Akshay lec5

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Inferences about a mean vector

$H_0: \mu = \mu_0$

$H_1: \mu \neq \mu_0$

$X_1, X_2, \ldots, X_n$ random sample from a normal pop^n.

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \sim t_{n-1}$$

$|t|$ exceeds specified % point, we reject null hypothesis

$$t^2 = \frac{(\bar{x} - \mu_0)^2}{s^2/n} = n(\bar{x} - \mu_0)(s^2)^{-1}(\bar{x} - \mu_0)$$

$$t^2 > t^2_{n-1}(\alpha/2)$$

C.I. Confidence Interval

$$\bar{x} - t_{n-1}(\alpha/2)\frac{s}{\sqrt{n}} \le \mu_0 \le \bar{x} + t_{n-1}(\alpha/2) \cdot s/\sqrt{n}$$

$$\underset{\sim}{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_p \end{bmatrix} \qquad \underset{\sim}{\mu_0} = \begin{bmatrix} \mu_{10} \\ \mu_{20} \\ \vdots \\ \mu_{p0} \end{bmatrix}$$

$S \to$ Covariance Matrix

$$T^2 = (\underset{\sim}{\bar{x}} - \underset{\sim}{\mu_0})' \left(\frac{1}{n} S\right)^{-1} (\underset{\sim}{\bar{x}} - \underset{\sim}{\mu_0}) \quad \text{Hotelling's } T^2$$

$$= n(\underset{\sim}{\bar{x}} - \underset{\sim}{\mu_0})' S^{-1} (\underset{\sim}{\bar{x}} - \underset{\sim}{\mu_0})$$

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If the squared distance is too large from μ_0 then null hypothesis will be rejected.

$$T^2 \sim \frac{(n-1)p}{n-p} F_{p,n-p}$$

Summary: let $X_1, X_2, \dots, X_n$ be a random sample from an $N_p(\underset{\sim}{\mu}, \Sigma)$ pop^n, Then $\bar{X}$ and S with

$$\alpha = P\left[T^2 > \frac{(n-1)p}{n-p} F_{p,n-p}(\alpha)\right].$$

$$= P\left[n(\underset{\sim}{\bar{x}}-\underset{\sim}{\mu})' S^{-1} (\underset{\sim}{\bar{x}}-\underset{\sim}{\mu}) > \frac{(n-1)p}{n-p} F_{p,n-p}(\alpha)\right]$$

$H_0: \mu = \mu_0$ if $T^2 = n(\underset{\sim}{\bar{x}}-\underset{\sim}{\mu_0})' S^{-1} (\underset{\sim}{\bar{x}}-\underset{\sim}{\mu_0}) > \frac{(n-1)p}{n-p} F_{p,n-p}(\alpha)$

$H_1: \mu \neq \mu_0$

null hypothesis is rejected

Example

$n=3$ $\quad x = \begin{bmatrix} 6 & 9 \\ 10 & 6 \\ 8 & 3 \end{bmatrix}$ evaluate the observed T^2 for $\underset{\sim}{\mu_0'} = [9 \;\; 5]$

Find its dist^n

$\bar{x} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$, $\quad S = \begin{bmatrix} 4 & -3 \\ -3 & 9 \end{bmatrix}$

$$S^{-1} = \frac{1}{4\times 9 - 9}\begin{bmatrix} 9 & 3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1/3 & 1/9 \\ 1/9 & 4/27 \end{bmatrix}$$

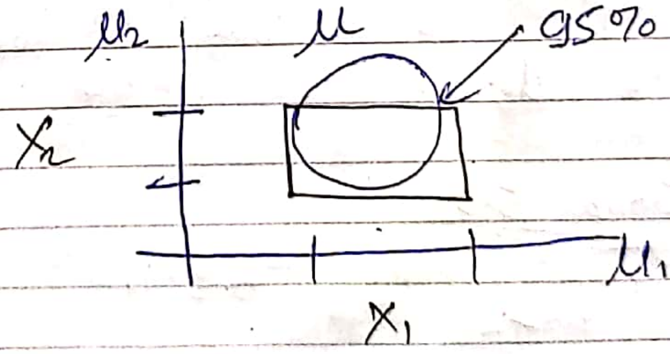
$$T^2 = 3\begin{bmatrix} 8-9 & 6-5 \end{bmatrix}\begin{bmatrix} 1/3 & 1/9 \\ 1/9 & 4/27 \end{bmatrix}\begin{bmatrix} 8-9 \\ 6-5 \end{bmatrix}$$

$$= 7/9$$

$$T^2 \sim \frac{(3-1)2}{3-2} \times F_{2,3-2}. \qquad \frac{(n-1)p}{n-p} F_{p,n-p}.$$

Confidence Regions. (33:18) minute

$\underset{\sim}{\theta} \Rightarrow$ population parameter



$\underset{\sim}{\theta}$ $\quad R(X)$

$R(X)$ is said to be a $100(1-\alpha)\%$ CR if.

$P(R(X)$ will cover the true value of $\theta) = 1-\alpha$

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$$P\left[n(\bar{\underset{\sim}{x}}-\underset{\sim}{\mu})' S^{-1}(\bar{\underset{\sim}{x}}-\underset{\sim}{\mu}) \le \frac{(n-1)p}{n-p} F_{p,n-p}(\alpha)\right] = 1-\alpha$$

for two variables region will be ellipse
for more than two it will be ellipsoid

$100(1-\alpha)\%$ confidence region for the mean of a p dimensional normall dist^n is the ellipsoid determined by all $\underset{\sim}{\mu}$ such that

$$n(\bar{\underset{\sim}{x}}-\underset{\sim}{\mu})' S^{-1}(\bar{\underset{\sim}{x}}-\underset{\sim}{\mu}) \le \frac{p(n-1)}{n-p} F_{p,n-p}(\alpha)$$

To find of $\underset{\sim}{\mu_0}$ is lies in region or not.

$$n(\bar{\underset{\sim}{x}}-\underset{\sim}{\mu_0})' S^{-1}(\bar{\underset{\sim}{x}}-\underset{\sim}{\mu_0}) \le \frac{p(n-1)}{n-p} F_{p,n-p}(\alpha)$$

if this is true it lies in CR otherwise it's doesn't.

Example 5.3

$n=42$ $\quad \bar{\underset{\sim}{x}} = \begin{bmatrix} 0.564 \\ 0.603 \end{bmatrix}$ $\quad S = \begin{bmatrix} 0.0144 & 0.0117 \\ 0.0117 & 0.0146 \end{bmatrix}$

$$S^{-1} = \begin{bmatrix} 203.018 & -163.391 \\ -163.391 & 200.228 \end{bmatrix}$$

Find 95% Confidence Region for $\underset{\sim}{\mu}$

given $\underset{\sim}{\mu_0} = \begin{bmatrix} 0.562 \\ 0.589 \end{bmatrix}$

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$p = 2$

$n = 42$

$$\frac{2(40)}{41} F_{2,40}(0.05) = \frac{2(40)}{40} * 3.23$$

$$= 6.62$$

$$42 (\underset{\sim}{\bar{x}} - \underset{\sim}{\mu_0})' S^{-1} (\underset{\sim}{\bar{x}} - \underset{\sim}{\mu_0}) = 1.30.$$

$1.30 \leq 6.62$ So μ_0 lies inside the CR.