ASSIGNMENT – 1 D.E.

S20180010017
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Q.1) parabola with focus at origin

$\Rightarrow y^2 = 4a(x+a)$ — ①

D.E. → $y' \Rightarrow$?

diff. w.r.t x

$2y\frac{dy}{dx} = 4a \quad \Big| \quad a = \frac{2yy'}{4}$

replacing it in ①

$y^2 = \cancel{4}\left(\frac{2yy'}{\cancel{4}}\right)\left(x + \frac{2yy'}{4}\right)$

$\Rightarrow y^{\cancel{2}} = 2\cancel{y}.y'\left(x + \frac{\cancel{2}yy'}{2}\right)$

$$\boxed{y = \frac{dy}{dx}\left(2x + y\frac{dy}{dx}\right)}$$

Q.2.)

(a) $2x(e^{x^2}+y^2) = (1+2y)e^{-y}\frac{dy}{dx}$

$\int 2x(e^{x^2})dx = \int \frac{(1+2y)}{e^{y+y^2}}dy$

using substitutions
$K = e^{x^2}$, $dK = e^{x^2}2x$
$h = e^{-(y+y^2)}$, $dh = -e^{(-y-y^2)}(1+2y)$

$\Rightarrow \int dK = \int dH$

$K = H + C$

$\rightarrow \boxed{e^{x^2} + e^{-y-y^2} = c}$

(b) $(x-y)^2\frac{dy}{dx} = a^2 \rightarrow \frac{dy}{dx} = \frac{c^2}{(x-y)^2}$

let $v = x-y \rightarrow dv = 1 + dy$

$1 - v' = \frac{a^2}{v^2}$

$\frac{dv}{dx} = 1 - \frac{a^2}{v^2} \rightarrow \frac{dv}{dx} = \frac{v^2-a^2}{v^2} \rightarrow \int \frac{v^2}{v^2-a^2}dv = \int dx$

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Q.2(b) $\int 1 + \frac{a^2}{v^2 - a^2} \, dv = x + C_1$

$\Rightarrow v - \frac{a^2}{2a} \ln \left| \frac{a+v}{a-v} \right| = x + C$

$\int \frac{1}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x+a}{x-a} \right|$

replace $v = x - y$.

$$\boxed{x - y - \frac{a}{2} \ln \left| \frac{x-y+a}{a-x+y} \right| = x + C}$$

Q.3. $x \tan \frac{y}{x} + x \sec^2\left(\frac{y}{x}\right) \frac{dy}{dx} = y \sec^2 y/x$

let $v = y/x \Rightarrow dy = v + x\,dv$

$\cancel{x} \tan v + \cancel{x} \cancel{\sec^2(v)} (v + x\,dv) = \cancel{x} v \cancel{\sec^2(v)}$

~~K tan~~

$\tan v + x \sec^2(v)\, dv = 0$

let $K = x \tan v$

$dK = \tan v + x \sec^2(v)\, dv$

$\Rightarrow dK = 0$

$\int dK = 0 \Rightarrow K = C$

$\Rightarrow x \tan(v) = C$

replace v

$$\boxed{x \tan\left(\frac{y}{x}\right) = C}$$

Q.4) $\dfrac{(e^x+2e^y-3)\,e^x}{(2e^x+e^y-3)\,e^y} = \dfrac{dy}{dx}$

let $e^x = X \quad e^x dx = dX$

let $e^y = Y \quad e^y dy = dY$

- $\dfrac{dY}{dX} = \dfrac{X+2Y-3}{2X+Y-3}$ | $X' = X-h$, $Y' = Y'-K$ | $dX' = dX$, $dY' = dY$

$\dfrac{dY'}{dX'} = \dfrac{(X-1)+2(Y-1)}{2(X-1)+Y-1} \Rightarrow \dfrac{X'+2Y'}{2X'+Y'}$ | $h=1$, $K=1$

let $VX' = Y' \quad dY' = X'dV + V$

$V + X'\dfrac{dV}{dX'} = \dfrac{1+2V}{2+V}$

$X'\dfrac{dV}{dX'} = \dfrac{1+2V}{2+V} - V = \dfrac{1-V^2}{2+V}$

$\int \dfrac{2+V}{1-V^2}\,dV = \int \dfrac{dX'}{X'}$

$\dfrac{3}{2}\int\dfrac{dV}{1-V} + \dfrac{1}{2}\int\dfrac{dV}{1+V} = \int\dfrac{dX'}{X'}$

$\Rightarrow -\dfrac{3}{2}\log(1-V) + \dfrac{1}{2}\log(1+V) = \log X' + C$

$\dfrac{(1+V)}{(1-V)^3} = CX'^2$

replacing V

$\dfrac{1+\frac{Y'}{X'}}{\left(1-\frac{Y'}{X'}\right)^3} = CX'^2$

replacing X', Y'

$\dfrac{1+\frac{Y-1}{X-1}}{\left(1-\frac{Y-1}{X-1}\right)^3} = C(X-1)^2$

$\Rightarrow \boxed{\dfrac{1+\frac{(e^y-1)}{(e^x-1)}}{\left(1-\frac{(e^y-1)}{(e^x-1)}\right)^3} = C(e^x-1)^2}$ Ans.

Q. 5)

$$\frac{dx}{dy} + \frac{x}{(1+y^2)} = \frac{e^{\tan^{-1}y}}{1+y^2}$$

$$I.F. = e^{\int \frac{1}{(1+y^2)} dy} \Rightarrow (e^{\tan^{-1}y})$$

$$x(e^{\tan^{-1}y}) = \int \frac{e^{(\tan^{-1}y)^2}}{1+y^2} dy$$

$$\Rightarrow x e^{\tan^{-1}y} = \frac{e^{(\tan^{-1}y)^3}}{3} + c$$

$$\boxed{x = \frac{e^{(\tan^{-1}y)^2}}{3} + c} \text{ Ans.}$$

Q.6. $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)e^x \sec y$ } multiply by $\cos y$

let $\sin y = t \rightarrow \cos y \, dy = dt$

$$\frac{dt}{dx} - \frac{t}{(1+x)} = (1+x)e^x$$

$$I.F. \Rightarrow e^{\int \frac{-1}{1+x} dx} \Rightarrow e^{-\log(1+x)} = (1+x)^{-1}$$

$$t(1+x)^{-1} = \int e^x \, dx$$

$$t(1+x)^{-1} = e^x + c$$

$$t = \left(\frac{e^x}{\cancel{1+x}} + \frac{c}{\cancel{1+x}}\right)(1+x)$$

substitute back t

$$\sin y = \left(\frac{e^x}{\cancel{1+x}} + \frac{c}{\cancel{1+x}}\right)(1+x)$$

$$\sin y = (e^x + c)(1+x) \quad \underline{\text{Ans.}}$$

Q.7.)

$$y(y^3 - x) + x(y^3 + x)\frac{dy}{dx} = 0$$

divide by y^3

$$\frac{(y^3 - x)}{y^2} + \frac{x(y^3 + x)}{y^3}\frac{dy}{dx} = 0$$

$\downarrow$ M_x $\qquad$ $\downarrow$ N_y

$$\frac{\partial M_x}{\partial y} = \frac{\partial N_y}{\partial x} = \frac{2x + y^3}{y^3}$$

$$F(x,y) \Rightarrow \int M_x dx \rightarrow \frac{-x^2}{2y^2} + xy + C_1 = C_2$$

$$\boxed{\frac{-x^2}{2y^2} + xy = C}$$

Q.8.)

$$(2xy^2 + y)dx + (x + 2x^2y - x^4y^3)dy = 0$$

$$(2xy + 1)y\,dx + (1 + 2xy + x^3y^3)x\,dy = 0$$

$$I.F \Rightarrow \frac{1}{Mx - Ny} \Rightarrow \frac{1}{xy(2xy + 1 - 2xy + x^3y^3 - 1)} = \frac{1}{x^4y^4}$$

$$\frac{(2xy^2 + y)\,dx}{x^4y^4} + \frac{(x + 2x^2y + x^4y^3)\,dy}{x^4y^4} = 0$$

$$\left(\frac{2}{x^3y^2} + \frac{1}{x^4y^3}\right)dx + \left(\frac{2}{x^2y^3} - \frac{1}{4} + \frac{1}{x^3y^4}\right)dy = 0$$

$\downarrow$ M_x $\qquad$ $\downarrow$ N_y

$$\frac{\partial M}{\partial y} \Rightarrow \frac{-4}{x^3y^3} - \frac{3}{x^4y^4}, \quad \frac{\partial N}{\partial x} = \frac{-4}{x^3y^3} + \frac{-3}{x^4y^4}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Q.8.

contd...

$$M_x = \frac{2}{x^3y^2} + \frac{1}{x^4y^3}$$

constant only (no - x)

$$\int \frac{2}{x^3y^2} + \frac{1}{x^4+y^3}\,dx + \int \frac{1}{y}\,dy = 0$$

y = constant

$$\frac{2}{(-2)\,x^2y^2} + \frac{1}{(-3)\,(x^3y^3)} - \ln y = c$$

$$\boxed{-\frac{1}{(xy)^2} + \frac{1}{3(xy)^3} + \ln y = c}$$

Q.9.)

$x^3y^3(2y\,dx + x\,dy) - (-5y\,dx + 7x\,dy) = 0$

$y(2x^3y^3 - 5)\,dx + x(x^3y^3 - 7)\,dy = 0$

$f(x,y).y.dx + f_2(x,y).x\,dy = 0$.

$I.F. \Rightarrow \frac{1}{M_x - N_y}$ if $M_x \neq N_y$.

$M_x - N_y \Rightarrow 2x^4y^4 - 5xy - x^4y^4 + 7xy$

$IF \Rightarrow \frac{1}{x^4y^4 + 2xy}$

$$\frac{(2x^3y^3-5)\cancel{y}\,dx}{(x^3y^3+2)x\cancel{y}} + \frac{(x^3y^3-7)\cancel{x}\,dy}{(x^3y^3+2)\cancel{x}y} = 0.$$

$$dx\frac{(\cancel{2}x^3y^3 + 2 + x^3y^3 - 7)}{(x^3y^3+2)x} + \frac{(x^3y^3-7)}{(x^3y^3+2)y}dy = 0$$

$$\frac{dx}{x} + \left(\frac{x^3y^3-7}{x^3y^3+2}\right)\left(\frac{dy}{y} + \frac{dx}{x}\right) = 0$$

$$\frac{dx}{x} + \left(\frac{x^3y^3-7}{x^3y^3+2}\right)\left(\frac{x\,dy + y\,dx}{xy}\right) = 0$$

let $v = xy$
$dv = x\,dy + y\,dx$.

$$\frac{dx}{x} + \left(\frac{v^3-7}{v^3+2}\right)\frac{dv}{v} = 0$$

$$\int\frac{dx}{x} + \int\frac{dv}{v}\left(\frac{v^3-7}{v^3+2}\right) = 0$$

Using partial fraction

$$\frac{t^3-7}{t(t^3+2)} = \frac{A}{t} + \frac{Bt^2+Ct+D}{t^3+2}$$

$t^3 = At^3 + 2A + Bt^3 + Ct^2 + Dt$

$A + B = 1 \quad 2A = -7,\ C = 0, D = 0$

$A = -7/2 \quad B = 9/2$

$$\ln(x) + -\frac{7}{2}\int\frac{dv}{v} + \frac{9}{2}\int\frac{t^2}{t^3+2}\,dt = 0$$

$$\ln x - \frac{7}{2}\ln(xy) + \frac{9}{2}\ln((xy)^3+2) = c$$

Q.10) $(x^2+x)p^2 + (x^2+x-2xy-y)p + y^2-xy = 0$, $p = \frac{dy}{dx}$

$x(x+1)p^2 + x(x+1)p - xyp - xyp - yp + y(y-x) = 0$

$x(x+1)p^2 + x(x+1)p - (x+1)yp - xyp - xy + y^2 = 0$

$(x+1)P[xp + x - y] - y[xp + x - y]$

$[(x+1)P - y][xp + x - y] = 0$

(i) $y = (x+1)P \rightarrow \frac{dy}{dx} = \frac{y}{x+1}$

$\Rightarrow \frac{dy}{y} = \frac{dx}{x+1}$

$\int \frac{dy}{y} = \int \frac{dx}{x+1}$

$\log y = \log x+1 + c$

$\Rightarrow \boxed{\left(\frac{y}{x+1}\right) = c}$

(ii) $y = x(P+1)$

$1 + \frac{dy}{dx} = \frac{y}{x}$

$\frac{dy}{dx} = \frac{y-x}{x}$

let $vx = y$ $\quad v + x\,dv = dy$

$\cancel{v} + x\frac{dv}{dx} = \frac{\cancel{x}v - \cancel{x}}{\cancel{x}} = \cancel{v} - 1$

$x\frac{dv}{dx} = -1$

$\int dv = -\int \frac{dx}{x}$

$\Rightarrow v = -\log(x) + c$

$\boxed{\frac{y}{x} + \log(x) = c}$

Q.11

$$4y^2 + p^3 = 2xyp \quad , \quad p = \frac{dy}{dx}.$$

$$2x = \frac{4y^2}{yp} + \frac{p^2}{y} \Rightarrow \frac{p^2}{y} + \frac{4y}{p}$$

diff. w.r.t y.

$$2\frac{dx}{dy} = -\frac{p^2}{y^2} + \frac{2p}{y}\frac{dp}{dy} + \frac{4}{p} - \frac{4y}{p^2}\frac{dp}{dy}$$

$$\frac{2}{p} = \frac{2dp}{dy}\left(\frac{p}{y} - \frac{2y}{p^2}\right) + \frac{4}{p} - \frac{p^2}{y^2}$$

$$\frac{2dp}{dy}\left(\frac{p}{y} - \frac{2y}{p^2}\right) + \left(\frac{2}{p} - \frac{p^2}{y^2}\right) = 0$$

$$\frac{2dp}{dy}\left(\frac{\cancel{p^3 - 2y^2}}{p^2 \cancel{y}}\right) = \left(\frac{\cancel{p^3 - 2y^2}}{py^{\cancel{2}}}\right)$$

$$\Rightarrow \frac{2dp}{dy}\frac{1}{p} = \frac{1}{y}$$

$$2\int \frac{dp}{p} = \int \frac{dy}{y}$$

$$\Rightarrow 2\log p = \log y = c$$

$$\boxed{\frac{p^2}{y} = c} \text{ Ans.}$$

Q.12)

$$p^2y + px^3 - x^2y = 0 \quad , x^2 = u \quad , y^2 = v, \quad p = \frac{dv}{du}$$

$du = 2x\,dx$ — ①

$dv = 2y\,dy$ — ②

using ① and ②:

$$P = \frac{dy}{dx} = \sqrt{\frac{u}{v}}\left(\frac{dv}{du}\right)$$

replacing P in original

$$\left(\sqrt{\frac{u}{v}}\frac{dv}{du}\right)^2 y + \frac{\sqrt{u}\,dv}{\sqrt{v}\,du} x^3 - x^2y = 0$$

$$\left(\frac{dv}{du}\right)^2 \times \frac{u}{v} + \frac{u^2}{\sqrt{v}}\left(\frac{dv}{du}\right) - u\sqrt{v} = 0$$

$$\rightarrow \left(\frac{dv}{du}\right)^2 + u\left(\frac{dv}{du}\right) - v = 0 \quad \Big| \text{ removing } \left(\frac{u}{\sqrt{v}}\right)$$

let $t = \dfrac{dv}{du}$

$$t^2 + ut - v = 0$$

$\boxed{\text{(i) solvable for } v \Rightarrow y^2.}$

$v = ut + t^2$

diff. w.r.t. u

$$\boxed{\frac{dv}{du} = 2t\frac{dt}{du} + t + u\frac{dt}{du}} \quad \text{since } \frac{dv}{du} = t$$

$$\cancel{t} = 2t\frac{dt}{du} + u\frac{dt}{du} + \cancel{t}$$

$$(2t + u)\frac{dt}{du} = 0$$

(i) $\dfrac{dt}{du} = 0$

$\rightarrow t = c$

$$\frac{dv}{du} = c \rightarrow v = uc_1 + c_2 \rightarrow \boxed{y^2 = c_1x^2 + c_2}$$

(ii) $2t + u = 0$

$$\frac{dv}{du} = \frac{-u}{2}$$

$$\int dv = -\int \frac{u}{2}\, du$$

$$v = -\frac{u^2}{4} + c$$

$$\boxed{4v^2 + u^2 = c}$$

$$\boxed{4y^4 + x^4 = c}$$ Ans.

Q. 13.)

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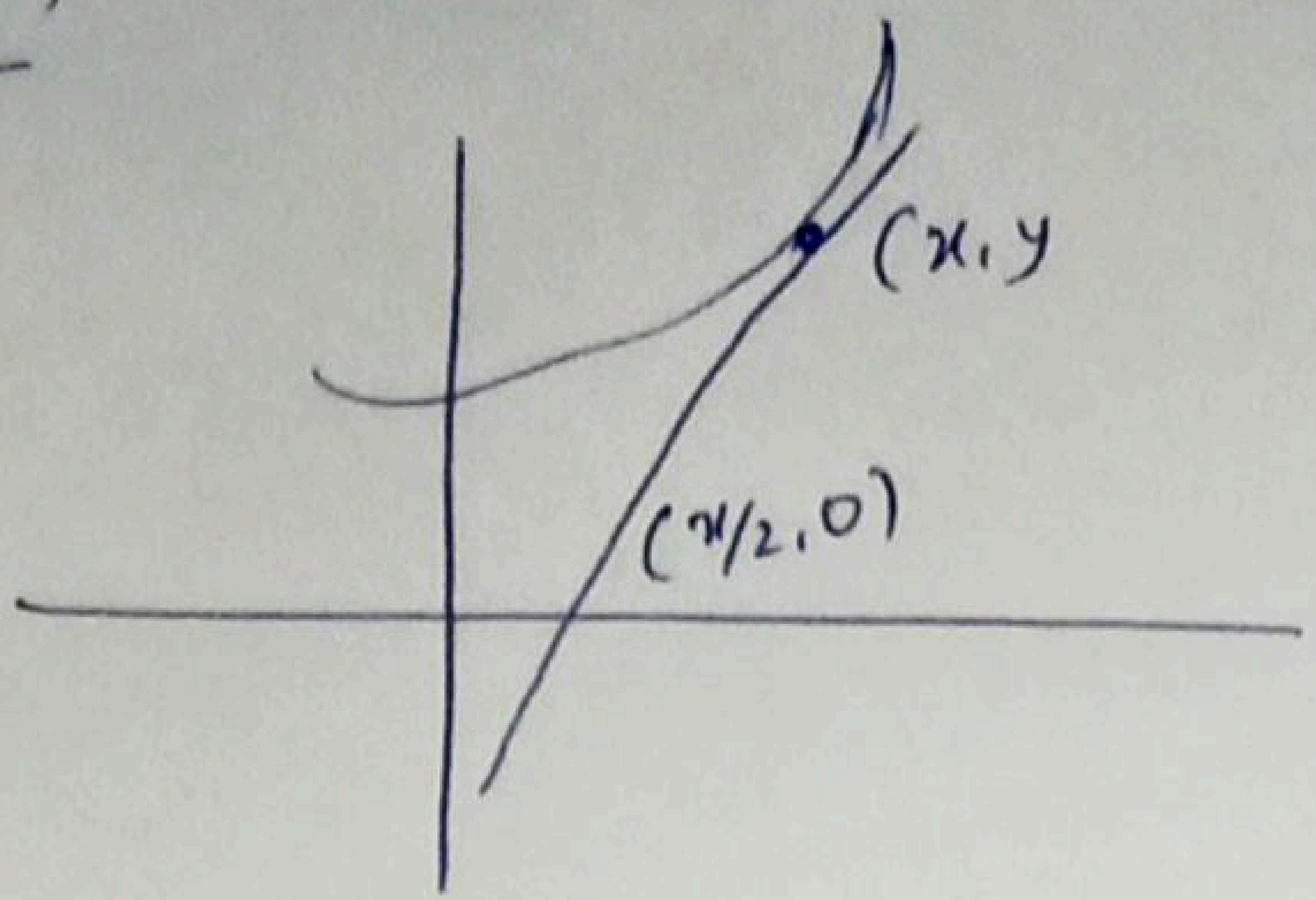
Q.14)



$$\frac{dy}{dx} = \frac{(y-0)}{(x - x/2)} = \frac{2y}{x}. \quad \Big| \; y(1) = 2$$

$$\int \frac{dy}{y} = 2\int \frac{dx}{x}$$

$$\Rightarrow \log(y) = 2\log(x) + c$$

$$y = cx^2 \quad \Big| \; y = 2, x = 1$$

$$\boxed{c = 2}$$

final equation $\Rightarrow \boxed{y = 2x^2}$

Q.15) Given Rate $\propto$ Infected, Rate $\propto$ Healthy.

$$\frac{dy}{dt} = y(N-y) \quad \Big| \; N = 4000$$

$$\frac{dy}{dt} = y(4000-y)$$

$$\int \frac{\partial y}{y(4000-y)} = \int dt.$$

$$\int \frac{1}{4000y} dy + \int \frac{1}{4000(4000-y)} dy = \int dt$$

using partial fractions

$$\frac{A}{y} + \frac{B}{4000-y} = \frac{1}{y(4000-y)}$$

$$A(4000) - Ay + By = 1$$

$$A - B = 0 \quad A = B$$

$$A \times 4000 = 1$$

$$A = \frac{1}{4000} = B.$$

$$\frac{1}{N}\int \frac{dy}{y} + \frac{1}{N}\int \frac{1}{N-y}\, dy = \int dt$$

$\Rightarrow$

$$\frac{1}{n}\log(y) + \frac{-1}{N}\log(N-y) = t$$

$$\log\left(\frac{y}{N-y}\right) = Nt$$

$$\frac{y}{N-y} = e^{Nt} + c$$

given $\rightarrow$ $t = 5,\ y = 100$

$$\frac{100}{4000-100} = e^{20000} + c$$

$$\boxed{C = \frac{1}{39} - e^{20000}}$$

eq^n $\rightarrow$ $\frac{y}{N-y} = e^{Nt} - e^{20000} + \frac{1}{39}$

find y when $\underline{t = 10}$.

$$\frac{y}{N-y} = e^{20000}(e^{20000} - 1) + \frac{1}{39}$$

$$\frac{y}{N-y} = \left(\frac{e^{20000}(e^{20000}-1) + \frac{1}{39}}{1}\right) \approx \underline{\text{unable to compute}}.$$

$$y \Rightarrow \frac{N\left(e^{20000} \times (e^{20000} - 1) + \frac{1}{39}\right)}{\left(e^{20000}(e^{20000} - 1) + \frac{1}{39} + 1\right)}$$

$\rightarrow$ extreamely long

$y \approx N = \underline{4000}$ students.