CFD Lab 1

Channel Entrance Length

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ME 4342 Computational Fluid Dynamics
Fall 2022
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Purpose

The purpose of this lab is to utilize numerical simulations on a laminar flow in a circular tube to study entrance lengths. Specifically, we wanted use analytical solution to build confidence with our model and subsequently use parametric studies to understand the entrance length's dependence on various flow parameters.

Geometry

Utilizing symmetry, a 2D model was used to replicate radially symmetric slice of the flow. No slip conditions are applied on the top and bottom solid walls, the inlet defined with a constant velocity profile and the outlet is defined with a zero-pressure condition.

Mesh Study

To determine optimal mesh sizing, a parametric sweep with element divider from 1 to 5 was used. In Figure 1, a rapid convergence in velocity values from element divider number of 2 onwards is demonstrated, and thus 3 was chosen to balance computational speed and numerical accuracy.

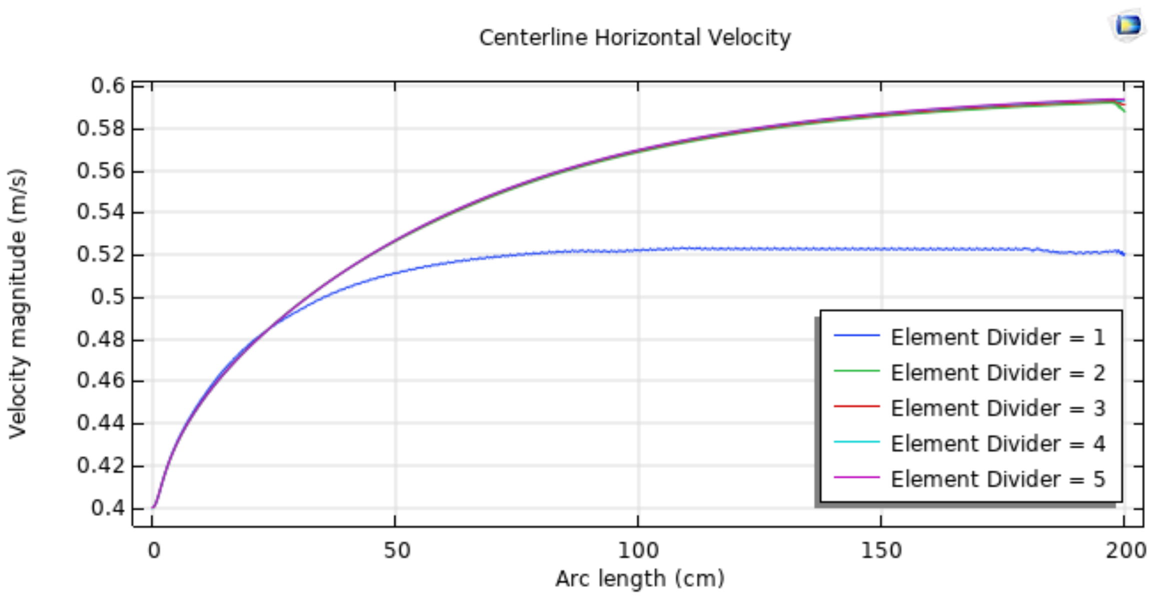

Figure 1: A comparison of the centerline horizontal velocity profiles with various initial mesh element sizes.

Results

To validate the model's accuracy, the error from x-momentum balance is first calculated and the result is normalized against in the inlet momentum:

$$e = \left[\int_0^H \sigma_x dx\right]_{inlet} + \left[\int_0^H \sigma_x dx\right]_{outlet} + \left[\int_0^L \sigma_x dx\right]_{top\ wall} + \left[\int_0^L \sigma_x dx\right]_{bottom\ wall} - \left[\left(\int_0^H \rho u^2 dx\right)_{outlet} - \left(\int_0^H \rho u^2 dx\right)_{inlet}\right] \tag{1}$$

$$\hat{e} = \frac{e}{M_{inlet}} \tag{2}$$

The resulting normalized error $\hat{e} \approx 0.009$, which was acceptable for studying low Reynolds number (Re < 1500) laminar flows. The next verification step was to compare fully developed horizontal velocity profiles Analytically, the fully developed velocity profile is determined by the following equation:

$$u(s) = \frac{6s(1-s)Q}{H}, \qquad Q = \left[\int_0^H u dx\right]_{inlet} \tag{3}$$

Where s is a normalized coordinate [0,1] of the channel height [0, H]. In Figure 2, the outlet velocity profile shows excellent agreement with the analytical profile.

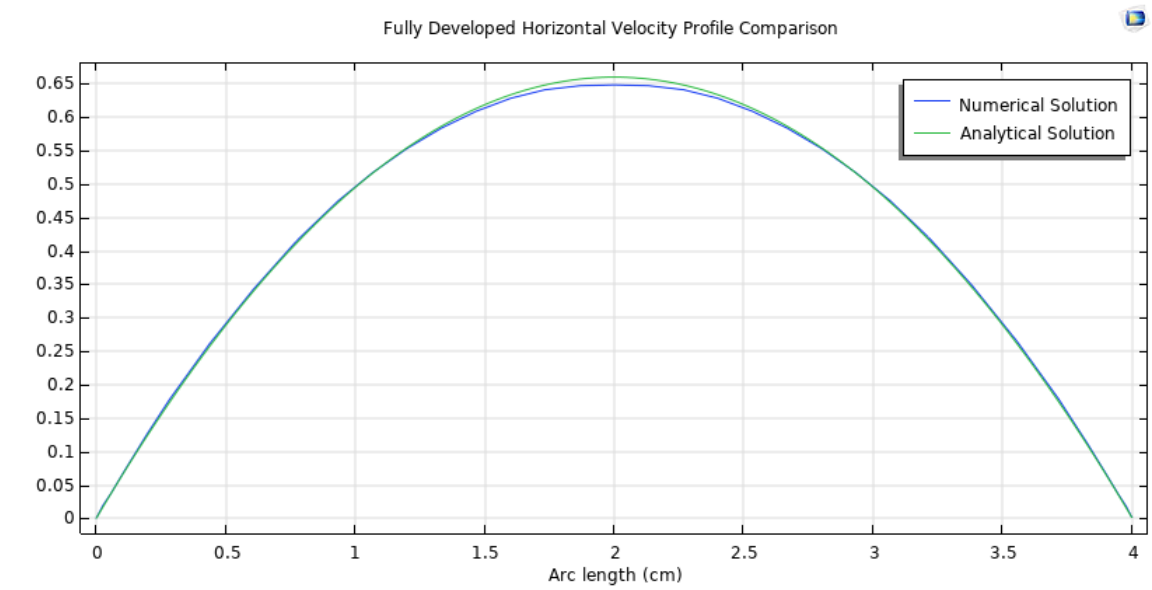

Figure 2: A comparison in the outlet horizontal velocity profile between the analytical solution and numerical solution provided. The result shows good agreement.

In Figure 3, horizontal velocity profiles at different locations along the channel length is shown. The effect of viscous shear along the no slip conditions of the top and bottom channel walls is clearly demonstrated as the uniform velocity profile transforms into the quadratic fully developed shape.

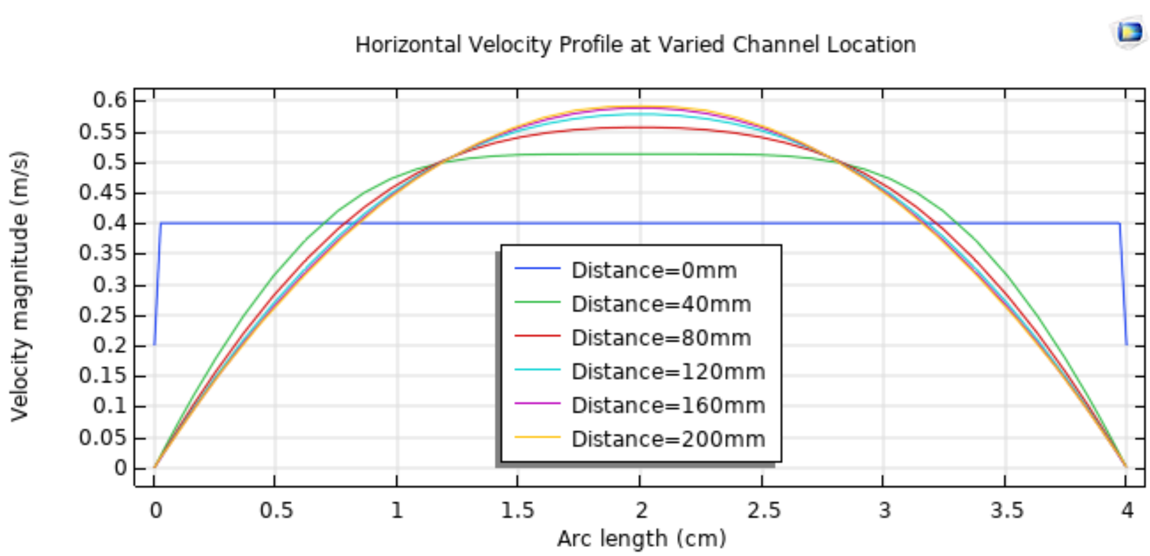

Figure 3: The horizontal velocity profiles at various spatial locations along channel length is shown, demonstrated a transition to a fully developed velocity profile.

The entrance length of the flow can be estimated using the following method:

$$L_{entrance} = \{l \,|u(l) > 0.99 \times 1.5u(0)\,\} \tag{4}$$

In Figure 4, using the above criteria the entrance length is beyond the total channel length and thus is not solvable using this current method. Vertical velocity components analytically are zero as all the flow is normal to the inlet surface, and any small vertical values are simply numerical errors. The average horizontal velocities at different spatial locations along the channel is also $\approx 0.4 m/s$.

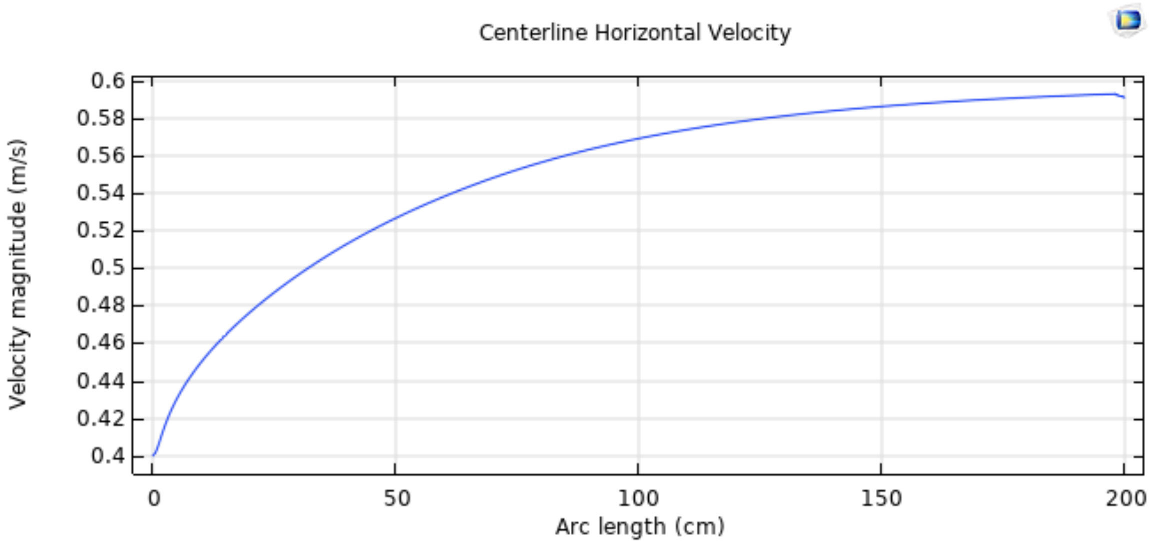

Figure 4: The horizontal velocity along the channel center line is shown.

The non-linear, continuous pressure field along the centerline of the flow channel is demonstrated at Figure 5. In a fully developed flow, the pressure variation should be linear.

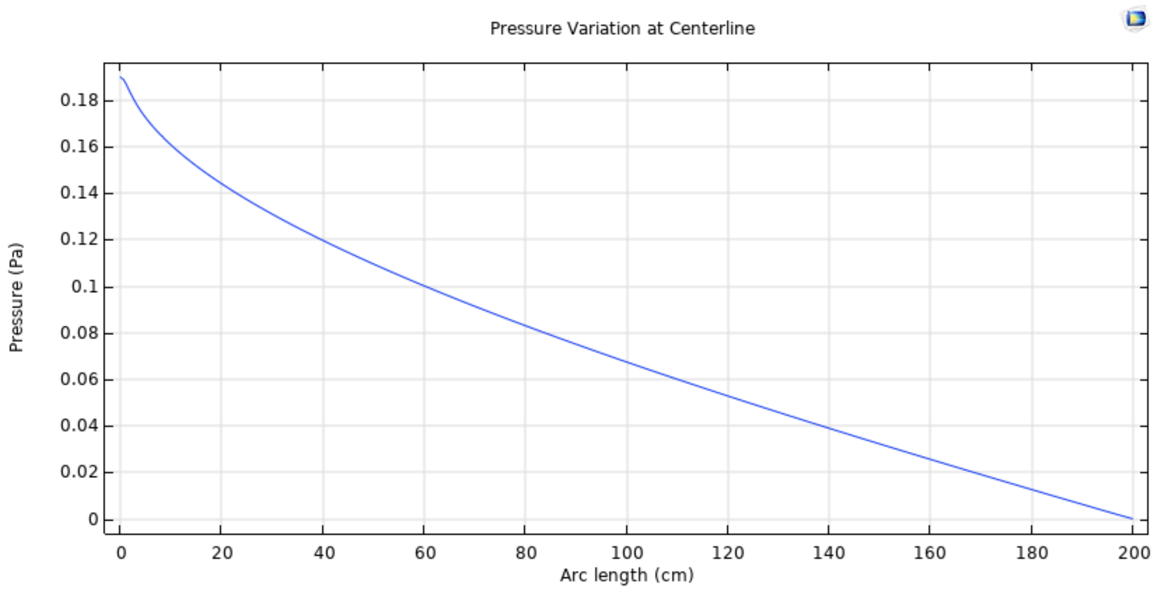

Figure 5: The pressure variation along the centerline of the channel.

Streamline analysis according to Bernoulli's equation is used to determine entry length as well. The streamline selected for analysis is the centerline. Excluding the gravity term, the total pressure calculated using the following equation and is plot against channel length:

$$P_{Bernoulli} = \frac{1}{2}\rho u^2 + P \tag{5}$$

Because the streamline is only valid in the entrance region, the length of the entrance length is defined as:

$$L_{entrance} = \{l |\, P_{Bernoulli}(l) < 0.99 P_{Bernoulli}(0)\,\} \tag{6}$$

Using this method, the estimate is $L_{entrance} \approx 67.8\ cm$.

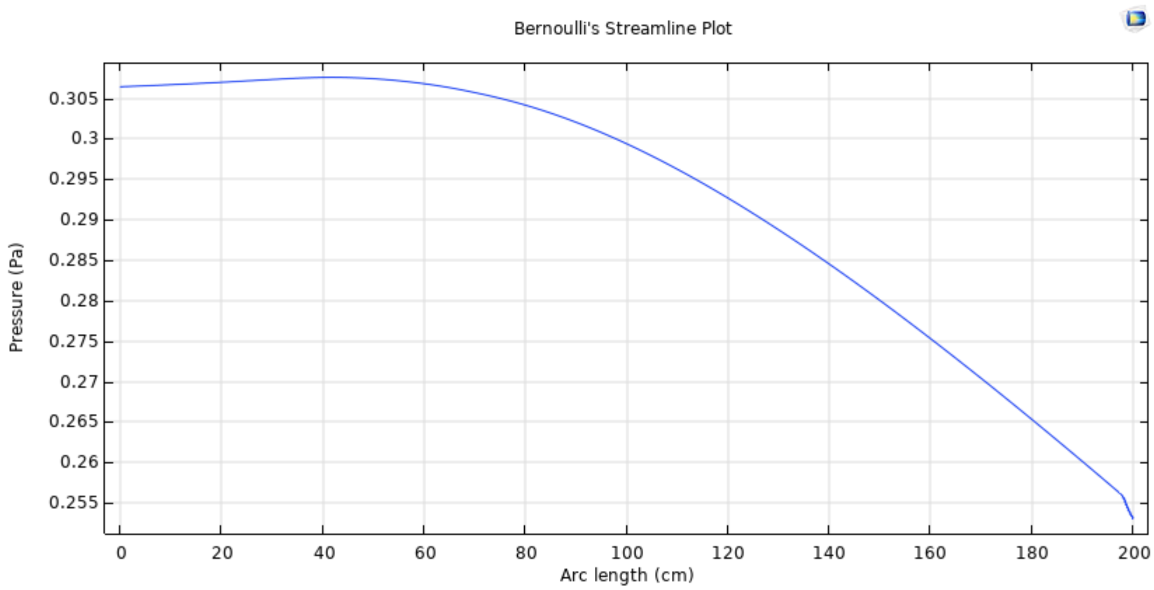

Figure 6: The Bernoulli constant is computed along channel length.

In the fully developed flow, the shear stress and pressure gradient are in equilibrium in momentum conservation equation, so the pressure gradient can be used to determine the entrance length:

$$L_{entrance} = \left\{ l \mid \frac{dP}{dx}(l) > 0.99 P_{Bernoulli}(L) \right\} \tag{7}$$

Ignoring the numerical instabilities near the outlet the estimate of $L_{entrance} \approx 195.8mm$.

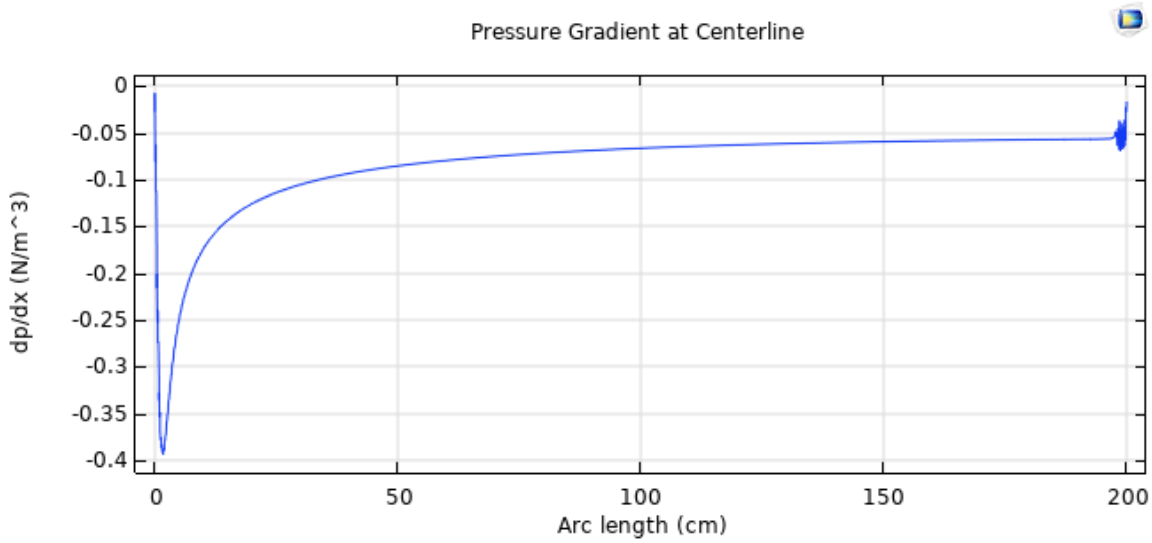

Figure 7: The pressure gradient along the centerline of the channel is shown.

Though the computation may be valid, discrepancies exist between different using Equation 4, 6 and 7. The most significant difference in estimation was using Bernoulli's equation. The streamline approximation is fundamentally flawed due some of the assumptions required, such as no viscosity. Furthermore, the recommended channel length was not long enough for an inlet velocity of 0.4*m/s* for both pressure gradient and velocity methods of estimating entrance lengths. The pressure gradient did not fully reach an asymptotic behavior yet and thus the estimate had significant errors. Similarly, the centerline horizontal velocity never satisfied Equation 4 for $U_0 = 0.4m/s$. Only after extending the channel length to 300cm, does it yield a final entrance length of $L_E(U_0 = 0.4) \approx 209.94cm$.

<u>Inlet Velocity Parametric Study</u>

To understand how boundary layers scale with inlet velocity, a parametric sweep between [0.04*m/s,* 0.4*m/s*] was done. Figure 8 shows Reynolds number plotted against L_e/H. Entrance lengths are determined using the centerline horizontal velocity method (Equation 4). The linear results agree well with the following empirical correlation:

$$(\frac{x_{fd,h}}{D})_{lam} \approx 0.05Re_D \tag{8}$$

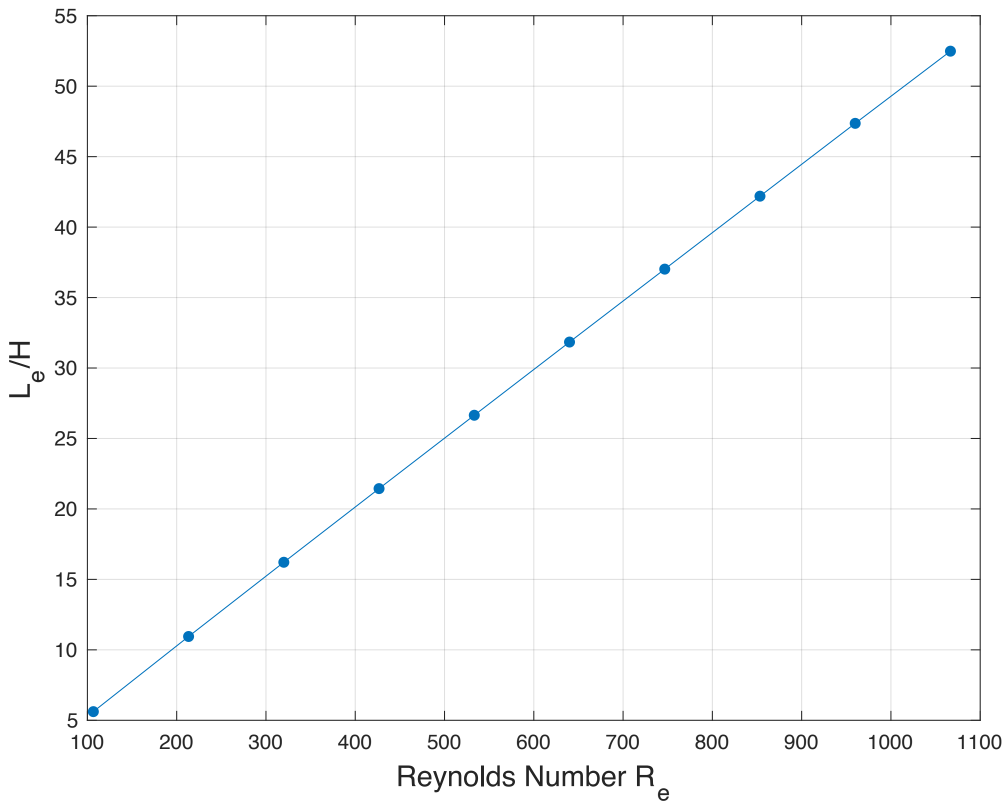

Figure 9: Non-dimensioned entrance lengths L_e/H shows a strong linear relationship against Reynolds number using centerline velocity determination method.

Fully Developed Flow

To increase computation speed when only the fully developed region is of interest, the inlet boundary condition is modified, and channel length is shorted to $12cm$. Instead of a uniform velocity profile, the analytical derived fully developed velocity profile is used. Figure 8 compares the inlet and outlet horizontal velocity profiles of the new model, showing excellent agreement. This indicates that the entire flow within the channel is fully developed.

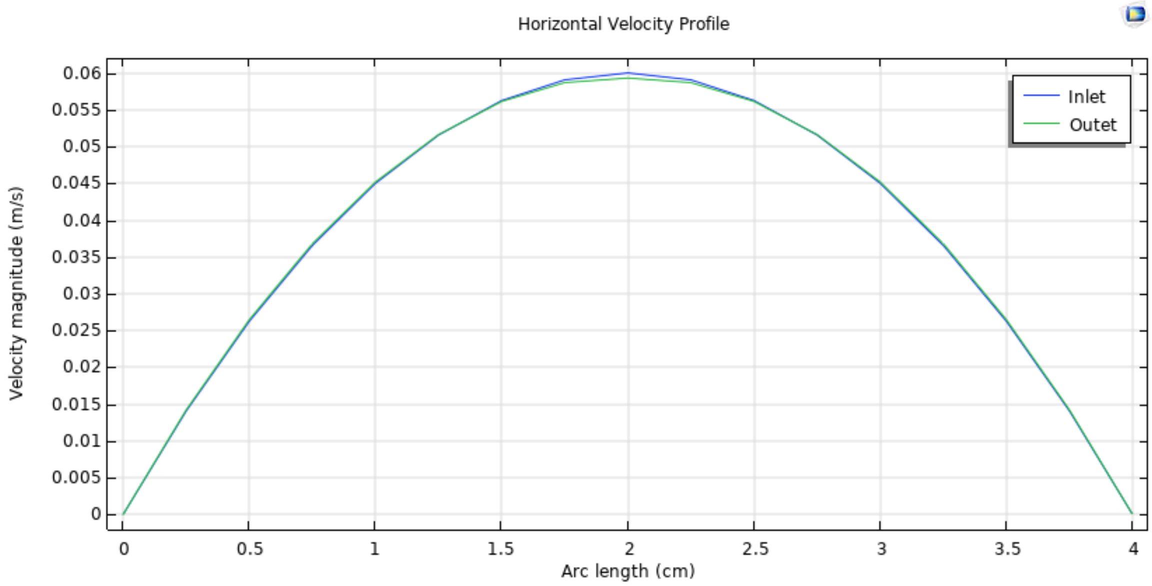

Figure 10: A comparison in the inlet and outlet velocity profiles in the fully developed flow study. The similarity in profile is a good indicator of fully developed flow.

Free vs Mapped Meshes

The fully developed flow model in the previous model was used to investigate the effects of higher order fluid discretization schemes. In Figure 9, when a second order Lagrange element was used to model the velocity components, the resulting profile is perfectly quadratic, meanwhile a first order element shows discontinuities at $H = 2mm$ (centerline). Although only three nodes were used to discretize the entire height of the channel, because the solution order matches the element order, a correct profile can be achieved.

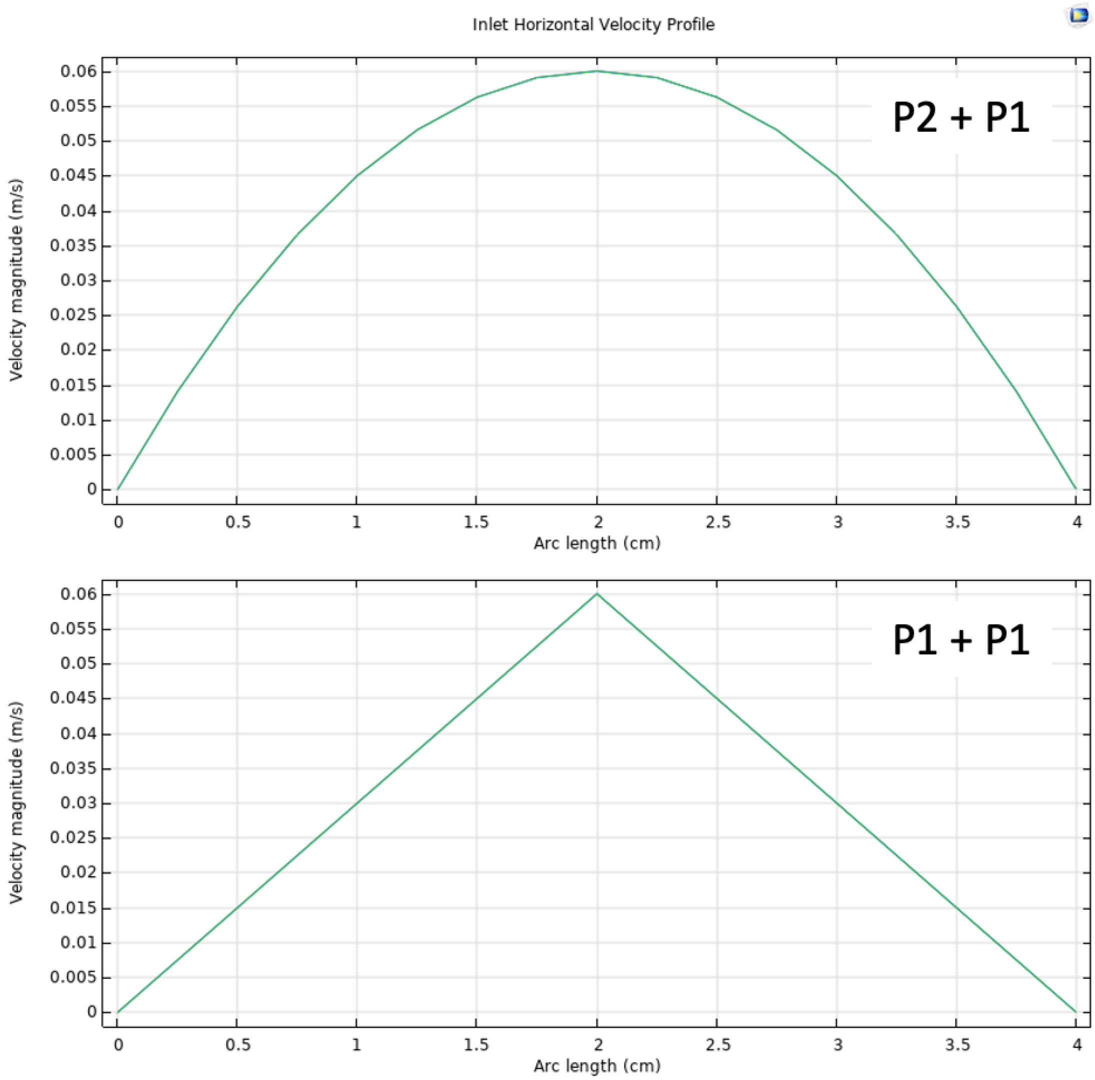

Figure 11: A comparison in inlet horizontal velocity profiles using different discretization schemes. A second order velocity discretization (P2 + P1) accurately captures the quadratic velocity profile, in comparison to the lower order models (P1 + P1).

Conclusions

I have learnt many new skills, especially on developing custom metrics to validate momentum conservation. Given this was our first COMSOL lab, essentially every Multiphysics feature was a new technique learnt through this lab. The total time spent is approximately 8 hours, in addition to the 6 hours of lab time spent completing the tasks in the lab manual.

Suggestions for Improvement

The lab can be streamlined by perhaps including a better COMSOL tutorial on the pseudo-language used to create and evaluate expressions. A significant amount of time was spent on understanding the syntax and interface of the software and less on actual analysis. I think it would also be interesting if we can investigate other physical parameters, such as temperature or density.