Indian Institute of Information Technology Chittoor, SriCity

Discrete Mathematics - Mid Semester 1 examination

Total marks: 45; Time: 1 hour 30 minutes

Instructions

- All the questions are to be attempted.
- All the answers should be written in formal way.
- All questions have the equal weightage.
- This is a closed book, closed notes exam.
- Calculators (non-programming) may be allowed in the exam hall.

1. Examine the validity of the following argument, using mathematical logic:

(a) There are three options of transportation from here to Chennai: By Bus, By Train and By Cab.
(b) For anybody who go by Bus, have to share with others.
(c) For anybody who go by Train, have to share with others.
(d) For anybody who dont want to share, have to go by Cab.

2. Prove that $((p \rightarrow r) \wedge (q \rightarrow r)) \leftrightarrow ((p \vee q) \rightarrow r)$ is a tautology (not using truth table).
3. If you are willing and inteligent, then you can solve this problem. If you are unable to solve the problem, then you are not inteligent. If you are not willing to solve the problem, then you are not sincere. You could not solve the problem. If you are a good student then you must be intelligent and sincere. Using mathematical logic, prove that, you are not a good student.
4. (3+2 marks) Prove that (not using truth table) $\neg(p \leftrightarrow q)$ and $(\neg p \leftrightarrow q)$ are logically equivalent. Find the Prenix Normal Form of the statement: $\exists x P(x) \rightarrow \forall x Q(x)$.
5. Obtain PCNF and PDNF of the given expression: $((p \vee q) \wedge (q \rightarrow r)) \leftrightarrow r$.
6. (3+2 marks) Find out the mistake(s) in the following steps and correct them.
   (a) ∀x(P (x) ∨ Q(x)), Given
   (b) P (c) ∨ Q(c), Universal instantiation
   (c) P (c), Simplification
   (d) ∨xP (x), Universal generalization
   (e) Q(c), Simplification from (b)
   (f) ∀xQ(x), Universal generalization
   (g) ∀xP (x) ∨ ∀xQ(x), Conjunction of (d) and (f)

7. (2+3 marks) Prove or disprove the following statements:

(a) For any two positive real numbers *x* and *y*, $\lceil x \rceil + \lceil y \rceil = \lceil x+y \rceil$.
(b) Prove that the integer *x+y* is an odd integer iff anyone of *x* and *y* is odd.

8. (3+2 marks) Prove the following:

(a) Prove that, $\sqrt{3}$ is irrational.
(b) Prove that, if *n* is a positive odd integer, then $\lceil \frac{3n}{2} - 2 \rceil = \frac{3(n-1)}{2}$. Note that, $\lceil x \rceil$ denotes the least integer greater than *x*.

9. (3+2 marks)

(a) Prove that, if $n = ab$, then either $n \leqslant \sqrt{a}$ or $n \leqslant \sqrt{b}$ for any positive integer *n*.
(b) Prove that, for any positive odd integer *n*, there exist a unique integer *k* such that, *n* can be expressed as a sum of *k-2* and *k+3*.