Akshay
Lec 6

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SCI Simultaneous Confidence Interval

$$\boxed{\begin{array}{c} \le \mu_1 \le \\ \cancel{\le} \mu_2 \le \end{array}}$$

95 %

We will know why $\mu = \mu_0$ was rejected.

$$\underset{\sim}{X} \sim N_p(\underset{\sim}{\mu}, \Sigma)$$

$$Z = a_1 x_1 + a_2 x_2 + \cdots + a_p x_p = a' \underset{\sim}{X}$$

$$E(Z) = \mu_Z = a'\mu, \quad \sigma_Z^2 = var(Z) = a'\Sigma a$$

$$Z \sim N(a'\mu, a'\Sigma a) \quad \text{Univariate}$$

$$Z_j = a_1 x_{j1} + a_2 x_{j2} + \cdots + a_p x_{jp}$$

from $x_{n \times p}$ we can get $z_1, z_2, \ldots, z_n$

$$\bar{Z} = a' \underset{\sim}{\bar{x}}, \quad s_Z^2 = a' S a$$

$\Rightarrow$ σ_Z^2 unknown, $100(1-\alpha)\%$ CI for $\mu_Z = a'\mu$

$$t = \frac{\bar{Z} - \mu_Z}{s_Z/\sqrt{n}} = \frac{\sqrt{n}(a'\bar{x} - a'\underset{\sim}{\mu})}{\sqrt{a'Sa}}$$

95% CI

$$\boxed{a'\underset{\sim}{\bar{x}} - t_{n-1}(\alpha/2)\frac{\sqrt{a'Sa}}{\sqrt{n}} \le a'\underset{\sim}{\mu} \le a'\underset{\sim}{\bar{x}} + t_{n-1}(\alpha/2)\frac{\sqrt{a'Sa}}{\sqrt{n}}}$$

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$$a' = [1, 0, 0, \cdots 0], \quad a' = [0, 1, 0, 0 \cdots 0]$$

together
0.95
$\times 0.95$

$$\begin{cases} 95\%\ CI & \le \mu_1 \le \\ 95\%\ CI & \le \mu_2 \le \end{cases}$$

$$a' = [1, -1, 0, \cdots 0], \quad \boxed{\le \underset{\sim}{\mu_1} - \underset{\sim}{\mu_2} \le}$$

$$\underset{\sim}{\bar{x}_1} - \sqrt{\frac{(n-1)p}{n-p} F_{p,(n-p)}(\alpha)} \times \sqrt{\frac{s_{11}}{n}} \le \mu_1 \le \underset{\sim}{\bar{x}_1} + \sqrt{\frac{(n-1)p}{n-p} F_{p,(n-p)}(\alpha)} \times \sqrt{\frac{s_{11}}{n}}$$

$\vdots$

$$\underset{\sim}{\bar{x}_j} - \sqrt{\quad} \times \sqrt{\frac{s_{jj}}{n}} \le \mu_j \le \underset{\sim}{\bar{x}_j} + \sqrt{\quad} \times \sqrt{\frac{s_{jj}}{n}}$$

$\vdots$

$$\le \mu_p \le$$

All hold simultaneously with confidence coefficient $1-\alpha$

$x_{n \times p} \to \underset{\sim}{\bar{x}_j}$, $S_{p\times p} \to s_{jj}$, n no of obsevation
$p \to X_1, \cdots X_p$.

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Bonferroni's Method

C_i denotes a confidence statement about μ_i

$$P(C_i \text{ true}) = 1-\alpha_i \qquad i=1,2,\dots,p$$

$$P(\text{all } C_i \text{ true}) = 1 - P[\text{at least one } C_i \text{ false}]$$

$$\geq 1 - \sum_{i=1}^{p} P(C_i \text{ false}) = 1 - \sum_{i=1}^{p} (1 - P(C_i \text{ true}))$$

$$= 1 - (\underbrace{\alpha_1 + \alpha_2 + \alpha_3 \dots + \alpha_p}_{\alpha})$$

$$= 1-\alpha$$

$$P\left[\bar{x}_i \pm t_{n-1}\left(\frac{\alpha}{2p}\right)\sqrt{\frac{s_{ii}}{n}} \text{ contains } \mu_i \ \forall\ i = 1,2,\dots p\right]$$

$$\geq 1 - \left(\frac{\alpha}{p} + \frac{\alpha}{p} + \dots + \frac{\alpha}{p}\right)$$

$$= 1-\alpha.$$

$$\bar{x}_1 - t_{n-1}\left(\frac{\alpha}{2p}\right)\sqrt{\frac{s_{11}}{n}} \leq \mu_1 \leq \bar{x}_1 + t_{n-1}\left(\frac{\alpha}{2p}\right)\sqrt{\frac{s_{11}}{n}}$$

$$\vdots$$

$$\bar{x}_j - t_{n-1}\left(\frac{\alpha}{2p}\right)\sqrt{\frac{s_{jj}}{n}} \leq \mu_j \leq \bar{x}_j + t_{n-1}\left(\frac{\alpha}{2p}\right)\sqrt{\frac{s_{jj}}{n}}$$

$$\vdots$$

$$\leq \mu_p \leq$$

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Summarization of CR

Case I

$\underset{\sim}{X} \sim N_p(\underset{\sim}{\mu}, \Sigma)$, Σ known.

$n(\bar{\underset{\sim}{x}} - \underset{\sim}{\mu})' \Sigma^{-1} (\bar{\underset{\sim}{x}} - \underset{\sim}{\mu}) \sim \chi^2_p$

$100(1-\alpha)\%$ CR $\quad n(\bar{\underset{\sim}{x}} - \underset{\sim}{\mu})' \Sigma^{-1} (\bar{\underset{\sim}{x}} - \underset{\sim}{\mu}) \leq \chi^2_p(\alpha)$

$H_0 : \underset{\sim}{\mu} = \mu_0$, $H_1 : \underset{\sim}{\mu} \neq \mu_0$

if $n(\bar{\underset{\sim}{x}} - \underset{\sim}{\mu_0})' \Sigma^{-1} (\bar{\underset{\sim}{x}} - \underset{\sim}{\mu_0}) \geq \chi^2_p(\alpha)$ reject H_0

Case 2

a) $\underset{\sim}{X} \sim N_p(\underset{\sim}{\mu}, \Sigma)$ Σ unknown — (1)

$n - p \geq 40$

$n(\bar{\underset{\sim}{x}} - \underset{\sim}{\mu})' S^{-1} (\bar{\underset{\sim}{x}} - \underset{\sim}{\mu})$ $\quad$ Reject H_0 when LHS $\geq$ RHS

χ^2_p

CR: $n(\bar{\underset{\sim}{x}} - \underset{\sim}{\mu})' S^{-1} (\bar{\underset{\sim}{x}} - \underset{\sim}{\mu}) \leq \chi^2_p(\alpha)$

b) even (1) is not following multivariate normal distribution rest are true, because multivariate CLT says for large n ie $n - p \geq 40$ below things follow χ^2_p.

Case 3

$\underset{\sim}{X} \sim N_p(\underset{\sim}{\mu}, \Sigma)$ Σ unknown $\quad n - p < 40$.

must

$n(\bar{\underset{\sim}{x}} - \underset{\sim}{\mu})' S^{-1} (\bar{\underset{\sim}{x}} - \underset{\sim}{\mu}) \quad \frac{(n-1)p}{n-p} F_{p,n-p}(\alpha)$ $\quad$ Reject H_0 if LHS $\geq$ RHS

CR: $n(\bar{\underset{\sim}{x}} - \underset{\sim}{\mu})' S^{-1} (\bar{\underset{\sim}{x}} - \underset{\sim}{\mu}) \leq \frac{(n-1)p}{n-p} F_{p,n-p}(\alpha)$