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Akshay

Lec 2

$$X_{n \times p} = \begin{bmatrix} X_{11} & - & X_{1p} \\ \vdots & \vdots & \vdots \\ X_{n1} & & X_{np} \end{bmatrix}_{n \times p}$$

$$X^* = \begin{bmatrix} x_{11}-\bar{x}_1 & x_{12}-\bar{x}_2 & \cdots & x_{1p}-\bar{x}_p \\ x_{21}-\bar{x}_1 & \vdots & & \vdots \\ & \vdots & & \vdots \\ x_{n1}-\bar{x}_1 & \vdots & & x_{np}-\bar{x}_p \end{bmatrix}$$

$$\bar{X}^* = \begin{bmatrix} \bar{x}_1 & \cdots & \bar{x}_p \\ \vdots & & \vdots \\ \bar{x}_1 & - & \bar{x}_p \end{bmatrix}$$

$$\bar{x}_i = \frac{1}{n}\sum_{j=1}^{n} x_{ji}$$

$$X^* = X - \bar{X}^*$$

$$S_{ik} = \frac{1}{n-1}\sum_{j=1}^{n}(\cancel{X}_{ji}-\cancel{\bar{X}}_i)(\cancel{X}_{jk}-\cancel{\bar{X}}_k)$$

$$S_{ik} = \frac{1}{n-1}\sum_{j=1}^{n}(x_{ji}-\bar{x}_i)(x_{jk}-\bar{x}_k)$$

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$$S = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1p} \\ s_{21} & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ s_{1p} & s_{2p} & \cdots & s_{pp} \end{bmatrix} \qquad s_{jk} = \frac{1}{n-1} \sum_{i=1}^{n} (x_{ij} - \bar{x}_j)(x_{ik} - \bar{x}_k)$$

$$(n-1)S = \underset{p \times n}{(X - \bar{X}^*)^T} \; \underset{n \times p}{(X - \bar{X}^*)}$$

$$= X^{*T} \cdot X^*$$

result $p \times p$

$$R = \begin{bmatrix} 1 & r_{12} & \cdots & r_{1p} \\ r_{12} & 1 & & r_{2p} \\ & & \ddots & \\ r_{1p} & & & 1 \end{bmatrix}_{p \times p}$$

$$D = \begin{bmatrix} s_{11} & 0 & 0 & 0 \\ 0 & s_{22} & & 0 \\ & & \ddots & \\ 0 & 0 & & s_{pp} \end{bmatrix}$$

$$R = D^{-1/2} \, S \, D^{-1/2}$$

$$S = D^{1/2} \, R \, D^{1/2}$$

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$X \to E(x)$, $var(X)$ $\quad E(X+Y) = E(X) + E(Y)$

matrix $\Rightarrow$

$$X_{p\times c} = [X_{ij}]$$

$$E(X_{p\times c}) = [E(X_{ij})]$$

$$E(A_{r\times p} X_{p\times c}) = A\,E(X)$$

$$\underset{\sim}{X} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_p \end{bmatrix} \qquad Cov(X) = Cov(X, X) = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1p} \\ \sigma_{21} & \sigma_{22} & & \\ \sigma_{p1} & \cdots & & \sigma_{pp} \end{bmatrix}$$

$$= E\left([X - E(X)][X - E(X)]^T\right)$$

$$\Downarrow$$

$$\underset{\sim}{\mu} = \begin{bmatrix} E(X_1) \\ \vdots \\ E(X_p) \end{bmatrix} = E(\underset{\sim}{X}) \qquad E[\underbrace{(\underset{\sim}{x} - \underset{\sim}{\mu})}_{p\times 1}\underbrace{(\underset{\sim}{x} - \underset{\sim}{\mu})^T}_{1\times p}]$$

$p\times p$ matrix

$$\Rightarrow \underset{\sim}{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_p \end{bmatrix}, \quad \underset{\sim}{Y} = \begin{bmatrix} y_1 \\ y_2 \\ y_q \end{bmatrix}$$

$$Cov(\underset{\sim}{X}, \underset{\sim}{Y}) = \begin{array}{c|c} & Y_1 \quad Y_2 \quad \cdots \quad Y_q \\ \hline X_1 & \\ X_2 & \\ \vdots & \\ X_p & \end{array}$$

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$$\underset{\sim}{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} \qquad \bar{\underset{\sim}{X}} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix}$$

$$= E\left[(\underset{\sim}{X} - \underset{\sim}{\mu_X})(\underset{\sim}{Y} - \underset{\sim}{\mu_Y})^T \right]$$

$$\underset{\sim}{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_p \end{bmatrix} = \begin{bmatrix} E(X_1) \\ E(X_2) \\ \vdots \\ E(X_p) \end{bmatrix}$$

$$\bar{\underset{\sim}{X}} = \begin{bmatrix} \bar{X}_1 \\ \bar{X}_2 \\ \bar{X}_p \end{bmatrix}$$

$$X_{n \times p} = \begin{bmatrix} X_{11} & - & - & X_{1p} \\ X_{21} & & - & \\ X_{n1} & - & - & X_{np} \end{bmatrix} \qquad \underset{\sim}{X} = \begin{bmatrix} X_1 \\ \vdots \\ X_p \end{bmatrix}$$

$X_{i.}$ = i^{th} observation on p variables.

$$= \begin{bmatrix} X_{1.} \\ X_{2.} \\ \vdots \\ X_{n.} \end{bmatrix}$$

joint Density function

$$f(\underset{\sim}{x}) = f(x_1, x_2, x_3, \ldots, x_p)$$

joint distribution

$$\underset{\sim}{X} \sim f(\underset{\sim}{x})$$

If they are independent their joint density function is given by

$$f(x_{1.}) \cdot f(x_{2.}) \cdot f(x_{3.}) \ldots f(x_{n.})$$

$$\cancel{f(x_{j.}) = f(x_{j1}) \; f(x_{j2}) \; \ldots \; f(x_{jp})}$$

$$f(x_{j.}) = f(x_{j1}, x_{j2}, x_{j3} \ldots x_{jp})$$

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Multivariate Normal Distribution.

$X \sim N(\mu, \sigma^2)$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$x \in (-\infty, \infty)$

$$\underset{\sim}{X} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_p \end{bmatrix} \sim N_p(\underset{\sim}{\mu}, \Sigma)$$

$$\left(\frac{x-\mu}{\sigma}\right)^2 = (x-\mu)(\sigma^2)^{-1}(x-\mu)$$

$$= (\underset{\sim}{x} - \underset{\sim}{\mu})\, \Sigma^{-1} (\underset{\sim}{x} - \underset{\sim}{\mu})$$

$$\sqrt{2\pi\sigma^2} = (2\pi)^{1/2} (\sigma^2)^{1/2}$$

$$= (2\pi)^{p/2} |\Sigma|^{1/2}$$

p dimensional normal density for the random vector $\underset{\sim}{X}' = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_p \end{bmatrix}'$

$$f(\underset{\sim}{x}) = \frac{1}{(2\pi)^{p/2} |\Sigma|^{1/2}} e^{-\frac{1}{2} \underbrace{(\underset{\sim}{x} - \underset{\sim}{\mu})'}_{1 \times p} \underbrace{\Sigma^{-1}}_{p \times p} \underbrace{(\underset{\sim}{x} - \underset{\sim}{\mu})}_{p \times 1}}$$

total 1×1 number

$-\infty < x < \infty$

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$N_p(\underset{\sim}{\mu}, \Sigma)$ Σ Covariance Matrix.

$$p\times p \begin{bmatrix} 25 & 23 & 0 \\ 21 & 0 & 23 \\ 12 & 17 & 0 \end{bmatrix} = \Sigma ?$$

Can any given matrix be covariance matrix?

Σ should be a <u>positive definite</u>. matrix
It means Σ should have it's inverse.

$$p=2, \quad \underset{\sim}{X} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \underset{\sim}{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix}$$

$$\rho_{12} = \frac{\sigma_{12}}{\sqrt{\sigma_{11}\sigma_{22}}}$$

$$\Sigma^{-1} = \frac{1}{\sigma_{11}\sigma_{22} - \sigma_{12}^2} \begin{bmatrix} \sigma_{22} & -\sigma_{12} \\ -\sigma_{12} & \sigma_{11} \end{bmatrix}$$

$$\downarrow$$

$$= \frac{1}{\sigma_{11}\sigma_{12}(1-\rho_{12}^2)}$$

$$f(x_1, x_2) = \frac{1}{2\pi\sqrt{\sigma_{11}\sigma_{22}(1-\rho_{12}^2)}} \times \exp\left\{\frac{-1}{2(1-\rho_{12}^2)}\left[\left(\frac{x_1-\mu_1}{\sqrt{\sigma_{11}}}\right)^2 + \left(\frac{x_2-\mu_2}{\sqrt{\sigma_{22}}}\right)^2 - 2\rho_{12}\left(\frac{x_1-\mu_1}{\sqrt{\sigma_{11}}}\right)\cdot\left(\frac{x_2-\mu_2}{\sqrt{\sigma_{22}}}\right)\right]\right\}$$