Linear Polarization-based Entanglement of a Single Photon, 2-Qubit Spatial Mode System

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***Abstract*—Quantum communications offers a revolutionary solution to future data security challenges. Photons, the building blocks of quantum communications, encode quantum information, such as polarization and momentum. We investigate the enhancement of communications protocols by treating the spatial modes of a single polarization-encoded photon as two separate qubits. In effect, we can recreate traditional 2-photon operations in an individual photon, improving the scability of quantum devices. To demonstrate the robustness of our approach, we created path entanglement between the spatial mode qubits in the polarization basis. By introducing a linear approach, we avoid low conversion efficiencies and spectral limitations associated with nonlinear processes like spontaneous parametric down conversion (SPDC). We explore extending the concept of two-photon Hong-Ou-Mandel (HOM) interference to demonstrate instinguishability between two spatial modes. Using the Python-based photonic quantum computing library Perceval, we leveraged a single photon source as the input, combined with beam splitters (BS), waveplates (WP), and polarizing beam splitters (PBS). The HOM effect is observed when both spatial modes bunch together at the port to form a photon coincidence. We used Perceval's Strong Linear Optical (SLOS) simulator to run 10,000 shots and measured equal probabilities at all four detectors, confirming consistent indistinguishability of the modes when the HWP is rotated $\pi/4$ radians. Finally by parameterizing the equivalent circuit in Qiskit with angles from $[0, 2\pi]$, we showed a clear violation of the Clauser-Horne-Shimony-Holt (CHSH) inequality, more definitively indicating successful entanglement.**

***Index Terms*—Single photon entanglement, Spatial modes, Polarization qubits**

I. INTRODUCTION

The dynamic use of photons as qubits allows for the integration of prior technology, such as fiber optics and quantum dots, into quantum computing hardware. Photonic quantum computers do not require extreme temperatures or severe pulses of energy to perform gate operations, unlike superconducting and ion-trap quantum computers.

Bell state entanglement is critical for quantum communication protocols, enabling information sharing between two parties. This paper explores producing a similar effect within a single photon, as opposed to two separate photons by considering its spatial modes as individual qubits. Experimentally, entanglement is created using a beam recombination technique inspired from a Mach-Zehnder interferometer, along with Hong-Ou-Mandel interference for verifying indistinguishability. This novel approach uses the vertical and horizontal polarization states within a single photon's spatial modes to encode separate qubits.

II. SINGLE PHOTON PATH ENTANGLEMENT

Single photon path entanglement involves the entanglement between spatial modes of a photon. When the polarized photon passes through the first beam splitter, the Fock spaces for the two output modes are entangled due to the photon being delocalized over multiple spatial locations simultaneously.

Let's denote the two possible paths following the first beamsplitter as spatial mode A and spatial mode B. The state can then be represented as $|\psi\rangle = \alpha |A\rangle + \beta |B\rangle)$, where α and β represent the probability amplitudes of the photon entering each path respectively.

However, since we are treating each mode as an individual qubit, we can write the photon state as a two-qubit system using polarization.

$$|\psi\rangle = \frac{1}{2}(|HH\rangle + |HV\rangle + |VH\rangle + |VV\rangle) \tag{1}$$

Our goal through the course of this paper will be to manipulate the spatial mode system such that the polarization state mirrors the third Bell state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|H\rangle_A + |V\rangle_A) \otimes |V\rangle_B = \frac{1}{\sqrt{2}}(|HV\rangle + |VH\rangle) \tag{2}$$

III. MATERIALS AND METHODS

The interferometer circuit was produced both computationally and experimentally using Perceval, Quandela's open-source photonic quantum computing toolkit, and Qiskit, IBM's quantum computing toolkit. The encoded qubits contained two distinct degrees of freedom: polarization and spatial modes. Linear components such as the single photon input source, beam splitters, waveplates, and polarizing beam splitters were used to create path entanglement. To ensure precision of these configurations, we used 3D-printed holders to secure the beam splitters, as well as a collimating lens to direct the beam.

To encode the input photon's polarization in the superposition state, we used a quarter wave-plate $(\lambda/4)$ rotated $0\,\deg$ about the optical axis, which acts as a Hadamard gate to the photon's polarization. We assumed that resultant

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modes A and B inherited the polarization from the input. We then used another quarter wave plate on mode B to remove its polarization from superposition, since the effects of two Hadamard gates cancel out. The critical component was the half-wave ($\lambda/2$) plate applied to mode B, which, when rotated by 45 degrees, acted as a Pauli-X gate, essential for creating the third Bell state.

Following the steps for encoding the spatial mode qubits, we can move into constructing the entangled Bell pair. The standard BSM procedure introduces two polarizing beam splitters (PBS) and involves a technique known as Hong-Ou-Mandel interference.

Traditionally, this effect occurs when two indistinguishable photons perfectly overlap at the PBS, causing them to exit from the same port 100% of the time. We consider extending this concept to two spatial modes as a verification of indistinguishability. Our research defines that each photon coincidence at the detector will correspond to perfectly overlapped spatial modes, indicating HOM interference. For consistency, we expect to see this effect across all four detectors when we run multiple shots.

IV. Theoretical Model

Mathematically, the Hong-Ou-Mandel effect can be analyzed for a single photon, two mode system using Fock states and annihilation and creation operators [5].

$$\hat{a}^\dagger |0\rangle_a = \frac{1}{\sqrt{2}}(|0\rangle_a + |1\rangle_a) = |+\rangle_a \tag{3}$$

$$\hat{b}^\dagger |0\rangle_b = \frac{1}{\sqrt{2}}(|0\rangle_b + |1\rangle_b) = |+\rangle_b \tag{4}$$

which represents the photon in superposition of being present and non-present for both spatial modes a and b.

Combining the state, we get

$$|+,+\rangle_{ab} = \hat{a}^\dagger \hat{b}^\dagger |0,0\rangle_{ab} \tag{5}$$

Next, we can define our output modes c and d. When mixed at a 50:50 PBS, the probability of detection at our output modes are equal. Thus,

$$\hat{a}^\dagger \hat{=} \frac{\hat{c}^\dagger + \hat{d}^\dagger}{2} \tag{6}$$

$$\hat{b}^\dagger \hat{=} \frac{\hat{c}^\dagger - \hat{d}^\dagger}{2} \tag{7}$$

with a phase shift introduced from the beam-splitter for the output, given mode b as the input.

We can combine these two operations as follows,

$$|+,+\rangle_{ab} = \hat{a}^\dagger \hat{b}^\dagger |0,0\rangle_{ab} = \frac{1}{2}(\hat{c}^\dagger + \hat{d}^\dagger)(\hat{c}^\dagger - \hat{d}^\dagger) |+,+\rangle_{cd} \tag{8}$$

given that

$$\hat{c}^{\dagger 2} |+,+\rangle_{cd} = \sqrt{2} |1,0\rangle_{cd} \tag{9}$$

According to this result, when one photon is inputted as a superposition of modes a and b, the modes interfere at a beam splitter such that the qubits are bunched together at the outputs.

V. Results and Discussion

Using Perceval's Strong Linear Optical Simulator (SLOS), we ran 10,000 shots and measured equal coincidences at all four detectors when the half wave plate angle was rotated 45 degrees. This confirms HOM interference and establishes the indistinguishability of the modes, a critical component of the standard Bell state measurement procedure. This sets us apart from previous research in the field which simply calculates correlations between measurements of the modes' polarization states, as opposed to rigorously defining indistinguishability as we have shown. In doing so, we fully capture the quality of entanglement of our system, establishing a framework for more complex protocols such as quantum key distribution (QKD). We further confirmed entanglement by demonstrating a violation of the Clauser-Horne-Shimony-Holt (CHSH) inequality in our Qiskit-replicated circuit. In the future, we will be exploring more robust experimental demonstrations, using waveguides and single-mode fibers to ensure mode alignment. Additionally, we will use quantum error correction models to combat mode-mismatch in potential hardware implementations.

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