A physical model for efficient ranking in networks

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Rankings and linear hierarchies

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The idea of rankings—pervasive!

Assumptions:

1. Competitors have some intrinsic quality (or vector of qualities).
2. Interactions can (stochastically) reveal differences in qualities.
3. Competitions are pair-wise. (Lee Sedol vs. AlphaGo; Astros vs. Dodgers)

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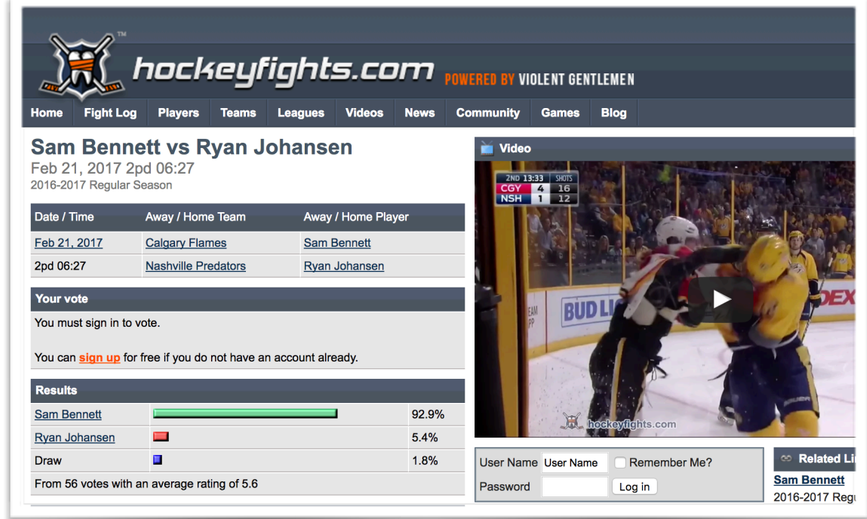
Systems of dominance

social

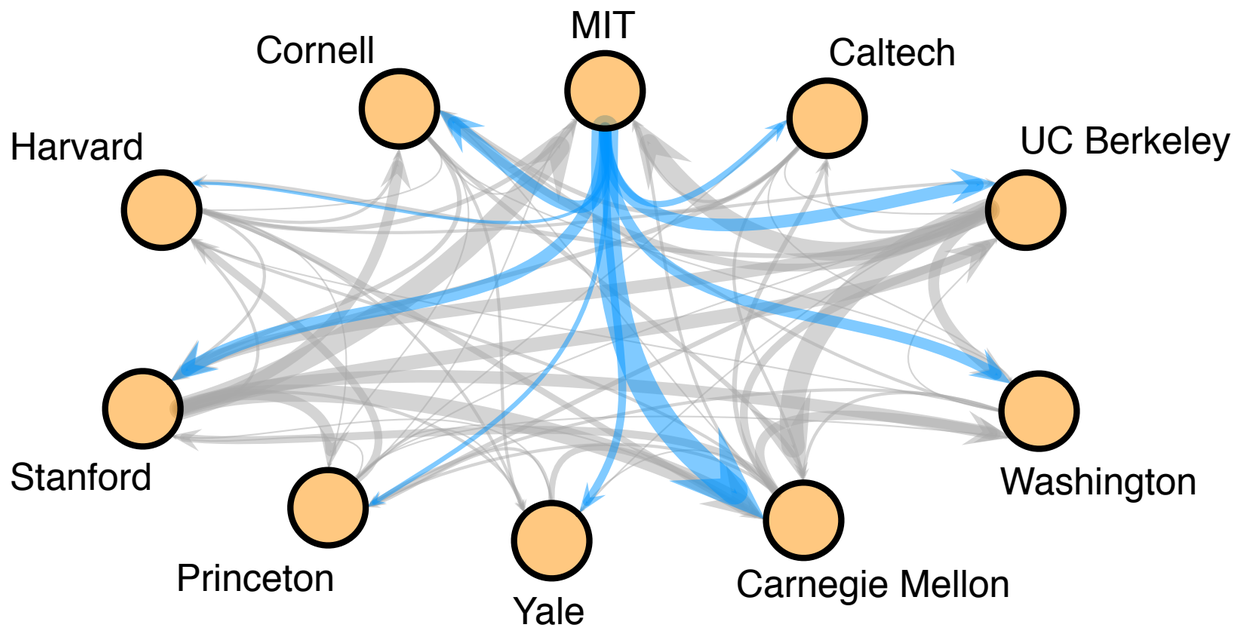
physical



mental

financial

Systems of endorsement



Clauset, Arbesman, Larremore. *Science Advances* 1, e1400005 (2015).

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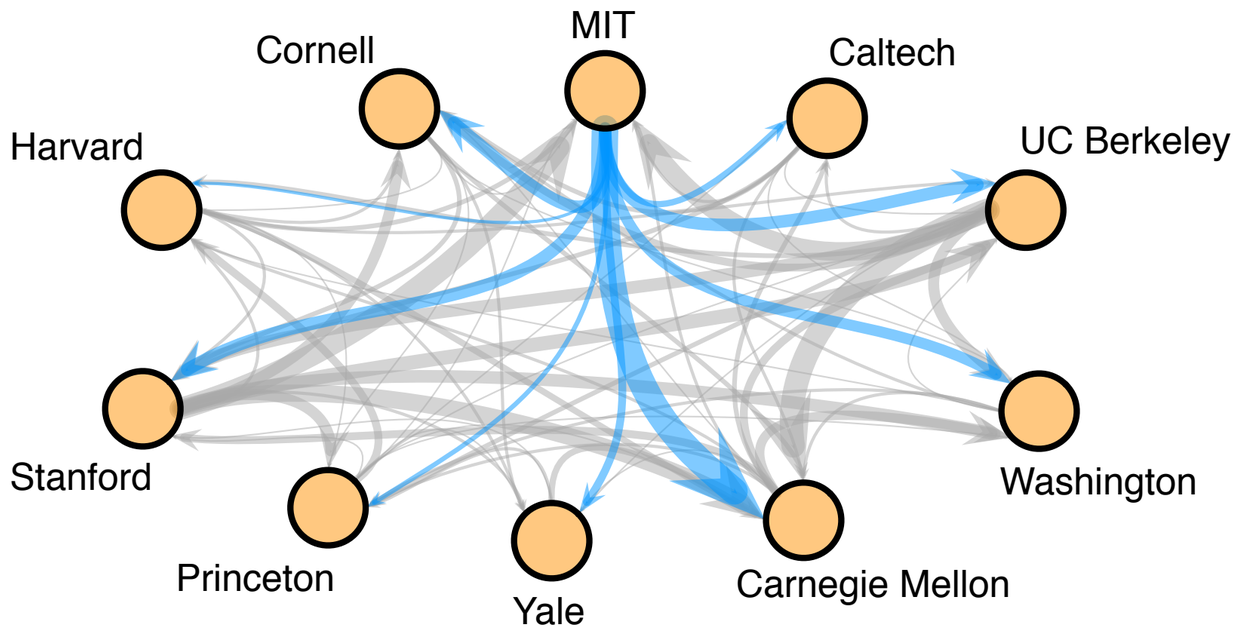

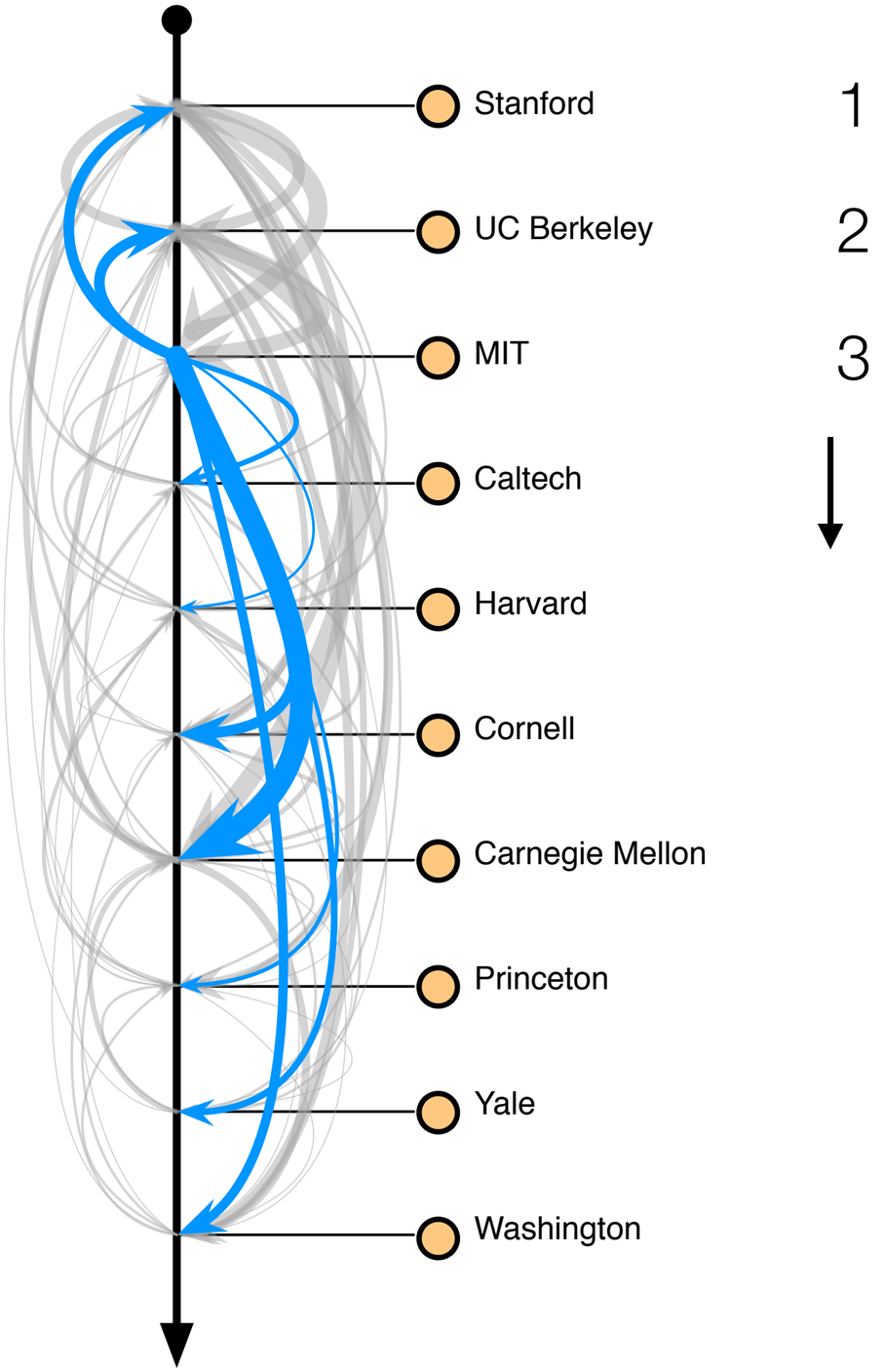

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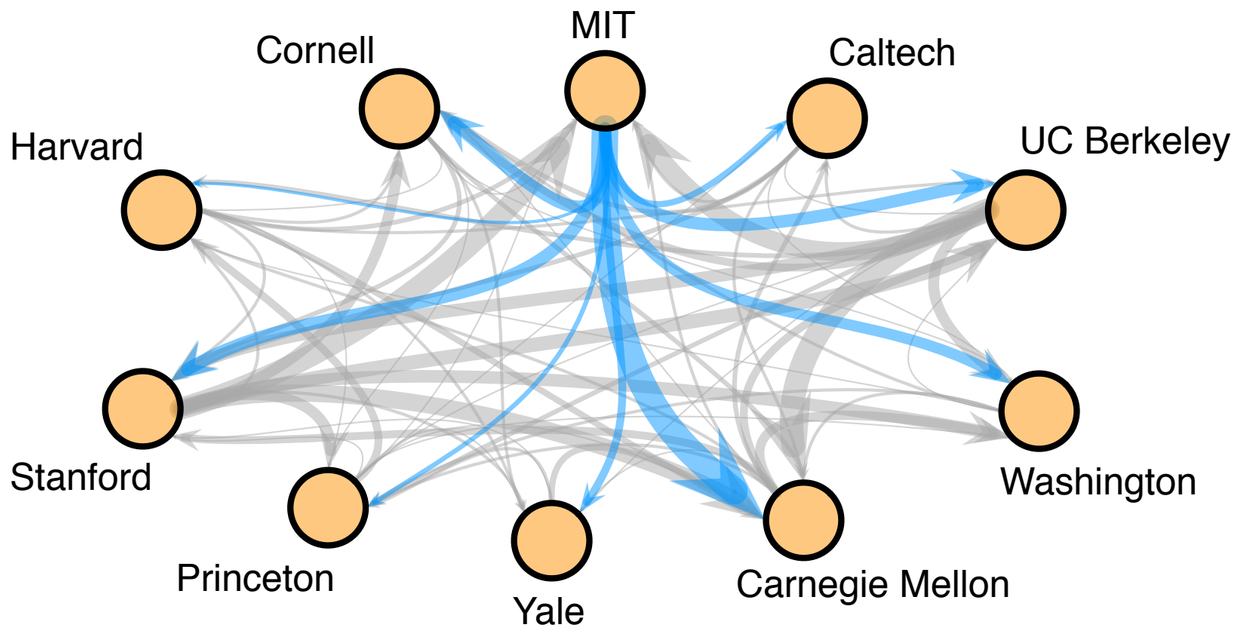

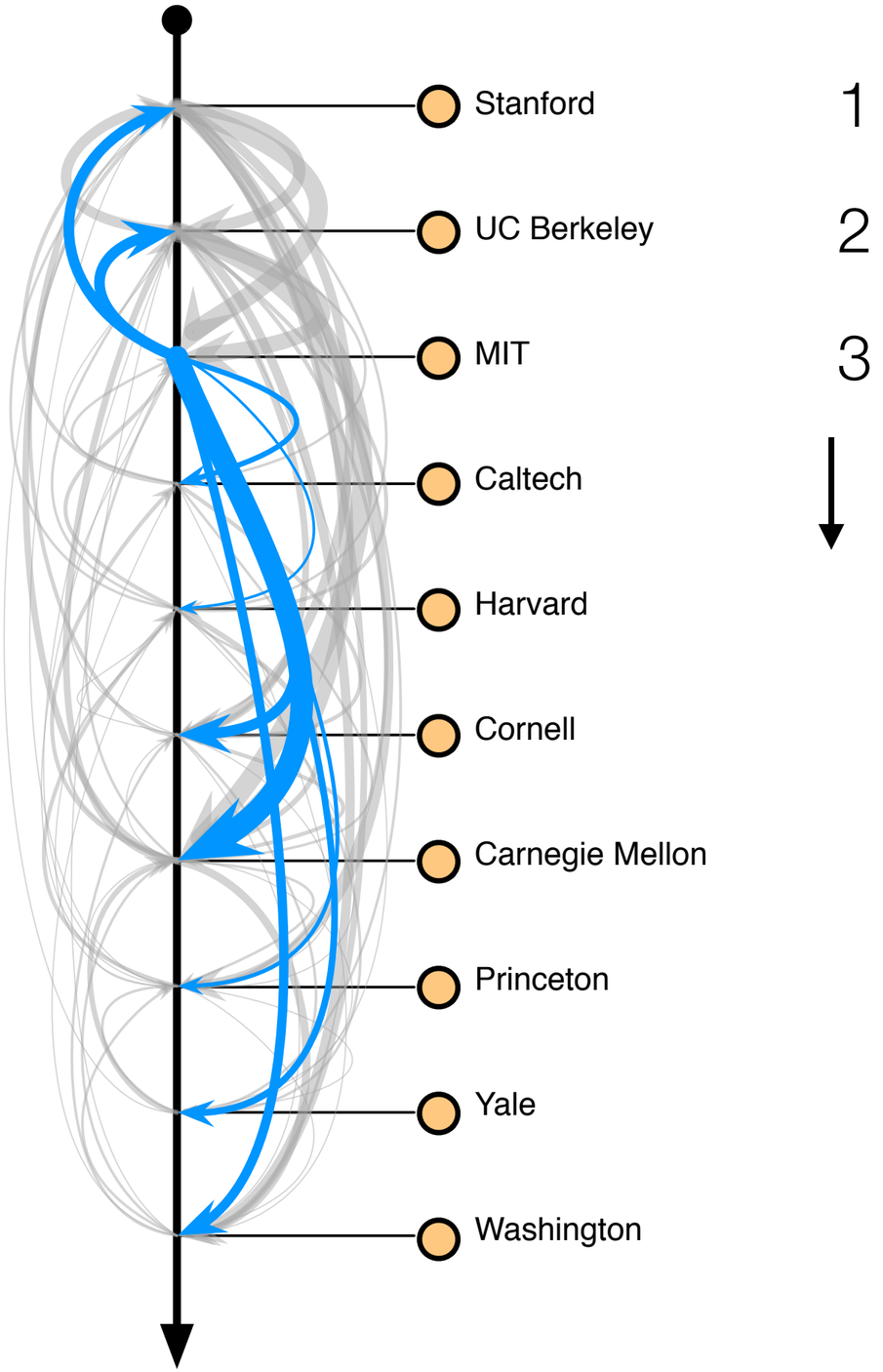

Assumptions:

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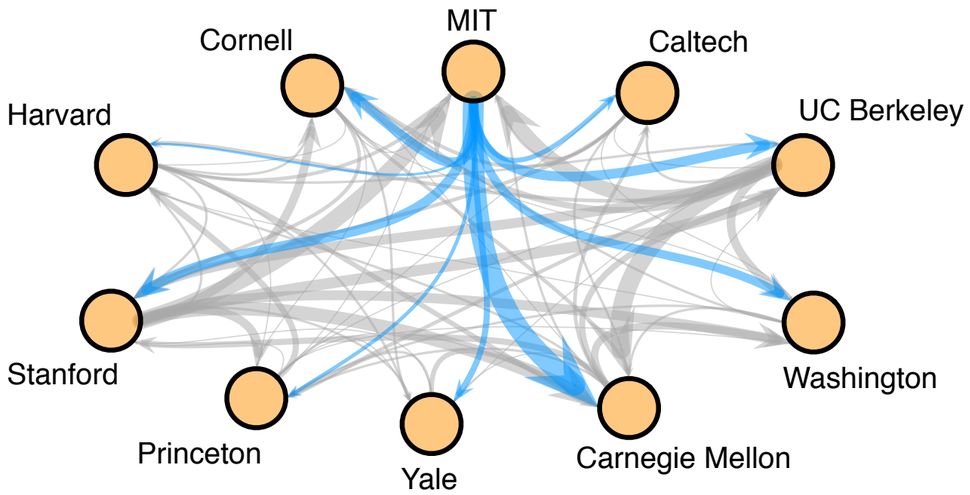
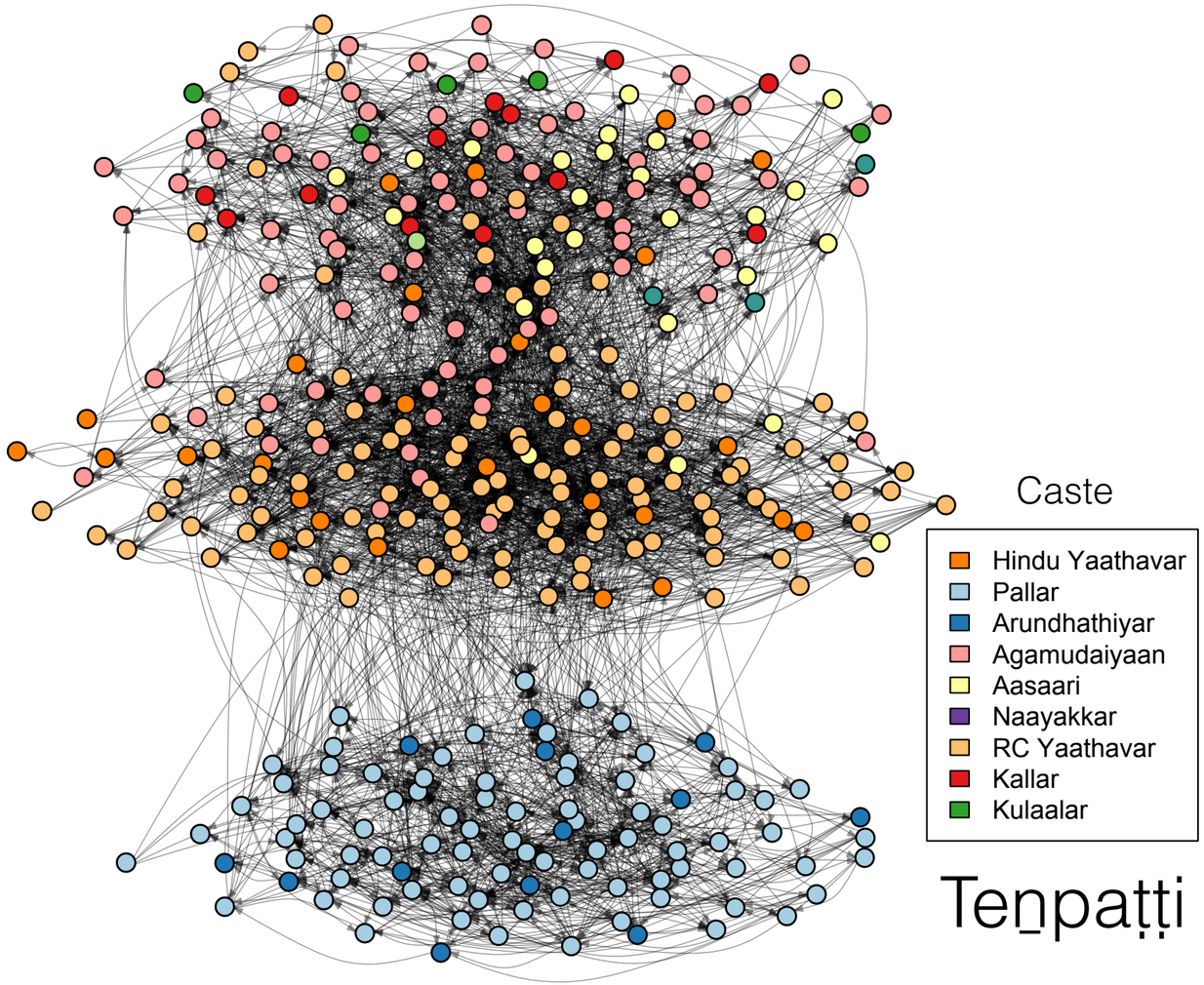
Systems of endorsement





Latent position can be revealed by **dominance** or **endorsement** interactions.

Clauset, Arbesman, Larremore. *Science Advances* (2015). Fig: Larremore, Hébert-Dufresne, Power. *Draft.* Data: Power. *Nature Human Behaviour* 1, 0057 (2017)

Win-Loss is not satisfactory: schedule matters

Beating the grandmaster counts for more than beating a novice.

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Ordinal rankings (order nodes; integer values)

Embeddings (place nodes; continuous values)

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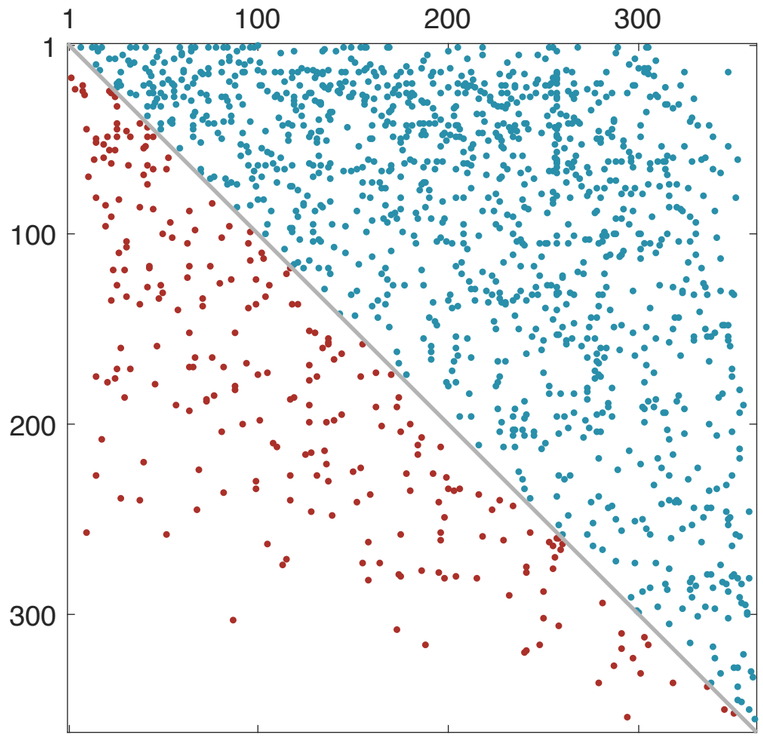
Minimum Violations Rank finds an order of nodes to minimize “upsets”:

- No ties (but see Agony methods), many minima, and provably NP hard.



minimum violation ranking: sort *A.*

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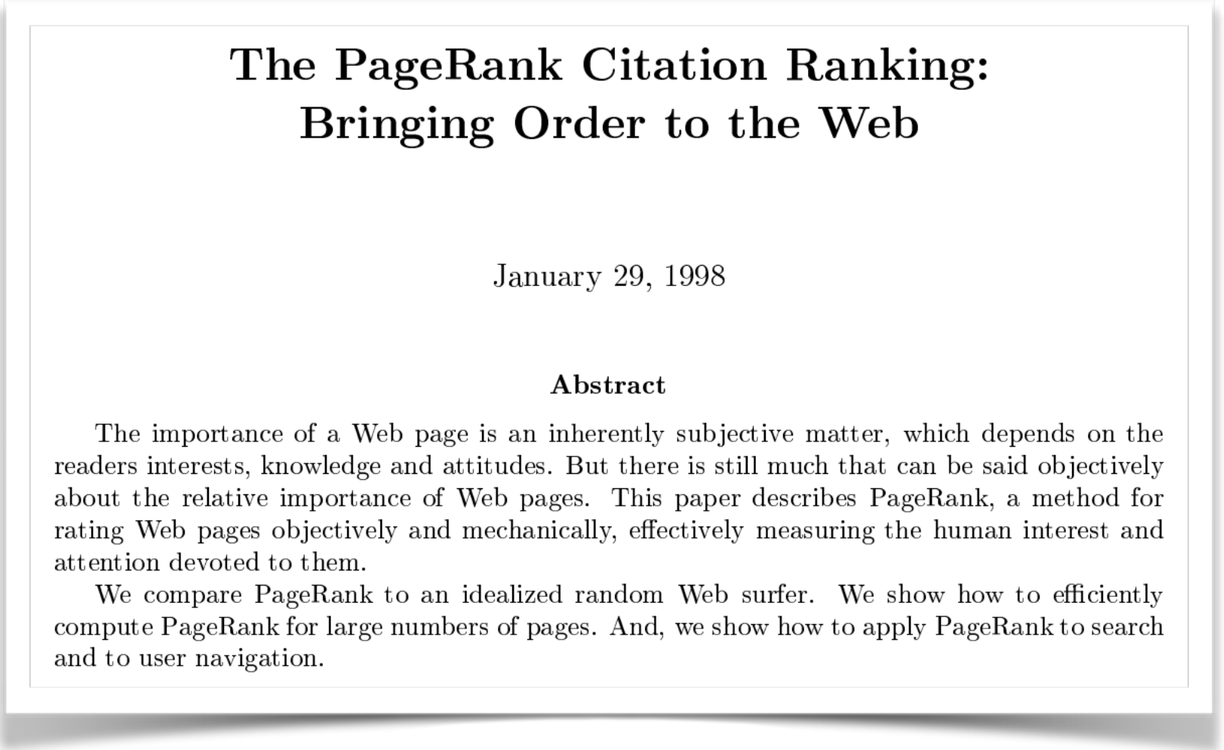

The PageRank Citation Ranking: Bringing Order to the Web

January 29, 1998

Abstract

The importance of a Web page is an inherently subjective matter, which depends on the readers interests, knowledge and attitudes. But there is still much that can be said objectively about the relative importance of Web pages. This paper describes PageRank, a method for rating Web pages objectively and mechanically, effectively measuring the human interest and attention devoted to them.

We compare PageRank to an idealized random Web surfer. We show how to efficiently compute PageRank for large numbers of pages. And, we show how to apply PageRank to search and to user navigation.

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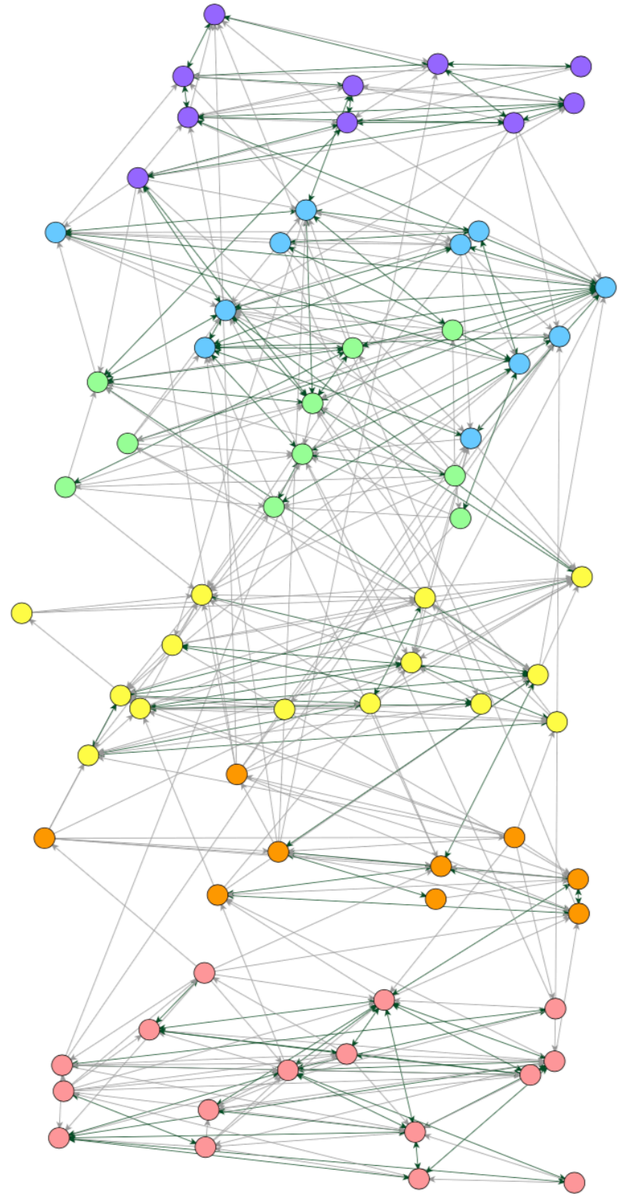

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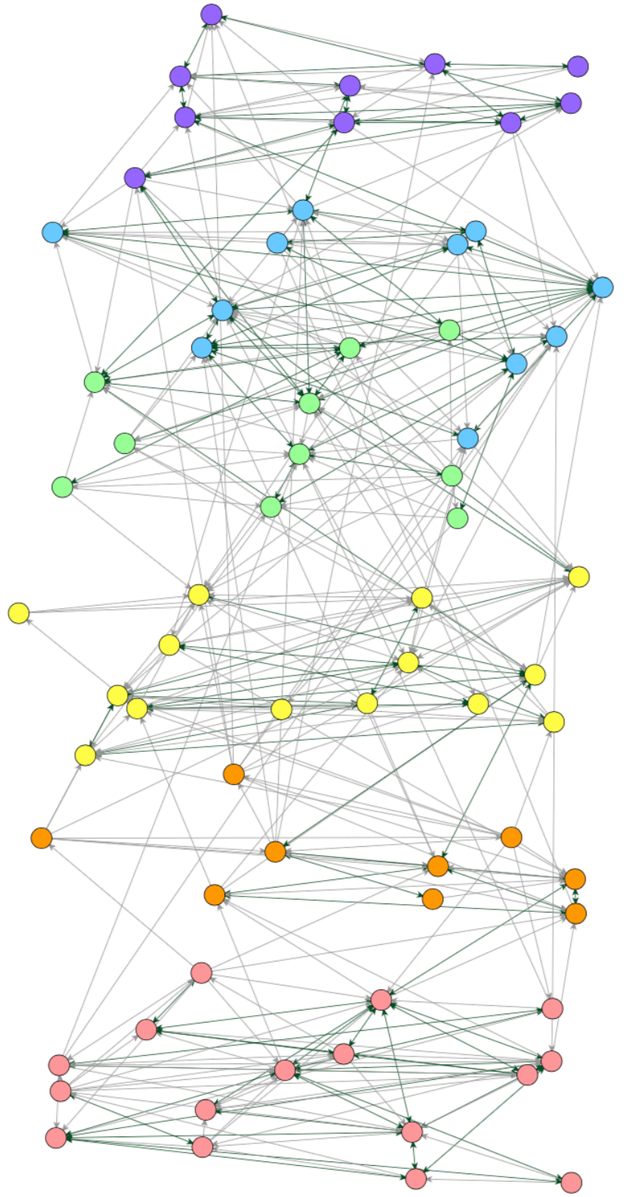
most species feed from narrow range in a 1-dim. space (~body size).

- Great for food webs. Inference models v slow for all but small networks.



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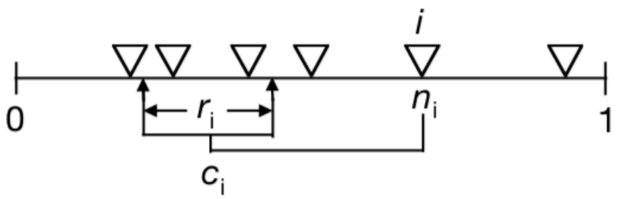
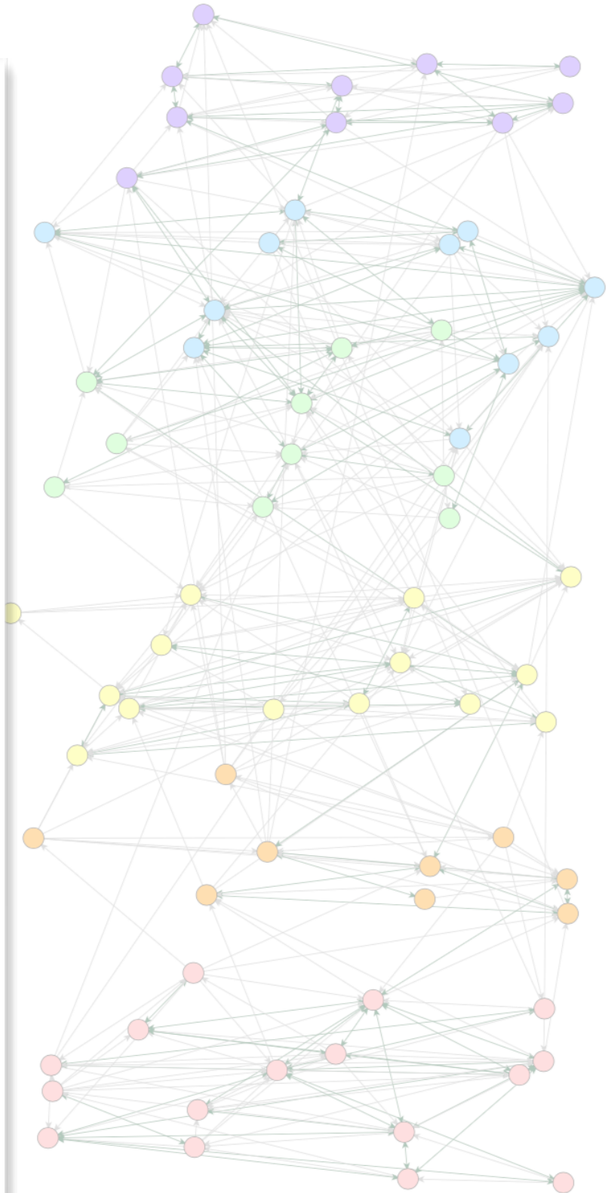


Figure 1 Diagram of the niche model. Each of ***S*** species (for example, ***S*** = 6, each shown as an inverted triangle) is assigned a 'niche value' parameter (n_i) drawn uniformly from the interval [0,1]. Species ***i*** consumes all species falling in a range (r_i) that is placed by uniformly drawing the centre of the range (c_i) from [r_i/2, n_i]. This permits looping and cannibalism by allowing up to half of r_i to include values $\geqslant n_i$. The size of r_i is assigned by using a beta function to randomly draw values from [0,1] whose expected value is 2***C*** and then multiplying that value by n_i [expected $E(n_i) = 0.5$] to obtain the desired ***C***. A beta distribution with $\alpha = 1$ has the form $f(x|1, \beta) = \beta(1-x)^{\beta-1}$, $0 < x < 1$, 0 otherwise, and $E(X) = 1/(1+\beta)$. In this case, $x = 1-(1-y)^{1/\beta}$ is a random variable from the beta distribution if y is a uniform random variable and β is chosen to obtain the desired expected value. We chose this form because of its simplicity and ease of calculation. The fundamental generality of species i is measured by r_i. The number of species falling within r_i measures realized generality. Occasionally, model-generated webs contain completely disconnected species or trophically identical species. Such species are eliminated and replaced until the web is free of such species. The species with the smallest n_i has $r_i = 0$ so that every web has at least one basal species.



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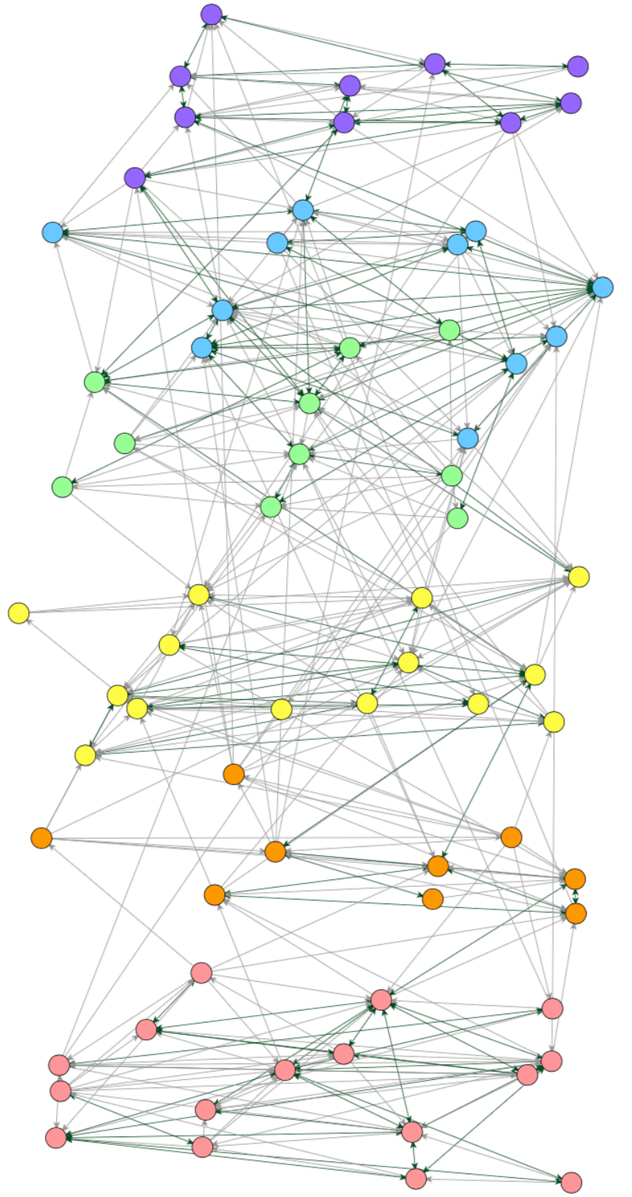
Discrete Choice Models (BTL, Thurstone V) embed products in a 1D space.

For BTL, outcome direction is simply:

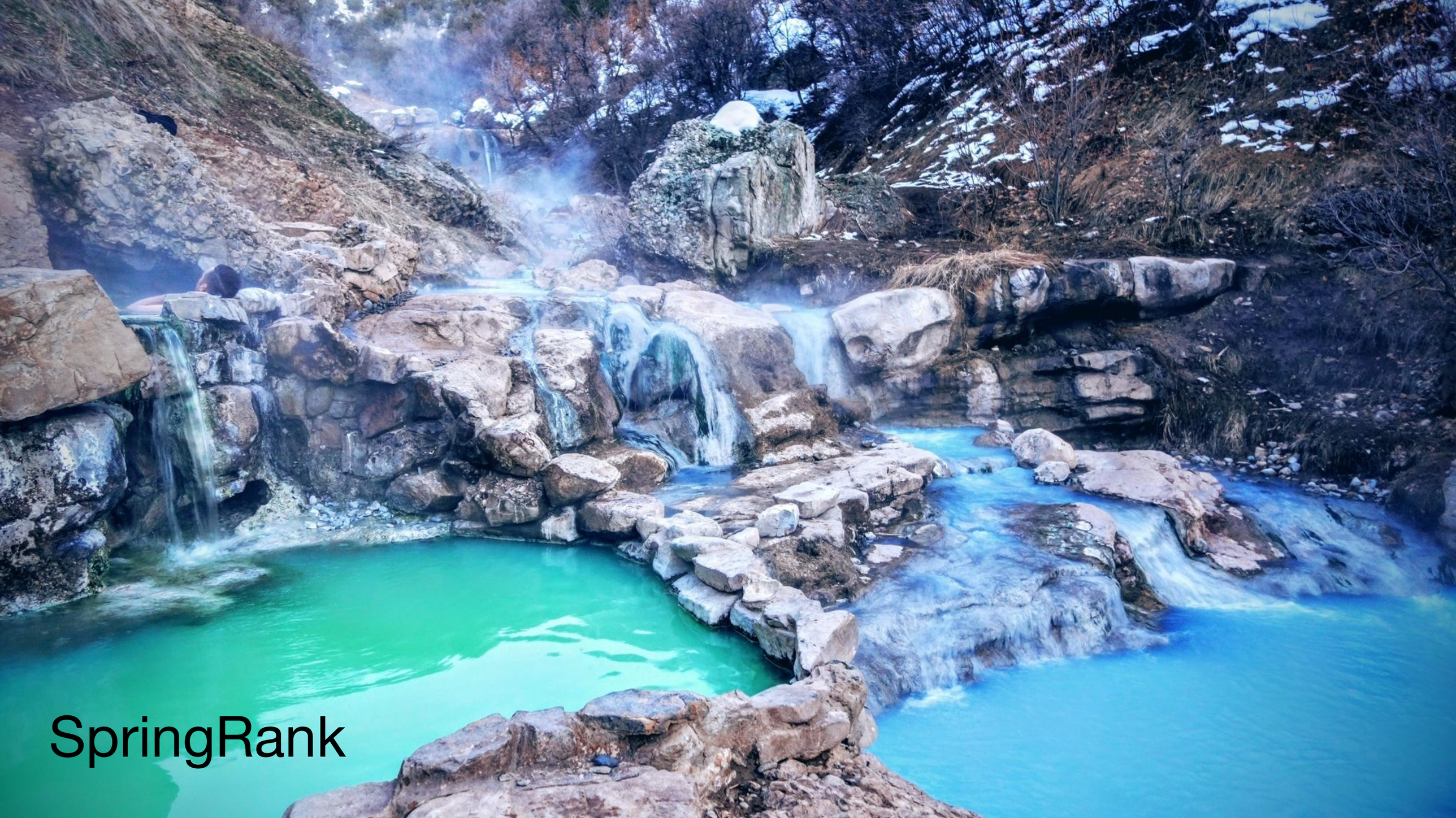
$$P(i \to j) = \frac{\gamma_i}{\gamma_i + \gamma_j}$$

- Provably avoids non-transitive properties.
- Great when lots of data per interaction.



Honestly? Best to google "BTL model." Williams & Martinez. Nature 404.6774 (2000). Ball, Newman. Network Science 1, 16-30 (2013)

The rest of this talk:
SpringRank—fast, interpretable, predictive.


SpringRank

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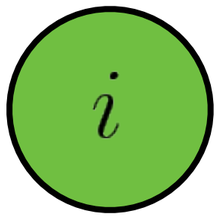
Each directed edge = directed spring



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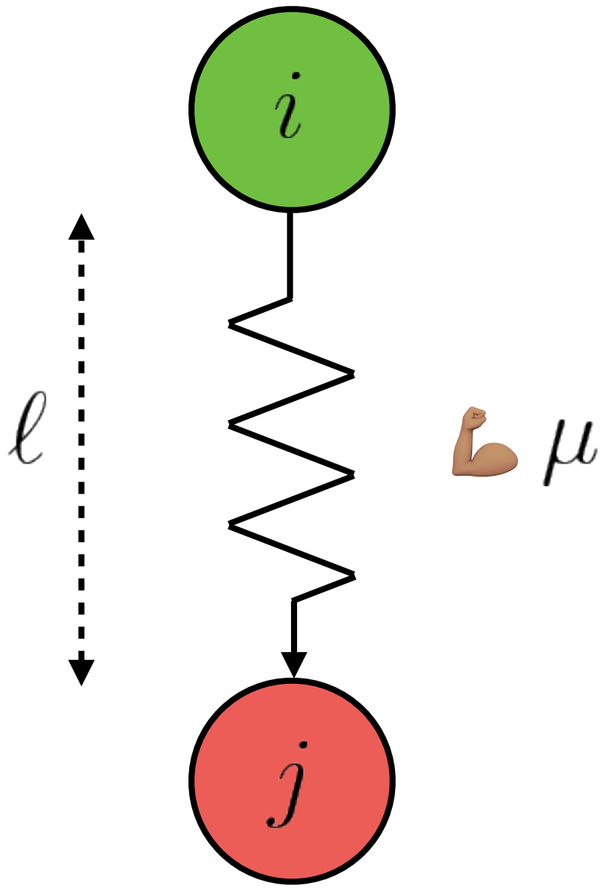

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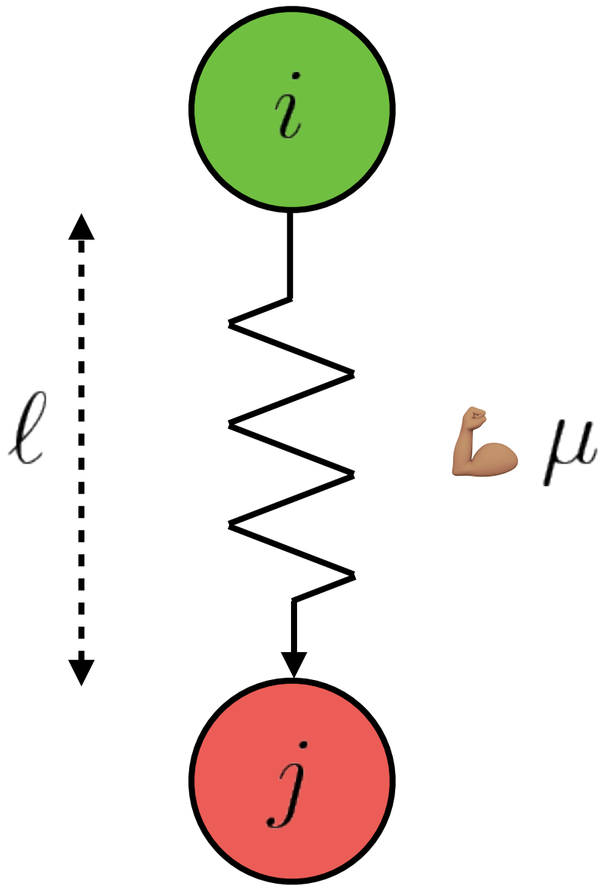

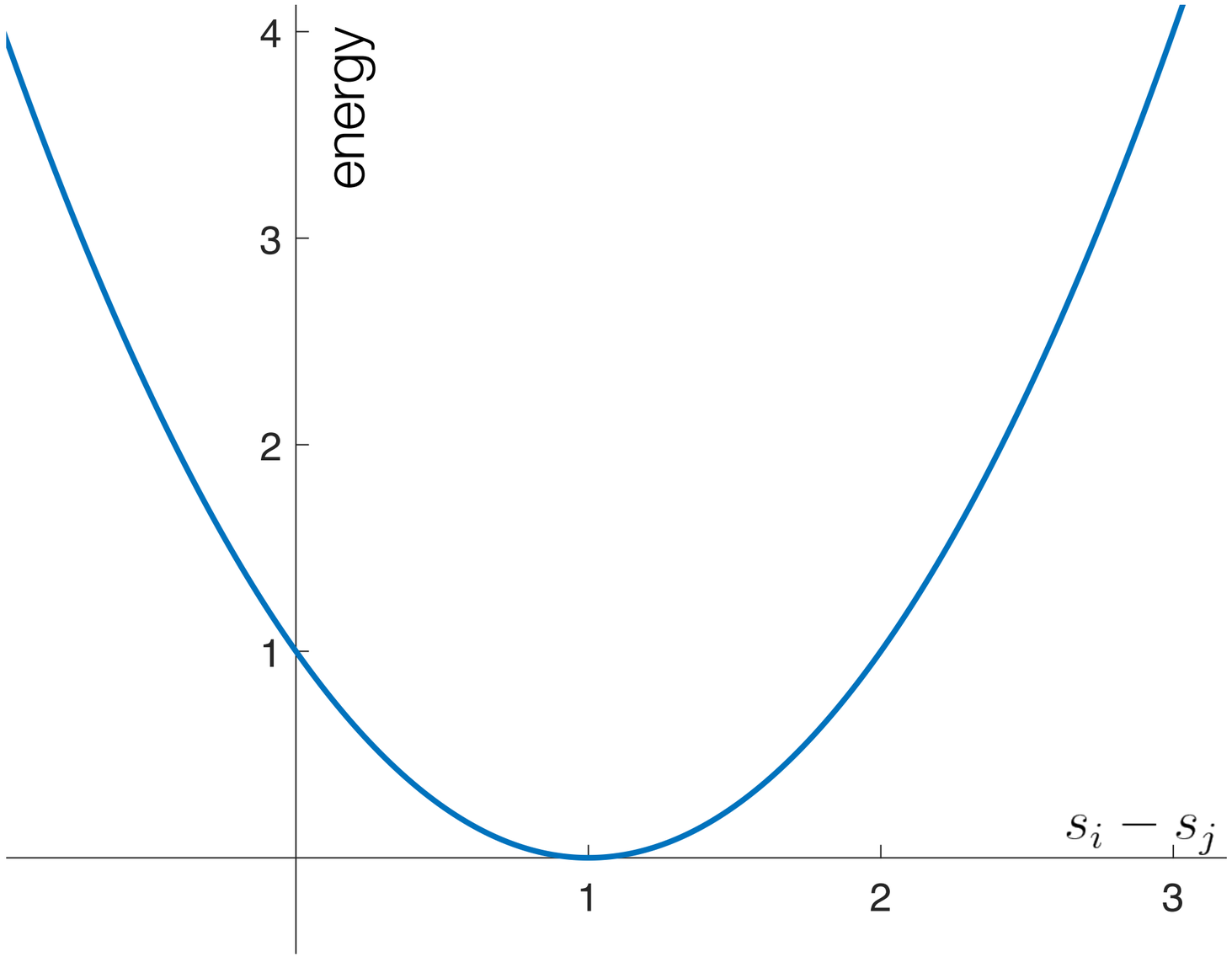

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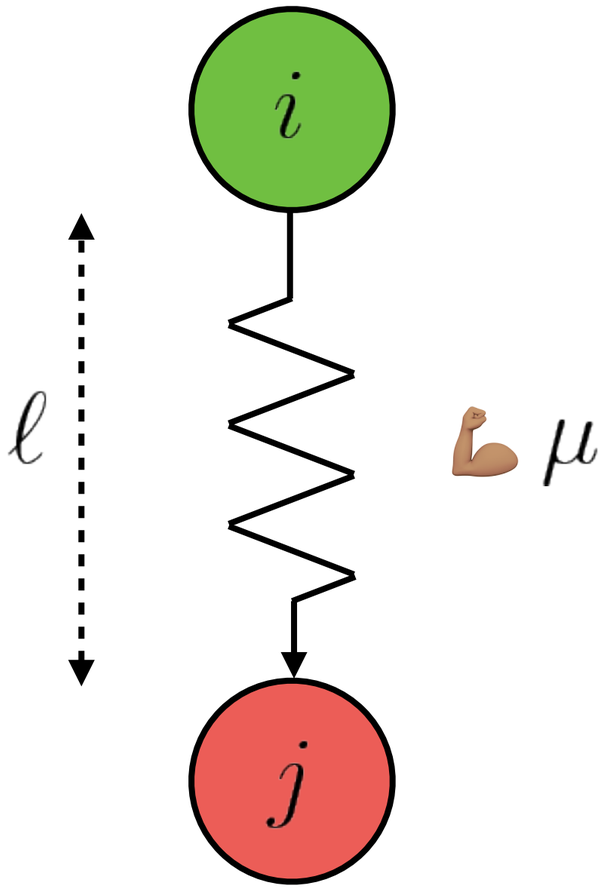

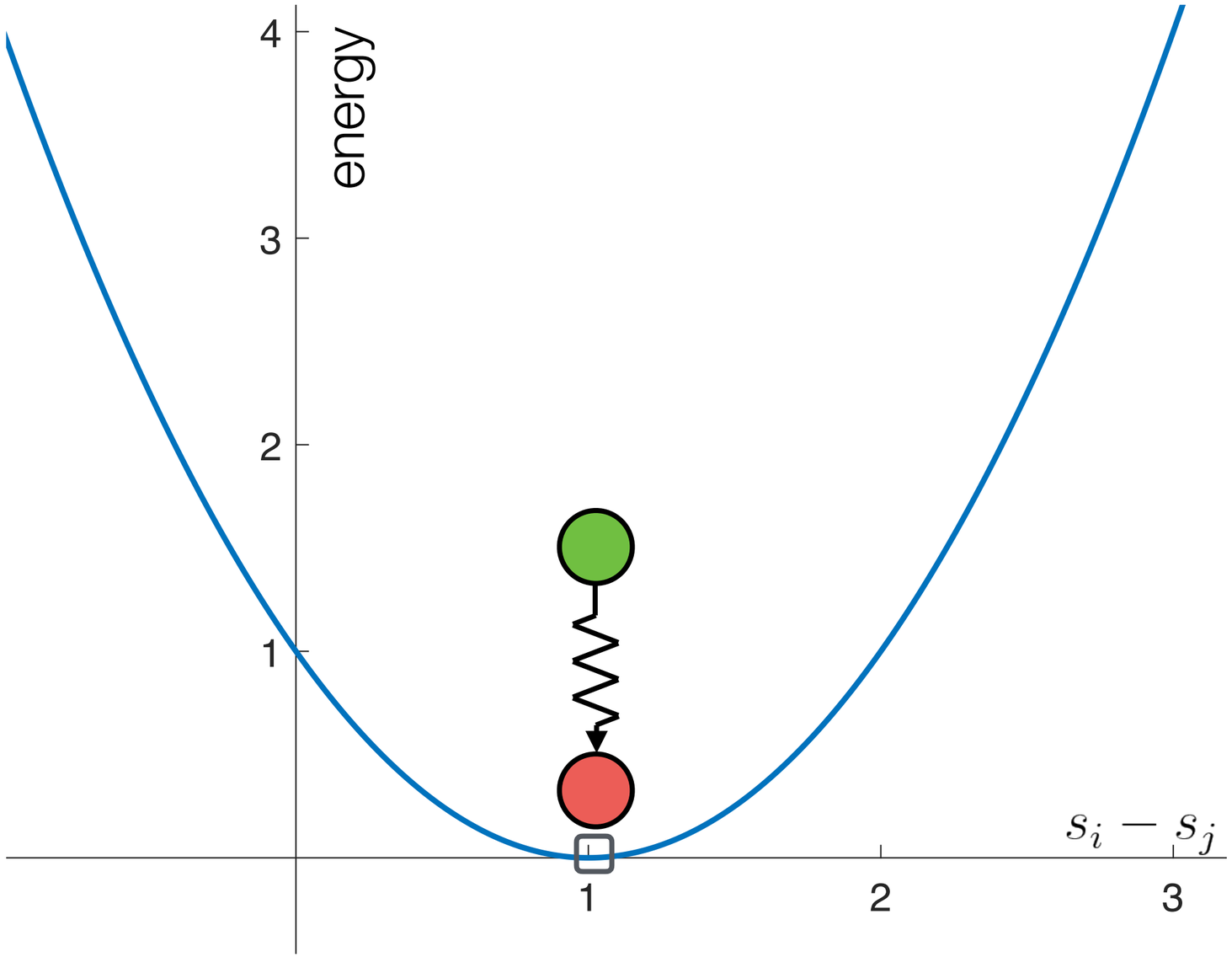

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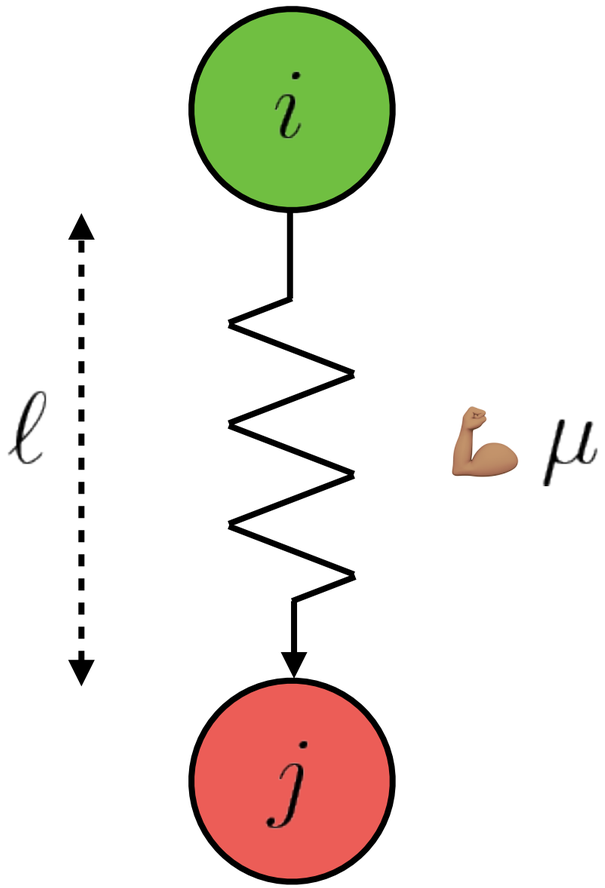

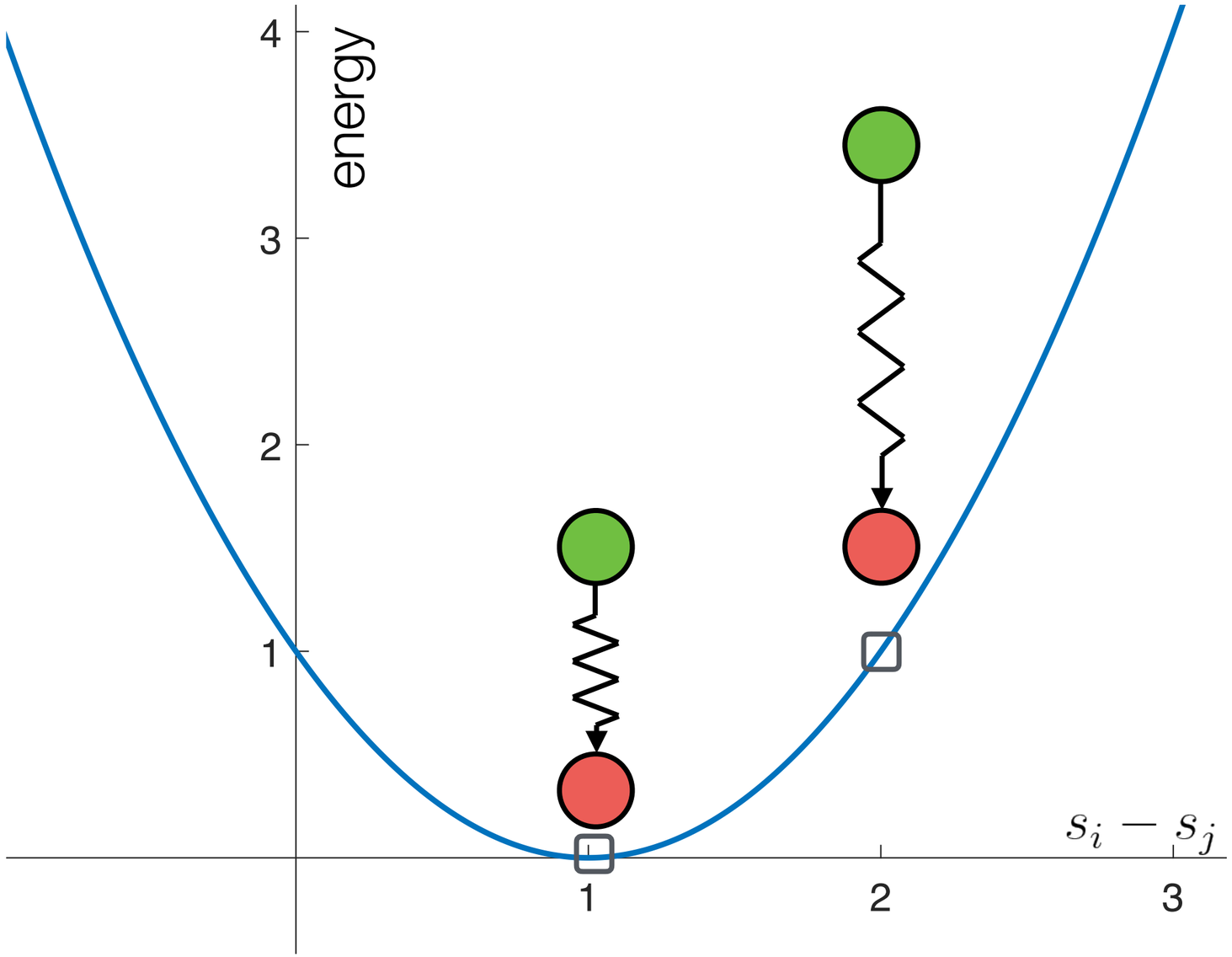

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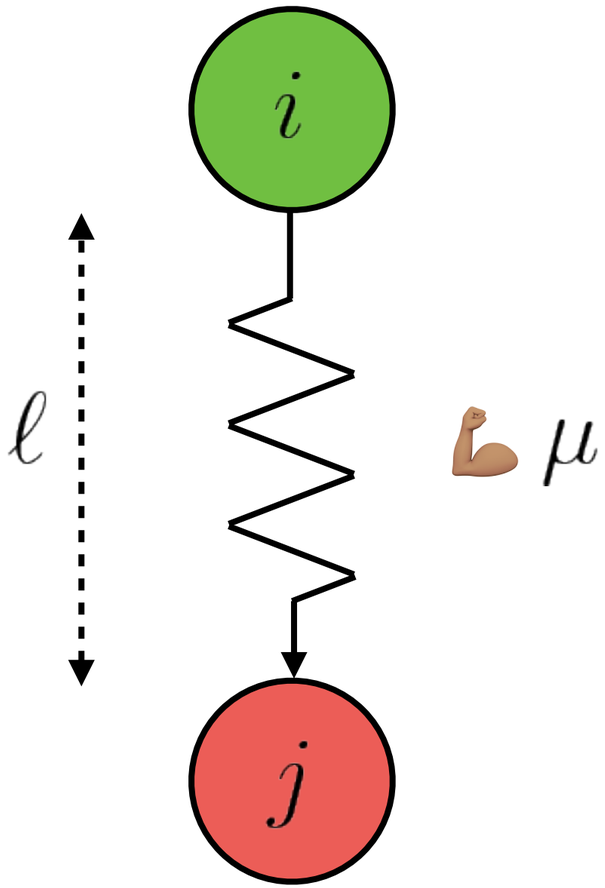

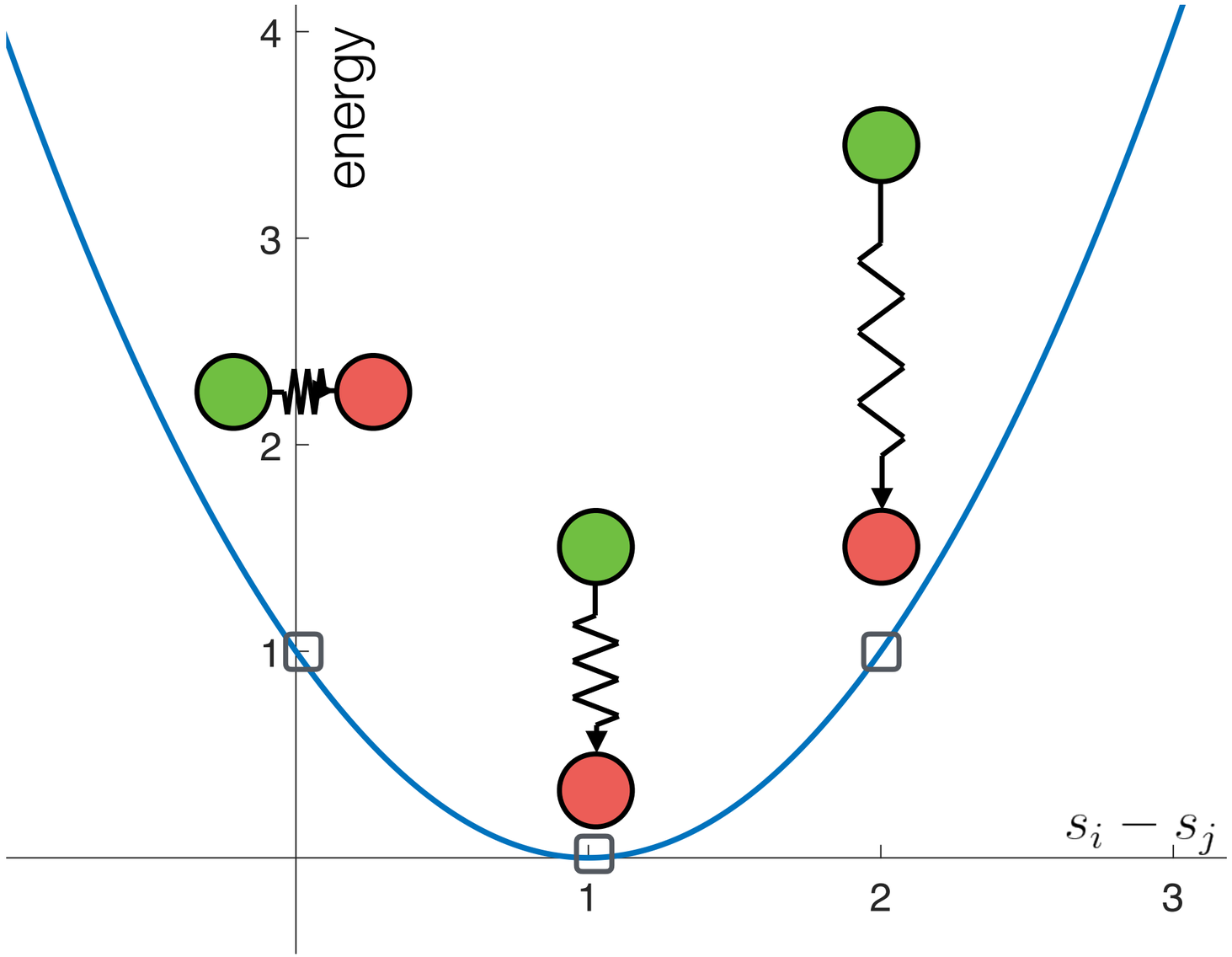

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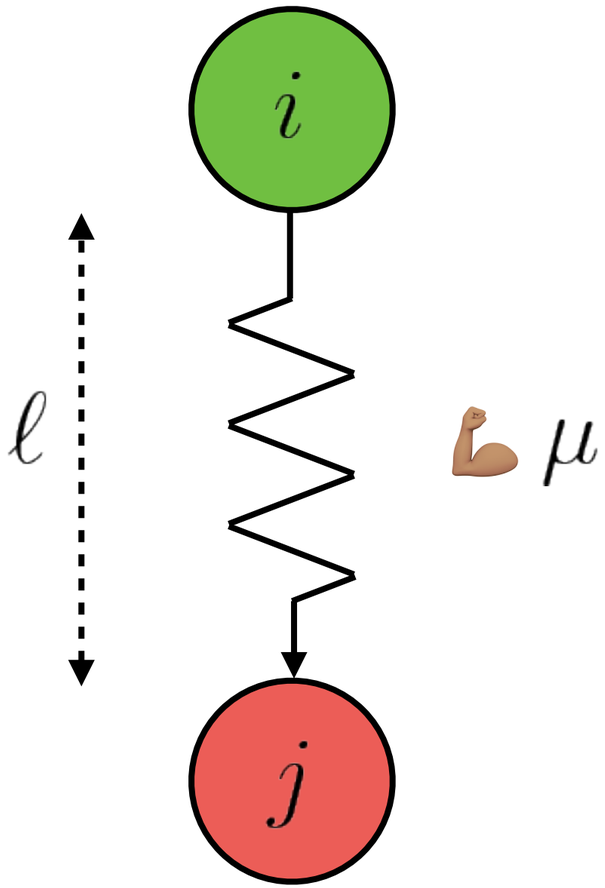

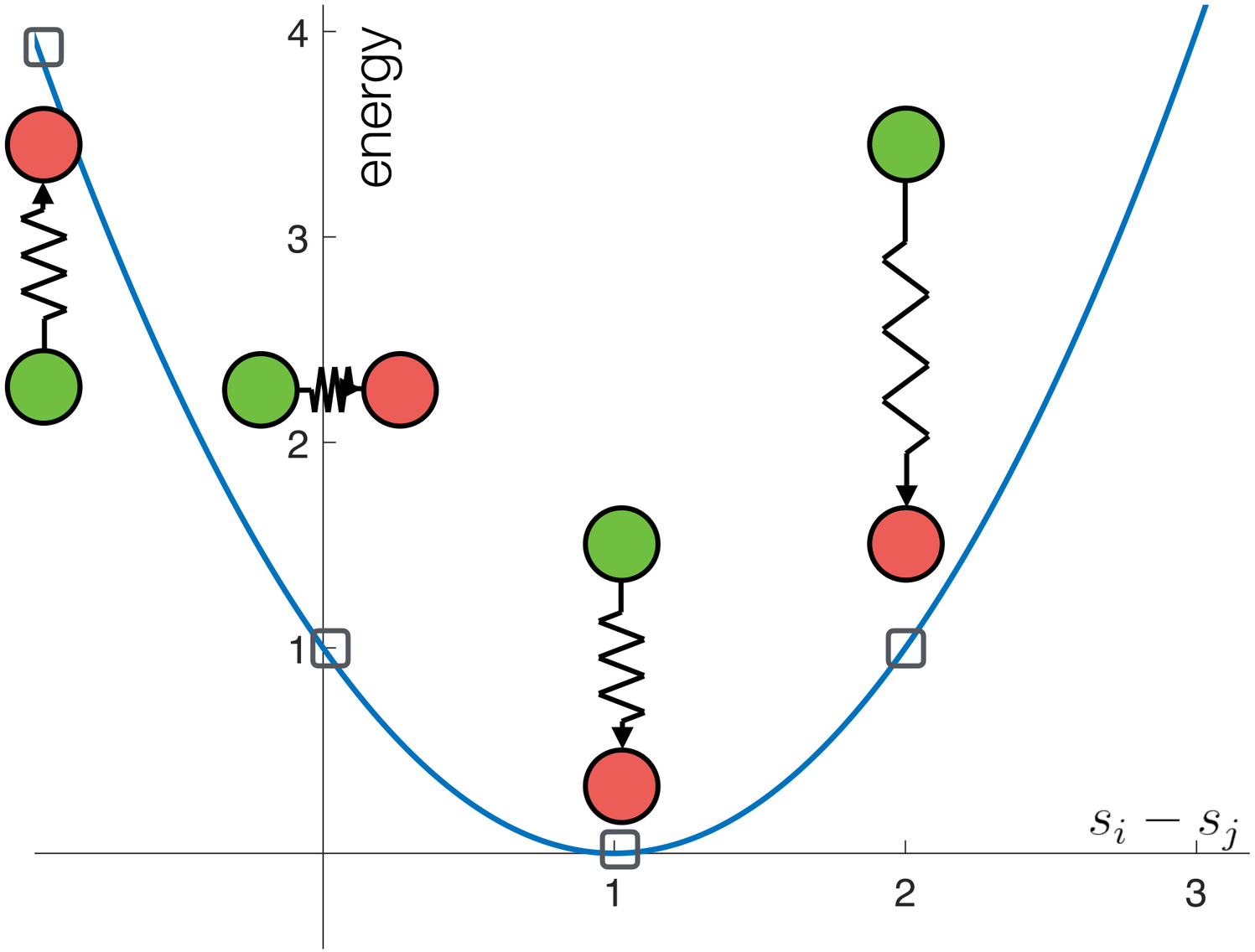

Relax and let the springs decide the ranks

$$H(s) = \frac{1}{2} \sum_{i,j=1}^{N} \mu_{ij} A_{ij} \left(s_i - s_j - \ell \right)^2$$

SpringRank Hamiltonian: energy of the system, given the positions *s*.

Because the springs are linear, the potential is quadratic.

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The SR Hamiltonian is *convex* in s.

$$\nabla H(s) = 0$$

The solution is unique…up to an additive constant. (Why?)

Derivatives work out nicely

$$0 = \frac{\partial H}{\partial s_i} = \sum_j A_{ij}\left(s_i - s_j - \ell\right) - A_{ji}\left(s_j - s_i - \ell\right)$$

NB: the matrix on the left is also called the *Network Laplacian matrix.*

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Rewrite as a linear algebra problem.

$$\left[D^{\text{out}} + D^{\text{in}} - \left(A + A^T\right)\right] s^* = \ell \left[D^{\text{out}} - D^{\text{in}}\right] \mathbf{1}$$

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[if *s* is a solution, then *s* + *k* is a solution for any constant *k;* eigenvalue 0, eigenvector **1**]

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3. Impose some regularization to prevent wandering.
   You could call it an external field.
   Or an undirected spring attached to the origin.

$$H_{\text{reg}}(s) = H(s) + \frac{\alpha}{2} \sum_{i=1}^{N} s_i^2$$

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now the eigenvector **1** has eigenvalue α

Dealing with translational invariance

$$\left[D^{\text{out}} + D^{\text{in}} - \left(A + A^T\right)\right] s^* = \ell \left[D^{\text{out}} - D^{\text{in}}\right] \mathbf{1}$$

1. Invert this matrix on the *n-1* dimensional subspace orthogonal to **1**. (pseudoinverse)

2. Fix a rank. Say s_1=0. Or min(*s*)=0. Or mean(*s*)=0.

3. Impose some regularization to prevent wandering.
   You could call it an external field.
   Or an undirected spring attached to the origin.

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$\alpha \to 0$

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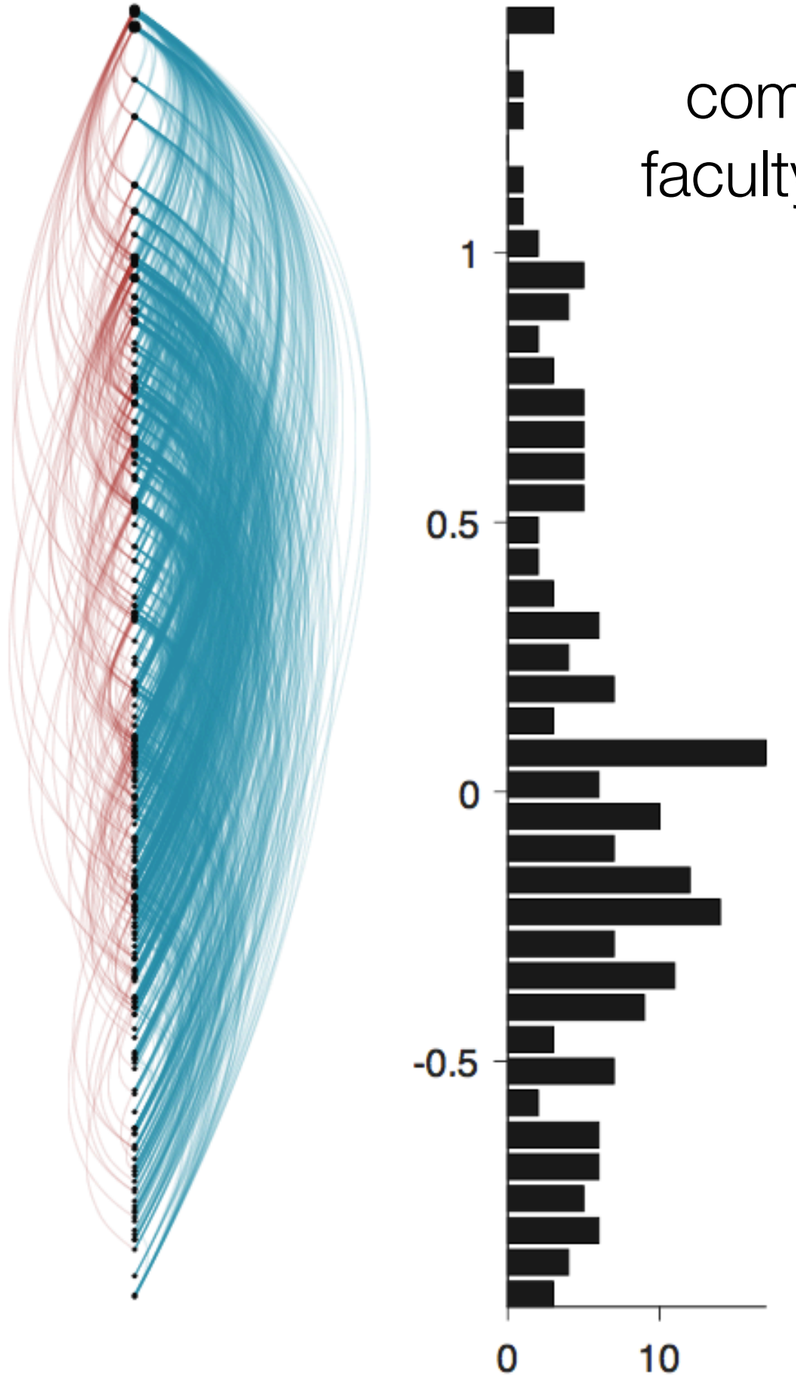
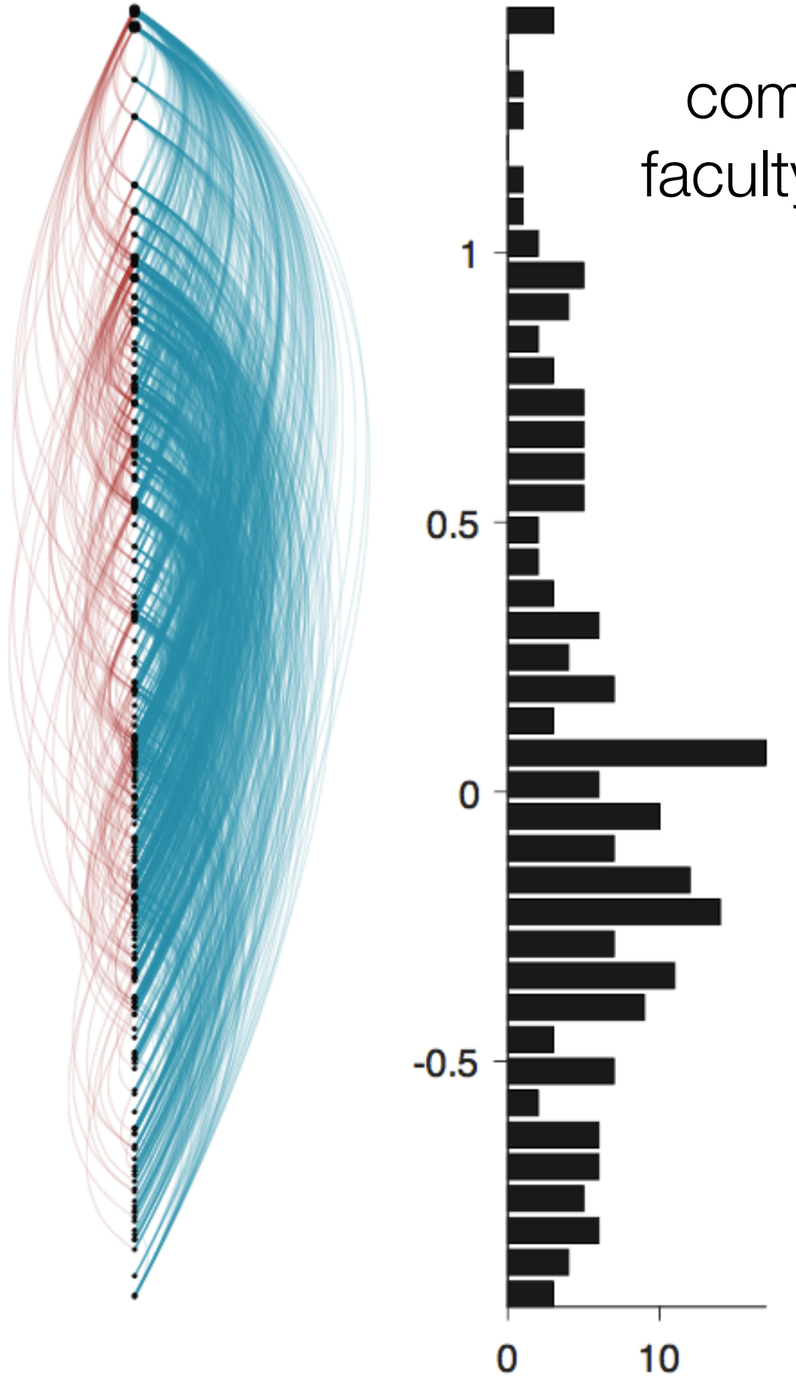
It works!



computer science
faculty hiring network

It works!

Real networks tend to be sparse…
our linear algebra problem is sparse…
we can use sparse iterative solvers…
millions of edges in **seconds.**



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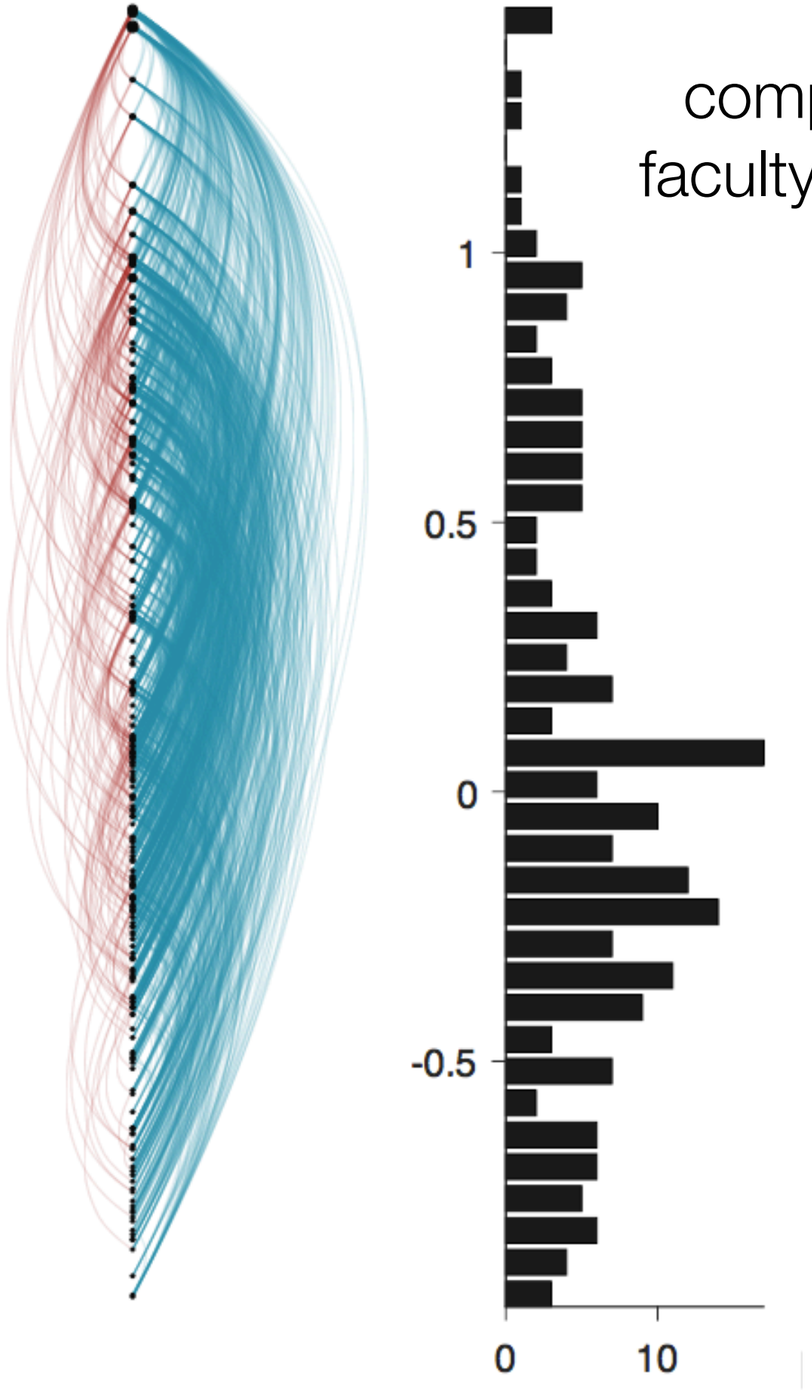

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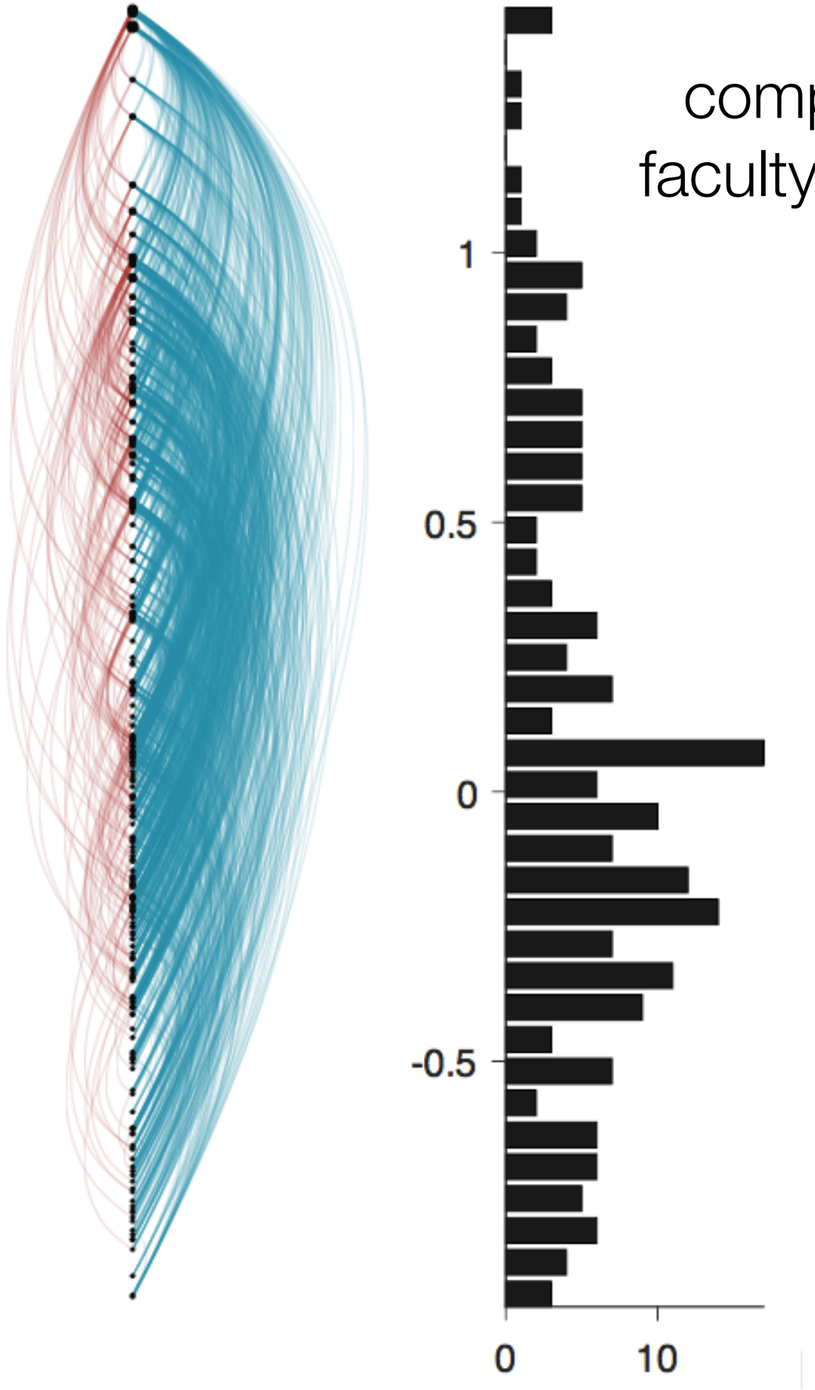
Note that node positions can be clumpy.



computer science faculty hiring network

How certain are we?

$$P(s) \propto e^{-\beta H(s)}$$

Boltzmann distribution: probability around ranks s

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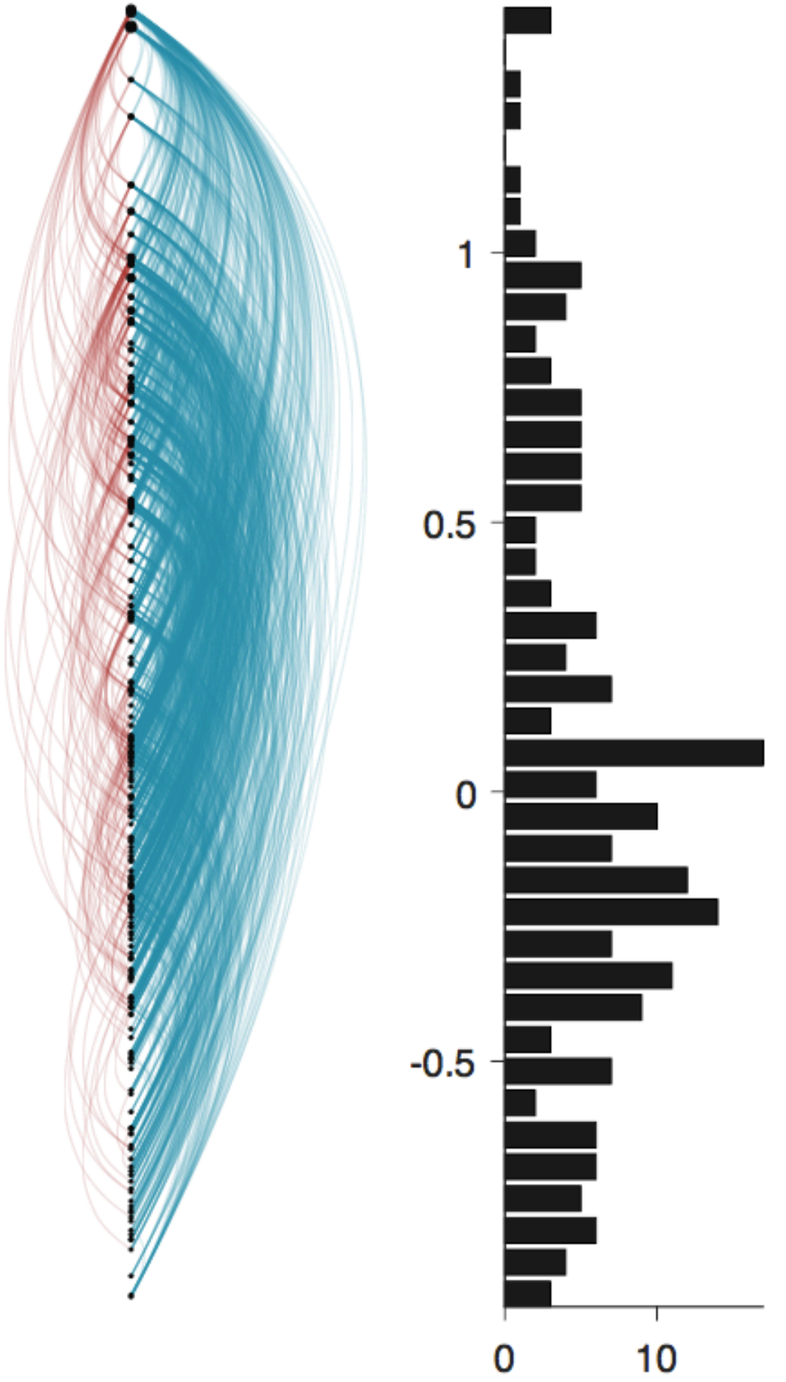

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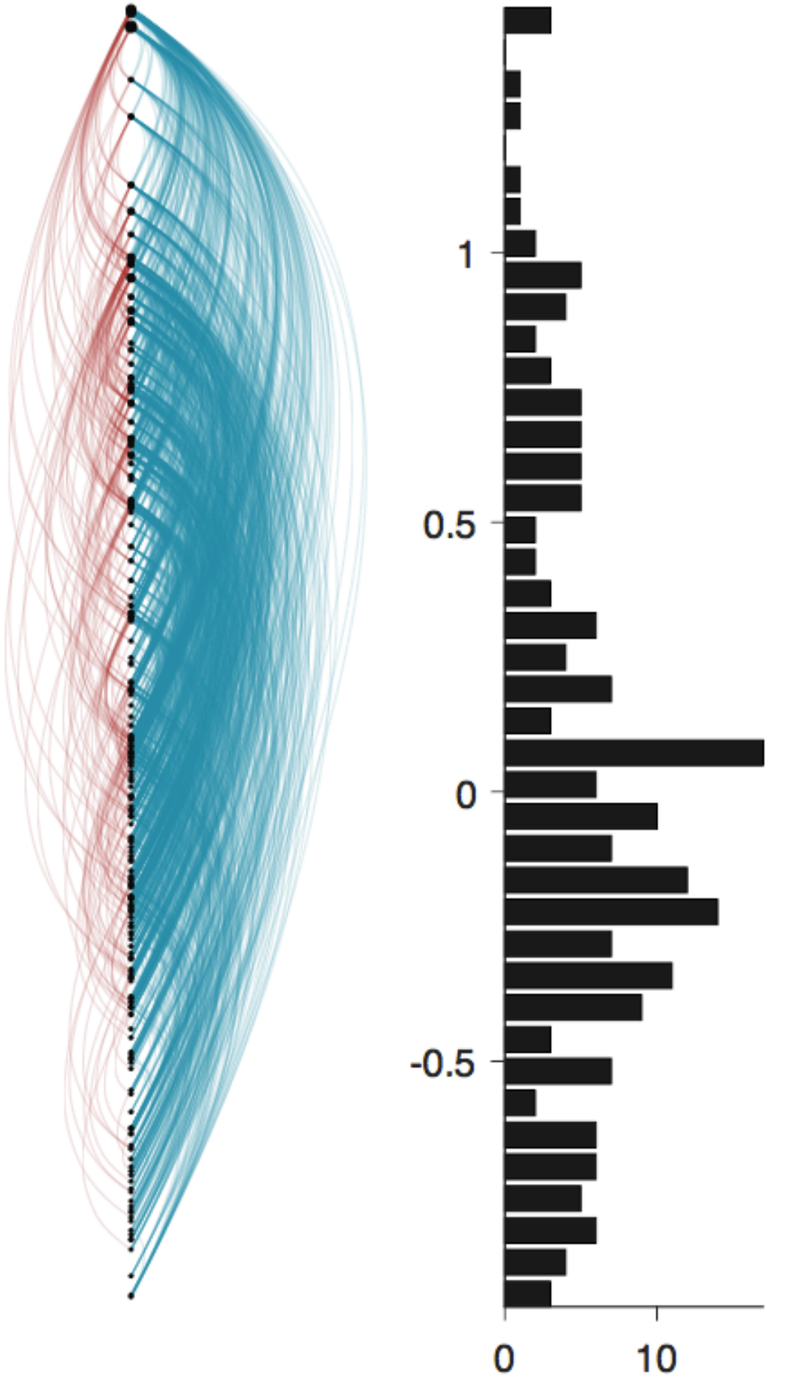

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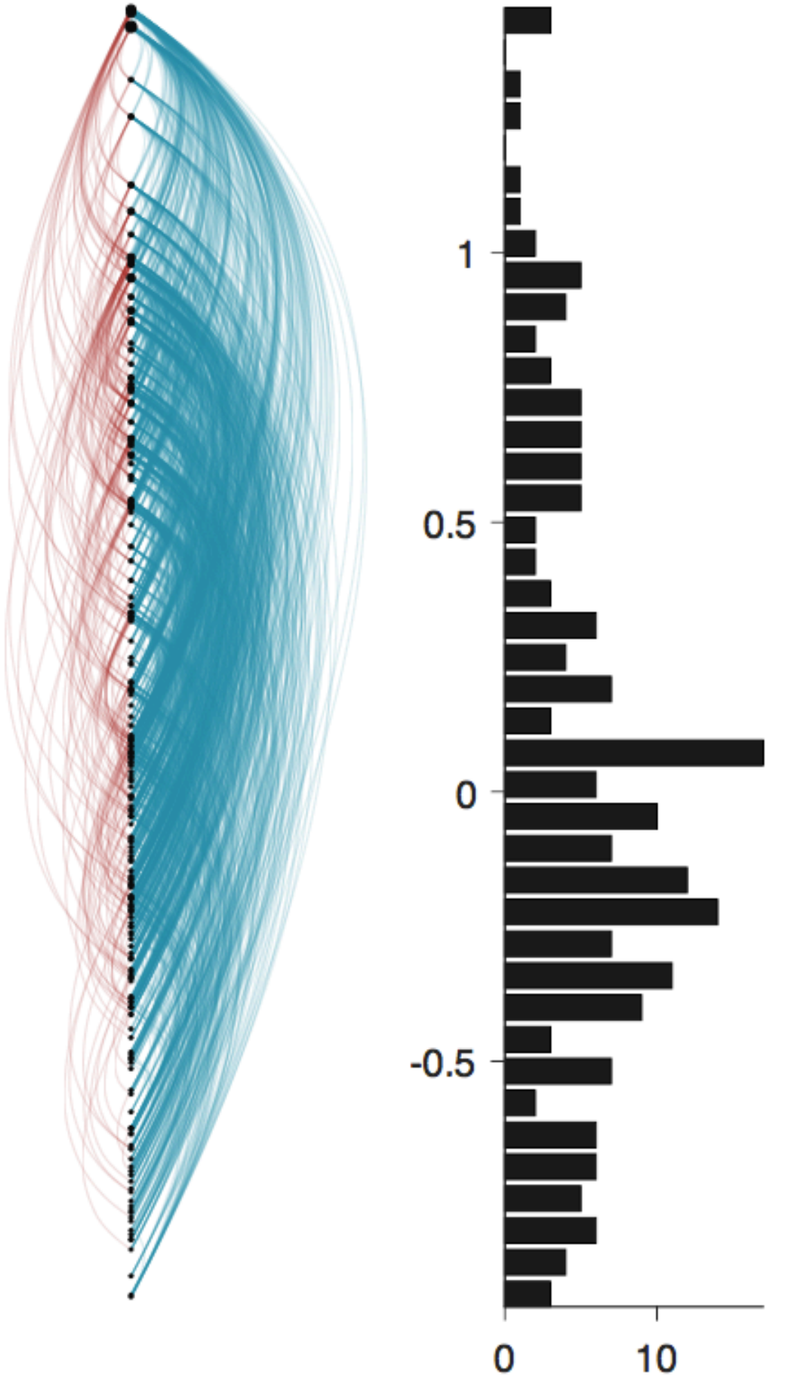

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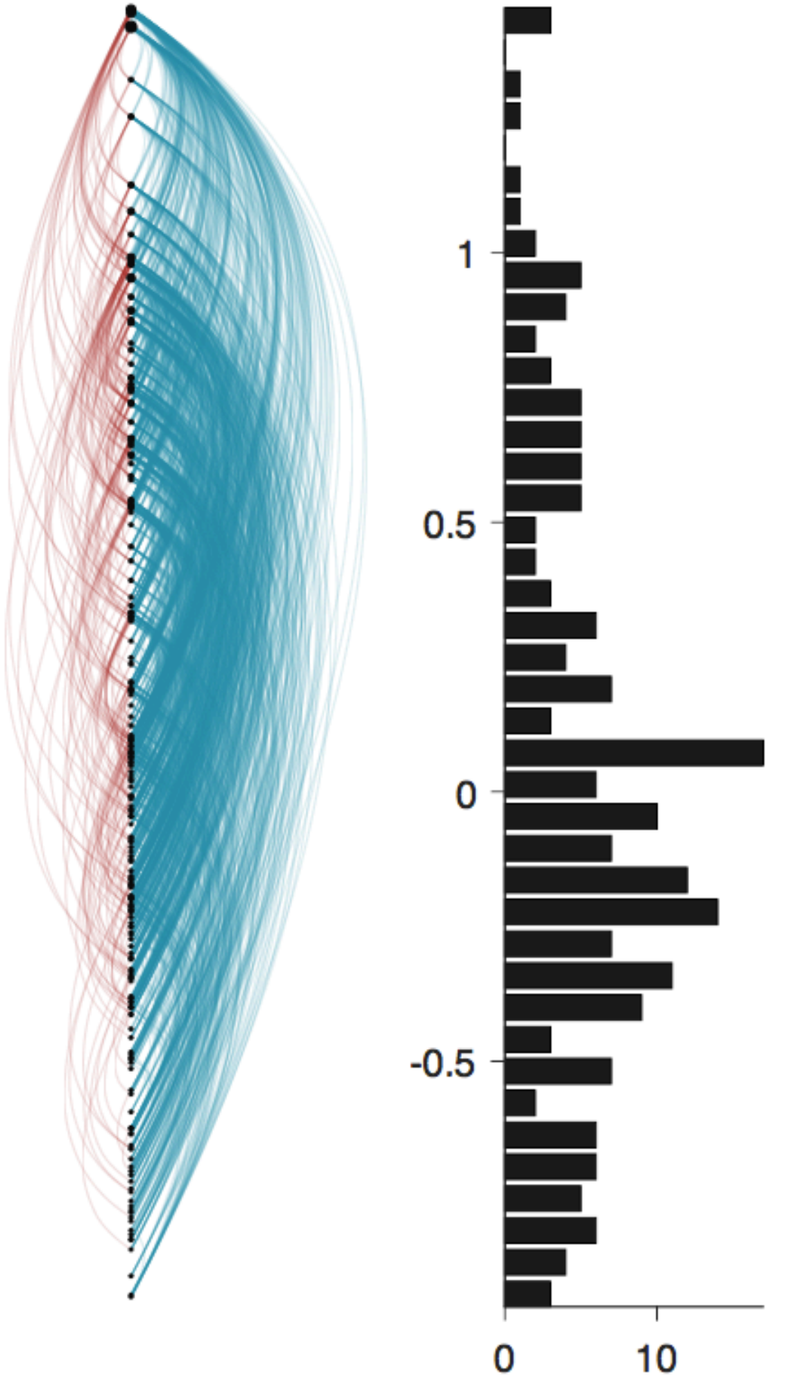
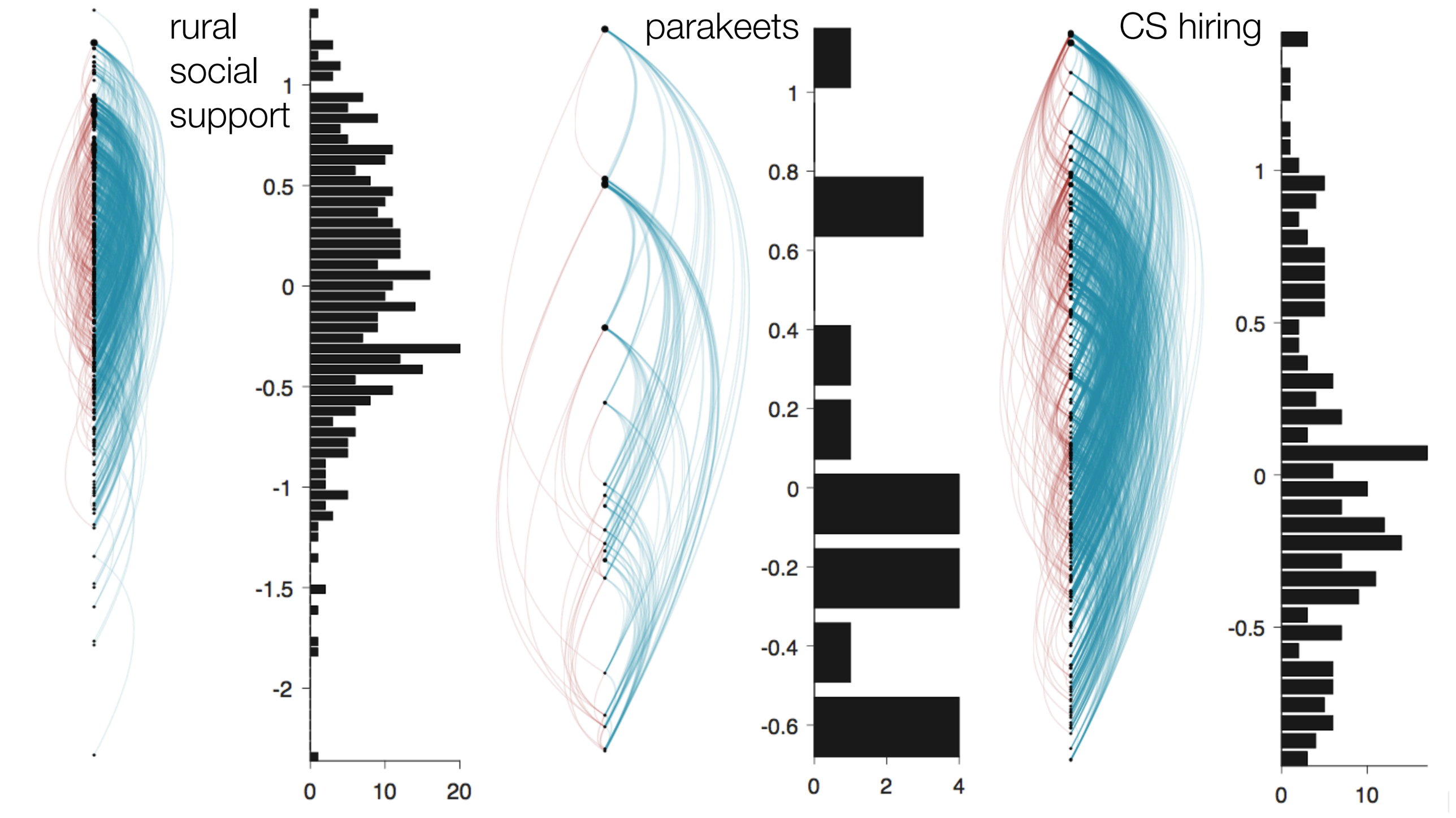
Comparison shows that mean rank is the SR solution,
and ranks are distributed according to n-variate Gaussian!

$\Sigma = \frac{1}{\beta}\left[D^{\text{out}} + D^{\text{in}} - \left(A + A^T + \alpha I\right)\right]^{-1}$ covariance matrix




rural
social
support
1
0.5
0
-0.5
-1
-1.5
-2
0
10
20
parakeets
1
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0.6
0.4
0.2
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4
CS hiring
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0.5
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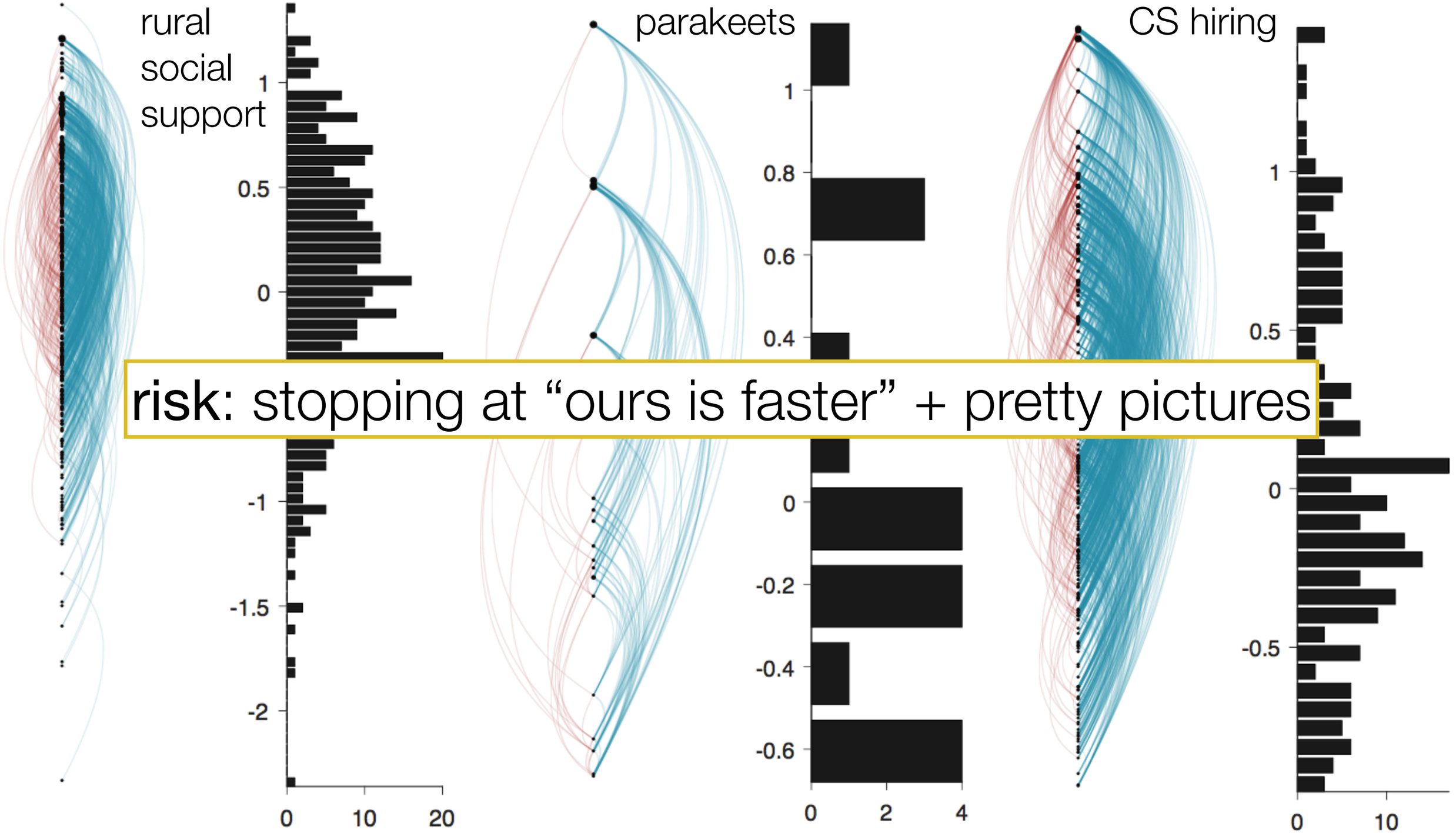
-0.5
0
10



risk: stopping at “ours is faster” + pretty pictures

Generative models

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Recall assumptions:

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*On the **second** day, a number of interactions occurred, whose outcomes were chosen by the Research Team in order to reflect the qualities, s.*

*On the **third** day, these interactions were written down as an adjacency matrix, and the Reviewers saw that it was good.* 😌

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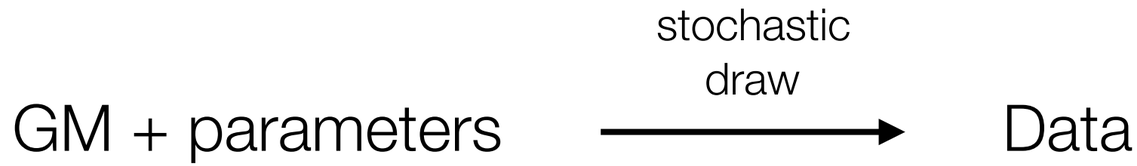

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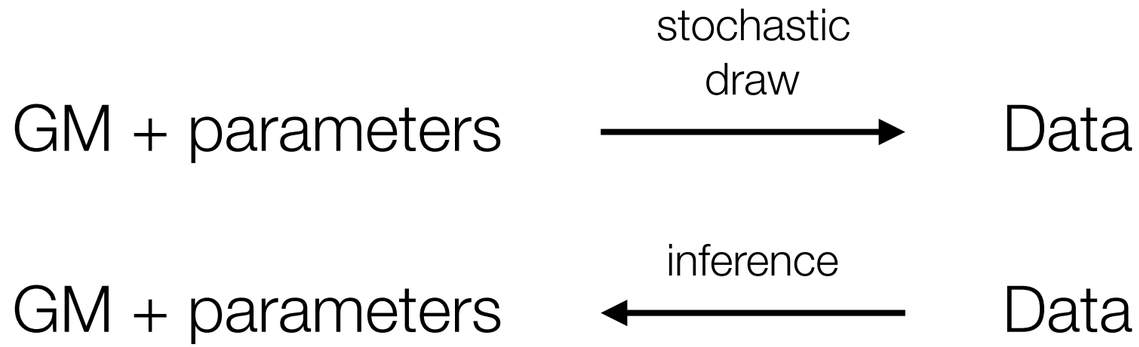

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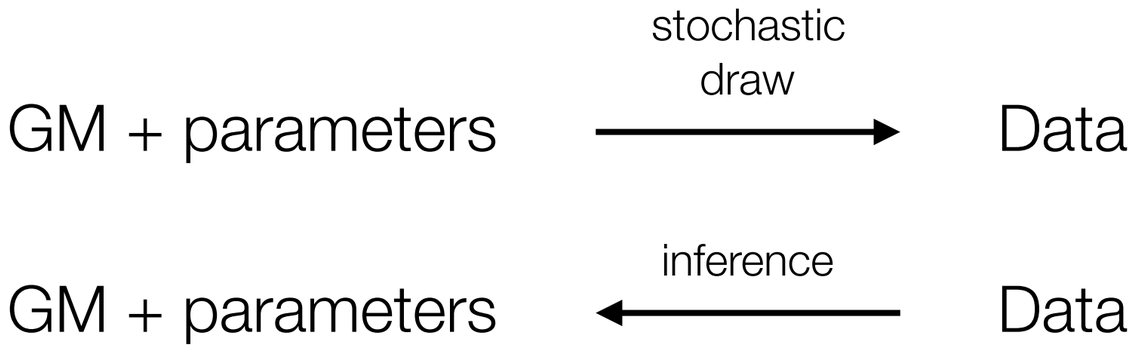

Let the expected number of edges from *i* to *j* be:

$$\mathrm{E}[A_{ij}] = c \exp\left[-\frac{1}{2}\beta(s_i - s_j - 1)^2\right]$$

And let the actual number of edges be drawn from a Poisson distribution with that mean.

Parameters…

$$\mathrm{E}[M] = \sum_{i,j}[A_{ij}] = c\sum_{i,j}\exp\left[-\frac{1}{2}\beta(s_i - s_j - 1)^2\right]$$

Thus, the total number of edges is proportional to *c*, meaning *c* controls for sparsity.

β specifies the disorder in the system; the extent to which edges respect the ranks *s*.

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The likelihood of observing/generating data *A*, given the parameters is:

$$P(A \mid s, \beta, c) = \prod_{i,j} \frac{\left[ce^{-\frac{\beta}{2}(s_i - s_j - 1)^2}\right]^{A_{ij}}}{A_{ij}!} \exp\left[-ce^{-\frac{\beta}{2}(s_i - s_j - 1)^2}\right]$$

Maximize the likelihood to find the ranks!

the usual tricks:

maximize the *log*; take derivatives & set to zero; chuck out additive constants.

$$\mathcal{L}(A \mid s, \beta) = -\beta H(s) - M \log \left[\sum_{i,j} e^{-\frac{\beta}{2}(s_i - s_j - 1)^2} \right]$$

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In other words: solving SpringRank is asymptotically equivalent to inferring *s* using the GM.

What about priors?

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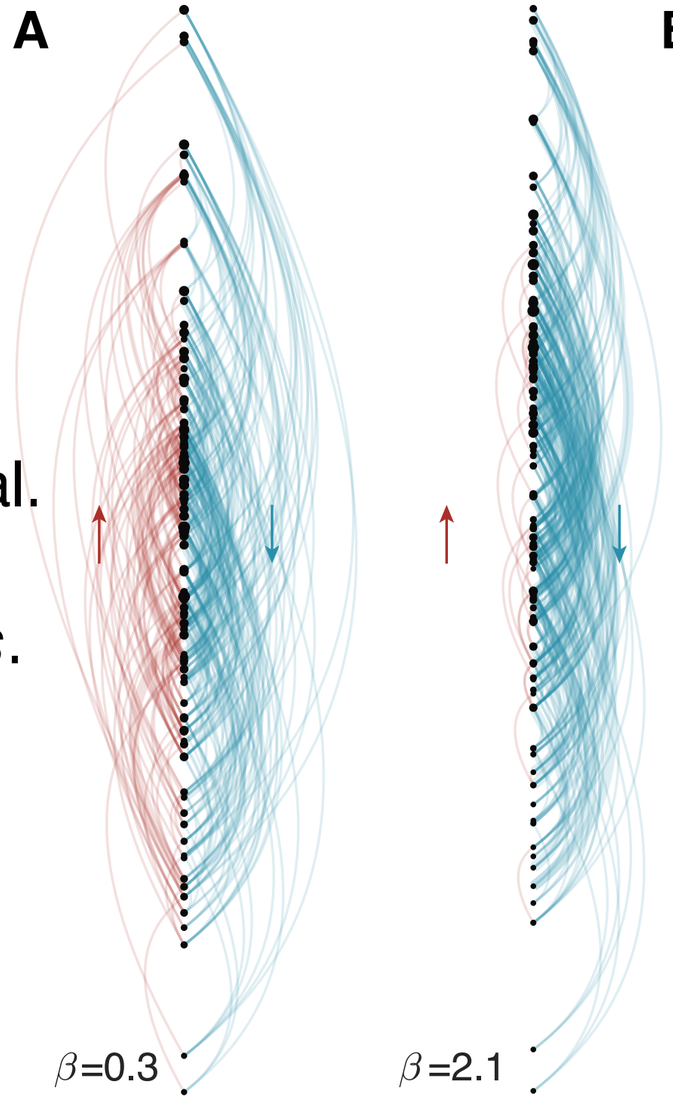
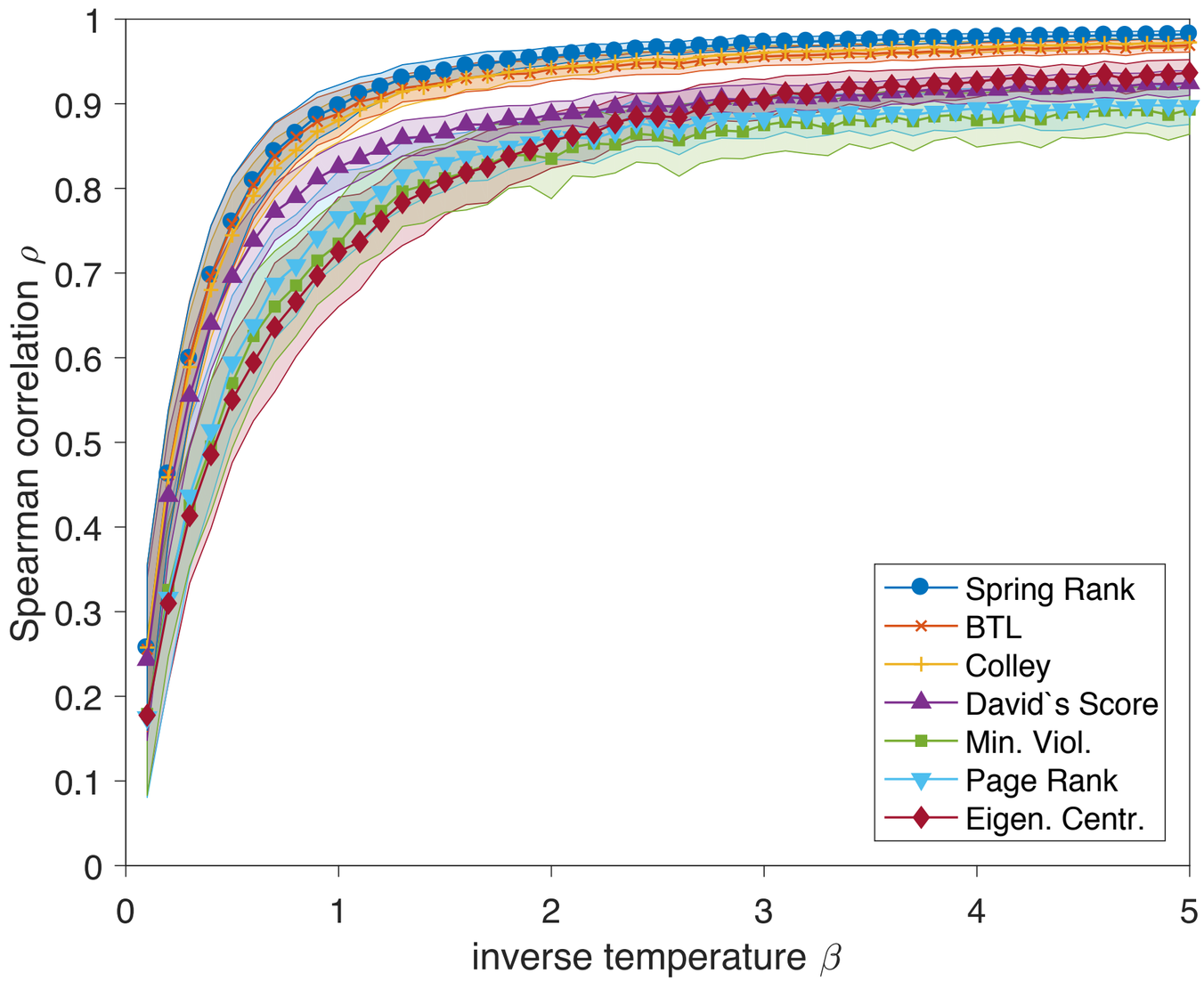
Put differently: a Gaussian prior over *s* corresponds to the regularization from before!

Consistency: Test SpringRank using GM synthetic data

Easy:

1. Plant *s* from **standard normal.**
2. Infer ranks.
3. Compare to *s*.





Consistency: Test SpringRank using GM synthetic data

Hard:

1. Plant *s* from **three normals.**
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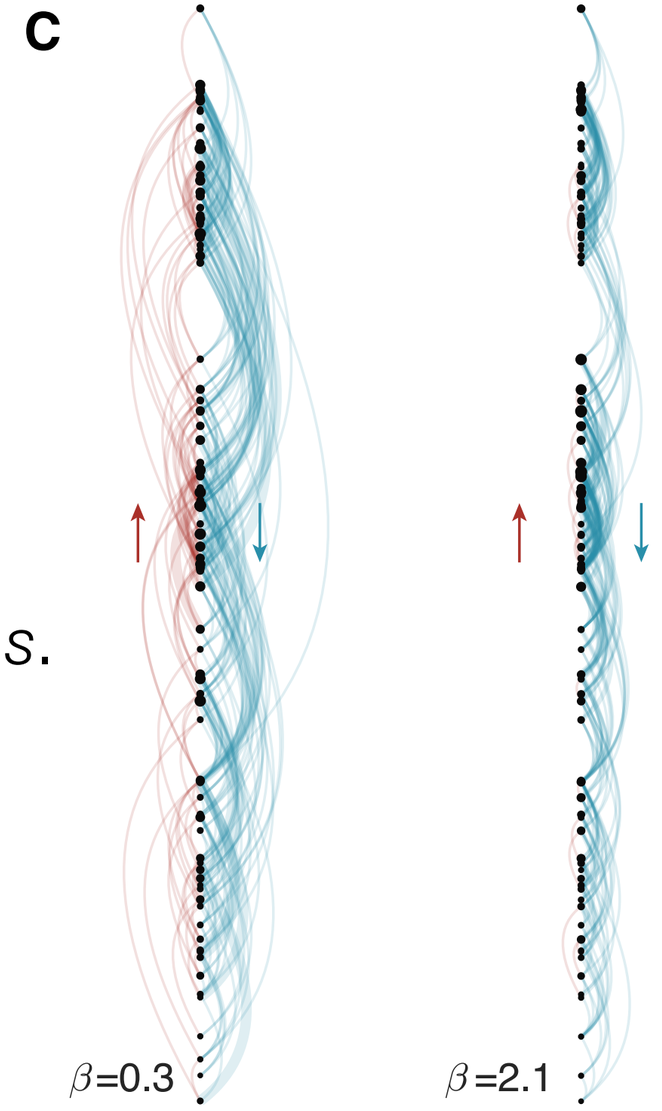

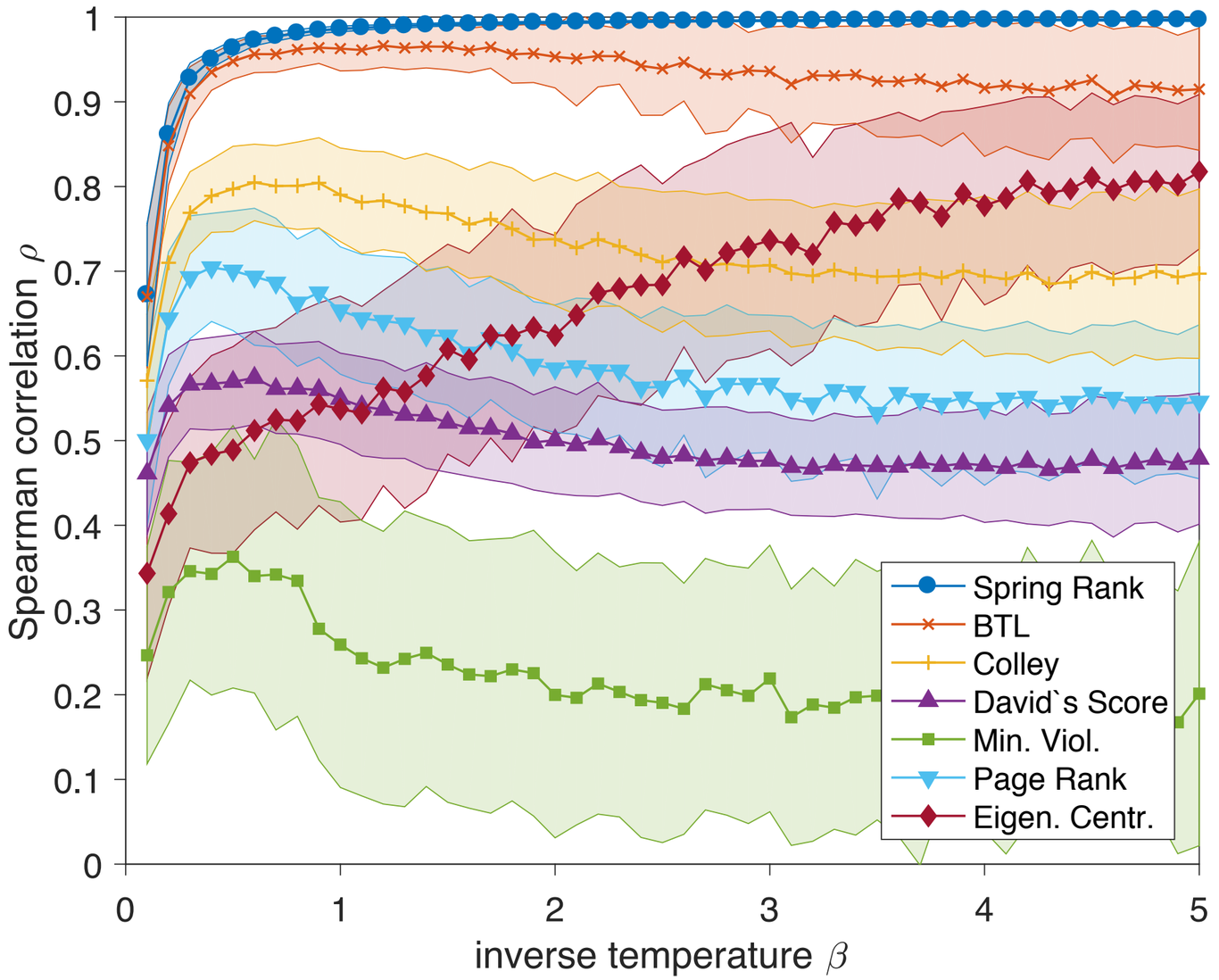

Edge prediction & cross validation

Cross validation: train on 80%, predict 20%

In a linear hierarchy the key quantity to predict is *edge direction,* given *edge existence*.

If *i* and *j* were to face off, who would win?

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SpringRank predicts edge direction based on the relative direction probabilities:

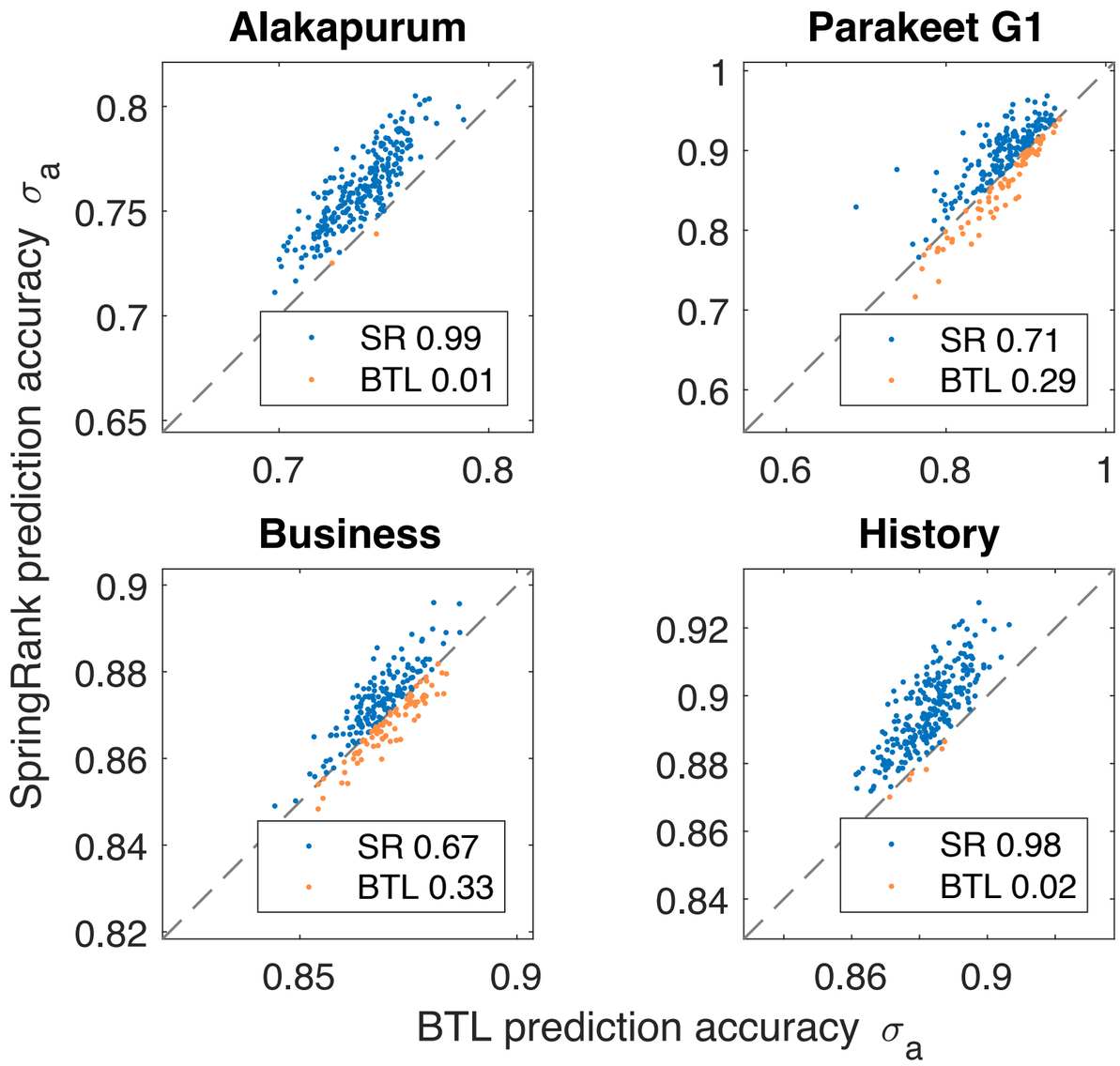
$$P_{ij}(\beta) = \frac{e^{-\beta H_{ij}}}{e^{-\beta H_{ij}} + e^{-\beta H_{ji}}} = \frac{1}{1 + e^{-2\beta(s_i - s_j)}}$$

Cross validation results

Accuracy:

$$\sigma_a = 1 - \frac{1}{2M} \sum_{i,j} |A_{ij} - (A_{ij} + A_{ji})\, P_{ij}|$$

Goal: maximize the number of correctly predicted edge directions



50 independent trials of 5-fold cross validation (250 folds)

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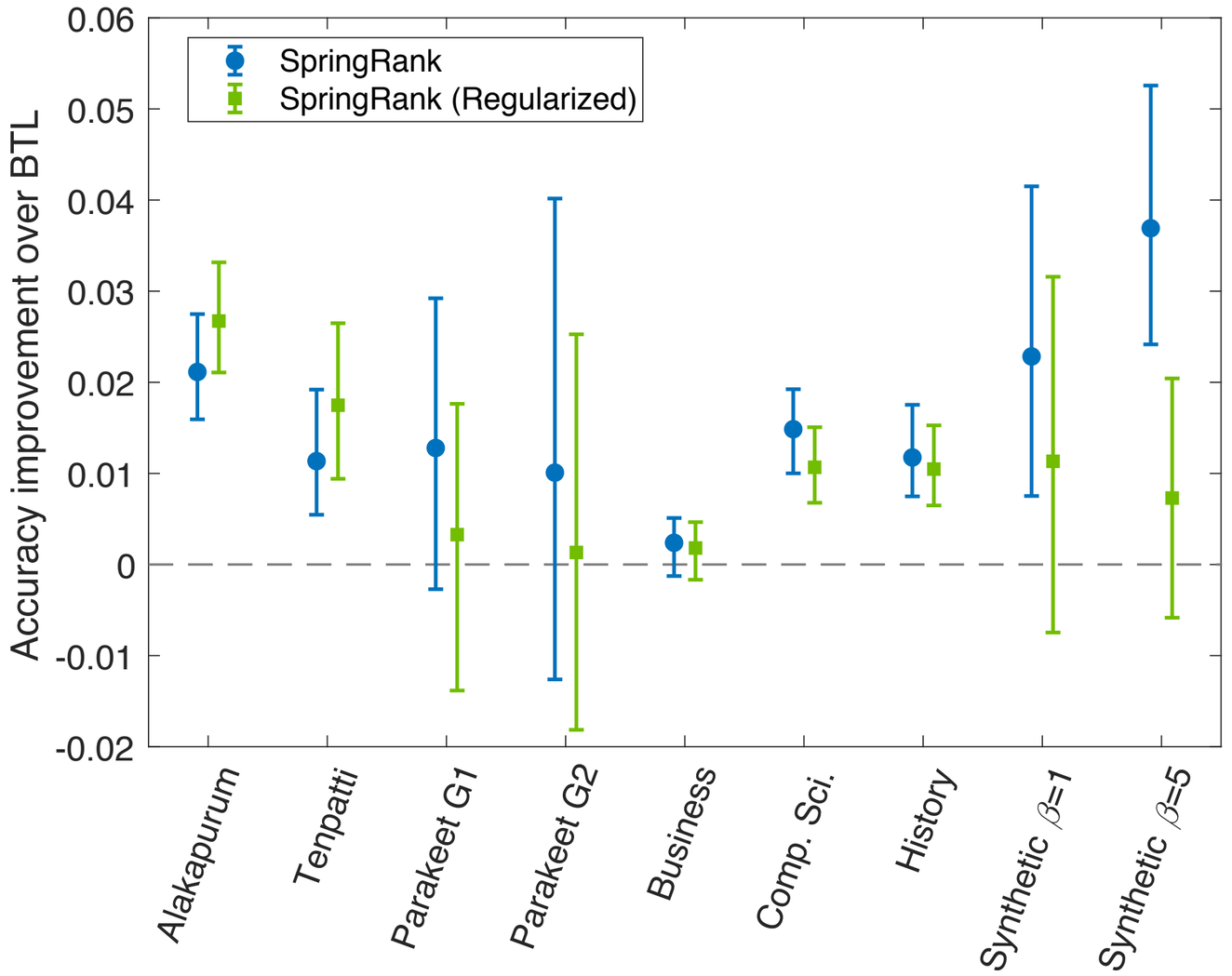
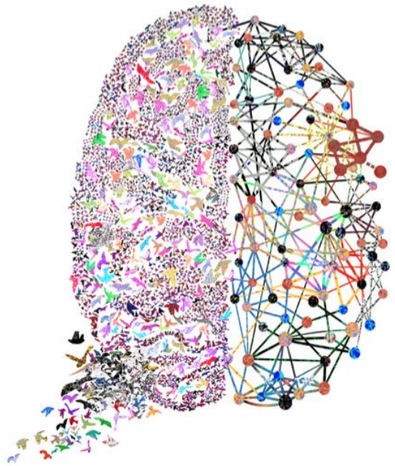


medians & quartiles; 50 independent trials of 5-fold cross validation (250 folds)

Conference on Complex Networks

COMPLENET '18

Hosted by Northeastern University Network Science Institute



BOSTON, MA APRIL 2018

Xindi Wang

netsi | Northeastern University
Network Science Institute

LEARNING TO PLACE OBJECTS: A NETWORK-BASED APPROACH

Xindi Wang

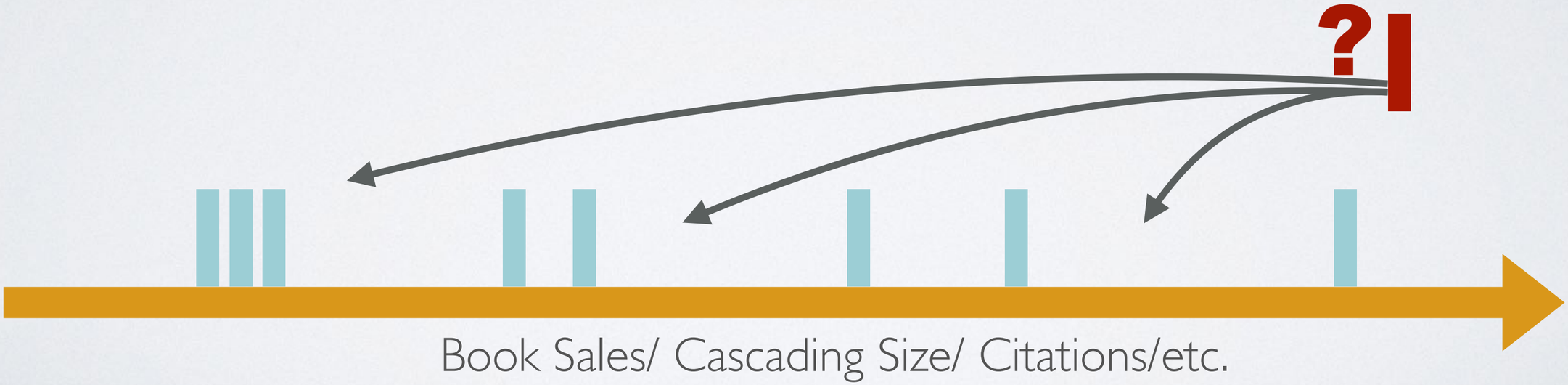
Onur Varol

Tina Eliassi-Rad

Albert-László Barabási

PROBLEM DEFINITION

Learning to place: Given a sequence of ordered items, where should we place a new item?



AUC

Method		AUC on Fiction	AUC on Biography
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Pairwise +	Cohen et al.	0.892	0.871
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But is there *really* a linear hierarchy?

If ranks are meaningful, then they are related to *outcomes,* i.e. the directions of edges.

Use the ground-state energy as a test statistic, under an edge-direction randomized null.

$$\frac{H(s^*)}{M} = \frac{1}{2M} \sum_i \left(d_i^{\text{in}} - d_i^{\text{out}} \right) s_i^* + \frac{1}{2}$$

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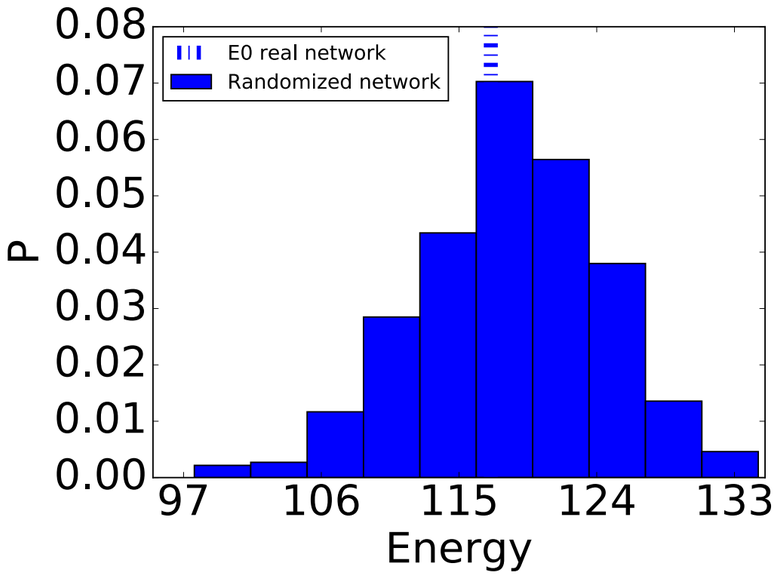

ER Network

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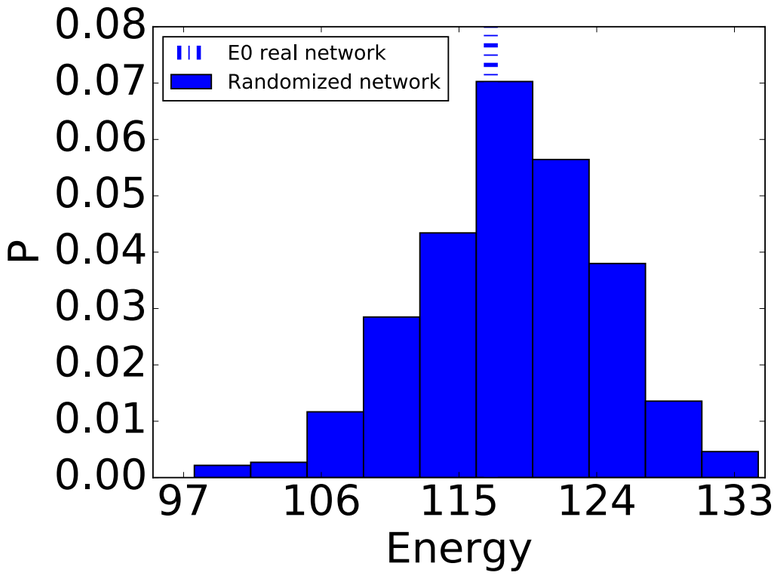
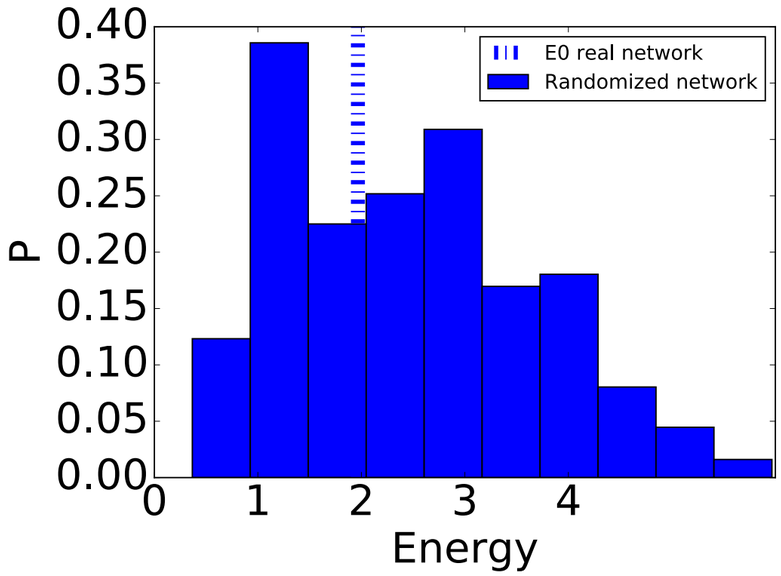


ER Network



Asian Elephants

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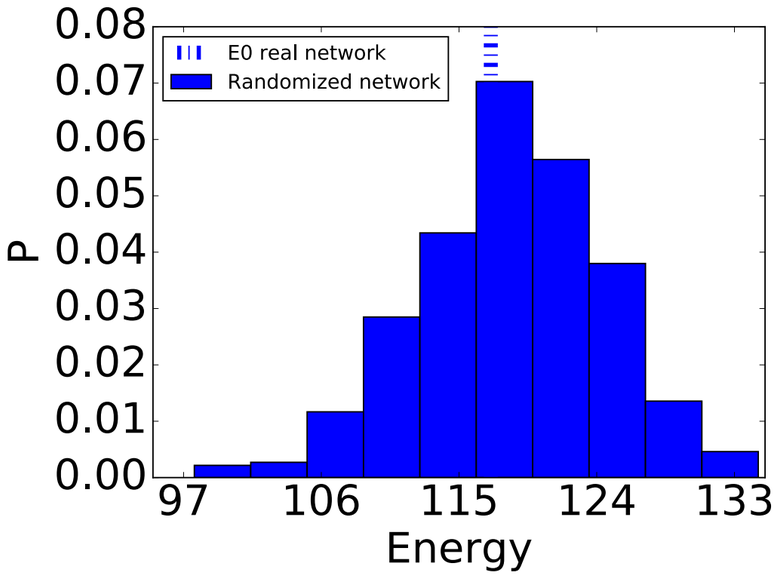
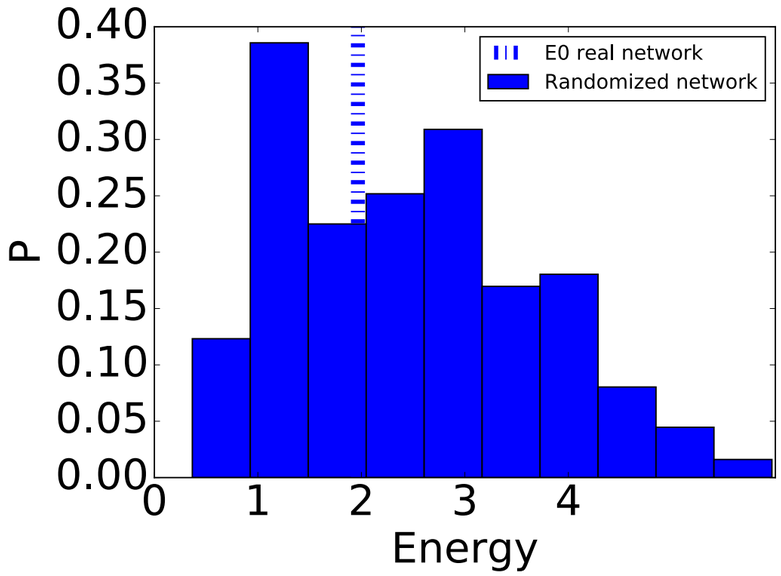
Use the ground-state energy as a test statistic, under an edge-direction randomized null.

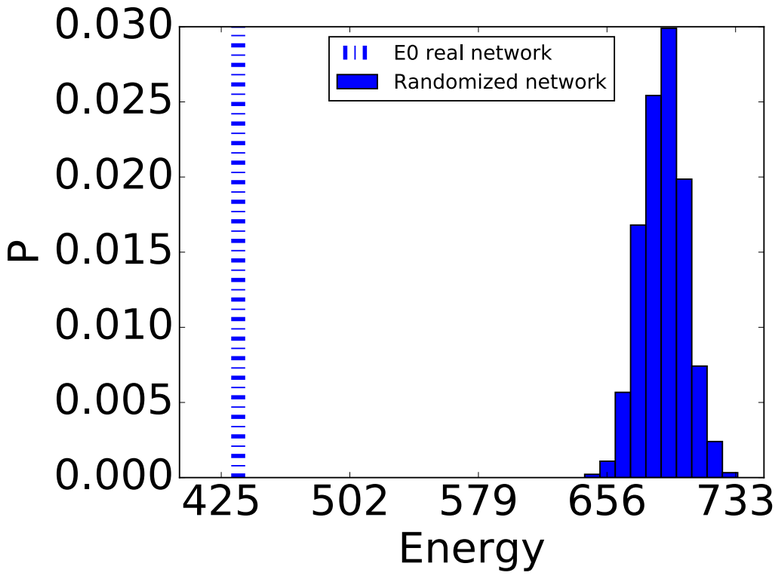
$$\frac{H(s^*)}{M} = \frac{1}{2M}\sum_i \left(d_i^{\text{in}} - d_i^{\text{out}}\right) s_i^* + \frac{1}{2}$$



ER Network



Asian Elephants



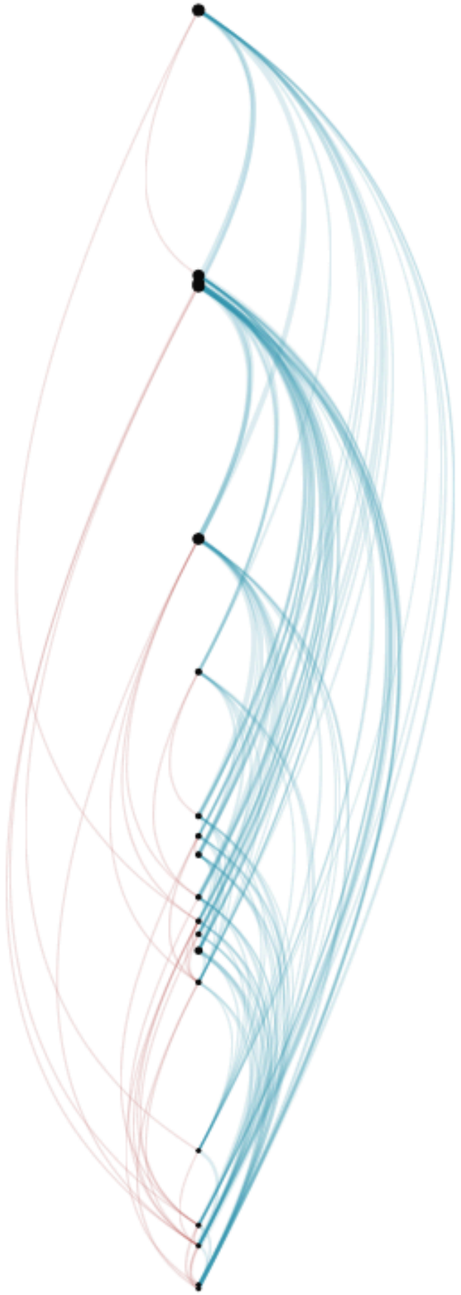
Social Support

Conclusions and extensions

1. SpringRank is a fast, scalable, *predictive* approach to ranking in directed networks. Works for Dominance or Endorsement.

2. It formalizes the idea that we can learn from not just the *direction* of edges, but the *existence* of edges as well.

3. There is a corresponding generative model whose ML solution is asymptotically equivalent. SpringRank approximates this ML solution.



Conclusions and extensions

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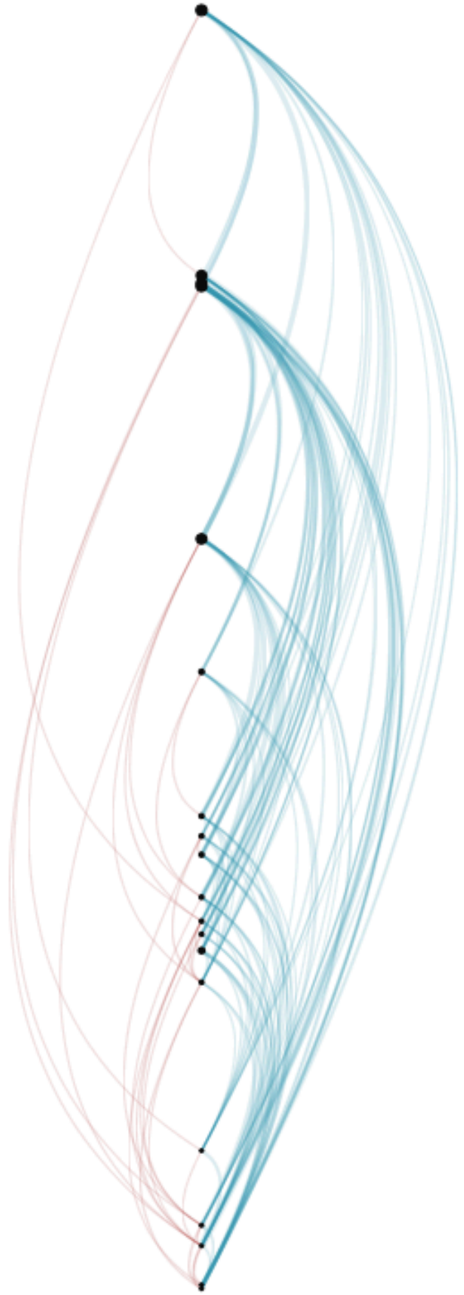
3. There is a corresponding generative model whose ML solution is asymptotically equivalent. SpringRank approximates this ML solution.

a. What if the hierarchy **branches**, like a Y?

b. What about other convex **spring potential functions**?

c. When might the **spring constants** depend on *i* or *j* ?



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A physical model for efficient ranking in networks
De Bacco*, Larremore*, and Moore. In Revision.
{preprint, python, matlab, viz} → danlarremore.com

NSF

Thank you

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Supplement: Non-transitive dice

- 3 (or more) dice {A,B,C}
- faces chosen so that they have the property:
  - A>B more than half the time.
  - B>C more than half the time.
  - C>A more than half the time (?!)

https://en.wikipedia.org/wiki/Nontransitive_dice

A great gift for your favorite nerd's desk!
Go to the Makerspace and laserbeam your own?