High-Energy Nuclear Physics

Measuring the v_2 elliptic flow coefficient in ultrarelativistic heavy ion collisions

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June 9, 2021

Abstract

One of the main focus of heavy ion research is the study of the flow anisotropies in large energy collisions – most prominently in Au+Au events at $\sqrt{s_{NN}} = 200$ GeV/c. I've measured the p_T traverse momentum dependence of the v_2 flow anisotropy coefficient using real, but simplified observational data from ultrarelativistic heavy ion experiments. At the end I've obtained similar results to those in the literature and showed that v_2 value saturates at $p_T \approx 2$ GeV/c with an approximate value around $v_2 \approx 0.3$.

I. Introduction

Through heavy-ion physics experiments – especially at the Relativistic Heavy Ion Collider (RHIC) – over the past few years it was observed, that in case of non-central heavy-ion collisions, a specific momentum space anisotropy so called *elliptic flow* is created. In these experiments mostly Au+Au collisions were observed and analysed (see eg. [1] or [2]).

This anisotropy can be observed through eg. the measurement of the angle distribution of outgoing particles and serves as a strong evidence for the existence of quark-gluon plasma (QGP), a dense form of matter with exotic properties that completely filled the universe in the very first moments after the Big Bang. The formation of QGP strongly affects, how the initial anisotropy during the collision is transferred to the final, observed state. Measuring the exact characteristics of this final „freeze-out" thus could give us precise insights about the early state of this anisotropy and the transport properties of the QGP too.

In a non-central heavy-ion collision only a fraction of the valence- and sea quarks participate in the actual collision itself. The so called „spectators" that do not fall into the participation zone are continues their travel down the pipe of the collider. Inside the zone an *elliptic* or almond shaped volume of QGP is created that thermalize extremely quickly and scatter particles around every direction. Since the results for the same circumstances (colliding material, energy, etc.) are governed by the orientation of the colliding beam lines, these measurements are usually categorized by their *centrality* [3].

One of the main interest for studies of the elliptic flow is the $n = 2$ azimuthal anisotropy coefficient v_2. This is the second-order coefficient in the Fourier expansion of the distribution of the outgoing particles and can be defined as

$$\frac{\mathrm{d}\,N}{\mathrm{d}\,\varphi} = v_0 \left[1 + 2v_1 \cos(\varphi) + 2v_2 \cos(2\varphi)\right], \tag{I.1}$$

where

$$\varphi = \varphi_0 - \Psi, \tag{I.2}$$

where φ_0 is the raw angle of the particle track, while Ψ is the angle of the reaction plane. In this project the determination of this quantity were in focus.

II. Project preliminaries

II.1. Task description

The goal of the project work was to explore the p_T transverse momentum dependence of the v_2 elliptic flow coefficient using real, but simplified observational data, presented us in ROOT `Tree` format[1,2]. The calculation of v_2 was needed to be done for a single, global centrality class in the interval of 0% to 92% as a function of the p_T transverse momentum, where

$$0 \text{ GeV/c} < p_T < 2 \text{ GeV/c}.$$

The width of the p_T bins were supposed to be 50 MeV/c, which implies the existence of 40 bins for the total of 2 GeV/c wide observational range.

The observed v_2^{obs} values needed to be corrected for the event plane resolution R to obtain the real v_2^{real} values of the measurements.

At last I've compared the results to real values (which could be seen eg. on the upper panel of Fig. (2) in [4]). It should be also noted that in our dataset only pions were present, thus all results should be treated accordingly when they're compared to values in the literature.

II.2. Solution strategy

The main goal of the project was to determine the $v_2(p_T)$ function that describes the connection between the transverse momentum and the elliptic flow parameter. The v_2 value is

[1] `https://root.cern/manual/trees/`
[2] `https://root.cern.ch/doc/master/classTTree.html`

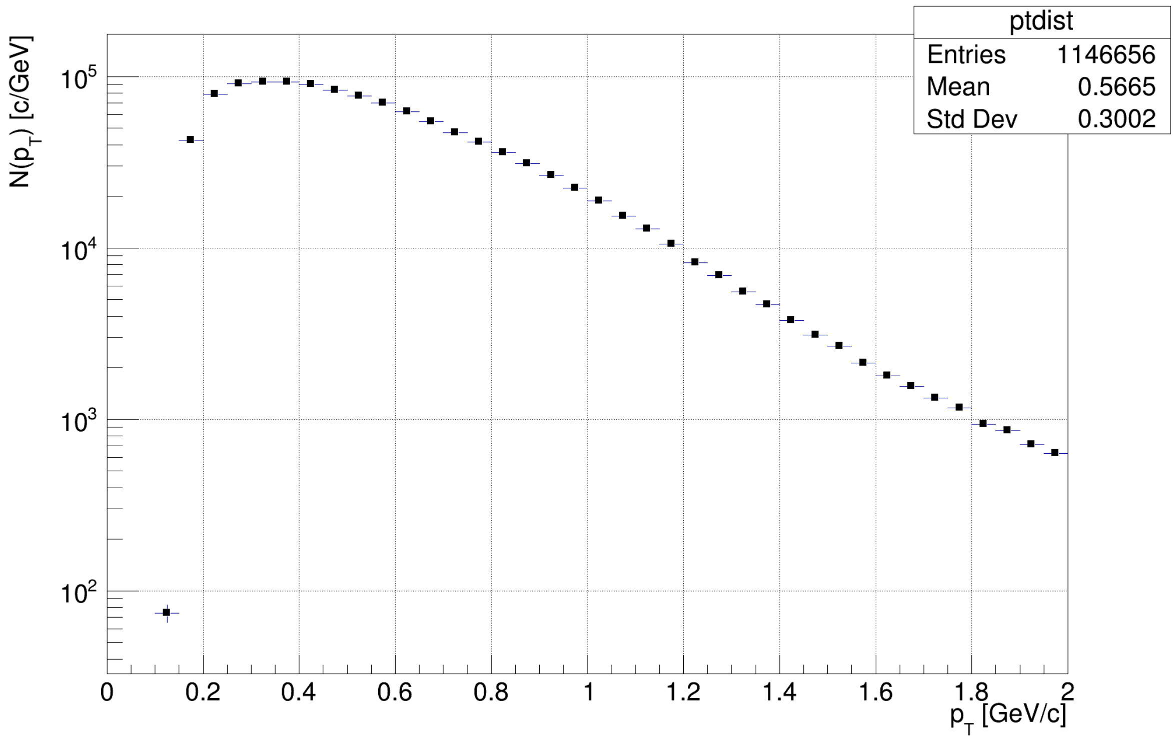


Figure 1. The p_T transverse momentum distribution in the experimental data. The reason for the interest in the transverse momentum can be simply understood. The momentum of the colliding particles are entirely parallel to the reaction plane, when they circulating inside the pipe of a collider. After the collision, spectator particles will travel down the pipe, while particles in the collision zone will scatter into every direction. This means that only particles in this interesting region will have a momentum component perpendicular to the reaction plane, thus studying focusing on particles with non-zero p_T values will ensure that only particles from the collision zone will be taken into account in our analysis.

proportional to the angle distribution, which relation can be expressed using the (I.1) definition as

$$\frac{\mathrm{d}\,N}{\mathrm{d}\,\varphi} \propto 1 + 2v_2 \cos\left(2\varphi\right). \tag{II.1}$$

The φ angle can be calculated according to (I.2), where the Ψ reaction plane angle is given in the provided dataset. The φ_0 raw angle can be simply obtained from the momentum values eg. as

$$\varphi_0 = \mathrm{atan2}\left(p_y, p_x\right) \tag{II.2}$$

At the end the φ values should be normalized into the interval of $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ eg. using the following simple algorithm:

$$
\varphi = \begin{cases} \varphi + 2\pi & \text{if } \varphi < -\pi \\ \varphi - 2\pi & \text{if } \varphi > \pi \\ \varphi + \pi & \text{if } \varphi < -\pi/2 \\ \varphi - \pi & \text{if } \varphi > \pi/2. \end{cases} \tag{II.3}
$$

Given the proportion above in (II.1), one can find the value of v_2^{obs} by fitting a

$$
f = a + 2 \cdot b \cos(2\varphi) \tag{II.4}
$$

parametrized trigonometric function on the angle distribution of particles. To explore the p_T dependence of the v_2 coefficient, one has to bin the p_T space, determine the angle distribution for all p_T bins and obtain the v_2 value for each of them by fitting the function defined in (II.4). The value of the v_2^{obs} parameter will be then calculated as

$$
v_2^{\text{obs}} = \frac{b}{a}. \tag{II.5}
$$

The bin width chosen for this project was 50 MeV/c. At the end this value has to be scaled by the event plane resolution to get the final value of v_2 as

$$
v_2 = \frac{v_2^{\text{obs}}}{R}. \tag{II.6}
$$

Event plane resolution is defined as a function of centrality and was given in a supplementary dataset. Since I was intended to work in a single centrality group, a smart average value of the event plane resolution should have been found. I've simply calculated the average of the resolution values, weighted by the widths of the centrality bins to obtain an approximate value of

$$
R \approx 0.3265
$$

for the average.

II.3. Project code

II.3.1 Initial base

A code base was provided for the project that contained some essentials to help us successfully complete the assignment. This package contained the framework that defines the structure of the data tree class, as well as lends a helping hand to easily process files encoded in this format.

The given code also contains a basic analysis routine that iterates over all events and particle tracks and thus allows us to collect all relevant entries from the dataset. Any further

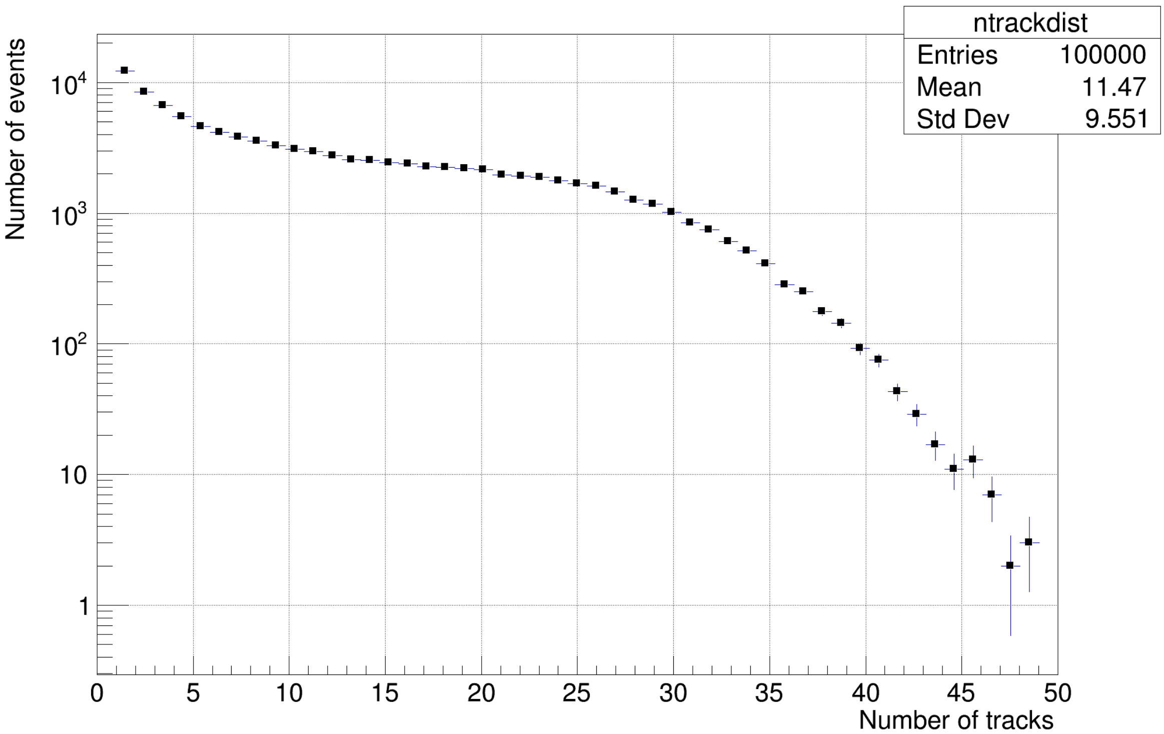


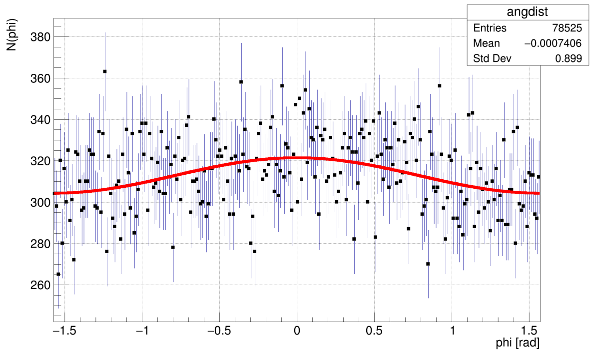
Figure 2. The histogram shows the number of events for different track numbers in the data. The average number of tracks per event are $11 - 12$ approximately.

project codes were advised to be built on top of this foundation.

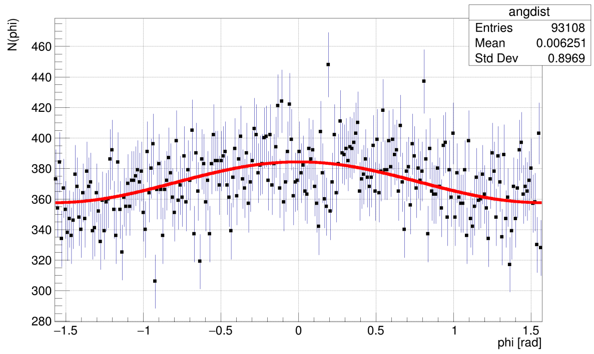
The provided data tree is organized into the following structure:

$$
\begin{aligned}
\text{Tree} = \Big\{ & \\
& \text{event}_1 : \left\{\text{track}_1^1, \text{track}_2^1, \ldots, \text{track}_{N_1}^1\right\}, \\
& \text{event}_2 : \left\{\text{track}_1^2, \text{track}_2^2, \ldots, \text{track}_{N_2}^2\right\}, \\
& \vdots \\
& \text{event}_n : \left\{\text{track}_1^n, \text{track}_2^n, \ldots, \text{track}_{N_n}^n\right\} \\
\Big\}. &
\end{aligned}
$$

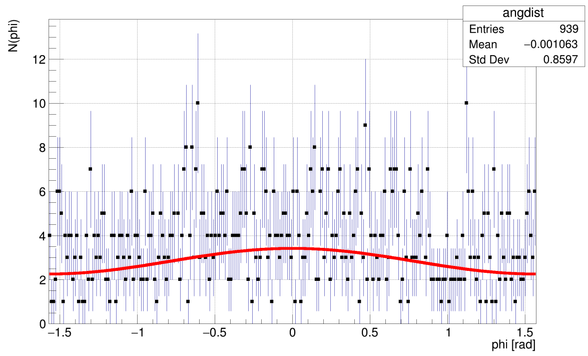
The provided dataset in our case contains 100 000 events, with a variable number of tracks in each of them. The average number of tracks for an event is $11-12$. Event-level properties data contain the centrality, z-vertex location, Ψ reaction plane angle and track number values, while track-level properties contain the particle momenta, energy, charge and some other – here not relevant – quantities.



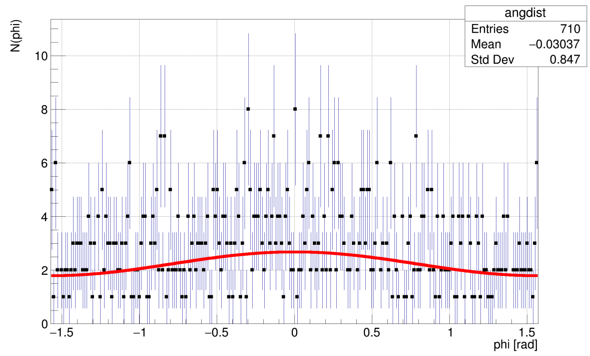
(a) p_T bin : $200 - 250\,[\mathrm{MeV/c}]$



(b) p_T bin : $350 - 400\,[\mathrm{MeV/c}]$



(c) p_T bin : $1800 - 1850\,[\mathrm{MeV/c}]$



(d) p_T bin : $1900 - 1950\,[\mathrm{MeV/c}]$

Figure 3. The fitted angle distributions for different p_T bins. The larger errors in the higher p_T regime can be attributed to the relatively small number of data in that interval. The difference in the number of data points between the high and low p_T regime is a 2 magnitudes.

II.3.2 Final code

The infamous ROOT library, used extensively by nuclear- and particle physics researchers all around the world for decades was utilized here to implement the final project code. However this software is considered to be heavily bloated, it contains all useful numerical methods and techniques that were needed to finish the task. Nonetheless to mention, that the data files were also stored in a ROOT format file that trivially ROOT can handle the most conveniently.

I've refractored almost the entirety of the analysis code, turning it into a functional code, instead of having a single `main()` method. This made the final code much more modular and clear to work with.

For the fitting of the angle distribution I've used the built-in method of ROOT for 1D histogram fitting, the `TH1::Fit()` function. This produced high precision results for lower p_T bins, but also high errors in the higher p_T regime. The results of some fits can be seen on (3). The upper two panel shows fits for angle distributions on low p_T values, while the bottom panels shows fits in the high p_T regime.

III. Results

The final results for the v_2 elliptic flow coefficient were obtained using the method in (II.2) and was processed and visualized using the ROOT library.

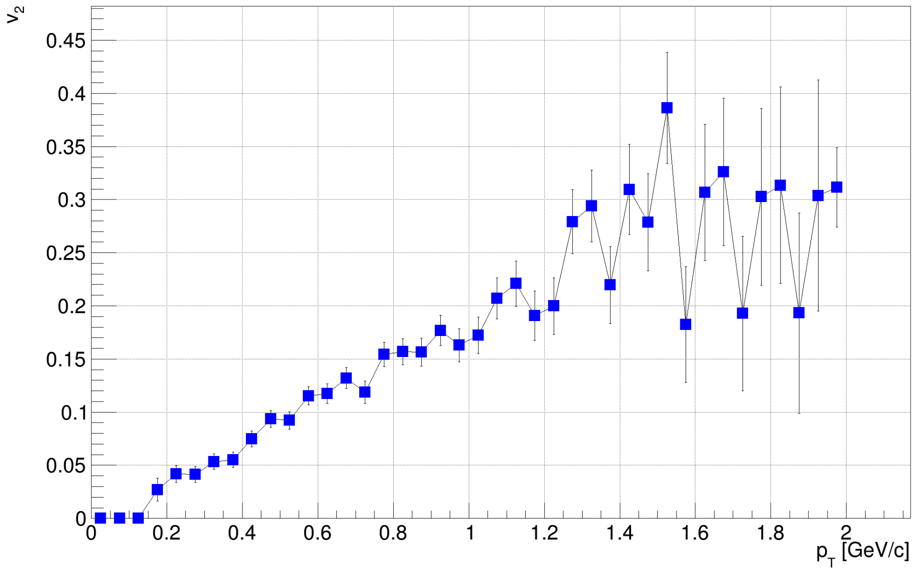


Figure 4. The v_2 elliptic flow coefficient as a function of the p_T transverse momentum for a single centrality class of $0\% - 92\%$.

For this numerous literature can be cited eg. Fig. (2) in [4] – as it was already mentioned here. This figure can be seen on Fig. (5) here in this paper. The very same characteristics can be observed on the obtained results in Fig. (3) and in Fig. (5). One of them that can be seen on both figures is that the $p_T - v_2$ curve saturates as it reaches $p_T \approx 1$ or 2 GeV/c, just like as it is mentioned in eg. [4] or [5]. The maximum value of v_2 is approximately 0.25, but the large errors in the high p_T regime makes it impossible to estimate it more precisely. Nonetheless this value is close to the ones on Fig. (5), where $v_2\,(p_T = 2\text{ GeV/c}) \approx 0.16$. The difference can be attributed to the wide centrality class used to calculate the R event plane resolution factor on.

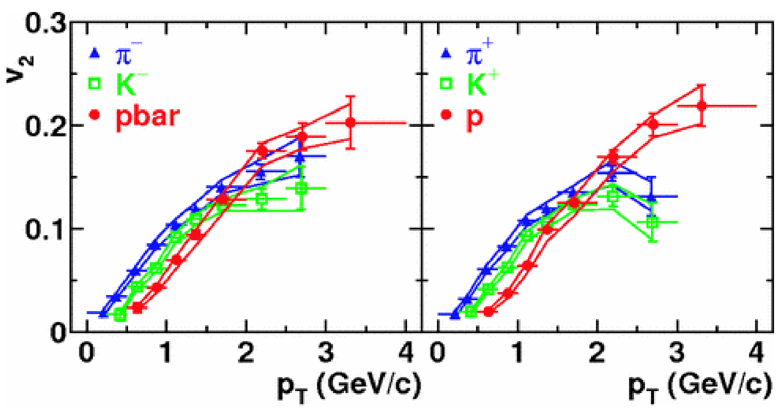


Figure 5. The two upper panels of Fig. (2) from [4]. The relevant lines here are those that belong to the pion measurements.

At the end I've obtained results consistent with values in the literature, which implies that the calculations here were probably correct. The errors of the v_2 values could be probably improved by choosing a more robust fitting method for the angle distribution, but it probably won't change the final implications of the results.

References

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