

1. **What do you want from this course?** (4 points) What skills do you hope to learn, what topics do you think we'll cover that will stick with you five, or ten years down the road? Write *at least three sentences* describing how you expect to benefit from taking this class. The reason for this question is twofold. First, it will help me understand what you want. Second, keep your answer to this question in mind throughout the semester as motivation if the going gets tough!

I hope to advance algorithms and improve my problem solving skills to understand, analyze and solve such problems that usually come in challenges, contests, interviews, etc. This will also help me in the future as this course will make us realize the bad coding mistakes people usually make and how to prevent ourselves from doing the same.

2. **New friends.** (16 points) Each of  $n$  users spends some time on a social media site. For each  $i = 1, \dots, n$ , user  $i$  enters the site at time  $a_i$  and leaves at time  $b_i \geq a_i$ . You are interested in the question: how many distinct pairs of users are ever on the site at the same time? (Here, the pair  $(i, j)$  is the same as the pair  $(j, i)$ ).

Example: Suppose there are 5 users with the following entering and leaving times:

User	Enter time	Leave time
1	1	4
2	2	5
3	7	8
4	9	10
5	6	10

Then, the number of distinct pairs of users who are on the site at the same time is three: these pairs are  $(1, 2)$ ,  $(4, 5)$ ,  $(3, 5)$ .

- (a) (3+3 pts) Given input  $(a_1, b_1), (a_2, b_2), \dots, (a_n, b_n)$  as above, there is a straightforward algorithm that takes about  $n^2$  time to compute the number of pairs of users who are ever on the site at the same time. (a) Give this algorithm and explain why it takes time about  $n^2$ . (b) Write the code in C programming language for this algorithm.

(a). Algorithm

Algorithm I: Time Complexity  $O(n^2)$

This Program will check how many unique pairs of students will be online at the same time based on their entry and exit times

The Algorithm Uses Two Nested Loops Which Go over  $N$  times

**Input:**

$n$  <= total no of students

people <= [(entry, exit), ...] Array defining a student's entry/exit time

no\_of\_pairs <= 0

**FOR**  $i$  from 0 to  $n - 1$ :

**FOR**  $j$  from  $(i+1)$  to  $n - 1$ :

**IF** (person[ $i$ ].entry < person[ $j$ ].exit and  
            person[ $j$ ].entry < person[ $i$ ].exit):

            Increment no\_of\_pairs by 1

(a). C Program:

```
#include <stdio.h>

typedef struct person{
    int entry;
    int exit;
} Person;

int main(int argc, char const *argv[]){
    int noOfPeople;
    scanf("%d", &noOfPeople);
    Person people[noOfPeople];

    for (int i = 0; i < noOfPeople; i++){
        scanf("%d %d", &people[i].entry, &people[i].exit);
    }
    int distinctPairs = 0;

    for (int i = 0; i < noOfPeople; i++){
        for (int j = i + 1; j < noOfPeople; j++){
            if(people[i].entry < people[j].exit && people[j].entry < people[i].exit){
                distinctPairs++;
            }
        }
    }
    printf("Possible distinct pairs are %d", distinctPairs);
    return 0;
}
```

(b) (5+5 pts) (a) Give an  $O(n \log(n))$ -time algorithm to do the same task and analyze its running time. (**Hint:** consider sorting relevant events by time). (b) Write the code in C programming language for this algorithm.

(b) . Prerequisite Algorithms:

### 1-Binary Search

Algorithm **BINARYSEARCH**: Time Complexity  $O(\log(n))$   
This Program will search for the greatest no. less than or equal to the Value given in the Sorted Array

```
BINARYSEARCH(Array, val)
l <= 0, r <= Array.Length -1

WHILE l < r:
    GET Middle Element and compare with val
    mid <= l + (r - l) / 2;
    IF val is less than Array[mid]:
        SEARCH for val in right half of sorted Array:
            l <= mid + 1
    ELSE
        SEARCH for val in left half of sorted Array:
            r <= mid
IF Array[l] == val:
    RETURN l
ELSE
    RETURN l - 1
```

## 2-Merge Sort

Algorithm **MERGESORT**: Time Complexity  $O(n \cdot \log(n))$

**MERGESORT**(arr[], l, r)

**IF** l == r

**RETURN**

**ELSE**

    Find the middle point to divide the array into two halves:

        middle m <= (l+r)/2

    Do **MERGESORT** for first half:

        Call **MERGESORT**(arr, l, m)

    Do **MERGESORT** for second half:

        Call **MERGESORT**(arr, m+1, r)

**MERGE** the two halves sorted in step 2 and 3:

        Call **MERGE**(arr, l, m, r)

Algorithm **MERGE**: Time Complexity  $O(n)$

**MERGE**(Array, l, m, r):

// Array1 is Array[l:m], Array2 is Array[m+1:r]

Create a Duplicate array of Total Length:

    Temp <= Array of Size [ Array1.Length + Array2.Length ]

    index <= 0

Initialize Index for both Array:

    i <= 0, j <= 0

**WHILE** index < Temp.Length:

**IF** any Array becomes empty

**IF** i == Array1.Length :

            Temp[index++] <= Array2[j++]

**IF** j == Array2.Length :

            Temp[index++] <= Array1[i++]

**CONTINUE**

INSERT the lower Element:

**IF** Array1[i] < Array2[j]:

    Temp[index++] <= Array1[i++]

**ELSE**

    Temp[index++] <= Array2[j++]

## (b) Algorithm

Algorithm II: Time Complexity  $O(n \cdot \log(n))$

This Program will check how many unique no of pairs of students will be available online based on their entry and exit times

The First Part of the Problem User MERGESORT which takes  $O(n \cdot \log(n))$  Time  
In Second Part we BINARYSEARCH the Value in Array over a loop of size  $n$  so net Complexity will be  $O(n) * O(\log(n)) \Rightarrow O(n \cdot \log(n))$

Input:

$n \leq$  Total no of students

Array defining a student structure with entry/exit time

people  $\leftarrow$  [(entry, exit), ...]

no\_of\_pairs  $\leftarrow$  0

Apply MERGESORT on people Array

**FOR**  $i$  from 0 to  $n - 1$ :

    GET Value When the  $i^{\text{th}}$  person Leaves

$val \leftarrow$  people[ $i$ ].exit

    Find How many people were logged in before  $i^{\text{th}}$  person left

$found\_index \leftarrow$  BINARYSEARCH(people,  $val$ )

    Add No. of Pairs of the respective person

    Increment no\_of\_pairs by  $found\_index - i$

## (b) C Program

```
#include <stdio.h>

typedef struct person {
    int entry;
    int exit;
} Person;

int binarySearchPersonEntry(Person Array[], int val, int start, int end)
{
    while (start <= end) {
        int m = start + (end - start) / 2;
        if (Array[m].entry == val)
            return m;
        if (Array[m].entry < val)
            start = m + 1;
        else
            end = m - 1;
    }
    return start - 1;
}
```

```

int compare(Person p1, Person p2){
    if (p1.entry < p2.entry) {
        return 1;
    }
    else {
        if (p1.exit < p2.exit) {
            return 1;
        }
        else {
            return 0;
        }
    }
}

void mergeSort(Person Array[], int start, int end)
{
    if (start == end) {
        return;
    }
    int mid = (start + end - 1) / 2;

    mergeSort(Array, start, mid);
    mergeSort(Array, mid + 1, end);

    // Merging Arrayay
    int length = end - start + 1;
    Person Temp[length];
    for (int i = 0; i < length; i++){
        Temp[i] = Array[i + start];
    }

    int i = 0, j = 0, m = mid - start + 1;

    for (int c = start; c <= end; c++){
        if(i >= m){
            Array[c] = Temp[m + j++];
        }
        else if (j + mid >= end)
        {
            Array[c] = Temp[i++];
        }
        else if (compare(Temp[i], Temp[m + j]))
        {
            Array[c] = Temp[i++];
        }
        else
        {
            Array[c] = Temp[m + j++];
        }
    }
}

```

```

int main(int argc, char const *argv[])
{
    int noOfPeople;
    scanf("%d", &noOfPeople);

    Person people[noOfPeople];

    for (int i = 0; i < noOfPeople; i++){
        scanf("%d %d", &people[i].entry, &people[i].exit);
    }

    mergeSort(people, 0, noOfPeople - 1);

    int distinctPairs = 0;

    int begin = 0, end = 0, current_online = 0;
    for (int i = 0; i < noOfPeople; i++){
        int k = binarySearchPersonEntry(people, people[i].exit - 1, i+1, noOfPeople-1) - i;
        distinctPairs = k;
    }
    printf("No. of distinct pairs are: %d.\n", distinctPairs);
}

```

3. **Proof of correctness.** (4+6 points) Consider the following algorithm that is supposed to sort an array of integers. (a) Provide a proof that this algorithm is correct. (**Hint:** you may want to use more than one loop invariant.), and (b) Write the code in C and validate through different inputs.

# Sorts an array of integers.

```

Sort(array A):
    for i = 1 to A.length:
        minIndex = i
        for j = i + 1 to A.length:
            if A[j] < A[minIndex]:
                minIndex = j
        Swap(A[i], A[minIndex])

```

# Swaps two elements of the array. You may assume this function is correct.

```

Swap(array A, int x, int y):
    tmp = A[x]
    A[x] = A[y]
    A[y] = tmp

```

**Inductive Hypothesis:** After iteration of  $i$  in the outer loop the array  $A[0 : i]$  will be sorted and all the Elements in  $A[i+1 : \text{end}]$  are greater than elements in  $A[0 : i]$ .

**Base Case:** At beginning of outer Loop  $i = 1$  The Array  $A[0:1]$  will contain single element which is already sorted since it is the only element in the Array.

**Inductive Step:** Suppose that the inductive hypothesis holds for  $i-1$  so  $A[0 : i-1]$  is sorted and all the elements in Array  $A[i : \text{end}]$  are larger than all elements in the Previous Array, after the  $i-1^{\text{th}}$  iteration. We want to show that Array  $A[0 : i]$  is sorted and contains the least  $i$  members of entire Array  $A$  after the  $i^{\text{th}}$  iteration.

Suppose that  $k^{th}$  element is the smallest integer in the remaining Unsorted Array  $A[i : end]$ . Then the effect of the inner loop is to swap the position of  $i^{th}$  element with  $k^{th}$  element, which will convert the existing array

A[0]	A[1]	...	A[i-1]	A[i]	...	A[k]	...	A[end]
------	------	-----	--------	------	-----	------	-----	--------

Into following:

A[0]	A[1]	...	A[i-1]	A[k]	...	A[i]	...	A[end]
------	------	-----	--------	------	-----	------	-----	--------

We can now claim that the following list is sorted:

A[0]	A[1]	A[2]	...	A[i-1]	A[k]
------	------	------	-----	--------	------

This is because the first  $i-1$  elements are already sorted and Since all the remaining elements were larger than all the elements in Array  $A[0 : i-1]$ . Hence  $A[k] > A[i-1]$ . Also Since  $A[k]$  was the smallest in the Array  $A[i : end]$  hence all the remaining elements  $A[i+1 : end]$  are larger than  $A[k]$  Satisfying the second condition. Since Both the conditions are satisfied and element  $A[k]$  is in the right place, this proves the claim.

Thus, After the  $i^{th}$  iteration completes the Array  $A[0 : i]$  is sorted and this establishes the inductive hypothesis for  $i$ .

Hence the Correctness of Algorithm is Proved.

#### (b)- C Program

```
#include <stdio.h>

void swap(int *a, int *b){
    *a = *a + *b;
    *b = *a - *b;
    *a = *a - *b;
}

void sort(int arr[], int length){
    for (int i = 0; i < length; i++){
        int pos = i;
        for (int j = i + 1; j < length; j++){
            if(arr[j] < arr[pos]){
                pos = j;
            }
        }
        if(i != pos){
            swap(&arr[i], &arr[pos]);
        }
    }
}

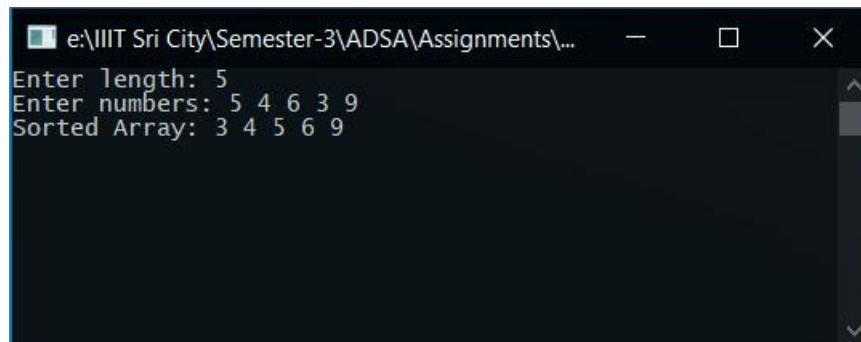
int main(){
    int n;
    printf("Enter length: ");
    scanf("%d", &n);
    int arr[n];
```

```

printf("Enter numbers: ");
for(int i = 0; i < n; i++){
    scanf("%d", &arr[i]);
}
sort(arr, n);
printf("Sorted Array: ");
for(int i = 0; i < n; i++){
    printf("%d ", arr[i]);
}
printf("\n");
}

```

Output



```

e:\IIIT Sri City\Semester-3\ADSA\Assignments\...
Enter length: 5
Enter numbers: 5 4 6 3 9
Sorted Array: 3 4 5 6 9

```

#### 4. Needlessly complicating the issue. (20 points)

- (a) (3pts) Give a linear-time (that is, an  $O(n)$ -time) algorithm for finding the minimum of  $n$  values (which are not necessarily sorted).

(A)- Algorithm

**Algorithm:** Time Complexity  $O(n)$

This Program will Search the Array for the least element and will return it  
The Algorithm Uses a Single Loop in which we will match each element  
with the minimum element and update the minimum element if turns out to be smaller

Since Every element is matched once the complexity will be  $O(n)$

Input:

$n \leq$  total no of students

$arr \leq$  [...] Array of Numbers

**DEFINE** first element as the least  
min = arr[0]

**FOR** i from 1 to n:  
  **IF** arr[i] is less than min:  
    **DEFINE** min as arr[i]  
    min <= arr[i]

**RETURN** min



(b)

Since The array is not sorted, hence the smallest element can lie at any position, Hence In the worst case the program/Algorithm has to check each and every element, If we do not check the value of any element there is probability that the unchecked element is the least element, which will lead to wrong output. So, a Minimum of  $n$  comparison are required in worst case to find least element in an Array.

(c) (3pts) Write the C program for this linear-time algorithm.

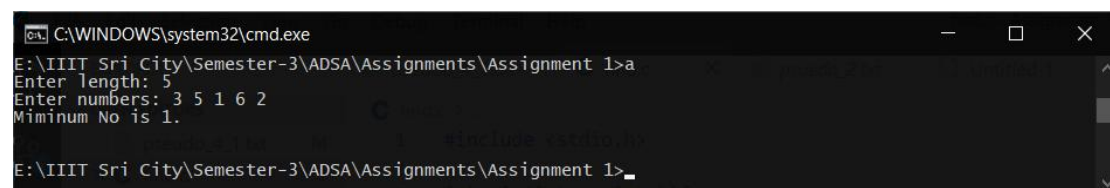
(c)- C Program

```
#include <stdio.h>

int findMinimum(int arr[], int length){
    int min = arr[0];
    for (int i = 0; i < length; i++){
        if (arr[i] < min) {
            min = arr[i];
        }
    }
    return min;
}

int main(){
    int n;
    printf("Enter length: ");
    scanf("%d", &n);
    int arr[n];
    printf("Enter numbers: ");
    for(int i = 0; i < n; i++){
        scanf("%d", &arr[i]);
    }
    printf("Mimumum No is %d.\n", findMinimum(arr, n));
    printf("\n");
}
```

Output:



```
C:\WINDOWS\system32\cmd.exe
E:\IIIT Sri City\Semester-3\ADSA\Assignments\Assignment 1>a
Enter length: 5
Enter numbers: 3 5 1 6 2
Mimumum No is 1.

E:\IIIT Sri City\Semester-3\ADSA\Assignments\Assignment 1>
```

Now consider the following recursive algorithm to find the minimum of a set of  $n$  items.

---

**Algorithm 1: findMinimum**

---

**Input:** List  $A = [a_1, \dots, a_n]$  of  $n$  items

**Output:**  $\min_i \{a_1, \dots, a_n\}$

**if**  $n=1$  **then**

**return** -----

$A_1 = A[0 : n/2]$

$A_2 = A[n/2 : n]$

**return**  $\min(\text{findMinimum}(A_1), \text{findMinimum}(A_2))$

---

(3pts) Fill in the blank in the pseudo-code: what should the algorithm return in the base case? Briefly argue that the algorithm is correct with your choice.

(d)-Algorithm

**Input:** List  $A = \{a_1, a_2, \dots, a_n\}$  of  $n$  items

**Output:**  $\min_i \{a_1, a_2, \dots, a_n\}$

**IF**  $n=1$  **then**

**RETURN**  $a_1$

$A_1 \leftarrow A[0 : n/2]$

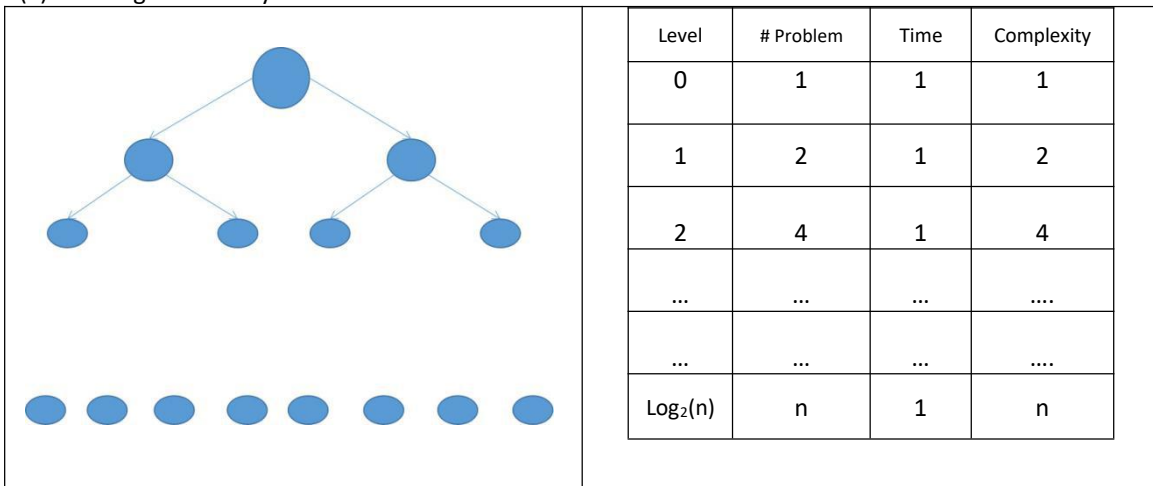
$A_2 \leftarrow A[n/2 : n]$

**RETURN**  $\min(\text{findMinimum}(A_1), \text{findMinimum}(A_2))$

The Algorithm will first compare the results of sub-problems of left and right arrays. When the base case occurs it will just return the element itself. This will create a Tree-like recursive call where each node will return the minimum element of its children and ultimately we will find the minimum of array.

(3pts) Analyze the running time of this recursive algorithm. How does it compare to your solution in part (a)?

(e)-Running Time Analysis



Since the process of Dividing and Conquering takes  $O(1)$  time we can find the net complexity of the Algorithm as  $1 + 2 + 4 + \dots + n/2 + n = 2n - 1 = O(n)$   
 Since This method Also takes Linear time it of same complexity as of previous Problem.

(6pts) Write the C program for this recursive algorithm.

(f)- C Program

```

#include <stdio.h>

int min(int a, int b){
    return ((a > b) ? b : a);
}

int findMinimum(int arr[], int start, int end){
    if(start == end){
        return arr[start];
    }
    int mid = start + (end - start) / 2;
    return(min(findMinimum(arr, start, mid), findMinimum(arr, mid+1, end)));
}

int main(){
    int n;
    printf("Enter length: ");
    scanf("%d", &n);
    int arr[n];
    printf("Enter numbers: ");
    for(int i = 0; i < n; i++){
        scanf("%d", &arr[i]);
    }
    printf("Mimumum No is %d.\n", findMinimum(arr,0, n-1));
    printf("\n");
}

```

```

C:\WINDOWS\system32\cmd.exe
E:\IIIT Sri City\Semester-3\ADSA\Assignments\Assignment 1>a
Enter length: 6
Enter numbers: 2 6 3 7 4 5
Mimumum No is 2.

E:\IIIT Sri City\Semester-3\ADSA\Assignments\Assignment 1>

```

##### 5. Recursive local-minimum-finding. (20 points)

- (a) Suppose  $A$  is an array of  $n$  integers (for simplicity assume that all integers are distinct). A *local minimum* of  $A$  is an element that is smaller than all of its neighbors. For example, in the array  $A = [1, 2, 0, 3]$ , the local minima are  $A[1] = 1$  and  $A[3] = 0$ .
- (2 points) Design a recursive algorithm to find a local minimum of  $A$ , which runs in time  $O(\log(n))$ .
  - (2 points) Prove formally that your algorithm is correct.
  - (2 points) Formally analyze the runtime of your algorithm.
  - (4 points) Write C program for your algorithm.

## (I) Algorithm

Algorithm: **FINDLOCALMINIMA** Time Complexity:  **$O(\log(n))$**

We compare middle element with its neighbors. If middle element is smaller than both of its neighbors, then we return it.

If the middle element is greater than its left neighbor, then there is always a local minima in left half.

If the middle element is greater than its right neighbor, then there is always a local minima in right half.

**FINDLOCALMINIMA**(arr, start, end, n):

GET index of middle element

mid  $\leftarrow$  (low + high) / 2

IF neighbour are there:

IF ((mid == 0 || arr[mid-1] > arr[mid]) and  
(mid == n-1 || arr[mid+1] > arr[mid]))

RETURN mid

IF Left Half Contains the minima search in left-Half  
otherwise search in right-half:

IF mid > 0 and arr[mid-1] < arr[mid]:

RETURN FINDLOCALMINIMA(arr, low, (mid -1), n)

ELSE

RETURN FINDLOCALMINIMA(arr, (mid + 1), high, n);

## (II) Correctness of Proof:

**Recursive Invariant:** If the middle element is smaller than both its neighbour than it is the local minima, If right neighbour element is smaller than the middle element then it is guaranteed that there will always be a local minima in right-half of the array, similarly if left neighbour element is smaller than the middle element then there must be local minima in the left-half of array

**Inductive Hypothesis:** If the recursive invariant is true then the algorithm will find

**Base Case:** When  $n = 1$  then the array has a single element hence it is a local minima.

**Inductive Step:** When we check the middle element there are following cases

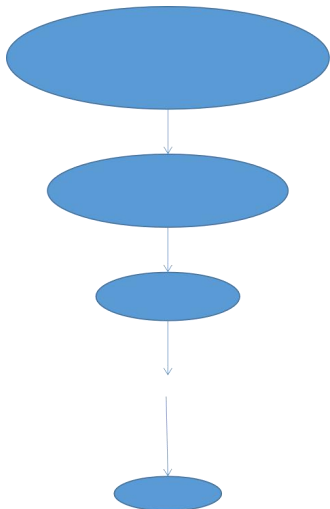
**Case I:** When the middle element is less than both of its neighbour, it will return the element.

**Case II:** If the middle element is greater than right neighbour, The recursive algorithm will search the right-half sub array A[mid+1: end], Now either "**Case I**" the middle of this element is the local minima or "**Case II**" The function will search right-half of this array or "**Case III**" The function will search left-half of this array recursively till solution is reached.

**Case III:** If the middle element is greater than left neighbour, The recursive algorithm will search the left-half sub array A[start: mid], Now either "**Case I**" the middle of this element is the local minima or "**Case II**" The function will search right-half of this array or "**Case III**" The function will search left-half of this array recursively till solution is reached.

Hence the algorithm will ultimately return a local minima.

(III)

	Level	# Problems	Time	Complexity
	0	1	1	1
	1	1	1	1
	2	1	1	1
	...	...	...	...
	...	...	...	...
	$\log_2(n)$	1	1	1

Since the process of Dividing and Conquering has conditional statements only it takes  $O(1)$  time and only a Single branch is formed we can find the net complexity of the Algorithm as  $1 + 1 + \dots \log_2(n) + 1$  times =  $O(\log(n))$

So This method takes Logarithmic time.

(IV)- C Program

```
#include <stdio.h>

int local_minima(int arr[], int low, int high, int length){
    int mid = low + (high - low)/2;
    if ((mid == 0 || arr[mid-1] > arr[mid]) && (mid == length-1 || arr[mid+1] > arr[mid]))
        return mid;
    else if (mid > 0 && arr[mid-1] < arr[mid])
        return local_minima(arr, low, (mid -1), length);
    return local_minima(arr, (mid + 1), high, length);
}

int main(){
    int n;
    printf("Enter length: ");
    scanf("%d", &n);
    int arr[n];
    printf("Enter numbers: ");
    for(int i = 0; i < n; i++){
        scanf("%d", &arr[i]);
    }
    printf("Local Mimima is %d.\n", local_minima(arr, 0, n-1, n));
    printf("\n");
}
```

- (b) Let  $G$  be a square  $n \times n$  grid of integers. A *local minimum* of  $A$  is an element that is smaller than all of its direct neighbors (diagonals don't count). For example, in the grid

$$G = \begin{bmatrix} 5 & 6 & 3 \\ 6 & 1 & 4 \\ 3 & 2 & 3 \end{bmatrix}$$

some of the local minima are  $G[1][1] = 5$  and  $G[2][2] = 1$ . You may once again assume that all integers are distinct.

- i. (2 points) Design a recursive algorithm to find a local minimum in  $O(n)$  time.
- ii. (2 points) We are not looking for a formal correctness proof, but please explain why your algorithm is correct.
- iii. (2 points) Give a formal analysis of the running time of your algorithm.
- iv. (4 points) Write C program for your algorithm.

(I)- Algorithm (Check Is Minima at given position)

**Algorithm:** ISMINIMA Time Complexity  **$O(1)$**

This Algorithm check the Neighbour of  $arr[x][y]$  and returns 1  
If it is smaller than all otherwise returns 0

```
ISMINIMA(arr[ ][ ], int x, int y, int len){
    check <= 1
    IF arr[x][y] is a local minima
        IF arr[x][y] is More than Top Element:
            check <= 0
        IF arr[x][y] is More than Bottom Element:
            check <= 0
        IF arr[x][y] is More than Left Element:
            check <= 0
        IF arr[x][y] is More than Right Element:
            check <= 0
    RETURN check;
}
```

Algorithm **FINDLOCALMINIMA2**: Time Complexity:  $O(n)$

This Program Finds the index for a local Minima in a 2D  $n \times n$  matrix  
It checks the boundry and middle row and columns and checks if the element in the center is Minimum, if not we recurse this problem for smaller quadrants.

**FINDLOCALMINIMA2**(A[ ][ ], Xi, Yi, Xn, n):

IF Xi is larger than or equal to Xn:

**RETURN** A[Yi][Yi]

Min<= arr[Yi][Xi]

MinX <= Yi

MinY <= Xi

MiddleX <= Xi + (Xn - Xi)/2

MiddleY <= Yi + (Xn - Yi)/2

length <= Xn - Xi

```

CHECK through the columns
FOR i from Xi to length:
    FOR j from Yi to length:
        check if local minima
        IF ISMINIMA(A, i, j, length)
            RETURN A[j][i]
        ELSE
            IF A[j][i] less than Min:
                min <= A[j][i]
            i += MiddleX

CHECK through the the rows
FOR i from Yi to length:
    FOR j from Xi to length:
        IF ISMINIMA(A, i, j, length)
            check FOR local minima
            RETURN A[j][i]
        else IF A[j][i] less than Min:
            min <= A[j][i]
    i += MiddleX

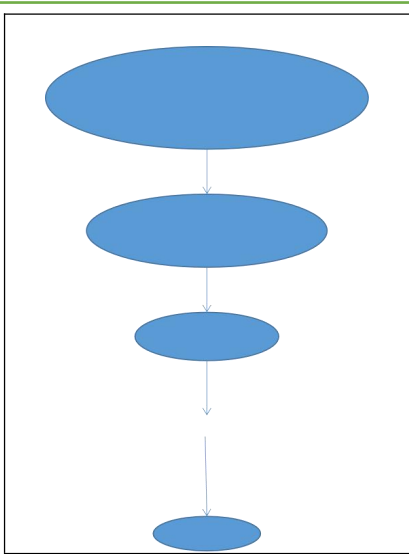
Search in Smaller Quadrants:
IF A[MinX][maxIndexX] > A[MinX][minIndex - 1]:
    IF MinX < MiddleY:
        RETURN FINDLOCALMINIMA2(arr, MinY - MiddleX, Yi, MinY, n);
    else:
        RETURN FINDLOCALMINIMA2(arr, MinY-MiddleX, Yi+MiddleY, MinY, n);

```

## (II)- Correctness Intuition

When We Check for Condition of minimum of boundary and middle row and column We also check the Adjacent (Top,Down,Left,Right) elements as well. In case it is not the minimum element, We find the minimum corner and recurse into that quadrant. Since We are keeping the minimum element each time in the quadrant the min value does not increases, we keep doing that until we find a local minima for the quadrant which will also be local minima for the entire matrix.

## (III) Running Time Analysis



Level	# Problems	Time	Complexity
0	1	n	c*n
1	1	n/2	c*n/2
2	1	n/4	c*n/4
...	...	...	...
...	...	...	...
Log <sub>2</sub> (n)	1	1	c

Since the process of Dividing and Conquering has conditional statements only it takes  $O(n)$  time and only a Single branch is formed we can find the net complexity of the Algorithm as  $c(1 + 2 + \dots n/2 + n)$  times  $= 2c*n - c = O(n)$   
So This method takes Linear time.

(IV) - C Program

```
#include <stdio.h>
#include <stdlib.h>

int isMinima(int *arr[], int x, int y, int len){
    int is_minima = 1;
    if(x + 1 <= len){
        if(arr[y][x + 1] < arr[y][x]){
            is_minima = 0;
        }
    }
    if(y + 1 <= len){
        if(arr[y + 1][x] < arr[y][x]){
            is_minima = 0;
        }
    }
    if(y - 1 >= 0){
        if(arr[y - 1][x] < arr[y][x]){
            is_minima = 0;
        }
    }
    if(x - 1 >= 0){
        if(arr[y][x - 1] < arr[y][x]){
            is_minima = 0;
        }
    }
    return is_minima;
}

int localMinima(int *arr[], int x0, int y0, int x1, int length){

    if(x0 >= x1){
        return arr[y0][x0];
    }

    int min = arr[y0][x0];
    int MinIx= x0, MinIy= y0;
    int midX = x0 + (x1 - x0)/2;
    int midY = y0 + (x1 - x0)/2, lastY = y0 + (x1 - x0);
    int len = x1 - x0;

    //Checking in the columns

    for(int i = x0; i <= x0 + len; i+=(x1-x0)/2){
        for(int j = y0; j <= y0 + len; j++){
            if(isMinima(arr, i, j, length)){
                return arr[j][i];
            }
        }
        else if(arr[j][i] < min){
            min = arr[j][i];
            MinIx= i;
            MinIy= j;
        }
    }
}
```



```

        }
    }
}

// Checking in the rows

for(int j = y0; j <= y0 + len; j+=(x1-x0)/2){
    for(int i = x0; i <= x0 + len; i++){
        if(isMinima(arr, i, j, length)){
            return arr[j][i];
        }
        else if(arr[j][i] < min){
            min = arr[j][i];
            MinIx= i;
            MinIy= j;
        }
    }
}

if(MinIx- 1 >= 0){
    if(arr[minIndY][minIndX] > arr[minIndY][MinIx- 1]){
        if(MinIy< midY){
            return localMinima(arr, MinIx- midX, y0, minIndX, length);
        }
        else
        {
            return localMinima(arr, MinIx- midX, y0 + midY, minIndX, length);
        }
    }
}
else if(MinIx+ 1 < length){
    if(arr[minIndY][minIndX] > arr[minIndY][MinIx+ 1]){
        if(MinIy< midY){
            return localMinima(arr, minIndX, y0, MinIx+ midX, length);
        }
        else
        {
            return localMinima(arr, minIndX, y0 + midY, MinIx+ midX, length);
        }
    }
}

else if(MinIy- 1 >= 0){
    if(arr[minIndY][minIndX] > arr[MinIy- 1][minIndX]){
        if(MinIx< midX){
            return localMinima(arr, x0, MinIy- midY, midX, length);
        }
        else
        {
            return localMinima(arr, midX, MinIy- midY, x1, length);
        }
    }
}
}

```

```

else if(MinIy+ 1 < length){
    if(arr[minIndY][minIndX] > arr[MinIy+ 1][minIndX]){
        if(MinIx< midX){
            return localMinima(arr, x0, MinIy+ midY, midX, length);
        }
        else
        {
            return localMinima(arr, midX, MinIy+ midY, x1, length);
        }
    }
}

return 0;
}

int main(){
    int n;
    printf("Enter length: ");
    scanf("%d", &n);
    int **arr = (void*)malloc(sizeof(int*) * n);
    for(int i = 0; i < n; i++){
        *(arr + i) = (void*)malloc(sizeof(int) * n);
    }
    for(int i = 0; i < n; i++){
        printf("Enter Row: ");
        for(int j = 0; j < n; j++){
            scanf("%d", &arr[i][j]);
        }
    }
    printf("Mimumum No is %d.\n", localMinima(arr, 0, 0, n-1, n));
    printf("\n");
}

```

## Output

```

E:\IIIT Sri City\Semester-3\ADSA\Assignments\Assignment 1>a
Enter length: 3
Enter Row: 5 6 3
Enter Row: 6 1 4
Enter Row: 3 2 3
Mimumum No is 5.
E:\IIIT Sri City\Semester-3\ADSA\Assignments\Assignment 1>\

```