ECE330 Semiconductor Physics

Lab # 2

Wavepacket Collision and Bound States

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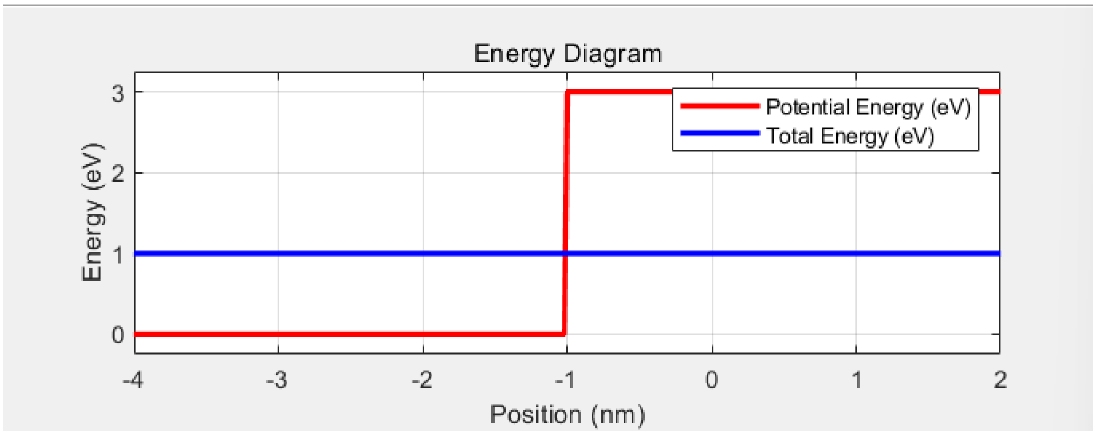
1 Wave packet Collision with Potential Step

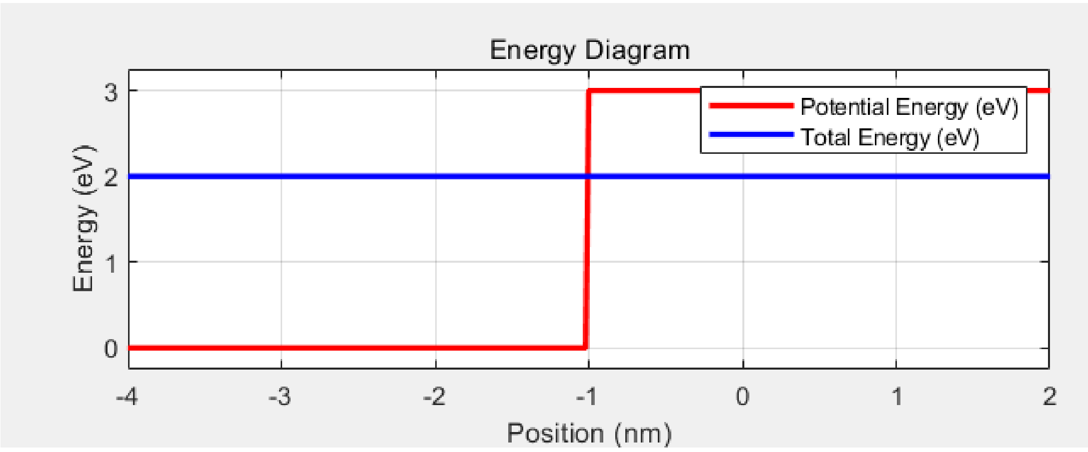
$$V = \begin{cases} 0\ \text{eV}, & x < -1\ \text{nm} \\ 3.0\ \text{eV}, & x \geq -1\ \text{nm} \end{cases}$$

(a)

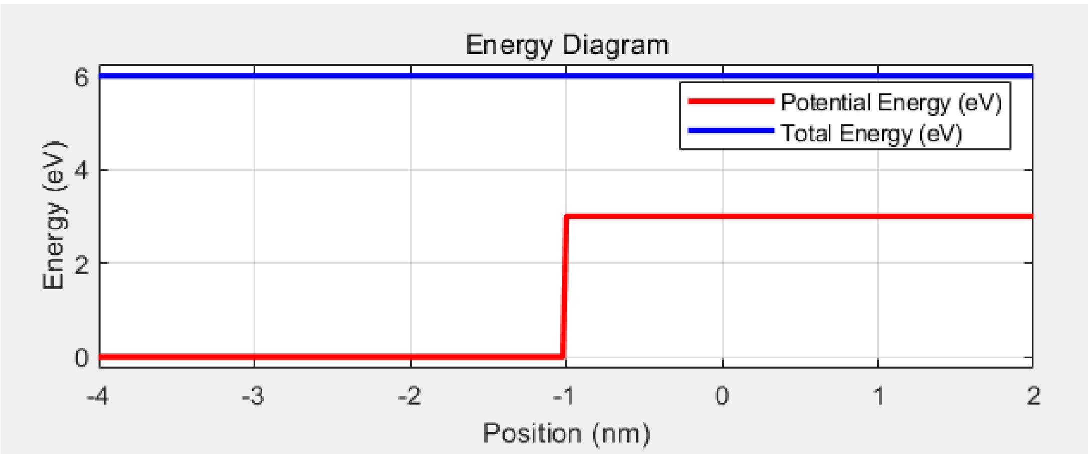
$\langle E \rangle$ = 1eV



$\langle E \rangle$ = 2eV



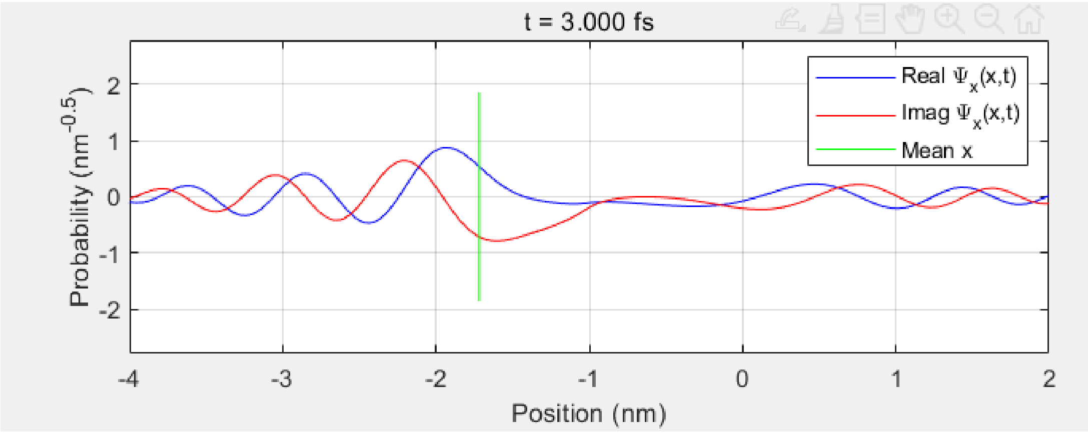
$\langle E \rangle$ = 6eV



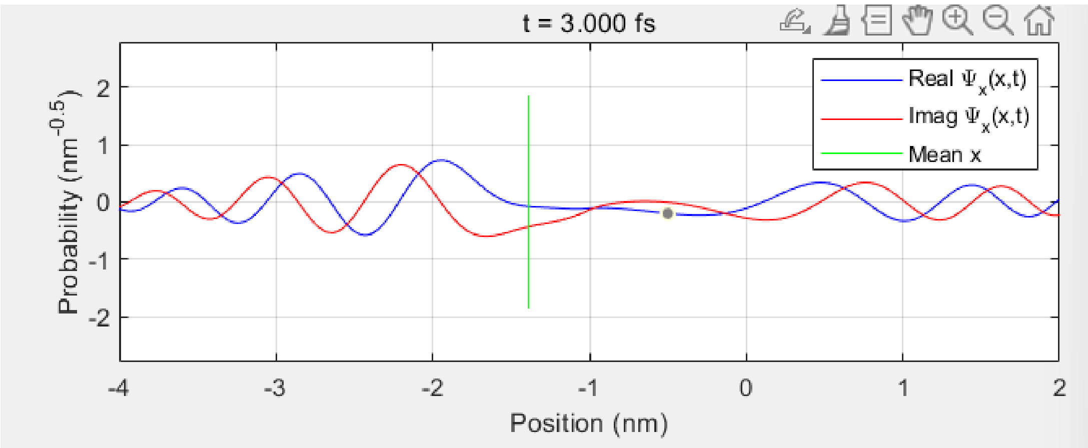
As the particle energy increases to 3 eV, it reaches the potential step, after which the particle energy increases beyond the potential step.

(b)

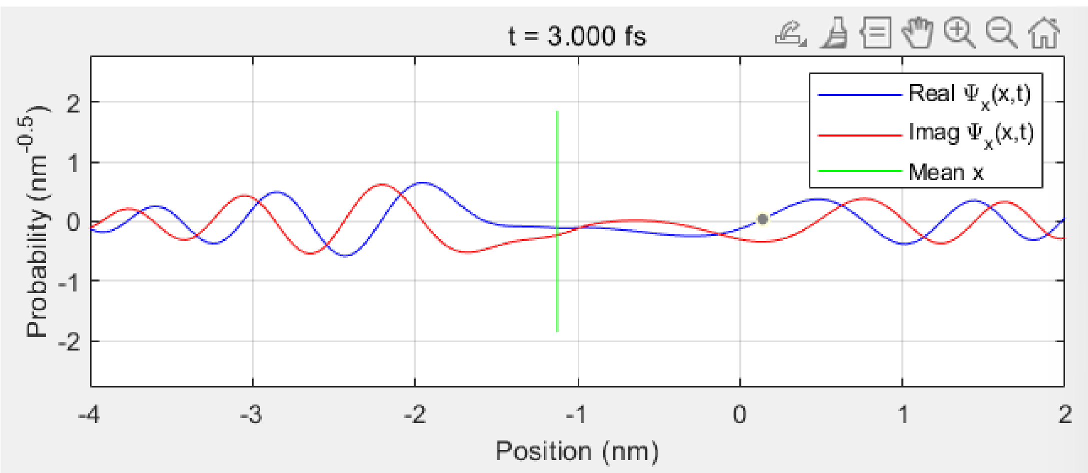
$\langle E \rangle$=1.0 eV



$\langle E \rangle$=2.0 eV

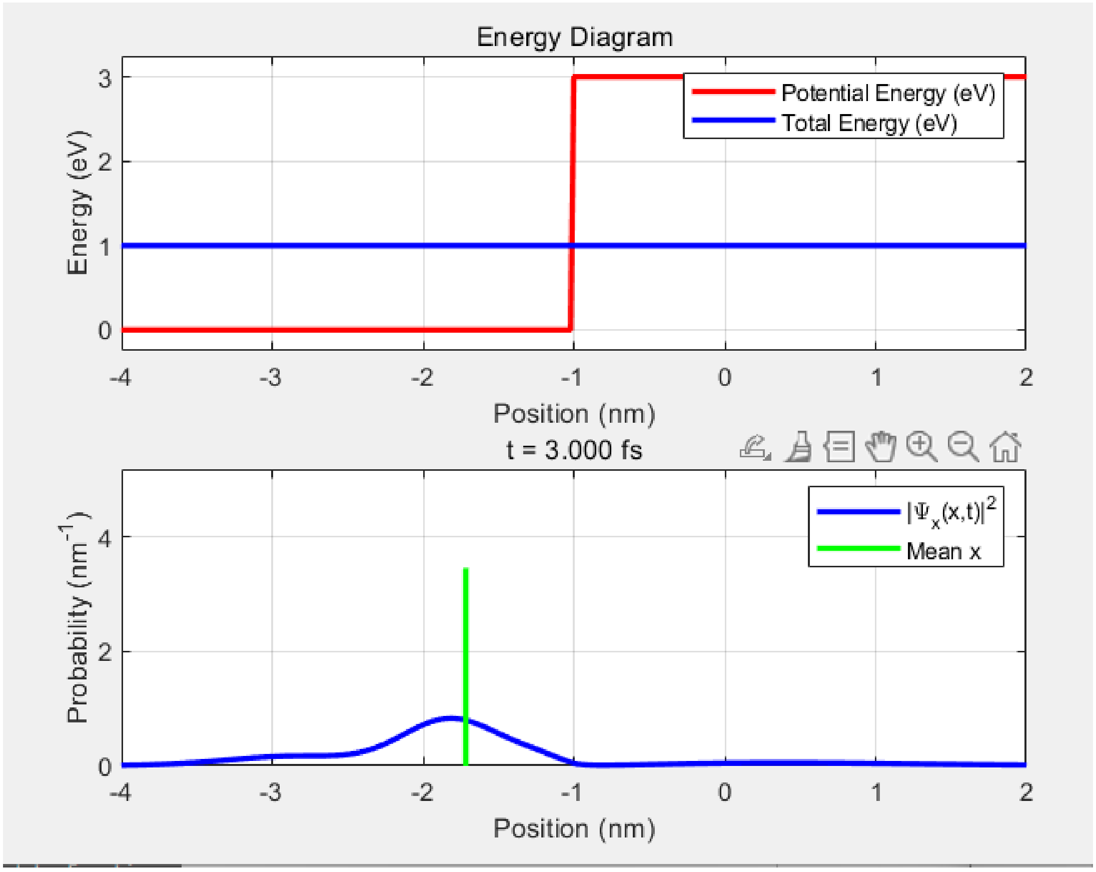


$\langle E \rangle$=2.5 eV

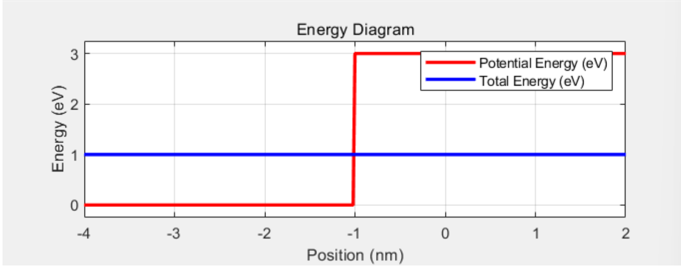


(c)

In classical physics, when potential energy is less than the step, it cannot penetrate through, or the region inside the barrier is classically forbidden, however, in quantum physics the wavefunction is continuous at the step and gets partially transmitted.



(d)

<table>
<tr><td>$\langle E \rangle = 1$ eV, $x = -0.1$ nm The average energy of the incident wave packet is lower than 3.0 eV</td><td>
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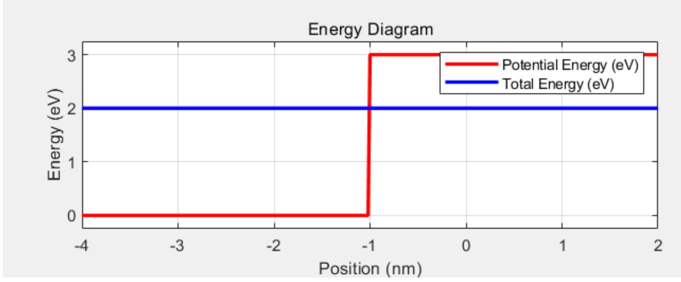
<tr><td>$\langle E \rangle = 2$ eV, $x = -0.1$ nm</td><td>
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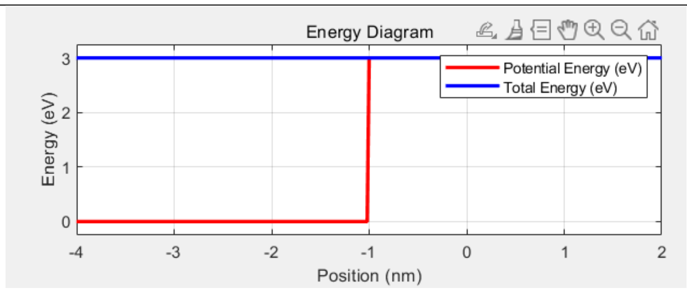
<tr><td>$\langle E \rangle = 3$ eV, $x = -0.1$ nm</td><td>
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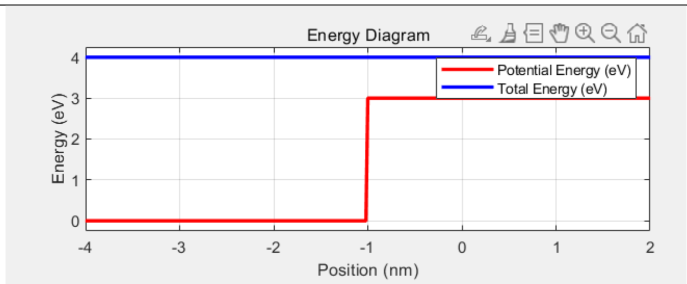
<tr><td>$\langle E \rangle = 4$ eV, $x = -0.1$ nm The average energy of the incident wave packet is higher than 3.0 eV</td><td>
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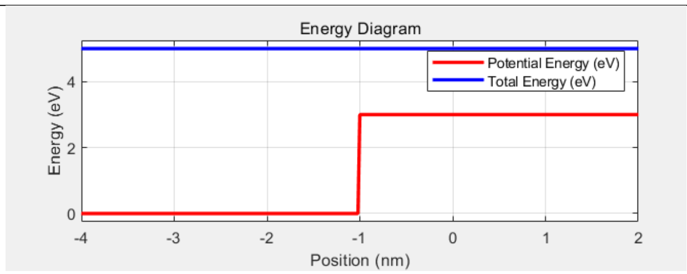
<tr><td>$\langle E \rangle = 5$ eV, $x = -0.1$ nm</td><td>
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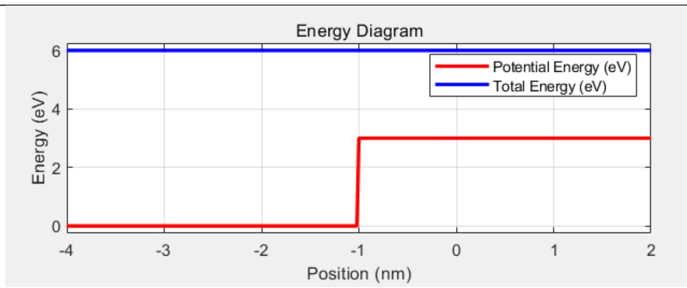
<tr><td>$\langle E \rangle = 6$ eV, $x = -0.1$ nm</td><td>
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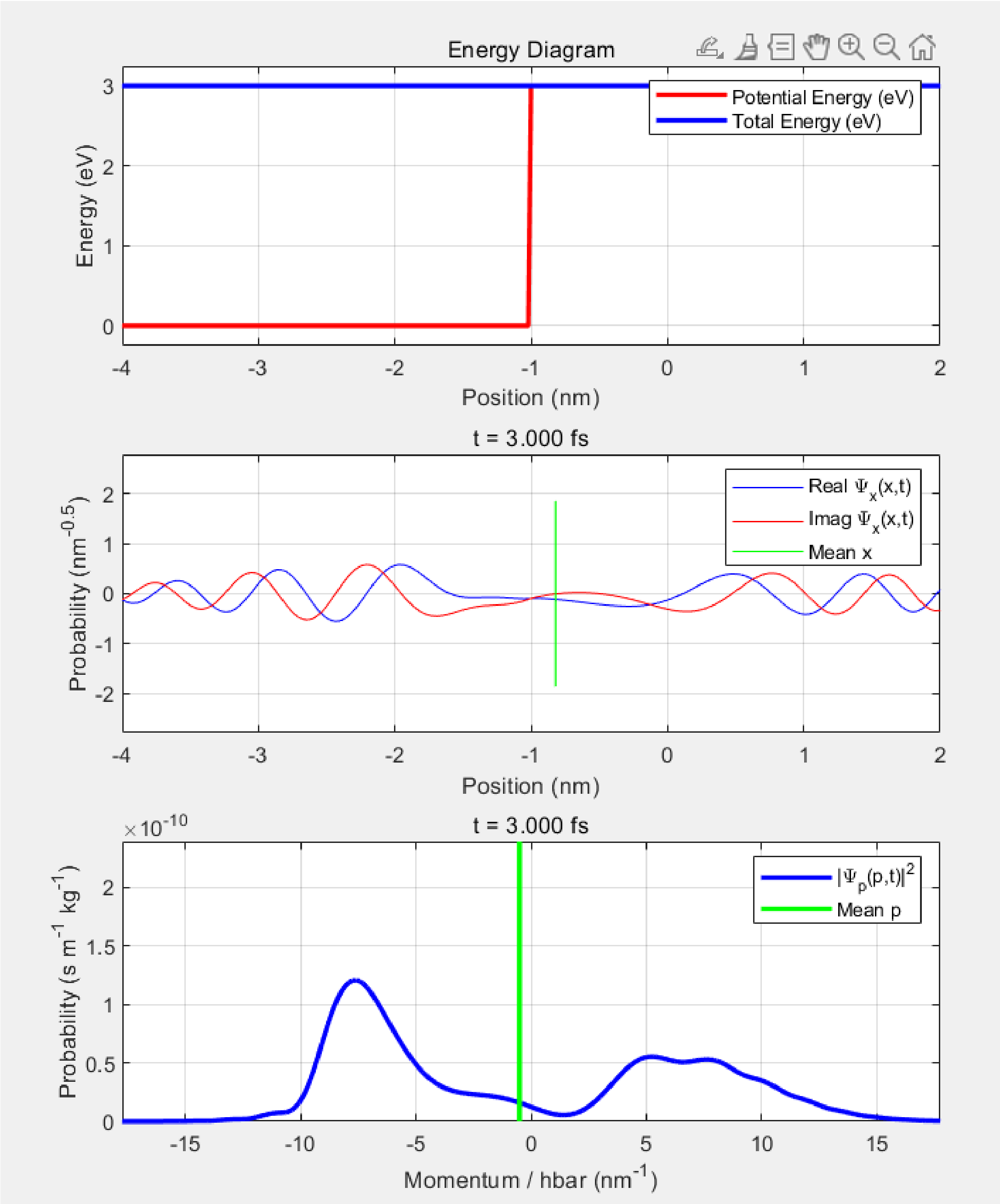
</table>

(e)



The waveform shows that this is a 50/50 transmission/reflection condition

(f)

Image of step potential position starting from -2 nm

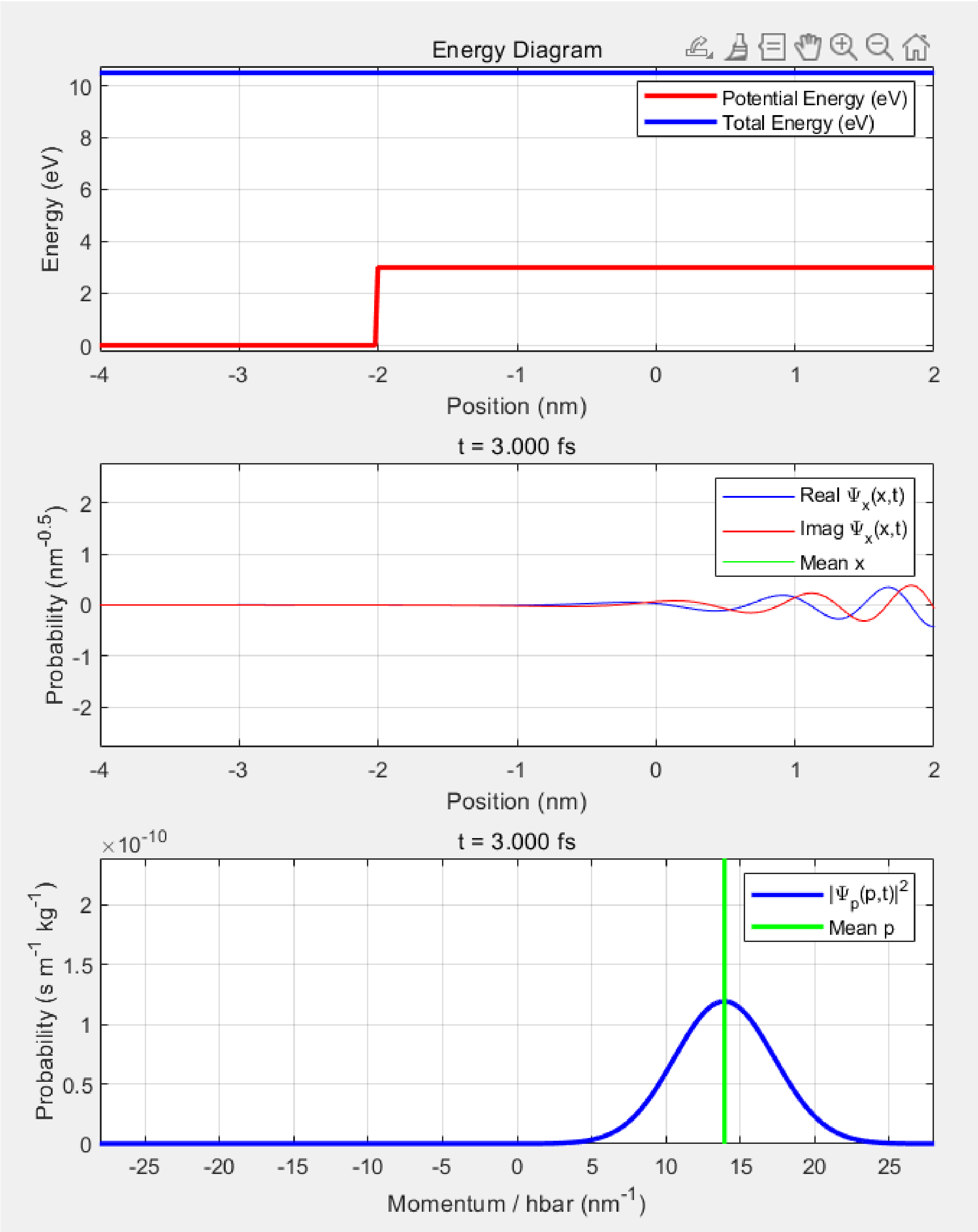


Image of step potential position starting from 0 nm

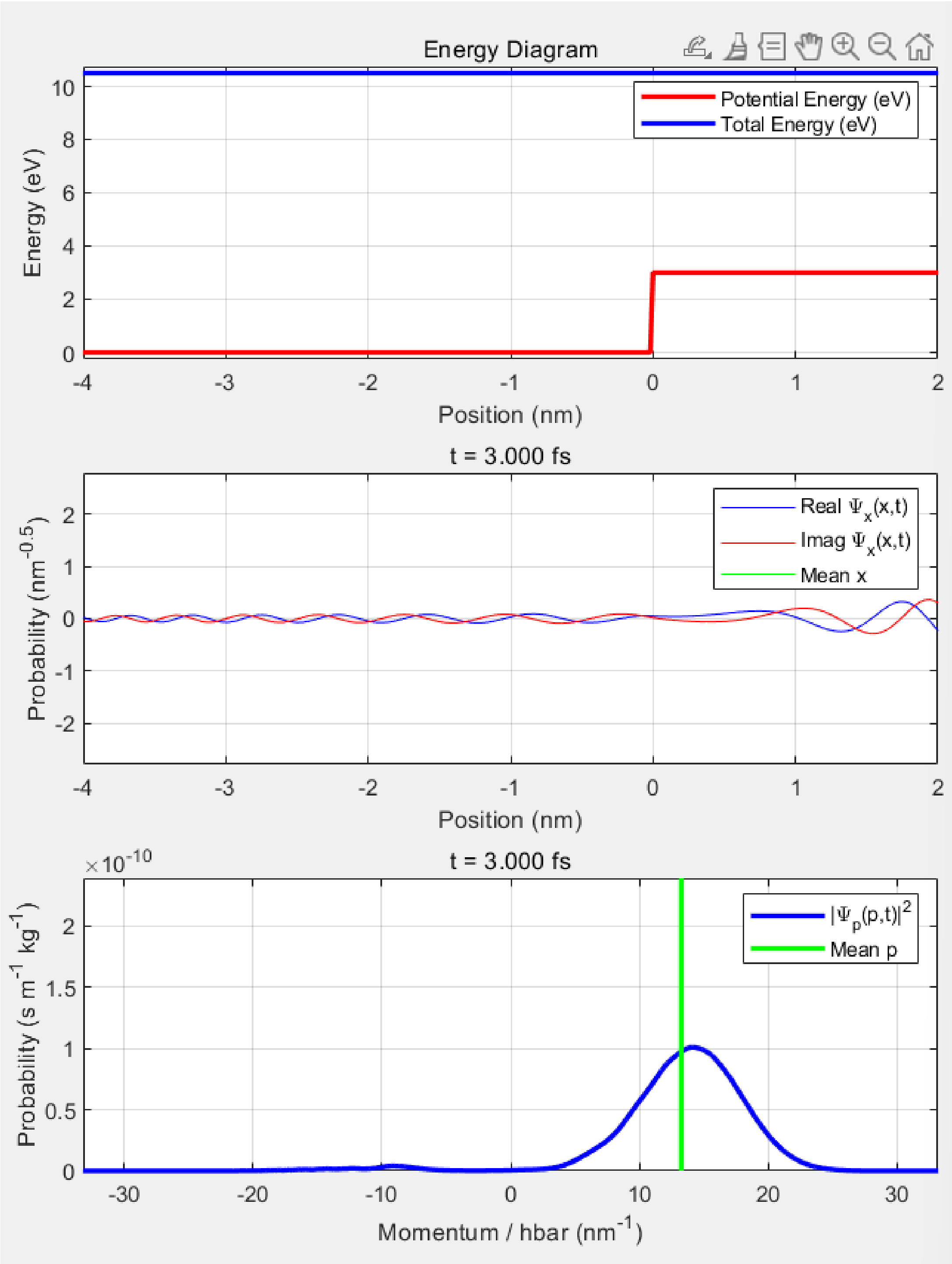
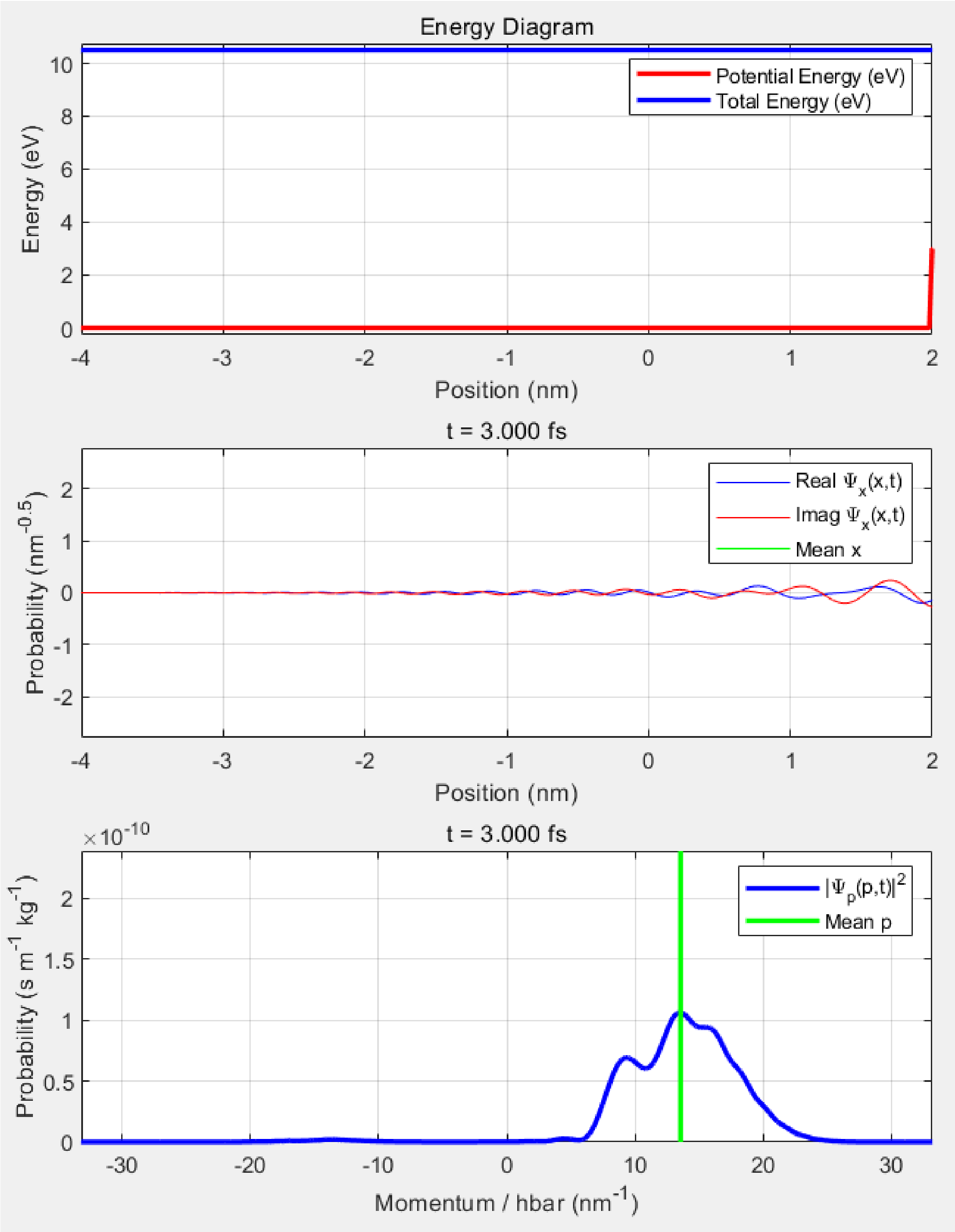


Image of step potential position starting from 2 nm

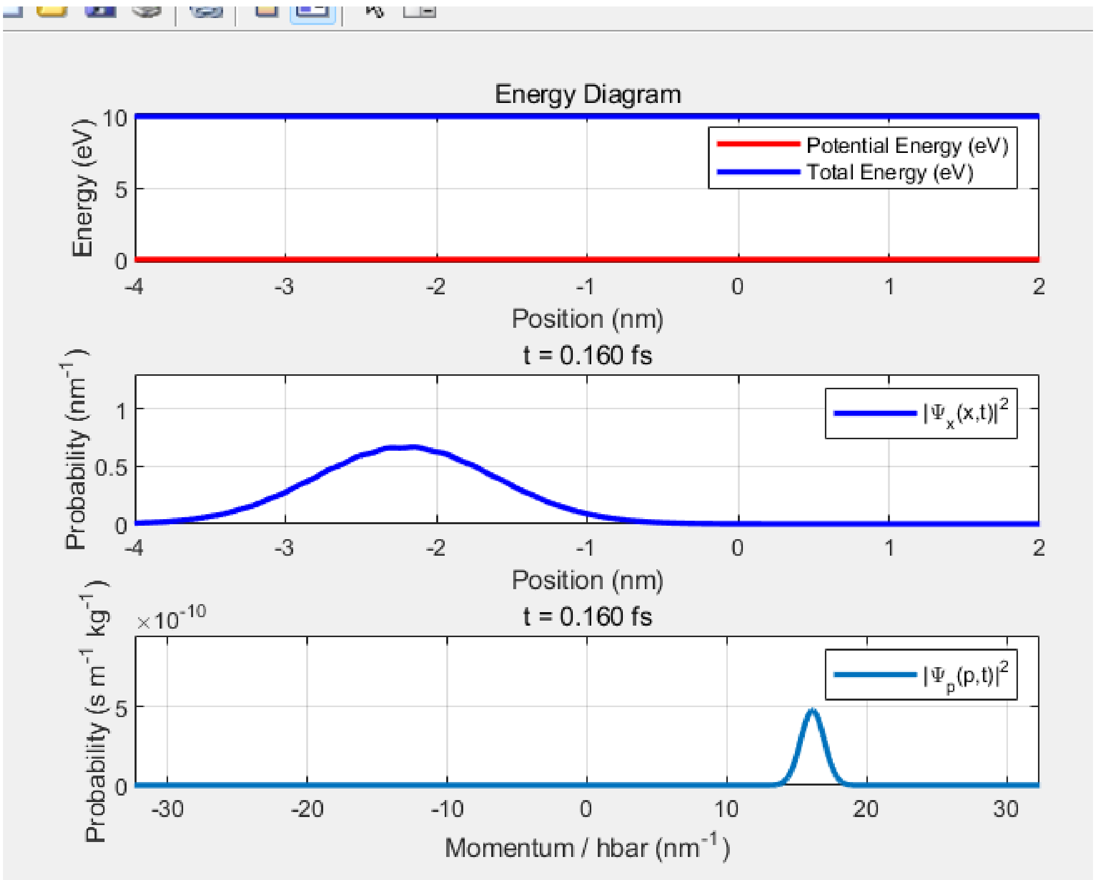


The particle velocity and average wavelength values also changes if changing the step potential position.

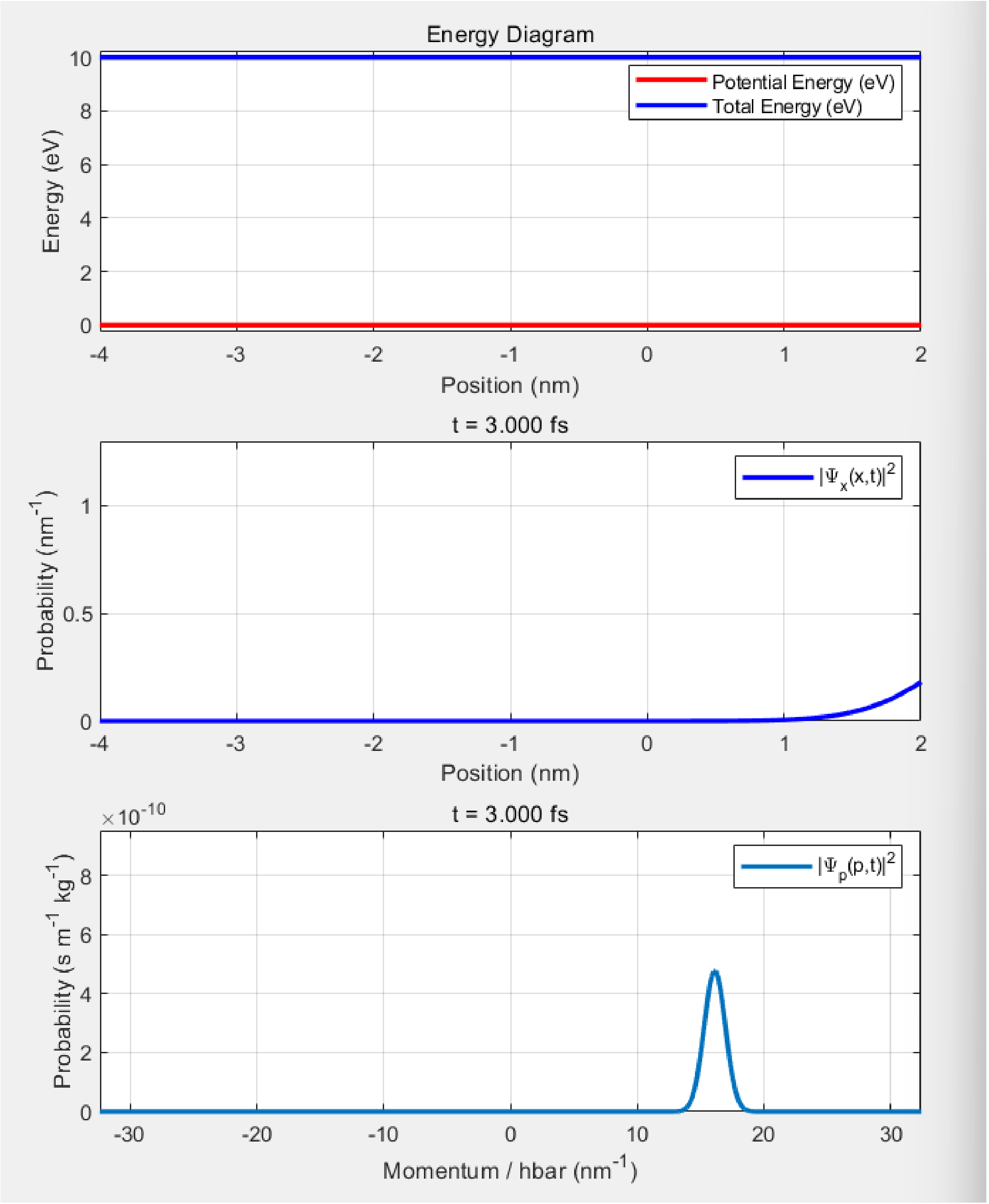
2 Wave packet Collision with Potential Barrier

$$V(x) = \begin{cases} 0\text{eV}, & x < 0 \text{ nm} \\ 17\text{eV}, & 0 \text{ nm} \le x < 0.5 \text{ nm} \\ 0\text{eV}, & 0.5 \text{ nm} \le x \quad (0.5 \text{ nm width}) \end{cases}$$

(a)



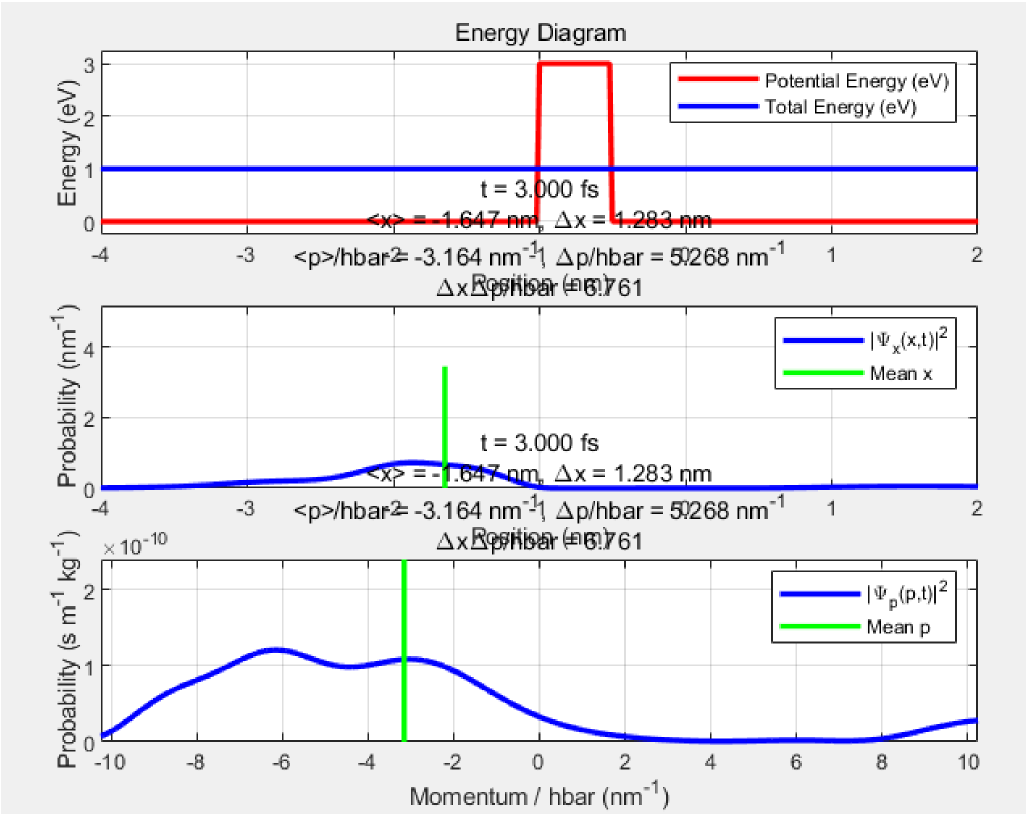
At $t = 0.16$ fs, wavefunction is between -3 and -1



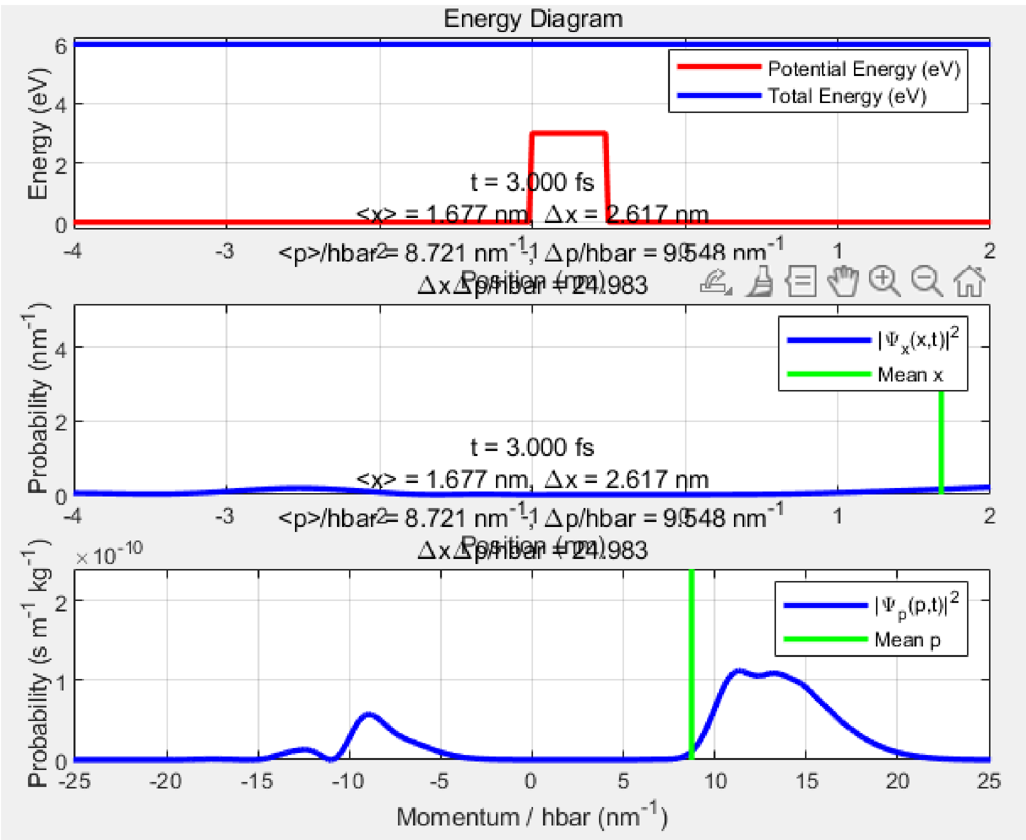
At $t = 3$ fs, The wave function wave moves to the right at a position greater than 1 as the simulation time increases

(b)

$\langle E \rangle = 1$ eV

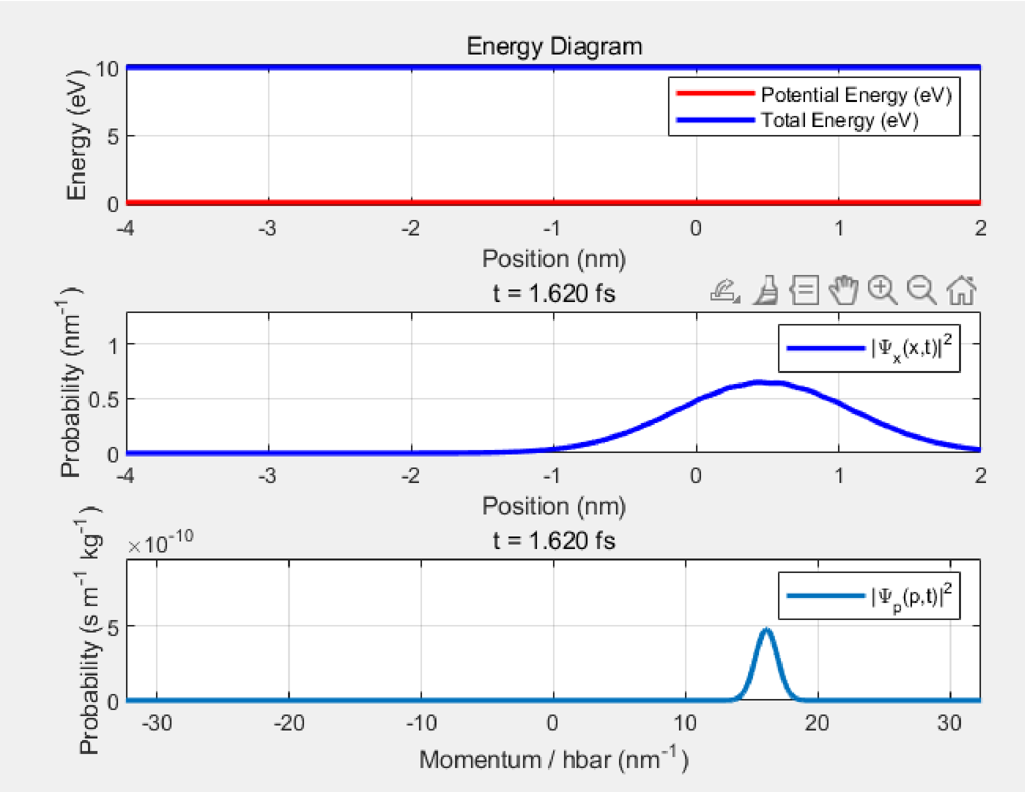


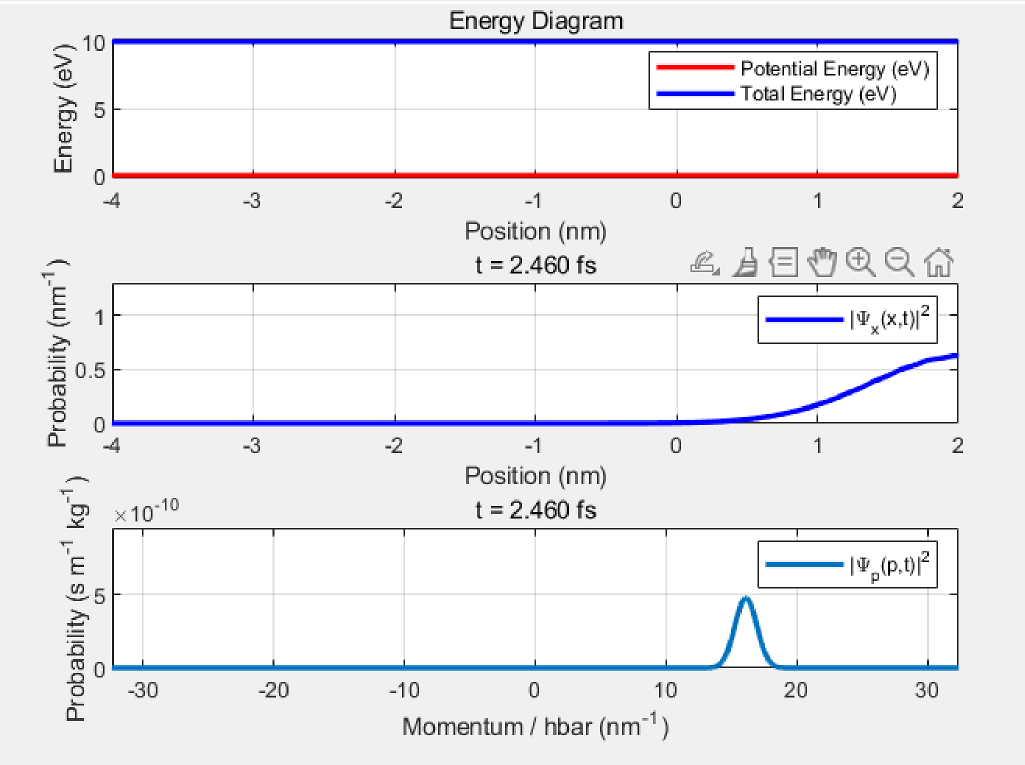
$\langle E \rangle = 6$ eV



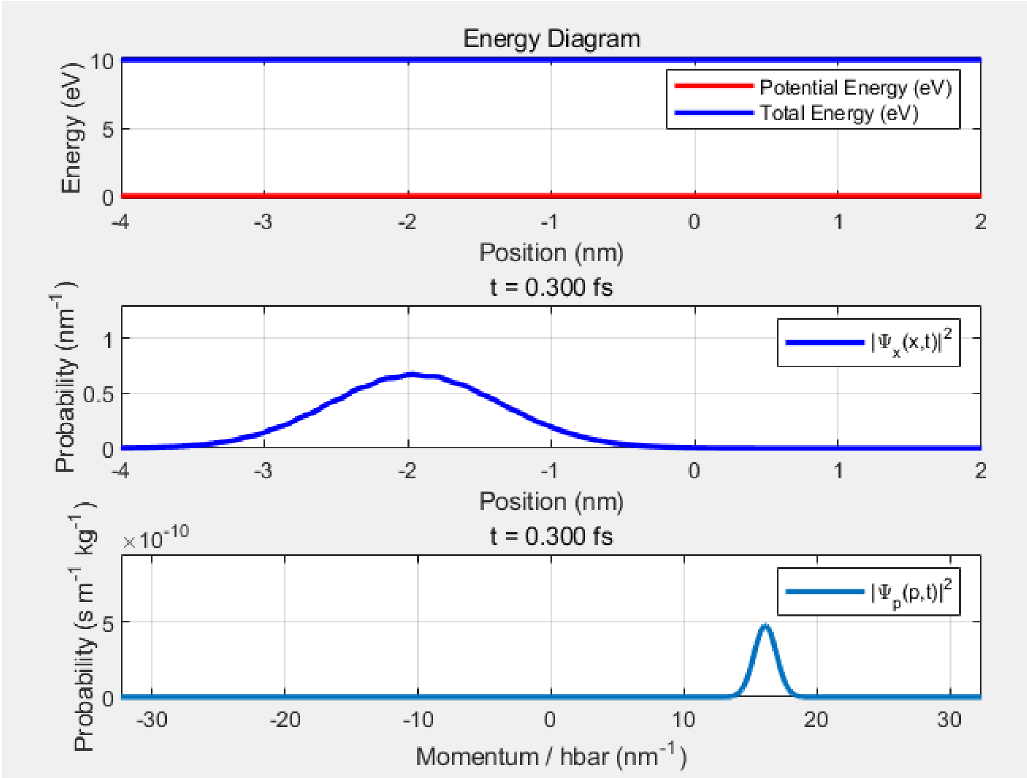
(c)

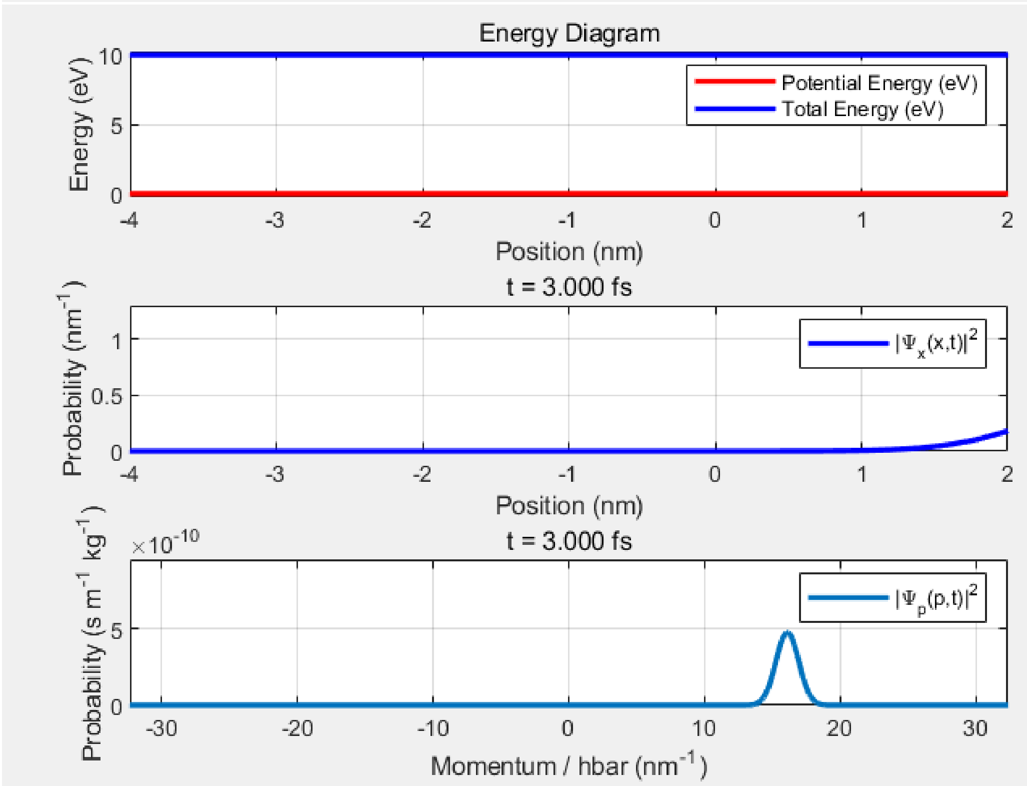
Wave packet evolution of barrier width 0.5 nm





Wave packet evolution of barrier width 0.05 nm

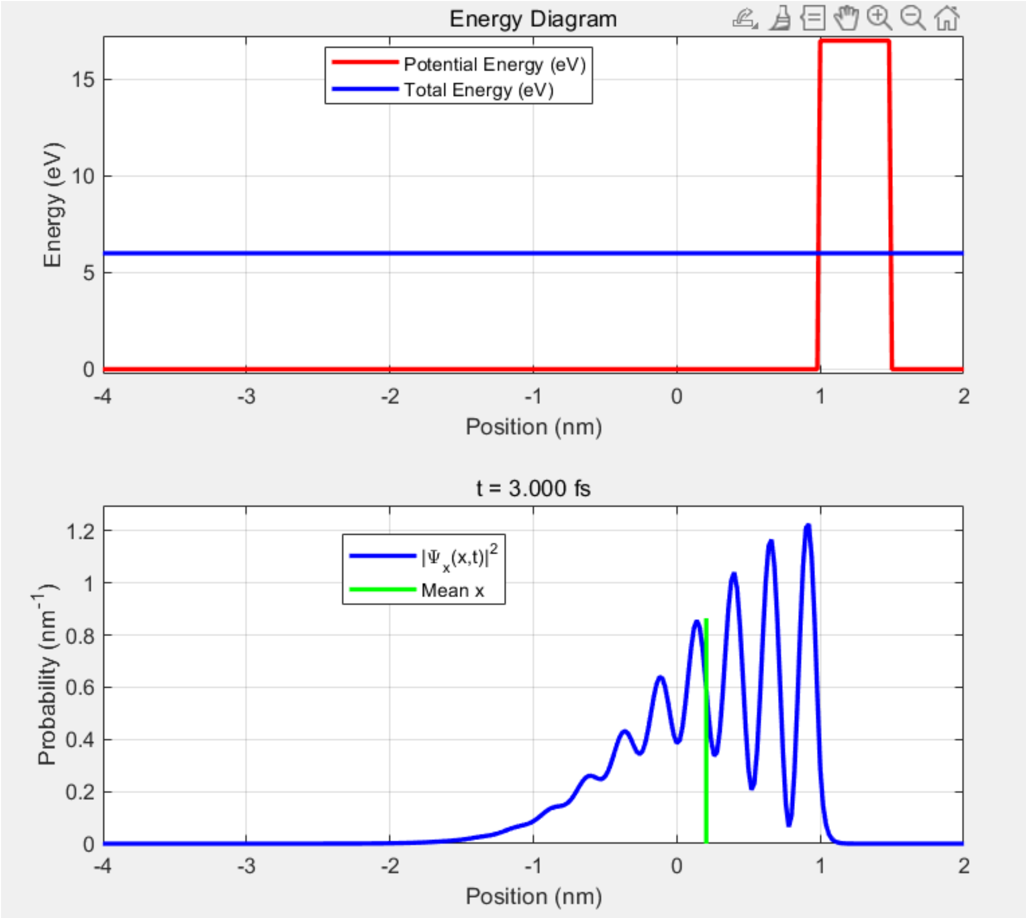




3 Bound Energy States of a Symmetric Quantum Well

(a)

When $a = 1$ nm, $A = 1$, $B = 1$, $E_o = 6.0$, $m = 9.11 * 10^{-31}$, $h = 6.626 * 10^{-34}$;



From simulation $\Psi(x)$ has max amplitude 1.2, and $1 > x > 0$, calculation $k = \sqrt{2 * m * (E + V_0)}/h = 6.4735 * 10^{-15}/6.626 * 10^{-34} = 0.977 * 10^{19}$.

$\Psi(x) = B\cos(kx) = \cos(k * 1 * 10^{-9}) = \cos(0.977 * 10^{19}) = 0.9988$

(b)

Due to the limiting effect of potential Wells, the energy of electrons or holes in each potential well moving perpendicular to the junction plane is no longer continuous, and only a series of discrete band values can be taken.

(c)：

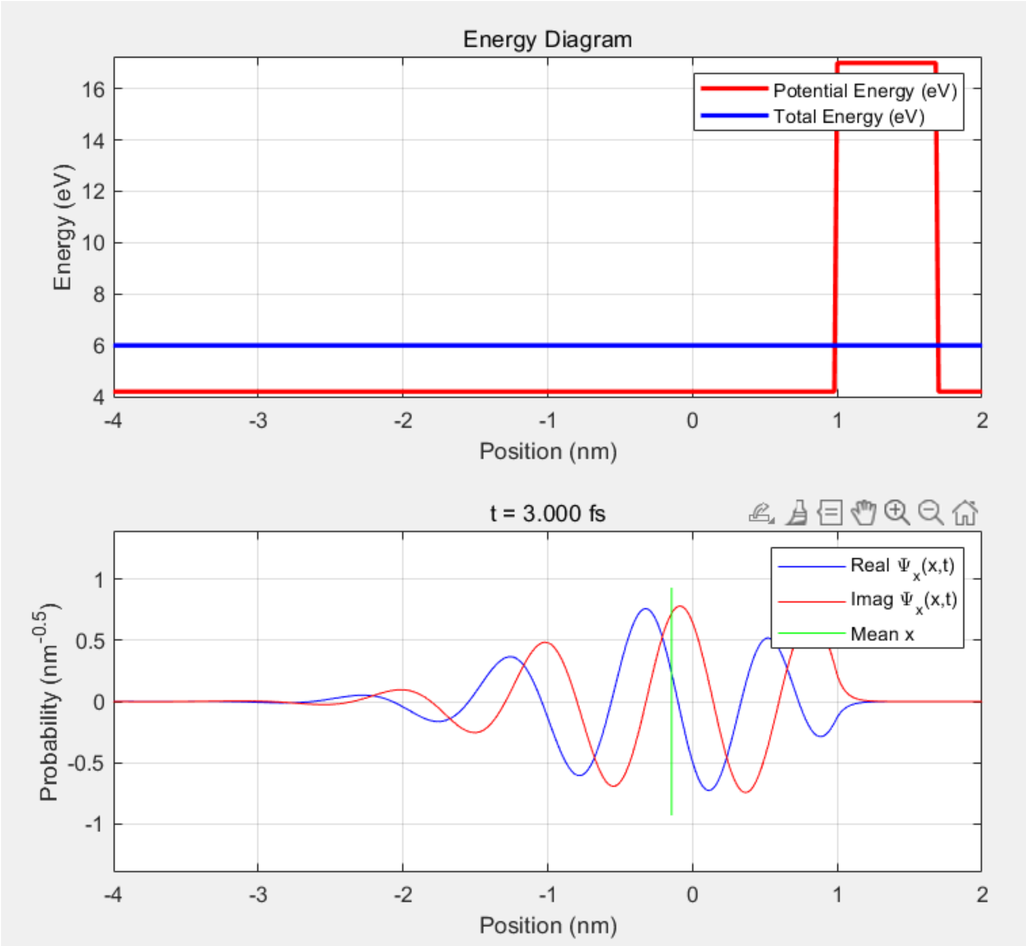
$x = a = 1\text{nm}$,

$z = k * a = (\sqrt{2 * m * (E + V_2)}/h) * 1 * 10^{-9} = 6.4735 * 10^{-15} * 1 * 10^{-9} = 6.4735 * 10^{-24}$

$z_0 = (a * \sqrt{2 * m * V_2})/h = (1 * 10^{-9} * 5.5654 * 10^{-15})/6.626 * 10^{-34} = 0.839 * 10^{10}$

$\tan(z) = \sqrt{(z_0/z)^2 - 1} = 1.2961 * 10^{33}$

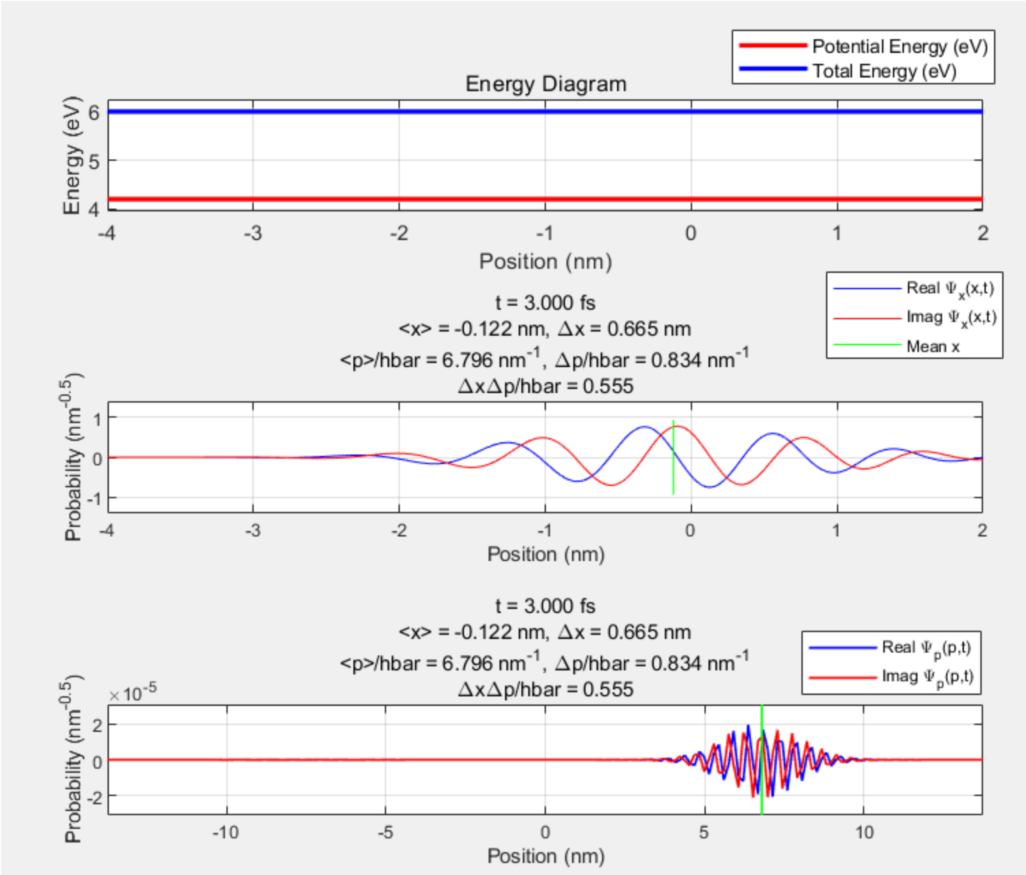
(d)



From simulation $\Psi(x)$ has max amplitude 0.76, and $1 > x > 0$, calculate $k = \sqrt{2 * m * (E_o + V1)}/h = 3.3064 * 10^{-15}/6.626 * 10^{-34} = 0.499 * 10^{19}$

$\Psi(x) = B\cos(kx) = \cos(k * 1 * 10^{-9}) = \cos(0.499 * 10^{10}) = 0.9839$

(e)



Potential Energy (eV)
Total Energy (eV)
Energy Diagram
Energy (eV)
Position (nm)
Real Ψx(x,t)
Imag Ψx(x,t)
Mean x
t = 3.000 fs
<x> = -0.122 nm, Δx = 0.665 nm
<p>/hbar = 6.796 nm-1, Δp/hbar = 0.834 nm-1
ΔxΔp/hbar = 0.555
Probability (nm-0.5)
Position (nm)
t = 3.000 fs
<x> = -0.122 nm, Δx = 0.665 nm
<p>/hbar = 6.796 nm-1, Δp/hbar = 0.834 nm-1
ΔxΔp/hbar = 0.555
Real Ψp(p,t)
Imag Ψp(p,t)
×10-5
Probability (nm-0.5)
Position (nm)