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Lec 11

Montgomery Book Useful

Test of Assumption

① Homoscedasticity. — $var(\hat{\epsilon}) = \sigma^2$ constant

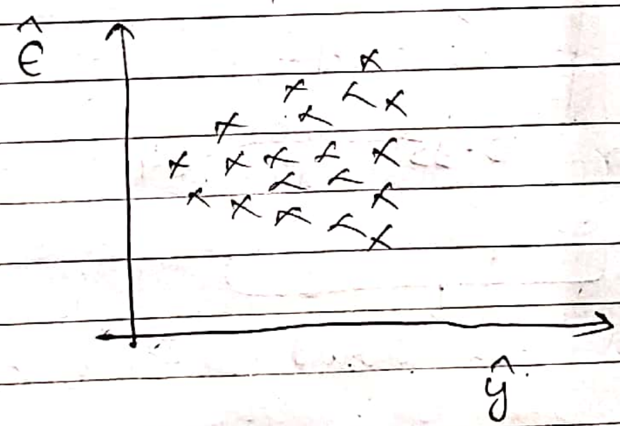
$Y = X\beta + \epsilon$

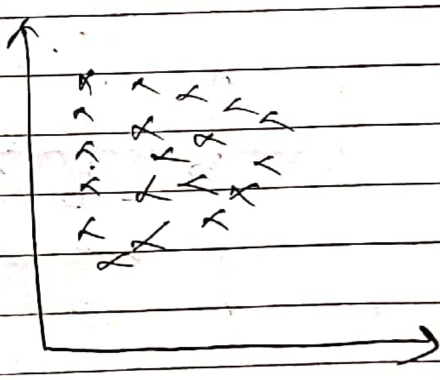
② Normality — $\epsilon_i \sim N(0, \sigma^2)$

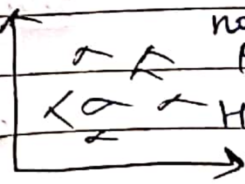
③ Uncorrelated Error. — $Cov(\epsilon_i, \epsilon_j) = 0 \quad i \neq j$

④ Independence of regressors

Test







Remedy

Variable Transformation $f(y) = X\beta + \epsilon$

y^2

$\sqrt{y}$

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<u>Normality</u> Q-Q plot | P-P plot

$x_1, x_2, \ldots x_n$

$x_{(1)}, x_{(2)}, \ldots, x_{(n)}$

j	$x_{(j)}$	$\frac{j-0.5}{n}$	z_j
1	$x_{(1)}$	0.05	-1.64
2	$x_{(2)}$	0.15	-1.036
3			
4			
⋮			
n			
10	$x_{(10)}$	0.95	1.64

Plot should be straight line

<u>Remedy</u> Transformation of y

Box cox method.

$$y^{(\lambda)} = \begin{cases} \dfrac{y^{\lambda}-1}{\lambda \cdot \dot{y}^{\lambda-1}}, & \lambda \neq 0 \\ \dot{y} \ln y, & \lambda = 0 \end{cases} \qquad \dot{y} = \ln^{-1}\left[\left(1/n \sum_1^n \ln y_i\right)\right]$$

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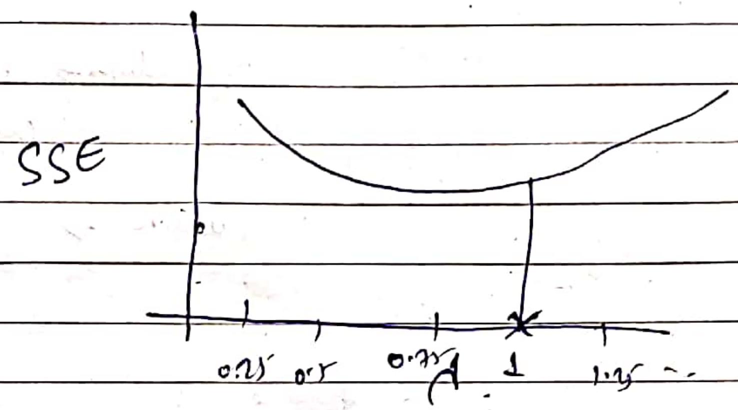


Test of auto Correlation (Durbin Watson test)
DW test

$$DW = 2(1-r)$$

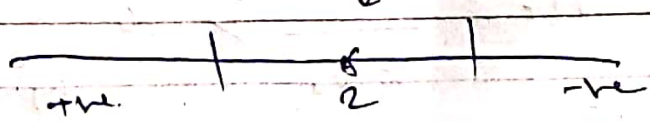
$$\hat{r} = \frac{\sum_{i=2}^{n} \hat{e}_i \hat{e}_{i-1}}{\sum_{i=1}^{n} \hat{e}_i^2}$$

$H_0: r = 0$
$H_1: r \neq 0$

$r = 0 \Rightarrow DW = 2 \rightarrow$ no "
$r > 0 \Rightarrow DW < 2 \rightarrow$ +ve Correlation
$r < 0 \Rightarrow DW > 2 \rightarrow$ -ve "

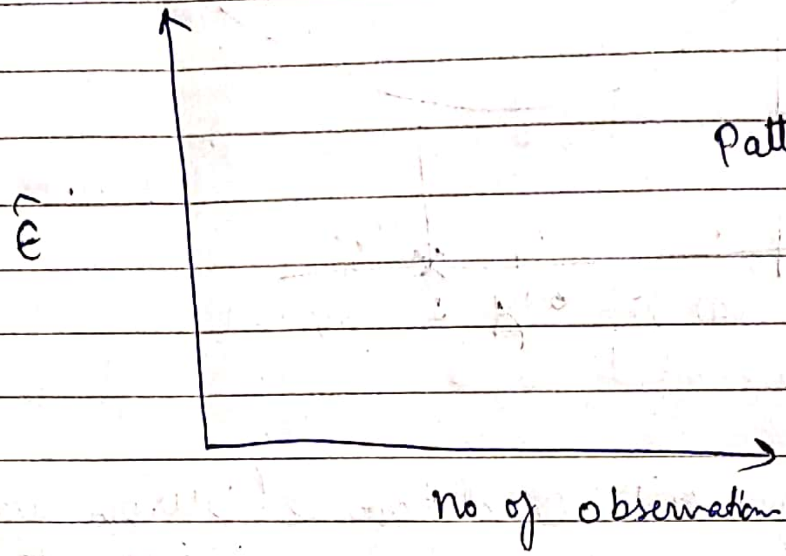
$r = 0.9$, $DW = 2 \times 0.1 = 0.2$

no autocorrelation



random → No autocorrelation

Pattern → yes.



Remedy: Not in the scope of syllabus.

Multicollinearity:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_r X_r + \epsilon$$

VIF - Variance Inflation factor

$$X_j \leftarrow \underline{X_1, X_2, \dots, X_{j-1}, X_{j+1}, \dots X_r}$$

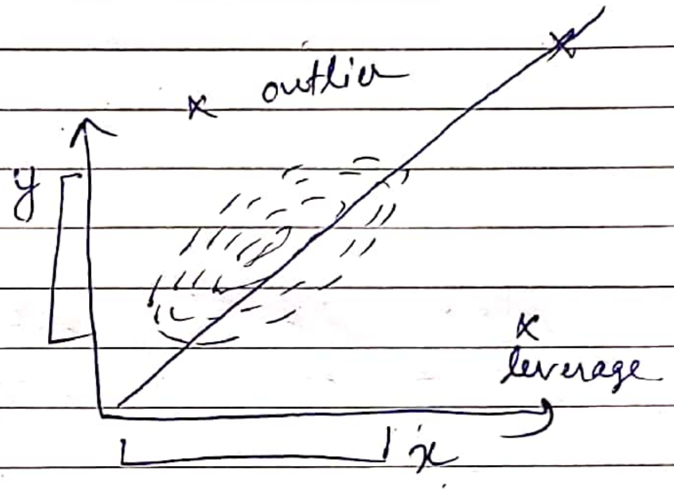
R_j^2

$$VIF = \frac{1}{1 - R_j^2} > 10 \qquad R_j^2 = 0.9$$

$X_1 \rightarrow VIF$
$X_2 \rightarrow VIF$
⋮
$X_r \rightarrow VIF$

Remedies:- PCA

Influencial Points



Outlier.

Outside the range of y

leverage

Outside the range of x

$$H = X(X^T X)^{-1} X^T$$

Summation is close to $r+1$

$$\begin{bmatrix} h_{11} & & & \\ & h_{22} & & \\ & & \ddots & \\ & & & h_{nn} \end{bmatrix}$$

$$h_{ii} > 2(r+1)/n$$

i ← leverage Point

Cook's distance