Date:
Page No.

Lec 10

$$E(\hat{\epsilon}'\hat{\epsilon}) = (n-r-1)\sigma^2$$

SS_R

$$Y = Z\beta + \epsilon$$

β σ^2

$$\hat{\sigma}^2 = s_\epsilon^2 = \frac{\hat{\epsilon}'\hat{\epsilon}}{n-r-1}$$

$$E\left(\frac{\hat{\epsilon}'\hat{\epsilon}}{n-r-1}\right) = \sigma^2, \quad E(s_\epsilon^2) = \sigma^2$$

$$\text{Cov}(\hat{\beta}) = \sigma^2 (Z'Z)^{-1} = s_\epsilon^2 (Z'Z)^{-1}$$

$$s_\epsilon^2 = \frac{SS_R}{n-r-1}$$

Sum square residual $= \hat{\epsilon}'\hat{\epsilon} = \sum_1^n \hat{\epsilon}_j$

$$\underset{\sim}{\hat{\beta}} = \begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \\ \vdots \\ \hat{\beta}_r \end{bmatrix} \sim N_{r+1}\left(\underset{\sim}{\beta}, \sigma^2 (Z'Z)^{-1}\right)$$

$\epsilon \sim N$

$Y \sim N$

CR of $\beta =$

Date:
Page No.

$X \sim N_p(\underset{\sim}{\mu}, \Sigma)$.

$$P\left(n(\bar{\underset{\sim}{X}} - \underset{\sim}{\mu})^T S^{-1} (\bar{\underset{\sim}{X}} - \underset{\sim}{\mu}) \leq \frac{(n-1)p}{n-p} F_{p,n-p}(\alpha)\right)$$

$$= 1-\alpha$$

$100(1-\alpha)\%$ CR of $\beta = P\left((\hat{\beta}-\beta)^T Z'Z (\hat{\beta}-\beta)\right) \leq s_\epsilon^2 (r+1) f_{r+1, n-r-1}(\alpha)$

$$= 1-\alpha$$

SCI $100(1-\alpha)\%$ for β_j

↳ Simultaneous Confidence Interval

$$\hat{\beta}_j \pm \sqrt{s_\epsilon^2 C_{jj}} \sqrt{(r+1) F_{r+1, n-r+1}(\alpha)}$$

$(Z'Z)^{-1} = C_{jj}$ $\quad C = \begin{bmatrix} C_{11} & & & \\ & C_{22} & & \\ & & \ddots & \\ & & & C \end{bmatrix}_{r+1, r+1}$

$Var(\hat{\beta}) = s_\epsilon^2 C_{jj}$

One at a time $100(1-\alpha)\%$ C.I for β_j

$$\hat{\beta}_j \pm t_{n-r-1}(\alpha/2) \sqrt{s_\epsilon^2 C_{jj}} \quad \forall j = 1, 2, \ldots n$$

$$\beta_j \sim N(\beta_j, s_\epsilon^2 C_{jj}) \Rightarrow \frac{\hat{\beta}_j - \beta}{\sqrt{s_\epsilon^2 C_{jj}}} \sim t_{n-r-1}$$

Date:

Page No.

Model adequecy test

$y = Z\beta + \epsilon \qquad \hat{y} = Z\hat{\beta}$

$\hat{\epsilon} = y - \hat{y}$

$$\sum_{i=1}^{n} (y_i - \bar{y})^2 = \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2 + \sum_{i=1}^{n} (y_i - \hat{y})^2$$

	SS_T or TSS	RSS	SS_R
dof →	$n-1$	r	$n-r-1$

$R^2 = \frac{RSS}{TSS}$ → Total sum of sqr.

$SS_R = s_\epsilon^2 (n-r-1)$

$$R^2 = \frac{RSS}{SS_T} = 1 - \frac{SS_R}{SS_T} = 1 - \frac{s_\epsilon^2 (n-r-1)}{s_y^2 (n-1)}$$

$R^2 \geq 0.75$

0.8 model is adequate

adding a variable increases R^2, which is not good.

R_a^2 adjusted $R^2 = 1 - \frac{SSR / n-r-1}{SST / n-1}$

$= 1 - \frac{S_\epsilon^2}{S_y^2}$

$R_a^2 \leq R^2$

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