Limitations on rotation and radius of NS

A presentation based on Glendenning's Compact Stars

Balázs Pál

Eötvös Loránd University

The structure of compact stars
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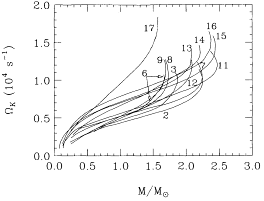
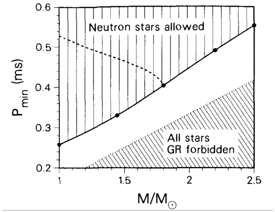
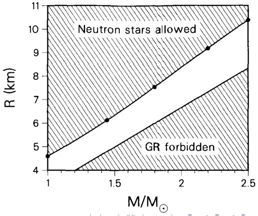
Overview

Types of limiting factors of rotation and radius

- Limits for single stars (e.g. Kepler frequency, gravitational wave instabilities etc.)
- Absolute limits (e.g. General Relativity, Le Chateliér's principle, Causality etc.)

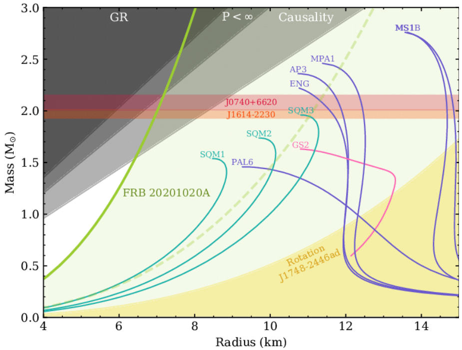






Absolute limits

- General relativity: the Einstein's equations should hold for the stellar structure
- Le Chatelier's principle: for small distortions, an NS in equilibrium will bounce back to an equilibrium ($\mathrm{d}p/\mathrm{d}\rho \geq 0$)
- Causality: Speed of sound in the NS should not be larger than the speed of light ($\sqrt{\mathrm{d}p/\mathrm{d}\epsilon} \leq 1$)



Kepler frequency in GR

- The Kepler angular velocity or Kepler frequency for short is the absolute limit for a star to remain stable. A rotational velocity above the Kepler frequency causes the star to start shredding along its equator due to centrifugal forces.
- The Newtonian equilibrium of gravitational and centrifugal force is

$$\frac{Mm}{R^2} = m\Omega^2 R \tag{1}$$

- A self-consistency condition for a fluid element moving along the equator of the star with V velocity with respect to the ZAMO:

$$\Omega_K = e^{\nu-\mu}\frac{V}{R+\omega(R)} \tag{2}$$

Kepler frequency in GR

- To only unknown in Eq. (2) is the V velocity of a fluid element along the equator. This can be determined by finding the extremum of

$$\mathrm{d}\tau = \int_{t_1}^{t_2} \mathrm{d}t \sqrt{e^{2\nu} - r^2 e^{2\mu} \left(\Omega - \omega\left(R\right)\right)^2} \tag{3}$$

- After solving the variational problem and expanding V, we get the solution as

$$V_{\pm} = \frac{R\omega'}{2\psi'} e^{\mu-\nu} \pm \sqrt{\frac{\nu'}{\psi'} + \left(\frac{R\omega'}{2\psi'} e^{\mu-\nu}\right)^2}, \tag{4}$$

where $\psi' = \mu' + 1/R$

- The physically interesting result is given by V_+, since that is the co-rotating case needed for Eq. (2).

Approximations of the Kepler frequency

- Approximating the rotating object with the Schwarzschild metric, which gives the same result as the Hartle–Thorn perturbative solution:

$$\Omega_K \approx \left[1 + \frac{\omega(R)}{\Omega_K} - 2\left(\frac{\omega(R)}{\Omega_K}\right)^2\right]^{-1/2} \sqrt{\frac{M}{R^3}} \tag{5}$$

- Approximating with a prefactor times the Newtonian formula:

$$\Omega_K \approx 0.65 \cdot \left(\frac{M}{R^3}\right)^{1/2} \tag{6}$$

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Approximations of the Kepler frequency

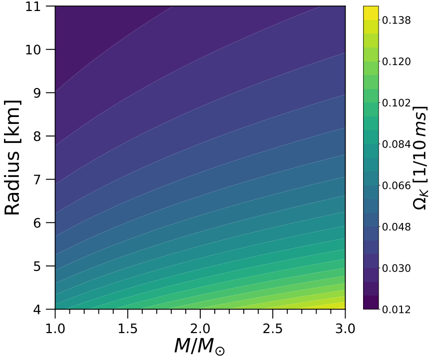

Figure 1: Contour plot of the Ω_K Kerpler frequency calculated from the approximation in Eq. (6).

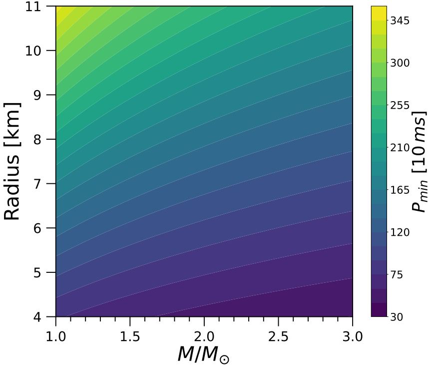

Figure 2: Contour plot of the P_{min} minimal rotation period calculated from the approximation in Eq. (6).