Rotating neutron stars

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The structure of compact stars
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Overview

Motivation

- Rotating objects are always problematic
- Rotation in relativistic conditions are even more problematic
- We still want to determine interesting attributes of compact rotating objects (stars), e.g. mass, angular momentum etc.
- We want to unveil the effects of rotation on the star's composition and inner structure

How to approach the problem?

1. Describe the frame-dragging effect
2. Derivate the Kepler-frequency
3. Discuss the consequences arising from the nature of different descriptions

Frame-dragging

Effect of rotation on metric tensor

- The metric tensor in case of a static, stationary star is described in the Schwarzschild metric with $G = c = 1$ contains only diagonal elements

$$g = d\tau^2 = e^{2\nu(r)} dt^2 - e^{2\lambda(r)} dr^2 - r^2 d\theta^2 - r^2 sin^2(\theta)\, d\varphi^2 \tag{1}$$

- Rotation however introduces off-diagonal elements too (where $\omega \neq 0$ is the angular velocity of local interial frames)

$$\begin{aligned} g = d\tau^2 = &\, e^{2\nu(r,\theta)} dt^2 - e^{2\lambda(r,\theta)} dr^2 \\ & - e^{2\mu(r,\theta)} \left[r^2 d\theta^2 + r^2 \sin^2(\theta) \cdot (d\varphi - \omega(r,\theta)\, dt)^2 \right] \end{aligned} \tag{2}$$

- Due to rotation the star deforms around the equator (centrifugal flattening) and loses spherical symmetry

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Frame-dragging

Effect of rotation on moving bodies

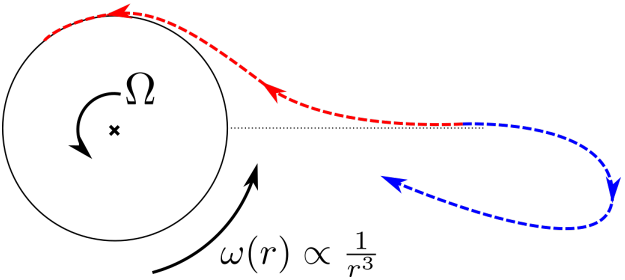

Figure 1: Frame-dragging effect over a massive rotating object on a free-falling body (red) and on a body that tries to move away from it (blue). Frame-dragging induces a force similar to the Coriolis–effect by „dragging" the compass of intertia in the direction of the star's rotation. Bodies falling towards the star's center are deflected in the direction of the rotation, while bodies moving away from it are deflected in the opposite direction.

Dimensional analysis of $\omega(r)$

- Since frame-dragging is caused by the rotation of the star, it will be proportional to the angular momentum. Since we're only interested in dimensions, we can write the classical formula for this:

$$J = I \cdot \Omega \propto MR^2\Omega \tag{3}$$

- Using CGS units to make dimensional analysis possible (since GR units won't make us any good obviously...), also using CGS, since astronomers likes it much better than SI for some reason

$$\begin{aligned}
[G] &= cm^3 \cdot g^{-1} \cdot s^{-2} = L^3 \cdot M^{-1} \cdot T^{-2} \\
[J] &= g \cdot cm^2 \cdot s^{-1} = M \cdot L^2 \cdot T^{-1} \\
[\omega] &= s^{-1} = T^{-1}
\end{aligned}$$

Dimensional analysis of $\omega(r)$

- Now substituting into (3) and expanding $\omega(r)$ we get the following:

$$\omega(r) \propto \frac{GJ^l}{c^m r^n} \propto L^{3+2l-m-n} T^{-2-l+m} M^{-1+l} \tag{4}$$

- Solving the system of equations for the exponents gives us the solution $l = 1$, $m = 2$ and $n = 3$:

$$\omega(r) \propto \frac{GJ}{c^2 r^3} \xRightarrow{G=1,\ c=1} \frac{J}{r^3} = \frac{I\Omega}{r^3}, \quad r \geq R \tag{5}$$

- The true relationship can be coined as

$$\omega(r) = \frac{2I\Omega}{r^2} \tag{6}$$

Amdahl's number

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