Linear Polarization-based Entanglement of a Single Photon, 2-Qubit Spatial Mode System

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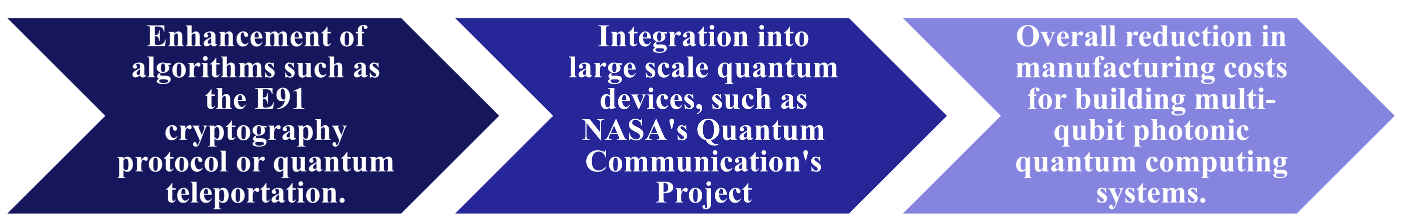
I. Introduction

Quantum communications stands at the forefront of providing enhanced data security by exploiting the laws of quantum mechanics. A major challenge in the field has been the costly process of manufacturing a substantial number of qubits – the building blocks of quantum information protocols. Previous photon entanglement techniques such as spontaneous parametric down conversion (SPDC) rely on a non-linear crystal which result in significant energy losses and low stability. Our research introduces a linear approach to produce entanglement ***within a single photon***, significantly bolstering the scalability of quantum communication systems. Additionally, by reducing the number of photons required for such protocols, our project paves the way for a more cost-effective production of qubits.

Our research question: Can we produce 2-qubit entanglement using the spatial modes of a single polarization-encoded photon such that it violates the Clauser-Horne-Shimony-Holte (CHSH) inequality?

Future Implications

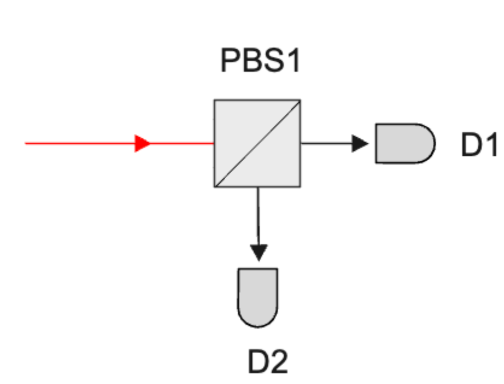


II. Procedure

1. **Define Circuit:** Create a circuit with four spatial modes.
2. **Apply Optics:**
    1. Place a quarter waveplate rotated 0° on mode 1 to create superposition in the polarization basis.
    2. Use a non-polarizing beam splitter to split the photon between modes 1 and 2, transferring the polarization state.
    3. Apply a quarter waveplate rotated 0° on mode 2 to remove its polarization from superposition, and a half-wave plate rotated 45° on mode 1 for Bell state encoding.
    4. Use non-polarizing beam splitters to recombine paths and polarizing beam splitters with ancilla modes 0 and 3 grouped with modes 1 and 2.
3. **Processor and Simulation:**
    1. Construct a processor with the circuit using the Strong Linear Optical (SLOS) simulator.
    2. Define the input Fock state $|0, 1, 0, 0\rangle$ and create a sampler.
    3. Run 10,000 shots, record results, and display with a Matplotlib bar graph.
    4. Repeat for half-wave plate angles in 45-degree increments.

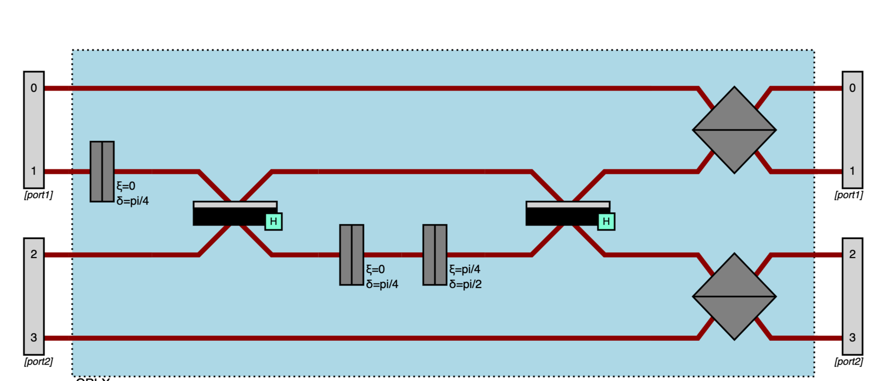
Hong-Ou-Mandel Effect

- A prerequisite for entanglement is "indistinguishability", which means the **qubits act as a unified system**
- With photons, we can accomplish this with Hong-Ou-Mandel (HOM) interference
- By the HOM effect, qubits are indistinguishable when both spatial modes **exit through the same port following a beam splitter**
    - Corresponds to one photon detection

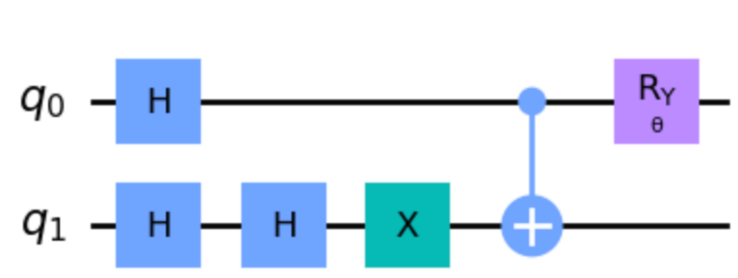


III. Simulation Techniques

Our research mainly relied on Perceval, a Python-based photonic quantum computing library developed by Quandela.



The **experimental interferometer set-up** above uses a Class 3R laser to supply the input photon. The spatial modes are split and recombined using beam splitters (BS) and mirrors, as well as a half-wave plate (HWP) to rotate the polarization in spatial mode B. The Hong-Ou-Mandel (HOM) effect takes place at the second BS and uses polarizing beam splitters (PBS) to measure photon coincidences at each output port. The PBS are secured by 3D-printed enclosures. Right is the Perceval-generated circuit.



Qiskit, IBM's quantum computing software development kit, was used to confirm the creation of the polarized two-qubit $|\Psi+\rangle$ state. Instead of depicting spatial mode interference, the circuit above applies gates to encode and rotate the polarization states of the qubits. The Hadamard gates correspond to quarter wave plates (QWP), the Pauli-X gate corresponds to the half-wave plate (HWP), and the controlled-NOT gate represents the HOM interference. A rotational Y gate was used to parameterize the circuit from angles between $[0, 2\pi]$.

IV. Data Analysis

Figure 1 and 3 were generated by a simulation of the interferometer above using Perceval, and Figure 2 was generated using Qiskit.

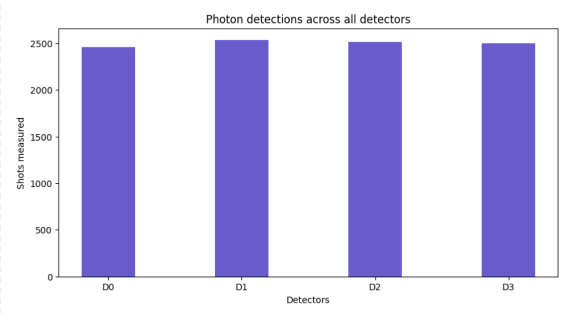


Figure 1: Roughly uniform readings across all four detectors, indicating successful HOM interference when $\xi = 0 + \pi(2k–1)/4$, $k \in \mathbb{Z}$. We observe indistinguishability at all ports, as spatial modes were bunched together to produce each photon coincidence.

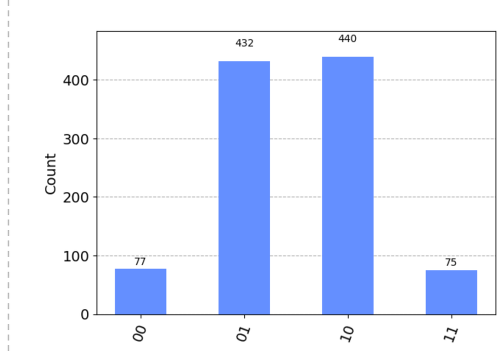


Figure 2: Noisy bit-flip model in Qiskit shows the simulated 0.01 error in measuring expected polarization states $|01\rangle$ and $|10\rangle$. These are the expected results for the third Bell state.

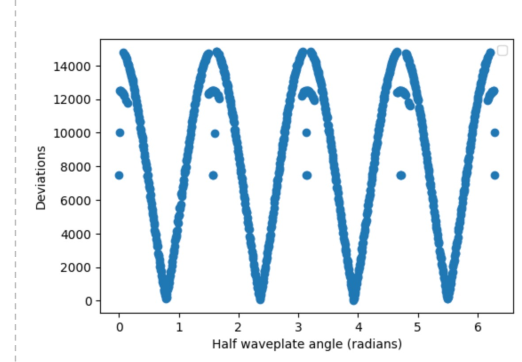


Figure 3: Total deviations from expected photon count of 2500 at each detector when the half waveplate is rotated by angles between $[0,2\pi]$. Minimal deviations occur when $\xi = 0 + \pi(2k–1)/4$, $k \in \mathbb{Z}$, since the 10,000 photon shots are uniformly distribution at each detector.

V. Theoretical Modelling

Mathematically, the Hong-Ou-Mandel effect can be analyzed for a single photon, two mode system using Fock states and annihilation and creation operators. When one photon is inputted as a superposition of modes *a* and *b*, the modes interfere at a beam splitter such that they are bunched together at the outputs. The Fock state at each output mode is therefore taken out of superposition and represented as a single photon detection, corresponding to bunched spatial mode qubits.

$$\hat{a}^\dagger\,|0\rangle_a = \tfrac{1}{\sqrt{2}}(|0\rangle_a + |1\rangle_a) = |+\rangle_a$$

$$\hat{b}^\dagger\,|0\rangle_b = \tfrac{1}{\sqrt{2}}(|0\rangle_b + |1\rangle_b) = |+\rangle_b$$

$$|+,+\rangle_{ab} = \hat{a}^\dagger\hat{b}^\dagger\,|0,0\rangle_{ab}$$

$$\hat{a}^\dagger = \frac{\hat{c}^\dagger + \hat{d}^\dagger}{\sqrt{2}}$$

$$\hat{b}^\dagger = \frac{\hat{c}^\dagger - \hat{d}^\dagger}{\sqrt{2}}$$

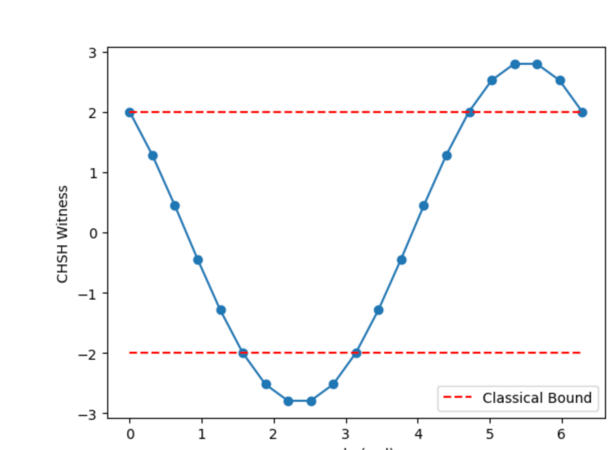
$$|+,+\rangle_{ab} = \hat{a}^\dagger\hat{b}^\dagger\,|0,0\rangle_{ab} = \tfrac{1}{2}(\hat{c}^\dagger + \hat{d}^\dagger)(\hat{c}^\dagger - \hat{d}^\dagger)\,|+,+\rangle_{cd}$$

$$= \tfrac{1}{2}(\hat{c}^{\dagger 2} - \hat{d}^{\dagger 2})\,|+,+\rangle_{cd} = \frac{|1,0\rangle_{cd} - |0,1\rangle_{cd}}{\sqrt{2}}$$

$$\hat{c}^{\dagger 2}\,|+,+\rangle_{cd} = \sqrt{2}\,|1,0\rangle_{cd}$$

VI. Results and Analyses

Waveplate rotation across optical axis	Photon coincidences at detector
$\xi = 0 + \pi k,\ k \in \mathbb{Z}$	$\|0, 0, 0, 1\rangle$: 10000
$\xi = 0 + \pi(2k - 1)/4,\ k \in \mathbb{Z}$	$\|1, 0, 0, 0\rangle$: 2500 $\|0, 1, 0, 0\rangle$: 2500 $\|0, 0, 1, 0\rangle$: 2500 $\|0, 0, 0, 1\rangle$: 2500
$\xi = 0 + \pi(2k - 1)/2,\ k \in \mathbb{Z}$	$\|1, 0, 0, 0\rangle$: 10000



- Successful Hong-Ou-Mandel (HOM) effect is indicated by **uniform detections across all ports, consistently demonstrating spatial modes are "bunched together".**
- This uniform detection confirms the indistinguishability between qubits, a **necessary condition for entanglement.**
- Unsuccessful HOM interference occurs when the HWP is rotated by multiples of **π and $\pi/2$,** resulting in all photons being detected at a single port.
- Successful HOM interference is observed at rotations of multiples **of $\pi/4$**, as seen by equal photon coincidences across all detectors.
- To definitively confirm entanglement, the **CHSH inequality** is calculated and observed through the Qiskit-generated graph on the right.
- Verification of entanglement is confirmed when parts of the curve **fall below y = –2 and above y = 2, aligning with Tsirelson's bound for a Bell state**.

VII. Conclusion

- Qubit entanglement in the $|\Psi+\rangle$ state can be created within an individual photon using its polarized spatial modes.
- The Hong-Ou-Mandel indistinguishability effect for modes was observed in Perceval as uniform photon detections across all ports when $\xi = 0 + \pi(2k–1)/4$, $k \in \mathbb{Z}$.
- The Clauser-Horne-Shimony-Holt (CHSH) inequality was violated in Qiskit, verifying entanglement.

References

1. Fiorentino, M., & Wong, F. (2004). Deterministic controlled-not gate for single-photon two-qubit quantum logic. *Physical Review Letters*, *93*(7). https://doi.org/10.1103/physrevlett.93.070502
2. Shafi, K. M., Gayatri, R., Padhye, A., & Chandrashekar, C. M. (2021). Bell-inequality in path-entangled single photon and purity test.

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