# Statistical Thinking and Probability Review

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# Statistics and Modeling

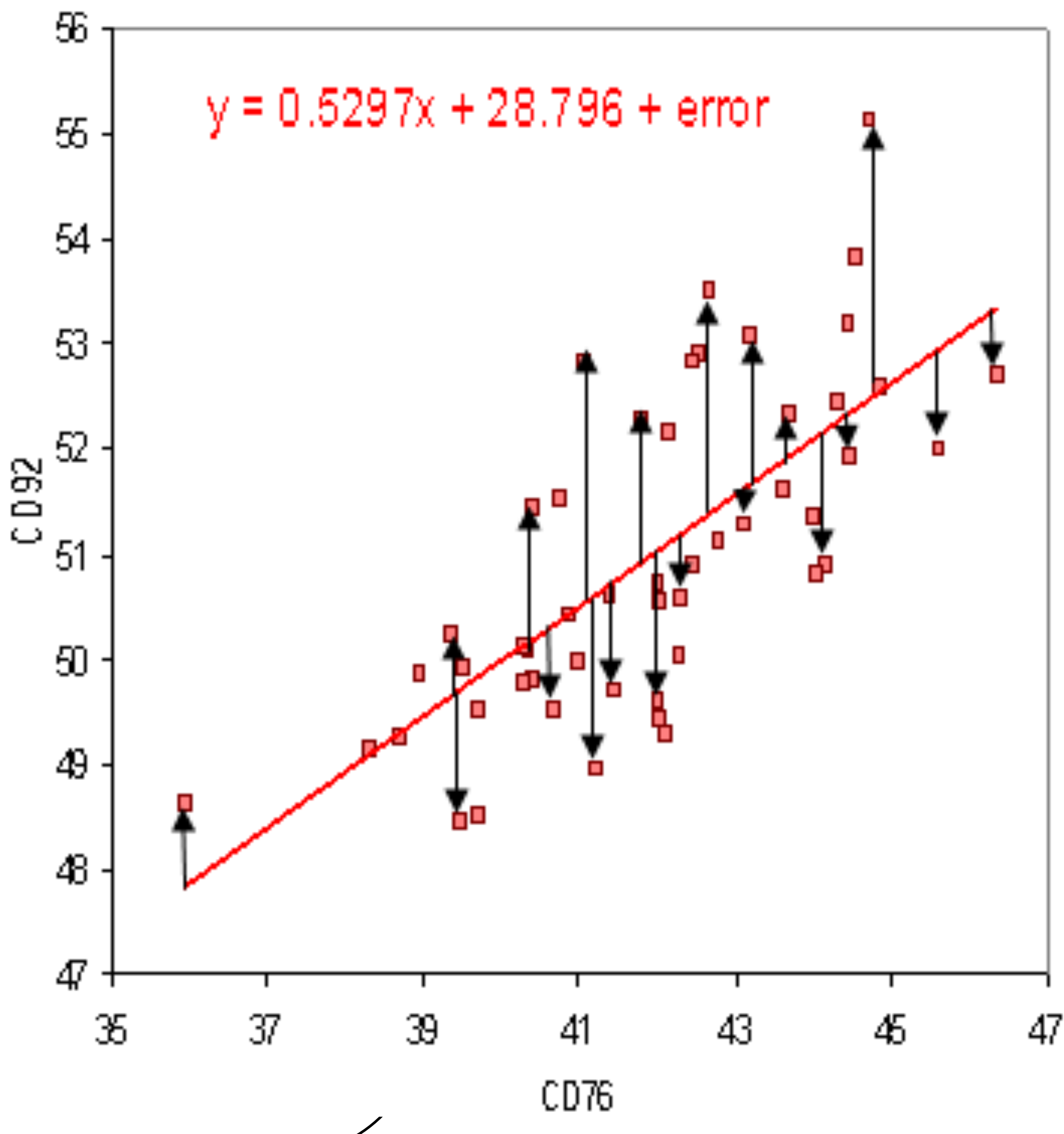
Most of the real life phenomenon are having more complicated processes? Need to estimate parameters from data. This is statistics.

Sometimes deterministic but too complex phenomenon

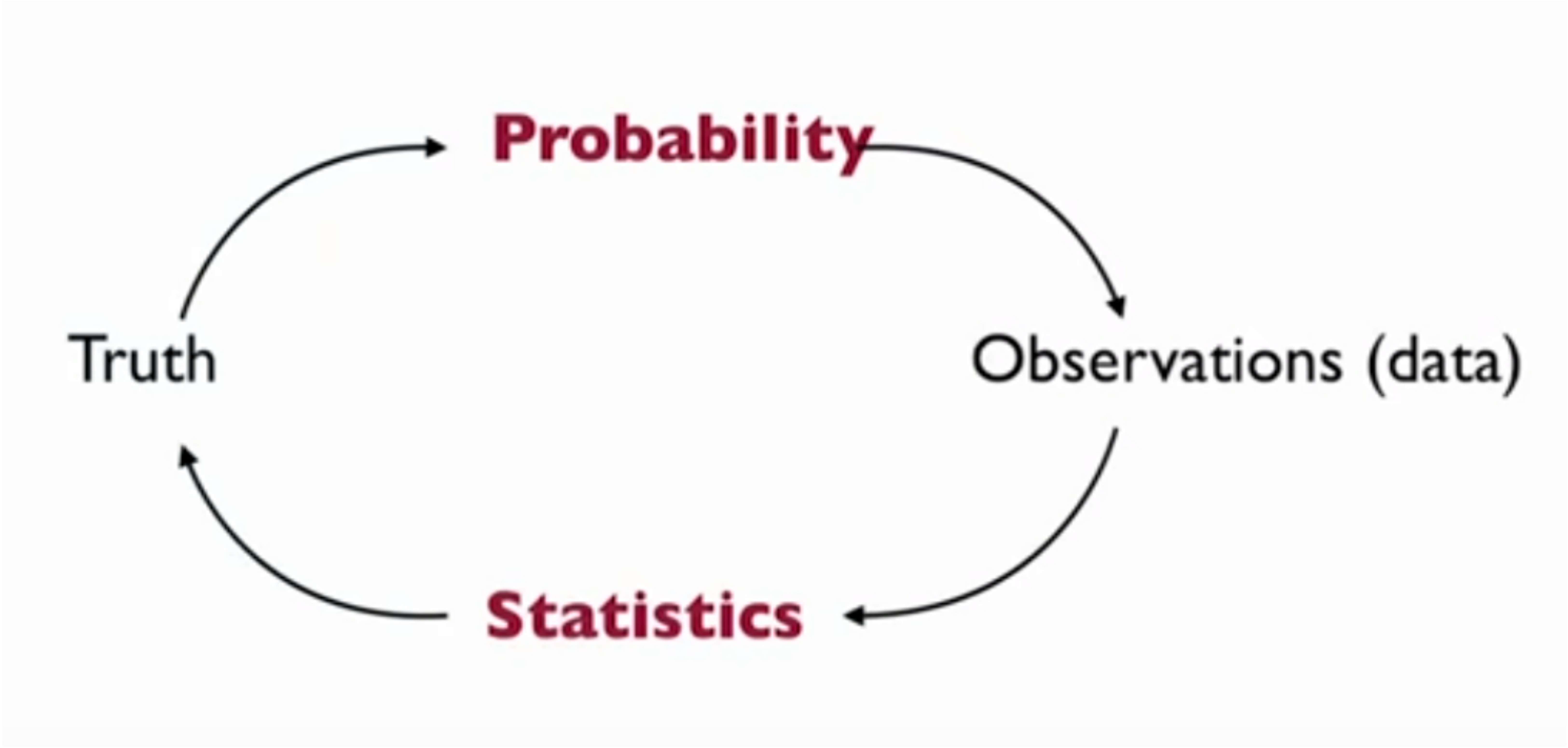
Statistical Modeling:

Complicated process = simple process + random noise.

Modeling consists in choosing simple process and noise distribution.



# Statistics vs Probability




Philippe Rigollet, MIT Open Courseware

# Statistics vs Probability

## Probability

Previous studies showed that the drug was 80% effective. Then we can anticipate for a study of 100 patients, in average 80 will be cured.

## Statistics

Observe that 78/100 patients were cured. We (will be able to) conclude that we are 95% confident that for other studies the drug will be effective on between 69.88% and 86.11% of patients.

# Why study statistics?

1. Data are everywhere
2. Statistical techniques are used to make many decisions that affect our lives
3. No matter what your career, you will make professional decisions that involve data. An understanding of statistical methods will help you make these decisions effectively

# Applications of statistical concepts in the business world

- Finance – correlation and regression, time series analysis
- Marketing – hypothesis testing, chi-square tests, nonparametric statistics
- Operating  management – hypothesis testing, estimation, time series analysis
- Research – Based on data

# Statistics

- The science of collecting, organizing, presenting, analyzing, and interpreting data to assist in making more effective decisions
- Statistical analysis – used to manipulate, summarize and investigate data, so that useful decision-making information results.

# Types of statistics

- **Descriptive statistics** – Methods of organizing, summarizing, and presenting data in an informative way
- **Inferential statistics** – The methods used to determine something about a population on the basis of a sample
  - Population –The entire set of individuals or objects of interest or the measurements obtained from all individuals or objects of interest
  - Sample – A portion, or part, of the population of interest

# Probability Review

# Notations

- Outcome Space/Sample Space ($\Omega$) - set of all possible outcomes ($\omega$).

Example: Tossing of two coins {HH,HT,TH,TT}

Life of a bulb $[0, \infty)$

- Event: Any subset of the sample space $\Omega$.

Example: Roll odd number in a die {1,3,5}

# Probability Axioms

Given a sample space S and an associated sigma algebra B, a probability function is a function P with domain B that satisfies

1. $P(A) \geq 0$ for all $A \in B$.
2. $P(\Omega)=1$.
3. If $A_1, A_2, \ldots \in B$ are pairwise disjoint, then $P(\cup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$

We can derive corollaries from the axioms:

$P(\phi)=0$

If $E \subseteq F$, then $P(E) \leq P(F)$

For any $A \subseteq \Omega$, $0 \leq P(A) \leq 1$

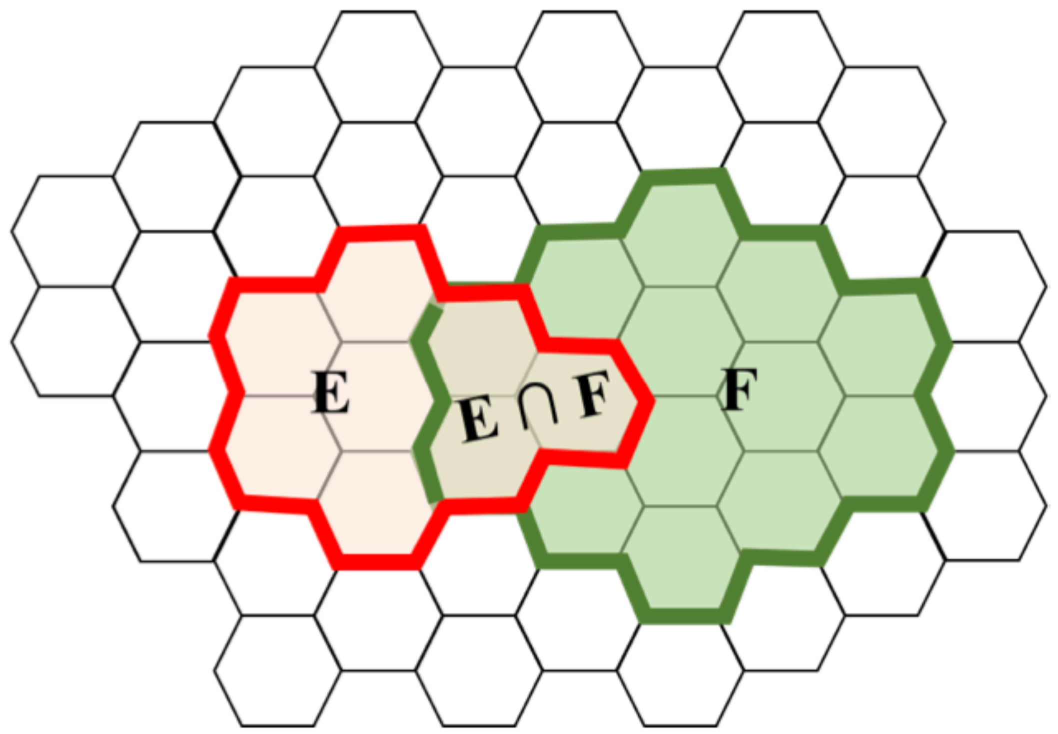
$A \cap B = \phi \Rightarrow P(A \cup B) = P(A) + P(B)$.

Ref: http://www.math.ntu.edu.tw/~hchen/teaching/StatInference/notes/lecture2.pdf
https://www2.stat.duke.edu/~cr173/Sta102_Sp16/Lec/Lec3.pdf

# Conditional Probability

- The conditional probability of event E given that event F has already happened is:

$P(E/F) = P(E \cap F)/ P(F), P(F)>0$




A is independent of B if $P(A/B)=P(A)$.

Disjointness and independence are different: If disjoint, $P(A \cap B)=0$.

Bayes Rule: $$P(A|B) = \frac{P(B|A)\cdot P(A)}{\sum_i P(B|A_i)\cdot P(A_i)}$$

Ref: https://ocw.mit.edu/courses/sloan-school-of-management/15-075j-statistical-thinking-and-data-analysis-fall-2011/lecture-notes/MIT15_075JF11_chpt02.pdf
https://web.stanford.edu/class/archive/cs/cs109/cs109.1178/lectureHandouts/040-cond-probability.pdf

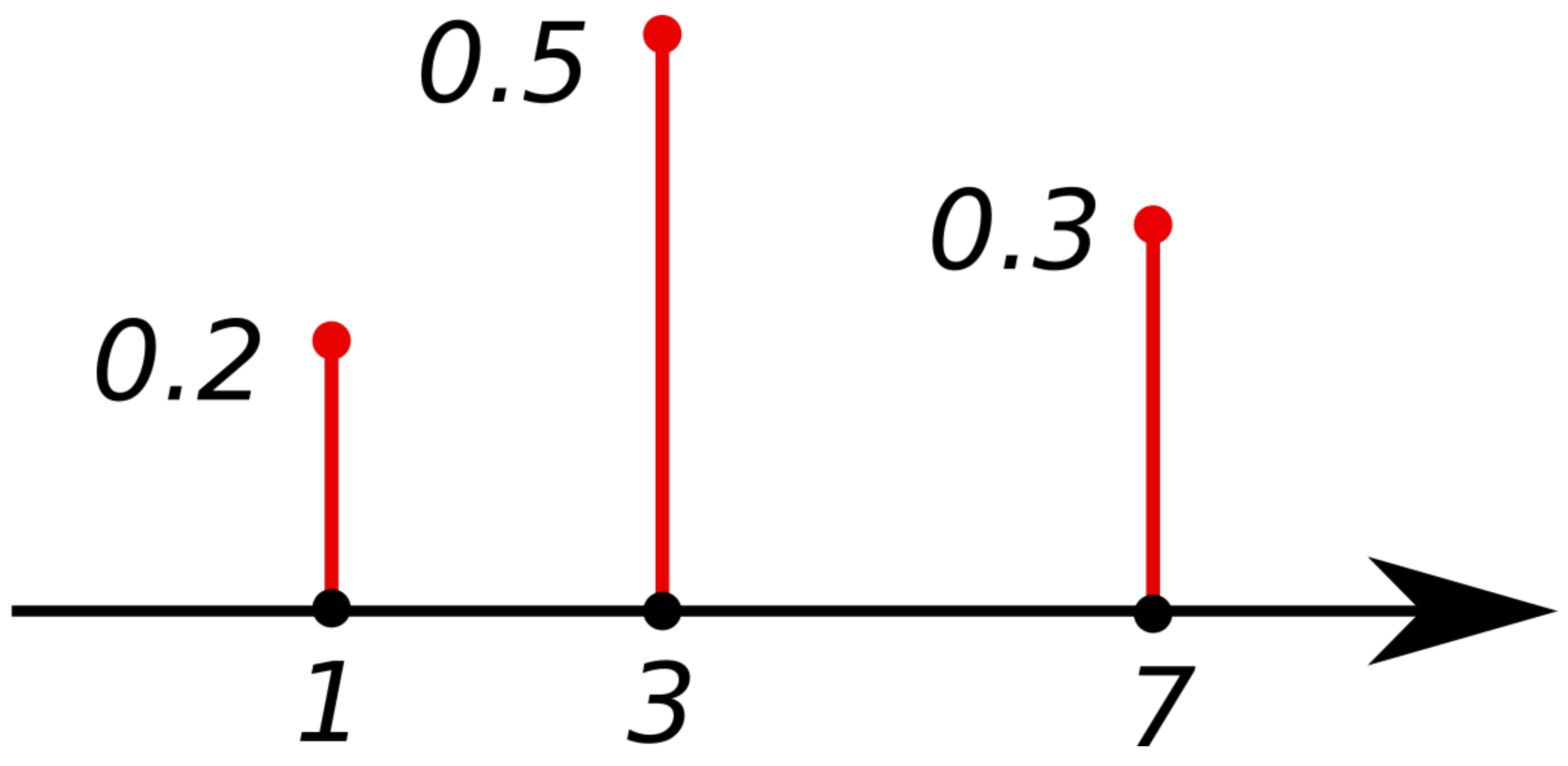
# Random Variables

A ***random variable***, usually written $X$, is a real valued function defined over sample space $\Omega$. It assigns numerical values to each outcome.
For example, X is number of toss before you get two consecutive heads. X can take values 0, 1, 2, . . .

Two Types: Discrete and Continuous
pmf pdf

Probability mass function: f(x) = P(X = x)

Probability density function: P(a ≤ X ≤ b) = $\int_a^b f(x)dx$



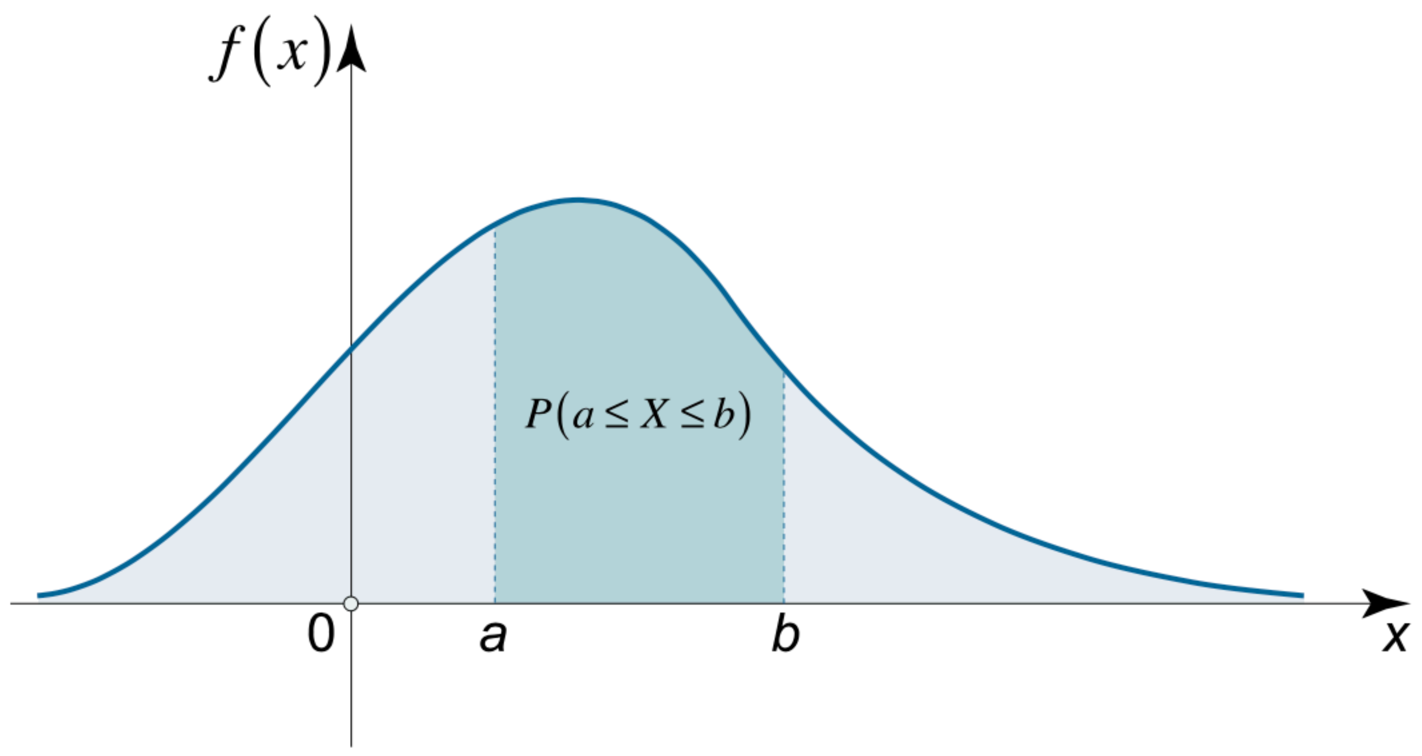



Figure 1.

Ref: wikipedia
math24.net

# Properties

Both discrete and continuous r.v.s have ***cumulative distribution function***. It provides the probability that the random variable *X* is less than or equal to any value *x*.

$F(x) = P(X \leq x)$

Expectation:
The expected value of a random variable X is E(X).

$$\begin{aligned} \mathbf{E}(X) &= \mu = \sum_x x f(x) \quad \text{(discrete)} \\ &= \int_x x f(x) dx \quad \text{(continuous).} \end{aligned}$$

Variance:
The expected value of the squared deviation from the mean of X, E(X)=μ.

$$\operatorname{Var}(X) = \mathrm{E}\left[(X - \mu)^2\right]$$

Ref: https://ocw.mit.edu/courses/sloan-school-of-management/15-075j-statistical-thinking-and-data-analysis-fall-2011/lecture-notes/MIT15_075JF11_chpt02.pdf
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