Test of Hypothesis

Mainak Thakur

21.09.2020

Introduction

Test of Hypothesis.

Learned till now: point and interval estimate.

You may encounter a different kind of problem where there are two competing claims about the value of a parameter and you have to determine which one is correct. For example, consider the designing of an air crew escape system. The system consists of an ejection seat and a rocket motor that powers the seat

Ref: Applied Statistics and Probability for Engineers (5th ed.) by Montgomery, D. C., & George, C. R..

Example

The rocket motor contains a propellant which should have a mean burning rate of 50 cm/sec in order for the ejection seat to work properly. Too high or low burning rates may lead to possible pilot injury.

The engineering question: Does the mean burning rate of the propellant equal 50 cm/sec, or is it some other value (higher/lower)?

Ref: Applied Statistics and Probability for Engineers (5th ed.) by Montgomery, D. C., & George, C. R..

Statistical Hypothesis

Many types of decision making problems can be ~~converted~~ formulated as hypothesis testing problems.

There is a close connection between hypothesis testing and confidence intervals.

Statistical Hypothesis: A statistical hypothesis is a statement about the parameters of one or more populations.

It may also be thought of as a statement about the probability dist^n.

Ref: Applied Statistics and Probability for Engineers (5th ed.) by Montgomery, D. C., & George, C. R..

Example

__Example: Aircrew escape system.__

Consider burning rate as a r.v. having a prob. dist^n. Our interest is in its mean, a parameter.

Formally,

$H_0 : \mu = 50$ cm/sec → null Hypothesis

$H_1 : \mu \neq 50$ cm/sec → Alternate "

Here it is two-sided alternative Hyp.

$H_0 : \mu = 50$ cm/sec	$H_0 : \mu = 50$ cm/sec
$H_1 : \mu < 50$ cm/sec.	$H_1 : \mu > 50$ cm/sec.

one-sided alternative Hyp.

__Note:__

Ref: Applied Statistics and Probability for Engineers (5th ed.) by Montgomery, D. C., & George, C. R..

Test of Hypothesis

Test of hypothesis.

A procedure leading to a decision about a ~~parameter~~ particular hypothesis is called a test of hypothesis.

Hyp. testing relies ~~on~~ information gathered from a random sample collected from the population of interest.

Information inconsistent with the Hyp leads to rejection of the hypothesis.

Truth ~~of~~ or falsity of the hyp. can never be known with certainty.

Testing of Hypothesis: taking a random sample, computing test statistic, make decision

Ref: Applied Statistics and Probability for Engineers (5th ed.) by Montgomery, D. C., & George, C. R..

Critical Region

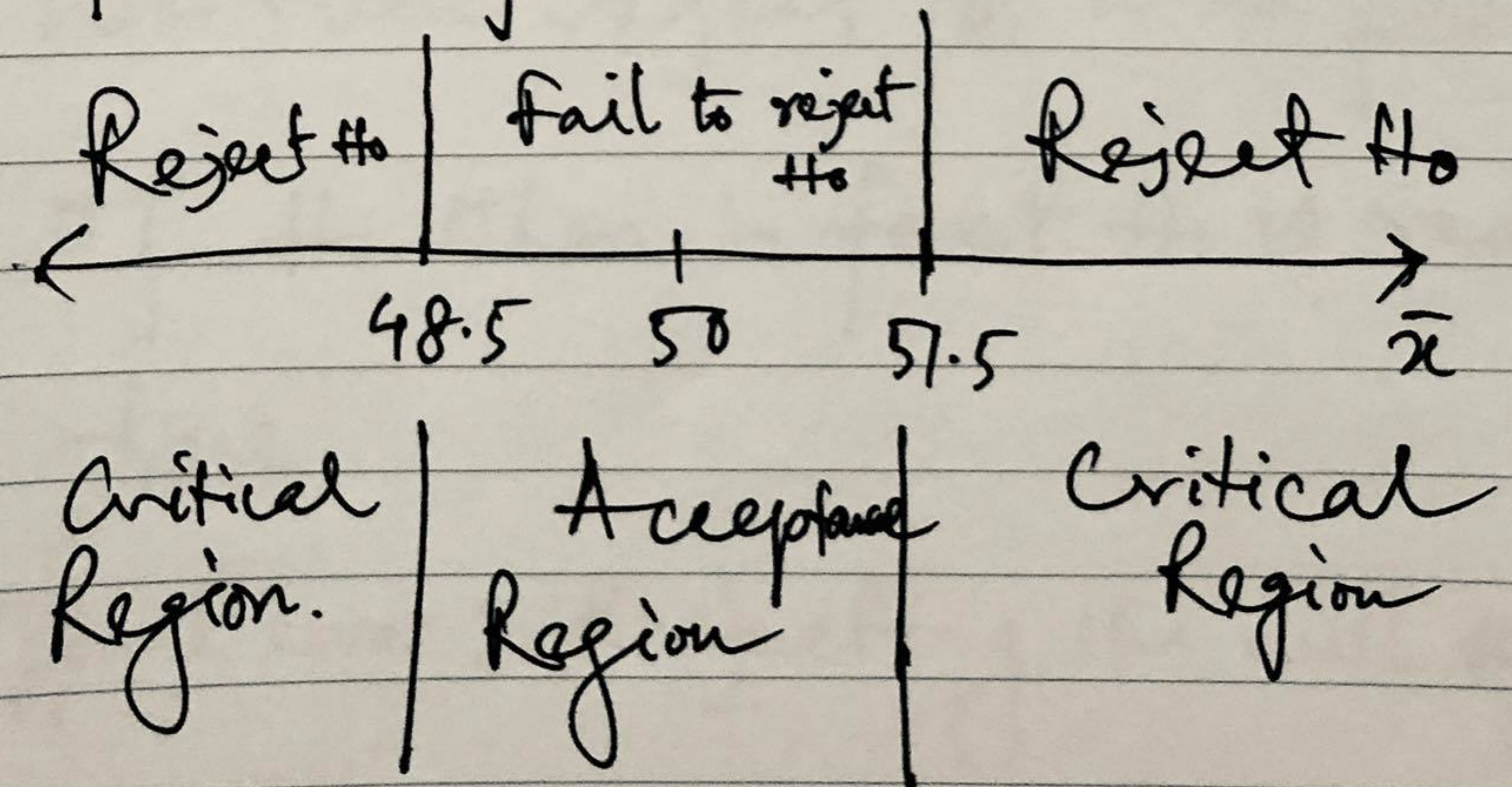
Consider,

$$H_0 : \mu = 50 \text{ cm/sec}$$

$$H_1 : \mu \neq 50 \text{ cm/sec}.$$

$n = 10$ specimens tested, $\bar{x}$ calculated.

Suppose, if $48.5 \leq \bar{x} \leq 51.5$, we will not reject H_0. If $\bar{x} < 48.5$ or $\bar{x} > 51.5$, we will reject H_0 in favour of H_1.



Ref: Applied Statistics and Probability for Engineers (5th ed.) by Montgomery, D. C., & George, C. R..

Errors

This decision procedure may lead to two wrong conclusions.

For example, ~~to~~ suppose true mean burning rate is really 50 cm/sec, but the randomly selected specimens give an observed value of $\bar{x}$ to be falling into the critical region. We then reject H_0 in favour of H_1 when in fact H_0 is really true.

Type I error: Rejecting the null Hyp. H_0 when it is true is defined as a Type I error.

Errors

On the other hand, for a different scenario if the true mean mean burning rate is different from 50 cm/sec. yet the sample mean $\bar{x}$ falls in the acceptance region. Then we fail to reject H_0 when it is really false.

Type II error: Failing to reject the null hypothesis when it is false is defined as a type II error.

Ref: Applied Statistics and Probability for Engineers (5th ed.) by Montgomery, D. C., & George, C. R..

Errors

Decision	Ho is true	Ho is false
Fail to reject Ho	No error	type II error
Reject Ho	Type I error	no error

~~α = P(type~~

α = P(type I error) = P(reject Ho when Ho is true).

(significance level)

(α- error)

(size of the test)

β = P(type II error) = P(fail to reject Ho when Ho is false).

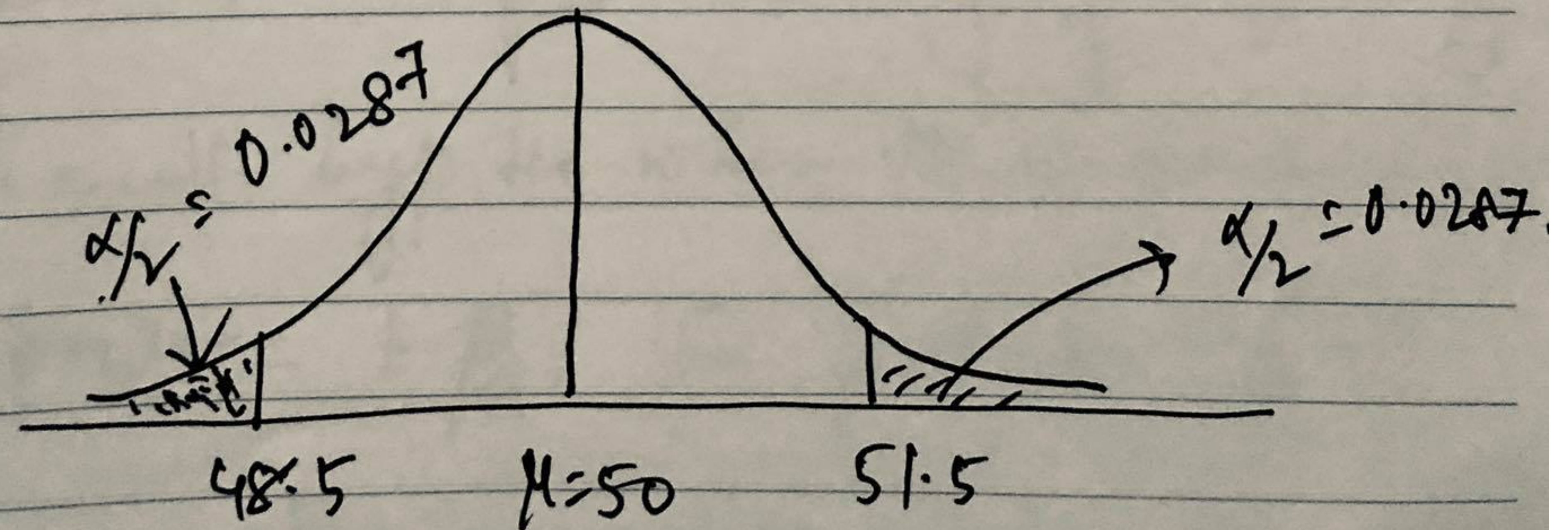
β- error.

Ref: Applied Statistics and Probability for Engineers (5th ed.) by Montgomery, D. C., & George, C. R..

Errors

Note: (A) The size of critical region and consequently the probability of Type I error α, can always be reduced by appropriate selection of the critical vals.

(B) Type I error and Type II error are related. A decrease in the probability of one results in the increase in the probability of the other, provided sample size remains same.



Ref: Applied Statistics and Probability for Engineers (5th ed.) by Montgomery, D. C., & George, C. R..

Power

Generally, α is controlled when an analyst selects critical values.
Since, the analyst can directly control α, the probability of wrongly rejecting H_0, rejection of H_0 is considered as a strong conclusion.

$\alpha = \underline{\underline{0.05}} / 0.1 / 0.01.$

Power: The power of a statistical test is the probability of rejecting the null hyp. H_0 when H_1 is true.

power $= 1-\beta$, the probability of correctly rejecting a false H_0.

P-values

P-values

DATE

One way to report the results of Hyp. test is to state that the null hypothesis was or was not rejected at a specified α-value or level of significance.

This fixed significance level approach has some disadvantages.

The P-value is the prob. that the test statistic will take on a value that is at least as extreme as the observed ~~as~~ value of the statistic when H_0 is true. P-value conveys much info about the weight of evidence against H_0, so a decision maker can draw a conclusion at any specified level of significance.

Ref: Applied Statistics and Probability for Engineers (5th ed.) by Montgomery, D. C., & George, C. R..

Example

P-value: The P-value is the smallest level of significance that would lead to rejection of the null Hypothesis H_0 with the given data.

Let, P-value = 0.038.

Then, H_0 would be rejected at any level of significance greater than or equal to 0.038.

$$P\text{-value} = 1 - P(48.7 < \bar{X} < 51.3)$$

$$= 1 - P\left(\frac{48.7-50}{2.5/\sqrt{16}} < Z < \frac{51.3-50}{2.5/\sqrt{16}}\right)$$

$$= 0.038.$$

Ref: Applied Statistics and Probability for Engineers (5th ed.) by Montgomery, D. C., & George, C. R..