Overview of Sampling Theory: Estimation

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Sampling Distribution

- Consider the probability distribution of a statistic T (Y_1, Y_2, . . . , Y_n) over repeated sampling. This probability distribution is called the sampling distribution of T .

- Let's consider the sample mean $\bar{Y}$. We know that for a population its mean μ is fixed and unknown. But the sample mean $\bar{Y}$ won't be the same for all different possible samples that can be drawn from the population. Although the characteristics of $\bar{Y}$ can be theoretically derived over samples by its probability distribution. This probability distribution of $\bar{Y}$ is called the sampling distribution of the statistic i.e. sample mean $\bar{Y}$.

Ref: http://mgmt.iisc.ac.in/CM/LectureNotes/statistical_inference_frequentist.pdf

Example

Example 1: Consider a population consisting of only four numbers 1, 2, 3 and 4 with 30% 1's, 40% 2's, 20% 3's and the remaining 10% 4's. That is the probability distribution in the population may be expressed in terms of the p.m.f. $p_Y(y)$ of the population random variable Y as follows:

y	1	2	3	4
$p_Y(y)$	0.3	0.4	0.2	0.1

Now consider drawing a random sample of size 2 from this population and the three statistics $\overline{Y} = (Y_1 + Y_2)/2$, $s_2^2 = 1/2 \sum_{i=1}^{2}(Y_i - \overline{Y})^2$ and $s_1^2 = \sum_{i=1}^{2}(Y_i - \overline{Y})^2$, where Y_1 and Y_2 are the two observations. The sampling distributions of these three statistics can be figured out by considering all possible samples of size 2 that can be drawn from this population, the corresponding probabilities of drawing each such sample, and the values of each of the statistics for every such sample

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Example

Possible Samples	{1,1}	{1,2}	{1,3}	{1,4}	{2,2}	{2,3}	{2,4}	{3,3}	{3,4}	{4,4}
Probability	0.09	0.24	0.12	0.06	0.16	0.16	0.08	0.04	0.04	0.01
$\overline{Y}$	1	1.5	2	2.5	2	2.5	3	3	3.5	4
s_2^2	0	0.25	1	2.25	0	0.25	1	0	0.25	0
s_1^2	0	0.5	2	4.5	0	0.5	2	0	0.5	0

Sampling Distribution of the Sample Mean $\overline{Y}$

$\overline{y}$	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$p_{\overline{Y}}(\overline{y})$	0.09	0.24	0.28	0.22	0.12	0.04	0.01

Sampling Distribution of s_1^2

s_2^2	0	0.5	2	4.5
prob.	0.30	0.44	0.20	0.06

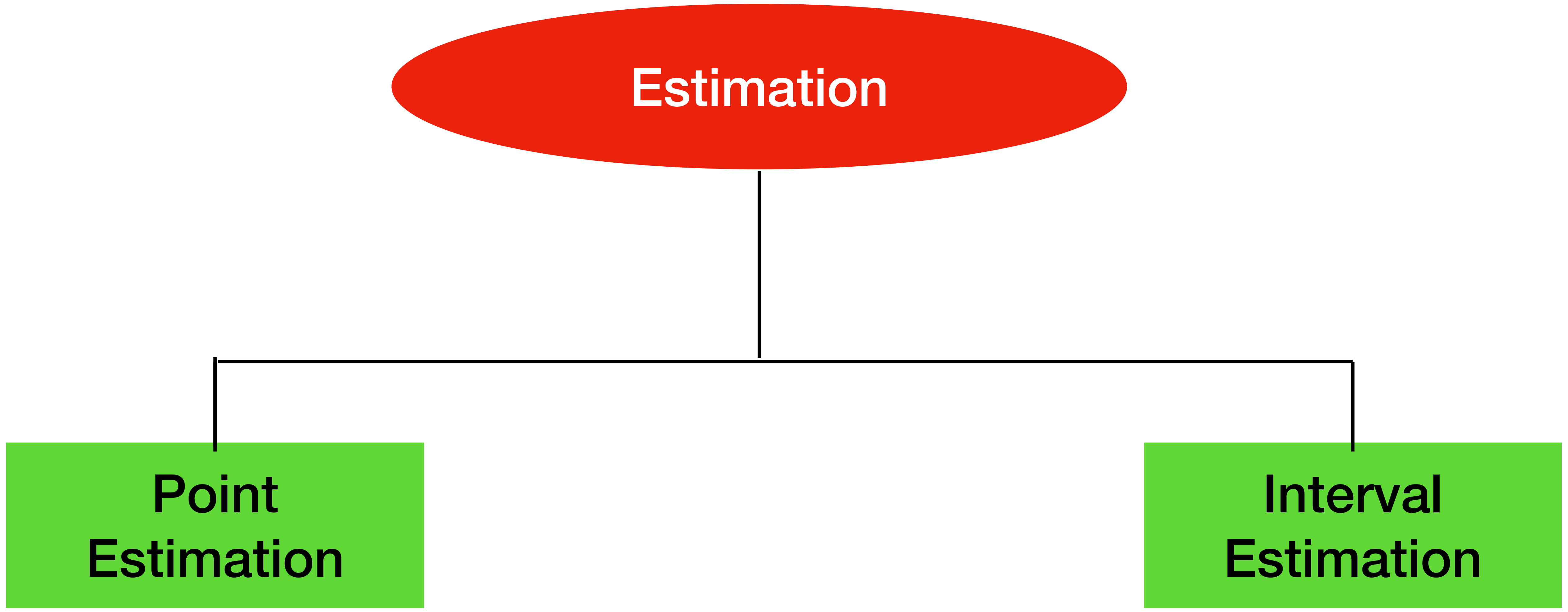
Sampling Distribution of s_2^2

s_2^2	0	0.25	1	2.25
prob.	0.30	0.44	0.20	0.06

Ref: http://mgmt.iisc.ac.in/CM/LectureNotes/statistical_inference_frequentist.pdf

Estimation

- Suppose population pdf $f(y/\theta)$ (or pmf $p(y/\theta)$)
- Random Sample: n i.i.d observations $Y_1, Y_2, \ldots, Y_n$ from population random variable Y
- How to estimate population parameter θ?



Ref: http://mgmt.iisc.ac.in/CM/LectureNotes/statistical_inference_frequentist.pdf

Point and Interval Estimation

- Point Estimation:

We need a single valued estimator of θ

- Interval Estimation:

We need an interval of values which is supposed to contain the true unknown value of θ

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Ref: Fundamentals of Statistics, Vol 1 by Gun, Gupta, Dasgupta

Point Estimation of Parameters

- Let θ be an unknown parameter of the distribution of a random variable Y.
- Want to estimate θ on the basis of a random sample $Y_1, Y_2, \ldots, Y_n$.
- Using a particular statistic T where T is itself a random variable.
- T is the estimator of θ. Value of T obtained from a given sample is its estimate.
- We want T to be a good estimator. The difference $|T - \theta|$ should be as small as possible.

Ref: Fundamentals of Statistics, Vol 1 by Gun, Gupta, Dasgupta

How “Good” your estimator is?

- In point estimation how to decide the nature of a “good” estimator?
- The kind of properties, characteristics or behavior a reasonable estimator should possess. What are the desirable criteria for “good” estimators.

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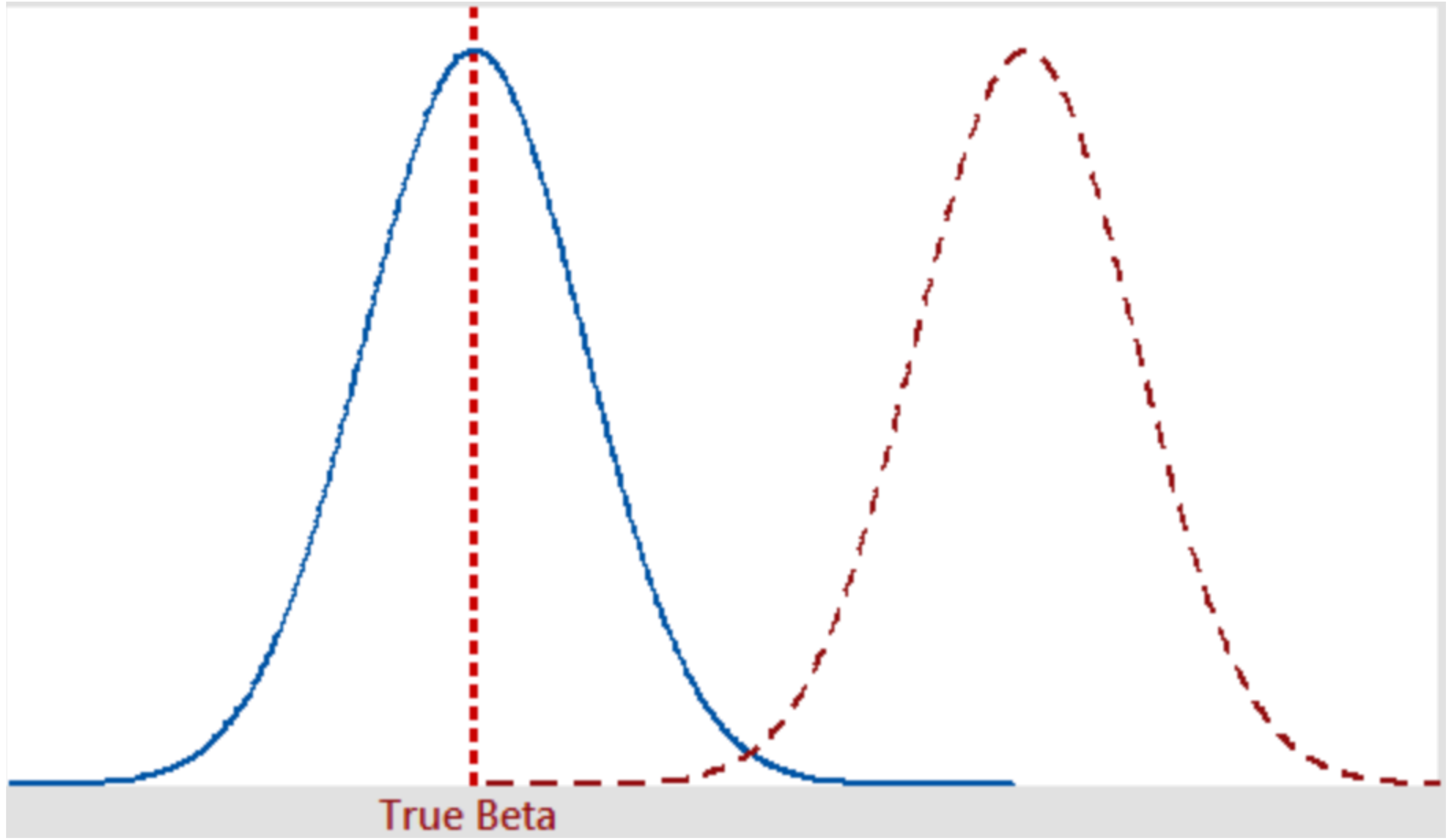
Ref: Fundamentals of Statistics, Vol 1 by Gun, Gupta, Dasgupta

Unbiasedness

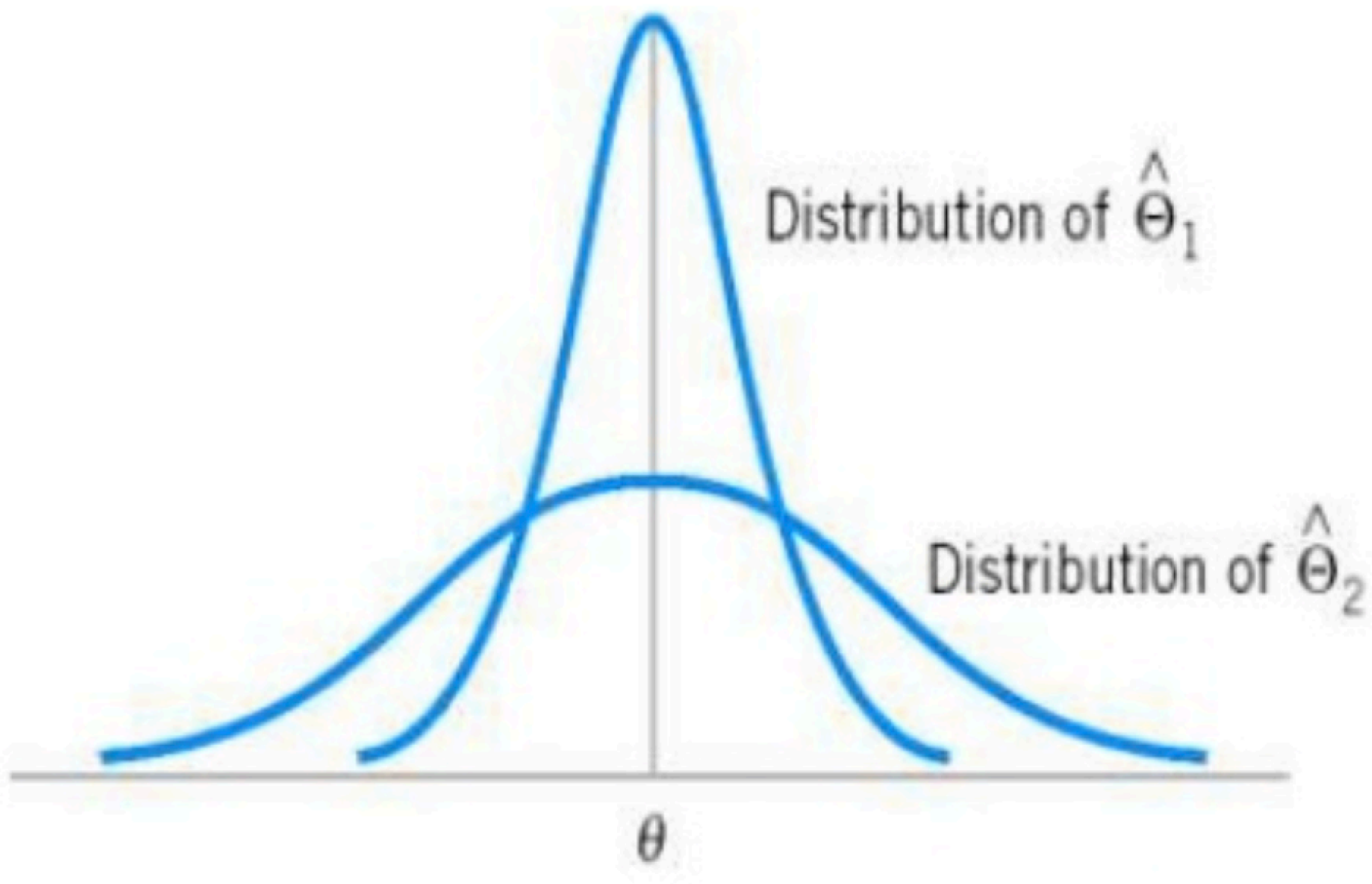
- Goodness can be defined in many ways.
- T is an estimator of θ and T has a sampling distribution
- What if the sampling distribution of T has a central tendency towards θ ?
- A statistic T is called unbiased if $E(T) = \theta$.
- $E(T) - \theta = b(\theta)$, where $b(\theta)$ is the bias of T.



Ref: Fundamentals of Statistics, Vol 1 by Gun, Gupta, Dasgupta
https://statisticsbyjim.com/regression/gauss-markov-theorem-ols-blue/

MVUE

- Minimum variance property: The sampling distribution of estimator T should also have a small dispersion
- Among all unbiased estimators, T should have the smallest variance.
- $Var(T) \leq Var(T')$ where T' is any other unbiased estimator
- A statistic T following these conditions is called MVUE



Ref: Fundamentals of Statistics, Vol 1 by Gun, Gupta, Dasgupta
https://www.slideshare.net/Tech_MX/theory-of-estimation

Example

- Random Sample: n observations $Y_1, Y_2, \ldots, Y_n$ from population random variable Y having mean μ.
- Sample mean: $\bar{Y}$= $Y_1 + Y_2 + \ldots + Y_n$
- E($\bar{Y}$) = $\dfrac{E(Y_1) + E(Y_2) + \ldots + E(Y_n)}{n}$ = $\dfrac{n\mu}{n}$ = μ, hence $\bar{Y}$ is an unbiased estimator of μ
- Suppose further that the observations are i.i.d and the population is normal with mean μ and variance σ^2. Here it can be shown that $\bar{Y}$ has the least variance among all unbiased estimators of μ. Hence, in this case, $\bar{Y}$ is MVUE of μ

Ref: Fundamentals of Statistics, Vol 1 by Gun, Gupta, Dasgupta