Tutorial

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CLT: Example

- A large freight elevator can transport a maximum of 9800 pounds. Suppose a load of cargo containing 49 boxes must be transported via the elevator. Experience has shown that the weight of boxes of this type of cargo follows a distribution with mean μ = 205 pounds and standard deviation σ = 15 pounds. Based on this information, what is the probability that all 49 boxes can be safely loaded onto the freight elevator and transported?

Ans: n = 49, μ = 205, σ = 15. The probability that the total weight of these 49 boxes is less than 9800 pounds is $P(S_n < 9800)$= $P(Z < \dfrac{9800 - 49 \times 205}{\sqrt{49} \times 15})$ = P(Z<-2.33) =0.0099

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Ref: http://www.stat.ucla.edu/~nchristo/introeconometrics/introecon_central_limit_theorem.pdf

Problem 1

- A machine produces screws having lengths normally distributed, around mean 6 cm and s.d. 1.4 cm. Find the probability of:

(a) getting a screw larger than 8 cm in length?
(b) getting a screw smaller than 5 cm in length?

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Solution

X (length) ~ N(6, 1.96)

a) P(X > 8) = ?

b) P(X < 5) = ?

$$Z = \frac{X-6}{1.4}$$

P(X > 8) = $P(\frac{X-6}{1.4} > \frac{8-6}{1.4})$ = P(Z > 1.429) = 1- P(Z<1.429) = 1−0.92364 = 0.07646

P(X<5) = P(Z < -0.7143) = P(Z > 0.7143) = 1 - P(Z < 0.7143) = 1 – 0.7625 = 0.2375.

Ref: http://www.cimt.org.uk/projects/mepres/alevel/stats_ch8.pdf

Problem 2. Let θ be a parameter. $\hat{\theta}_1$ and $\hat{\theta}_2$ are two unbiased estimators of θ. Another estimator $\hat{\theta}_3 = K_1 \hat{\theta}_1 + K_2 \hat{\theta}_2$ is suggested. You want this new estimator to be unbiased as well. Under what condition, this is possible?

Soln:

$$E(\hat{\theta}_3) = k_1 E(\hat{\theta}_1) + k_2 E(\hat{\theta}_2)$$

$$= k_1 \theta + k_2 \theta$$

$$= (k_1 + k_2)\,\theta$$

That means the linear comb. of the two unbiased estimators is again an unbiased ~~estimator~~ estimator iff $k_1 + k_2 = 1$.

Ref: https://personal.utdallas.edu/~sxe062000/solhw53522.pdf

Problem 3. Suppose $\hat{\theta}$ is an unbiased estimator of θ and $Var(\hat{\theta}) > 0$.

Can we say then that $\hat{\theta}^2$ is an unbiased estimator of θ^2?

Sol^n: $E(\hat{\theta}) = \theta$.

$$E(\hat{\theta}^2) = E(\hat{\theta} - \theta + \theta)^2$$

$$= E\left[(\hat{\theta} - \theta)^2 + 2.\theta.(\hat{\theta} - \theta) + \theta^2\right]$$

$$= E[\hat{\theta} - \theta]^2 + \underbrace{0}_{E} + \theta^2$$

$$= \underbrace{Var(\hat{\theta})}_{>0} + \theta^2.$$

$$\neq \theta^2.$$

Hence, $\hat{\theta}^2$ is a biased estimator of θ^2.

Ref: https://personal.utdallas.edu/~sxe062000/solhw53522.pdf

Problem 4. Let $X_1, X_2, \ldots, X_n$ are i.i.d random samples taken from a $U(\theta, \theta+1)$ population. Is $\bar{X}$ unbiased estimator for θ?

find $MSE(\bar{X})$.

Soln. $E(X_i) = \theta + \frac{1}{2}$, $Var(X_i) = \frac{1}{12}$, $i = 1(1)n$

$$Bias(\bar{X}) = E(\bar{X}) - \theta = \left(\theta + \frac{1}{2}\right) - \theta = \frac{1}{2}.$$

$$MSE(\bar{X}) = Var(\bar{X}) + [Bias(\bar{X})]^2$$

$$= \frac{1}{12n} + \frac{1}{4} = \frac{3n+1}{12n}.$$

Ref: https://math.hawaii.edu/~grw/Classes/2016-2017/2017Spring/Math472_1/Assignment03.nb.html