



# Christ Church Grammar School

Semester Two Examination, 2021  
Question/Answer booklet

## MATHEMATICS APPLICATIONS UNITS 3&4

### Section One: Calculator-free

## SOLUTIONS

WA student number: In figures

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In words \_\_\_\_\_

\_\_\_\_\_

Your name \_\_\_\_\_

\_\_\_\_\_

### Time allowed for this section

Reading time before commencing work: five minutes  
Working time: fifty minutes

Number of additional  
answer booklets used  
(if applicable):

### Materials required/recommended for this section

#### *To be provided by the supervisor*

This Question/Answer booklet

Formula sheet

#### *To be provided by the candidate*

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

### Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
<b>Total</b>					<b>100</b>

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

**Section One: Calculator-free****35% (52 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

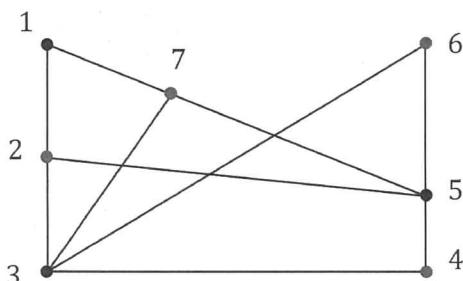
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**Question 1****(6 marks)**

- (a) A connected planar graph has 17 edges and 12 faces. Determine the number of vertices this graph has. (2 marks)

<b>Solution</b>
Using Euler's formula $v + 12 - 17 = 2 \Rightarrow v = 7$
Hence graph has 7 vertices.
<b>Specific behaviours</b>
✓ correct use of Euler's formula ✓ correct number of vertices

- (b) The vertices in the following graph can be split into two distinct groups to demonstrate that the graph is bipartite. List the vertices in each group. (2 marks)



<b>Solution</b>
Groups are $\{1, 3, 5\}$ and $\{2, 4, 6, 7\}$
<b>Specific behaviours</b>
✓ marks alternate vertices/redraws ✓ correctly lists groups

- (c) Determine the number of edges that must be added to a tree with 4 vertices so that it becomes a complete graph with 4 vertices. (2 marks)

<b>Solution</b>
$K_4$ has $4 \times 3 \div 2 = 6$ edges.
Tree with 4 vertices has 3 edges.
Hence add $6 - 3 = 3$ edges.
<b>Specific behaviours</b>
✓ edges in $K_4$ ✓ correct number to add

**Question 2****(5 marks)**

A sequence is defined as  $T_{n+1} = T_n + a$  with  $T_1 = b$ , so that  $T_4 = 40$  and  $T_5 = 43$ .

- (a) Determine the value of the constant  $a$  and the value of the constant  $b$ . (2 marks)

Solution
$a = 43 - 40 = 3$
$40 - 3 - 3 - 3 = 31$ and so $b = 31$ .
Specific behaviours
✓ value of $a$
✓ value of $b$

The sequence can also be written in the form  $T_n = an + k$ .

- (b) Determine the value of the constant  $k$ . (1 mark)

Solution
$\begin{aligned} T_n &= 31 + (n - 1)(3) \\ &= 31 + 3n - 3 \\ &= 3n + 28 \end{aligned}$
Hence $k = 28$ .
Specific behaviours
✓ value of $k$

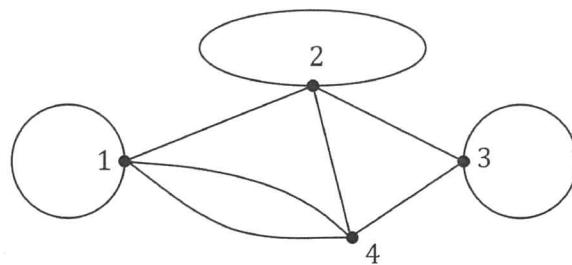
- (c) Determine the value of  $n$  so that  $T_n = 250$ . (2 marks)

Solution
$\begin{aligned} 3n + 28 &= 250 \\ 3n &= 222 \\ n &= 74 \end{aligned}$
Specific behaviours
✓ forms equation and simplifies ✓ value of $n$

**Question 3**

(7 marks)

A company runs sightseeing boat trips from several terminals throughout a harbour city. In the graph below, the numbered vertices represent terminals, and the edges represent trips either between the terminals or that start and finish at the same terminal.



- (a) State two reasons why the graph above is not simple. (2 marks)

Solution
The graph has loops and multiple edges.
Specific behaviours
✓ states loops ✓ states multiple edges

- (b) Construct an adjacency matrix from the graph. (2 marks)

Solution
$\begin{bmatrix} 1 & 1 & 0 & 2 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 2 & 1 & 1 & 0 \end{bmatrix}$
Specific behaviours
✓ symmetrical matrix ✓ correct adjacency matrix

- (c) Describe two conditions necessary for the existence of a semi-Eulerian trail in a graph and state the length of such a trail in the above graph. (3 marks)

Solution
The graph must (i) be connected; and (ii) have exactly two odd vertices
Length of trail is 9 edges.
Specific behaviours
✓ one condition for semi-Eulerian trail ✓ second condition for semi-Eulerian trail ✓ length of trail

**Question 4****(10 marks)**

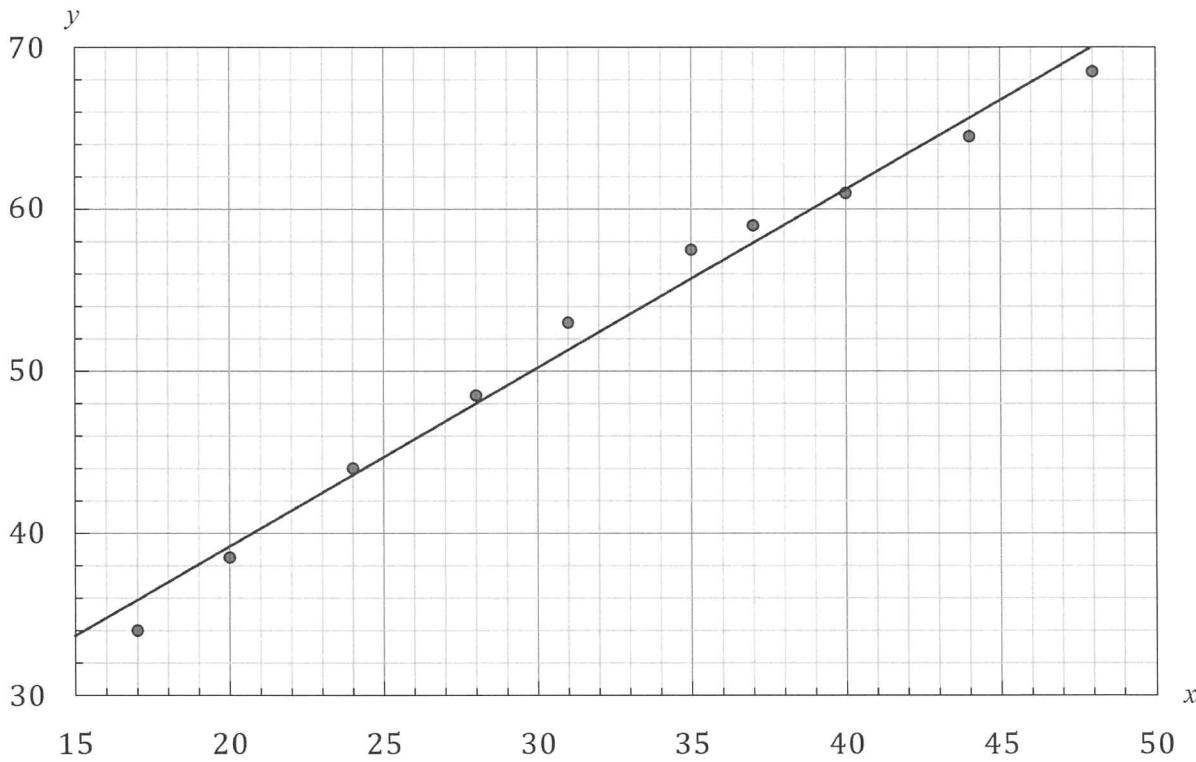
An analyst collected data from a sample of ten trees in a plantation, as shown in the table below.

The variables  $x$  and  $y$  are the diameter of a tree in centimetres and the daily water use of the tree in litres respectively.

$\hat{y}$  is the predicted daily water use in litres, calculated using the least-squares line  $\hat{y} = 1.1x + 17.1$ , and  $R$  is the residual.

$x$	17	20	24	28	31	35	37	40	44	48
$y$	34.0	38.5	44.0	48.5	53.0	57.5	59.0	61.0	64.5	68.5
$\hat{y}$	35.8	39.1	43.5	47.9	51.2	55.6	57.8	-	65.5	69.9
$R$	-1.8	-0.6	0.5	0.6	-	1.9	1.2	-	-1.0	-1.4

Nine of the data points  $(x, y)$  are shown on this scatterplot:



- (a) Add the missing point to the scatterplot.

Solution
See graph: (37, 59)
Specific behaviours
✓ correctly plots point

(1 mark)

- (b) Explain how information from the table can be used to draw the least-squares line on the scatterplot and hence draw this line. (2 marks)

Solution
Plot at least two points using $(x, \hat{y})$ and draw line through these points.
Specific behaviours
✓ explains use of $(x, \hat{y})$ ✓ correct line on graph

- (c) Determine the value of the residual  $R$  when:

(i)  $x = 31$ .

<b>Solution</b>	
$R = 53.0 - 51.2 = 1.8$	
<b>Specific behaviours</b>	
✓ correct residual	

(1 mark)

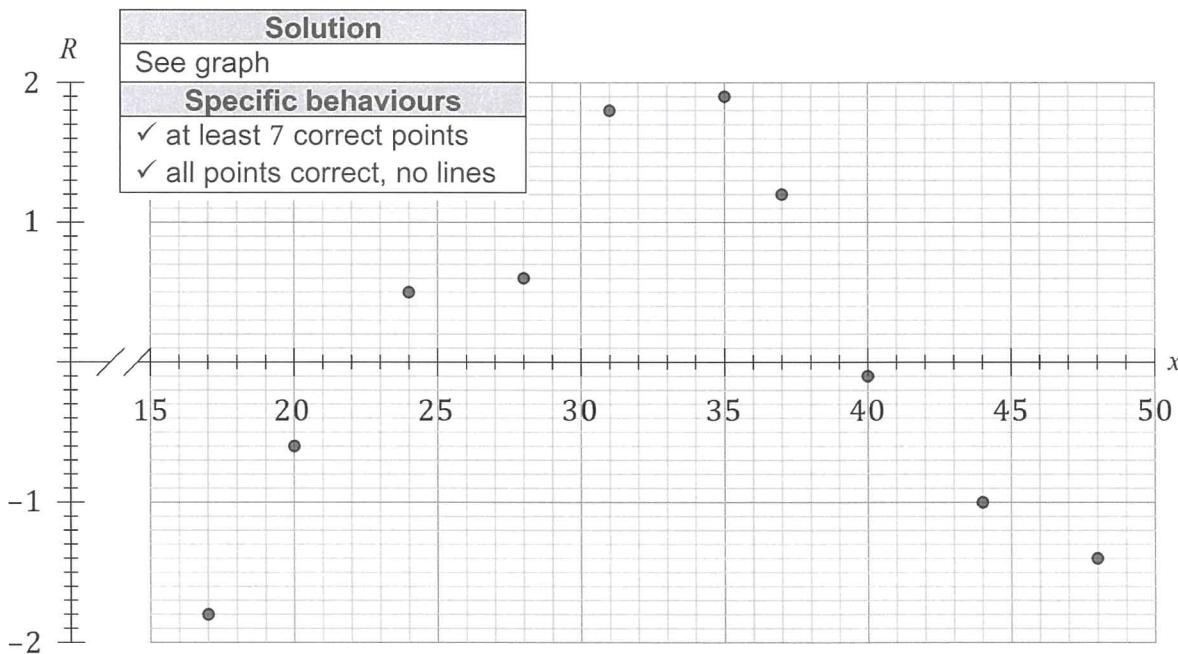
(ii)  $x = 40$ .

<b>Solution</b>	
$\hat{y} = 1.1(40) + 17.1 = 61.1$	
$R = 61.0 - 61.1 = -0.1$	
<b>Specific behaviours</b>	
✓ calculates $\hat{y}$	
✓ correct residual	

(2 marks)

- (d) Construct a residual plot on the axes below.

(2 marks)



- (e) Comment on the appropriateness of fitting a linear model to the data. Justify your answer.

(2 marks)

**Solution**

A linear model is not appropriate as a clear pattern is evident in the residual plot.

**Specific behaviours**

- ✓ states not appropriate
- ✓ refers to pattern evident in residual plot

**Question 5**

(5 marks)

A relay team consists of four cadets who must each be assigned to one of the four sections of an assault course in order to minimise their overall time.

The table shows the least time, in minutes, that each cadet has previously taken to complete the different sections.

	Section			
	1	2	3	4
Drew	33	26	32	34
Eve	27	26	29	28
Faye	30	27	30	32
Gem	31	30	27	30

- (a) Show use of the Hungarian algorithm to determine the optimum assignment of cadets, writing the assignment in the table below. (4 marks)

Solution (rows first)				
Reduce rows				
$\begin{bmatrix} 7 & 0 & 6 & 8 \\ 1 & 0 & 3 & 2 \\ 3 & 0 & 3 & 5 \\ 4 & 3 & 0 & 3 \end{bmatrix}$				
Reduce columns and cover zeros				
$\begin{bmatrix} 6 & 0 & 6 & 6 \\ 0 & 0 & 3 & 0 \\ 2 & 0 & 3 & 3 \\ 3 & 3 & 0 & 1 \end{bmatrix}$				
Use algorithm ( $\pm 2$ )				
$\begin{bmatrix} 4 & 0 & 4 & 4 \\ 0 & 2 & 3 & 0 \\ 0 & 0 & 1 & 1 \\ 3 & 5 & 0 & 1 \end{bmatrix}$				
Specific behaviours				
<ul style="list-style-type: none"> <li>✓ reduce rows and columns</li> <li>✓ cover zeros with 3 lines</li> <li>✓ apply algorithm</li> <li>✓ correct assignment</li> </ul>				

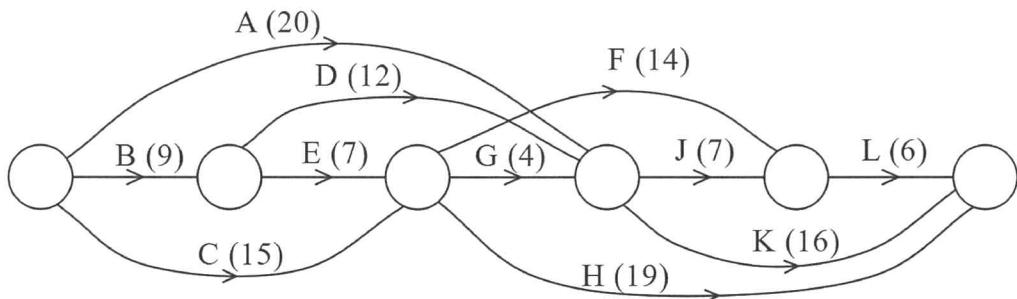
Section	1	2	3	4
Cadet	Faye	Drew	Gem	Eve

- (b) State the least overall time for the team to complete the relay. (1 mark)

Solution	
Time is $30 + 26 + 27 + 28 = 111$ minutes.	
Specific behaviours	
✓ correct time	

**Question 6****(6 marks)**

The network below represents the durations and interdependencies of the 11 activities required to complete a project. For example, activity H has a duration of 19 days and cannot commence until activities C and E are complete.



- (a) Determine the minimum completion time for the project.

(2 marks)

**Solution**

Earliest start times

$$A, B, C = 0; \quad D, E = 9; \quad F, G, H = 16; \quad J, K = 21; \quad L = 30; \quad \text{End} = 37.$$

Minimum completion time is 37 days.

**Specific behaviours**

- ✓ evidence of EST's
- ✓ correct minimum completion time

- (b) Determine which of the non-critical activities has the greatest float time and state the earliest start time and latest start time for this activity.

(2 marks)

**Solution**

Activity J. It has EST: day 21 and LST: day 24.

**Specific behaviours**

- ✓ states activity
- ✓ states EST and LST

- (c) Proposed changes to the project will decrease the duration of activity D by 4 days. Determine the impact this will have on the critical path of the project and its minimum completion time.

(2 marks)

**Solution**

The critical path will change and there will be three of them.  
(AK, BEGK, BEFL Type equation here.).

Minimum completion time will decrease by 1 day (to 36 days).

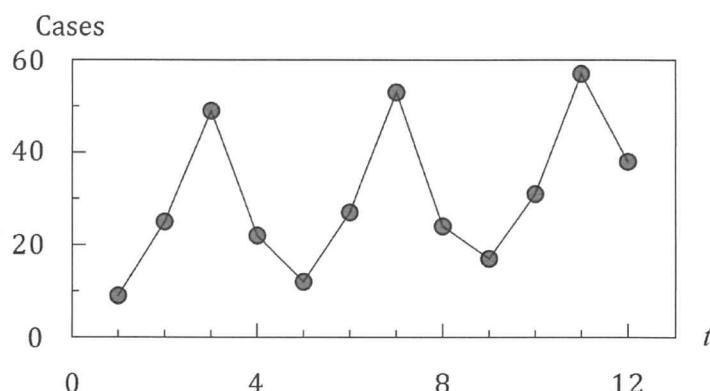
**Specific behaviours**

- ✓ states critical path changes and indicates there will be two
- ✓ NOT
- ✓ NOT

**Question 7****(6 marks)**

A clinic recorded the number of cases of influenza that presented each quarter, and an extract from the data is shown in the table and graph below.

Year	Cases per quarter			
	1	2	3	4
2015	9	25	49	22
2016	12	27	53	24
2017	17	31	57	38



- (a) Describe the trend and seasonality of the data.

(2 marks)

**Solution**

There is an increasing/upward trend.

cycle of 4; Cases are highest (peaks) in the third quarter and lowest (troughs) in the first quarter of each year.

**Specific behaviours**

✓ indicates direction of trend

✓ indicates features common to all years

- (b) Calculate the 4-point centred moving average for the number of cases that presented in the second quarter of 2017. (2 marks)

**Solution**

$$\frac{24}{2} + 17 + 31 + 57 + \frac{38}{2} = 136$$

$$\frac{136}{4} = 34$$

Moving average is 34 cases.

**Specific behaviours**

✓ indicates correct method

✓ correct moving average

The 3-point moving average for the number of cases that presented in the first quarter of 2015 is 18.

- (c) Determine the number of cases that presented in the fourth quarter of 2014. (2 marks)

**Solution**

$$\frac{9 + 25 + x}{3} = 18$$

$$34 + x = 54$$

$$x = 20$$

Hence 20 cases presented that quarter.

**Specific behaviours**

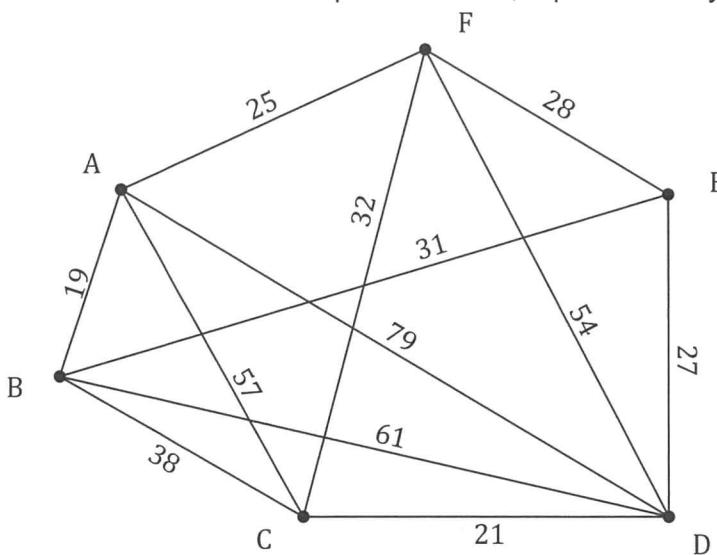
✓ indicates correct equation

✓ correct number of cases

**Question 8**

(7 marks)

The edge weights on the graph below represent the time, in milliseconds, to send a data packet between routers on a computer network, represented by the vertices.



- (a) Determine the minimum time to send a data packet from router A to router D and state, in order, the routers on this path. (3 marks)

Solution
Routers on path: ABED
Minimum time: $19 + 31 + 27 = 77$ milliseconds.
Specific behaviours
✓ evidence of checking times for at least two paths
✓ correct path
✓ correct minimum time

- (b) Explain, with justification, why the graph in this question is Hamiltonian. (2 marks)

Solution
The graph contains a cycle that visits all vertices. For example, the cycle ABCDEFA.
Specific behaviours
✓ explanation using <b>cycle</b> and <b>all vertices</b>
✓ example of Hamiltonian cycle in graph

- (c) State, with reasoning, the least number of edges that must be removed from the graph so that it is no longer Hamiltonian. (2 marks)

Solution
2 edges. By removing any 2 of the 3 edges from vertex E the graph will become semi-Hamiltonian.
Specific behaviours
✓ correct number
✓ reasoning



# Christ Church Grammar School

## Semester Two Examination, 2021 Question/Answer booklet

### MATHEMATICS APPLICATIONS UNITS 3&4

#### Section Two: Calculator-assumed

## SOLUTIONS

WA student number: In figures

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In words \_\_\_\_\_

\_\_\_\_\_

Your name \_\_\_\_\_

\_\_\_\_\_

#### Time allowed for this section

Reading time before commencing work: ten minutes

Number of additional  
answer booklets used  
(if applicable):

Working time: one hundred minutes

#### Materials required/recommended for this section

##### *To be provided by the supervisor*

This Question/Answer booklet

Formula sheet (retained from Section One)

##### *To be provided by the candidate*

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

#### Important note to candidates

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## Structure of this paper

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Section Two: Calculator-assumed	13	13	100	98	65
<b>Total</b>					<b>100</b>

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**Section Two: Calculator-assumed****65% (98 Marks)**

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

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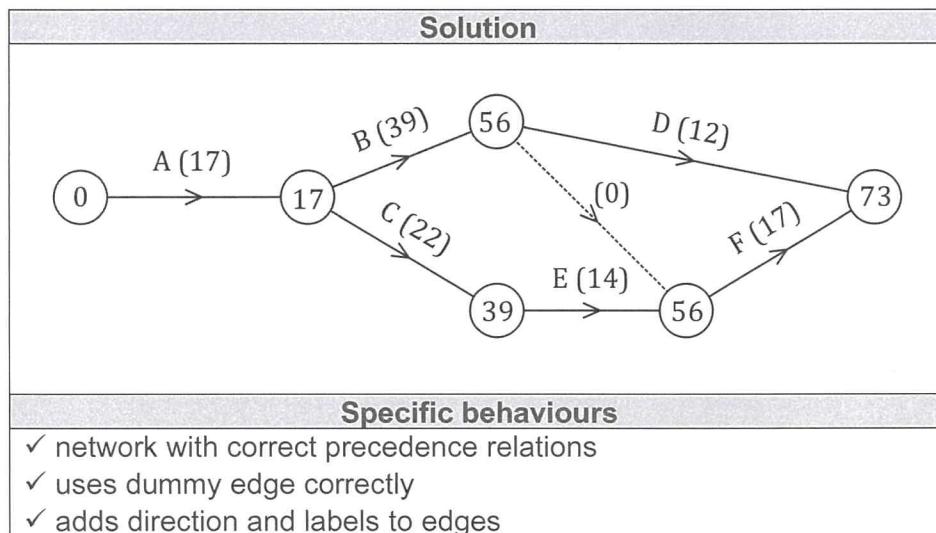
**Question 9****(5 marks)**

The table below shows the duration and immediate predecessors for all the activities required to complete a project.

Activity	A	B	C	D	E	F
Duration (minutes)	17	39	22	12	14	17
Immediate predecessors	—	A	A	B	C	B, E

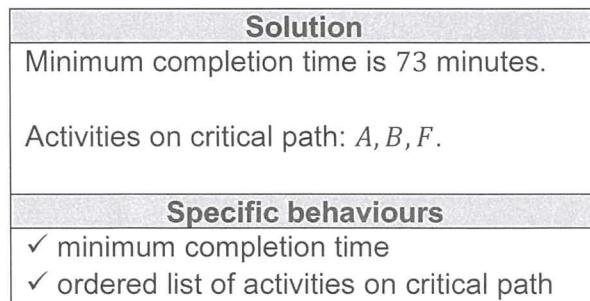
- (a) Construct a project network to show all the above information.

(3 marks)



- (b) Determine the minimum completion time for the project and list, in order, the activities that lie on the critical path.

(2 marks)



**Question 10**

(7 marks)

A builder bought a scissor lift for \$58 000 and for accounting purposes will depreciate its value by 23% annually.

- (a) Show that the value of the lift after one year will be \$44 660.

(1 mark)

Solution
$\$58\ 000 \times 0.77 = \$44\ 660$
Specific behaviours
✓ clearly shows any correct method

- (b) Write a recursive rule to calculate the value of the lift,  $T_n$  in dollars, after  $n$  years.

(2 marks)

Solution
$T_{n+1} = 0.77 \times T_n, \quad T_0 = 58\ 000$
Specific behaviours
✓ recursive part ✓ initial term using $T_0$

- (c) Calculate the value of the lift after 7 years.

(1 mark)

Solution
$T_7 = \$9308.14$
Specific behaviours
✓ correct value (or rounded to nearest \$)

- (d) Deduce the  $n^{\text{th}}$  term rule for the value of the lift after  $n$  years.

(1 mark)

Solution
$T_n = 58\ 000(0.77)^n$
Specific behaviours
✓ correct rule

- (e) Determine the least number of years that must pass for the lift to depreciate by more than 95% of its original value.

(2 marks)

Solution
5% of value will remain, so value will be \$2900. $T_{11} = 3272, \quad T_{12} = 2520.$
Least number of years is 12.
Specific behaviours
✓ indicates required value ✓ correct number of years

**Question 11****(9 marks)**

The body length and eye diameter of eight tropical fish of the same species are shown in the table below.

Body length, $L$ cm	6.3	11.9	3.8	8.2	9.1	10.5	5.2	7.5
Eye diameter, $D$ mm	9.3	8.6	9.9	9.1	9.0	8.7	9.8	9.4

- (a) Use your calculator to graph the above data and hence describe the direction and strength of the linear association between the variables. (1 mark)

**Solution**

The association is strong and negative.

**Specific behaviours**

✓ states strong and negative

- (b) Determine the coefficient of determination for the linear association and interpret its value in context. (2 marks)

**Solution**

$$r^2 = 0.950$$

95% of variation in the eye diameter can be explained by the variation in body length.

**Specific behaviours**

✓  $r^2$ , to at least 2 decimal places

✓ correct interpretation of coefficient

- (c) Determine the equation of the least-squares line to predict  $D$  from  $L$  and interpret the slope of the line in context. (3 marks)

**Solution**

$$D = -0.17L + 10.55$$

The slope means that for every 1 cm increase in length of a fish, the eye diameter is expected to decrease by 0.17 mm.

**Specific behaviours**

✓ equation, using given variables

✓ indicates as one variable increases, the other decreases

✓ quantifies relationship, using units

- (d) Predict the eye diameter of another tropical fish of the same species that has a body length of 5.6 cm and give two reasons that support the validity of this prediction. (3 marks)

**Solution**

$$D(5.6) = 9.6 \text{ mm}$$

Prediction is valid because correlation is strong, and it does not involve extrapolation.

**Specific behaviours**

✓ calculates diameter

✓ states strong correlation

✓ states interpolation or no extrapolation

**Question 12****(8 marks)**

A researcher observed a large number of mice, noting for each one its gender and the paw that it used to react to a stimulus placed in front of it. The numbers in each category are shown in the table below.

Mice	Left-pawed	Right-pawed
Male	39	63
Female	23	60

The researcher is interested in whether there is an association between the variables.

- (a) Name one of the variables in the study and classify it as numerical or categorical.

(2 marks)

Solution
Name: Gender or Paw.
Classifies as Categorical.
Specific behaviours
✓ names one variable
✓ classifies variable

- (b) Determine the percentage of left-pawed mice that were male.

(2 marks)

Solution
$\frac{39}{39 + 23} = \frac{39}{62} = 62.9\%$
Specific behaviours
✓ indicates correct total for category
✓ correct percentage (whole number ok)

- (c) Use the above data to complete the following table so that it shows column percentages rounded to the nearest whole number.

(2 marks)

Mice (%)	Left-pawed	Right-pawed
Male	63	51
Female	37	49

Solution
See table
Specific behaviours
✓ both columns add to 100
✓ correct table

- (d) Explain whether the percentaged table suggests the presence of an association between the variables.

(2 marks)

Solution
An association is evident between the variables as the pairs of percentages in each row (i.e., for each gender) are quite different.
Specific behaviours
✓ states association
✓ explanation using different percentages

**Question 13**

(7 marks)

A reducing balance loan is defined by the recurrence relation  $T_{n+1} = k \times T_n - m$ ,  $T_1 = p$ , where  $T_n$  is the balance of the loan in dollars at the start of month  $n$ . The relation was used to create the following spreadsheet.

Month $n$	Balance of loan at start of month $n$	Monthly interest	Monthly repayment	Loan balance carried forward
1	12 500.00	92.50	395.00	12 197.50
2	12 197.50	90.26	395.00	11 892.76
3	11 892.76	88.01	395.00	$Q$
4		$R$		$S$

- (a) Determine the annual percentage interest rate that applies to the loan. (2 marks)

Solution
$92.50 \div 12500 = 0.0074$
$0.0074 \times 12 \times 100 = 8.88\% \text{ pa}$
Specific behaviours
✓ calculates monthly rate
✓ correct annual rate as percentage

- (b) State the value of each of the constants  $k$ ,  $m$  and  $p$  in the recurrence relation. (2 marks)

Solution
$k = 1.0074$
$m = 395, \quad p = 12 500$
Specific behaviours
✓ value of $k$
✓ value of $m$ and value of $p$

- (c) Determine the value of  $Q$ , the value of  $R$  and the value of  $S$  shown in the spreadsheet. (3 marks)

Solution
$Q = 11892.76 + 88.01 - 395.00 = \$11 585.77$
$R = 11 585.77 \times 0.0074 = \$85.73$
$S = 11 585.77 + 85.73 - 395.00 = \$11 276.50$
Specific behaviours
✓ value of $Q$
✓ value of $R$
✓ value of $S$

**Question 14**

(9 marks)

The table below shows the number of new clients attracted by a business during the first three weeks of an advertising campaign, together with some derived values to assist in the analysis of the time series data.

Week	Day ( $n$ )		New Clients	Weekly mean	Percentage of weekly mean
1	Mon	1	49	<b>A</b>	98.4
	Tue	2	53		106.4
	Wed	3	53		106.4
	Thu	4	49		<b>B</b>
	Fri	5	45		90.4
2	Mon	6	58	60.6	95.7
	Tue	7	66		108.9
	Wed	8	62		102.3
	Thu	9	63		104.0
	Fri	10	54		89.1
3	Mon	11	68	70.2	96.9
	Tue	12	<b>C</b>		106.8
	Wed	13	72		102.6
	Thu	14	71		101.1
	Fri	15	65		92.6

- (a) Calculate the value of **A**, the value of **B** and the value of **C** in the table.

(3 marks)

Solution
$A = (49 + 53 + 53 + 49 + 45) \div 5 = 49.8$
$B = 49 \div 49.8 = 98.4\%$
$C = 70.2 \times 106.8\% = 75$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ value of <b>A</b></li> <li>✓ value of <b>B</b></li> <li>✓ value of <b>C</b></li> </ul>

- (b) Determine the seasonal index for Mondays.

(2 marks)

Solution
$I_{MON} = (98.4 + 95.7 + 96.9) \div 3$ $= 291 \div 3$ $= 97\%$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ uses correct indices</li> <li>✓ calculates index as percent or decimal</li> </ul>

- (c) The least-squares line to predict the deseasonalised number of new clients  $c$  from the day is  $c = 1.83n + 45.6$ . Stating any assumptions made, determine the best estimate for the number of new clients expected on Friday of Week 4. (4 marks)

**Solution**

Friday of Week 4 is day 20.

$$\begin{aligned} c &= 1.83(20) + 45.6 \\ &= 82.2 \end{aligned}$$

$$\begin{aligned} I_{FRI} &= (90.4 + 89.1 + 92.6) \div 3 \\ &= 272.1 \div 3 \\ &= 90.7\% \end{aligned}$$

$$\begin{aligned} C &= 82.2 \times 0.907 \\ &= 74.6 \end{aligned}$$

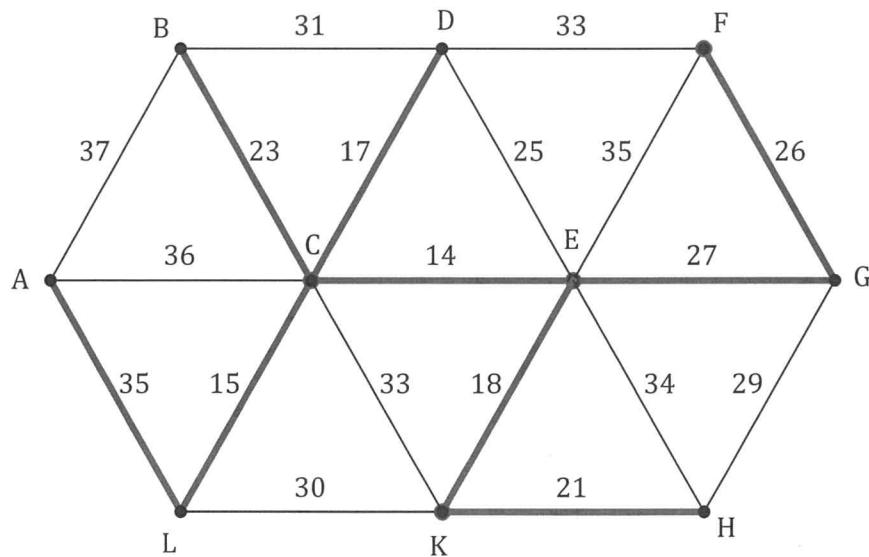
Expect 75 new clients, assuming that the existing trend and seasonality continues into the future.

**Specific behaviours**

- ✓ uses correct day to calculate deseasonalised number
- ✓ calculates seasonal index for Friday
- ✓ uses index to calculate whole number estimate
- ✓ states assumption

**Question 15****(8 marks)**

Ten mains-powered smoke alarms must be installed in a building. The edge weights in the graph below represent the length of cable, in metres, required between adjacent alarms.



- (a) Clearly identify the minimum spanning tree on the graph above. (3 marks)

**Solution**

See graph

**Specific behaviours**

any tree;  any spanning tree;  minimum spanning tree

- (b) Determine the cost of installing the cabling between the alarms using the minimum spanning tree, given that each metre of cabling will cost \$21.50. (2 marks)

**Solution**

Sum of edges:

$$L = 35 + 15 + 23 + 17 + 14 + 18 + 21 + 27 + 26 = 196 \text{ m}$$

Total cost:

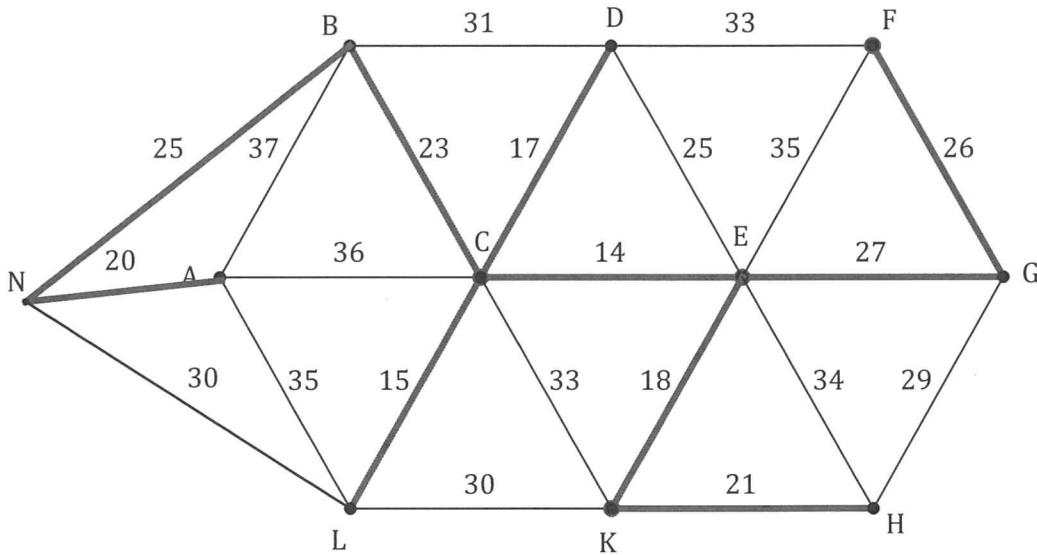
$$C = 196 \times 21.50 = \$4214$$

**Specific behaviours**

sum of edges

calculates cost

- (c) Explain how your answer to part (b) will change if smoke alarm  $N$  is added to the system with cable lengths of 20, 25 and 30 metres to alarms  $A$ ,  $B$  and  $L$  respectively. (A copy of the graph from the previous page is shown below if you wish to use it.) (3 marks)



#### Solution

The minimum spanning tree changes so that edge  $AL$  no longer used but edges  $AN$  and  $BN$  are now included.

The edge sum increases by 10 m to 206 m.

Total cost increases by \$215 to \$4429.

#### Specific behaviours

- ✓ states change to edges
- ✓ calculates increase in (or new) sum of edges
- ✓ calculates increase in (or new) cost

**Question 16**

(8 marks)

Saltwater flows steadily into a tank, where it is mixed with existing water. An overflow spout on the tank allows excess water to flow out. The salt concentration in the tank can be modelled by  $S_{n+1} = 0.95S_n + 950$ ,  $S_0 = 1000$ , where  $S_n$  is the concentration, in parts per million, after saltwater has been flowing into the tank for  $n$  minutes.

- (a) Determine
- $S_5$
- .

Solution
$S_5 = 5072 \text{ ppm}$
Specific behaviours
✓ correct value

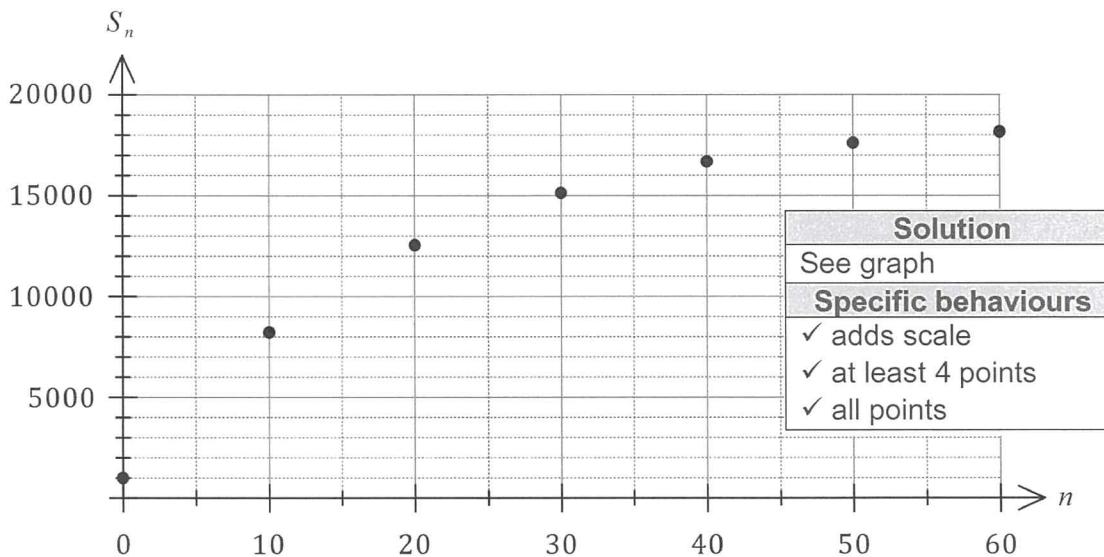
(1 mark)

- (b) Determine the value of
- $n$
- for
- $S_n$
- to first exceed 10 000 ppm.

Solution
$n = 14$
Specific behaviours
✓ correct value

(1 mark)

- (c) Plot points to show the salt concentration at 10 minute intervals on the axes below, after first adding an appropriate scale to the vertical axis. (3 marks)



- (d) Describe the feature of the plotted points that indicates the salt concentration will eventually reach a steady-state.

(1 mark)

Solution
Describes 'levelling off' or 'increasing at a decreasing rate', etc.
Specific behaviours
✓ reasonable description

- (e) Determine, with justification, the steady-state salt concentration.

(2 marks)

Solution
$S = 0.95S + 950 \Rightarrow S = 19\ 000 \text{ ppm}$
Specific behaviours
✓ equation, or explanation using term(s) with large $n$ ✓ correct concentration

**Question 17**

(7 marks)

Loans are offered by lender A at a rate of 10.32% per annum compounded quarterly and from lender B at a rate of 10.22% per annum compounded daily.

- (a) Calculate the total amount that must be repaid if \$1500 is borrowed from lender A for nine months. (2 marks)

<b>Solution</b>	
$A = 1500 \left(1 + \frac{10.32}{4 \times 100}\right)^3$ $A = 1619.12$ <p>Hence \$1619.12 must be repaid.</p>	Financial calculator $N = 3, I\% = 10.32, PV = -1500$ $PMT = 0, P/Y = 4, C/Y = 4$ Solve for $FV = 1619.12$ Hence \$1619.12 must be repaid.
<b>Specific behaviours</b>	
<ul style="list-style-type: none"> <li>✓ shows compound interest formula or values used in financial calculator</li> <li>✓ correct amount</li> </ul>	

- (b) Calculate the total interest that will be charged on a loan of \$90 000 from lender B for 30 days. (2 marks)

<b>Solution</b>	
$A = 90\ 000 \left(1 + \frac{10.22}{365 \times 100}\right)^{30}$ $A = 90\ 759.08$ <p>Hence \$759.08 interest charged.</p>	Financial calculator $N = 30, I\% = 10.22, PV = -90000$ $PMT = 0, P/Y = 365, C/Y = 365$ Solve for $FV = 90\ 759.08$ Hence \$759.08 interest charged.
<b>Specific behaviours</b>	
<ul style="list-style-type: none"> <li>✓ shows compound interest formula or values used in financial calculator</li> <li>✓ correct amount of interest</li> </ul>	

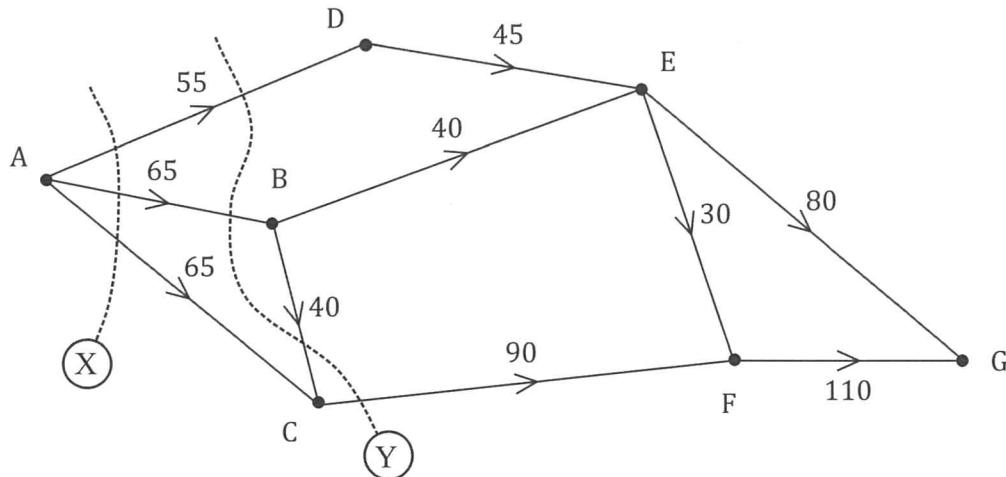
- (c) Minimising loan interest is the primary goal for a borrower. Calculate the effective interest rate for each lender and hence recommend which should be chosen. (3 marks)

<b>Solution</b>	
Lender A: $N = 4, APR = 10.32, EFF = 10.7263$ .	
Lender B: $N = 365, APR = 10.22, EFF = 10.7589$ .	
Hence interest will be minimised by choosing lender A (as their effective interest rate is less than that of lender B).	
<b>Specific behaviours</b>	
<ul style="list-style-type: none"> <li>✓ effective interest rate for A</li> <li>✓ effective interest rate for B</li> <li>✓ uses effective rates to choose lender</li> </ul>	

**Question 18**

(7 marks)

When a city bypass is closed, traffic that would normally use it is forced to flow through main roads in the city. The edge weights on the directed graph below show the maximum number of vehicles per minute that can travel between junctions (represented by vertices) without causing congestion in the city.



- (a) Determine the value of cut  $X$  and the value of cut  $Y$ . (2 marks)

**Solution**

Cut  $X = 55 + 65 + 65 = 185$  vpm. Cut  $Y = 55 + 65 + 0 + 90 = 210$  vpm.

**Specific behaviours**

- ✓ value of cut  $X$
- ✓ value of cut  $Y$

- (b) Determine the maximum flow of vehicles per hour from  $A$  to  $G$ . (3 marks)

**Solution**

$ADEG = 45$ ;  $ABEG = 35$ ;  $ABEFG = 5$ ;  $ABCAG = 25$ ;  $ACFG = 65$ ; Total = 175.

OR

Minimum cut is  $45 + 40 + 90 = 175$ .

Maximum hourly flow is  $175 \times 60 = 10\,500$  vehicles per hour.

**Specific behaviours**

- ✓ indicates systematic listing or several cuts
- ✓ correct maximum flow per minute
- ✓ correct maximum flow per hour

- (c) City engineers recommend taking steps to improve traffic flow between junctions  $D$  and  $E$ . Determine, with reasoning, the maximum increase in the hourly flow of vehicles from  $A$  to  $G$  that their plan could achieve. (2 marks)

**Solution**

Spare capacity along  $ADEFG$  is  $10 \cdot 0 \cdot 25 \cdot 15$ , so increase by 10.

OR

Minimum cut not using  $DE$  is 185, so  $185 - 175 = 10$ .

Maximum possible increase is  $10 \times 60 = 600$  vehicles/hour.

**Specific behaviours**

- ✓ reasoning
- ✓ correct increase

**Question 19****(8 marks)**

A fund with a balance of \$475 000 is used to create an annuity, from which regular withdrawals of \$9800 are to be made at the end of each quarter. Interest at a rate of 7.2% per annum is added to the fund quarterly, just before each withdrawal.

- (a) Use one or more calculations to show that the balance of the fund after one withdrawal is \$473 750. (2 marks)

<b>Solution</b>	
$475\ 000 \times 0.072 \div 4 = 8550$	$7.2 \div 4 = 1.8$
$475\ 000 + 8550 - 9800 = 473\ 750$	$475\ 000 \times 1.018 - 9800 = 473\ 750$
<b>Specific behaviours</b>	
<ul style="list-style-type: none"> <li>✓ indicates adjustment for interest for one quarter</li> <li>✓ calculation: interest less withdrawal or recursive type</li> </ul>	

**NOTE; given error in question – full marks given for  $\times 1.8 - 9500$ ; a number of variations in solution paid if working shown; full marks awarded for crossed out working if it was correct**

- (b) Write a recurrence relation to calculate the balance  $T_n$  after the  $n^{\text{th}}$  withdrawal. (2 marks)

<b>Solution</b>	
$T_{n+1} = 1.018 \times T_n - 9800$ , $T_0 = 475000$ .	
<b>Specific behaviours</b>	
<ul style="list-style-type: none"> <li>✓ correct recursive part</li> <li>✓ correct statement for <math>T_0</math></li> </ul>	

- (c) Determine the total interest earned by the fund during its first year of operation. (2 marks)

<b>Solution</b>	
$T_4 = 469\ 863.37$	
( 471 096.16 also paid correct)	
$469\ 863.37 + 4 \times 9800 - 475\ 000 = 34\ 063.37$	
Total interest is \$34 063.37.	
( 34 096.16 also paid correct)	
<b>Specific behaviours</b>	
<ul style="list-style-type: none"> <li>✓ indicates closing balance</li> <li>✓ correct total interest</li> </ul>	

- (d) The amount of the regular withdrawal from the fund can be modified so that from the outset, the fund is a perpetuity rather than an annuity. Determine the withdrawal required for this to occur. (2 marks)

<b>Solution</b>	
Withdrawal is interest for one quarter: $475\ 000 \times 0.072 \div 4 = 8550$	
Amount of withdrawal is \$8550.	
<b>Specific behaviours</b>	
<ul style="list-style-type: none"> <li>✓ indicates withdrawal must be interest for one quarter</li> <li>✓ correct amount</li> </ul>	

**Question 20**

(7 marks)

An electronics store advertises a TV for sale at a price of \$5950. Rather than pay this amount in full, the store offers customers a no deposit reducible balance loan with 24 monthly payments of \$281.20. The first repayment is due one month after the customer makes the purchase, just after interest for the month is added to the loan balance.

- (a) Determine the total interest paid by customers who buy the TV using the loan. (2 marks)

Solution
Repay: $24 \times 281.20 = \$6748.80$
Interest: $6748.80 - 5950 = \$798.80$
Specific behaviours
✓ calculates total repaid ✓ calculates interest

- (b) Determine the annual percentage interest rate that applies to the loan. (2 marks)

Solution
Using Financial app: $N = 24, PV = 5950, PMT = -281.20,$ $FV = 0, P/Y = 12, C/Y = 12$
Solve for $I\% = 12.4$ , so rate is 12.4% per annum.
Specific behaviours
✓ shows correct values for financial calculator ✓ correct annual rate

A customer decides to buy the TV using a similar reducible balance loan to that offered by the store but financed by their bank at an annual interest rate of 13.98% and over 36 months.

- (c) Determine their monthly repayment. (2 marks)

Solution
Using Financial app with: $N = 36, PV = 5950, I\% = 13.98,$ $FV = 0, P/Y = 12, C/Y = 12$
Solve for $PMT = -203.30$ , so their repayment is \$203.30 per month.
Specific behaviours
✓ shows correct values for financial calculator ✓ correct repayment

- (d) Determine the total interest that the bank will charge on the loan over the 36 months. (1 mark)

Solution
Using Financial app: $\sum INT = -1368.77$ .
Using $36 \times 203.30 - 5950 = 1368.80$ .
Total of \$1368.77 or \$1368.80 in interest.
Specific behaviours
✓ correct amount

**Question 21****(8 marks)**

For each quarter in the 12 years from 2006 to 2017, a city library calculated its mean number of users per day and used seasonal indices to deseasonalise the data. A snapshot of the data for the year 2015 is shown in the table below.

Quarter	1	2	3	4
Mean daily users	415.4	465.4	462.6	407.0
Deseasonalised mean daily users	436.5	442.7	431.7	439.8

The trend line for the deseasonalised mean daily users is  $\hat{n} = 722 - 8.1t$  where  $t$  is the quarter and  $t = 1$  corresponds to the first quarter of 2006.

- (a) State, with justification, whether the mean number of users per day was increasing or decreasing over the 12 years. (1 mark)

Solution
Decreasing - slope of trend line is negative
Specific behaviours
✓ states decreasing, justifies with slope

- (b) Calculate the seasonal indices and enter them in the following table. (2 marks)

Quarter	1	2	3	4
Seasonal Index	0.952	1.051	1.072	0.925

Solution
See table: Decimals to at least 2 dp (or equivalent as percentage). $415.4 \div 436.5 = 0.952$ ,    1.051,    1.072,    0.925
Specific behaviours
✓ at least two correct and indices sum to 4 ✓ all correct

- (c) State, with justification, in which quarter the library was usually busiest during the period that data was collected. (2 marks)

Solution
Third quarter - the seasonal index is highest for this quarter.
Specific behaviours
✓ correct quarter ✓ justifies using indices

- (d) Determine an estimate, to the nearest whole number, for the mean daily users the library experienced in the first quarter of 2017. (3 marks)

Solution
$t = 45$
$\hat{n} = 722 - 8.1(45) = 357.5$
$357.5 \times 0.952 \approx 340$
Estimate there were 340 users.
Specific behaviours
✓ correct value of $t$ ✓ calculates $\hat{n}$ ✓ seasonalises to obtain correct estimate