Method of moments & Maximum Likelihood Estimation

Tutorial-2

Course: Statistical Data Analysis

IIIT Sricity, 2020

Estimation of parameter from Binomial Distribution

Associated to Bernouli trials

x = total number of "successes" (pass or fail, heads or tails etc.)

p = probability of a success on an individual trial

n = number of trials

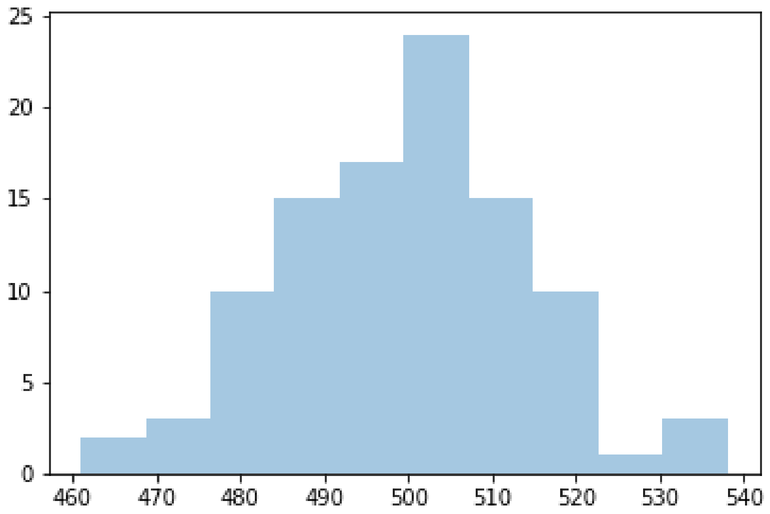
- To estimate '$\tilde{p}$'

from Binomial Distribution

Sampled from the Binomial distribution with described parameter

- $\bar{x}=n\tilde{p}$

- sample 100 points from a binomial distribution with known parameters.



$f(x,n,p)= {}^{n}C_{x}\, p^{x}(1-p)^{(n-x)}$

Mean= np

Variance= np(1-p)

n=1000

p=0.5

Mean=500

Variance= 250

Estimation of parameter for sample from Binomial Distribution

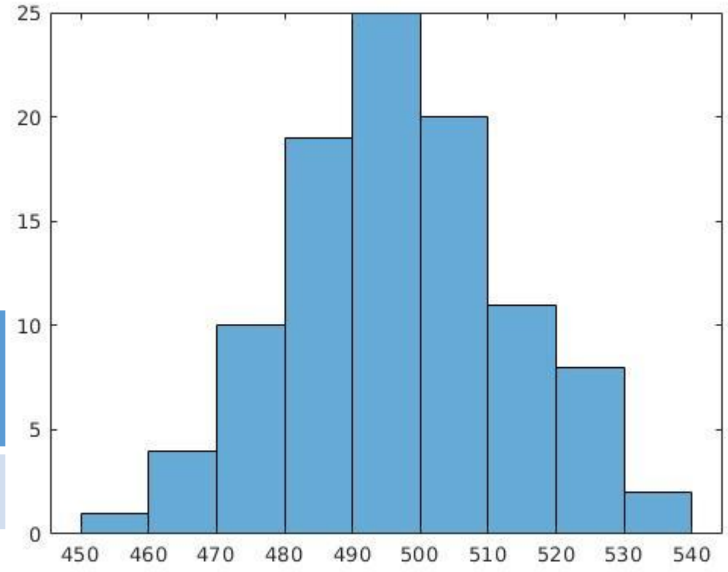
Compute moment of 1st order from 100 samples:

$\bar{x} = n\tilde{p} = 496.8$

$\tilde{p} = (496.8/n) = 0.496$

Sample Size	100	200	300	400	500
Value of '$\tilde{p}$'	0.496	0.503	0.501	0.5007	0.501

- Actual probability of a success on an individual trial= 0.5



Histogram of 100 samples

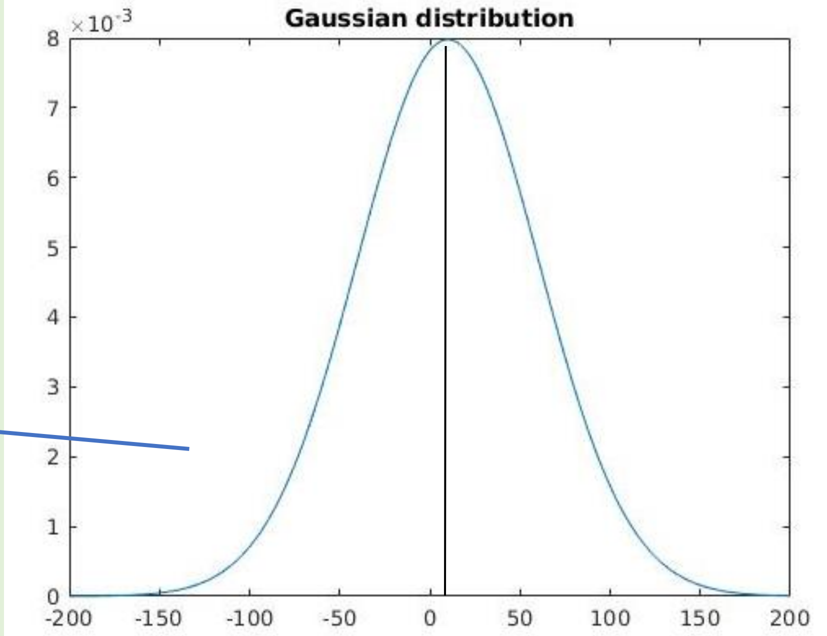
Note:

Consistency of the estimator:
As sample size increases the estimation becomes more accurate.

Estimation of parameter using sample from Gaussian Distribution

- Data sampled from Gaussian distribution.
- Used 1000 samples.
- To be estimated Mean and variance
- Mean= μ = 1st moment
- 1st moment= 8.36 = μ

Sample size	1000	2000	5000	10000	15000
Mean	8.36	11.64	9.11	10.22	9.96



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

μ :mean , σ standard deviation
μ =10,σ= 50

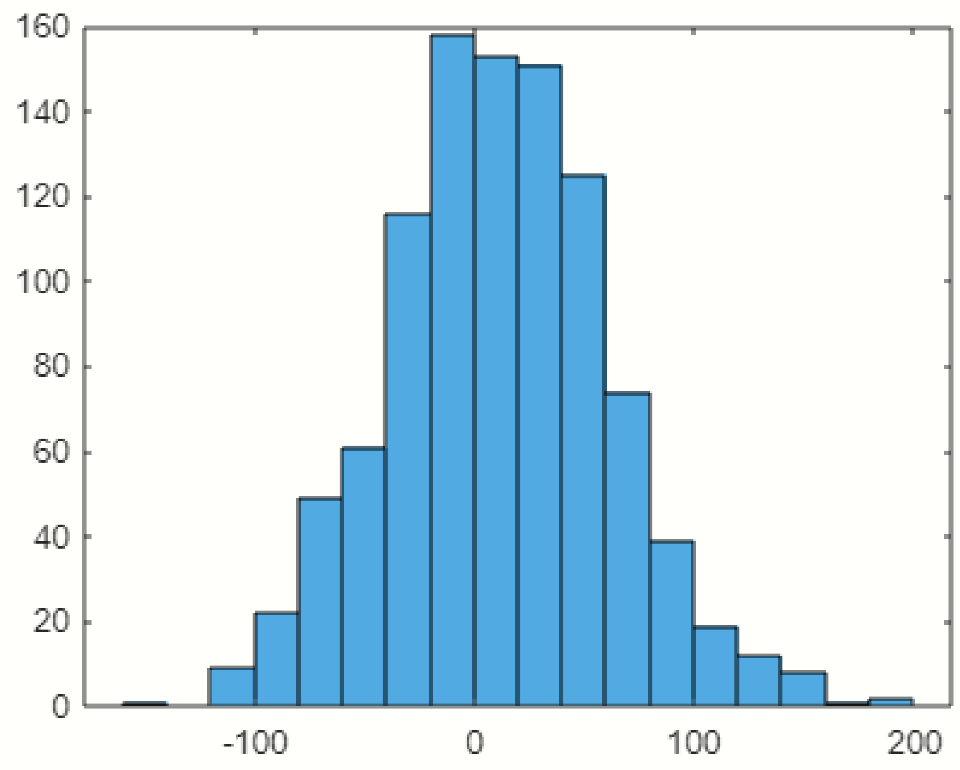
Estimation of parameter using sample from Gaussian Distribution

For estimation of variance:

Use

- **Var(X)= E(X^2)- {E(X)}^{2}**

- Estimation for 1000 sample: Var(X)= 2524



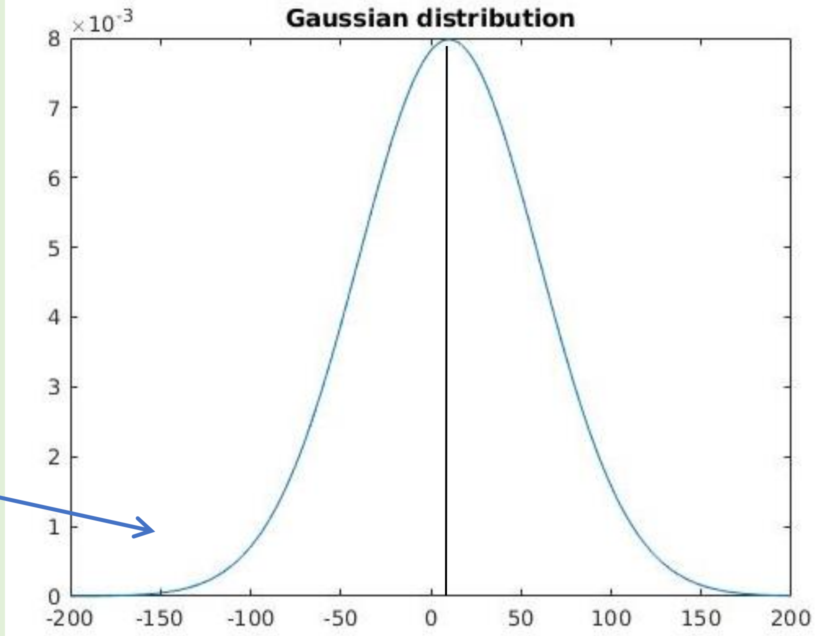
Histogram of 1000 samples

Sample size	1000	2000	5000	10000	15000
Variance	2524	2500	2489	2528	2513

Maximum Likelihood Estimation of parameter using sample from Gaussian Distribution

- Data sampled from Gaussian distribution.
- Used 1000 samples.
- To estimate Mean.
- We used s.d= 50 for the 100 samples.
- Likelihood function= $f(x_1,x_2,\ldots.x_n | \mu)$, where f(x) given by
- Assuming the samples are from i.i.d

We have $f(x_1| \mu)\ f(x_2| \mu) \ldots f(x_n| \mu)$



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

μ :mean , σ standard deviation

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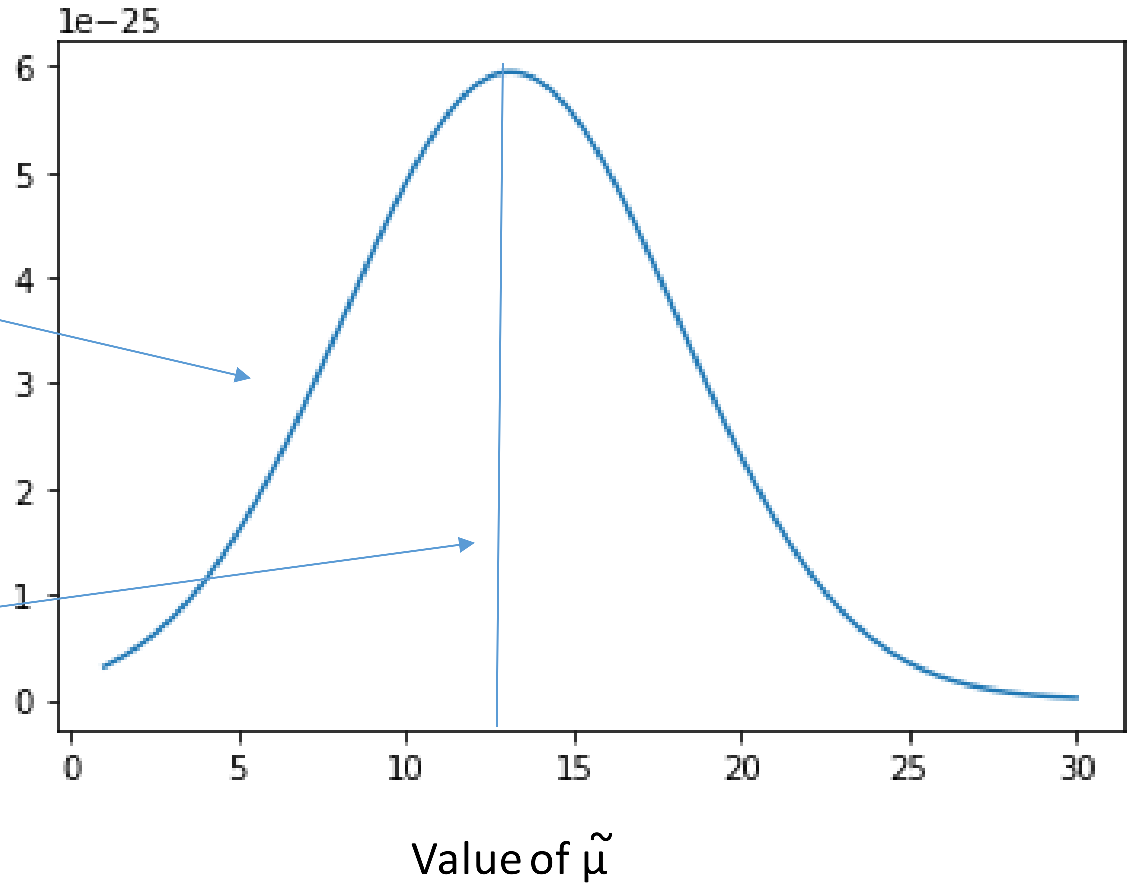
Maximum Likelihood Estimation of parameter using sample from Gaussian Distribution

- Sample mean=13.12
- $\tilde{\mu}$ =13.119, Likelihood estimation gives maximum value

Likelihood estimate of $\tilde{\mu}$ for 100 samples

$\tilde{\mu}$= 13.119



- To estimate population variance (σ^2)

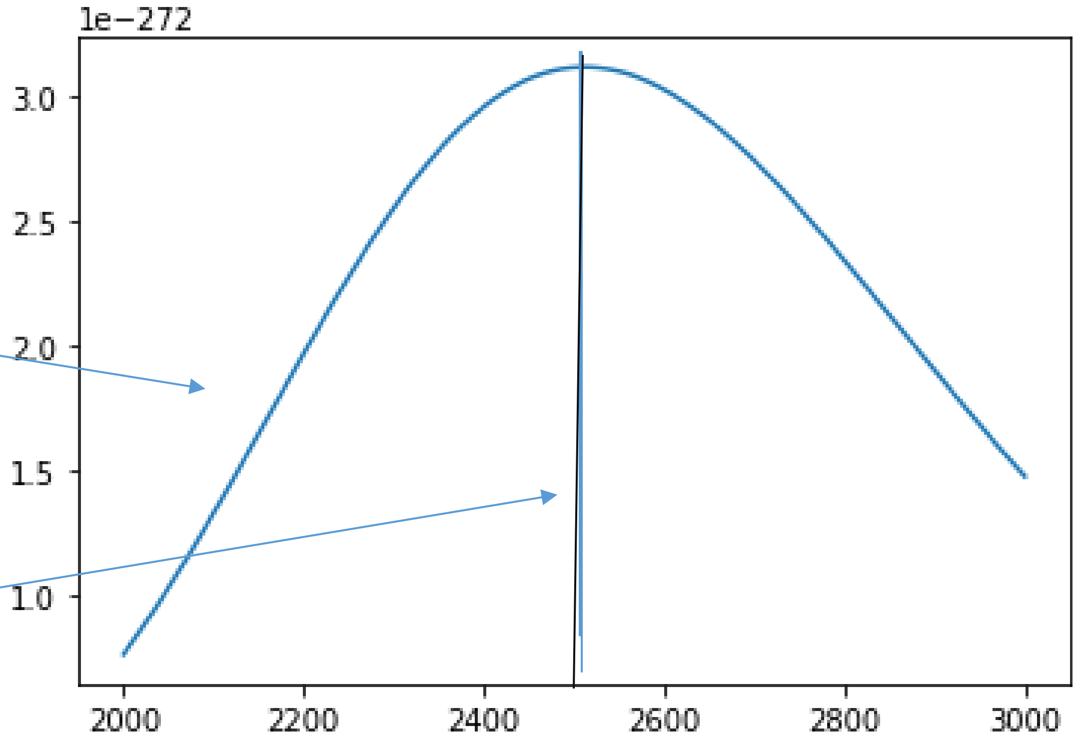
For the 100 samples,
taken, μ=10 (known)

Likelihood estimate
of σ^2 for 100 samples

$$\frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2 = 2501.66$$

- Likelihood function maximizes at variance value 2501.6.

2501.6



Values of variance