Confidence Interval 3

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Ex 1

<u>Example 1.</u> An automatic filling machine is used to fill bottles with liquid detergent. A random sample of 20 bottles results in a sample variance of fill volume of $s^2 = 0.0153$ (fluid ounce)^2. If the variance of fill volume is too large, an unacceptable proportion of bottles will be under- or overfilled. We will assume that the fill volume is approximately normally distributed. Find 95% CI for the process variance.

Ref: Applied Statistics and Probability for Engineers (5th ed.) by Montgomery, D. C., & George, C. R..

Solution

Sol^n $n = 20,\ 1-\alpha = 0.95,\ \frac{\alpha}{2} = 0.025$

CI: $\dfrac{(n-1)s^2}{\chi^2_{\alpha/2,\,n-1}} \le \sigma^2 \le \dfrac{(n-1)s^2}{\chi^2_{1-\alpha/2,\,n-1}}$

$$\frac{19 \times 0.0153}{\chi^2_{0.025,19}} \le \sigma^2 \le \frac{19 \times 0.0153}{\chi^2_{0.975,19}}$$

$$\frac{19 \times 0.0153}{32.85} \le \sigma^2 \le \frac{19 \times 0.0153}{8.91}$$

$$0.0088 \le \sigma^2 \le 0.0326$$

Ref: Applied Statistics and Probability for Engineers (5th ed.) by Montgomery, D. C., & George, C. R..

Ex 2

Example 2: Tensile strength tests were performed on two different grades of aluminium spars used in manufacturing the wing of a commercial transport aircraft. From past experience, the s.ds of tensile strengths are assumed to be known. The data obtained are as follows: $n_1 = 10$, $\bar{x}_1 = 87.6$, kg/mm^2 $\sigma_1 = 1$, $n_2 = 12$, $\bar{x}_2 = 74.5$, and $\sigma_2 = 1.5$. Assume independent normal populations. If μ_1 and μ_2 denote the true mean tensile strengths for the two grades of spars, find 90% CI for $(\mu_1 - \mu_2)$.

Ref: Applied Statistics and Probability for Engineers (5th ed.) by Montgomery, D. C., & George, C. R..

Solution

$\underline{\text{Sol}^n}$: $\quad 1-\alpha = 0.90, \quad \alpha = 0.10, \quad \alpha/2 = 0.05$

$$\text{CI}: \bar{x}_1 - \bar{x}_2 - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \leq \mu_1 - \mu_2$$

$$\leq \bar{x}_1 - \bar{x}_2 + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$87.6 - 74.5 - 1.645\sqrt{\frac{1^2}{10} + \frac{(1.5)^2}{12}} \leq \mu_1 - \mu_2$$

$$\leq 87.6 - 74.5 + 1.645\sqrt{\frac{(4)^2}{10} + \frac{(1.5)^2}{12}}$$

$$12.22 \leq \mu_1 - \mu_2 \leq 13.98 \text{ kg/mm}^2.$$

Ex 3

Example 3: An article in the journal Hazardous Waste and Hazardous Materials reported the results of an analysis of the weight of calcium in standard cement and cement doped with lead. Reduced level of calcium would indicate that the hydration mechanism in the cement is blocked and would allow water to attack various locations in cement structure. Ten samples of standard cement had an average weight percent calcium

Ref: Applied Statistics and Probability for Engineers (5th ed.) by Montgomery, D. C., & George, C. R..

Ex 3

of $\bar{x}_1 = 90.0$, with a sample s.d. of $s_1 = 5.0$, while 15 samples of the lead-doped cement had an average weight percent calcium of $\bar{x}_2 = 87.0$, with a sample s.d. of $s_2 = 4.0$. Assume that weight percent calcium is normally distributed and find a 95% CI for the difference in means, $\mu_1 - \mu_2$, for the two types of cement. Furthermore, assume that both normal populations have the same s.d.

Ref: Applied Statistics and Probability for Engineers (5th ed.) by Montgomery, D. C., & George, C. R..

Solution

Solⁿ. The pooled estimate of the common s.d. is found ~~using~~ as follows: $s_p^2 = \dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$

$$= \frac{9\times(5.0)^2 + 14\times(4.0)^2}{10+15-2}$$

$$= 19.52$$

$\Rightarrow s_p = \sqrt{19.52} = 4.4.$

95% CI: $\bar{x}_1 - \bar{x}_2 - t_{0.025,23}\, s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \le \mu_1 - \mu_2 \le \bar{x}_1 - \bar{x}_2 + t_{0.025,23}\, s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$.

$t_{0.025,23} = 2.069$

$-0.72 \le \mu_1 - \mu_2 \le 6.72.$

CI for Ratio for Variances

CI for Ratio of Variances of two Populations

Let $X_{11}, X_{12}, \ldots, X_{1n_1}$ be ~~an~~ random i.i.d. sample from a normal population with mean μ_1 and variance σ_1^2, and let $X_{21}, X_{22}, \ldots, X_{2n_2}$ be ~~an~~ i.i.d random sample from a second normal population with mean μ_2 and variance σ_2^2. Assume that both normal pop^{n}s are independent. Let S_1^2 and S_2^2 be the sample vars. Then the ratio

$$F = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F(n_1 - 1, n_2 - 1).$$

Ref: Applied Statistics and Probability for Engineers (5th ed.) by Montgomery, D. C., & George, C. R..

CI for Ratio for Variances

CI for $\frac{\sigma_1^2}{\sigma_2^2}$

$$f = \frac{S_2^2/\sigma_2^2}{S_1^2/\sigma_1^2} \sim F_{n_2-1,\, n_1-1}$$

$$\Rightarrow P\left(f_{1-\alpha/2;\, n_2-1,\, n_1-1} \leq f \leq f_{\alpha/2;\, n_2-1,\, n_1-1}\right) = 1-\alpha.$$

$\downarrow$

replace f with $\frac{S_2^2/\sigma_2^2}{S_1^2/\sigma_1^2}$.

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CI for Ratio for Variances

CI for $\frac{\sigma_1^2}{\sigma_2^2}$

If s_1^2 and s_2^2 are the sample vars of iid random samples of sizes n_1 and n_2, respectively, from two independent normal populations with unknown variances σ_1^2 and σ_2^2, then a $100(1-\alpha)\%$ CI for the ratio σ_1^2/σ_2^2 is:

$$\frac{s_1^2}{s_2^2} f_{1-\alpha/2;\, n_2-1,\, n_1-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{s_1^2}{s_2^2} f_{\alpha/2;\, n_2-1,\, n_1-1}$$

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