Home Work #   <u>**Radicals**</u>

In 2^3, 2 is called the base and 3 is the exponent.

<u>**Rules of Exponents:**</u> (For any real numbers a, b, m, and n)

$a^m \cdot a^n = a^{m+n}$ $2^3 \cdot 2^5 = 2^{3+5} = 2^8$

$(ab)^m = a^m b^m$ $(5*6)^3 = 5^3 * 6^3$ Note: $(a+b)^m$ **is not** $a^m + b^m$

$\dfrac{a^m}{a^n} = a^{m-n}$, $a \neq 0$ $\dfrac{3^5}{3^2} = 3^{5-2} = 3^3$

$(a^m)^n = a^{mn}$, $a \neq 0$, $(2^3)^5 = 2^{15}$

$a^{-n} = \dfrac{1}{a^n}$, $a \neq 0$ $3^{-5} = \dfrac{1}{3^5}$

$\dfrac{1}{a^{-n}} = a^n$, $a \neq 0$ $\dfrac{1}{2^{-3}} = 2^3$

$a^0 = 1$, $a \neq 0$, $5^0 = 1$

$\sqrt{}$ is called a radical. Fractional exponents are also valid.

$$\sqrt[n]{a} = a^{1/n} \text{ , so } \sqrt[2]{9} = 9^{1/2} = 3, \quad \sqrt[3]{8} = 8^{1/3} = 2$$

<u>Class Examples:</u>

1) $\dfrac{2^5 \; 2^3}{2^2}$ 2) $\dfrac{2^4 \; 2^{-3} \; 3^4}{2^2 \; 3^2}$ 3) $\dfrac{(2^5)^3}{(2^7)^2}$

4) $\sqrt[3]{8^5}$ 5) $\left(\sqrt[2]{8}\right)^{\frac{2}{3}}$ 6) $\left(\sqrt[4]{81^{-3}}\right)$

7) $\left(\dfrac{1}{8}\right)^{\frac{2}{3}}$

8) $2^{300} - 2^{299} = ?$

9) $\dfrac{4^{1000}}{2^{2000}} = ?$

10) If $M = 3^{2^{2^{2}}}$ and $D = (((3)^2)^2)^2$, what is $\dfrac{M}{D}$?

Solve for x:

11) $2 \cdot 2^{2x} = 8^x$

12) $x^{\frac{2}{3}} = 25$

13) Which number is the largest: a) 2^{30} b) 3^{20} c) 5^{10}

14) Simplify: $\left(\sqrt[3]{x^2} \cdot \sqrt[6]{x^4}\right)^{-3}$

15) Simplify: $\dfrac{(4^{\frac{2}{3}})(2^{\frac{1}{6}})(3^{\frac{3}{2}})}{(2^{\frac{-1}{2}})(3^{\frac{1}{2}})}$

16) What is the value of the expression $\dfrac{2017^2 + 11(2017) - 42}{2014}$?

17) $4\sqrt{18} + 2\sqrt{50}$

18) Rationalize: $\dfrac{1}{\sqrt[3]{25}}$

Homework Problems: (Do not use calculator)

1) $\sqrt{6}\ \sqrt{15}\ \sqrt{10}$

2) $\sqrt{\left(\dfrac{56}{126}\right)}$

3) $\dfrac{\sqrt{9}\ \sqrt{5}}{\sqrt{20}}$

4) $\left(\sqrt[3]{81}\right)^{\frac{3}{2}}$

5) $(64)^{\frac{-4}{3}}$

6) $\left(\dfrac{4}{9}\right)^{\frac{-3}{2}}$

7) $\sqrt[5]{100000^{3}}$

8) $\sqrt[4]{\left(\dfrac{1}{16}\right)^{-3}}$

9) $\sqrt[4]{1600}$

10) $\sqrt[3]{27^{5}}$

11) $\left(\sqrt[2]{27}\right)^{\frac{2}{3}}$

12) $\left(\sqrt[4]{256^{-3}}\right)$

13) $\sqrt[3]{\left(\dfrac{36000}{243}\right)}$

14) $81^{-(2^{-2})}$

15) $\sqrt[4]{81}\ \sqrt{81}$

16) $\left(\dfrac{1}{8}\right)^{\frac{5}{3}}$

17) $\sqrt{\left(2\dfrac{14}{25}\right)}$

Find all real x in each of the following (problems 18 – 25):

18) $x^6 = 64$

19) $X = (-27)^{\frac{-2}{3}}$

20) $X = \sqrt[3]{\left(\dfrac{-1}{8}\right)}$

21) $x^3 = 64$

22) $(X)^{\frac{5}{3}} = 243$

23) $9^{2x} = 27^{3x+1}$

24) $2\,(2^{3x}) = 32^x$

25) $2\,(2^{2x}) = 4^x + 64$

26) Simplify: $2^{200} - 2^{199}$

27) Find the difference: $4^{2^3} - ((4)^2)^3$

28) Simplify: $(4^{-1} - 3^{-1})^{-1}$

29) If $2^4 \times 3^8 = n \times 6^4$, then find n.

30) If $a \# b = 4^a\,2^{-b}$, find y such that $y \# 2 = 4$

31) If $2^{1998} - 2^{1997} - 2^{1996} + 2^{1995} = k\,2^{1995}$, what is k ?

32) $\dfrac{(4^{2003})\,(3^{2002})}{(6^{2002})\,(2^{2003})} = ?$

33) $\sqrt[3]{(4^5 + 4^5 + 4^5 + 4^5)}$

34) Arrange the following in increasing order:
$16^{25},\ 8^{100},\ 3^{500},\ 4^{400},\ 2^{600}$

35) What is the value of $\dfrac{\sqrt{45}+2\sqrt{15}}{\sqrt{128}+2\sqrt{24}}$

Express your answer as a common fraction in simplest radical form.

36) What is the value of the expression shown? Express your answer as a common fraction.

$$\cfrac{1}{1+\cfrac{2}{1+\cfrac{3}{1+\cfrac{5}{1+7}}}}$$

37) What value of x yields the minimum value of the sum

$|x\text{-}2^0| + |x - 2^1| + |x\text{-}2^2| + |x\text{-}2^3| + \ldots + |x\text{-}2^{10}|$

38) If $\sqrt{7-\sqrt{2+\sqrt{n}}} = 2,$ what is the value of n?

39) Let $x > 0$ and $y > 0$. Suppose that $xy^2 = 6$, and $x^2y^6 = 72$. What is the value of xy? Express your answer in simplest radical form.

40) If $(7^2 + 24^2)^4 \times (5^2 + 10^2)^5 \times (75^2 + 100^2)^6 = 5^n$, what is the value of n? Must show all steps. No calculator.

41) Rationalize $\dfrac{1}{\sqrt[3]{25}}$

Radicals: (More Class Examples) - Not Home Work

1) $\left(\sqrt[3]{125} \right)^{\frac{3}{2}}$

2) If $C^d = 3$, find $C^{4d} - 5$

3) If $5^x - 5^{x-1} = \dfrac{4}{125}$, find x

4) Simplify: $\sqrt[3]{(4^8 + 4^8 + 4^8 + 4^8)}$

5) Solve for x: $2\,(2^{3x}) = 64^x$

6) Solve for x: $2\,(2^{2x}) = 4^x + 256$

7) Rationalize the denominator: $\dfrac{1}{\sqrt{2}}$

8) Rationalize the denominator: $\dfrac{1}{1 + \sqrt{2}}$

9) What integer n has the property that 5^{96} is greater than n^{72} and 5^{96} is less than $(n+1)^{72}$?

10) What is the greatest common factor of 20! And 200,000 ?

11) Solve for x: $(2^5 - 2^3)\,(2^4 - 2^2) \div 9 = 2^{x+1}$

12) The largest integer x such that 4^x divides exactly 10! is ?

13) Rationalize: $\dfrac{1}{\sqrt[3]{9}}$