

$$\begin{bmatrix} X_{11} & \dots & X_{1p} \\ X_{21} & \dots & X_{2p} \\ X_{31} & \dots & \\ \vdots & & \\ X_{n1} & \dots & X_{np} \end{bmatrix}$$

n items, p random variables.

Sample mean for first variable:

$$\bar{X}_1 = \frac{1}{n} \sum_{j=1}^n X_{j1}$$

$$\bar{X}_k = \frac{1}{n} \sum_{j=1}^n X_{jk} \quad k = 1, 2, 3, \dots, p$$

Sample variance on first variable:

$$S_1^2 = \frac{1}{n} \sum_{j=1}^n (X_{j1} - \bar{X}_1)^2$$

$$S_k^2 = \frac{1}{n} \sum_{j=1}^n (X_{jk} - \bar{X}_k)^2 \quad k = 1, 2, \dots, p$$

$S_k^2 = S_{kk}$ \Rightarrow Sample variance. just representation

Sample standard deviation $\sqrt{S_{kk}}$

Linear association between the measurement of variables 1 and 2 is provided by Sample covariance.

$$S_{12} = \frac{1}{n} \sum_{j=1}^n (x_{j1} - \bar{x}_1)(x_{j2} - \bar{x}_2)$$

$$S_{ik} = \frac{1}{n} \sum_{j=1}^n (x_{ji} - \bar{x}_i)(x_{jk} - \bar{x}_k)$$

Covariance = Sample variance
when $i=k$

$$S_{ik} = S_{ki} \quad \forall i, k$$

Sample Correlation Coefficient (does not depend on the unit of measurement)

$$r_{ik} = \frac{S_{ik}}{\sqrt{S_{ii}} \sqrt{S_{kk}}} = \frac{\frac{1}{n} \sum_{j=1}^n (x_{ji} - \bar{x}_i)(x_{jk} - \bar{x}_k)}{\sqrt{\frac{1}{n} \sum_{j=1}^n (x_{ji} - \bar{x}_i)^2} \sqrt{\frac{1}{n} \sum_{j=1}^n (x_{jk} - \bar{x}_k)^2}}$$

$$r_{ik} = r_{ki} \quad \forall k, i \quad k, i \in [1, p]$$

$$\text{Standardised } x_{ji} = \frac{(x_{ji} - \bar{x}_i)}{\sqrt{s_{ii}}}$$

value of $r \in [-1, 1]$

r measure strength of linear association

- ① $r \approx 0$, lack of linear association b/w components
- ② $r < 0$, tendency of one value in pair to be larger than its avg and tendency of other to be smaller than its avg
- ③ $r > 0$, either both are larger or both are smaller.
- ④ r_{ik} remains unchanged if measurement of i^{th} variable changed to $y_{ji} = ax_{ji} + b$ $\forall j \in [1, n]$ and k^{th} variable changed to $y_{jk} = cx_{jk} + d$ $j \in [1, n]$
 Provided constants a and c should have same sign

$$W_{kk} = \sum_{j=1}^n (x_{jk} - \bar{x}_k)^2$$

$$W_{ik} = \sum_{j=1}^n (x_{ji} - \bar{x}_i)(x_{jk} - \bar{x}_k)$$

$$i, k \in [1, p]$$

Arrays of descriptive statistics

Sample means: $\bar{X} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \vdots \\ \bar{x}_p \end{bmatrix}$

Sample variances
and covariances.

$$S_n = \begin{bmatrix} s_{11} & s_{12} & \dots & s_{1p} \\ & s_{22} & & s_{2p} \\ & & \ddots & \\ s_{p1} & & & s_{pp} \end{bmatrix}$$

n here represent, n is
deployed as divisor

Sample Correlation

$$R = \begin{bmatrix} 1 & r_{12} & \dots & r_{1p} \\ r_{21} & 1 & & \\ & & \ddots & \\ r_{p1} & & & 1 \end{bmatrix}$$