Discrete Mathematics - Assignment 1

Deadline: September 5th, 2017, 5PM
Maximum marks: 30

1. Let S be the set of all students of UG1 in IIITS. Let D(x) denote that "x is good at DM" and let C(x) denote that "x is good at computer language C." Express the following sentences using C (x), D(x), quantifiers, and logical connectives:

(a) (1 mark) If any student of UG1 is good in DM then he/she is good in C as well.

(b) (1 mark) There may be some students in UG1 who are good in C but not good in DM.

2. (2+2+2+2 marks) Prove that the following pairs of statements are equivalent, (i) using truth tables and (ii) without using truth tables:

(a) $(p \rightarrow q) \wedge (\neg p \rightarrow \neg q)$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$.

(b) $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$.

3. (2 mark) Prove that the following statement is a tautology: $((p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)) \rightarrow r$.

4. (2 marks) Find the negation of the following expression: $\exists y(\forall x \exists z A(x, y, z) \vee \exists x \forall z B(x, y, z))$.

5. (3 marks) Determine whether the following argument is valid or not:

If any student is good in DM then he/she has good logical ability.
If any student is good in CP then he/she has good logical ability.
If any student has good logical ability then he/she can get a good job.
There may be some students who are not good in CP but can get a good job.

6. (2 marks) Find out the mistake(s) in the following steps.

(a) ∀x(P (x) ∨ Q(x)), [Given]
(b) P (c) ∨ Q(c), [Universal instantiation]
(c) P (c), [Simplification]
(d) ∨xP (x), [Universal generalization]
(e) Q(c), [Simplification from (b)]
(f) ∀xQ(x), [Universal generalization]
(g) ∀xP (x) ∨ ∀xQ(x), [Conjunction of (d) and (f)]

7. (2 marks) Prove that the sum of any two rational numbers is a rational number.

8. (5 marks) During a murder investigation, you have gathered the following clues:

1. If the knife is in the store room, then we saw it when we cleared the store room;

2. The murder was committed at the basement or inside the apartment;

3. If the murder was committed at the basement, then the knife is in the yellow dust bin;

4. We did not see a knife when we cleared the store room;

5. If the murder was committed outside the building, then we are unable to find the knife;

6. If the murder was committed inside the apartment, then the knife is in the store room.

The questions are: Where is the knife? Where was the murder committed? Solve this puzle mathematically.

9. (2 marks) Prove by contrapositive approach: *If 5n^2 + 2 is an odd number then n is an odd number.*

10. (2 marks) Prove that if $x+y$ is even for two integers x and y, then either x and y or both are even or both are odd numbers.