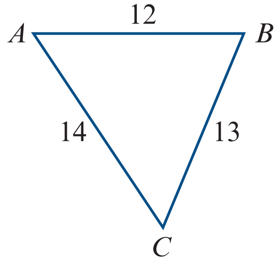
ISBN 978-1-009-25891-3

Review

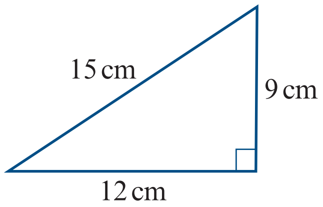
5 Write an expression that should be used to find the value of $\cos x$.



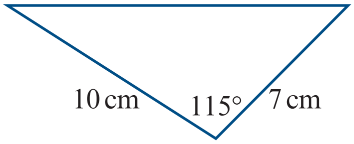
6 Write an expression that should be used to find angle C.



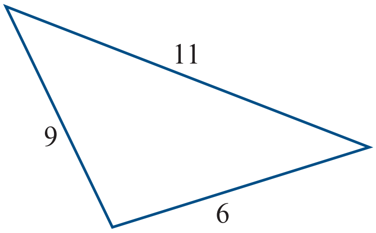
7 Find the area of the given triangle.



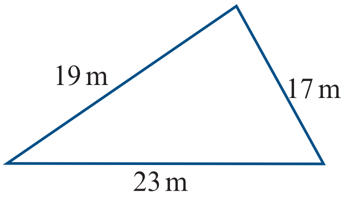
8 Find the area of the triangle shown, correct to two decimal places.



9 Find the area of the triangle shown, correct to two decimal places.

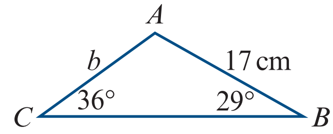


10 Find the area of the triangle shown, correct to one decimal place.

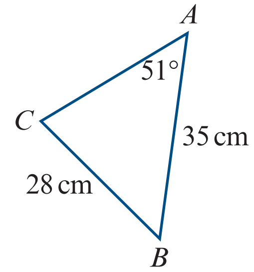


ISBN 978-1-009-25891-3

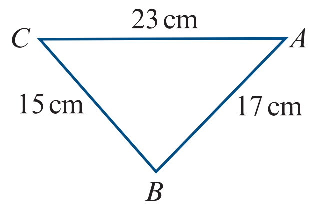
11 Find the length of side b, correct to two decimal places.



12 Find the angle C, correct to one decimal place.

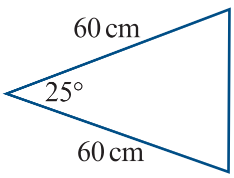


13 Find the smallest angle in the triangle shown, correct to one decimal place.



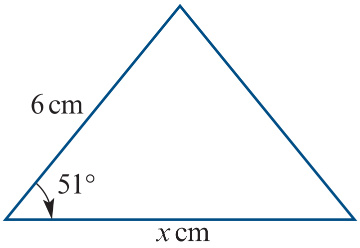
14 A car travelled 30 km east, then travelled 25 km on a bearing of 070°. How far was the car from its starting point? Answer correct to two decimal places.

15 A pennant flag is to have the dimensions shown. What area of cloth will be needed for the flag? Answer correct to one decimal place.



16 Find the area of an equilateral triangle with sides of 8 m, correct to one decimal place.

17 If the area of the triangle is 15 cm^2, find the value of x.

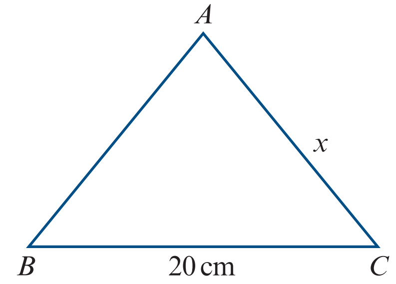


18 A level block of land is triangular, having dimensions 28 m by 18 m by 20 m. Because of its shape, a real estate company decides to plant trees on the block instead of selling it. Trees can be obtained locally from a nursery for $4.65 each. It is ideal that one tree be planted every 3.6 m^2 of land. How many trees are required to fill the area and what will be the total cost to fill the block of land with the trees?

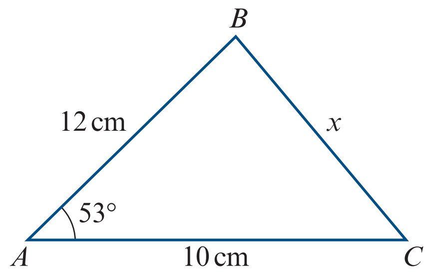
ISBN 978-1-009-25891-3

Review

19 In ΔABC, $BC = 20$ cm, $\sin A = 0.4$ and $\sin B = 0.2$. Without using a calculator, determine the length of AC.



20 Find the value of x.



Extended-response questions

1 One group of bushwalkers left a road running north–south to walk along a bearing of 060°. A second group of walkers left the road from a point 3 km further north. They walked on a bearing of 110°. The two groups met at the point C, where their paths intersected.

a Find the angle at which their paths met.

b Find the distance walked by each group, correct to two decimal places.

c If the bushwalkers decided to return to the road by walking back along the path that the second group of walkers had taken, what bearing should they follow?

2 A yacht P left port and sailed 45 km on a bearing of 290°. Another yacht Q left the same port but sailed for 54 km on a bearing of 040°.

a What was the angle between their directions?

b How far apart were they at that stage (correct to two decimal places)?

3 A triangular shade cloth must have sides of 5 m, 6 m and 7 m to cover the required area of a children's playground.

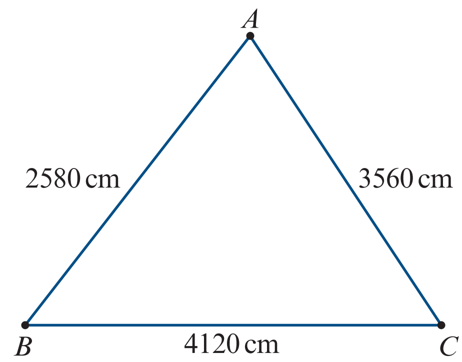
a What angle is required in each of the corners of the shade cloth (correct to one decimal place)?

b The manufacturer charges according to the area of the shade cloth. What is the area of this shade cloth (correct to two decimal places)?

c The cost of shade cloth is \$29 per square metre. What will be the cost of this shade cloth?

ISBN 978-1-009-25891-3

4 The diagram below (not drawn to scale) shows a school oval consisting of three walls AB, BC and AC.



a Determine the size of the angle BAC to the nearest degree.

b Determine the size of the angle ABC.

c The section ABC needs to be covered with artificial lawn. The cost of the material is \$48 per square metre. Determine the total cost of installing the lawn.

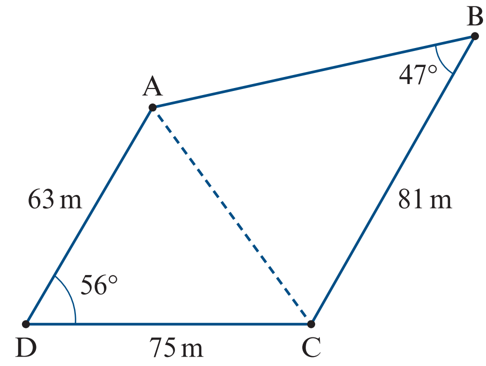
5 The diagram shows a farmer's block of land having the shape of a quadrilateral labelled $ABCD$. Given that $AD = 63$ m, $DC = 75$ m and $BC = 81$ m. Also $\angle ADC = 56°$ and $\angle ABC = 47°$.

a Find the length of AC, correct to the nearest m.

b Find the area of triangle ADC.

c Determine the size of the acute angle BAC, correct to the nearest degree.

d Find the area of the block $ABCD$.



ISBN 978-1-009-25891-3

10 Linear graphs and models

In this chapter

Syllabus references

Topics: Straight-line graphs and their applications; Piece-wise linear graphs and step graphs

Subtopics: 2.3.3 – 2.3.6, 2.3.9 – 2.3.10

Many everyday situations can be described and investigated using a linear graph and its equation. Examples include the depreciating value of a newly purchased car, and the short-term growth of a newly planted tree. In this chapter, you will revise the properties of linear graphs and their equations and apply these ideas to modelling linear growth and decay in the real world.

ISBN 978-1-009-25891-3

10A Drawing straight-line graphs

Plotting straight-line graphs

Relations defined by equations such as:

$$y = 1 + 2x \qquad y = 3x - 2 \qquad y = 10 - 5x \qquad y = 6x$$

are called *linear* relations because they generate *straight-line graphs*.

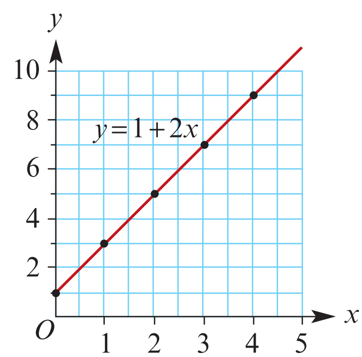
For example, consider the relation $y = 1 + 2x$. To plot this graph, we can form a table.

x	0	1	2	3	4
y	1	3	5	7	9

We can then plot the values from the table on a set of axes, as shown opposite.

The points appear to lie on a straight line.

A ruler can then be used to draw in this straight line to give the graph of $y = 1 + 2x$.



Example 1 Constructing a graph from a table of values

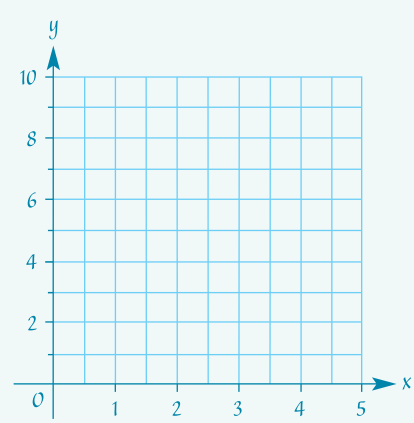
Plot the graph of $y = 8 - 2x$ by forming a table of values of y using $x = 0, 1, 2, 3, 4$.

Solution

1 Set up table of values.
When $x = 0$, $y = 8 - 2 \times 0 = 8$.
When $x = 1$, $y = 8 - 2 \times 1 = 6$, and so on.

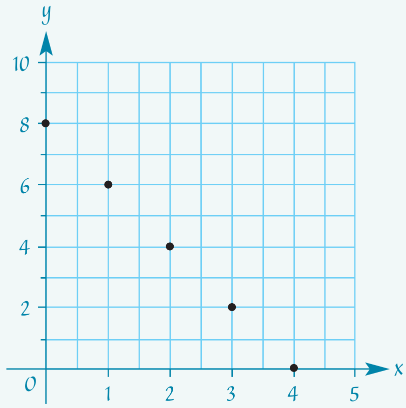
x	0	1	2	3	4
y	8	6	4	2	0

2 Draw, label and scale a set of axes to cover all values.

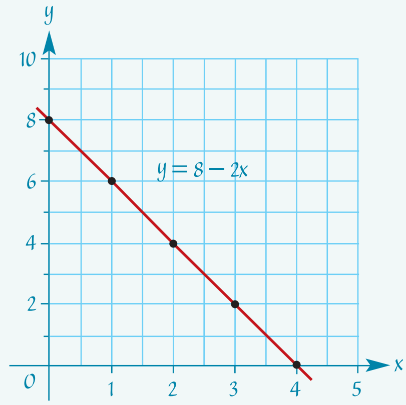


ISBN 978-1-009-25891-3

3 Plot the values in the table on the graph by marking with a dot (•). The first point is (0, 8). The second point is (1, 6), and so on.



4 The points appear to lie on a straight line. Use a ruler to draw in the straight line. Label the line $y = 8 - 2x$.



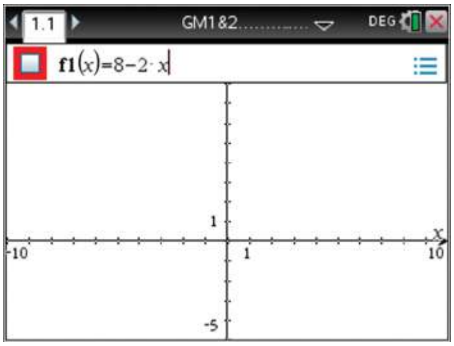
A CAS calculator can also be used to draw straight-line graphs, although it can take some fiddling around with scaling to get the exact graph you want. However, one advantage of using a CAS calculator is that, when drawing the graph, it automatically generates a table of values for you.

How to draw a straight-line graph and show a table of values using the TI-Nspire CAS

Use a CAS calculator to draw the graph $y = 8 - 2x$ and show a table of values.

Steps

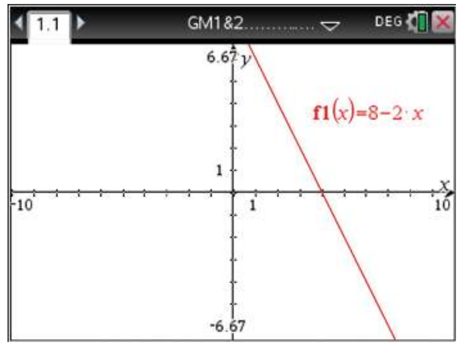
1 Start a new document (ctrl + N) and select **Add Graphs**.



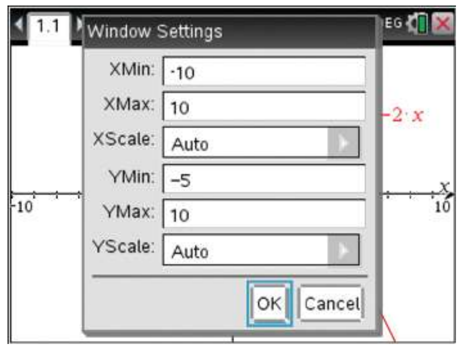
ISBN 978-1-009-25891-3

2 Type in the equation as shown. Note that f1(x) represents the y. Press enter to obtain the graph below.

Hint: If the function entry line is not visible, press tab.

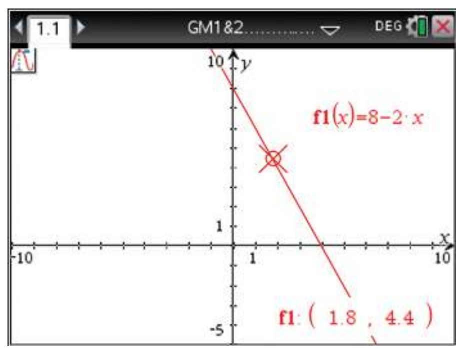


3 Change the window setting to see the key features of the graph. Use menu>**Window/Zoom**>**Window Settings** and edit as shown. Use the tab key to move between the entry lines. Press enter when finished editing the settings. The re-scaled graph is shown below.

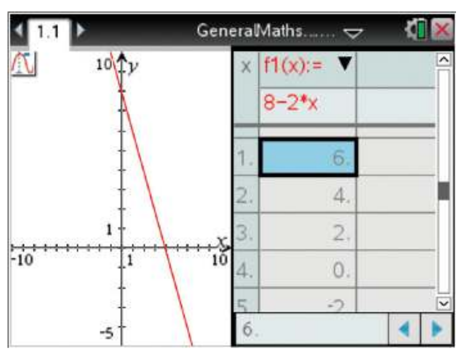


4 To show values on the graph, use menu>**Trace**>**Graph Trace** and then use the ◄► arrows to move along the graph.

Note: Press esc to exit the **GraphTrace** tool.



5 To show a table of values, press ctrl + T. Use the ▲▼ arrows to scroll through the values in the table.



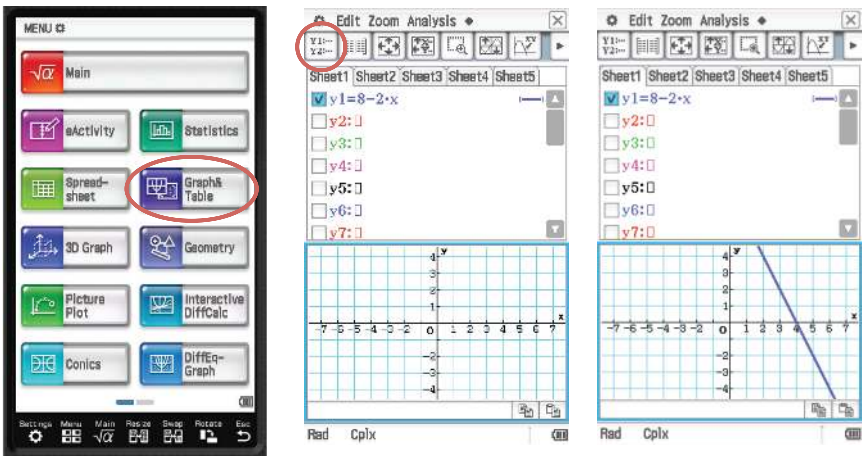
Cambridge Senior Maths for Western Australia
Mathematics Applications 1&2
ISBN 978-1-009-25891-3

How to draw a straight-line graph and show a table of values using the ClassPad

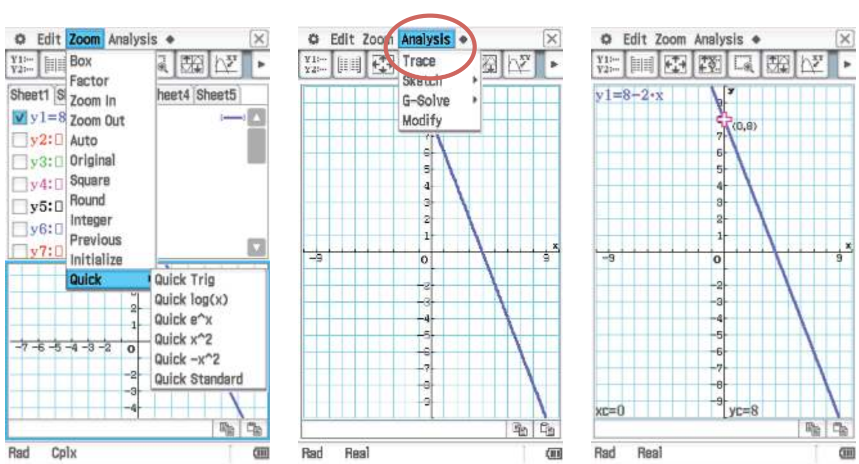
Use a CAS calculator to draw the graph of $y = 8 - 2x$ and show a table of values.

Steps

1. Open the **Graphs and Table** application.
2. Enter the equation into the graph editor window by typing **8 − 2*x*** and press **EXE**.
3. Tap the icon to plot the graph.

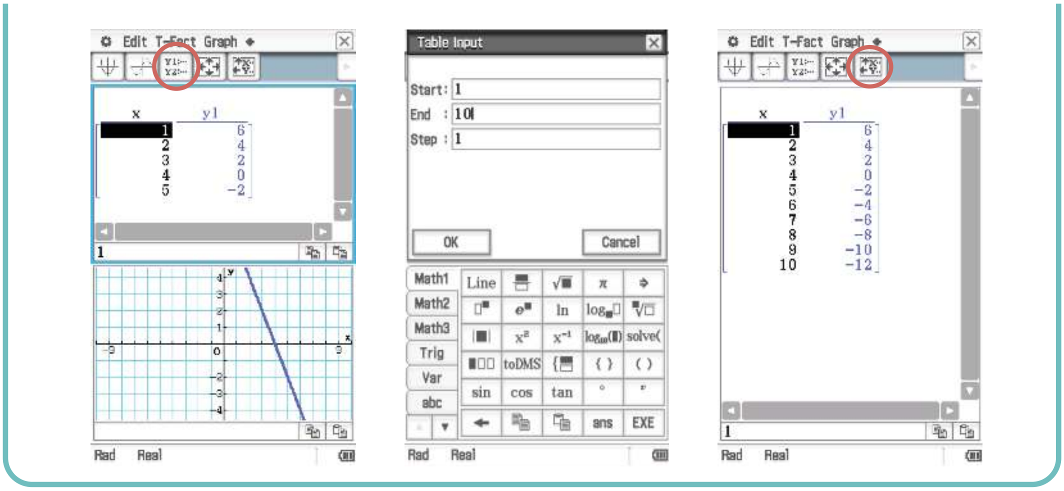


4. To adjust the graph screen go to **Zoom > Quick > Quick Standard**.
5. Tap resize from the toolbar to increases the size of the graph window.
6. Select **Analysis>Trace** to place a cursor on the graph. The coordinates of the point will be displayed at the location of the cursor. E.g. (0, 8).
7. Use the cursor key to move the cursor along the line.



8. Tap the icon from the toolbar to display a table of values.
9. Tap the icon from the toolbar to open the **Table Input** dialog box. The values displayed in the table can be adjusted by changing the values in this window.

ISBN 978-1-009-25891-3



Exercise 10A

Plotting by hand

Example 1

1 Plot the graph of the linear equations below by first forming a table of values of y for $x = 0, 1, 2, 3, 4$.

a $y = 1 + 2x$ **b** $y = 2 + x$ **c** $y = 10 - x$ **d** $y = 9 - 2x$

Using a calculator

CAS

2 Use your CAS calculator to plot a graph for the window given and generate a table of values.

a $y = 4 + x$
$-5 \le x \le 5$
$-1 \le y \le 10$

b $y = 2 + 3x$
$-1 \le x \le 5$
$-1 \le y \le 20$

c $y = 10 + 5x$
$-1 \le x \le 5$
$-1 \le y \le 40$

d $y = 5x$
$-5 \le x \le 5$
$-25 \le y \le 25$

e $y = -5x$
$-5 \le x \le 5$
$-25 \le y \le 25$

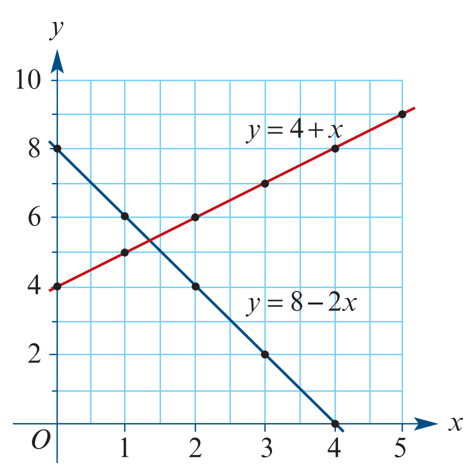
f $y = 100 - 5x$
$-1 \le x \le 25$
$-25 \le y \le 125$

Conceptual understanding

3 Two straight-line graphs, $y = 4 + x$ and $y = 8 - 2x$, are plotted as shown opposite.

a Reading from the graph of $y = 4 + x$, determine the missing coordinates: (0, ?), (2, ?), (?, 7), (?, 9).

b Reading from the graph of $y = 8 - 2x$, determine the missing coordinates: (0, ?), (1, ?), (?, 4), (?, 2).



ISBN 978-1-009-25891-3

10B Determining the slope of a straight line

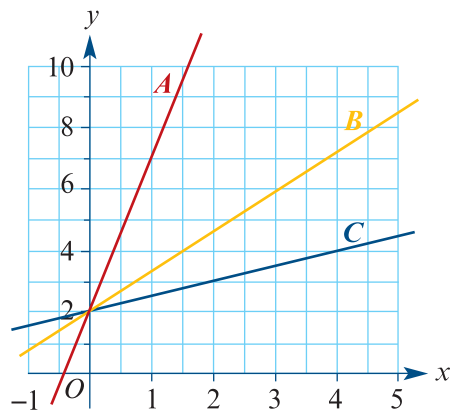
Positive and negative slopes

One thing that makes one straight-line graph look different from another is its steepness or slope. Another name for slope is **gradient**.

For example, the three straight lines on the graph opposite all cut the y-axis at $y = 2$, but they have quite different slopes.

Line A has the steepest slope while Line C has the gentlest slope. Line B has a slope somewhere in between.

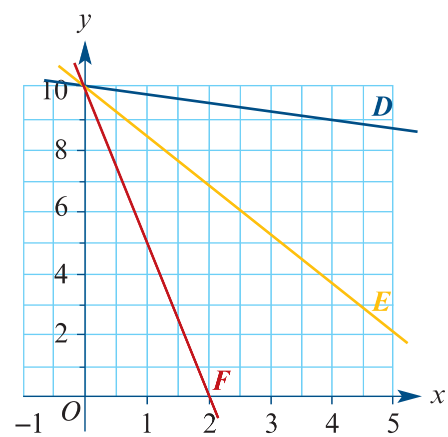
In all cases, the lines have **positive slopes**; that is, they rise from left to right.



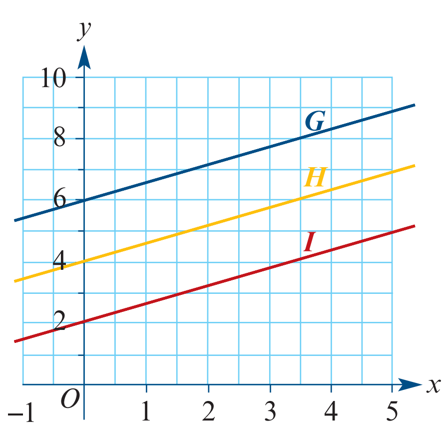
Similarly, the three straight lines on the graph opposite all cut the y-axis at $y = 10$, but they have quite different slopes.

In this case, Line D has the gentlest slope while Line F has the steepest slope. Line E has a slope somewhere in between.

In all cases, the lines have **negative slopes**; that is, they fall from left to right.



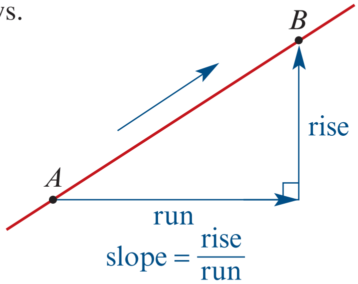
By contrast, the three straight lines G, H, I on the graph opposite cut the y-axis at different points, but they all have the *same* slope.



ISBN 978-1-009-25891-3

Determining the slope

When talking about the **slope of a straight line**, we want to be able to do more than say that it has a gentle positive slope. We would like to be able to give the slope a value that reflects this fact. We do this by defining the slope of a line as follows.



First, two points A and B on the line are chosen.
As we go from A to B along the line, we move:

- up by a distance called the **rise**
- and across by a distance called the **run**.

The slope is found by dividing the rise by the run.

Example 2 Finding the slope of a line from a graph: positive slope

Find the slope of the line through the points (1, 4) and (4, 8).

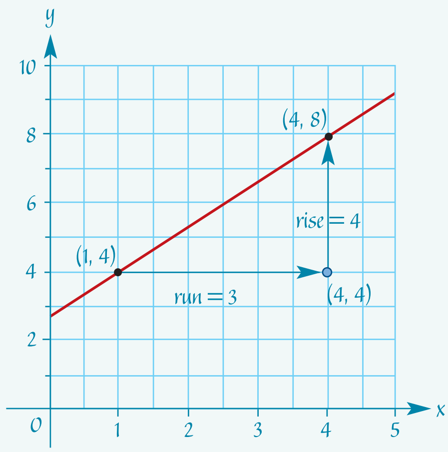
Solution

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{rise} = 8 - 4 = 4$$

$$\text{run} = 4 - 1 = 3$$

$$\therefore\ \text{slope} = \frac{4}{3}$$



Note: To find the 'rise', look at the y-coordinates.
To find the 'run', look at the x-coordinates.

ISBN 978-1-009-25891-3

Example 3 **Finding the slope of a line from a graph: negative slope**

Find the slope of the line through the points (0, 10) and (4, 2).

Solution

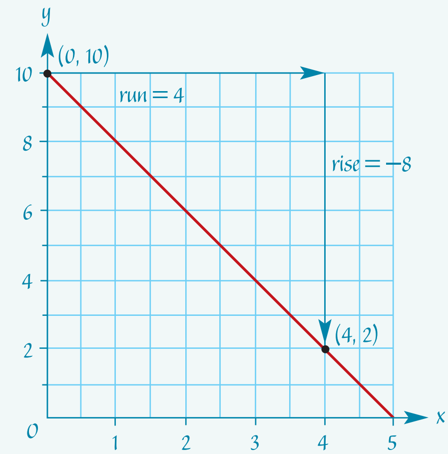
$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{rise} = 2 - 10 = -8$$

$$\text{run} = 4 - 0 = 4$$

$$\therefore\ \text{slope} = \frac{-8}{4} = -2$$

Note: In this example, we have a negative 'rise' which represents a 'fall'.

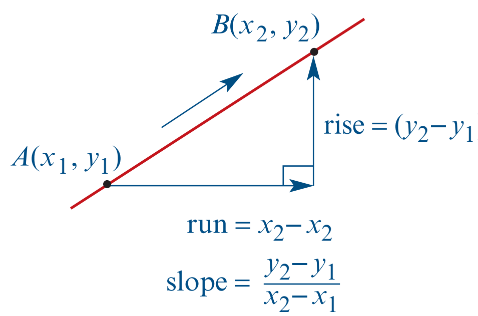


A formula for finding the slope of a line

While the 'rise/run' method for finding the slope of a line will always work, some people prefer to use a formula for calculating the slope. The formula is derived as follows.

Label the coordinates of point A: (x_1, y_1).

Label the coordinates of point B: (x_2, y_2).



By definition: $\text{slope} = \dfrac{\text{rise}}{\text{run}}$

From the diagram:

$$\text{rise} = y_2 - y_1$$

$$\text{run} = x_2 - x_1$$

By substitution: $\text{slope} = \dfrac{y_2 - y_1}{x_2 - x_1}$

ISBN 978-1-009-25891-3

Example 4 Finding the slope of a line using the formula for the slope

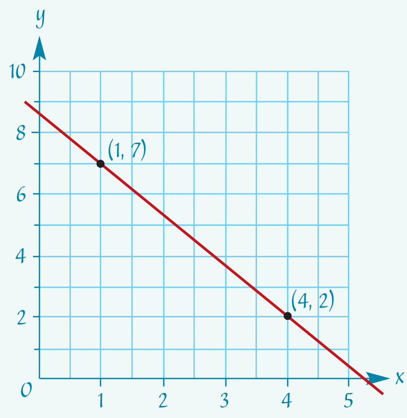
Find the slope of the line through the points (1, 7) and (4, 2) using the formula for the slope of a line. Give your answer correct to two decimal places.

Solution

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

Let $(x_1, y_1) = (1, 7)$ and $(x_2, y_2) = (4, 2)$.

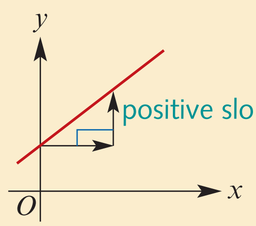
$$\text{slope} = \frac{2-7}{4-1} = -\frac{5}{3}$$



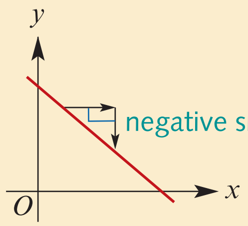
Note: To use this formula it does not matter which point you call (x_1, y_1) and which point you call (x_2, y_2), the rule still works.

The slope of straight-line graphs

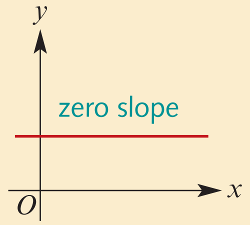
A straight-line graph that rises from left to right is said to have a **positive slope** (positive rise).



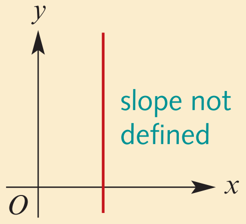
A straight-line graph that falls from left to right is said to have a **negative slope** (negative rise).



A straight-line graph that is horizontal has **zero slope** ('rise' = 0).



The slope is **undefined** for a straight-line graph that is vertical ('run' = 0).

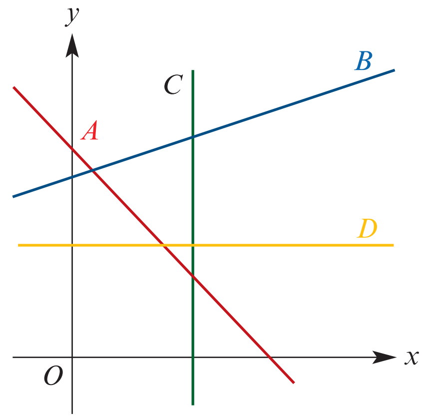


ISBN 978-1-009-25891-3

Exercise 10B

Basic ideas

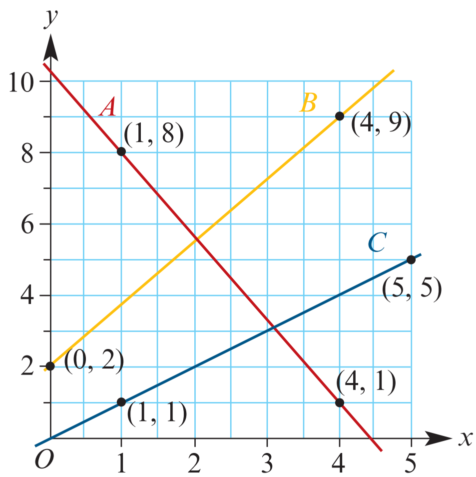
1 Without calculation, identify the slope of each of the straight-line graphs A, B, C and D as: positive, negative, zero, or undefined.



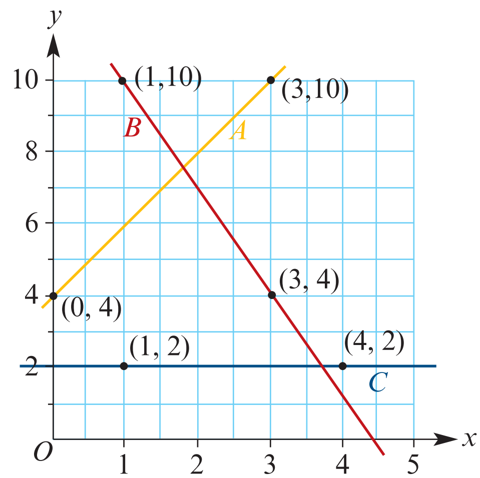
Calculating slopes of lines

Example 2
Example 3

2 Find the slope of each of the lines (A, B, C) shown on the graph below.



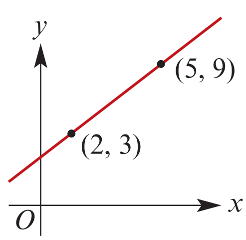
3 Find the slope of each of the lines (A, B, C) shown on the graph below.



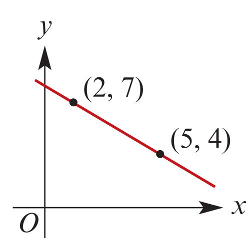
Example 4

4 Find the slope of each of the lines shown.

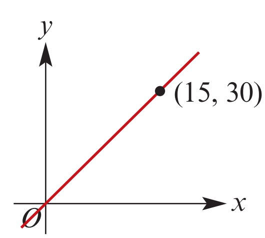
a



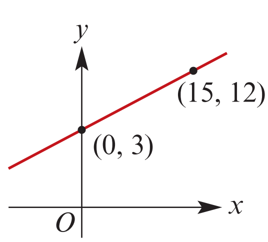
b



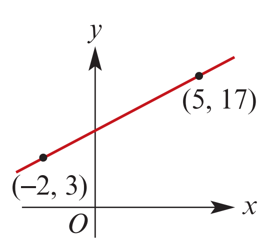
c



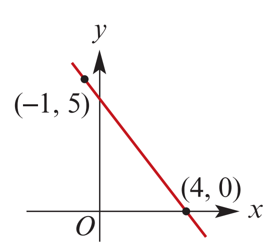
d



e



f



ISBN 978-1-009-25891-3

10C The intercept–slope form of the equation of a straight line

Determining the intercept and slope of a straight-line graph from its equation

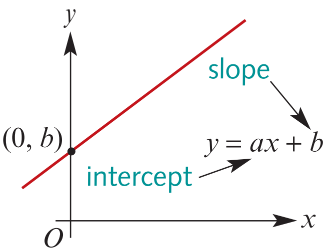
When we write the equation of a straight line in the form:[1]

$$y = ax + b$$

we are using what is called the **slope–intercept form of the equation** of a straight line.

We call $y = ax + b$ the intercept-slope form of the equation of a straight line because:

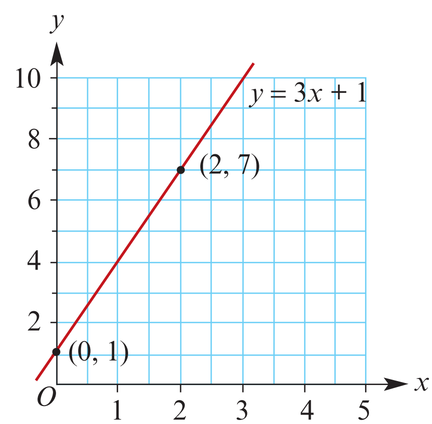
- a = the **slope** of the graph
- b = the ***y*-intercept** of the graph.



The slope–intercept form of the equation of a straight line is useful in modelling relationships in many practical situations. It is also the form used in **bivariate** (two-variable) statistics.

An example of the equation of a straight line written in slope–intercept form is $y = 3x + 1$.

Its graph is shown opposite.



From the graph we see that the:

$$y\text{-intercept} = 1$$

$$\text{slope} = \frac{7-1}{2-0} = \frac{6}{2} = 3$$

That is:

- the y-intercept corresponds to the (*constant*) term in the equation (intercept = 1)
- the slope is given by the *coefficient of x* in the equation (slope = 3).

[1] Note: You may be used to writing the equation of straight line as $y = mx + c$. However, when we are using a straight-line graph to model (represent) real world phenomena, we tend to use 'a' for the slope or gradient and 'b' for the y-intercept (rather than 'm' and 'c') and write the equation as $y = ax + b$. This is particularly true in performing statistical computations where your calculator will use 'a' for slope and 'b' for the y-intercept. You will see this later in the book, so it is worth making the change now.

ISBN 978-1-009-25891-3

Intercept–slope form of the equation of a straight line

If the equation of a straight line is in the slope–intercept form:

$$y = ax + b$$

then: a = the slope of the graph

b = y-intercept of the graph (where the graph cuts the y-axis)

Example 5 Finding the slope and y-intercept of a line from its equation

Write down the slope and y-intercept of each of the straight-line graphs defined by the following equations.

a $y = 9x - 6$ **b** $y = 10 - 5x$ **c** $y = -2x$ **d** $y - 4x = 5$

Solution

For each equation:

1 Write the equation. If it is not in slope–intercept form, rearrange the equation.

2 Write down the slope and y-intercept. When the equation is in slope–intercept form, the value of:
a = the slope (the coefficient of x)
b = the y-intercept (the constant term)

a $y = 9x - 6$
slope $= 9$, y-intercept $= -6$

b $y = 10 - 5x$ or $y = -5x + 10$
slope $= -5$, y-intercept $= 10$

c $y = -2x$ or $y = -2x + 0$
slope $= -2$, y-intercept $= 0$

d $y - 4x = 5$ or $y = 4x + 5$
slope $= 4$, y-intercept $= 5$

Example 6 Writing down the equation of a straight line given its slope and y-intercept

Write down the equations of the straight lines with the following y-intercepts and slopes.

a slope $= 6$, y-intercept $= 9$ **b** slope $= -5$, y-intercept $= 2$

c slope $= 2$, y-intercept $= -3$

Solution

The equation of a straight line is $y = ax + b$. In this equation, a = slope and b = y-intercept.
Form an equation by inserting the given values of the slope and the y-intercept for a and b in the standard equation $y = ax + b$.

a slope $= 6$, y-intercept $= 9$
equation: $y = 6x + 9$

b slope $= -5$, y-intercept $= 2$
equation: $y = -5x + 2$

c slope $= 2$, y-intercept $= -3$
equation: $y = 2x - 3$

ISBN 978-1-009-25891-3

Sketching straight-line graphs

Because only two points are needed to draw a straight line, all we need to do is find two points on the graph and then draw a line passing through these two points. When the equation of a straight line is written in slope–intercept form, one point on the graph is immediately available: the y-intercept. A second point can then be quickly calculated by substituting a suitable value of x into the equation.

When we draw a graph in this manner, we call it a *sketch graph*.

Example 7 Sketching a straight-line graph from its equation

Sketch the graph of $y = 2x + 8$.

Solution

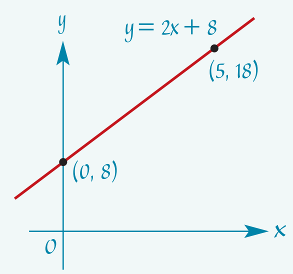
1 Write the equation of the line. $y = 2x + 8$

2 As the equation is in slope–intercept form, the y-intercept is given by the constant term. Write the y-intercept. y-intercept $= 8$

3 Find a second point on the graph. Choose an x-value (not 0) that makes the calculation easy: $x = 5$ would be suitable. When $x = 5, y = 2(5) + 8 = 18$

$\therefore$ (5, 18) is a point on the line.

4 To sketch the graph:
- draw a set of labelled axes
- mark in the two points with coordinates
- draw a straight line through the points
- label the line with its equation.



Exercise 10C

Finding slope and intercept of a straight-line graph from its equation

Example 5

1 Write down the slope and y-intercept of each of the straight lines with the following equations.

a $y = 5 + 2x$	**b** $y = 6 - 3x$	**c** $y = 15 - 5x$	**d** $y + 3x = 10$
e $y = 3x$	**f** $4y + 8x = -20$	**g** $x = y - 4$	**h** $x = 2y - 6$
i $2x - y = 5$	**j** $y - 5x = 10$	**k** $2.5x + 2.5y = 25$	**l** $y - 2x = 0$
m $y + 3x - 6 = 0$	**n** $10x - 5y = 20$	**o** $4x - 5y - 8 = 7$	**p** $2y - 8 = 2(3x - 6)$

ISBN 978-1-009-25891-3

Finding the equation of a straight-line graph given its slope and y-intercept

Example 6

2 Write down the equation of a line that has:

a slope = 5, y-intercept = 2 **b** slope = 10, y-intercept = 5

c slope = 4, y-intercept = −2 **d** slope = −3, y-intercept = 12

e slope = −5, y-intercept = −2 **f** slope = −0.4, y-intercept = 1.8

g slope = −2, y-intercept = 2.9 **h** slope = −0.5, y-intercept = −1.5

Sketching straight-line graphs from their equation

Example 7

3 Sketch the graphs of the straight lines with the following equations, clearly showing the y-intercepts and the coordinates of one other point.

a $y = 2x + 5$ **b** $y = 5x + 5$ **c** $y = 20 - 2x$

d $y = -10 + 10x$ **e** $y = 4x$ **f** $y = 16 - 2x$

10D Finding the equation of a straight-line graph from its intercept and slope

We have learned how to construct a straight-line graph from its equation. We can also determine the equation from a graph. In particular, if the graph shows the y-intercept, it is a relatively straightforward procedure.

Finding the equation of a straight-line graph from its slope and y-intercept

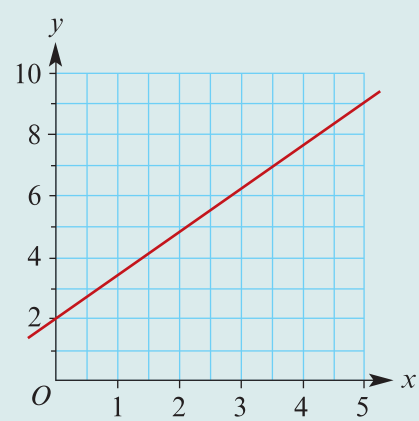
To find the equation of a straight line in slope–intercept form ($y = ax + b$) from its graph:

1. identify the y-intercept (b)
2. use two points on the graph to find the slope (a)
3. substitute these two values into the standard equation $y = ax + b$.

Note: This method *only works* when the graph *scale includes* $x = 0$.

Example 8 Finding the equation of a line: intercept–slope method

Determine the equation of the straight-line graph shown opposite.



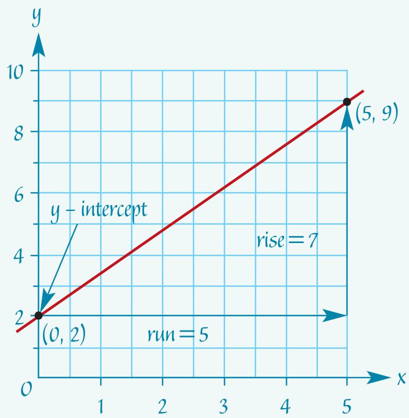
ISBN 978-1-009-25891-3

Solution

1 Write the general equation of a line in slope–intercept form. $y = ax + b$

2 Read the y-intercept from the graph. y-intercept $= 2$ so $b = 2$

3 Find the slope using two well-defined points on the line, for example, (0, 2) and (5, 9).



$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{7}{5} = 1.4 \text{ so } a = 1.4$$

4 Substitute the values of a and b into the equation. $y = 1.4x + 2$

5 Write your answer. $y = 1.4x + 2$ is the equation of the line.

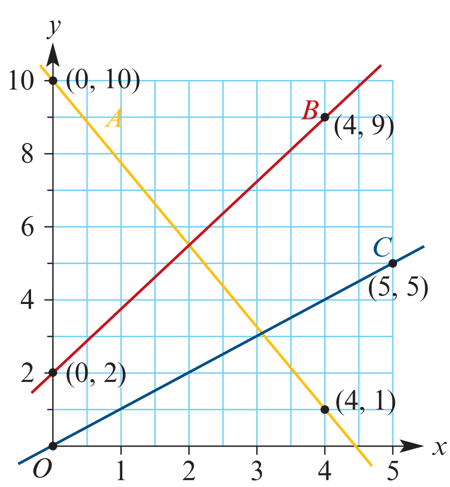
Skill-sheet

Exercise 10D

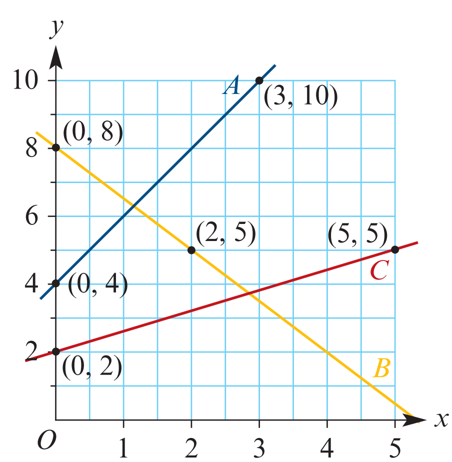
Finding the equation a line from its graph using the slope–intercept method

Example 8

1 Find the equation of each of the lines (A, B, C) shown on the graph below.



2 Find the equations of each of the lines (A, B, C) shown on the graph below.



ISBN 978-1-009-25891-3

10E Finding the equation of a straight-line graph using two points on the graph

Unfortunately, not all straight-line graphs show the y-intercept. When this happens, we have to use the two-point method for finding the equation of the line.

Finding the equation of a straight-line graph using two points

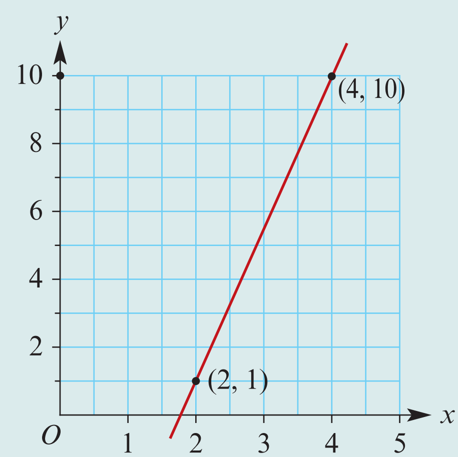
The general equation of a straight-line graph is $y = ax + b$.

1. Use the coordinates of the two points to determine the slope a.
2. Substitute this value for the slope into the equation. There is now only one unknown, b.
3. Substitute the coordinates of one of the two points on the line into this new equation and solve for the unknown b.
4. Substitute the values of a and b into the general equation $y = ax + b$ to obtain the equation of the straight line.

Note: This method works in *all* circumstances.

Example 9 Finding the equation of a straight-line using two points on the graph

Find the equation of the line that passes through the points (2, 1) and (4, 10).



Solution

1. Write down the general equation of a straight-line graph.

   $y = ax + b$

2. Use the coordinates of the two points on the line to find the slope a.

   Note: If the slope of the line does not simplify to a terminating decimal it is acceptable to leave its value as a simplified fraction. You are encouraged to do this to avoid unnecessary rounding.

   $\text{slope} = \dfrac{\text{rise}}{\text{run}} = \dfrac{10 - 1}{4 - 2} = \dfrac{9}{2} = 4.5$

   so $a = 4.5$

3. Substitute the value of a into the general equation.

   $y = 4.5x + b$

ISBN 978-1-009-25891-3

4 To find the value of b, substitute the coordinates of one of the points on the line (either will do) and solve for b.

Using the point (2, 1):
$1 = 4.5(2) + b$
$1 = 9 + b$
$b = -8$

5 Substitute the values of a and b into the general equation $y = ax + b$ to find the equation of the line.

Thus, the equation of the line is:
$y = 4.5x - 8$

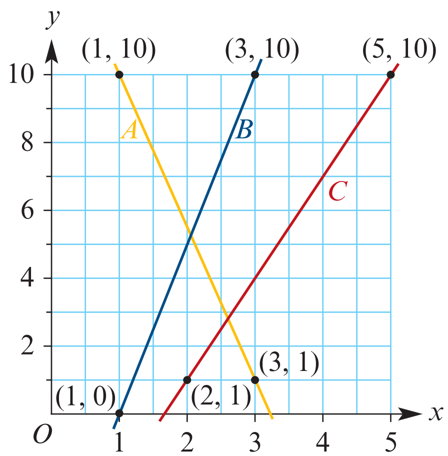
Skill-sheet

Exercise 10E

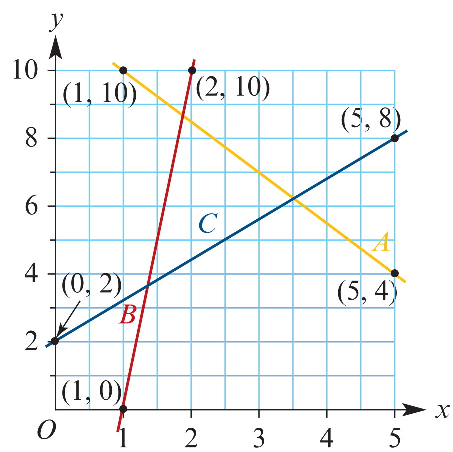
Finding the equation of a line given any two points on its graph

Example 9

1 Find the equation of each of the lines (A, B, C) on the graph below. Write your answers in the form $y = ax + b$.



2 Find the equations of each of the lines (A, B, C) on the graph below. Write your answers in the form $y = ax + b$.



10F Finding the equation of a straight-line graph from two points using a CAS calculator

While the slope–intercept method of finding the equation of a line from its graph is relatively quick and easy to apply, using the two-point method to find the equation of a line can be time consuming. An alternative to using either of these methods is to use the line-fitting facility of your CAS calculator. You will meet this method again when you study the topic ‘Investigating relationships between two numerical variables’ later in the year.

ISBN 978-1-009-25891-3

How to find the equation of a line from two points using the TI-Nspire CAS

Find the equation of the line that passes through the two points (2, 1) and (4, 10).

Steps

1 Write the coordinates of the two points. Label one point A, the other B.

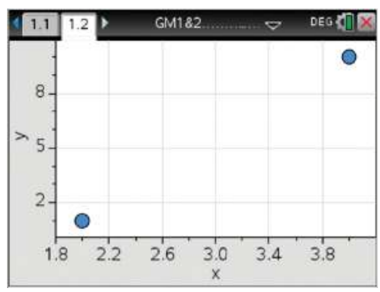
2 Start a new document (ctrl + N) and select **Add Lists & Spreadsheet**.
Enter the coordinate values into lists named ***x*** and ***y***.

3 Plot the two points on a scatterplot. Press ctrl + N and select **Add Data & Statistics**.
(or press on and arrow to the Data & Statistics icon and press enter)
Note: A random display of dots will appear – this is to indicate list data is available for plotting. It is not a statistical plot.

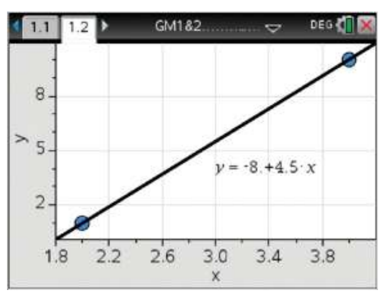
4 To construct a scatterplot:

a Press tab and select the variable ***x*** from the list. Press enter to paste the variable ***x*** to the ***x***-axis.

b Press tab again and select the variable ***y*** from the list. Press enter to paste the variable ***y*** to the ***y***-axis axis to generate the required scatter plot.



5 Use the **Regression** command to draw a line through the two points and determine its equation.
Press menu **>Analyze>Regression>Show Linear (a+bx)** and enter to complete the task.
Correct to one decimal place, the equation of the line is: $y = -8.0 + 4.5x$.



6 Write your answer.

The equation of the line is $y = -8 + 4.5x$ or $y = 4.5x - 8$.

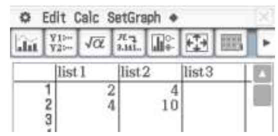
ISBN 978-1-009-25891-3

How to find the equation of a line from two points using the ClassPad

Find the equation of the line that passes through the two points (2, 4) and (4, 10).

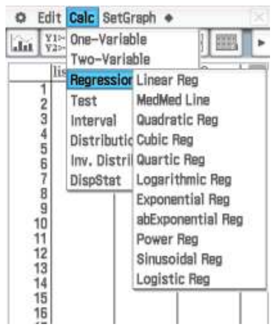
Steps

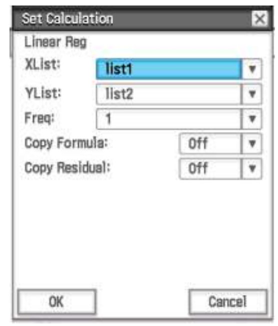
1 Open the **Statistics** application and enter the coordinate values into the lists as shown.



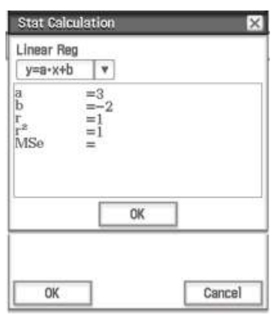
2 To find the equation of the line $y = ax + b$ that passes through the two points:

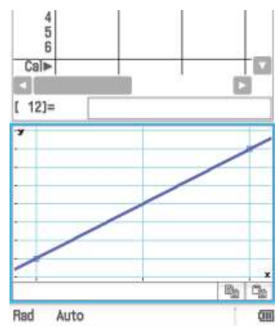
- Select **Calc** from the menu bar
- Select **Regression** and **Linear Reg**
- Ensure that the **Set Calculation** dialog box is set as shown
- Press OK.





3 The results are given in a **Stat Calculation** dialog box.





4 Write your answer.

The equation of the line is $y = 3x - 2$.

Note: Tapping **OK** will automatically display the graph window with the line drawn through the two points. This confirms that the line passes through the two points.

ISBN 978-1-009-25891-3

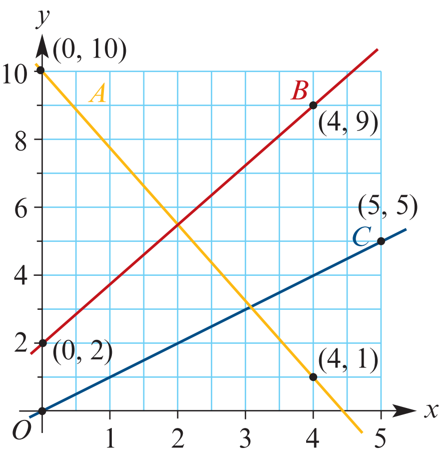
Exercise 10F

Using a CAS calculator to find the equation of a line from two points

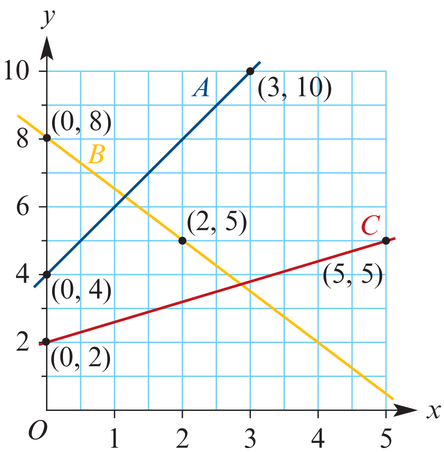
Note: This exercise repeats Exercises 10D and 10E, but this time using a CAS calculator.

CAS

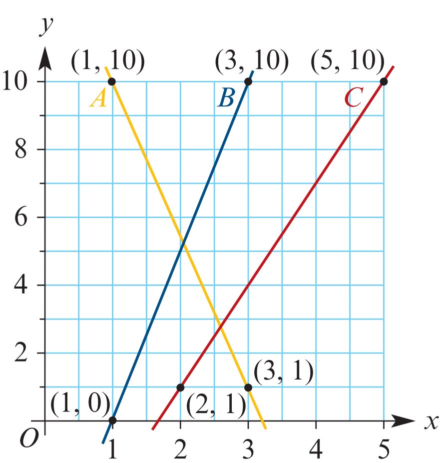
1 Use a CAS calculator to find the equation of each of the lines (*A*, *B*, *C*) shown on the graph below. Write your answers in the form $y = ax + b$.



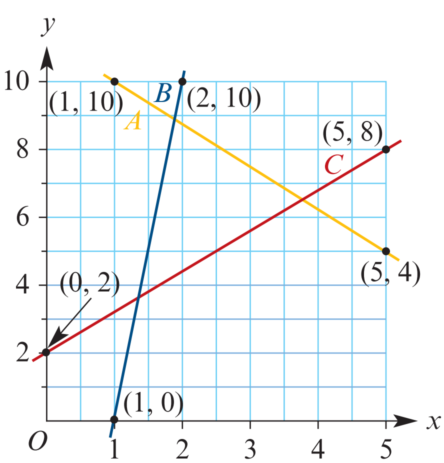
2 Use a CAS calculator to find the equation of each of the lines (*A*, *B*, *C*) shown on the graph below. Write your answers in the form $y = ax + b$.



3 Use a CAS calculator to find the equation of each of the lines (*A*, *B*, *C*) shown on the graph below. Write your answers in the form $y = ax + b$.



4 Use a CAS calculator to find the equation of each of the lines (*A*, *B*, *C*) shown on the graph below. Write your answers in the form $y = a + bx$.



ISBN 978-1-009-25891-3

10G Linear modelling

Many real life relationships between two variables can be described mathematically by linear (straight-line) equations. This is called *linear modelling*.

These linear models can be then used to solve problems such as finding the time taken to fill a partially filled swimming pool with water, estimating the depreciating value of a car over time or describing the growth of a plant over time.

Modelling plant growth with a linear equation

Some plants, such as tomato plants, grow remarkably quickly.

When first planted, the height of this plant was 5 cm.

The plant then grows at a constant rate of 6 cm per week for the next 10 weeks.

From this information, we can now construct a mathematical model that can be used to chart the growth of the plant over the following weeks and predict its height at any time during the first 10 weeks after planting.

Start   After 1 week   After 2 weeks

Constructing a linear model

Let h be the height of the plant (in cm).

Let t be the time (in weeks) after it was planted.

For a linear growth model we can write:

$$h = a\,t + b$$

where:

- a is the constant rate at which the plant's height increases each week; in this case, 6 cm per week (in graphical terms, the slope of the line).
- b is the initial height of the plant; in this case, 5 cm (in graphical terms, the y-intercept)

Thus we can write: $h = 6t + 5$ for $0 \le t \le 10$

The graph for this model is plotted on the next page.

ISBN 978-1-009-25891-3

Three important features of the linear model $h = 6t + 5$ for $0 \le t \le 10$ should be noted:

- The *h-intercept* gives the height of the plant at the start; that is, its height when $t = 0$. The plant was 5 cm tall when it was first planted.
- The *slope* of the graph gives the growth rate of the plant. The plant grows at a rate of 6 cm per week; that is, each week the height of the plant increases by 6 cm.
- The graph is only plotted for $0 \le t \le 10$. This is because the model is only valid for the time when the plant is growing at the constant rate of 6 cm a week.

Note: The expression $0 \le t \le 10$ is included to indicate the range of number of weeks for which the model is valid. In more formal language this would be called the domain of the model.

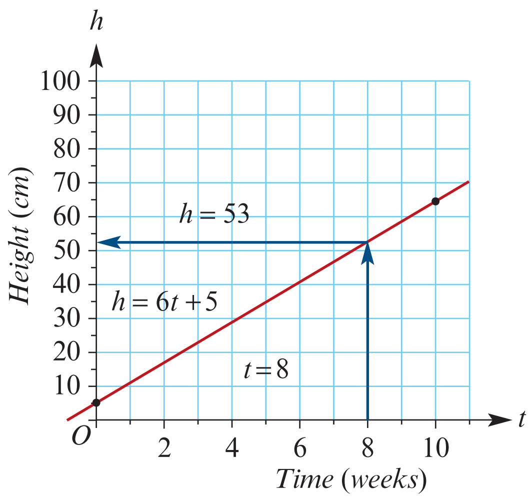
Using a linear model to make predictions

To use the mathematical model to make predictions, we simply substitute a value of t into the model and evaluate.

For example, after eight weeks growth ($t = 8$), the model predicts the height of the plant to be:

$$h = 6(8) + 5 = 53 \text{ cm}$$

This value could also be read directly from the graph, as shown below.



Exercise 10G-1

Constructing and analysing linear models

1 A tree is 910 cm tall when first measured. For the next five years its height increases at a constant rate of 16 cm per year.
Let h be the height of the tree (in cm).
Let t the time in years after the tree was first measured.

- **a** Write down a linear model in terms of h and t to represent this situation.
- **b** Sketch the graph showing the coordinates of the h-intercept and its end point.
- **c** Use the model to predict the height of the tree 4.5 years after it was first measured.

ISBN 978-1-009-25891-3

2 An empty 20 L cylindrical beer keg is to be filled with beer at a constant rate of 5 litres per minute.
Let V be the volume of beer in the keg after t minutes.

a The beer keg is filled in 4 minutes, write down a linear model in terms of V and t to represent this situation.

b Sketch the graph showing the coordinates of the V-intercept and its end point.

c Use the model to predict the volume of beer in the keg after 3.2 minutes.

3 A home waste removal service charges \$80 to come to your property. It then charges \$120 for each cubic metre of waste it removes. The maximum amount of waste that can be removed in one visit is 8 cubic metres.
Let c be the total charge for removing w cubic metres of waste.

a Write down a linear model in terms of c and w to represent this situation.

b Sketch the graph showing the coordinates of the c-intercept and its end point.

c Use the model to predict the cost of removing 5 cubic metres of waste.

4 A motorist fills the tank of her car with unleaded petrol, which costs \$1.57 per litre. Her tank can hold a maximum of 60 litres of petrol. When she started filling her tank, there was already 7 litres in her tank.
Let c be the cost of adding v litres of petrol to the tank.

a Write down a linear model in terms of c and v to represent this situation.

b Sketch the graph of showing the coordinates of the c-intercept and its end point.

c Use the model to predict the cost of filling the tank of her car with petrol.

5 A business buys a new photocopier for \$25 000. It plans to depreciate its value by \$4000 per year for five years, at which time it will be sold.
Let V be the value of the photocopier after t years.

a Write down a linear model in terms of V and t to represent this situation.

b Sketch the graph showing the coordinates of the V-intercept and its end point.

c Use the model to predict the depreciated value of the photocopier after 2.6 years.

6 A swimming pool when full contains 10 000 litres of water. Due to a leak, it loses on average 200 litres of water per day.
Let V be the volume of water remaining in the pool after t days.

a The pool continues to leak. How long will it take to empty the pool?

b Write down a linear model in terms of V and t to represent this situation.

c Sketch the graph showing the coordinates of the V-intercept and its end point.

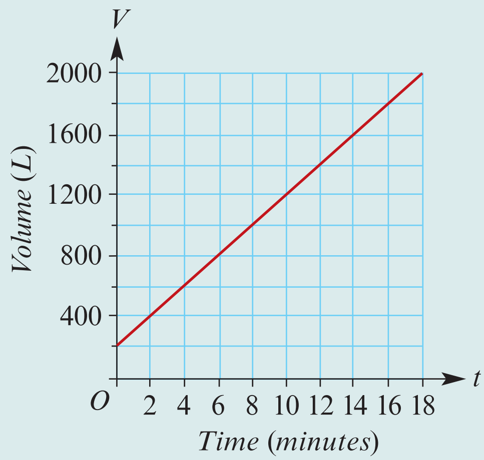
d Use the model to predict the volume of water left in the pool after 30 days.

ISBN 978-1-009-25891-3

Interpreting and analysing the graphs of linear models

Example 10 Graphs of linear models with a positive slope

Water is pumped into a partially full tank. The graph gives the volume of water V (in litres) after t minutes.



a How much water is in the tank at the start ($t = 0$)?

b How much water is in the tank after 10 minutes ($t = 10$)?

c The tank holds 2000 L. How long does it take to fill?

d Find the equation of the line in terms of V and t.

e Use the equation to calculate the volume of water in the tank after 15 minutes.

f At what rate is the water pumped into the tank; that is, how many litres are pumped into the tank each minute?

Solution

a Read from the graph (when $t = 0$, $V = 200$). 200 L

b Read from the graph (when $t = 10$, $V = 1200$). 1200 L

c Read from the graph (when $V = 2000$, $t = 18$). 18 minutes

d The equation of the line is $V = at + b$. $V = at + b$

b is the V-intercept. Read from the graph. $b = 200$

a is the slope. Calculate using two points on the graph, say (0, 200) and (18, 2000).

$$a = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{2000 - 200}{18 - 0}$$
$$= 100$$

Note: You can use your calculator to find the equation of the line if you wish.

$\therefore\ V = 100t + 200\ \ (t \geq 0)$

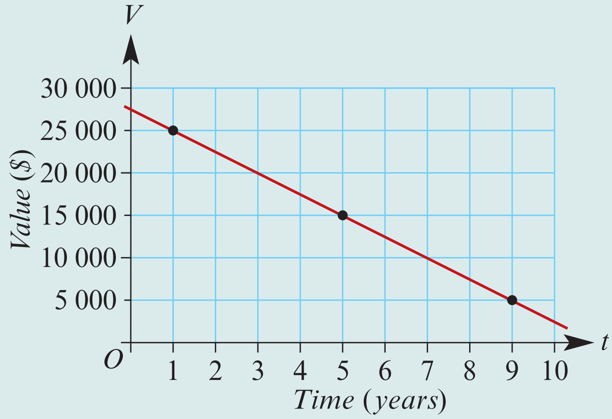
e Substitute $t = 15$ into the equation. Evaluate. $V = 100(15) + 200 = 1700$ L

f The rate at which water is pumped into the tank is given by the slope of the graph, 100 (from **d**). 100 L/min

ISBN 978-1-009-25891-3

Example 11 Graphs of linear models with a negative slope

The value of new cars depreciates with time. The graph shows how the value V (in dollars) of a new car depreciates with time t (in years).



a What was the value of the car when it was new?

b What was the value of the car when it was 5 years old?

c Find the equation of the line in terms of V and t.

d At what rate does the value of the car depreciate with time; that is, by how much does its value decrease each year?

e When does the equation predict the car will have no (zero) value?

Solution

a Read from the graph (when $t = 0$, $V = 27\,500$).

$\$27\,500$

b Read from the graph (when $t = 5$, $V = 15\,000$).

$\$15\,000$

c The equation of the line is $V = at + b$.

- b is the V-intercept. Read from the graph.
- a is the slope. Calculate using two points on the graph, say $(1, 25\,000)$ and $(9, 5000)$.

Note: You can use your calculator to find the equation of the line if you wish.

$V = at + b$

$b = 27\,500$

$$a = \text{slope} = \frac{25\,000 - 5000}{1 - 9}$$

$$= -2500$$

$\therefore V = -2500t + 27\,500$

or $V = 27\,500 - 2500t$ for $t \geq 0$

d The slope of the line is -2500, so the car depreciates in value by $2500 per year.

$\$2500$ per year

e Substitute into the equation and solve for t.

$0 = -2500t + 27\,500$

$2500t = 27\,500$

$$\therefore t = \frac{27\,500}{2500} = 11 \text{ years}$$

ISBN 978-1-009-25891-3

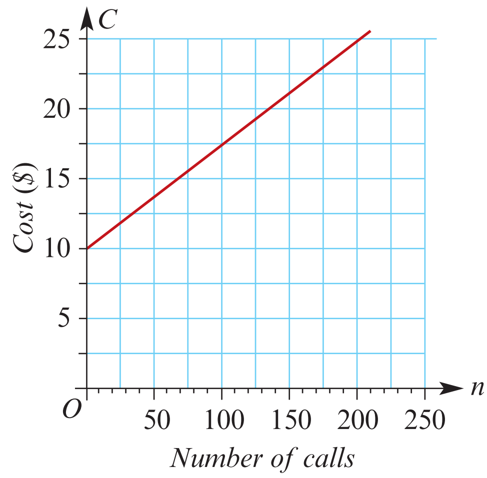
Exercise 10G-2

Interpreting the graphs of linear models in their context

Example 10

1 A phone company charges a monthly service fee plus the cost of calls. The graph opposite gives the total monthly charge, C dollars, for making n calls. This includes the service fee.

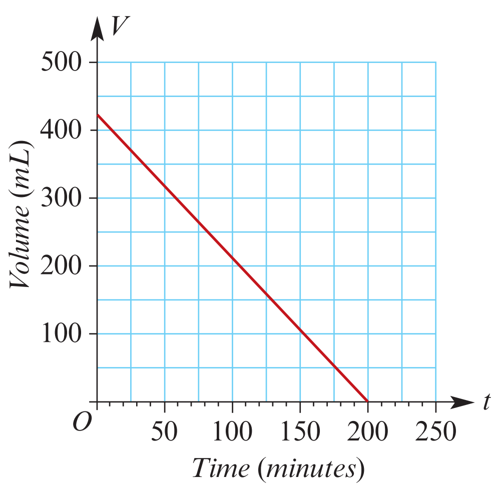
- **a** How much is the monthly service fee ($n = 0$)?
- **b** How much does the company charge if you make 100 calls a month?
- **c** Find the equation of the line in terms of C and n.
- **d** Use the equation to calculate the cost of making 300 calls in a month.
- **e** How much does the company charge per call?



Example 11

2 The graph opposite shows the volume of saline solution, V mL, remaining in the reservoir of a saline drip after t minutes.

- **a** How much saline solution was in the reservoir at the start?
- **b** How much saline solution remains in the reservoir after 40 minutes? Read the result from the graph.
- **c** How long does it take for the reservoir to empty?
- **d** Find the equation of the line in terms of V and t.
- **e** Use the equation to calculate the amount of saline solution in the reservoir after 115 minutes.
- **f** At what rate (in mL/minute) is the saline solution flowing out of the drip?



ISBN 978-1-009-25891-3

3 The graph opposite can be used to convert temperatures in degrees Celsius (C) to temperatures in degrees Fahrenheit (F).

a Find the equation of the line in terms of F and C.

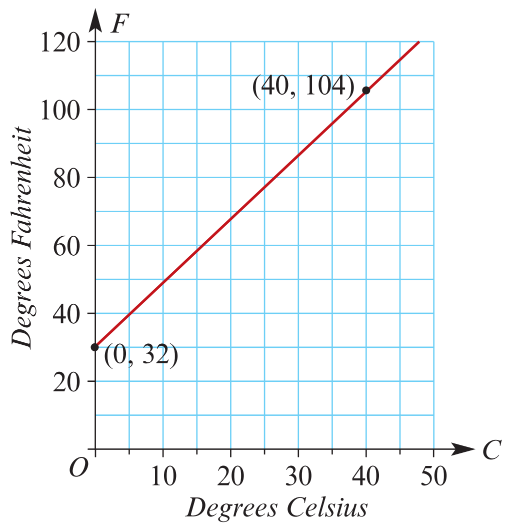
b Use the equation to calculate the temperature in degrees Fahrenheit when the temperature in degrees Celsius is:

i 50°C **ii** 150°C

iii −40°C

c Complete the following sentence by filling in the box.

When the temperature in Celsius increases by 1 degree, the temperature in Fahrenheit increases by $\boxed{}$ degrees.



Piecewise linear graphs

Sometimes a situation requires two linear graphs to obtain a suitable model. The graphs we use to model such situations are called **piecewise linear graphs**.

Example 12 Constructing a piecewise linear graph model

The amount, C dollars, charged to supply and deliver $x\,\text{m}^3$ of crushed rock is given by the equations:

$$C = 40x + 50 \qquad (0 \le x < 3)$$

$$C = 30x + 80 \qquad (3 \le x < 8)$$

a Use the appropriate equation to determine the cost to supply and deliver the following amounts of crushed rock.

i $2.5\,\text{m}^3$

ii $3\,\text{m}^3$

iii $6\,\text{m}^3$

b Use the equations to construct a piecewise linear graph for $0 \le x \le 8$.

ISBN 978-1-009-25891-3

Solution

a **1** Write the equations.

$C = 40x + 50 \ (0 \le x < 3)$

$C = 30x + 80 \ (3 \le x \le 8)$

2 Then, in each case:
- choose the appropriate equation.
- substitute the value of x and evaluate.
- write your answer.

i When $x = 2.5$

$C = 40(2.5) + 50 = 150$

Cost for 2.5 m^3 of crushed rock is \$150.

ii When $x = 3$

$C = 30(3) + 80 = 170$

Cost for 3 m^3 of crushed rock is \$170.

iii When $x = 6$

$C = 30(6) + 80 = 260$

Cost for 6 m^3 of crushed rock is \$260.

b The graph has two line segments.

1 Determine the coordinates of the end points of both lines.

$x = 0 : C = 40(0) + 50 = 50$

$x = 3 : C = 40(3) + 50 = 170$

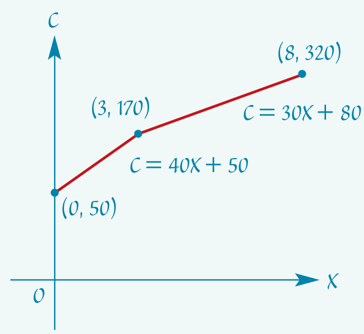
$x = 3 : C = 30(3) + 80 = 170$

$x = 8 : C = 30(8) + 80 = 320$

2 Draw a set of labelled axes and mark in the points with their coordinates.

3 Join up the end points of each line segment with a straight line.

4 Label each line segment with its equation.



Step-graphs

A **step-graph** can be used to represent information that is constant for particular intervals. One example of this is the cost of sending an airmail letter.

The table on the right shows that the cost of sending a letter is constant for intervals of mass. For example, a letter will cost \$2.00 to send if it has a mass that is over 50 g up to 100 g.

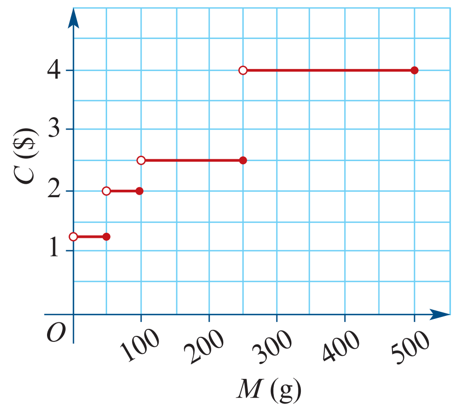
Mass	Cost
Up to 50 g	\$1.20
Over 50 g and up to 100 g	\$2.00
Over 100 g and up to 250 g	\$2.50
Over 250 g and up to 500 g	\$4.00

We must be careful when considering the cost of letters that have a mass at the end points of the intervals.

ISBN 978-1-009-25891-3

A letter that has a mass of exactly 100 g belongs in the second interval because this interval includes masses up to 100 g. It does not belong in the third interval because this is for masses over 100 g, not equal to 100 g.

Each interval of mass from the table is shown as a horizontal line segment on the graph. The end points are open circles if the mass is *not* included in the interval and closed circles if the mass *is* included.



Example 13 Constructing a step-graph

The entry fee for a music competition depends on the age of the competitor, as shown in the table on the right.

Age of competitor	Entry fee
under 4 years	$5.00
5 years – under 10 years	$8.00
10 years – under 15 years	$15.00
15 years – under 20 years	$25.00
20 years and over	$30.00

a What is the entry fee for a competitor who is 14 years and 9 months?

b Is the circle for the horizontal line segment that **ends** at 20 years open or closed?

c Sketch the step-graph that shows the *entry fee* against the *age of competitor*.

Solution

a 14 years and 9 months is over 10 years and under 15 years.

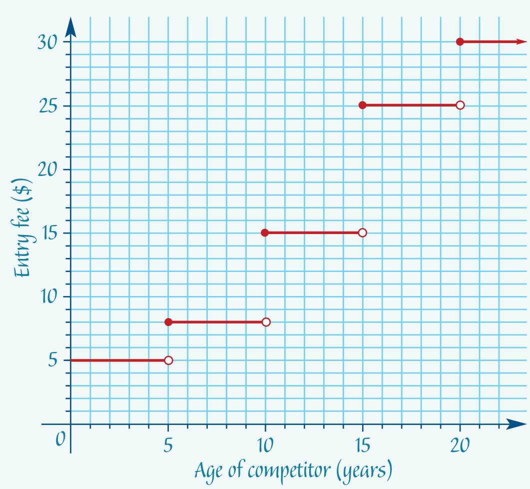
The entry fee for a competitor who is 14 yrs and 9 months old is $15.00.

b The interval that ends at 20 years is 15 years – under 20 years.

Since 'under 20 years' does not include exactly 20 years, the circle will be open.

c For each interval of age, draw an appropriate line segment. The upper end of each interval will have an open circle.

Note: *The interval '20 years and over' does not have an exact end point. An arrow can be used to indicate that this interval extends beyond the graph.*

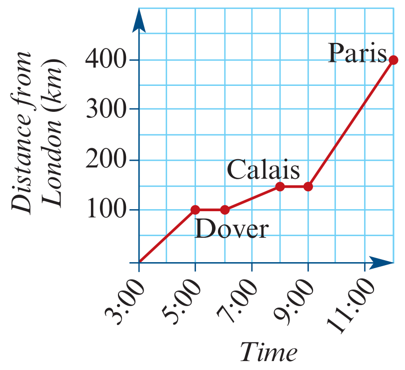


ISBN 978-1-009-25891-3

Exercise 10G-3

Example 12

1 The graph shows a man's journey by train, boat and train from London to Paris.



a At what time does he:

i arrive at Dover?

ii arrive in Paris?

iii leave Calais?

iv leave Dover?

b For how long does he stop at Dover?

c At what time is he exactly halfway between Calais and Paris?

Using a piecewise linear graph to model and analyse practical situations

2 An empty tank is being filled from a mountain spring. For the first 30 minutes, the equation giving the volume, V, of water in the tank (in litres) at time t minutes is:

$$V = 15t \quad (0 \le t \le 30)$$

After 30 minutes, the flow from the spring slows down. For the next 70 minutes, the equation giving the volume of water in the tank at time, t, as given by the equation:

$$V = 10t + 150 \quad (30 < t \le 100)$$

a Use the appropriate equation to determine the volume of water in the tank after:

i 20 minutes **ii** 30 minutes **iii** 60 minutes **iv** 100 minutes

b Use the equations to construct a piecewise linear graph for $0 \le t \le 100$.

3 For the first 25 seconds of the journey of a train between stations, the speed, S, of the train (in metres/second) after t seconds is given by:

$$S = 0.8t \quad (0 \le t \le 25)$$

For the next 180 seconds, the train travels at a constant speed of 20 metres/second as given by the equation:

$$S = 20 \quad (25 < t \le 205)$$

Finally, after travelling for 205 seconds, the driver applies the brakes and the train comes to rest after a further 25 seconds as given by the equation:

$$S = 184 - 0.8t \quad (205 < t \le 230)$$

a Use the appropriate equation to determine the speed of the train after:

i 10 seconds **ii** 60 seconds **iii** 180 seconds **iv** 210 seconds

b Use the equations to construct a piecewise linear graph for $0 \le t \le 230$.

ISBN 978-1-009-25891-3

Using a step-graph to model and analyse practical situations

Example 13

4 The postal rates for letters for a particular country are shown in this table. Sketch a step-graph to represent this information.

Weight not over	Rate	Weight not over	Rate
60 g	34c	350 g	$1.22
100 g	48c	450 g	$1.38
150 g	62c	500 g	$1.56
200 g	76c	750 g	$2.56
250 g	90c	1000 g	$3.40
300 g	$1.06		

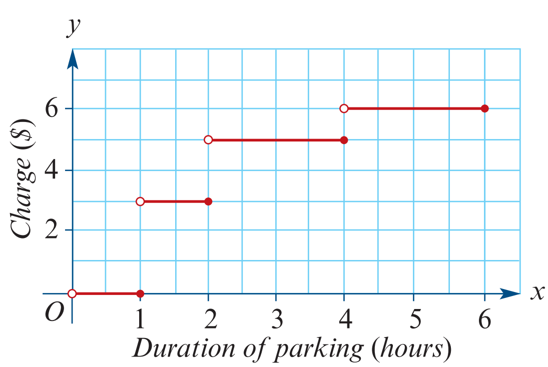
5 A multistorey car park has tariffs as shown. Sketch a step-graph showing this information.

Time in car park, t	Tariff
$0 < t \leq 2$	$5.00
$2 < t \leq 3$	$7.50
$3 < t \leq 4$	$11.00
$4 < t \leq 8$	$22.00

6 Suppose that Australia Post charged the following rates for airmail letters to Africa: $1.20 up to 20 g; $2.00 over 20 g and up to 50 g; $3.00 over 50 g and up to 150 g. Sketch a graph to represent this information.

7 This step-graph shows the charges for a market car park.

a How much does it cost to park for 40 minutes?
b How much does it cost to park for 2 hours?
c How much does it cost to park for 3 hours?



ISBN 978-1-009-25891-3

Key ideas and chapter summary

Assign-ment

Slope of a straight-line graph

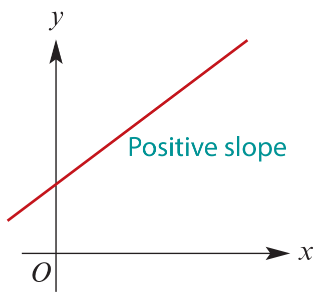
Slope of a straight-line graph is defined to be:

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

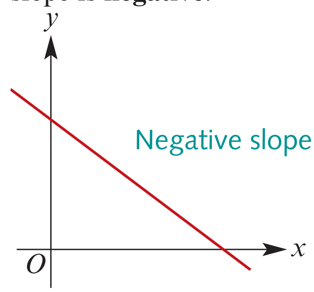
where (x_1, y_1) and (x_2, y_2) are two points on the line.

Positive and negative slope

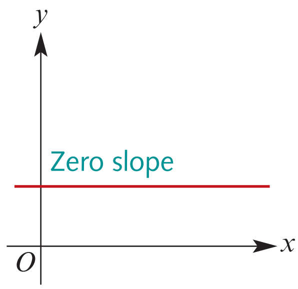
If the line rises to the right, the slope is **positive**.



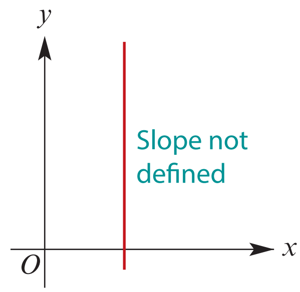
If the line falls to the right, the slope is **negative**.



If the line is horizontal, the slope is **zero**.



If the line is vertical, the slope is **undefined**.



Equation of straight-line graph: the intercept–slope form

The equation of a straight line can take several forms.
The **slope–intercept form** is:

$$y = ax + b$$

where a is the **slope** and b is the ***y*-intercept** of the line.

Linear model

A **linear model** has a linear equation or relation of the form:

$$y = ax + b \quad \text{where} \quad c \le x \le d$$

where a, b, c and d are constants.

ISBN 978-1-009-25891-3

Piecewise linear graphs — **Piecewise linear graphs** are used in practical situations where more than one linear equation is needed to model the relationship between two variables.

Step-graphs — A **step graph** consists of one or more horizontal line segments. They can be used to graphically represent situations where the value of one variable is constant for intervals of another variable.

Skills check

Having completed this chapter you should be able to:

- recognise a linear equation written in slope–intercept form
- determine the slope and y-intercept of a straight-line graph from its equation
- determine the slope of a straight line from its graph
- determine the y-intercept of a straight line from its graph (if shown)
- determine the equation of a straight line, given its graph
- construct a linear model to represent a practical situation using a linear equation or a straight-line graph
- interpret the slope and the intercept of a straight-line graph in terms of its context and use the equation to make predictions
- construct a piecewise linear graph used to model a practical situation
- construct and interpret a step-graph used to model a practical situation.

Short-answer questions

1 The equation of a straight line is $y = 3x + 4$. When $x = 2$, state the value of y.

2 The equation of a straight line is $y = 4x + 5$. State the y-intercept.

3 The equation of a straight line is $y = 10 - 3x$. State the slope.

4 The equation of a straight line is $y - 2x = 3$. State the gradient.

5 Find the slope of the line passing through the points (5, 8) and (9, 5).

6 Sketch the graph of $y = 5x + 4$, stating any two points on the line.

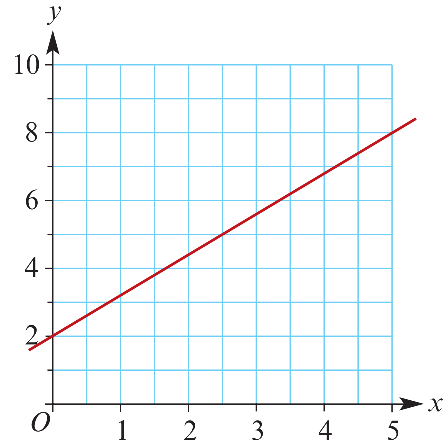
7 Sketch the graph of $y = -3x + 15$, stating any two points on the line.

 ISBN 978-1-009-25891-3

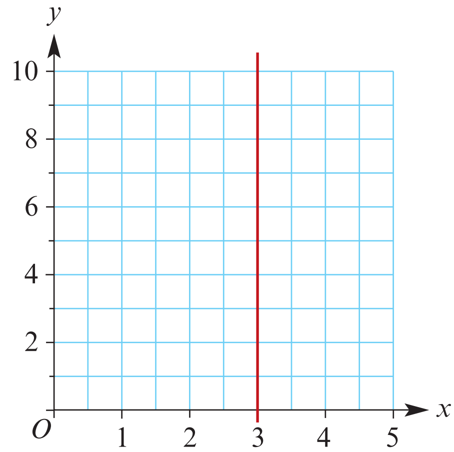
Questions 8 and 9 relate to the following graph

8 State the y-intercept.

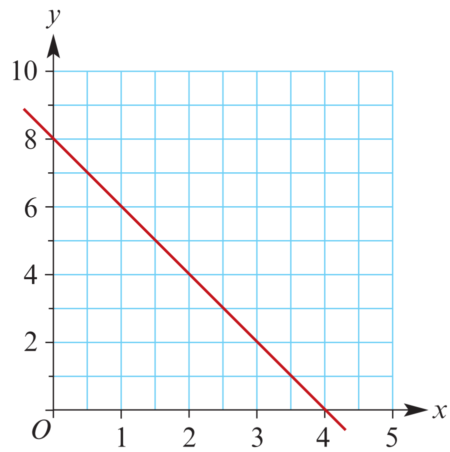
9 State the slope.



10 State the slope of the line in the graph shown.

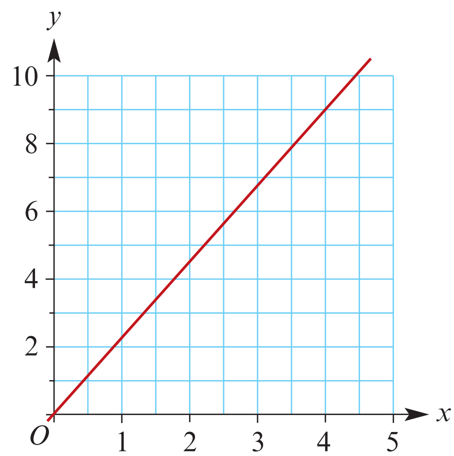


11 State the equation of the graph shown.



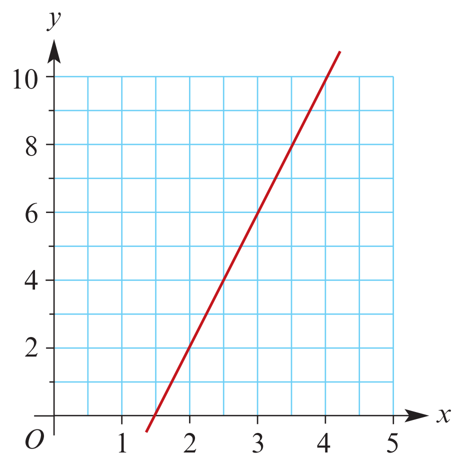
ISBN 978-1-009-25891-3

12 State the equation of the graph shown.

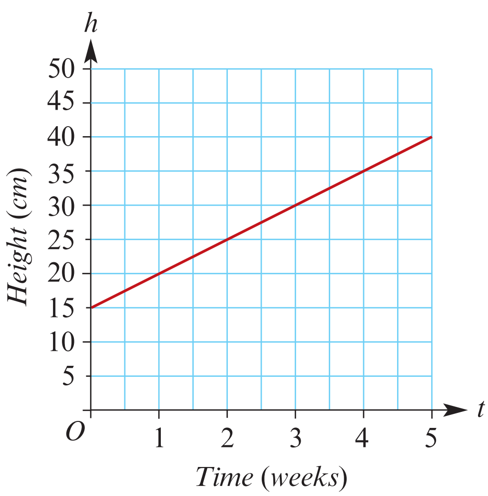


13 Does the point $(1, 5)$ lie on the line $y = 10x - 5$?

14 Express the equation of the line graph shown in the form $y = ax + b$.



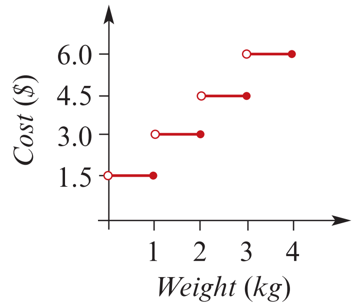
15 The graph opposite shows the height of a small sapling, h, as it increases with time, t. State the growth rate.



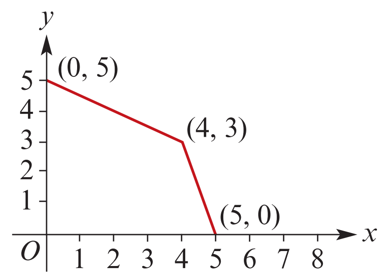
ISBN 978-1-009-25891-3

Review

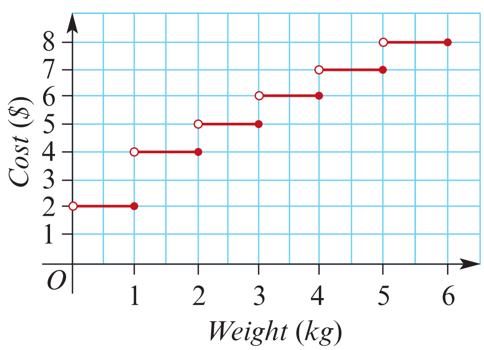
16 The graph shows the cost of posting parcels of various weights. How much will a parcel weighing 3 kilograms cost?



17 Determine the piecewise function represented by the diagram shown.



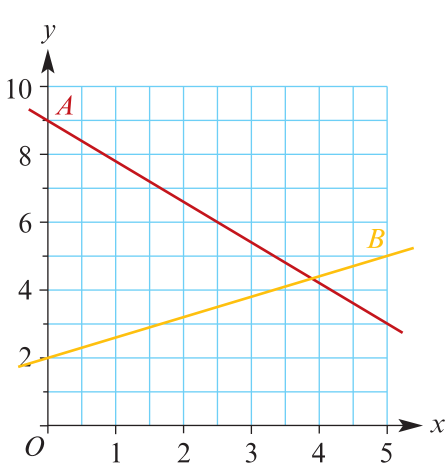
18 The graph shows the cost of posting parcels of various weights. A person posts two parcels, one weighing 3 kg and the other 1.5 kg. If each parcel is charged for separately, find the cost of sending the two parcels.



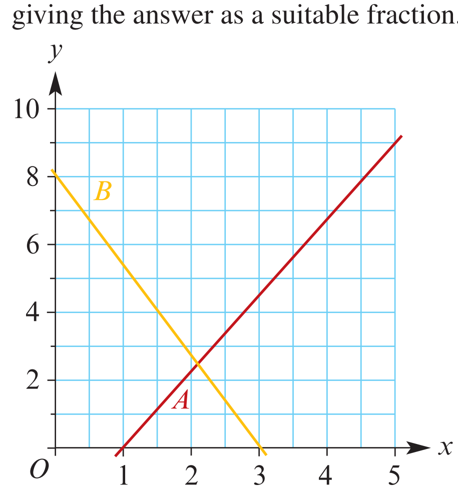
19 Plot the graphs of these linear relations by hand.

a $y = 5x + 2$ **b** $y = 12 - x$ **c** $y = 4x - 2$

20 Find the slope of each of the lines A and B shown on the graph below.



21 Find the slope of each of the lines A and B shown on the graph below, giving the answer as a suitable fraction.



ISBN 978-1-009-25891-3

22 A linear model for the amount C, in dollars, charged to deliver w cubic metres of builder's sand is given by $c = 110w + 95$ for $0 \leq w \leq 7$.

- **a** Use the model to determine the total cost of delivering 6 cubic metres of sand.
- **b** When the initial cost of \$95 is paid, what is the cost for each additional cubic metre of builder's sand?

23 **a** Sketch the piecewise linear graph defined by the rules:

$$y = x + 2 \quad 0 < x \leq 2$$

$$y = 5 \quad 2 < x \leq 4$$

$$y = -\frac{1}{2}x \quad 4 < x \leq 6$$

- **b** What is the value of y when $x = 5$?

24 An olive farm sells bottles of olive oil to visitors. The cost per bottle depends on the number of bottles purchased, as shown in the table below.

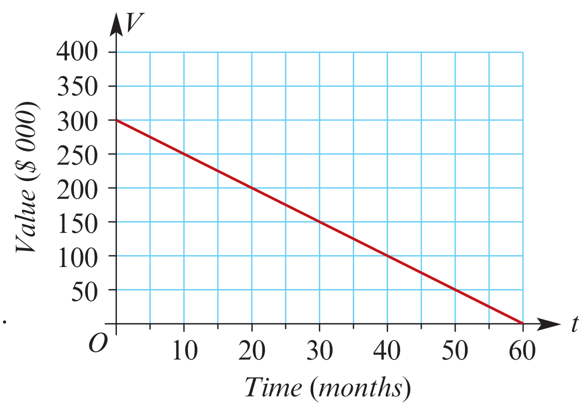
Number of bottles	Price per bottle
Up to 4	\$6.50
5 up to 9	\$6.00
10 up to 14	\$5.50
15 up to 20	\$5.00

- **a** What is the price per bottle if exactly 15 bottles of olive oil are purchased?
- **b** How much will it cost to purchase 6 bottles of olive oil?
- **c** Show the information from the table on a step-graph.

Extended-response questions

1 A new piece of machinery is purchased by a business for \$300 000. Its value is then depreciated each month using the following graph.

- **a** What is the value of the machine after 20 months?
- **b** When does the line predict that the machine has no value?
- **c** Find the equation of the line in terms of value V and time t.
- **d** Use the equation to predict the value of the machine after 3 years.
- **e** By how much does the machine depreciate in value each month?



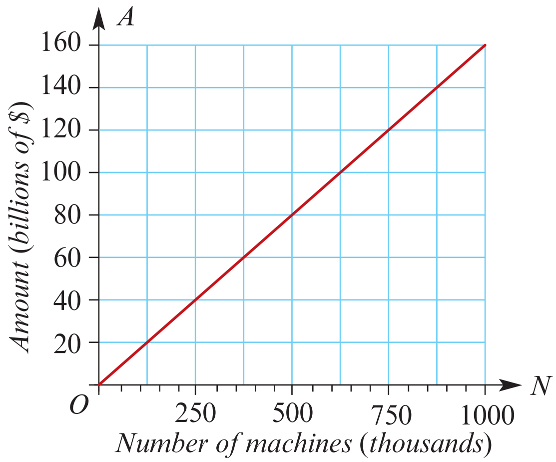
ISBN 978-1-009-25891-3

2 The amount of money transacted through ATMs has increased with the number of ATMs available. The graph charts this increase.

a What was the amount of money transacted through ATMs when there were 500 000 machines?

b Find the equation of the line in terms of amount of money transacted, A, and number of ATMs, N. (Leave A in billions and N in thousands).

c Use the equation to predict the amount transacted when there were 600 000 machines.



d If the same rule applies, how much money is predicted to be transacted through ATM machines when there are 1 500 000 machines?

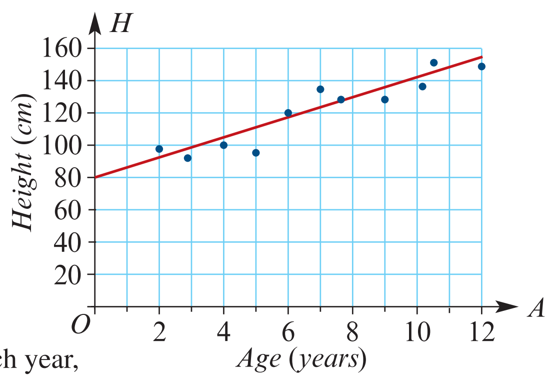
e By how much does the amount of money transacted through ATMs increase with each 1000 extra ATMs?

3 The heights, H, of a number of children are shown plotted against age, A. Also shown is a line of best fit.

a Find the equation of the line of best fit in terms of H and A.

b Use the equation to predict the height of a child aged 3.

c Complete the following sentence by filling in the box. The equation of the line of best fit tells us that, each year, children's heights increase by $\boxed{}$ cm.



4 To conserve water one charging system increases the amount people pay as the amount of water used increases. The charging system is modelled by:

$$C = 0.4x + 5 \ (0 \le x < 30) \qquad C = 1.6x - 31 \ (x \ge 30)$$

C is the charge in dollars and x is the amount of water used in kilolitres (kL).

a Use the appropriate equation to determine the charge for using:

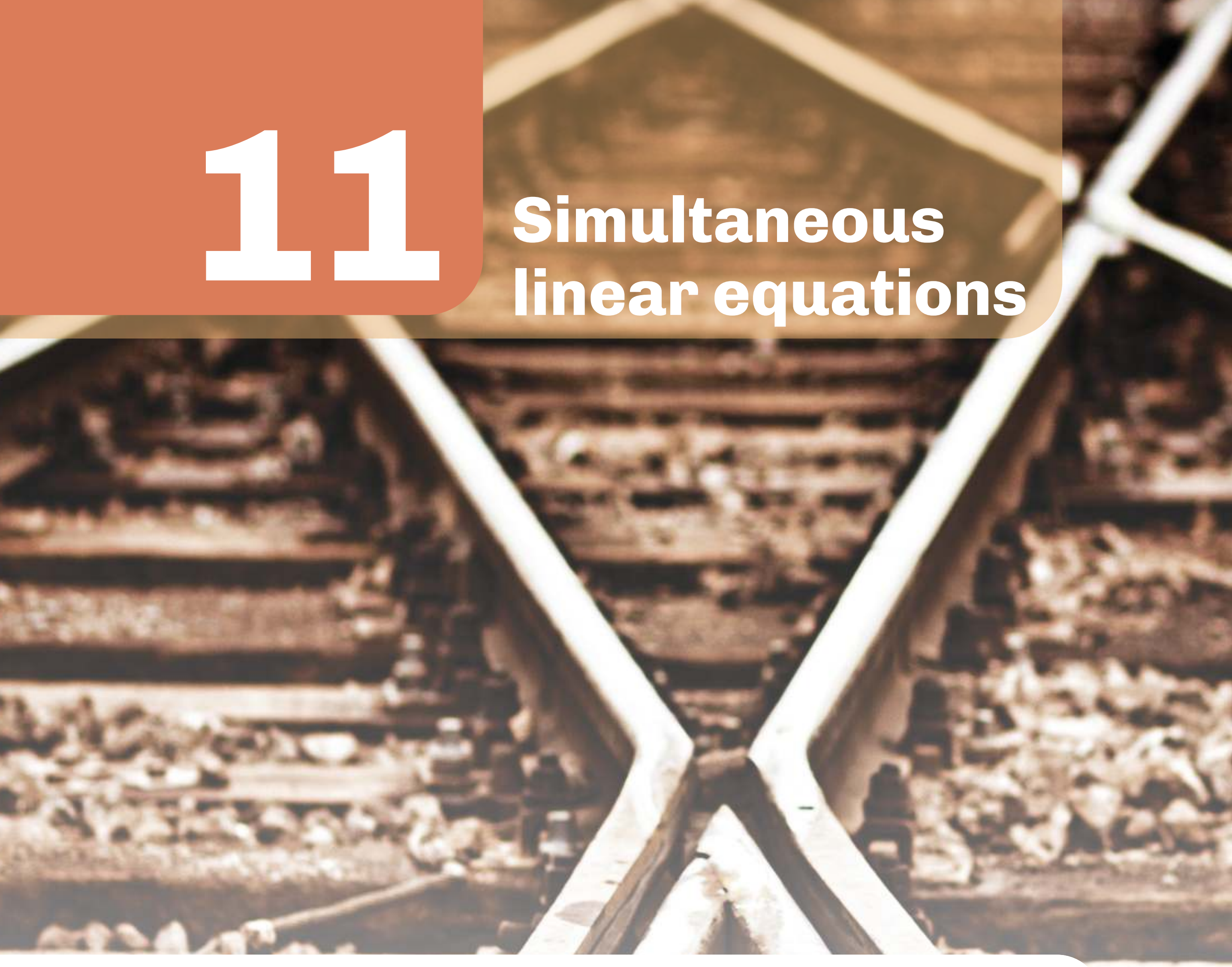
i 20 kL **ii** 30 kL **iii** 50 kL

b How much does a kilolitre of water cost when you use:

i less than 30 kL? **ii** more than 30 kL?

c Use the equations to construct a segmented graph for $0 \le x \le 50$.

ISBN 978-1-009-25891-3



11 Simultaneous linear equations

In this chapter

Syllabus references

Topic: Simultaneous linear equations and their applications

Subtopics: 2.3.7, 2.3.8

Simultaneous equations in this chapter are two equations, each containing two unknown letters. We must use both equations to find the value of the unknown letters through different methods such as elimination, substitution or graphical. Without realising it, we all employ simultaneous linear equations in our everyday lives.

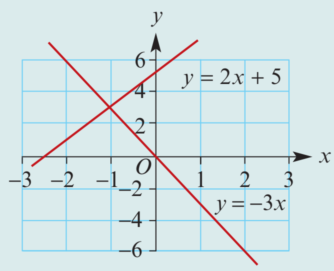
ISBN 978-1-009-25891-3

11A Finding the point of intersection of two linear graphs

Two straight lines will always intersect unless they are parallel. The point at which two straight lines intersect can be found by sketching the two graphs on the one set of axes and then reading off the coordinates at the point of intersection. When we find the *point of intersection*, we are said to be **solving the equations simultaneously**.

Example 1 Finding the point of intersection of two linear graphs

The graphs of $y = 2x + 5$ and $y = -3x$ are shown.
Write their point of intersection.



Solution

From the graph it can be seen that the point of intersection is $(-1, 3)$.

Note: A CAS calculator can also be used to find the point of intersection.

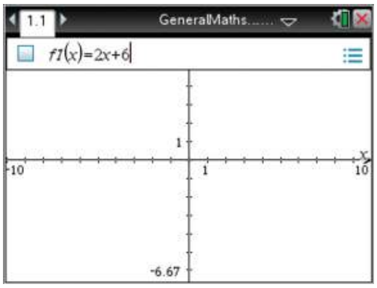
ISBN 978-1-009-25891-3

How to find the point of intersection of two linear graphs using the TI-Nspire CAS

Use a graphics calculator to find the point of intersection of the simultaneous equations $y = 2x + 6$ and $y = -2x + 3$.

Steps

1 Start a new document ([ctrl] + [N]) and select **Add Graphs**.

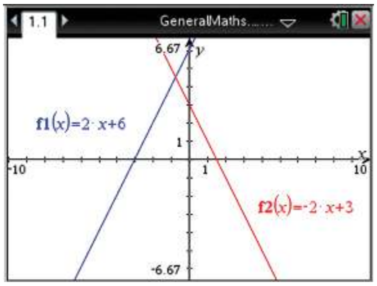


2 Type in the first equation as shown. Note that $f1(x)$ represents the y. Press ▼ and the edit line will change to $f2(x)$ and the first graph will be plotted. Type in the second equation and press [enter] to plot the second graph.

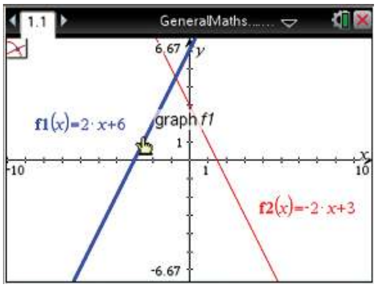
Hint: If the entry line is not visible press [tab].

Hint: To see all entered equations move the cursor onto the ☰ and press [del].

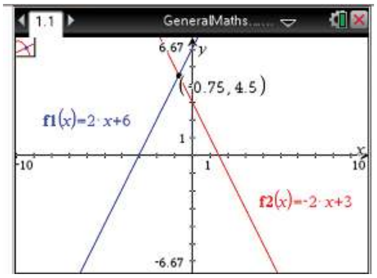
Note: To change window settings, press [menu]**>Window/Zoom>Window Settings** and change to suit. Press [enter] when finished.



3 To find the point of intersection, press [menu]**>Geometry>Points & Lines>Intersection Point(s)**.
Move the cursor to one of the graphs until it flashes, press , then move to the other graph and press . The solution will appear.
Alternatively, use [menu]**>Analyze Graph>Intersection**.



4 Press [enter] to display the solution on the screen. The coordinates of the point of intersections are $x = -0.75$ and $y = 4.5$.



Note: you can also find the intersection point using [menu]**>Analyze Graph>Intersection**.

ISBN 978-1-009-25891-3

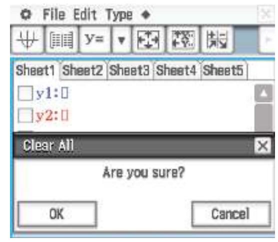
How to find the point of intersection of two linear graphs using the ClassPad

Use a graphics calculator to find the point of intersection of the simultaneous equations $y = 2x + 6$ and $y = -2x + 3$.

Steps

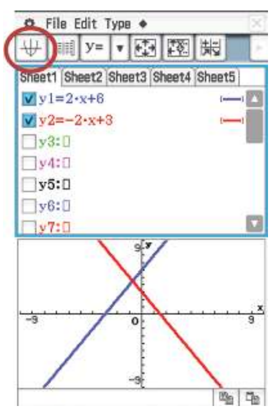
1 Open the built-in **Graphs and Tables** application. Tapping Menu from the icon panel (just below the touch screen) will display the Application menu if it is not already visible.

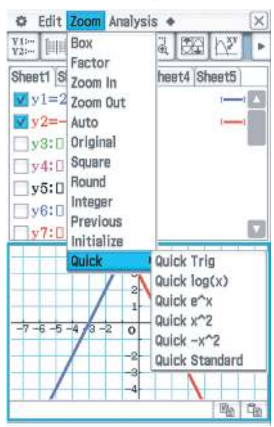
2 If there are any equations from previous questions go to **Edit Clear** all and tap **OK**.

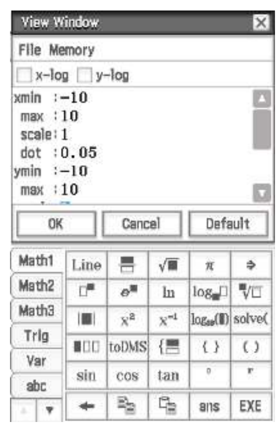


3 Enter the equations into the graph editor window. Tick the boxes. Tap the icon to plot the graphs.

4 To adjust the graph window, tap **Zoom, Quick Standard**. Alternatively, tap the icon and complete the **View Window** dialog box.

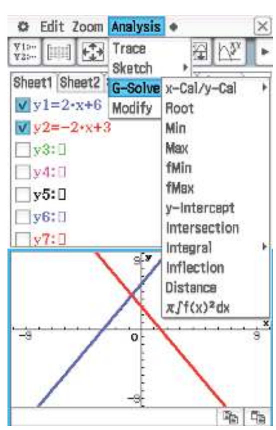


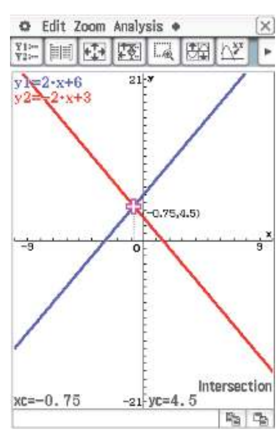




5 Solve by finding the point of intersection. Select **Analysis** from the menu bar, then **G-solve**, then **Intersect**. Tap Resize to view graph only.

The required solution is displayed on the screen: $x = -0.75$ and $y = 4.5$.





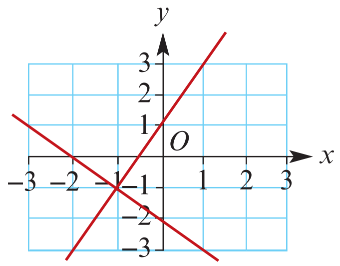
ISBN 978-1-009-25891-3

Exercise 11A

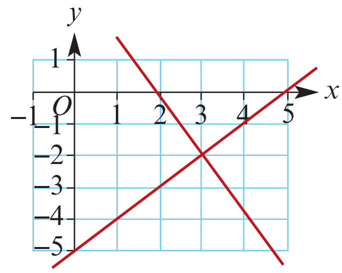
Example 1

1 State the point of intersection for each of these pairs of straight lines.

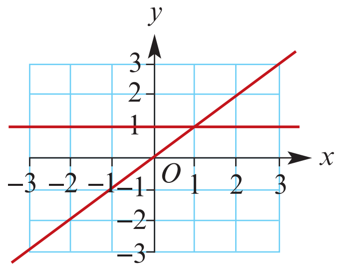
a



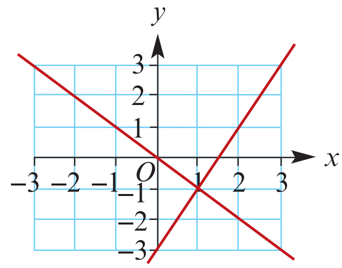
b



c



d



2 Using a CAS calculator, find the point of intersection of each of these pairs of lines.

CAS

a $y = x - 6$ and $y = -2x$

b $y = x + 5$ and $y = -x - 1$

c $y = 3x - 2$ and $y = 4 - x$

d $y = x - 1$ and $y = 2x - 3$

e $y = 2x + 6$ and $y = 6 + x$

f $x - y = 5$ and $y = 2$

g $x + 2y = 6$ and $y = 3 - x$

h $2x + y = 7$ and $y - 3x = 2$

i $3x + 2y = -4$ and $y = x - 3$

j $y = 4x - 3$ and $y = 3x + 4$

k $y = x - 12$ and $y = 2x - 4$

l $y + x = 7$ and $2y + 5x = 5$

m $y = 2x + 3$ and $y = 2x - 7$

ISBN 978-1-009-25891-3

11B Solving simultaneous linear equations algebraically

When solving simultaneous equations algebraically, there are two methods that can be used: **substitution** or **elimination**. Both methods will be demonstrated here.

Method 1: Substitution

When solving simultaneous equations by substitution, the process is to substitute one variable from one equation into the other equation.

Substitution is useful if one equation is in the form $y = ax + b$, where a and b are constants.

Example 2 Solving simultaneous equations by substitution

Solve the pair of simultaneous equations $y = 5 - 2x$ and $3x - 2y = 4$.

Solution

1 Number the two equations as (1) and (2).	$y = 5 - 2x \quad (1)$ $3x - 2y = 4 \quad (2)$
2 Substitute the y-value from equation (1) into equation (2).	substitute (1) into (2) $3x - 2(5 - 2x) = 4$
3 Expand the brackets and then collect like terms.	$3x - 10 + 4x = 4$ $7x - 10 = 4$
4 Solve for x. Add 10 to both sides of the equation. Divide both sides of the equation by 7.	$7x - 10 + 10 = 4 + 10$ $7x = 14$ $\frac{7x}{7} = \frac{14}{7}$ $\therefore x = 2$
5 To find y, substitute $x = 2$ into equation (1).	Substitute $x = 2$ into (1). $y = 5 - 2(2)$ $y = 5 - 4$ $\therefore y = 1$
6 Check by substituting $x = 2$ and $y = 1$ into equation (2).	$\text{LHS} = 3(2) - 2(1)$ $= 6 - 2 = 4 = \text{RHS}$
7 Write your solution.	$x = 2, y = 1$

ISBN 978-1-009-25891-3

Example 3 Solving simultaneous equations by substitution

Solve the pair of simultaneous equations $y = x + 5$ and $y = -3x + 9$.

Solution

1 Number the two equations as (1) and (2).

$$y = x + 5 \quad (1)$$
$$y = -3x + 9 \quad (2)$$

2 Both equations are expressions for y, so they can be made equal to each other.

$$x + 5 = -3x + 9$$

3 Solve for x.

Add $3x$ to both sides of the equation.

$$x + 5 + 3x = -3x + 9 + 3x$$
$$4x + 5 = 9$$

Subtract 5 from both sides of the equation.

$$4x + 5 - 5 = 9 - 5$$
$$4x = 4$$

Divide both sides of the equation by 4.

$$\frac{4x}{4} = \frac{4}{4}$$
$$\therefore x = 1$$

4 Find y by substituting $x = 1$ into either equation (1) or equation (2).

Substitute $x = 1$ into (1).

$$y = 1 + 5$$
$$\therefore y = 6$$

5 Check by substituting $x = 1$ and $y = 6$ into the other equation.

$$\text{LHS} = 6$$
$$\text{RHS} = -3(1) + 9$$
$$= -3(1) + 9$$
$$= -3 + 9$$
$$= 6$$

6 Write your answer.

$$x = 1, y = 6$$

ISBN 978-1-009-25891-3

Method 2: Elimination

When solving simultaneous equations by elimination, one of the unknown variables is eliminated by the process of adding or subtracting multiples of the two equations.

Example 4 Solving simultaneous equations by elimination

Solve the pair of simultaneous equations $x + y = 3$ and $2x - y = 9$.

Solution

1 Number the two equations.
On inspection, it can be seen that if the two equations are added, the variable y will be eliminated as the y-coefficients have equal but opposite signed values.

$$x + y = 3 \quad (1)$$
$$2x - y = 9 \quad (2)$$

2 Add equations (1) and (2).

$$(1) + (2): 3x = 12$$

3 Solve for x. Divide both sides of the equation by 3.

$$\frac{3x}{3} = \frac{12}{3}$$
$$\therefore x = 4$$

4 Substitute $x = 4$ into equation (1) to find the corresponding y value.

Substitute $x = 4$ into (1).

$$4 + y = 3$$

5 Solve for y. Subtract 4 from both sides of the equation.

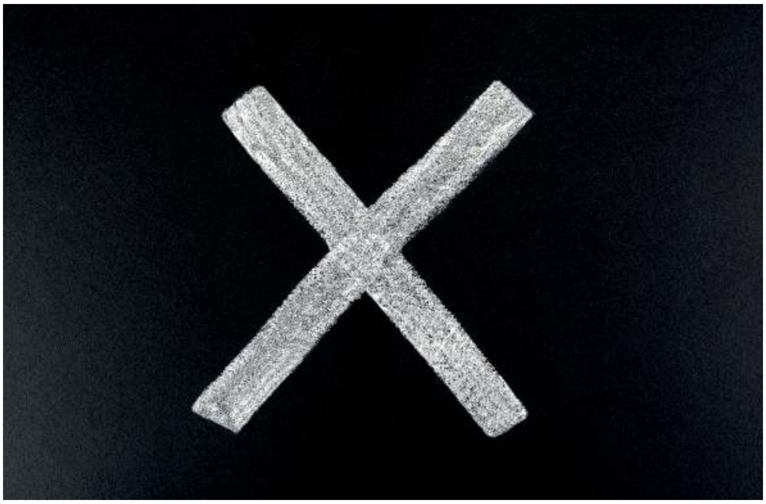
$$4 + y - 4 = 3 - 4$$
$$\therefore y = -1$$

6 Check by substituting $x = 4$ and $y = -1$ into equation (2).

$$\text{LHS} = 2(4) - (-1)$$
$$= 8 + 1 = 9 = \text{RHS}$$

7 Write your answer.

$$x = 4, y = -1$$



ISBN 978-1-009-25891-3

Example 5 Solving simultaneous equations by elimination

Solve the pair of simultaneous equations $3x + 2y = 2.3$ and $8x - 3y = 2.8$.

Solution

1 Label the two equations (1) and (2).

$$3x + 2y = 2.3 \quad (1)$$
$$8x - 3y = 2.8 \quad (2)$$

2 Multiply equation (1) by 3 and equation (2) by 2 to give $6y$ in both equations.

Remember: Each term in equation (1) must be multiplied by 3 and each term in equation (2) by 2.

$$(1) \times 3 \quad 9x + 6y = 6.9 \quad (3)$$
$$(2) \times 2 \quad 16x - 6y = 5.6 \quad (4)$$

3 Add equation (4) to equation (3) to eliminate $6y$.

$$(3) + (4) \quad 25x = 12.5$$

4 Solve for x. Divide both sides of the equation by 25.

$$\frac{25x}{25} = \frac{12.5}{25}$$
$$x = 0.5$$

5 To find y, substitute $x = 0.5$ into equation (1).

$$3(0.5) + 2y = 2.3$$
$$1.5 + 2y = 2.3$$

6 Solve for y. Subtract 1.5 from both sides of the equation.

$$1.5 + 2y - 1.5 = 2.3 - 1.5$$
$$2y = 0.8$$

7 Divide both sides of the equation by 2.

$$\frac{2y}{2} = \frac{0.8}{2}$$
$$y = 0.4$$

8 Check by substituting $x = 0.5$ and $y = 0.4$ into equation (1).

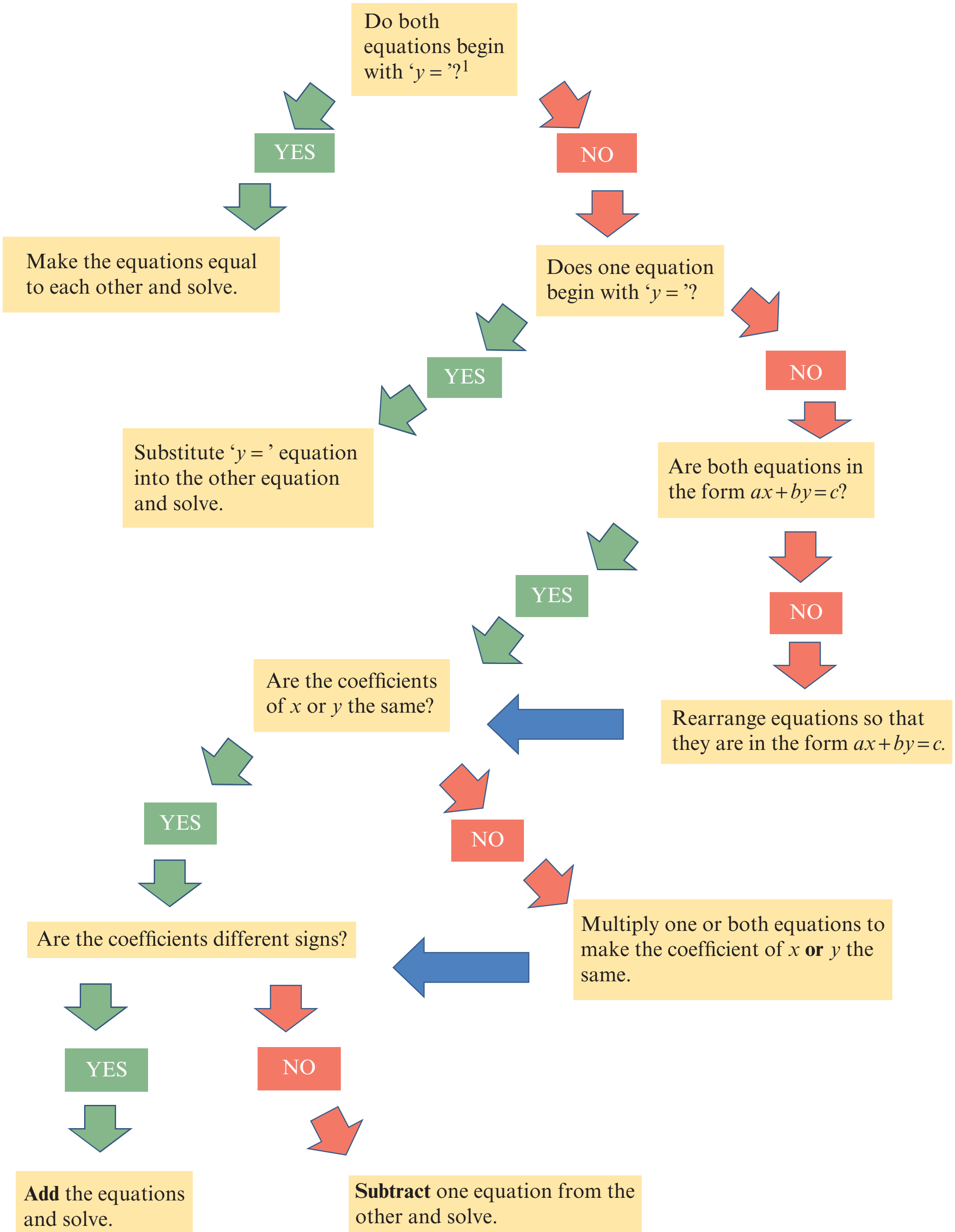
$$\text{LHS} = 3(0.5) + 2(0.4)$$
$$= 2.3 = \text{RHS}$$

9 Write your answer.

$$x = 0.5, y = 0.4$$

ISBN 978-1-009-25891-3

Deciding whether to use the substitution or elimination method



[1] This chart assumes the equation uses x and y as the variables but it could use a and b or other symbols.

ISBN 978-1-009-25891-3

Exercise 11B

Example 2–5

1 Solve the following pairs of simultaneous equations by any algebraic method (elimination or substitution).

a $y = x - 1$
$3x + 2y = 8$

b $y = x + 3$
$6x + y = 17$

c $x + 3y = 15$
$y - x = 1$

d $x + y = 10$
$x - y = 8$

e $2x + 3y = 12$
$4x - 3y = 6$

f $3x + 5y = 8$
$x - 2y = -1$

g $2x + y = 11$
$3x - y = 9$

h $2x + 3y = 15$
$6x - y = 11$

i $3p + 5q = 17$
$4p + 5q = 16$

j $4a + 3b = 7$
$6a - 3b = -27$

k $3f + 5g = -11$
$-3f - 2g = 8$

l $4x - 3y = 6$
$5y - 2x = 4$

2 Solve the following pairs of simultaneous equations by any suitable method.

a $y = 6 - x$
$2x + y = 8$

b $3x + 2y = 0$
$-3x - y = 3$

c $3x + y = 4$
$y = 2 - 4x$

d $3x + 5y = 9$
$y = 3$

e $2x + 3y = 5$
$y = 7 - 2x$

f $4x + 3y = -28$
$5x - 6y = -35$

11C Solving simultaneous linear equations using CAS

How to solve a pair of simultaneous linear equations using the TI-Nspire CAS

Solve the following pair of simultaneous equations:

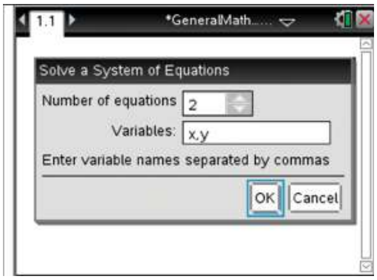
$$24x + 12y = 36 \qquad 45x + 30y = 90$$

Steps

1 Start a new document and select **Add Calculator**.

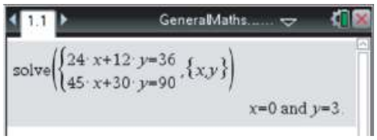
2 Press menu **>Algebra>Solve System of Equations>Solve System of Equations** and complete the pop-up screen as shown (the default settings are for two equations with variables x and y).

A simultaneous equation template will be pasted to the screen.



3 Enter the equations as shown into the template. Use the tab key to move between entry boxes.

4 Press enter to display the solution, $x = 0$ and $y = 3$.



5 Write your answer.

$x = 0, y = 3$

ISBN 978-1-009-25891-3

How to solve a pair of simultaneous linear equations algebraically using the ClassPad

Solve the following pair of simultaneous equations:

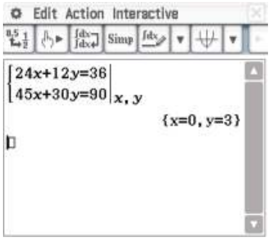
$$24x + 12y = 36 \qquad 45x + 30y = 90$$

Steps

1 Open the built-in **Main** application.

- **a** Press Keyboard on the front of the calculator to display the built-in keyboard.
- **b** Tap the simultaneous equations icon:
- **c** Enter the information

$$\begin{cases} 24x + 12y = 36 \\ 45x + 30y = 90 \end{cases}\Bigg|_{x,\,y}$$



2 Press $\boxed{\text{EXE}}$ to display the solution, $x = 0$ and $y = 3$.

3 Write your answer. $x = 0, y = 3$

Exercise 11C

CAS

1 Solve the following simultaneous equations using a CAS calculator.

a $2x + 5y = 3$
$x + y = 3$

b $3x + 2y = 5.5$
$2x - y = -1$

c $3x - 8y = 13$
$-2x - 3y = 8$

d $2h - d = 3$
$8h - 7d = 18$

e $2p - 5k = 11$
$5p + 3k = 12$

f $5t + 4s = 16$
$2t + 5s = 12$

g $2m - n = 1$
$2n + m = 8$

h $15x - 4y = 6$
$-2y + 9x = 5$

i $2a - 4b = -12$
$2b + 3a - 2 = -2$

j $3y = 2x - 1$
$3x = 2y + 1$

k $2.9x - 0.6y = 4.8$
$4.8x + 3.1y = 5.6$

ISBN 978-1-009-25891-3

11D Practical applications of simultaneous equations

Simultaneous equations can be used to solve problems in real situations. It is important to define the unknown quantities with appropriate variables before setting up the equations.

Example 6 Using simultaneous equations to solve a practical problem

Tickets for a movie cost \$19.50 for adults and \$14.50 for children. Two hundred tickets were sold giving a total of \$3265. How many children's tickets were sold?

Solution

Strategy: Using the information given, set up a pair of simultaneous equations to solve.

1 Choose appropriate variables to represent the cost of an adult ticket and the cost of a child ticket.	Let a be the number of adults' tickets sold and c be the number of children's tickets sold.
2 Write two equations using the information given in the question. Label the equations as (1) and (2). Note: The total number of adult and children's tickets is 200, which means that $a + c = 200$.	$a + c = 200 \quad (1)$ $19.5a + 14.5c = 3265 \quad (2)$
3 Rearrange equation (1) to make a the subject.	$a = 200 - c \quad (3)$
4 Substitute a from (3) into equation (2).	$19.5(200 - c) + 14.5c = 3265$
5 Expand the brackets and then collect like terms.	$3900 - 19.5c + 14.5c = 3265$ $3900 - 5c = 3265$
6 Solve for c. Subtract 3900 from both sides of the equation. Divide both sides of the equation by −5.	$3900 - 5c - 3900 = 3265 - 3900$ $-5c = -635$ $\frac{-5c}{-5} = \frac{-635}{-5}$ $\therefore c = 127$
7 To solve for a, substitute $c = 127$ into equation (1).	$a + 127 = 200 \quad (1)$
8 Subtract 127 from both sides.	$a + 127 - 127 = 200 - 127$ $a = 73$
9 Check by substituting, $a = 73$ and $c = 127$ into equation (2).	$127 + 73 = 200$
10 Write your solution.	127 children's tickets were sold.

ISBN 978-1-009-25891-3

Example 7 Using simultaneous equations to solve a practical problem

The perimeter of a rectangle is 48 cm. If the length of the rectangle is three times the width, determine its dimensions.

Solution

Strategy: Using the information given, set up a pair of simultaneous equations to solve.

1 Choose appropriate variables to represent the dimensions of width and length.

Let w = width
l = length

2 Write two equations from the information given in the question. Label the equations as (1) and (2).

Remember: The perimeter of a rectangle is the distance around the outside and can be found using $2w + 2l$.

Note: If the length, l, of a rectangle is three times its width, w, then this can be written as $l = 3w$.

$2w + 2l = 48 \quad (1)$
$l = 3w \quad (2)$

3 Solve the simultaneous equations by substituting equation (2) in equation (1).

Substitute $l = 3w$ into (1).
$2w + 2(3w) = 48$

4 Expand the brackets.

$2w + 6w = 48$

5 Collect like terms.

$8w = 48$

6 Solve for w. Divide both sides by 8.

$$\frac{8w}{8} = \frac{48}{8}$$
$\therefore w = 6$

7 Find the corresponding value for l by substituting $w = 6$ into equation (2).

Substitute $w = 6$ into (2).
$l = 3(6)$
$\therefore l = 18$

8 Give your answer in the correct units.

The dimensions of the rectangle are width 6 cm and length 18 cm.

ISBN 978-1-009-25891-3

Example 8 Using simultaneous equations to solve a practical problem

Mark buys 3 roses and 2 gardenias for \$15.50.
Peter buys 5 roses and 3 gardenias for \$24.50.
How much did each type of flower cost?

Solution

Strategy: Using the information given, set up a pair of simultaneous equations to solve.

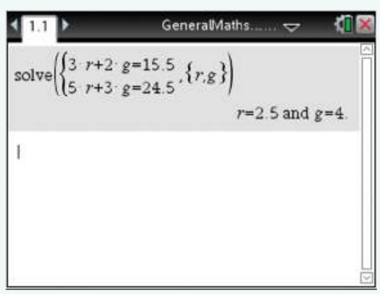
1 Choose appropriate variables to represent the cost of roses and the cost of gardenias.

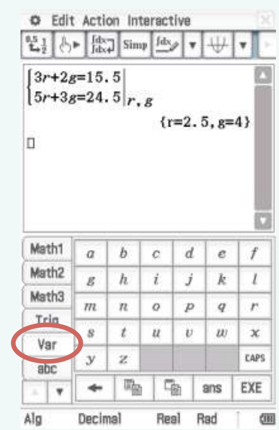
Let r be the cost of a rose and g be the cost of a gardenia.

2 Write equations using the information given in the question. Label the equations (1) and (2).

$3r + 2g = 15.5 \quad (1)$
$5r + 3g = 24.5 \quad (2)$

3 Use your CAS calculator to solve the two simultaneous equations.





4 Write down the solutions.

$r = 2.50$ and $g = 4$

5 Check by substituting $r = 2.5$ and $g = 4$ into equation (2).

$\text{LHS} = 5(2.50) + 3(4)$
$= 12.5 + 12 = 24.5 = \text{RHS}$

6 Write your answer with the correct units.

Roses cost \$2.50 each and gardenias cost \$4 each.

ISBN 978-1-009-25891-3

Exercise 11D

Example 6–8

1 Jessica bought 5 textas and 6 pencils for \$12.75, and Tom bought 7 textas and 3 pencils for \$13.80.

a Using t for texta and p for pencil, find a pair of simultaneous equations to solve.

b How much did one pencil and one texta cost each?

2 Peter buys 50 litres of petrol and 5 litres of motor oil for \$109. His brother Anthony buys 75 litres of petrol and 5 litres of motor oil for \$146.

a Using p for petrol and m for motor oil, write down a pair of simultaneous equations to solve.

b How much do a litre of petrol and a litre of motor oil cost each?

3 Six oranges and ten bananas cost \$7.10. Three oranges and eight bananas cost \$4.60.

a Write down a pair of simultaneous equations to solve.

b Find the cost each of an orange and a banana.

4 The weight of a box of nails and a box of screws is 2.5 kg. Four boxes of nails and a box of screws weigh 7 kg. Determine the weight of each box.

5 An enclosure at a wildlife sanctuary contains wombats and emus. If the number of heads totals 28 and the number of legs totals 88, determine the number of each species present.

6 The perimeter of a rectangle is 36 cm. If the length of the rectangle is twice its width, determine its dimensions.

7 The sum of two numbers x and y is 52. The difference between the two numbers is 8. Find the values of x and y.

8 The sum of two numbers is 35 and their difference is 19. Find the numbers.

9 Bruce is 4 years older than Michelle. If their combined age is 70, determine their individual ages.

10 A boy is 6 years older than his sister. In three years time he will be twice her age. What are their present ages?

ISBN 978-1-009-25891-3

11 A chocolate thickshake costs \$2 more than a fruit smoothie. Jack pays \$27 for 3 chocolate thickshakes and 4 fruit smoothies. How much do a chocolate thickshake and a fruit smoothie cost each?

12 In 4 years time a mother will be three times as old as her son. Four years ago she was five times as old as her son. Find their present ages.

13 The registration fees for a mathematics competition are \$1.20 for students aged 8–12 years and \$2 for students 13 years and over. One hundred and twenty-five students have already registered and an amount of \$188.40 has been collected in fees. How many students between the ages of 8 and 12 have registered for the competition?

14 A computer company produces 2 laptop models: standard and deluxe. The standard laptop requires 3 hours to manufacture and 2 hours to assemble. The deluxe model requires $5\frac{1}{2}$ hours to manufacture and $1\frac{1}{2}$ hours to assemble. The company allows 250 hours for manufacturing and 80 hours for assembly over a limited period. How many of each model can be made in the time available?

15 A chemical manufacturer wishes to obtain 700 litres of a 24% acid solution by mixing a 40% solution with a 15% solution. How many litres of each solution should be used?

16 In a hockey club there are 5% more boys than there are girls. If there is a total of 246 members in the club, what is the number of boys and the number of girls?

17 The owner of a service station sells unleaded petrol for \$1.42 and diesel fuel for \$1.54. In five days he sold a total of 10 000 litres and made \$14 495. How many litres of each type of petrol did he sell? Give your answer to the nearest litre.

18 James had \$30 000 to invest. He chose to invest part of his money in an account earning 5% interest and the remaining amount in an account earning 8% interest. Overall he earned \$2100 in interest. How much did he invest at each rate?

19 The perimeter of a rectangle is 120 metres. The length is one and a half times the width. Calculate the width and length.

20 Three classes, A, B and C, in a school are such that class A has two thirds the number of students that class B has and class C has five sixths the number of students in class B. If the total number of all pupils in the three classes is 105, how many are there in each class?

ISBN 978-1-009-25891-3

11E Problem solving and modelling

Exercise 11E

1 As part of its urban renewal strategy, Camtown council makes $\frac{1}{4}$ hectare of land available for building middle-income homes. The project manager decides to build 10 houses on blocks of varying sizes.

There are five small blocks, three medium-sized blocks and two large blocks. The medium-sized blocks are $100\,\text{m}^2$ larger than the small blocks and the large blocks are $200\,\text{m}^2$ larger than the medium-sized ones.

What are the sizes of the blocks?

Note: $1 \text{ hectare} = 10\,000\,\text{m}^2$

2 A shop sells fruit in two types of gift boxes: standard and deluxe. Each standard box contains 1 kg of peaches and 2 kg of apples and each deluxe box contains 2 kg of peaches and 1.5 kg of apples. On one particular day the shop sold 12 kg of peaches and 14 kg of apples in gift boxes. How many containers of each kind of box were sold on the day?

3 A person's total body water is dependent on their gender, age, height and weight. Total body water (TBW) for males and females can be found by the following formulas:

$$\underset{\text{(litres)}}{\textit{Male TBW}} = 2.447 - 0.095\,16 \times \underset{\text{(years)}}{\textit{age}} + 0.1074 \times \underset{\text{(cm)}}{\textit{height}} + 0.3362 \times \underset{\text{(kg)}}{\textit{weight}}$$

$$\underset{\text{(litres)}}{\textit{Female TBW}} = -2.097 + 0.1069 \times \underset{\text{(cm)}}{\textit{height}} + 0.2466 \times \underset{\text{(kg)}}{\textit{weight}}$$

a What is the TBW for a female of height 175 cm and weight 62 kg? Give your answer correct to two decimal places.

b Calculate the TBW, correct to two decimal places, for a 45-year-old male of height 184 cm and weight 87 kg.

ISBN 978-1-009-25891-3

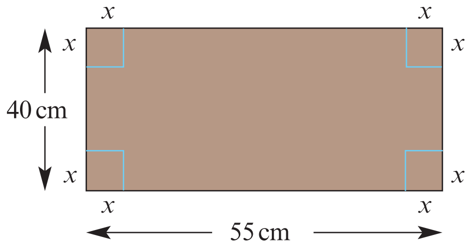
c A healthy 27-year-old female has a TBW of 32 and weighs 62 kg. What is her height, to the nearest cm?

d What would be the TBW, correct to two decimal places, for a 78-year-old man of height 174 cm and weight 80 kg?

e Over a period of a week, the 78-year-old man's weight rapidly increases to 95 kg due to fluid retention. What is his new TBW, correct to two decimal places?

f Construct a table showing the TBW for a 22-year-old male of height 185 cm with weights in increments of 5 kg from 60–120 kg. Give you answers correct to two decimal places.

4 A cardboard storage box is made by cutting equal squares of side length x, from a piece of cardboard measuring 55 cm by 40 cm.



a The volume, V, of the box is given by:

$V = length \times width \times height$

Find an expression for the volume in terms of x.

b If the volume of the box is to be 7000 cm^3, find two possible values for the height of the box, correct to two decimal places.

ISBN 978-1-009-25891-3

Review

Key ideas and chapter summary

Assign-ment

Simultaneous equations

Two straight lines will always intersect, unless they are parallel. At the point of intersection the two lines will have the same coordinates. When we find the point of intersection, we are solving the equations simultaneously. **Simultaneous equations** can be solved graphically, algebraically or by using a CAS calculator.

Example:

$$3x + 2y = 6$$
$$4x - 5y = 12$$

are a pair of simultaneous equations.

Skills check

Having completed the current chapter you should be able to:

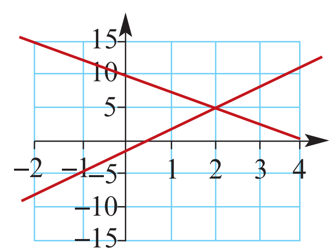
- solve simultaneous equations graphically, algebraically and with a CAS calculator.

Short-answer questions

1 Is the point (2, 10) a solution to the pair of simultaneous equations?

$$y = 5x$$
$$y = 2x + 6$$

2 State the point of intersection of the lines shown in the diagram.



3 Is (−6, 1) a solution to the pair of simultaneous equations?

$$2x + 3y = -6$$
$$x + 3y = 0$$

4 Find the point of intersection of the following pairs of lines.

a $y = x + 2$ and $y = 6 - 3x$

b $y = x - 3$ and $2x - y = 7$

c $x + y = 6$ and $2x - y = 9$

ISBN 978-1-009-25891-3

5 Solve the following pairs of simultaneous equations.

a $y = 5x - 2$ and $2x + y = 12$

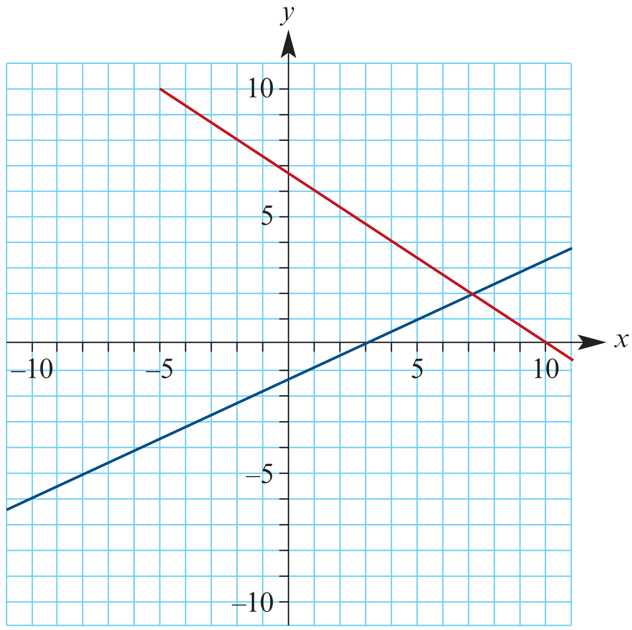
b $x + 2y = 8$ and $3x - 2y = 4$

c $2p - q = 12$ and $p + q = 3$

d $3p + 5q = 25$ and $2p - q = 8$

e $3p + 2q = 8$ and $p - 2q = 0$

6 State the point of intersection of the following two lines.



7 Find the value of y if $x = 2$ is a solution of the pair of simultaneous equations $y = x + 3$ and $2x + y = 9$.

8 Find the value of x if $y = 4$ is a solution of the pair of simultaneous equations $2x + y = 10$ and $x + y = 7$.

9 Use the solve facility on your calculator to solve the simultaneous equations. CAS

$$x + y = 50$$
$$2x + 3y = 120$$

10 Solve the following pair of simultaneous equations.

$$4x + y = 10$$
$$2x + y = 4$$

11 Solve the following pair of simultaneous equations.

$$2a + b = 7$$
$$5a - 2b = 22$$

ISBN 978-1-009-25891-3

Review

12 Solve the following pair of simultaneous equations.

$$3x + 4y = 23$$
$$2x + 3y = 16$$

13 Solve the following simultaneous equations.

$$y = 2x + 1$$
$$2x + 3y = 27$$

14 Solve the following simultaneous equations.

$$x - 3y = 8$$
$$2x + 5y = 5$$

CAS

15 Using the graphical facility on your calculator solve the following simultaneous equations.

$$y = 2x + 5$$
$$x + y = 11$$

16 Seven footballs (x) and three soccer balls (y) cost a total of \$314. Four footballs and five soccer balls cost a total of \$255. Write down a pair of simultaneous equations involving x and y. Hence use your CAS calculator to find the cost of each football and each soccer ball.

Extended-response questions

1 Two families went to the theatre. The first family bought tickets for 3 adults and 5 children and paid \$73.50. The second family bought tickets for 2 adults and 3 children and paid \$46.50.

a Write down two simultaneous equations that could be used to solve the problem.

b What was the cost of an adult's ticket?

c What was the cost of a child's ticket?

2 The perimeter of a rectangle is 10 times the width. The length is 9 metres more than the width. Find the width of the rectangle.

3 A secondary school offers three languages: French, Indonesian and Japanese. In year 9, there are 105 students studying one of these languages. The Indonesian class has two-thirds the number of students that the French class has and the Japanese class has five-sixths the number of students of the French class. How many students study each language? (No student is studying more than one language).

ISBN 978-1-009-25891-3

4 Ten years ago, Ai was 12 times as old as Tisa and in ten years' time, Ai will be twice as old as Tisa. Find their present ages.

5 At Perth Concert Hall, 1000 tickets were sold in the lead up to a concert. Adult tickets cost \$8.50, children's cost \$4.50, and a total of \$7300 was collected. How many tickets of each kind were sold?

6 The admission fee at the Royal Show is \$3.50 for children and \$5 for adults. On a certain day, 2200 people enter the Royal Show and \$8750 is collected. Two simultaneous equations can be written from this information.
One of the equations is $3.5x + 5y = 8750$

- **a** Explain clearly what the term $5y$ represents in this situation.
- **b** Write down the second equation.
- **c** Solve the pair of equations to determine the number of children and the number of adults who attended the show.

7 Anya is x years old and Haru is y years old.
Last year, Haru was 6 times as old as Anya.

- **a** Form an equation using x and y and show that it simplifies to $y = 6x - 5$.
- **b** In 19 years' time, Haru will be twice as old as Anya. Form another equation in x and y and show that it simplifies to $y = 2x + 19$.
- **c** Hence find the present ages of Anya and Haru.

ISBN 978-1-009-25891-3

Glossary

A

Angle of depression [p. 356] The angle between the horizontal and a direction *below* the horizontal.

Angle of elevation [p. 356] The angle between the horizontal and a direction *above* the horizontal.

Arc [p. 131] The part of a circle between two given points on the circle. The length of the arc of a circle is given by $s = r\left(\frac{\theta}{180}\right)\pi$, where r is the radius of the circle and θ is the angle in degrees subtended by the arc at the centre of the circle.

Area [p. 121] The area of a shape is a measure of the region enclosed by its boundaries, measured in square units.

Area formulas [p. 121] Formulas used to calculate the areas of regular shapes, including squares, rectangles, triangles, parallelograms, trapeziums, kites, rhombi and circles.

B

Back-to-back stem plot [p. 300] A type of stem plot used to compare two sets of data, with a single stem and two sets of leaves (one for each group).

Bar chart [p. 255] A statistical graph used to display the frequency distribution of categorical data, using vertical bars.

Bearing [p. 361] *See* Three-figure bearing.

BIDMAS [p. 2] An aid for remembering the order of operations: **B**rackets first; **I**ndices (powers, square roots); **M**ultiplication and **D**ivision, working left to right; **A**ddition and **S**ubtraction, working left to right.

Bivariate data [p. 415] Data where each observation involves recording information about two variables for each person or thing.

Boxplot [p. 292] A graphical representation of a five-number summary showing outliers if present. *See* Outliers.

C

Capacity [p. 137] The amount of substance that an object can hold.

Categorical data [p. 251] Data generated by a categorical variable. Even if numbers, for example, house numbers, categorical data *cannot* be used to perform meaningful numerical calculations.

Categorical variable [p. 252] Variables that are used to represent characteristics of individuals, for example, place of birth, house number. Categorical variables come in types, nominal and ordinal.

Circumference [p. 129] The circumference of a circle is the length of its boundary. The circumference, C, of a circle with radius, r, is given by $C = 2\pi r$.

Column matrix [p. 210] A matrix with only one column.

ISBN 978-1-009-25891-3

Composite shape [p. 125] A shape that is made up of two or more basic shapes.

Compound interest [pp. 62, 73] Under compound interest, the interest paid on a loan or investment is credited or debited to the account at the end of each period. The interest earned for the next period is based on the principal plus previous interest earned. The amount of interest earned increases each year.

Compounding period [p. 68] The time period between compound interest calculations and payments, for example, a day, a month, or a year.

Connections [p. 215] A matrix can be used to record various types of connections, such as social communications and roads directly connecting towns.

Continuous data [p. 252] Measurements of a variable that can take any value (usually within a range) to any fraction or any number of decimal places.

Cosine ratio (cos θ) [p. 342] In right-angled triangles, the ratio of the side adjacent to a given angle (θ) to the hypotenuse.

Cosine rule [p. 389] In non-right-angled triangles, a rule used to find:

- the third side, given two sides and an included angle
- an angle, given three sides.

For triangle ABC and side a, the rule is:

$$a^2 = b^2 + c^2 - 2bc\ \cos A$$

or $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

Similar rules exist for sides b and c.

Credit [p. 71] An advance of money from a financial institution, such as a bank, that does not have to be paid back immediately but which attracts interest after an interest-free period.

D

Data [p. 251] Information collected about a variable.

Diameter [p. 129] The distance from one side of a circle (or sphere) to the opposite side through the centre; it is equal to twice the radius.

Directed numbers [p. 3] Positive and negative numbers.

Discount [p. 41] A reduction in the price of an item usually expressed as percentage decrease, which may also be expressed as a fraction of the price or an amount subtracted from the price; also known as mark down.

Discrete data [p. 252] Data that is counted rather than measured and can take only specific numerical values, usually but not always whole numbers.

Discrete variable [p. 252] A numerical variable that represents a quantity that is determined by counting, for example, the number of people waiting in a queue is a discrete variable.

Distribution [p. 254] The pattern in a set of data values. It reflects how frequently different data values occur.

Dividend [p. 95] The financial return to shareholders on a share of a company. Dividends can be specified as the number of dollars each share receives or as a percentage of the current share price, called the dividend yield.

Dot plot [p. 275] A dot plot consists of a number line with each data point marked by a dot. When several data points have the same value, the points are stacked on top of each other.

E

Elements of a matrix [p. 209] The numbers or symbols displayed in a matrix.

Elimination method [p. 450] When solving simultaneous equations by elimination, one of the unknown variables is eliminated by the process of adding or subtracting multiples of the two equations.

Equal matrices [p. 245] Two matrices are equal when they have the same numbers in the same positions.

Exchange rate [p. 13] For a currency, the rate used to convert it into another currency.

F

Five-number summary [p. 292] A list of the five key points in a data distribution: the minimum value (Min), the first quartile (Q_1), the median (M), the third quartile (Q_3) and the maximum value (Max).

ISBN 978-1-009-25891-3

Flat-rate interest [p. 71] The interest rate calculated from the total interest paid, divided by the term of the loan or investment.

Formula [p. 177] A mathematical relation connecting two or more variables, for example, $C = 5t + 20, P = 2L + 2W, A = \pi r^2$.

Frequency [p. 254] The number of times a value or a group of values occurs in a data set. Sometimes known as the count.

Frequency table [p. 254] A listing of the values that a variable takes in a data set along with how often (frequently) each value occurs. Frequency can also be recorded as a percentage.

G

Gradient of a straight line [p. 410] *See* Slope of a straight line.

Gross salary [p. 81] The amount someone is paid for their employment before tax is deducted.

Grouped data [p. 262] Where there are many different data values, data is grouped in intervals such as 0–9, 10–19, . . .

GST [p. 44] GST (goods and services tax) is a tax, currently at the rate of 10%, that is added to most purchases of goods and services.

H

Heron's rule (Heron's formula) [p. 122] A rule for calculating the area of a triangle from its three sides.

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \frac{1}{2}(a + b + c)$ and is called the semi-perimeter.

Hire purchase [p. 71] A financial agreement where the purchaser hires an item and in return makes periodic repayments at an agreed rate of interest. At the end of the agreement, the item becomes the property of the purchaser.

Histogram [p. 265] A statistical graph used to display the frequency distribution of a numerical variable; suitable for medium- to large-sized data sets.

Hypotenuse [p. 110] The longest side in a right-angled triangle.

I

Identity matrix (I) [p. 241] A matrix that behaves like the number one in arithmetic, represented by the symbol I. Any matrix multiplied by an identity matrix remains unchanged.

Inflation [p. 77] The tendency of prices to increase with time, resulting in the loss of purchasing power.

Intercept-slope form of the equation of a straight line [p. 415] A linear equation written in the form $y = ax + b$, where a and b are constants. In this equation, a represents the slope and b represents the y-intercept. For example, $y = 5 - 2x$ is the equation of a straight line with y-intercept of 5 and the slope of -2.

Interest [p. 47] An amount of money paid (earned) for borrowing (lending) money over a period of time. It may be calculated on a simple or compound basis.

Interest rate [p. 47] The rate at which interest is charged or paid. It is usually expressed as a percentage of the money owed or lent.

Interquartile range (IQR) [p. 285] A summary statistic that measures the spread of the middle 50% of values in a data distribution. It is defined as IQR $= Q_3 - Q_1$.

L

Linear equation [p. 203] An equation that has a straight line as its graph. In linear equations, the unknown values are always to the power of 1, for example, $y = 2x - 3, y + 3 = 7, 3x = 8$.

Location of distribution [p. 274] Two distributions are said to differ in location if the values of the data in one distribution is generally larger than the values of the data in the other distribution.

Loss *See* profit.

M

Mark-down [p. 41] *See* discount.

Mark-up [p. 42] An increase in the price of an item usually expressed as percentage increase, or the calculation of the price of an item as a fixed increase from its cost.

ISBN 978-1-009-25891-3

Matrix [p. 207] *A* rectangular array of numbers or symbols set out in rows and columns within square brackets. (Plural – matrices.)

Matrix addition [p. 217] Matrices are added by adding the elements that are in the same positions.

Matrix multiplication [p. 225] The process of multiplying a matrix by a matrix.

Matrix subtraction [p. 217] Matrices are subtracted by subtracting the elements that are in the same positions.

Maximum (Max) [p. 292] The largest value in a set of numerical data.

Mean ($\bar{x}$) [p. 280] The balance point of a data distribution. The mean is given by $\bar{x} = \dfrac{\Sigma x}{n}$, where Σx is the sum of the data values and n is the number of data values.

Median (*M*) [p. 282] The midpoint of an ordered data set that has been divided into two equal parts, each with 50% of the data values. It is equal to the middle value (for an odd number of data values) or the average of the two middle values (for an even number of data values). It is a measure of the centre of the distribution.

Minimum (Min) [p. 292] The smallest value in a set of numerical data.

Minimum monthly balance [p. 103] The lowest amount that a bank account contains in a given calendar month.

Modal category or interval [pp. 256, 264] The category or data interval that occurs most frequently in a data set.

Mode [p. 256] The most frequently occurring value in a data set. There may be more than one.

N

Negative slope [pp. 410, 413] A straight-line graph with a negative slope represents a decreasing y-value as the x-value increases. For the graph of a straight line with a negative slope, y decreases at a constant rate with respect to x.

Negatively skewed distribution [p. 273] A data distribution that has a long tail to the left. In negatively skewed distributions, the majority of data values fall to the right of the mean.

Net salary [p. 81] The amount of pay left after tax has been deducted.

Network [p. 215] A set of points called vertices and connecting lines called edges, enclosing and surrounded by areas called faces.

Nominal data [p. 251] Type of categorical data, such as gender, in which categories are given names ('nominations') or labels rather than taking a numerical value.

Nominal variable [p. 252] A categorical variable that generates data values that can be used by name only, for example, eye colour: blue, green, brown.

Non-linear equation [p. 203] An equation with a graph that is *not* a straight line. In non-linear equations, the unknown values are not all to the power of 1, for example, $y = x^2 + 5, 3y^2 = 6, b^3 = 27$.

Normal distribution [p. 315] Data that have a bell shape and are symmetric.

Numerical data [p. 252] Data obtained by measuring or counting some quantity. Numerical data can be discrete (for example, the *number* of people waiting in a queue) or continuous (for example, the *amount of time* people spent waiting in a queue).

O

Order of a matrix [p. 208] An indication of the size and shape of a matrix, written as $m \times n$, where m is the number of rows and n is the number of columns.

Order of operations [p. 2] The sequence in which arithmetical operations should be carried out. *See* BODMAS.

Ordinal data [p. 251] Type of categorical data, such as clothing size, in which categories are given labels that can be arranged in order, such as numbers or letters.

Ordinal variable [p. 252] A categorical variable that generates data values that can be used to both name and order, for example, house number.

Outliers [p. 277] Data values that appear to stand out from the main body of a data set. Using box plots, possible outliers are defined as data values greater than $Q_3 + 1.5 \times \text{IQR}$ or less than $Q_1 - 1.5 \times \text{IQR}$.

ISBN 978-1-009-25891-3

P

Parallel boxplot [p. 300] A statistical graph in which two or more boxplots are drawn side by side. Used to compare distributions in terms of shape, centre and spread.

Percentage [p. 18] The number as a proportion of one hundred, indicated by the symbol %. For example, 12% means 12 per one hundred.

Percentage change [p. 42] The amount of the increase or decrease of a quantity expressed as a percentage of the original value.

Percentage frequency [p. 254] Frequency of a value or group of values, expressed as a percentage of the total frequency.

Perimeter [p. 121] The distance around the edge of a two-dimensional shape.

Positive slope [pp. 410, 413] A positive slope represents an increasing y-value with increase in x-value. For the graph of a straight line with a positive slope, y increases at a constant rate with respect to x.

Positively skewed distribution [p. 273] A data distribution that has a long tail to the right. In positively skewed distributions, the majority of data values fall to the left of the mean.

Price-to-earnings ratio [p. 94] A measure of the profit of a company, given by the *current share price/profit per share*. A lower value of the price-to-earnings ratio may indicate a better investment.

Principal (P) [p. 47] The initial amount borrowed, lent or invested.

Profit [p. 94] A financial gain, such as the amount left over when costs have been subtracted from revenue leaving a positive number. If the number left is negative, that is called a loss.

Pronumeral [p. 2] A symbol (usually a letter) that stands for a numerical quantity or variable.

Purchasing power [p. 77] A measure of how much a specific good or service money can buy at different times (due to inflation, for instance), or which different currencies can buy.

Pythagoras' theorem [p. 110] A rule for calculating the third side of a right-angled triangle given the length of the other two sides. In triangle ABC, the rule is: $a^2 = b^2 + c^2$, where a is the length of the hypotenuse.

Q

Quantile [p. 326] The value below which a certain percentage of data values are expected to fall.

Quartiles (Q_1, Q_2, Q_3) [p. 285] Summary statistics that divide an ordered data set into four equal-sized groups, each containing 25% of the scores.

R

Radius [p. 129] The distance from the centre of a circle (or sphere) to any point on its circumference (or surface); equal to half the diameter.

Range (R) [p. 308] The difference between the smallest (minimum) and the largest (maximum) values in a data set: a measure of spread.

Ratio [p. 27] A fraction, or two numbers in the form $a{:}b$, used to numerically compare the values of two or more quantities.

Right angle [p. 110] An angle equal to 90°.

Rise [p. 411] *See* Slope of a straight line.

Rounding [p. 6] Shortening a number by removing the last digit and adjusting the digit before it by the rules for rounding.

Row matrix [p. 210] A matrix with only one row.

Run [p. 411] *See* Slope of a straight line.

S

Scalar multiplication [p. 220] The multiplication of a matrix by a number.

Scale factor (areas) [p. 153] The scale factor, k^2, by which the area of a two-dimensional shape is scaled (increased or decreased) when its linear dimensions are scaled by a factor of k.

Scale factor (volumes) [p. 164] The scale factor, k^3, by which the volume of a solid shape is scaled (increased or decreased) when its linear dimensions are scaled by a factor of k.

Scientific notation [p. 7] A number expressed as a number between 1 and 10 and multiplied by a power of 10.

Shares [p. 94] A share is a unit of ownership of a company.

ISBN 978-1-009-25891-3

Significant figures [p. 9] The digits of a number that give its value to a required level of accuracy.

Similar figures [p. 153] Figures that have exactly the same shape but differ in size.

Similar triangles [p. 160] Different sized triangles in which the corresponding angles are equal. The ratios of the corresponding pairs of sides are always the same.

Simple interest [pp. 51, 58] Interest that is calculated for an agreed period and paid only on the original amount invested or borrowed. Also called flat-rate interest.

Simultaneous linear equations [p. 444] Two or more linear equations in two or more variables, for values that are common solutions to all equations. For example, $3x - y = 7$ and $x + y = 5$ are a pair of simultaneous linear equations in x and y, with the common solution $x = 3$ and $y = 2$.

Sine ratio (sin θ) [p. 342] In right-angled triangles, the ratio of the side opposite a given angle (θ) to the hypotenuse.

Sine rule [p. 379] In non-right-angled triangles, a rule used to find:

- an unknown side, given the angle opposite and another side and its opposite angle
- an unknown angle, given the side opposite and another side and its opposite angle.

For triangle ABC the rule is:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

Skewness [p. 273] Lack of symmetry in a data distribution. It may be positive or negative.

Slope of a straight line [p. 411] The ratio of the increase in the dependent variable (y) to the increase in the independent variable (x) in a linear equation. Also known as the gradient. The slope (or gradient) of a straight-line graph is defined to be:

$$\text{slope} = \frac{\text{rise}}{\text{run}}.$$

SOH–CAH–TOA [p. 342] A memory aid for remembering the trigonometric ratio rules.

Solution [p. 192] A value that can replace a variable and make an equation or inequality true.

Spread of a distribution [p. 274] A measure of the degree to which data values are clustered around some central point in the distribution. Measures of spread include the standard deviation (s), the interquartile range (IQR) and the range (R).

Square matrix [p. 210] A matrix with the same number of rows as columns.

Standard deviation (*s*) [p. 286] A summary statistic that measures the spread of the data values around the mean. It is given by

$$s = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n - 1}}.$$

Standardised score [p. 332] See z-score.

Stem plot (stem-and-leaf plot) [p. 276] A method for displaying data in which each observation is split into two parts: a 'stem' and a 'leaf'. A stem plot is an alternative display to a histogram; suitable for small- to medium-sized data sets.

Substitution method [p. 448] When solving simultaneous equations by substitution, the process is to substitute one variable from one equation into the other equation.

Summary statistics [p. 280] Statistics that give numerical values to special features of a data distribution, such as centre and spread. Summary statistics include the mean, median, range, standard deviation and IQR.

Surface area [p. 147] The total of the areas of each of the surfaces of a solid.

Symmetric distribution [p. 273] A data distribution in which the data values are evenly distributed around the mean. In a symmetric distribution, the mean and the median are equal.

T

Tangent ratio (tan θ) [p. 342] In right-angled triangles, the ratio of the side opposite a given angle θ to the side adjacent to the angle.

Term [p. 193] One value in an algebraic expression.

Three-figure bearing: [p. 361] An angular direction, measured clockwise from north and written with three digits, for example, 060°, 324°. Also called a true bearing.

ISBN 978-1-009-25891-3

Total surface area (TSA) [p. 149] The total surface area (TSA) of a solid is the sum of the surface areas of all of its faces.

Trigonometric ratios [p. 342] In right-angled triangles, the ratios

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

True bearing [p. 361] *See* Three-figure bearing.

U

Undefined [p. 413] Has no meaning; has no value. The slope, or gradient, of a vertical line is undefined because $\frac{rise}{run}$ gives a zero denominator.

Unit cost [p. 32] The cost or price of a single item.

V

Variable [p. 251] A quantity that can have many different values in a given situation. Symbols such as x, y and z are commonly used to represent variables.

Volume [p. 135] The volume of a solid is a measure of the amount of space enclosed within it, measured in cubic units.

Volume formulas [p. 135] Formulas used to calculate the volumes of solids, including cubes, cuboids, prisms, pyramids, cylinders, cones and spheres.

Y

***y*-intercept** [p. 415] The point at which a graph cuts the y-axis.

Z

***z*-score** [p. 332] A standardised score used to compare scores in a normal distribution, it measures how many standard deviations a value is from the mean.

Zero matrix (*O*) [p. 218] A matrix that behaves like zero in arithmetic, represented by the symbol O. Any matrix with zeros in every position is a zero matrix.

Zero slope [p. 413] A horizontal line has zero slope. The equation of this line has the form $y = c$ where c is any constant.

ISBN 978-1-009-25891-3

Answers

Chapter 1

Exercise 1A

1 **a** 37 **b** 5 **c** 50 **d** 7 **e** 7.2 **f** 48 **g** 34.5 **h** 4.5 **i** 0.6 **j** −0.9

2 **a** 12.53 **b** −27 **c** 31.496 **d** 1

3 **a** 27 **b** $2x - 14$ **c** $50 - 10y$ **d** $6w$ **e** $k^2 + 8k$ **f** 30 **g** $2x + 14$ **h** 22 **i** $3x - 5$ **j** $-2 - 2x$

Exercise 1B

1 **a** −1 **b** −4 **c** −16 **d** 3 **e** −26 **f** −25 **g** −12 **h** 22 **i** 22 **j** 32 **k** 28 **l** 10 **m** −6 **n** −13

2 **a** −12 **b** 24 **c** 2.5 **d** −5 **e** 36 **f** −12 **g** −7 **h** 60 **i** −1 **j** 60 **k** 6 **l** 19 **m** −7 **n** −38 **o** 34 **p** 160

Exercise 1C

1 **a** 10 000 **b** 343 **c** 5 **d** 2 **e** 64 **f** 20 736 **g** 3 **h** 13 **i** 1000 **j** 4 **k** 2

2 **a** 26 **b** 15 **c** 37 **d** 5 **e** 79 **f** 4

Exercise 1D

1 **a** 87 **b** 606 **c** 3 **d** 34

2 **a** 6800 **b** 46 800 **c** 80 000 **d** 300

3 **a** 53 467 **b** 3 800 000 **c** 789 000 **d** 0.009 21 **e** 0.000 000 103 **f** 2 907 000 **g** 0.000 000 000 003 8 **h** 21 000 000 000

4 **a** 7.92×10^5 **b** 1.46×10^7 **c** 5.0×10^{11} **d** 9.8×10^{-6} **e** $1.456\,97 \times 10^{-1}$ **f** 6.0×10^{-11} **g** $2.679\,886 \times 10^6$ **h** 8.7×10^{-3}

5 **a** 6×10^{24} **b** 4×10^7 **c** 1×10^{-10} **d** 1.5×10^8

6 **a** 5 **b** 6 **c** 1 **d** 3 **e** 2 **f** 1 **g** 2 **h** 4

7 **a** 4.9 **b** 0.0787 **c** 1506.9 **d** 6

8 **a** 0.0033 **b** 0.148 68 **c** 317 **d** 335

9 **a** 1.56 **b** 0.025 **c** 0.03 **d** 1.8823 **e** 17.668 **f** 0.2875

10 **a** 15.65 **b** 4.69 **c** 39.14

Exercise 1E

1 **a** €506.98 EUR **b** $3571.15 USD **c** R$511.62 BRL **d** $2219.03 AUD **e** $3056.95 AUD **f** $5855.04 AUD

2 **a** 158 mm **b** 589.169 km **c** 364.6 cm **d** 13.5 cm^2

3 7.86 m **4** 3000 kg

5 2 250 000 litres **6** 31 trays

7 17 245.20 BWP **8** €328.32 EUR

9 $159.09

10 221 euros

11 Abe $265 NZD, Ren £137.50 GBP

12 **a** HK$3267 **b** $11.16

13 **a** $277.50 **b** $142.70 **c** $57.72

14 **a** $0.51 **b** $1 = €0.81 **c** €1.33

ISBN 978-1-009-25891-3

15 3157.50 SGD

16 379.23 AUD

17 393.02 AUD

18 858.61 AUD

Exercise 1F

1 **a** 25% **b** 40% **c** 15% **d** 70% **e** 19% **f** 79% **g** 215% **h** 3957% **i** 7.3% **j** 100%

2 **a** **i** $\frac{1}{4}$ **ii** 0.25

b **i** $\frac{1}{2}$ **ii** 0.5

c **i** $\frac{3}{4}$ **ii** 0.75

d **i** $\frac{17}{25}$ **ii** 0.68

e **i** $\frac{23}{400}$ **ii** 0.0575

f **i** $\frac{34}{125}$ **ii** 0.272

g **i** $\frac{9}{2000}$ **ii** 0.0045

h **i** $\frac{3}{10\,000}$ **ii** 0.0003

i **i** $\frac{13}{200\,000}$ **ii** 0.000 065

j **i** 1 **ii** 1

3 **a** \$114 **b** \$110 **c** 25.5 m **d** \$1350 **e** 1.59 cm **f** 264 **g** €0.161 **h** \$4570 **i** \$77 700 **j** \$19 800

4 80% **5** 37.5%

6 95.6% **7** 83.33%

8 20% **9** 37.5%

10 65.08%

11 **a** \$150 **b** 33.3%

12 **a** \$20.00 **b** 20%

13 150%

Exercise 1G

1 **a** \$37 **b** \$148

2 **a** \$4.50; \$85.49 **b** \$18.90; \$170.10 **c** \$74.85; \$424.15 **d** \$49.80; \$199.20 **e** \$17.99; \$61.96 **f** \$5.74; \$17.21 **g** \$164.73; \$434.28 **h** \$19.05; \$44.45 **i** \$330; \$670

3 **a** \$425.25 **b** \$699.13 **c** \$227.50 **d** \$656.25 **e** \$215.25

4 **a** \$12.95 **b** \$202.95

5 Decreasing \$60 by 8%

6 14 840 **7** 21.95%

8 26 880 km

9 **a** 13% **b** 26% **c** 6% **d** 24% **e** 18% **f** 23%

10 7.08%

11 **a** 19% **b** 33% **c** 45% **d** 20% **e** 33% **f** 16%

12 **a** 25% **b** 40% **c** 7.5%

Exercise 1H

1 35 : 15

2 **a** 80 : 40 **b** 70 : 9 **c** 80 : 120 **d** 40 : 4 **e** 40 : 4 : 80

Exercise 1I

1 **a** 4 : 5 **b** 2 : 9 **c** 2 : 5 : 3 **d** 1 : 3 **e** 3 : 1 **f** 20 : 3 **g** 9 : 4

2 **a** 12 : 5 **b** 1 : 20 **c** 3 : 8 **d** 25 : 3 **e** 3 : 100 : 600 **f** 100 000 : 100 : 1 **g** 4 : 65 **h** 50 : 10 : 2 : 1

3 **a** 5 **b** 72 **c** 120 **d** 5000 **e** 24

4 **a** False **b** False 3 : 4 = 15 : 20 **c** True **d** False 60 : 12 = 15 : 3 = 5 : 1 **e** False. The girl would be 8. **f** True

5 **a** 100 : 60 : 175 : 125 : 125

b 20 : 12 : 35 : 25 : 25

c 300 g rolled oats, 180 g coconut, 525 g flour, 375 g brown sugar, 375 g butter, 9 tbsp water, 6 tbsp golden syrup, 3 tsp bicarb soda

Exercise 1J

1 **a** \$15.60 **b** 84 seconds **c** \$885 **d** 10 kilometres

2 7 red, 28 yellow

3 **a** 270 km **b** 225 km **c** 30 km **d** 105 km **e** 330 km **f** 67.5 km

4 73 g cone for \$2

5 Brand A

6 51 eggs

7 **a** 550 kilometres **b** 17 litres

Chapter 1 review

Short-answer questions

1 **a** 11 **b** 10 **c** 7 **d** 14 **e** 49 **f** –5 **g** 1 **h** 30 **i** 2

2 **a** 125 **b** 3 **c** 6 **d** 2.83 **e** 4 **f** –4 **g** 8.64 **h** 11.66

3 **a** 2.945×10^3 **b** 5.7×10^{-2} **c** 3.69×10^5 **d** 8.509×10^2

ISBN 978-1-009-25891-3

4 **a** 7500 **b** 0.001 07 **c** 0.456

5 **a** 8.9 **b** 0.059 **c** 800

6 **a** 7.15 **b** 598.2 **c** 4.079

7 **a** $264.67 USD **b** $185.14 AUD

8 **a** $105.76 USD **b** £66.57 GBP

9 **a** 0.75 **b** 0.4 **c** 0.275

10 **a** $\frac{1}{10}$ **b** $\frac{1}{5}$ **c** $\frac{11}{50}$

11 **a** 24 **b** $10.50 **c** $13.25

12 **a** $51.90 **b** $986.10

13 $862.50 **14** 20% **15** 46%

16 Melissa 5.13%, Jody 4.41%

17 **a** False **b** False **c** False **d** True

18 18 cups

19 Brand A **20** $1.21 **21** $20 **22** 301 km

Extended-response questions

1 **a** $1.23 **b** $1.17 **c** $1.25 **d** B, A, C

2 **a** He ignored the weight and compared the prices.
b They are all equally priced.

3 **a** 1486.62 AUD
b £4.50
c £9.44 loss

4 **a** 4.78%
b discuss with teacher, 79.4%
c 11 800

Chapter 2

Exercise 2A

1 **a** 48.8% **b** 24.4% **c** 11.1% **d** 9.2%
e 33.3% **f** 25%

2 **a** $2600 **b** $200 **c** $200 **d** $1.80
e $1865 **f** $20 000

3 **a** $86.40 **b** $180 **c** $0.58 **d** $73.08

4 **a** $291.20 **b** $626.40 **c** $68 **d** $6318

5 **a** $1865.50 **b** $11.14 **c** $27.72
d $10 282 **e** $847.70 **f** $2631.20

6 **a** 24% **b** 20%

7 **a** $60 **b** $50 **c** $71.43 **d** $88

8 **a** $13.30 **b** $62.22 **c** $104.50
d $4993.90

9 $212.75 **10** $92.07

11 **a** $12.13 **b** $6.76 **c** $98.55 **d** $39.50

12 **a** $152.90 **b** $2945.80
c $10 835 **d** $1534.50

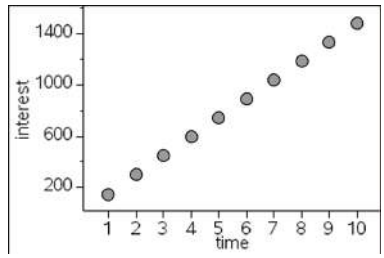
13 $2180.91 **14** $3635.45

15 **a** $289.97 **b** $29.00

Exercise 2B

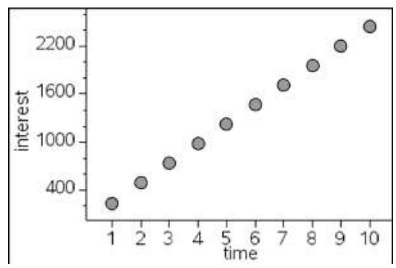
1 **a** $80 **b** $300 **c** $600 **d** $384.38
e $4590 **f** $324.38 **g** $29.95 **h** $14.43
i $6243.75

2 **a**



b $742.5

3 **a**



b $980.50

4 **a** $600 **b** $932.10 **c** $1243.50
d $2411.25 **e** $2832

5 **a** $12 000 **b** $32 000

6 **a** $1950 **b** $11 950

7 $1243.50 **8** $9041.10

9 **a** $118.75 **b** $2750 **c** $2463.19
d $24 000 **e** $1983.63
f $13 617.92

10 **a** $4500 **b** $13.33

11 $3.12 **12** $1.88

13 **a** March $650.72, April $650.72, May $900.72
b $6.88

14 Average daily balance = $1325, Interest earned = $37.76

15 8 days, 7 days, 5 days, 11 days with $36.78 of interest earned

16 **a** $13 600, $14 500, $9500 with daily average balance of $12 261.29
b Interest earned = $416.88

17 **a**

Date	Balance ($)
10 April	19 000
17 April	22 000
23 April	12 000

b Interest payable to Rebecca = $123.30

ISBN 978-1-009-25891-3

Exercise 2C

1 12%

2 15%

3 6.5 years

4 354 days

5 $1210

6 $45 552

7 **a** $180 **b** $780 **c** 3 years
d $1051.60 **e** 7% **f** $1335.15
g $4500 **h** $4650 **i** 5%
j $3698.63 **k** 3 years **l** $220.50
m $1448.28 **n** $1500.78

8 4 years

9 20 years

10 **a** $18 000
b $3000

Exercise 2D

1 **a** $4466.99
b $966.99

2 **a** $9523.42
b $2523.42

3 Difference: CI − SI = $ 202.61

4 **a** $1552.87
b $302.87

5 **a** $1338.23
b $338.23

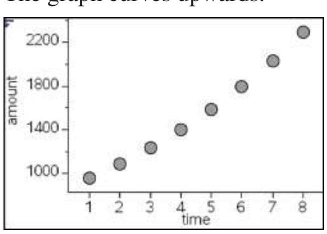
6 Difference: CI − SI = $105.10

7 **a**

1	962.625
2	1090.17
3	1234.62
4	1398.21
5	1583.47

b $1583.47; $733.47

c The graph curves upwards.

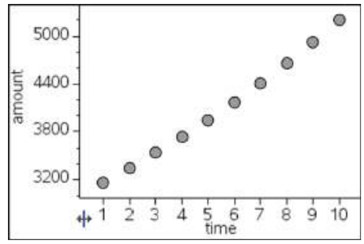


8 **a**

1	3169.5
2	3348.58
3	3537.77
4	3737.66
5	3948.83

b $3737.66 ; $737.66

c The graph curves upwards.



9 **a** 0.3%
b $10 181.36
c $27 846.69

10 **a** 0.2%
b $6482.97
c $1482.97

11 **a** **i** $18 696.04
ii $18 696.64
iii $18 697.13
b 60 cents

Exercise 2E-1

1 **a** $366
b $36

2 **a** $26.40
b $324

3 **a** $1210.20
b $313.20

Exercise 2E-2

1 **a** $54.57
b $110.29
c $452.00
d $45.99

2 **a** No interest on either card, choose either
b A: no interest, B: $20.93, choose A
c A: $36.48, B: $52.72, choose A
d A: $229.12, B: $219.38, choose B

3 **a** $877.69
b $962.30

4 $3060.46

5 $1485.73

Exercise 2F

1 **a** $3.59 **b** $3.72

2 **a** $851.40 **b** $896.52

3 **a** $2.62 **b** $7.10

4 **a** $680 359.31 **b** $1 113 531.40

5 **a** $148 818.78 **b** $58 917.67

6 **a** $598.48 **b** $263.30 **c** $69.04

ISBN 978-1-009-25891-3

Exercise 2G

1 *See table* at bottom of page*

2 a See spreadsheet** **b** $621.98

c $748.86

3 a

Wilson's weekly budget	
Salary	+ $833.60
Rent	– $150.00
Food	– $100.00
Bills	– $120.00
Pocket money	– $10.00
Savings	– $140.00
Surplus	$313.60

b $313.60 **c** allow $10.77 **d** $89.08

4

	A	B
1	Wilson's weekly budget	
2	Salary	833.6
3	Rent	-150
4	Food	-100
5	Bills	-120
6	Pocket money	-10
7	Savings	-140
8	Surplus	=SUM(B2:B7)
9		

5 Answers will vary

6 a Total income = $34 590.64,
Total expenditure = $31 907.39

b 7.76%

c ~ 6 years

d Fixed: Rent, Discretionary: Entertainment

7 a Jack saves $126, Kelly saves $129

b Jack saves $4536, Kelly saves $4644.
No, both fall short of their target

c Jack needs $26.78 more monthly,
Kelly needs $23.78 more monthly

d Petrol, repairs, Insurance etc..

8 a $224.20 **b** $501.40 **c** $135.46

9 $151.20 **10** $58.80 **11** $795.80

12 $928 **13** $624.20

14 a Monday = 6, Tuesday = 7, Wednesday = 4.5,
Friday = 9.5, Total = 27

b $334.80

15 $147 **16** $216 **17** $626.35

18 $16.50 **19** $584.20

Exercise 2H-1

1 $8206.52 **2** $7700.56 **3** $10 831.16

4 NIL **5** $11 273.12

Exercise 2H-2

1 $457.95

2 $683.70 **3** $710.50 **4** $579.20

5 a $3681.12 **b** $729.70 **c** $572

Exercise 2I

1 a 0.5% **b** $12 500

2 a A: 8.8, B: 10 **b** A

3 a $0.50 **b** $1.00

c 500 Alpha oil, 250 Omega mining

d $612.50

4 a $250 **b** 10.9%

5 $14 970

6 Optimus Prime Inc

*

	Hours worked												
	M	T	W	T	F	S	S	Total	Rate/hr	Gross salary	Tax rate	Tax	Net salary
Penny	0.0	5.0	0.0	6.0	0.0	2.0	2.0	15.0	$12.50	$187.50	13.8%	$25.88	$161.33
Wilson	7.5	7.5	7.5	7.5	7.5	0.0	0.0	37.5	$26.75	$1003.13	16.9%	$169.53	$833.60
Vimbai	5.5	8.0	6.5	0.0	3.5	2.0	2.0	27.5	$31.85	$875.88	16.9%	$148.02	$727.85
Brendon	6.5	0.0	7.5	7.5	6.5	5.5	0.0	33.5	$25.80	$864.30	16.9%	$146.07	$718.23
Keith	7.5	0.0	7.5	7.5	7.5	0.0	5.5	35.5	$29.65	$1052.58	16.9%	$177.89	$874.69

**

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1		Hours worked												
2		M	T	W	T	F	S	S	Total	rate/hr	Gross Salary	Tax rate	Tax	Nett Salary
3	Penny	0	5	0	6	0	2	2	=SUM(B3:H3)	12.5	=I3*J3	0.138	=L3*K3	=K3-M3
4	Wilson	7.5	7.5	7.5	7.5	7.5	0	0	=SUM(B4:H4)	26.75	=I4*J4	0.169	=L4*K4	=K4-M4
5	Vimbai	5.5	8	6.5	0	3.5	2	2	=SUM(B5:H5)	31.85	=I5*J5	0.169	=L5*K5	=K5-M5
6	Brendon	6.5	0	7.5	7.5	6.5	5.5	0	=SUM(B6:H6)	25.8	=I6*J6	0.169	=L6*K6	=K6-M6
7	Keith	7.5	0	7.5	7.5	7.5	0	5.5	=SUM(B7:H7)	29.65	=I7*J7	0.169	=L7*K7	=K7-M7
8														

ISBN 978-1-009-25891-3

Exercise 2K

1 C3 = 90, C4 = 60, C5 = 140, C6 = 66, D3 = 9, D4 = 6, D5 = 14, D6 = 6.60

2 F2 = 29, F3 = 19, F4 = 35, F5 = 31, F6 = 12, G2 = 72.5, G3 = 47.5, G4 = 87.5, G5 = 77.5, G6 = 30

3 D2 = 70, D3 = 75, D4 = 128, D5 = 300, D6 = 160

4 a A = \$1590 b 15%, \$339
c \$1630.50

5 a A = \$355, B = \$480
b C = \$200 c 39.6%

Chapter 2 review

Short-answer questions

1 \$12 2 \$24 3 \$69
4 \$133.65 5 \$289.97 6 7.89
7 \$80 8 \$1165 9 6.67%
10 \$3080.83 11 \$4375.82 12 \$11 374.92
13 \$3087 14 \$2.39 15 60
16 \$8400 17 9.3% 18 \$24 660
19 \$4660 20 7.8%

21 a Rabbit Easter Eggs b 0.7%

22 \$137.50

23 a \$1809.09 b \$180.91

24 \$791.89

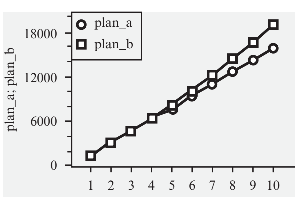
25 a \$451.39 b \$468.47

26 a \$220 b 12.9% to 1 d.p.

Extended-response questions

1 a \$612.50
b i \$437.50 ii \$87.50 iii \$765.63 iv 25%

2 a & b



c i Plan A ii Plan B

3 a \$11 000 b 9.2%

4 a 12.5%
b i \$190 ii 27.1%
c \$155.20 d Credit card

5 a \$171.04 b \$470.36 c \$775.28

Chapter 3

Exercise 3A

1 a 4.9 cm b 83.1 cm c 24 mm
d 2.4 mm e 15.8 mm f 7.4 cm
g 6.4 cm h 141.4 mm i 15.4 m

2 2.9 m 3 3.8 m 4 5.3 m
5 48.88 km 6 15 km 7 12.81 km
8 20 cm 9 9.4 m 10 61.717 m
11 4.24 cm 12 103 m

Exercise 3B

1 a 4.243 cm b 5.20 cm

2 a 10.77 cm b 11.87 cm
c 6.40 cm

3 a 27.73 mm b 104.79 mm

4 9.54 cm

5 a i 8.5 cm ii 9.1 cm
b i 10.6 cm ii 3.8 cm

6 17 cm 7 13 cm 8 25 cm
9 Yes it will fit 10 8.02 m 11 17.55 m

Exercise 3C

1 a i 60 cm ii 225 cm^2
b i 22.4 cm ii 26.1 cm^2
c i 312 cm ii 4056 cm^2
d i 44 cm ii 75 cm^2

2 a 56.2 m^2 b 16.7 m^2 c 103.6 cm^2
d 73.8 cm^2 e 28 cm^2 f 35.9 cm^2
g 29.9 m^2 h 31.3 m^2

3 100 m^2 4 63 375 m^2
5 40 tiles 6 4 L

7 a 5 cm^2 b 125 cm^2 c 40 cm^2

8 30.88 m^2

9 a 252 m^2 b 273 m^2

Exercise 3D

1 a i 31.4 cm ii 78.5 cm^2
b i 53.4 cm ii 227.0 cm^2
c i 49.6 mm ii 196.1 mm^2
d i 1.3 m ii 0.1 m^2

2 a i 25.71 cm ii 39.27 cm^2
b i 1061.98 mm ii 14 167.88 mm^2
c i 203.54 cm ii 2551.76 cm^2
d i 53.70 mm ii 150.80 mm^2

3 62.83 cm^2

4 a 343.1 cm^2 b 34.9 m^2 c 19.2 cm^2
d 177 377.5 mm^2

ISBN 978-1-009-25891-3

5 a 1051.33 m b 37 026.55 m^2
6 a 6 m b 3.4 m^2
7 4241 cm^2
8 30.91 m^2
9 8.73 cm
10 8.19 m

Exercise 3E

1 a 125 cm^3 b 49 067.8 cm^3
c 3685.5 cm^3 d 3182.6 mm^3
e 29 250 cm^3 f 0.3 m^3
g 6756.2 cm^3 h 47.8 m^3
2 424 cm^3
3 516 cm^3
4 24 L
5 a 20 319.82 cm^3 b 20 L
6 228 cm^3

Exercise 3F

1 a 9500.18 cm^3 b 16.36 m^3
c 59.69 m^3 d 2356.19 mm^3
2 a 153.94 cm^3 b 705.84 m^3
c 102.98 cm^3 d 1482.53 cm^3
3 393 cm^3
4 7.87 m^3
5 0.02 L
6 18 263.13 cm^3
7 2791 m^3

Exercise 3G

1 a 26.67 cm^3 b 420 m^3
c 24 m^3 d 68.64 cm^3
2 213.333 cm^3
3 1 694 000 m^3
4 a 335.6 cm^3 b 66.6 cm^3
5 3937.5 cm^3

Exercise 3H

1 a 523.60 mm^3 b 229.85 mm^3
c 7238.23 cm^3
2 a 179.59 cm^3 b 11 494.04 cm^3
c 33.51 cm^3
3 a 8578.64 cm^3 b 7679.12 cm^3
c 261.80 cm^3 d 4.09 m^3
4 44 899 mm^3
5 14 L

Exercise 3I

1 a 1180 cm^2 b 40 m^2
c 383.3 cm^2 d 531 cm^2
e 2107.8 cm^2 f 176.1 m^2
2 a 3053.63 cm^2 b 431.97 cm^2
c 277.59 m^2 d 7.37 m^2
e 242.53 cm^2 f 24.63 m^2
g 235.62 m^2 h 146.08 m^2
3 15 394 cm^2
4 a 1.08 m^2 b 6 m
5 0.28 m^2

Exercise 3J

1 a i $\frac{3}{1}$ or $k = 3$ ii $\frac{9}{1}$ or $k^2 = 9$
b i $\frac{2}{1}$ or $k = 2$ ii $\frac{4}{1}$ or $k^2 = 4$
2 a Similar, $\frac{3}{1}$ or $k = 3$ b Similar, $\frac{2}{1}$ or $k = 2$
c Not similar
3 a Not similar b Similar, $\frac{4}{1}$ or $k = 4$
c Not similar d Similar $\frac{1}{3}$ or $k = \frac{1}{3}$
e Similar $\frac{3}{2}$ or $k = \frac{3}{2}$
4 $\frac{4}{1}$
5 a 3 cm b $\frac{9}{1}$
6 112 cm^2
7 864 cm^2
8 1.67
9 a 36 km b 3 cm
10 14.4 cm

Exercise 3K

1 a SSS b AA c SAS or SSS or AA
2 a $x = 27$ cm, $y = 30$ cm
b $x = 26$ m, $y = 24$ m
3 a 28 cm, 35 cm b 119 cm
4 a AA b $\frac{1}{2}$ c 2 m
5 1.8 m
6 72 cm^2
7 29.4 cm^2

Exercise 3L

1 27 times
2 a $\frac{4}{1}$ b $\frac{64}{1}$
3 $\frac{27}{1}$
4 a 9 cm b $\frac{125}{1}$
5 a Scaled up b 27
c 3240 cm^3
6 a 6 cm b 27: 64
7 a 3 cm b Height = 12 cm, base = 16 cm
8 a 1 : 4 b 1 : 8

Exercise 3M

1 a 1.23 m^3 b 56 boards c 15 m^2
d 0.32 m^3 e 3.84 m^2
2 a 1.12 ha b 413.34 m
3 24 hectares
4 a 29.7 m^2 b 0.39 cm^2
5 a 10 cm b 39 cm, 61 cm
6 a 28.27 cm^2 b 2544.7 cm^3
c 2.545 L d 2827.4 cm^2
7 a 5 times b 32 768 000 cm^3
c 614 400 cm^2 d 7 times

ISBN 978-1-009-25891-3

Chapter 3 review

Short-answer questions

1 a 58 cm b 30 m
2 36 m 3 68 cm
4 a 9.22 cm b 9 cm
5 a 140 cm^2 b 185 cm^2
6 37.5 m^2
7 a 31.42 cm b 75.40 cm
8 a 78.54 cm^2 b 452.39 cm^2
9 a 373.85 cm^2 b 2.97 m^2
c 0.52 litres
10 31 809 litres
11 a 514 718 540 km^2 b 1.098×10^{12} km^3
12 a 376.99 cm^3 b 377 mL
13 6.4 m
14 a 30 m^2 b 15 m^2 c 5.83 m d 69.97 m^2
e 421.94 m^2
15 33.32 m^3
16 4 17 $\frac{1}{4}$
18 a 50.27 cm b 228.53 cm^2
19 both equal 25.13 m

Extended-response questions

1 a 154.30 m^2 b 101.70 m
2 a 61.54 m b 140 m^2 c 120 m^3 d 128 m^2
3 13.33 m
4 a 15.07 m b 1.89 m^3
5 a $\frac{1.96}{1}$ or 1 : 1.96 b $\frac{2.744}{1}$ or 1 : 2.744
c 63 cm^3
6 2048 cm^3 7 18.71 cm
8 a 400 m
b 400 m, 406 m, 412 m, 419 m, 425 m, 431 m
c Each starting point should be 6 m apart except for distance between 3rd and 4th runners which is 7 m.

Chapter 4

Exercise 4A

1 a \$1400 b \$1500 c \$1425
2 380 km
3 a \$10.50 b \$14.40 c \$30
4 a i $C = 157.08$ cm ii $A = 1963.50$ cm^2
b i $C = 18.85$ mm ii $A = 28.27$ mm^2
c i $C = 33.93$ cm ii $A = 91.61$ cm^2
d i $C = 45.24$ m ii $A = 162.86$ m^2
5 a $P = 14$ b $P = 46$ c $P = 23$
6 a $A = 4$ b $A = 14.25$
c $A = 16.74$
7 a i $V = 7420.70$ cm^3
ii $A = 1839.84$ cm^2
b i $V = 8181.23$ mm^3
ii $A = 1963.50$ mm^2
c i $V = 10.31$ m^3 ii $A = 22.90$ m^2
d i $V = 1047.39$ cm^3 ii $A = 498.76$ cm^2
8 a 10°C b −17.8°C
c 100°C d 33.3°C
9 a \$2400.00 b \$180.00
c \$375.00 d \$2014.50
10 a i 15 points ii 37 points
iii 68 points
b Greenteam
11 a 13 b 23 c 101
12 a i 1 h 50 min ii 2 h 56 min
iii 3 h 6 min iv 2 h 38 min
b 5:15 p.m.
13 a 100.53 cm^2 b 123.76 cm^2
c 268.08 cm^3 d 16.76 cm^3

Exercise 4B

1 $x = \frac{20}{7}$ 2 $u = 25$ 3 $h = 9$
4 $m = 0.0001$ 5 $u = 20$ 6 $a = 7$
8 $s = 50$ 9 $m = 4$
9 a 312 500 J b $v = 10$ m/s
10 a 22.86 b 51 kg
11 a 25 Ns b 40 kg
12 a 2.43 s b 1.99 s

Exercise 4C

1

x	40	41	42	43	44	45
C(\$)	86	88.15	90.3	92.45	94.6	96.75

x	46	47	48	49	50
C(\$)	98.9	101.05	103.2	105.35	107.5

2

r	0	0.1	0.2	0.3	0.4	0.5
C	0	0.628	1.257	1.885	2.513	3.142

r	0.6	0.7	0.8	0.9	1.0
C	3.770	4.398	5.027	5.655	6.283

3

n	50	60	70	80	90	100
C(\$)	49	50.8	52.6	54.4	56.2	58

n	110	120	130
C(\$)	59.8	61.6	63.4

ISBN 978-1-009-25891-3

4

$M(kg)$	60	65	70	75	80	85	90
$E(kJ)$	650	695	740	785	830	875	920

$M(kg)$	95	100	105	110	115	120
$E(kJ)$	965	1010	1055	1100	1145	1190

5

n	3	4	5	6
S	180°	360°	540°	720°

n	7	8	9	10
S	900°	1080°	1260°	1440°

6

P		T			
		1	2	3	4
M	1	1	2	3	4
	2	2	4	6	8
	3	3	6	9	12
	4	4	8	12	16

7

H		Z			
		1	2	3	4
R	1	3	$\frac{3}{2}$	1	$\frac{3}{4}$
	2	4	2	$\frac{4}{3}$	1
	3	5	$\frac{5}{2}$	$\frac{5}{3}$	$\frac{5}{4}$
	4	6	3	2	$\frac{6}{4}=\frac{3}{2}$

8 a

n	0	1	2	3	4	5
$E(\$)$	680	740	800	860	920	980

n	6	7	8	9	10
$E(\$)$	1040	1100	1160	1220	1280

b 6 cars

9

T (years)	1	2	3	4	5
$I(\$)$	450	900	1350	1800	2250

T (years)	6	7	8	9	10
$I(\$)$	2700	3150	3600	4050	4500

10

t(years)	5	10	15	20	25
$A(\$)$	6535	8541	11 162	14 589	19 067

11 a

C		p					
		0	2	4	6	8	10
t	0	0	0.4	0.8	1.2	1.6	2
	2	0.1	0.5	0.9	1.3	1.7	2.1
	4	0.2	0.6	1	1.4	1.8	2.2
	6	0.3	0.7	1.1	1.5	1.9	2.3
	8	0.4	0.8	1.2	1.6	2	2.4
	10	0.5	0.9	1.3	1.7	2.1	2.5

b \$1.20

12 a

I		T				
		1	2	3	4	5
R	3	150	300	450	600	750
	3.2	160	320	480	640	800
	3.4	170	340	510	680	850
	3.6	180	360	540	720	900
	3.8	190	380	570	760	950
	4	200	400	600	800	1000

b \$760.00

Exercise 4D

1 a $x = 9$ **b** $y = 15$ **c** $t = 5$ **d** $m = 6$
e $g = 6$ **f** $f = 19$ **g** $f = -3$ **h** $v = -5$
i $x = -1$ **j** $g = 1$ **k** $b = 5$ **l** $m = -2$
m $y = 6$ **n** $e = 3$ **o** $h = -5$ **p** $a = -4$
q $t = -10$ **r** $s = -11$ **s** $k = 7$ **t** $n = 4$
u $a = 8$ **v** $b = 21$

2 a $x = 3$ **b** $g = 9$ **c** $n = 4$ **d** $x = -8$
e $j = -4$ **f** $m = 7$ **g** $f = 5.5$ **h** $x = 3.5$
i $y = 5$ **j** $s = -3$ **k** $b = -5$ **l** $d = -4.5$
m $r = 12$ **n** $q = 30$ **o** $x = 48$ **p** $t = -12$
q $h = 40$ **r** $m = 21$ **s** $a = 2$ **t** $f = -2$
u $a = 6$ **v** $y = 40$ **w** $r = 8$ **x** $x = 5$
y $m = 8$ **z** $x = 6.5$

3 a $y = 4$ **b** $x = 11$ **c** $g = 2$ **d** $x = 3$
e $x = 0.5$ **f** $m = 1.2$ **g** $a = 18$ **h** $r = 13$

4 a $x = 5$ **b** $a = 3$ **c** $b = 9$ **d** $y = 3$
e $x = 0$ **f** $c = -4$ **g** $f = -2$ **h** $y = -5$

5 a $a = 2$ **b** $b = 6$ **c** $w = 2$ **d** $c = 2$
e $y = 7$ **f** $f = 2$ **g** $h = 5$ **h** $k = \frac{4}{3}$
i $g = 8.5$ **j** $s = 20$ **k** $t = 2.2$ **l** $y = 2$
m $x = -2$ **n** $g = 37$ **o** $p = 2$

Exercise 4E

1 a $P = 27 + x$ **b** $P = 4x$
c $P = 2a + 2b$ **d** $P = 19 + y$

2 a $P = 22 + m$ **b** 8 cm

3 a $P = 4y$ **b** 13 cm

4 a $n + 7 = 15$ **b** 8

5 6 **6** 59

7 a $P = 4x + 12$ **b** 18 cm
c 24 cm, 18 cm

8 78 tickets **9** 25 invitations

10 Anne \$750, Barry \$250

11 30 km

12 a 47 min
b Ben 9.4 km, Amy 7.8 km

ISBN 978-1-009-25891-3

Exercise 4F

1 **a** $C = 0.5x + 0.2y$ **b** \$16.50

2 **a** $C = 40x + 25y$ **b** \$13 875

3 **a** $C = 1.6x + 1.4y$ **b** \$141.20

4 **a** $C = 1.75x + 0.7y$ **b** \$52.15

5 **a** $C = 3.5x + 5y$ **b** \$312

6 **a** $C = 30x + 60y$ **b** \$3480

7 **a** $N = x + y$ **b** $V = 0.5x + 0.2y$ **c** \$37.90

8 5 balls

9 George 16, Maria 21

10 6.67 m

Chapter 4 review

Short-answer questions

1 17

2 −7

3 414

4 16

5 3

6 38

7 113.10 cm^2

8 $x = 6$

9 −24

10 $v = 3$

11 $k = -3$

12 \$247.50

13 1.38

14 $h = \dfrac{2A}{a+b}$

15 **a** $x = 10$ **b** $x = 11$ **c** $x = 8$ **d** $x = 6$ **e** $x = 1$ **f** $x = 7$ **g** $x = -6$ **h** $x = 11$ **i** $x = 3$ **j** $x = 3$ **k** $x = 15$ **l** $x = -24$

16 **a** $P = 40$ **b** $P = 130$

17 **a** $A = 30$ **b** $A = 54$

18 94.25 cm

19 424.12 cm^2

20 **a**

x	−20	−15	−10	−5	0
y	−716	−551	−386	−221	−56

x	5	10	15	20	25
y	109	274	439	604	769

b $x = 10$ **c** $x = -5$

21

k		g				
		−2	−1	0	1	2
h	−2	−6	−5	−4	−3	−2
	−1	−4	−3	−2	−1	0
	0	−2	−1	0	1	2
	1	0	1	2	3	4
	2	2	3	4	5	6

22 1 **23** 5

Extended-response questions

1 **a** \$57 **b** 7 hours

2 **a**

n	60	70	80	90	100	110
C	55	60	65	70	75	80

n	120	130	140	150	160
C	85	90	95	100	105

b \$105

3 **a**

P		c					
		0	1	2	3	4	5
a	0	0	42	84	126	168	210
	1	89	131	173	215	257	299
	2	178	220	262	304	346	388
	3	267	309	351	393	435	477
	4	356	398	440	482	524	566
	5	445	487	529	571	613	655

b \$309.00

4 **a** $C = 80 + 45h$ **b** \$215

5 **a** 5% **b** \$315.40 **c** 6.5 hours

6 **a** Fixed cost **b** \$1950 **c** 243 km **d** 192.5 km

Chapter 5

Exercise 5A

1 **a** 3×4 **b** **i** 16 **ii** 3 **iii** 5 **c** 22 **d** 18

2 **a** **i** 2×3 **ii** 6, 7
b **i** 1×3 **ii** 2, 6
c **i** 3×2 **ii** −4, 5
d **i** 3×1 **ii** 9, 8
e **i** 2×2 **ii** 15, 12
f **i** 3×4 **ii** 20, 5

3 **a** B **b** D **c** E

4,5 **a** 9 **b** 2 **c** 3 **d** 10 **e** 8

ISBN 978-1-009-25891-3

6 a 32 students **b** 3×4

c 22 year 11 students preferred football.

7 a $A\ 4 \times 3$, $B\ 2 \times 1$, $C\ 1 \times 2$, $D\ 2 \times 5$

b $a_{32} = 4$, $b_{21} = -5$ $c_{11} = 8$, $d_{24} = 7$

8 a i 75 ha **ii** 300 ha **iii** 200 ha

b 350 ha

c i Farm Y uses 0 ha for cattle.

ii Farm X uses 75 ha for sheep.

iii Farm X uses 150 ha for wheat.

d i f_{23} **ii** f_{12} **iii** f_{21}

e 2×3

Exercise 5B

1 a i
$$\begin{array}{cc} \begin{matrix} A & B \end{matrix} & \\ \begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix} & \begin{matrix} A \\ B \end{matrix} \end{array}$$

ii
$$\begin{array}{cc} \begin{matrix} A & B & C \end{matrix} & \\ \begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} & \begin{matrix} A \\ B \\ C \end{matrix} \end{array}$$

iii
$$\begin{array}{cc} \begin{matrix} A & B & C \end{matrix} & \\ \begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix} & \begin{matrix} A \\ B \\ C \end{matrix} \end{array}$$

iv
$$\begin{array}{cc} \begin{matrix} A & B & C \end{matrix} & \\ \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix} & \begin{matrix} A \\ B \\ C \end{matrix} \end{array}$$

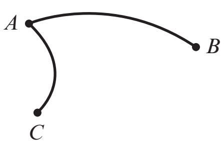
v
$$\begin{array}{cc} \begin{matrix} A & B & C \end{matrix} & \\ \begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} & \begin{matrix} A \\ B \\ C \end{matrix} \end{array}$$

vi
$$\begin{array}{cc} \begin{matrix} A & B & C \end{matrix} & \\ \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \\ 2 & 2 & 0 \end{bmatrix} & \begin{matrix} A \\ B \\ C \end{matrix} \end{array}$$

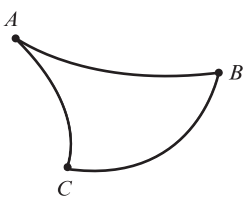
b The number of roads directly connected to B.

2 a Many answers are possible. Examples:

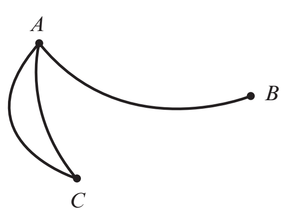
i



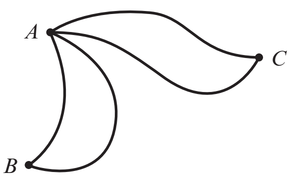
ii



iii



iv



b The number of roads directly connected to town A.

3 a
$$\begin{array}{cc} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

b Compare the sums of the rows (or columns). The person with the highest total has met the most people.

c Person B

d Person C

Exercise 5C

1 a $\begin{bmatrix} 9 & 10 \\ 6 & 3 \end{bmatrix}$ **b** $\begin{bmatrix} 7 & 8 \\ 13 & 3 \end{bmatrix}$

c $\begin{bmatrix} 3 & 5 \\ 7 & 0 \end{bmatrix}$ **d** $\begin{bmatrix} 9 \\ 8 \end{bmatrix}$

e $\begin{bmatrix} 3 & 4 \\ 1 & 6 \end{bmatrix}$ **f** $\begin{bmatrix} 4 & -2 \\ 3 & 9 \end{bmatrix}$

g $\begin{bmatrix} 12 & 7 \end{bmatrix}$ **h** $\begin{bmatrix} 0 & 0 \end{bmatrix}$

i $\begin{bmatrix} 0 & 0 \end{bmatrix}$ **j** $\begin{bmatrix} -2 & 2 & 3 & -9 \end{bmatrix}$

2 a $\begin{bmatrix} 8 & 5 \\ 3 & 7 \end{bmatrix}$ **b** $\begin{bmatrix} 8 & 5 \\ 3 & 7 \end{bmatrix}$

c $\begin{bmatrix} -2 & -9 \\ 1 & 1 \end{bmatrix}$ **d** $\begin{bmatrix} 2 & 9 \\ -1 & -1 \end{bmatrix}$

e Not possible **f** $\begin{bmatrix} 3 & 7 \\ 5 & -2 \\ 4 & -1 \end{bmatrix}$

g Not possible **h** $\begin{bmatrix} -9 & 3 \\ 3 & -2 \\ -2 & 15 \end{bmatrix}$

3
$$\begin{array}{cc} & \begin{matrix} \text{Liberal} & \text{Labor} & \text{Democrat} & \text{Green} \end{matrix} \\ \begin{matrix} \text{Men} \\ \text{Women} \end{matrix} & \begin{bmatrix} 43 & 42 & 10 & 5 \\ 37 & 37 & 17 & 9 \end{bmatrix} \end{array}$$

ISBN 978-1-009-25891-3

4 a
$$\begin{array}{r} \\ \textit{Weight (kg)} \\ \textit{Height (cm)} \end{array}\begin{array}{c} \begin{array}{cccc} \textit{Aida} & \textit{Bianca} & \textit{Chloe} & \textit{Donna} \end{array} \\ \begin{bmatrix} 6 & 8 & -2 & 7 \\ 5 & 8 & 7 & 6 \end{bmatrix} \end{array}$$

b Bianca

c Bianca

6 a
$$\begin{array}{r} \\ \textit{Gymnastics rings} \\ \textit{Parallel bars} \end{array}\begin{array}{c} \textit{Wins} \\ \begin{bmatrix} 3 \\ 2 \end{bmatrix} \end{array}$$

b
$$\begin{array}{r} \\ \textit{Gymnastics rings} \\ \textit{Parallel bars} \end{array}\begin{array}{c} \$ \\ \begin{bmatrix} 150 \\ 100 \end{bmatrix} \end{array}$$

Exercise 5D

1 a $\begin{bmatrix} 14 & -2 \\ 8 & 18 \end{bmatrix}$ **b** $\begin{bmatrix} 0 & -10 \\ 25 & 35 \end{bmatrix}$

c $\begin{bmatrix} -64 & 12 \\ -6 & -14 \end{bmatrix}$ **d** $\begin{bmatrix} 2.25 & 0 \\ -3 & 7.5 \end{bmatrix}$

e $\begin{bmatrix} 18 & 21 \end{bmatrix}$ **f** $\begin{bmatrix} -12 \\ 30 \end{bmatrix}$

g $\begin{bmatrix} 2 & 3 & 0 \\ 0 & 1\frac{1}{2} & \frac{1}{2} \end{bmatrix}$ **h** $\begin{bmatrix} -3 & -6 & 8 \end{bmatrix}$

2 a $\begin{bmatrix} 9 & -12 \\ 6 & 15 \end{bmatrix}$ **b** $\begin{bmatrix} 2 & 28 \\ -6 & -28 \end{bmatrix}$

c $\begin{bmatrix} 1 & -32 \\ 8 & 33 \end{bmatrix}$ **d** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

e $\begin{bmatrix} 21 & 18 \\ 3 & -12 \end{bmatrix}$

3 a $\begin{bmatrix} 79 & -31 \\ 68 & -36 \end{bmatrix}$ **b** $\begin{bmatrix} -121 & 50 \\ -84 & 103 \end{bmatrix}$

c $\begin{bmatrix} 13 & -2 \\ 36 & 53 \end{bmatrix}$ **d** $\begin{bmatrix} 69 & -27 \\ 60 & -30 \end{bmatrix}$

4 a $\begin{bmatrix} 3 \\ 7 \\ 4 \end{bmatrix}$ **b** $\begin{bmatrix} 0 & 5 & 3 & 3 \end{bmatrix}$ **c** $\begin{bmatrix} 6 \\ 14 \\ 8 \end{bmatrix}$

d $\begin{bmatrix} 0 & 15 & 9 & 9 \end{bmatrix}$

5 a
$$\begin{array}{r} \\ \textit{Store A} \\ \textit{Store B} \\ \textit{Store C} \end{array}\begin{array}{c} \begin{array}{ccc} \textit{Clothing} & \textit{Furniture} & \textit{Electronics} \end{array} \\ \begin{bmatrix} 6 & 2 & 9 \\ 5 & 1 & 9 \\ 4 & -1 & 5 \end{bmatrix} \end{array}$$

b
$$\begin{array}{r} \\ \textit{Store A} \\ \textit{Store B} \\ \textit{Store C} \end{array}\begin{array}{c} \begin{array}{ccc} \textit{Clothing} & \textit{Furniture} & \textit{Electronics} \end{array} \\ \begin{bmatrix} 1.8 & 0.6 & 2.7 \\ 1.5 & 0.3 & 2.7 \\ 1.2 & 0 & 1.5 \end{bmatrix} \end{array}$$

Exercise 5E

1 a Defined, 2×1, $\begin{bmatrix} 38 \\ 19 \end{bmatrix}$

b Not defined **c** Defined, 3×1, $\begin{bmatrix} 17 \\ 32 \\ -10 \end{bmatrix}$

d Not defined

e Defined, 2×2, $\begin{bmatrix} 42 & 14 \\ 21 & 7 \end{bmatrix}$

f Not defined

g Not defined

h Defined, 3×2, $\begin{bmatrix} 15 & 5 \\ 24 & 8 \\ -3 & -1 \end{bmatrix}$

2 a 1×2 and 2×1, [38]

b 1×2 and 3×1, not defined

c 1×3 and 3×1, [1]

d 1×3 and 2×1, not defined

e 1×4 and 4×1, [2]

f 1×4 and 3×1, not defined

3 a i $\begin{bmatrix} 6 & 9 \end{bmatrix}$ **ii** $\begin{bmatrix} 10 & 15 \end{bmatrix}$

iii $\begin{bmatrix} 16 & 24 \end{bmatrix}$ **iv** $\begin{bmatrix} 16 & 24 \end{bmatrix}$

b i $\begin{bmatrix} 30 \\ 24 \end{bmatrix}$ **ii** $\begin{bmatrix} 35 \\ 28 \end{bmatrix}$ **iii** $\begin{bmatrix} 35 \\ 28 \end{bmatrix}$

c i $\begin{bmatrix} 4 & 6 \end{bmatrix}$ **ii** $\begin{bmatrix} 15 \\ 12 \end{bmatrix}$ **iii** [22]

iv [132] **v** [132]

4,5 a $\begin{bmatrix} 22 \\ 33 \end{bmatrix}$ **b** $\begin{bmatrix} 64 \\ 53 \end{bmatrix}$ **c** $\begin{bmatrix} 0 & -8 \\ 4 & 2 \end{bmatrix}$

d $\begin{bmatrix} -4 & -3 \\ -14 & -20 \end{bmatrix}$ **e** $\begin{bmatrix} 8 & 3 \\ 5 & 2 \end{bmatrix}$

f $\begin{bmatrix} 16 & 14 \\ 16 & 14 \end{bmatrix}$ **g** $\begin{bmatrix} 31 \\ 35 \\ 21 \end{bmatrix}$ **h** $\begin{bmatrix} 11 \\ 1 \\ 7 \end{bmatrix}$

i [83] **j** [21] **k** [8] **l** [4]

m [30] **n** [36] **o** $\begin{bmatrix} 3 & 3 \end{bmatrix}$

ISBN 978-1-009-25891-3

6 a $\begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$ **b** $\begin{bmatrix} 23 & 34 \\ 31 & 46 \end{bmatrix}$

c No

7 a $\begin{bmatrix} 104 & 70 \\ 80 & 54 \end{bmatrix}$ **b** $\begin{bmatrix} 9 & 8 \\ 7 & 6 \end{bmatrix}$

c $\begin{bmatrix} 17 & 17 \\ 13 & 13 \end{bmatrix}$ **d** $\begin{bmatrix} 8 & 6 \\ 4 & 2 \end{bmatrix}$

e $\begin{bmatrix} 14 & 14 \\ 6 & 6 \end{bmatrix}$

8 a $\begin{bmatrix} 376 & 118 & 154 & 420 \\ 643 & 117 & 281 & 523 \end{bmatrix}$ **b** $\begin{bmatrix} 1292 \end{bmatrix}$

c $\begin{bmatrix} -496 & 752 & 976 & -224 \\ -310 & 470 & 610 & -140 \\ -744 & 1128 & 1464 & -336 \\ 558 & -846 & -1098 & 252 \end{bmatrix}$

d $\begin{bmatrix} -131 & -264 & 176 \\ 467 & 62 & 535 \\ 697 & 279 & 406 \end{bmatrix}$

9 a i $\begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$ **ii** $\begin{bmatrix} 37 & 54 \\ 81 & 118 \end{bmatrix}$

iii $\begin{bmatrix} 199 & 290 \\ 435 & 634 \end{bmatrix}$

b i $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ **ii** $\begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$ **iii** $\begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}$

c i $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **ii** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **iii** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

d i $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **ii** $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ **iii** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

e i $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **ii** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **iii** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Exercise 5F

1 5800 kJ

2 $\begin{array}{r c} & \begin{array}{cc} \textit{Wheels} & \textit{Seats} \end{array} \\ \begin{array}{r} \textit{Smith} \\ \textit{Jones} \end{array} & \begin{bmatrix} 14 & 13 \\ 12 & 9 \end{bmatrix} \end{array}$

3 $[\,110\,]$

4 a $\begin{array}{c} \begin{array}{ccc} \textit{Quiche} & \textit{Soup} & \textit{Coffee} \end{array} \\ \begin{bmatrix} 18 & 12 & 64 \end{bmatrix} \end{array}$

b $\begin{array}{r c} & \$ \\ \begin{array}{r} \textit{Quiche} \\ \textit{Soup} \\ \textit{Coffee} \end{array} & \begin{bmatrix} 5 \\ 8 \\ 3 \end{bmatrix} \end{array}$ **c** \$378

5 a $\begin{array}{c} \begin{array}{cccc} \textit{Chips} & \textit{Pastie} & \textit{Pie} & \textit{Sausage roll} \end{array} \\ \begin{bmatrix} 90 & 84 & 112 & 73 \end{bmatrix} \end{array}$

b $\begin{array}{r c} & \$ \\ \begin{array}{r} \textit{Chips} \\ \textit{Pastie} \\ \textit{Pie} \\ \textit{Sausage roll} \end{array} & \begin{bmatrix} 4 \\ 5 \\ 5 \\ 3 \end{bmatrix} \end{array}$ **c** \$1559

6 a 1720 **b** \$990

7 a $\begin{array}{r c} & \textit{Hrs} \\ \begin{array}{r} I \\ J \\ K \end{array} & \begin{bmatrix} 10 \\ 7 \\ 12 \end{bmatrix} \end{array}$ **b** $\begin{array}{r c} & \textit{Av. Hrs} \\ \begin{array}{r} I \\ J \\ K \end{array} & \begin{bmatrix} 2.5 \\ 1.75 \\ 3 \end{bmatrix} \end{array}$

c $\begin{array}{r c} & \begin{array}{cccc} M & Tu & W & Th \end{array} \\ \textit{Hrs} & \begin{bmatrix} 6 & 11 & 5 & 7 \end{bmatrix} \end{array}$

d $\begin{array}{r c} & \begin{array}{cccc} M & Tu & W & Th \end{array} \\ \textit{Av. Hrs} & \begin{bmatrix} 2 & 3.7 & 1.7 & 2.3 \end{bmatrix} \end{array}$

8 a $\begin{array}{r c} & \textit{Total score} \\ \begin{array}{r} E \\ F \\ G \\ H \end{array} & \begin{bmatrix} 446 \\ 415 \\ 329 \\ 409 \end{bmatrix} \end{array}$ **b** $\begin{array}{r c} & \textit{Av. score} \\ \begin{array}{r} E \\ F \\ G \\ H \end{array} & \begin{bmatrix} 89.2 \\ 83 \\ 65.8 \\ 81.8 \end{bmatrix} \end{array}$

c $\begin{array}{r c} & \begin{array}{ccccc} T1 & T2 & T3 & T4 & T5 \end{array} \\ \textit{Test total} & \begin{bmatrix} 319 & 307 & 324 & 292 & 357 \end{bmatrix} \end{array}$

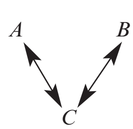
d $\begin{array}{r c} & \begin{array}{ccccc} T1 & T2 & T3 & T4 & T5 \end{array} \\ \textit{Test av.} & \begin{bmatrix} 79.75 & 76.75 & 81 & 73 & 89.25 \end{bmatrix} \end{array}$

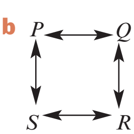
Exercise 5G

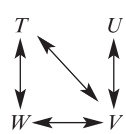
1 D communicates with A, yet A does not communicate with D.

2 a $\begin{array}{c c} \begin{array}{ccc} A & B & C \end{array} & \\ \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} & \begin{array}{l} A \\ B \\ C \end{array} \end{array}$ **b** $\begin{array}{c c} \begin{array}{cccc} D & E & F & G \end{array} & \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} & \begin{array}{l} D \\ E \\ F \\ G \end{array} \end{array}$

c $\begin{array}{c c} \begin{array}{cccc} J & K & L & M \end{array} & \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} & \begin{array}{l} J \\ K \\ L \\ M \end{array} \end{array}$

3 a 

b 

c 

ISBN 978-1-009-25891-3

4 a

$$Q = \begin{array}{c} \begin{array}{cccc} C & E & K & R \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ C \\ E \\ K \\ R \end{array}$$

b Remy communicates with 3 people.

c i

$$Q^2 = \begin{array}{c} \begin{array}{cccc} C & E & K & R \end{array} \\ \begin{bmatrix} 3 & 1 & 1 & 2 \\ 1 & 2 & 2 & 1 \\ 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 3 \end{bmatrix} \end{array} \begin{array}{c} \\ C \\ E \\ K \\ R \end{array}$$

ii Add column E to get 6 ways.

iii $E \to R \to C$

$E \to R \to E$

$E \to C \to E$

$E \to R \to K$

$E \to C \to K$

$E \to C \to R$

5 a

$$R = \begin{array}{c} \begin{array}{cccc} E & F & G & H \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ E \\ F \\ G \\ H \end{array}$$

b Three roads directly connected to Fields.

c i

$$R^2 = \begin{array}{c} \begin{array}{cccc} E & F & G & H \end{array} \\ \begin{bmatrix} 2 & 1 & 1 & 2 \\ 1 & 3 & 2 & 1 \\ 1 & 2 & 3 & 1 \\ 2 & 1 & 1 & 2 \end{bmatrix} \end{array} \begin{array}{c} \\ E \\ F \\ G \\ H \end{array}$$

ii 7

iii $F \to G \to E$

$F \to E \to F$

$F \to G \to F$

$F \to H \to F$

$F \to E \to G$

$F \to H \to G$

$F \to G \to H$

Exercise 5H

1 $\begin{bmatrix} 7 & 5 \\ 5 & 1 \end{bmatrix}$

2 $\begin{bmatrix} 3 & -3 \\ 0 & 0 \end{bmatrix}$

3 $\begin{bmatrix} 6 & -1 \\ 4 & 2 \end{bmatrix}$

4 $\begin{bmatrix} 3 & 0 \\ 5 & -7 \end{bmatrix}$

5 Yes

6 Yes

7 $B = \begin{bmatrix} 6 & 10 \\ -6 & -2 \end{bmatrix}$

8 $C = \begin{bmatrix} 7 & 4 \\ -5 & 8 \end{bmatrix}$

9 $D = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$

10 $P = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$

11 $p = \frac{1}{4}, q = 2, r = 1.$

12 $c = 7$

Exercise 5I

1 Multiplying by a scalar matrix has the same result as multiplying by a scalar.

2 a $\begin{bmatrix} 52 \\ 64 \\ 44 \end{bmatrix}$ Third quarter costs.

b $F = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

c 1×4

d Pre-multiply with a 1×3 matrix.

e $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$

Chapter 5 review

Short-answer questions

1 2×3 **2** 5 **3** 3 **4** 5

5 2 **6** $\begin{bmatrix} 12 & 4 \\ 5 & 3 \end{bmatrix}$ **7** $\begin{bmatrix} 2 & 8 \\ 3 & 3 \end{bmatrix}$

8 $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **9** $\begin{bmatrix} 10 & -4 \\ 2 & 0 \end{bmatrix}$ **10** $\begin{bmatrix} 19 & 10 \\ 9 & 6 \end{bmatrix}$

11 2×3 **12** 3×2 **13** RP **14** $\begin{bmatrix} 34 \\ 50 \end{bmatrix}$

15 $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **16** 2×4 **17** 1

18 $\begin{bmatrix} 38 & 34 & 47 & 54 \end{bmatrix}$

19 2×1

ISBN 978-1-009-25891-3

20
$$\begin{array}{cc} \begin{matrix} P & Q & R \end{matrix} & \\ \begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 3 \\ 0 & 3 & 0 \end{bmatrix} & \begin{matrix} P \\ Q \\ R \end{matrix} \end{array}$$

21 a $\begin{bmatrix} 9 & 3 \\ 12 & 6 \end{bmatrix}$ **b** $\begin{bmatrix} 3 & 6 \\ 11 & 8 \end{bmatrix}$

c $\begin{bmatrix} -3 & 4 \\ 3 & 4 \end{bmatrix}$ **d** $\begin{bmatrix} 6 & 7 \\ 15 & 10 \end{bmatrix}$

e $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **f** $\begin{bmatrix} 7 & 21 \\ 14 & 32 \end{bmatrix}$

g $\begin{bmatrix} 20 & 10 \\ 45 & 19 \end{bmatrix}$ **h** $\begin{bmatrix} 13 & 5 \\ 20 & 8 \end{bmatrix}$

i $\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$

Extended-response questions

1 a 40 pigs **b** 320 sheep **c** Farm A

2 a 21 pies **b** \$2 **c** $\begin{bmatrix} 104 \\ 103 \end{bmatrix}$

d Value of sales for each shop

e Shop A, \$104

3 a
$$\begin{array}{cc} & \begin{matrix} \textit{Hours walking} & \textit{Hours jogging} \end{matrix} \\ \begin{matrix} \textit{Patsy} \\ \textit{Geoff} \end{matrix} & \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix} \end{array}$$

b
$$\begin{array}{cc} & \begin{matrix} \$ & kJ \end{matrix} \\ \begin{matrix} \textit{Walking} \\ \textit{Jogging} \end{matrix} & \begin{bmatrix} 2 & 1500 \\ 3 & 2500 \end{bmatrix} \end{array}$$

c
$$\begin{array}{cc} & \begin{matrix} \$ & kJ \end{matrix} \\ \begin{matrix} \textit{Patsy} \\ \textit{Geoff} \end{matrix} & \begin{bmatrix} 11 & 8500 \\ 12 & 9500 \end{bmatrix} \end{array}$$

4 a $\begin{bmatrix} 360 & 50 \end{bmatrix} \begin{bmatrix} 12 \\ 6 \end{bmatrix} = \begin{bmatrix} 4620 \end{bmatrix}$

b $\begin{bmatrix} 400 & 150 \end{bmatrix} \begin{bmatrix} 10 \\ 5 \end{bmatrix} = \begin{bmatrix} 4750 \end{bmatrix}$ **c** 2.81%

Chapter 6

Exercise 6A

1 a Nominal **b** Ordinal
c Ordinal **d** Nominal

2 a Categorical **b** Numerical
c Categorical **d** Numerical
e Categorical **f** Categorical

3 a Nominal **b** Ordinal
c Numerical (discrete)

4 a Discrete **b** Discrete
c Continuous **d** Continuous
e Discrete

Exercise 6B

1 a Nominal

b

	Frequency	
Sex	Number	%
female	5	33.3
male	10	66.7
Total	15	100.0

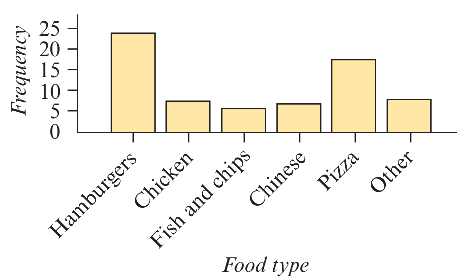
2 a Ordinal

b

	Frequency	
Shoe size	Number	%
7	3	15
8	7	35
9	4	20
10	3	15
11	2	10
12	1	5
Total	20	100

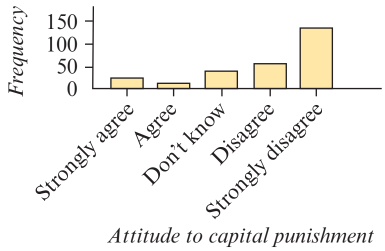
3 a 69; 8.7%, 26.1% **b** Nominal
c 7 students **d** 10.1%
e Hamburger

f

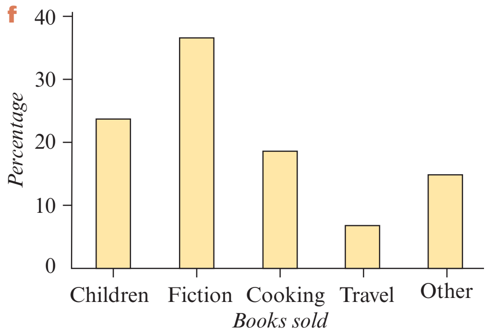


ISBN 978-1-009-25891-3

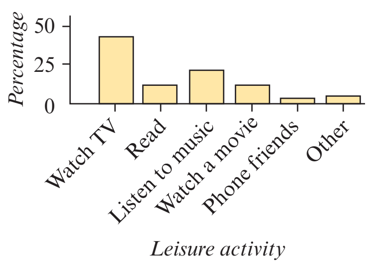
4 **a** 53; 16.4%, 20.7% **b** Ordinal
c 21 people **d** 50.4%
e Strongly disagree
f



5 **a** 38.4%, 6.5%, 100.0% **b** Nominal
c 89 books **d** 22.8%
e 232 books
f



6 **a** 200 students **b** Nominal
c 4% **d** Watch TV
e



Exercise 6C

1 69, hamburgers, 26.1%

2 Strongly disagreed, 20.7%, 16.4%

3 A group of 200 students were asked how they prefer to spend their leisure time. The most popular response was using the internet and digital games (42%), followed by listening to music (23%), reading (13%), watching TV or going to a movie (12%) and phoning friends (4%). The remaining 6% said 'other'. Watching TV for this group of students was clearly the most popular leisure time activity.

4 A group of 579 employees from a large company were asked about the importance to them of the salary that they earned in the job. The majority of employees said that it was important (56.8%), or very important (33.5%). Only a small number of employees said that it was somewhat important (7.8%) with even fewer saying that it was not at all important (1.9%). Salary was clearly important to almost all of the employees in this company.

Exercise 6D

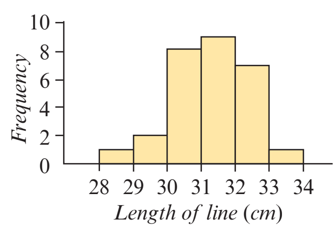
1

Number of magazines	Frequency	
	Number	Percent
0	4	26.7
1	4	26.7
2	3	20.0
3	2	13.3
4	1	6.7
5	1	6.7
Total	15	100.0

2

Amount of money ($)	Frequency	
	Number	Percent
0.00–4.99	13	65
5.00–9.99	3	15
10.0–14.99	2	10
15.00–19.99	1	5
20.00–24.99	1	5
Total	20	100

3 **a** **i** 2 students **ii** 3 students
iii 8 students
b **i** 32.1% **ii** 39.3% **iii** 89.3%
c

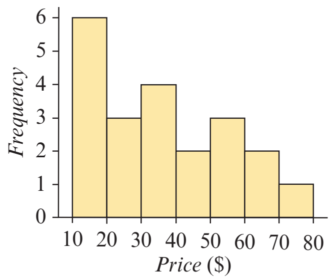


4 **a** 4 students **b** 2 children
c 5 students **d** 28 students

5 **a** 0 students **b** 48 students
c 60–69 marks **d** 33 students

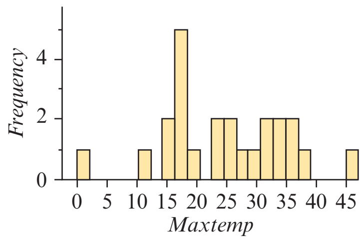
ISBN 978-1-009-25891-3

6 a



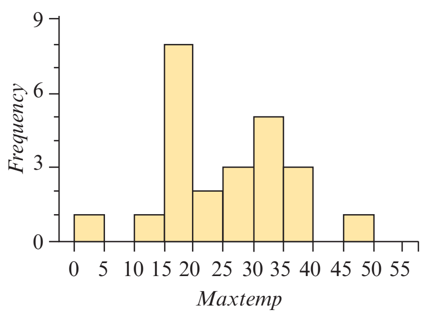
b **i** \$30–\$39 **ii** 4 books **iii** \$10–\$19

7 a



b **i** 11°C **ii** 1 city

c



d **i** 2 cities **ii** 15°C–19°C

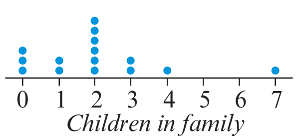
Exercise 6E

1 a Positively skewed **b** Negatively skewed
c Approximately symmetric

2 a Location **b** Neither **c** Both

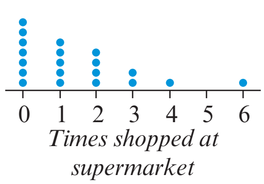
Exercise 6F

1 a



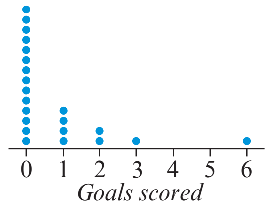
b 2 children

2 a



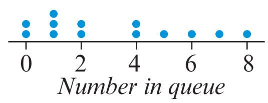
b 7 people

3 a



b 0 goals
c Positively skewed with a possible outlier. The player who kicked six goals.

4 a



b Around 12:25 p.m.

5 a English marks

1	7
2	3 3 6 8
3	2 5 5 8 9
4	3 4 6 6 9
5	0 2 8 9
6	1 4 5 6 9
7	5 8 9 9
8	3 3 4 9
9	2 3 4 7

5|0 represents 50 marks

b 21 students **c** 17 marks

6 a 40 people
b Approximately symmetric
c 21 people

7 a Battery time (hours)

0	4
1	7 9
2	0 1 2 4 5 6 6 7 7 8
3	0 0 1 1 3 3 4 7
4	0 1 6

2|5 represents 25 hours

b 9 batteries

8 a Homework time (minutes)

0	0
1	0 0 4 5 5 6 9
2	0 0 1 3 7 8 9
3	3 7 9
4	6
5	6
6	3
7	0

4|6 represents 46 minutes.

b 2 students
c Positively skewed

ISBN 978-1-009-25891-3

9 a Price ($)

2	5 8
3	5 6 9
4	5 6 9
5	2
6	8
7	5 5 6 8 9
8	2 4
9	5
10	
11	
12	
13	
14	9
15	

16|4 represents $164 (truncated).

b Approximately symmetric with an outlier ($149)

Exercise 6G

1 a mean = 5; median = 5

b mean = 5; median = 4.5

c mean = 15; median = 15

d mean = 101; median = 99.5

e mean = 2.8; median = 2.1

2 a $M = 9$, IQR = 10.5, $R = 21$

b $M = 6.5$, IQR = 8, $R = 11$

c $M = 27$, IQR = 7, $R = 12$

d $M = 106.5$, IQR = 4.5, $R = 8$

e $M = 1.2$, IQR = 1.1, $R = 2.7$

3 a $M = 57$ mm; IQR = 59 − 49.5 = 9.5 mm

b $M = 27.5$ hours; IQR = 33 − 23 = 10 hours

4 a $\bar{x} = 12.5$ ha, $M = 7.4$ ha

b The median, as it is typical of more suburbs. The median is not affected by the outlier.

5 a $\bar{x} = \$393\,243$, $M = \$340\,000$

b The median, as it is typical of more apartment prices.

6 $\bar{x} = 365.8$, $s = 8.4$, $M = 366.5$, IQR = 12.5, $R = 31$

7 $\bar{x} = 214.8$, $s = 35.4$, $M = 207.5$, IQR = 42, $R = 145$

8 $\bar{x} = 3.5$ kg, $s = 0.6$ kg, $M = 3.5$ kg, IQR = 1 kg, $R = 2.4$ kg

9 a i $\bar{x} = 6.79$, $M = 6.75$

ii IQR = 1.45, $s = 0.93$

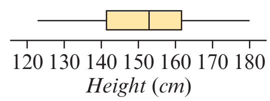
b i $\bar{x} = 13.54$, $M = 7.35$

ii IQR = 1.80, $s = 18.79$

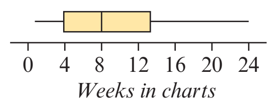
c The error does not affect the median or interquartile range very much. It almost doubles the mean and increases the standard deviation by a factor of over 20.

Exercise 6H

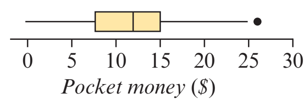
1



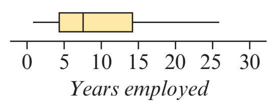
2



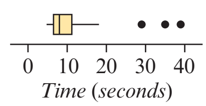
3



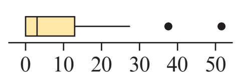
4



5

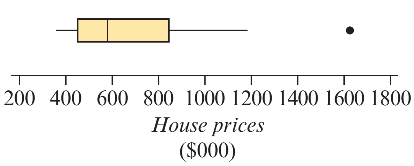


6 a



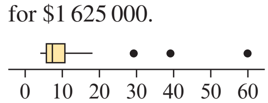
b There are two possible outliers; the people who borrowed 38 and 52 books respectively.

7 a



b There was one outlier, the unit which sold for $1 625 000.

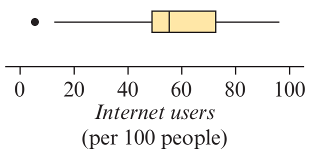
8 a



b There are three possible outliers, the three children who took 29, 39, and 60 seconds respectively to complete the puzzle.

ISBN 978-1-009-25891-3

9 a



b There is one possible outlier, Afghanistan, that recorded extremely low percentages of internet users (5.45%). At 12.52%, India is just above the outlier cutoff of 12.05%.

Exercise 6I

Note: The written reports should only be regarded as sample reports. There are many ways of writing the same thing.

1 a Females: $M = 34$ years, IQR $= 28$ years
Males: $M = 25.5$ years, IQR $= 13$ years

b Report: The median age of the females ($M = 34$ years) was higher than the median age of males ($M = 25.5$ years). The spread of ages of the females (IQR $= 28$ years) was greater than the spread of ages of the males (IQR $= 13$ years). In conclusion, the median age of the females admitted to the hospital on that day was higher than the males. Their ages were also more variable.

2 a Class A: 6; Class B: 2

b Class A: $M = 76.5$ marks, IQR $= 30.5$ marks;
Class B : $M = 78$ marks, IQR $= 12$ marks

c Report: The median mark for Class A ($M = 76.5$) was lower than the median mark for Class B ($M = 78$). The spread of marks for Class A (IQR $= 30.5$) was greater than the spread of marks of Class B (IQR $= 12$). In conclusion, Class B had a higher median mark than Class A and their marks were less variable.

3 a

Japan		Australia
3	0	2 3 3 3 4 4
9 7 6 5 5	0	5 5 6 7 7 8 9
4 4 2	1	1 1 4
9 7 5	1	5 7
3 3 2 2	2	1 3
9 8 6	2	
2	3	3
	3	
4	4	

6|2 represents 26 1|5 represents 15

b Japanese A: $M = 17$ days, IQR $= 16.5$ days;
Australians : $M = 7$ days, IQR $= 10.5$ days

c Report: The median time spent away from home by the Japanese ($M = 17$ days) was much higher than the median time spent away from home by the Australians ($M = 7$ days). The spread in the time spent away from home by the Japanese (IQR $= 16.5$ days) was also greater than the time spent away from home by the Australians (IQR $= 10.5$). In conclusion, the median time spent away from home by the Japanese was longer than the Australians and the time they spent away from home more variable.

4 a Year 12: $M = 5.5$ hours, IQR $= 4.5$ hours;
Year 8: $M = 3$ hours, IQR $= 2.5$ hours
(values can vary slightly)

b Report: The median homework time for the year 12 students ($M = 5.5$ hours/week) was higher than the median homework time for year 8 students ($M = 3$ hours/week). The spread in the homework time for the year 12 students (IQR $= 4.5$ hours/week) was also greater than the year 8 students (IQR $= 2.5$ hours/week). In conclusion, the median homework time for the year 12 students was higher than the year 8 students and the homework time was more variable.

5 a Males: $M = 22\%$, IQR $= 15\%$;
females : $M = 19\%$, IQR $= 14\%$ (values can vary a little)

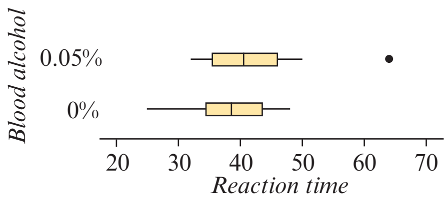
b Report: The median smoking rate for males ($M = 22\%$) was higher than for females ($M = 19\%$). The spread in smoking rates for males (IQR $= 15\%$) was similar to females (IQR $= 14\%$). In conclusion, median smoking rates were higher for males than females but the variability in smoking rates was similar.

6 a Before: $M = 26$, IQR $= 4$,
outlier $= 45$; After : $M = 30$, IQR $= 6$,
outliers $= 48$ & 52 (values can vary a little)

b Report: The median number of sit ups before the fitness class ($M = 26$) was lower than after the fitness class ($M = 30$). The spread in number of sit ups before the fitness class (IQR $= 4$) was less than after the fitness class (IQR $= 6$). There was one outlier before the fitness class, the person who did 45 sit ups. There were two outliers after the fitness class, the person who did 48 sit ups and the person who did 52 sit ups. In conclusion, the median number of sit ups increased after taking the fitness class and there was more variability in the number of sit ups that could be done.

ISBN 978-1-009-25891-3

7 a

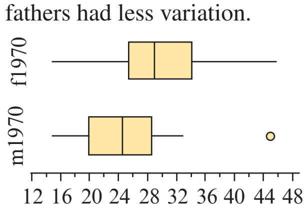


b Report: The median time is slightly higher for the 0.05% blood alcohol group (M = 40.5) than for the 0% blood alcohol group (M = 38.5). The spread in time is also slightly higher for the 0.05% blood alcohol group (IQR = 9.5) than for 0% blood alcohol (IQR = 9.0). There was one outlier, the person with 0.05% blood alcohol who had a very long time of 64 seconds.

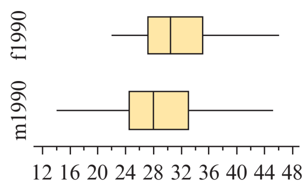
In conclusion, the median time was longer for the 0.05% blood alcohol group than for the 0% blood alcohol group but the variability in times was similar.

Exercise 6J

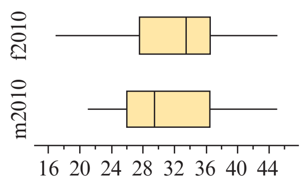
1 a The age of fathers is consistently higher than that of mothers. The median ages in 1970, 1990 and 2010 are 29, 31 and 33.5 for fathers respectively, whilst the median ages for mothers are 23.5, 28 and 31. The variability of fathers stayed consistent, whilst the variability in ages for mothers grew and even surpassed that of fathers over the course of the study. Fathers having IQR of 9, 9 and 9.5 whilst mothers recorded IQR of 8.5, 9 and 10.5 each year. In conclusion, the median age of fathers was higher than that of mothers, and on average the age of fathers had less variation.



b The median age for mothers has grown significantly with each survey, increasing from 23.5 to 28 to 31 years in 1970,1990 and 2010 respectively. The variability in ages has also increased, with the IQR for mothers increasing from 8.5 to 9 to 10.5 respectively. That is, the median age of mothers has increased and the variability in their ages has also increased.



c The median age for fathers has increased over the course of the study, going from 29 to 31 to 33.5 years in 1970, 1990 and 2010 respectively. Their spread has remained consistent however, with the IQR being measured at 9, 9 and 9.5 in the respective surveys. The median age of fathers has increased, whilst the spread has stayed constant.



d Report

The median age for mothers has increased steadily over the years, from 23.5 in 1970, to 28 in 1990 and 31 in 2010. The spread in ages for mothers was the same in 1970 (IQR = 8.5) and 1990 (IQR = 9), but increased in 2010 (IQR = 10.5). In 1970, a mother of age 45 was considered an outlier, but in 1990 and 2010, the age of 45 was not unusual enough to be an outlier.

The median age for fathers has increased steadily over the years, from 29 in 1970, to 31 in 1990 and to 33.5 in 2010. The spread in ages for fathers has remained reasonably steady during this time (1970: IQR = 9; 1990: IQR = 9; 2010: IQR = 9.5)

The difference in age between mothers and fathers has changed little over time. Fathers have typically been older than mothers by the same amount (1970: 5.5 y; 1990: 3 y; 2010: 4 y)

Chapter 6 review

Short-answer questions

1 2% **2** 6%

3 40 to less than 50

4 40 to less than 50

5 43 **6** 8

7 11 **8** 10

9 mean 184.35; standard deviation = 8.0

10 3 **11** 80%

ISBN 978-1-009-25891-3

12 30

13 9

14 10

15 25%

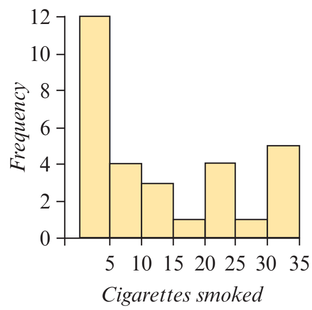
16 company 3

17 company 1

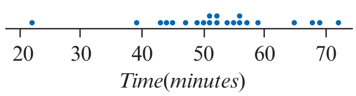
18 **a** Discrete **b** Ordinal

19 **a** Categorical **b** 7.5%

20



21 **a**



b Time (minutes)

2	2
3	9
4	3 4 5 7 9
5	0 1 1 2 2 4 5 6 6 7 9
6	5 8 9
7	2

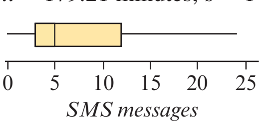
4|7 represents 47 minutes

c M = 52 minutes, Q_1 = 47 minutes, Q_3 = 57 minutes

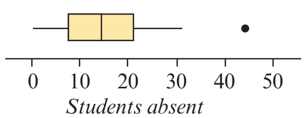
22 $\bar{x}$ = \$283.57, s = \$122.72, M = \$267.50, IQR = \$90, R = \$495

23 $\bar{x}$ = 179.21 minutes, s = 14.61 minutes

24



25 **a**



b 14.5 students **c** 27.8%

Extended-response questions

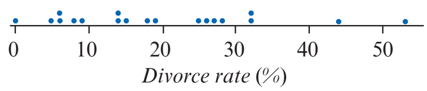
1 **a** Numerical

b Divorce rate (%)

0	0 5 6 6 8 9
1	4 4 5 8 9
2	5 6 7 8
3	2 2
4	4
5	3

3|2 represents 32%

c

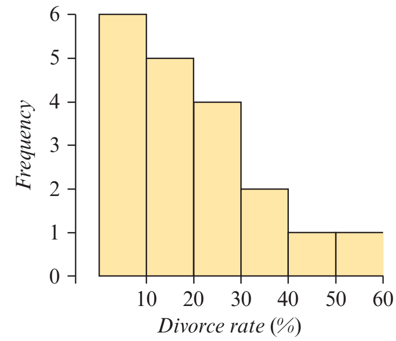


d Positively skewed

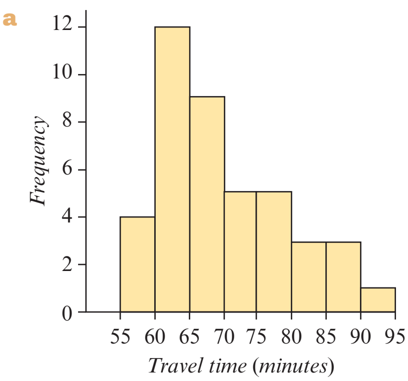
e 21.1%

f $\bar{x}$ = 20.05%, M = 18%

g **i** Positively skewed **ii** 5 countries



2 **a**

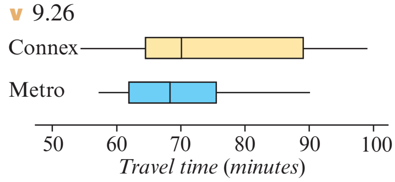


i 9 days **ii** Positively skewed **iii** 38.1%

b $\bar{x}$ = 69.60 minutes, s = 9.26 minutes, Min = 57 minutes, Q_1 = 62 minutes, M = 68 minutes, Q_3 = 76 minutes, Max = 90 minutes

c **i** 69.60 **ii** 68 **iii** 33, 14 **iv** 76 **v** 9.26

d



e The median travel times for Connex (M = 70 minutes) is larger than the median travel times for Metro (M = 68 minutes). The spread of times is also longer for Connex (IQR = 24 minutes) compared to Metro (IQR = 14). Both median travel times and variability in travel times was less for Metro than for Connex.

ISBN 978-1-009-25891-3

Chapter 7

Exercise 7A

1 a 114 and 154 b 94 and 174 c 74 and 194 d 154 e 94 f 74 g 134

2 a 68% b 99.7% c 16% d 2.5% e 0.15% f 50%

3 a i 84% ii 50% iii 47.5% b 25

4 a i 99.7% ii 2.5% iii 81.5% b 800

5 a i 50% ii 34% iii 81.5% b 1994

Exercise 7B

1 a 81.8% b 28.0% c 34.5% d 21.2%

2 a 48.4% b 1.6% c 41.5% d 23.8%

3 a i 81.9% ii 164 b 13 c 138

Exercise 7C

1 a 26.6 b 80.0 c 24.9 d 4.5

2 $p = 43.2$

3 $p = 29.7$

4 84

5 a 11.1 sec b 192

6 199 mm

7 24.4 cm

8 a 511 g b 535 g c Between 511 g and 535 g

Exercise 7D

1 a $z = 1$ b $z = 2$ c $z = -1$ d $z = 0$ e $z = -3$ f $z = 0.5$

2 a 120 b 116 c 142 d 100 e 72 f 50

3 a–b

Subject	*z-score*	*Rating*
English	2.25	Top 2.5%
Biology	3	Top 0.15%
Chemistry	0	Exactly average
Further Maths	1.1	Top 16%
Psychology	−2.25	Bottom 2.5%

4 a 0.2 b 46.5 kg c 2.5% d 34% e 16% f 97.5%

Chapter 7 review

Short-answer questions

1 True

2 68%

3 True

4 68%

5 0.9088

6 0.1056

7 63.1

8 20.3

9 1.4

10 SD = 4

11 81.5%

12 208

13 788

14 13

15 389.5 hours

16 28 kg

Extended-response questions

1 a 95% b 50% c 0% d 34%

2 a 202 cm b 68% c 16%

3 a 84% b 0.95 cm

4 a 59% b $Q_3 = 67.1$, $Q_1 = 50.9$, IQR = 16.2 c 74.4

5 a 140 b 105

Chapter 8

Exercise 8A

1 Answers are in order: hypotenuse, opposite, adjacent.

a 13, 5, 12 b 10, 6, 8 c 17, 8, 15 d 25, 24, 7 e 10, 8, 6 f 13, 12, 5

2 Answers are in order: $\sin\theta, \cos\theta, \tan\theta$.

a $\frac{5}{13}, \frac{12}{13}, \frac{5}{12}$ b $\frac{3}{5}, \frac{4}{5}, \frac{3}{4}$

c $\frac{8}{17}, \frac{15}{17}, \frac{8}{15}$ d $\frac{24}{25}, \frac{7}{25}, \frac{24}{7}$

e $\frac{4}{5}, \frac{3}{5}, \frac{4}{3}$ f $\frac{12}{13}, \frac{5}{13}, \frac{12}{5}$

ISBN 978-1-009-25891-3

3 **a** 0.4540 **b** 0.7314 **c** 1.8807 **d** 0.1908
e 0.2493 **f** 0.9877 **g** 0.9563 **h** 1.1106
i 0.9848 **j** 0.7638 **k** 5.7894 **l** 0.0750

Exercise 8B

1 **a** $\sin\theta$, 20.74 **b** $\cos\theta$, 20.76
c $\tan\theta$, 32.15 **d** $\cos\theta$, 8.24
e $\tan\theta$, 26.63 **f** $\sin\theta$, 7.55
g $\sin\theta$, 17.92 **h** $\tan\theta$, 15.59
i $\cos\theta$, 74.00 **j** $\tan\theta$, 17.44
k $\sin\theta$, 32.72 **l** $\sin\theta$, 37.28

2 **a** 78.05 **b** 25.67 **c** 8.58 **d** 54.99
e 21.32 **f** 11.59 **g** 30.67 **h** 25.38
i 63.00 **j** 62.13 **k** 4.41 **l** 15.59

3 **a** 12.8 **b** 28.3 **c** 38.5 **d** 79.4
e 16.2 **f** 15.0 **g** 14.8 **h** 37.7
i 59.6

Exercise 8C

1 **a** 28.8° **b** 51.1° **c** 40.9° **d** 30.0°
e 45.0° **f** 45.0° **g** 60.0° **h** 68.2°
i 33.0° **j** 73.0° **k** 17.0° **l** 30.0°
m 45.0° **n** 26.6° **o** 30.0° **p** 70.0°

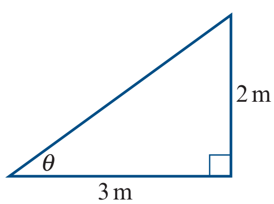
2 **a** 32.2° **b** 59.3° **c** 28.3° **d** 55.8°
e 46.5° **f** 48.6° **g** 53.1° **h** 58.8°
i 22.6° **j** 53.1° **k** 46.3° **l** 22.6°
m 32.2° **n** 41.2° **o** 48.2°

3 **a** 36.9° **b** 67.4° **c** 53.1° **d** 67.4°
e 28.1° **f** 43.6°

Exercise 8D

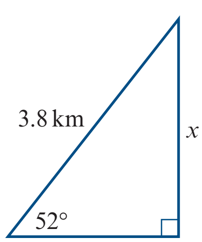
1 6.43 m **2** 21.0° **3** 10 m **4** 16 m

5 **a**



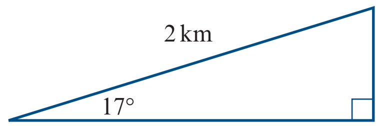
b 33.7°

6 **a**



b 3.0 km

7 **a**



b **i** Horizontal distance 1.91 km
ii Height 0.58 km

8 70.5° **9** 78.1 m **10** 5.77 m

Exercise 8E

1 413 m **2** 11 196 m **3** 33 m
4 164.8 m **5** 244 m **6** 14°
7 **a** 44.6 m **b** 36°
8 **a** 16.2 m **b** 62°
9 **a** 35 m **b** 64 m **c** 29 m
10 507 m

Exercise 8F

1 **a** 025° **b** 110° **c** 210° **d** 280°
2 **a** 25° **b** 7.61 km
3 **a** 236° **b** 056°
4 130°
5 **a** 4.2 km **b** 230°
6 **a** 10 km, 15 km **b** 5 km
c 8.7 km **d** 10 km
e 319°, 13.2 km
7 **a** 12.9 km **b** 15.3 km **c** 17.1 km
d 42° **e** 138°, 23.0 km

Chapter 8 review

Short-answer questions

1 $\dfrac{12}{13}$
2 $24\cos(36°)$
3 $17\tan(62°)$
4 $\dfrac{95}{\sin(46°)}$
5 $\dfrac{20}{\tan(43°)}$
6 $\cos^{-1}\left(\dfrac{15}{19}\right)$
7 53.1°

ISBN 978-1-009-25891-3

8 120°

9 210°

10 35.87 cm

11 117.79 cm

12 4°

13 a 65, 72, 97 b $\frac{65}{97}$

14 11 cm

15 7.83 cm

16 26.6°

17 8.66°

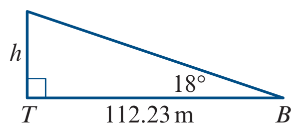
18 604 m

19 237°

Extended-response questions

1 a 50.95 m b 112.23 m

c



height of tree = 36.47 m

2 x = 4.45 cm, y = 4.55 cm, z = 13.46 cm

3 a 24.85 m b 4.14 m

4 a XXX b 50 km c 293°

Chapter 9

Exercise 9A

1 a 102 cm^2 b 40 cm^2 c 24 cm^2
d 52 cm^2 e 17.5 cm^2 f 6 cm^2

2 a 25.7 cm^2 b 65.0 cm^2
c 26.0 cm^2 d 32.9 cm^2
e 130.5 cm^2 f 10.8 cm^2

3 a 36.0 km^2 b 9.8 m^2 c 23.5 cm^2
d 165.5 km^2 e 25.5 cm^2 f 27.7 cm^2

4 a iv b iii c i d ii

5 a 10 cm^2 b 23.8 cm^2 c 63.5 cm^2
d 47.3 m^2 e 30 m^2 f 30.1 m^2
g 100.9 km^2 h 21.2 km^2 i 6 km^2

6 224 cm^2

7 1124.8 cm^2

8 150.4 km^2

9 3500 cm^2

10 a 6 m^2 b 4.9 m^2 c 6.9 m^2

11 a 33.83 km^2 b 19.97 km^2
c 53.80 km^2

12 a 43.30 cm^2 b 259.81 cm^2

13 a i 12 km^2 ii 39 km^2 iii 21 km^2
b 29.6°

Exercise 9B

1 a $a = 15, b = 14, c = 13$
b $a = 19, b = 18, c = 21$
c $a = 31, b = 34, c = 48$

2 a $C = 50°$ b $A = 40°$ c $B = 105°$

3 a 5.94 b 12.08 c 45.11 d 86.8°
e 44.4° f 23.9°

4 a 41.0° b 53.7° c 47.2° d 50.3°

5 a 19.60 b 30.71 c 55.38 d 67.67

6 a 4.45 b 16.06 c 67.94 d 67.84

7 a $c = 10.16, B = 50.2°, C = 21.8°$
b $b = 7.63, B = 20.3°, C = 39.7°$
c $a = 52.22, c = 61.01, C = 37°$
d $b = 34.65, c = 34.23, C = 54°$

8 39.09 9 43.2° 10 49.69

11 $a = 31.19, b = 36.56, A = 47°$

12 $A = 27.4°, C = 22.6°, c = 50.24$

13 $a = 154.54, b = 100.87, C = 20°$

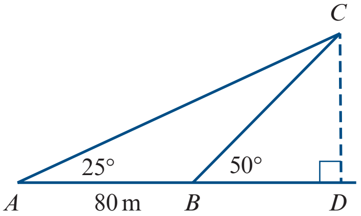
14 2.66 km from A, 5.24 km from B

15 409.81 m

16 a 26.93 km from naval ship, 20.37 km from other ship
b 1.36 h (1 h 22 min)

17 a Airport A b 90.44 km
c Yes

18 a



b 130° c 25° d 145.01 m
e 61.28 m

ISBN 978-1-009-25891-3

Exercise 9C

1 a 36.72 b 47.62 c 12.00 d 14.55
e 29.95 f 11.39

2 17.41 3 27.09 4 51.51

5 a 33.6° b 88.0° c 110.7° d 91.8°
e 88.3° f 117.3°

6 50.5° 7 63.2° 8 40.9°

9 $B = 46.6°$ 10 $B = 73.2°$ 11 33.6°

12 19.08 km 13 a 39.6° b 310°

14 a 60° b 42.51 km 15 5.27 km

16 11.73 km 17 4.63 km 18 45.83 m

Exercise 9D

Answers below are approximate, based upon measured quantities. Student answers may vary.

a i 106.5 km ii 177.9 km

b No c 1 h 34 min d $351

e i 067° T ii 228° T iii 024° T

f Fly 177.9 km on a bearing of 228° T

g 19° h 84.6 km (using 19°)

i 3084 km^2

Chapter 9 review

Short-answer questions

1 46.3°

2 $\dfrac{12\sin 100°}{\sin 50°}$

3 $\dfrac{3\sin 60°}{5}$

4 $\sqrt{21^2 + 29^2 - 2(21)(29)\cos 47°}$

5 $\dfrac{7^2 + 5^2 - 6^2}{2(7)(5)}$

6 $\cos^{-1}\left(\dfrac{14^2 + 13^2 - 12^2}{2(14)(13)}\right)$

7 54 cm^2

8 31.72 cm^2

9 26.98

10 158.6 m^2

11 14.02 cm

12 76.3°

13 $A = 40.7°$

14 54.17 km

15 760.7 cm^2

16 27.7 m^2

17 6.43 cm

18 49 trees, with cost $227.85

19 10 cm

20 9.98 cm

Extended-response questions

1 a 50°
b First group 3.68 km, second group 3.39 km
c 290°

2 a 110° b 81.26 km

3 a 44.4°, 57.1°, 78.5° b 14.7 m^2 c $426.21

4 a 83° b 59.0° c $218 615 284.90

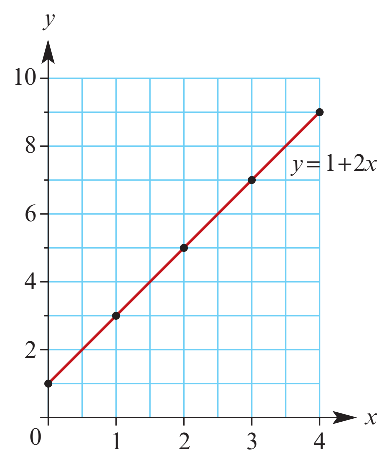
5 a 66 m b 1958.6 m^2 c 64°
d 4456.7 m^2

Chapter 10

Exercise 10A

1 a

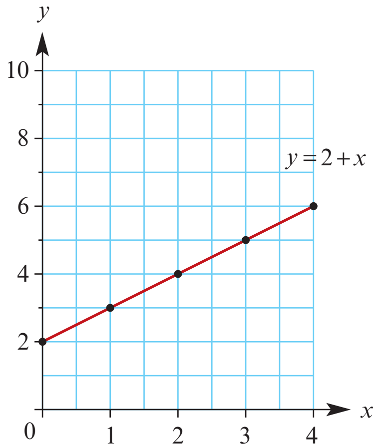
x	0	1	2	3	4
y	1	3	5	7	9



ISBN 978-1-009-25891-3

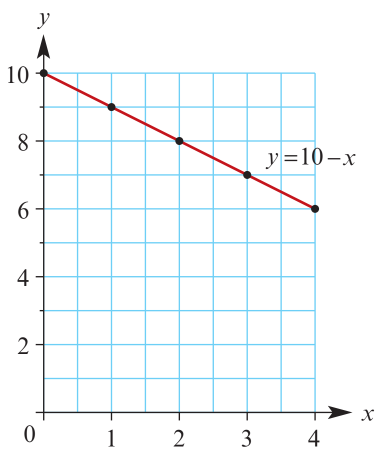
b

x	0	1	2	3	4
y	2	3	4	5	6



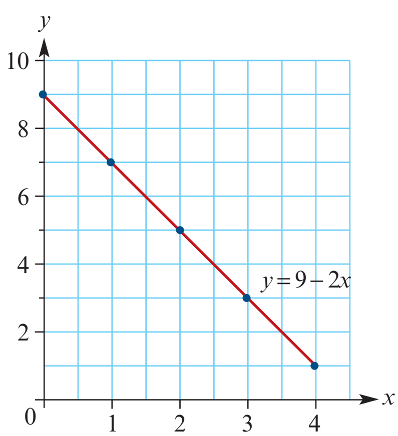
c

x	0	1	2	3	4
y	10	9	8	7	6

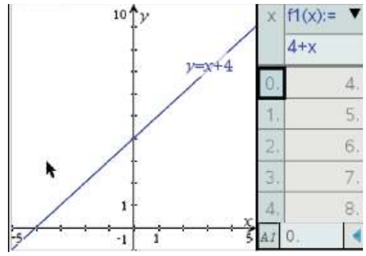


d

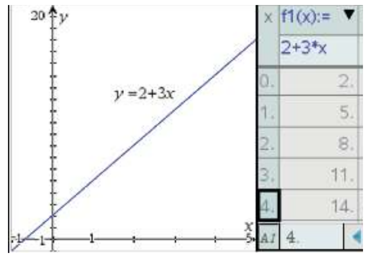
x	0	1	2	3	4
y	9	7	5	3	1



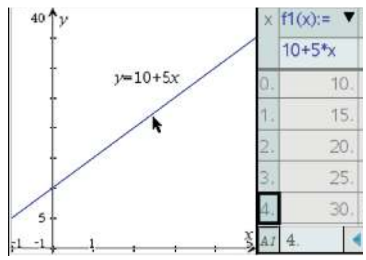
2 a



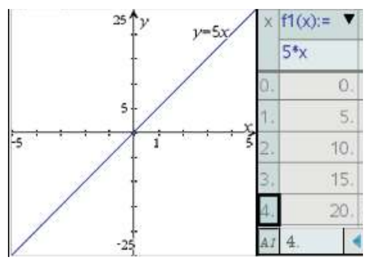
b



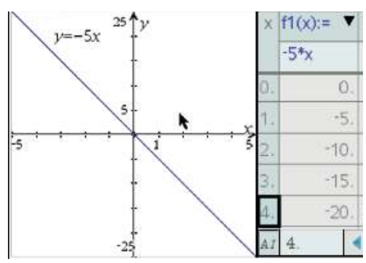
c



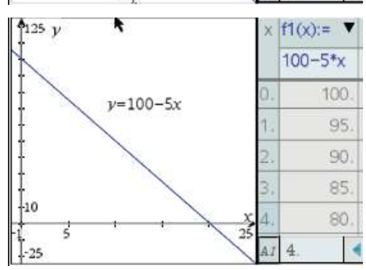
d



e



f



3 a $(0, 4), (2, 6), (3, 7), (5, 9)$

b $(0, 8), (1, 6), (2, 4), (3, 2)$

ISBN 978-1-009-25891-3

Exercise 10B

1 A negative, B positive, C not defined, D zero

2 $A\ \frac{-7}{3}$, $B\ \frac{7}{4}$, $C\ 1$

3 $A\ \ 2$, $B\ -3$, $C\ \ 0$

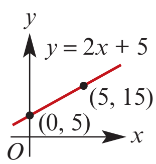
4 a 2 **b** -1 **c** 2 **d** 0.6
e 2 **f** -1

Exercise 10C

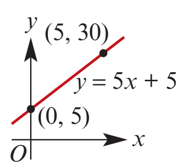
1 a slope $= 2$, y-intercept $= 5$
b slope $= -3$, y-intercept $= 6$
c slope $= -5$, y-intercept $= 15$
d slope $= -3$, y-intercept $= 10$
e slope $= 3$, y-intercept $= 0$
f slope $= -2$, y-intercept $= -5$
g slope $= 1$, y-intercept $= 4$
h slope $= 0.5$, y-intercept $= 3$
i slope $= 2$, y-intercept $= -5$
j slope $= 5$, y-intercept $= 10$
k slope $= -1$, y-intercept $= 10$
l slope $= 2$, y-intercept $= 0$
m slope $= -3$, y-intercept $= 6$
n slope $= 2$, y-intercept $= -4$
o slope $= 0.8$, y-intercept $= -3$
p slope $= 3$, y-intercept $= -2$

2 a $y = 5x + 2$ **b** $y = 10x + 5$
c $y = 4x - 2$ **d** $y = -3x + 12$
e $y = -5x - 2$ **f** $y = -0.4x + 1.8$
g $y = -2x + 2.9$ **h** $y = -0.5x - 1.5$

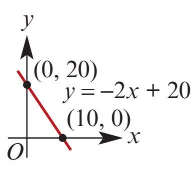
3 a



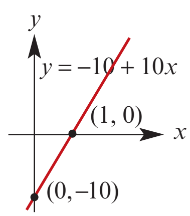
b



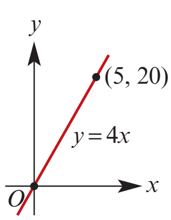
c



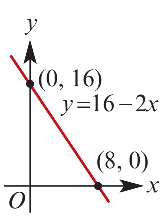
d



e



f



Exercise 10D

1 A: $y = -2.25x + 10$ B: $y = 1.75x + 2$
C: $y = x$

2 A: $y = 2x + 4$ B: $y = -1.5x + 8$
C: $y = 0.6x + 2$

Exercise 10E

1 A: $y = -4.5x + 14.5$ B: $y = 5x - 5$
C: $y = 3x - 5$

2 A: $y = -1.5x + 11.5$ B: $y = 10x - 10$
C: $y = 1.2x + 2$

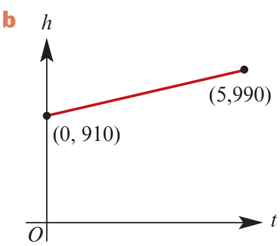
Exercise 10F

This exercise repeats Exercises 10D and 10E, so the answers are the same as the answers in the above exercises. The exact method will depend on the calculator used by the student.

Exercise 10G-1

1 a $h = 16t + 910$ for $0 \le t \le 5$

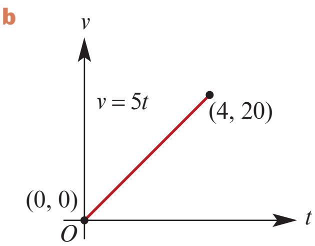
b



c 982 cm

2 a $V = 5t + 0$ for $0 \le t \le 4$

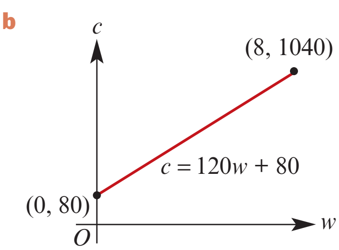
b



c 16 litres

3 a $c = 120w + 80$ for $0 \le w \le 8$

b

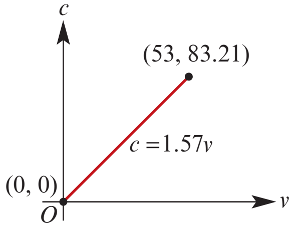


c \$680

ISBN 978-1-009-25891-3

4 **a** $c = 1.57v, \quad \text{for } 0 \le v \le 53$

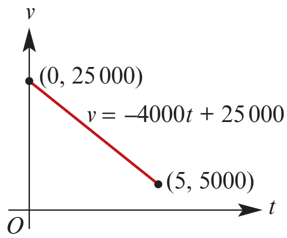
b



c \$83.21

5 **a** $v = -4000t + 25\,000, \text{ for } 0 \le t \le 5$

b

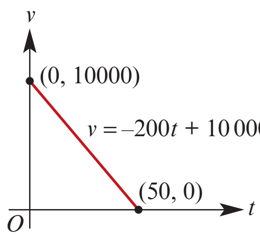


c \$14 600

6 **a** 50 days

b $V = -200t + 10\,000, \quad \text{for } 0 \le t \le 50$

c



d 4000 litres

Exercise 10G-2

1 **a** \$10 **b** \$17.50 **c** $C = 0.075n + 10$
d \$32.50 **e** \$0.075 (7.5 cents)

2 **a** 500 mL **b** Slightly below 400 mL
c 200 minutes **d** $V = -2.5t + 500$
e 212.5 mL **f** 2.5 mL/min

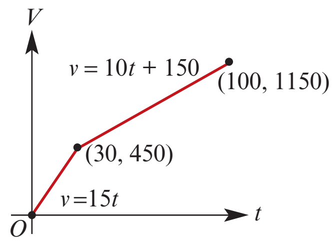
3 **a** $F = 32 + 1.8C$ (or as more commonly written: $F = \frac{9}{5}C + 32$)
b **i** 122°F **ii** 302°F **iii** −40°F
c 1.8

Exercise 10G-3

1 **a** **i** 5 : 00 **ii** 12 : 00 **iii** 9.00 **iv** 6 : 00
b 1hr
c 10 : 30

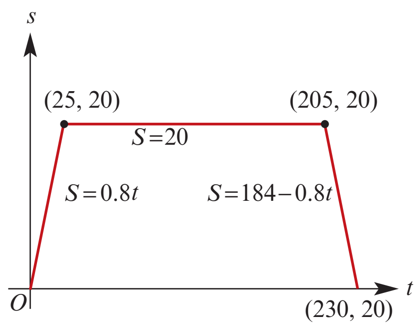
2 **a** **i** 300 L **ii** 450 L **iii** 750 L
iv 1150 L

b

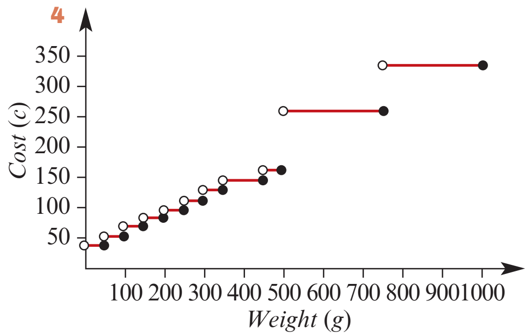


3 **a** **i** 8 m/s **ii** 20 m/s **iii** 20 m/s **iv** 16 m/s

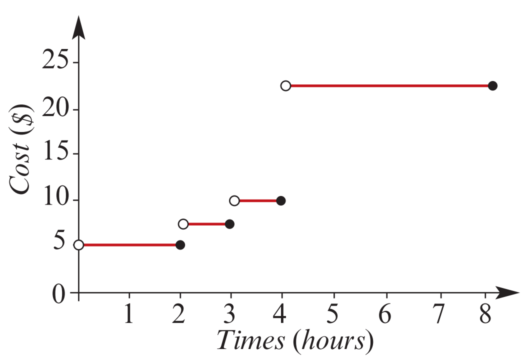
b



4

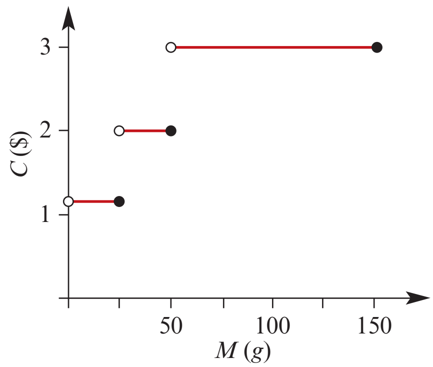


5



ISBN 978-1-009-25891-3

6



7 a free **b** $3.00 **c** $5.00

Chapter 10 review

Short-answer questions

1 10

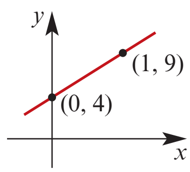
2 5

3 −3

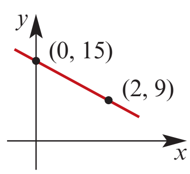
4 2

5 −0.75

6



7



8 2

9 1.2

10 not defined

11 $y = -2x + 8$

12 $y = 2.25x$

13 yes

14 $y = 4x - 6$

15 5 cm/week

16 $4.50

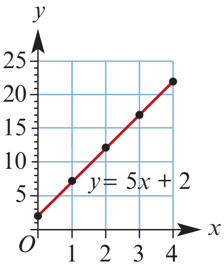
17 $y = -\frac{1}{2}x + 5 \quad 0 \le x \le 4$

$y = -3x + 15 \quad 4 \le x \le 5$

18 $9.00

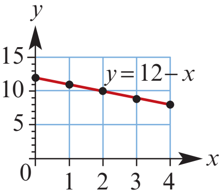
19 a

x	0	1	2	3	4
y	2	7	12	17	22



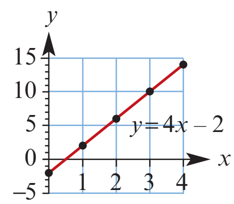
b

x	0	1	2	3	4
y	12	11	10	9	8



c

x	0	1	2	3	4
y	−2	2	6	10	14

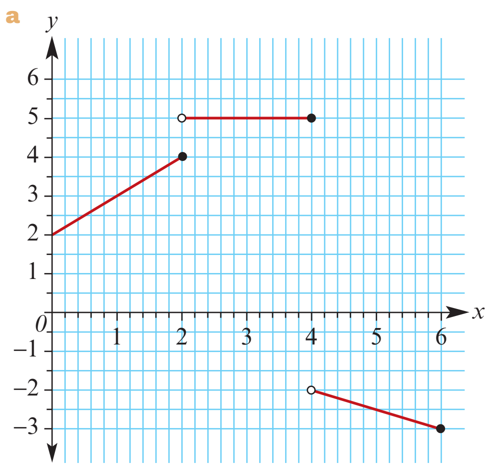


20 A −1.2, B 0.6

21 $A\ \frac{9}{4}$, $B\ -\frac{8}{3}$

22 a $755 **b** $110

23 a

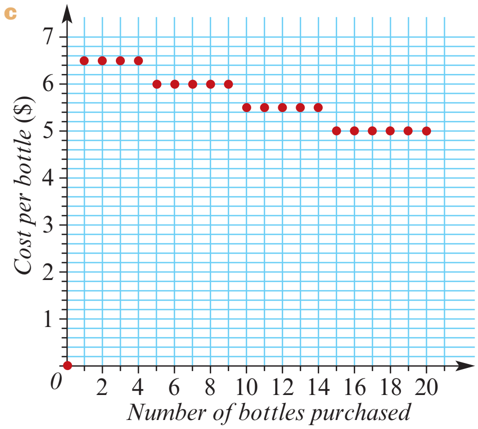


b $y = -\frac{5}{2}$

ISBN 978-1-009-25891-3

24 a \$5.00 **b** \$36.00

c



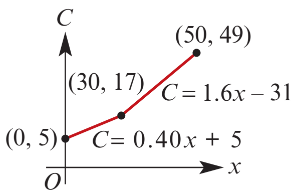
Extended-response questions

1 a \$200 000
b After 60 months (5 years)
c $V = -5t + 300$ **d** \$120 000
e \$5000

2 a \$80 billion
b $A = 0.16N$ (with A in billions, N in thousands)
c \$96 billion **d** \$240 billion
e \$0.16 billion

3 a $H = 6.25A + 80$ **b** 98.75 cm
c 6.25

4 a i \$13 **ii** \$17 **iii** \$49
b i \$0.40 (40 cents) **ii** \$1.60
c



Chapter 11

Exercise 11A

1 a $(-1, -1)$ **b** $(3, -2)$
c $(1, 1)$ **d** $(1, -1)$

2 a $(2, -4)$ **b** $(-3, 2)$ **c** $(1.5, 2.5)$
d $(2, 1)$ **e** $(0, 6)$ **f** $(7, 2)$
g $(0, 3)$ **h** $(1, 5)$ **i** $(0.4, -2.6)$
j $(7, 25)$ **k** $(-8, -20)$ **l** $(-3, 10)$
m No intersection, lines are parallel

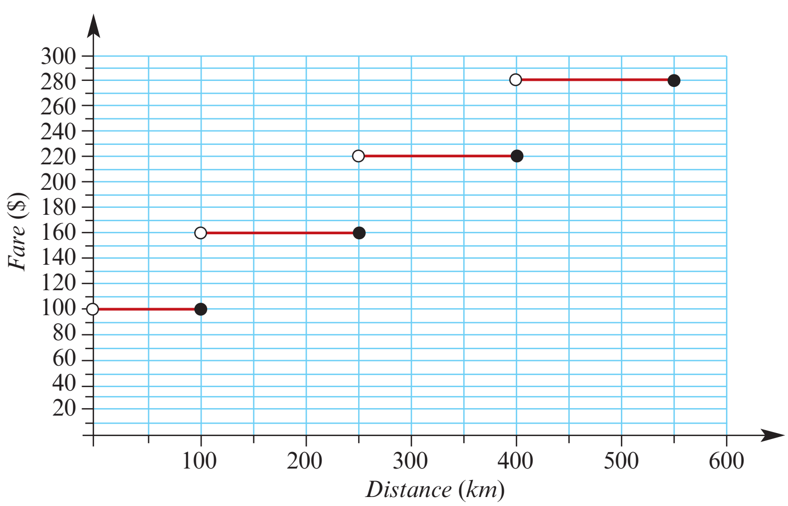
Exercise 11B

1 a $x = 2, y = 1$ **b** $x = 2, y = 5$
c $x = 3, y = 4$ **d** $x = 9, y = 1$
e $x = 3, y = 2$ **f** $x = 1, y = 1$
g $x = 4, y = 3$ **h** $x = 2.4, y = 3.4$
i $p = -1, q = 4$ **j** $a = -2, b = 5$
k $f = -2, g = -1$ **l** $x = 3, y = 2$

2 a $x = 2, y = 4$ **b** $x = -2, y = 3$
c $x = -2, y = 10$ **d** $x = -2, y = 3$
e $x = 4, y = -1$ **f** $x = -7, y = 0$

Exercise 11C

1 a $x = 4, y = -1$ **b** $x = \frac{1}{2}, y = 2$
c $x = -1, y = -2$ **d** $h = \frac{1}{2}, d = -2$
e $p = 3, k = -1$ **f** $t = \frac{32}{17}, s = \frac{28}{17}$
g $m = 2, n = 3$ **h** $x = \frac{4}{3}, y = \frac{7}{2}$
i $a = -1.5, b = 2.25$ **j** $x = \frac{1}{5}, y = -\frac{1}{5}$
k $x = 1.5, y = -0.6$, to 1 d.p.



ISBN 978-1-009-25891-3

Exercise 11D

1 a $5t + 6p = 12.75$ and $7t + 3p = 13.80$
b Texta \$1.65, pencil \$0.75

2 a $50p + 5m = 109$ and $75p + 5m = 146$
b Petrol \$1.48/L, motor oil \$7/L

3 a $6a + 10b = 7.10$ and $3a + 8b = 4.60$
b Banana 35c, orange 60c

4 Nails 1.5 kg, screws 1 kg

5 12 emus, 16 wombats

6 6 cm, 12 cm

7 22, 30

8 8, 27

9 Bruce 37, Michelle 33

10 Boy is 9, sister is 3

11 Chocolate thickshake \$5, fruit smoothie \$3

12 Mother 44, son 12

13 77 students

14 10 standard, 40 deluxe

15 252 litres (40%), 448 litres (15%)

16 126 boys, 120 girls

17 7542 litres unleaded, 2458 litres diesel

18 \$10 000 at 5%, \$20 000 at 8%

19 Width 24 m, length 36 m

20 28, 42, 35

Exercise 11E

1 160 m^2, 260 m^2, 460 m^2

2 4 deluxe, 4 standard

3 a 31.90 L b 47.18 L c 176 cm d 40.61 L
e 45.65 L
f

weight	60	65	70	75	80
TBW	40.39	42.08	43.76	45.44	47.12

weight	85	90	95	100
TBW	48.80	50.48	52.16	53.84

weight	105	110	115	120
TBW	55.52	57.20	58.89	60.57

4 a $V = (55 - 2x)(40 - 2x)x$
b 10 cm or 5.462 cm

Chapter 11 review

Short-answer questions

1 Yes

2 (2,5)

3 No

4 a (1,3) b (4,1) c (5,1)

5 a $x = 2,\ y = 8$ b $x = 3,\ y = 2.5$
c $p = 5,\ q = -2$ d $p = 5,\ q = 2$
e $p = 2, q = 1$

6 (7,2)

7 $y = 5$

8 $x = 3$

9 $x = 30, y = 20$

10 $x = 3, y = -2$

11 $a = 4, b = -1$

12 $x = 5, y = 2$

13 $x = 3, y = 7$

14 $x = 5, y = -1$

15 $x = 2, y = 9$

16 $7x + 3y = 314$, $4x + 5y = 255$
$x = \$35$, $y = \$23$
Football \$35, Soccer ball \$23

Extended-response questions

1 a $3a + 5c = 73.5$ and $2a + 3c = 46.5$
b \$12 c \$7.50

2 3 m

3 Indonesian 28; French 42; Japanese 35

4 Ai 34 years, Tisa 12 years

5 700 adults, 300 children

6 a money raised from adults' admission fees
b $x + y = 2200$
c 1500 children, 700 adults

7 c Anya 6 years, Haru 31 years

ISBN 978-1-009-25891-3

Appendix A TI-Nspire CAS CX with OS4.0

Keystroke actions and short cuts for the TI-Nspire CAS CX

esc : removes menus and dialogue boxes

ctrl + esc : undo last move

shift + esc : redo last move

tab : move to next entry box (field)

ctrl + tab : switch applications in split screen

Navpad (Touchpad)

ctrl : accesses secondary (blue) commands

ctrl + ▲ : displays page sorter

ctrl + ◀ : displays previous page

ctrl + ▶ : displays next page

on : displays icon page to select applications, mode, My Documents and start a new document

menu : options for each application

ctrl + menu : contextual menus (same as right mouse click)

: mouse pointer (cursor). Selects items.

ctrl + : grab

del : backspace, deletes a character

: catalogue

: 2D maths template

ctrl + ÷ : adds fraction template

enter : completes commands and displays results

ISBN 978-1-009-25891-3

Mode Settings

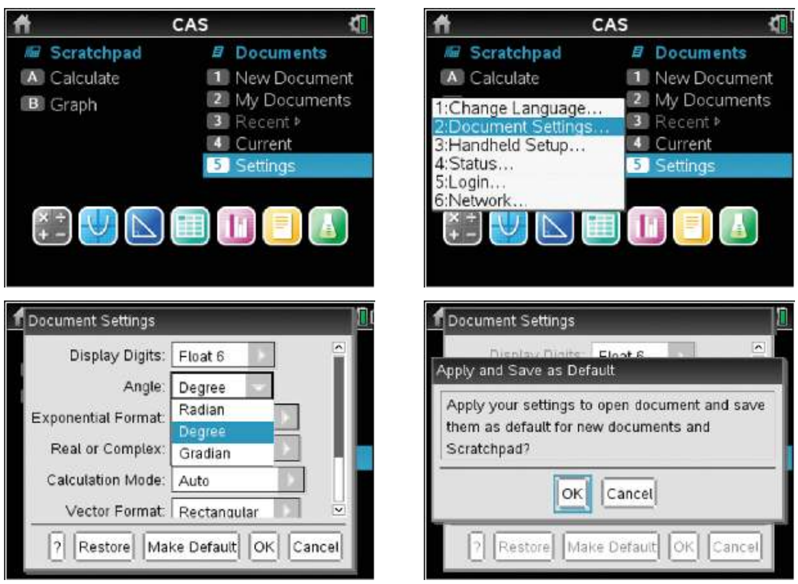
How to set in Degree mode

For this subject it is necessary to set the calculator to **Degree** mode right from the start. This is very important for the Trigonometry topic. The calculator will remain in this mode unless you change the setting again or update the operating system.

Press ⌂on and move to **Settings>Document Settings**, arrow down to the **Angle** field, press ▶ and select **Degree** from the list then arrow down to the **Make Default** tab. Select **OK** to accept the change.

Note that there is a separate settings menu for the **Graphs** and **Geometry** pages. These settings are accessed from the relevant pages. For Mathematics it is not necessary for you to change these settings.

Note: When you start your new document you will see **DEG** in the top status line.



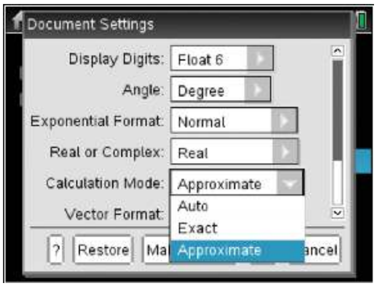
How to set in Approximate (Decimal) mode

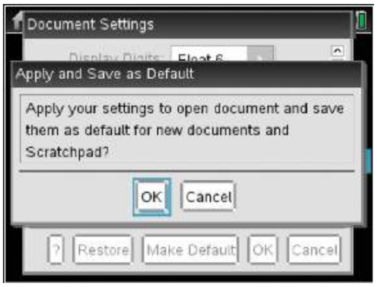
For this subject it is useful to set the calculator to **Approximate (Decimal)** mode right from the start. The calculator will remain in this mode unless you change the setting again or update the operating system.

Press ⌂on and move to **Settings>Document Settings**, arrow down to the **Calculation Mode** field, press ▶ and select **Approximate** from the list then arrow down to the **Make Default** tab. Select **OK** to accept the change.

Note: You can make both the **Degree** and **Approximate Mode** selections at the same time if desired.

ISBN 978-1-009-25891-3





The home screen is divided into two main areas – **Scratchpad** and **Documents**.

All instructions given in the text, and in the Appendix, are based on the **Documents** platform.

Documents

Documents can be used to access all the functionality required for this subject including all calculations, graphing, statistics and geometry.

Starting a new document

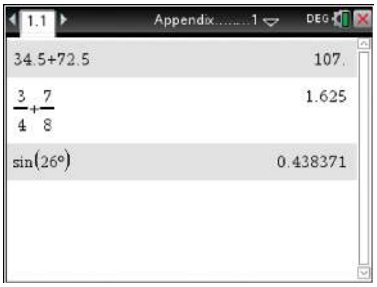
1. To start a new document, press [⌂ on] and select **New Document**.
2. If prompted to save an existing document move the cursor to **No** and press [enter].

   Note: Pressing [ctrl] + [N] will also start a new document.

A: Calculator page - this is a fully functional CAS calculation platform that can be used for calculations such as arithmetic, algebra, finance, trigonometry and matrices. When you open a new document select **Add Calculator** from the list.

ISBN 978-1-009-25891-3

1 You can enter fractions using the fraction template if you prefer. Press ctrl ÷ to paste the fraction template and enter the values. Use the tab key or arrows to move between boxes. Press enter to display the answer. Note that all answers will be either whole numbers or decimals because the mode was set to approximate (decimal).



2 For problems that involve angles (e.g. evaluate sin(26°)) it is good practice to include the degree symbol even if the mode is set to degree (DEG) as recommended.

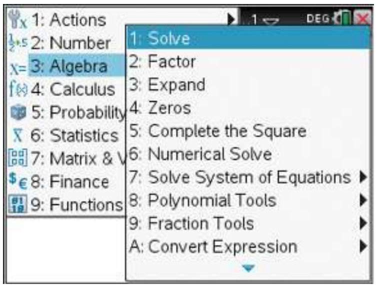
Note: If the calculator is accidentally left in radian (RAD) mode the degree symbol will override this and compute using degree values.

The degree symbol can be accessed using ?!▸. Alternatively select from the **Symbols** palette ctrl.To enter trigonometry functions such as *sin, cos*, press the trig key or just type them in with an opening parenthesis.

Solving equations

Using the **Solve command** Solve $2y + 3 = 7$ for y.

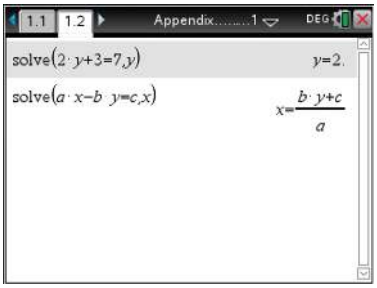
In a **Calculator** page press menu**>Algebra>Solve** and complete the **Solve** command as shown opposite. You must include the variable you are making the subject at the end of the command line.



Hint: You can also type in **solve(** directly from the keypad but make sure you include the opening bracket.

Literal equations such as $ax - by = c$ can be solved in a similar way.

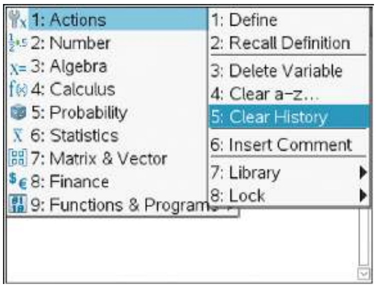
Note that you must use a multiplication sign between two letters.



ISBN 978-1-009-25891-3

Clearing the history area

Once you have pressed enter the computation becomes part of the **History** area. To clear a line from the History area, press ▲ repeatedly until the expression is highlighted and press enter. To completely clear the History area, press menu**>Actions>Clear History** and press enter again.

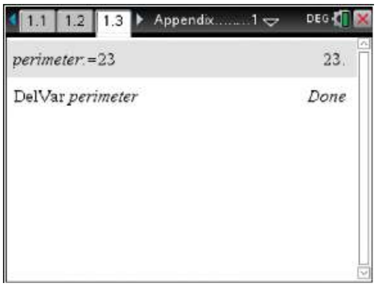


Alternatively press ctrl+menu to access the contextual menu.

It is also useful occasionally to clear any previously stored values. Clearing **History** does not clear stored variables.

Pressing menu**>Actions>Clear a-z...** will clear any stored values for single letter variables that have been used.

Use menu**>Actions>Delete Variable** if the variable name is more than one letter. For example, to delete the variable perimeter, then use **DelVar *perimeter***.



Note: When you start a new document any previously stored variables are deleted.

How to construct parallel boxplots from two data lists

Construct parallel boxplots to display the pulse rates of 23 adult females and 23 adult males.

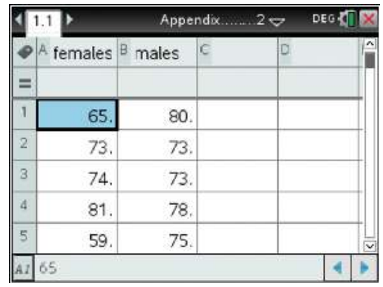
Pulse rate (beats per minute)	
Females	Males
65 73 74 81 59 64 76 83 95 70 73 79 64 77 80 82 77 87 66 89 68 78 74	80 73 73 78 75 65 69 70 70 78 58 77 64 76 67 69 72 71 68 72 67 77 73

ISBN 978-1-009-25891-3

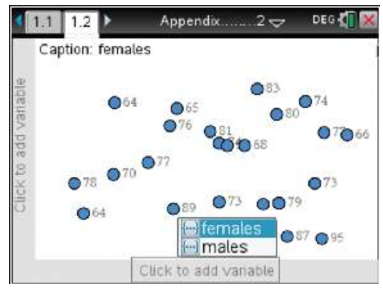
Steps

1 Start a new document: [ctrl]+[N].

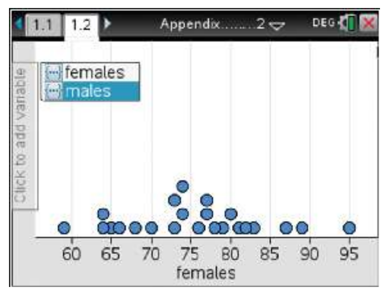
2 Select **Add Lists & Spreadsheet**. Enter the data into lists called ***females*** and ***males*** as shown.



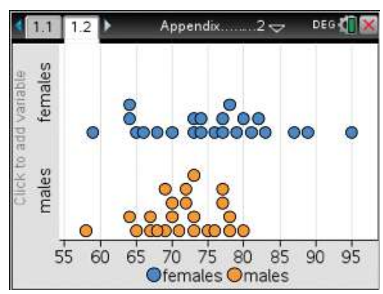
3 Statistical graphing is done through the **Data & Statistics** application.
Press [ctrl]+[I] and select **Add Data & Statistics** (or press [on] and arrow to and press [enter]).
Note: A random display of dots will appear – this is to indicate list data is available for plotting. It is not a statistical plot.



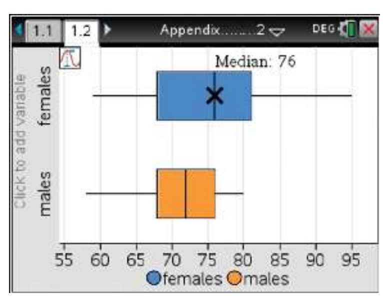
a Press [tab], or navigate and click on the "Click to add variable" box to show the list of variables. Select the variable, ***females***. Press [enter] or [click] to paste the variable to the x-axis. A dot plot is displayed by default as shown.



b To add another variable to the x-axis press [menu]**>Plot Properties>Add X Variable**, then [enter]. Select the variable ***males***. Parallel dot plots are displayed by default.



c To change the plots to box plots press [menu]**>Plot Type>Box Plot**, then press [enter]. Your screen should now look like that shown below.



Use ▼ to trace the other plot.
Press [esc] to exit the **Graph Trace** tool.

ISBN 978-1-009-25891-3

4 Data analysis

Use menu**>Analyze>Graph Trace** and use the cursor arrows to navigate through the key points. Alternatively just move the cursor over the key points. Starting at the far left of the plots, we see that, for females, the

- minimum value is 59: **MinX** = **59**
- first quartile is 68: **Q1**= **68**
- median is 76: **Median** = **76**
- third quartile is 81: **Q3** = **81**
- maximum value is 95: **MaxX** = **95**

and for males, the

- minimum value is 58: **MinX** = **58**
- first quartile is 68: **Q1** = **68**
- median is 72: **Median** = **72**
- third quartile is 76: **Q3** = **76**
- maximum value is 80: **MaxX** = **80**

ISBN 978-1-009-25891-3

Appendix

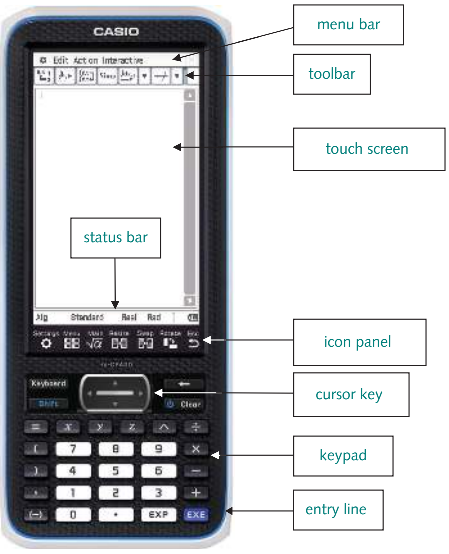
B Casio ClassPad II

Operating system

Written for operating system 2.0 or above.

Terminology

Some of the common terms used with the ClassPad are:



The menu bar

The toolbar

The touch screen contains the work area where the input is displayed on the left and the output is displayed on the right. Use your finger or stylus to tap and perform calculations.

The icon panel contains seven permanent icons that access settings, applications and different view settings. Press **escape** to cancel a calculation that causes the calculator to freeze.

The cursor key works in a similar way to a computer cursor keys.

The keypad refers to the hard keyboard.

ISBN 978-1-009-25891-3

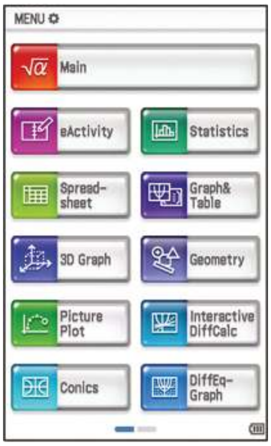
Calculating

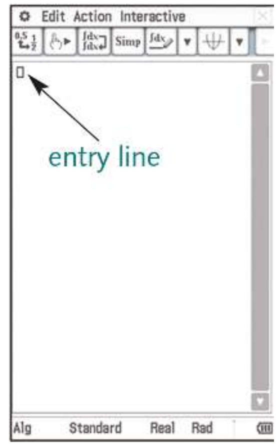
Tap Menu from the **icon panel** to display the application menu if it is not already visible.

Tap Main to open the **Main** application.

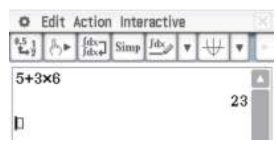
Note: There are two application menus. Alternate between the two by tapping on the screen selector at the bottom of the screen.

1 The main screen consists of an entry line which is recognised by a flashing vertical line (cursor) inside a small square. The history area, showing previous calculations, is above the entry line.

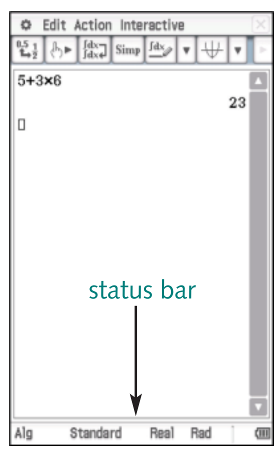




2 To calculate, enter the required expression in the entry line and press EXE. For example, if we wish to evaluate $5 + 3 \times 6$, type the expression in the entry line and press EXE.
You can move between the entry line and the history area by tapping or using the cursor keys (i.e. ▼, ◀, ▲, ▶).



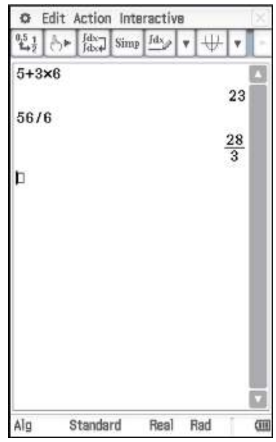
3 The ClassPad gives answers in either exact form or as a decimal approximation. Tapping settings in the **status bar** will toggle between the available options.

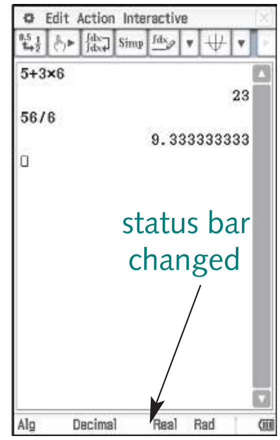


ISBN 978-1-009-25891-3

4 For example, if an exact answer is required for the calculation 56 ÷ 6, the **Standard** setting must be selected.

5 If a decimal approximation is required, change the **Standard** setting to **Decimal** by tapping it and press **EXE**.

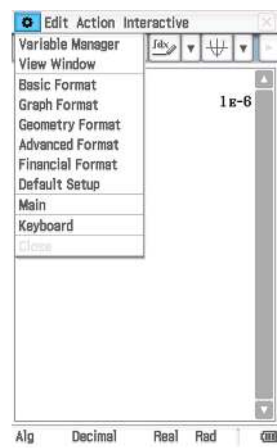




Extremely large and extremely small numbers

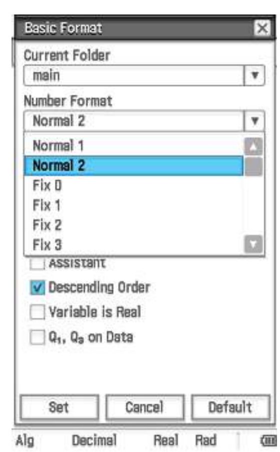
When solving problems that involve large or small numbers the calculator's default setting will give answers in scientific form.

For example, one millionth, or 1/1000000, in scientific form is written as 1×10^{-6} and the calculator will present this as 1E-6.



To change this setting, tap on the settings icon and select **Basic Format.**

Under the Number Format select **Normal 2** and tap SET.

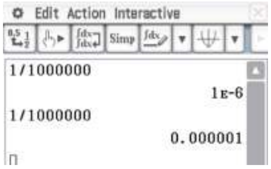


ISBN 978-1-009-25891-3

In the Main screen type 1/1000000 and press **EXE**.

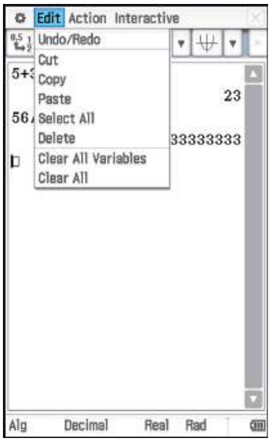
The answer will now be presented in decimal form 0.000001

This setting will remain until the calculator is reset.



Clearing the history screen

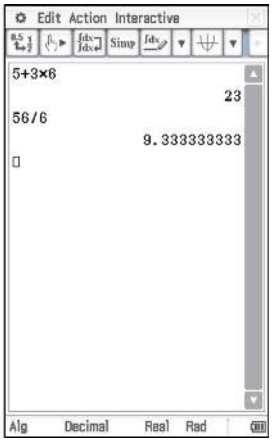
To clear the **Main** application screen, select **Edit** from the menu bar and then tap **Clear All**. Confirm your selection by tapping **OK**. The entire screen is now cleared. To clear the entry line only, press **Clear** on the front calculator.



Clearing variables

To clear stored variable values, select **Edit** from the menu bar and then tap **Clear All Variables**. Confirm your selection by tapping OK.

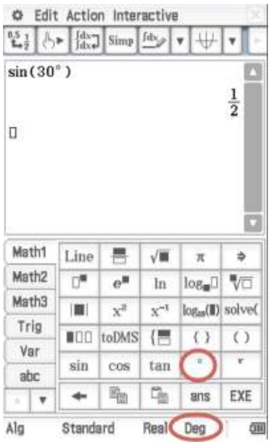
The variables are cleared but the history created on the main screen is kept.



ISBN 978-1-009-25891-3

Degree mode

When solving problems in trigonometry, your calculator should be kept in **Degree** mode. In the main screen, the status bar displays the angle mode.



To change the angle mode, tap on the angle unit in the status bar until **Deg** is displayed.

In addition, it is recommended that you always insert the degree symbol after any angle. This overrides any mode changes and reminds you that you should be entering an angle, not a length.

The degree symbol is found in the Math1 keyboard.

ISBN 978-1-009-25891-3