Part. 1, Coding

Q1:

1. Compute the mean vectors mi, (i=1,2) of each 2 classes

```
## Your code HERE
# classify training data into two classes
class1 = np.empty((0,2))
class2 = np.empty((0,2))
# give initial zeroes
for i in range(y_train.size):
    # class1
    if y_train[i]==0:
        class1 = np.append(class1, np.array([x_train[i]]), axis=0)
    # class2
    else:
        class2 = np.append(class2, np.array([x_train[i]]), axis=0)

m1 = class1.mean(axis=0).reshape(-1,1)
m2 = class2.mean(axis=0).reshape(-1,1)
```

Loading...

```
[ ] print(f"mean vector of class 1: {m1}", f"mean vector of class 2: {m2}")
```

```
mean vector of class 1: [[2.47107265]
 [1.97913899]] mean vector of class 2: [[1.82380675]
 [3.03051876]]
```

Q2:

2. Compute the Within-class scatter matrix SW

```
## Your code HERE

# give initial zeroes
sw = np.zeros((2,2))
# class1
for x in class1:
    sw += np.dot(x.reshape(-1,1)-m1, (x.reshape(-1,1)-m1).T)
    # class2
for x in class2:
    sw += np.dot(x.reshape(-1,1)-m2, (x.reshape(-1,1)-m2).T)
```

```
[ ] assert sw.shape == (2, 2)
    print(f"Within-class scatter matrix SW: {sw}")
```

```
Within-class scatter matrix SW: [[140.40036447  -5.30881553]
 [ -5.30881553 138.14297637]]
```

Q3:

3. Compute the Between-class scatter matrix SB

```
[ ] ## Your code HERE


    # total sb
    sb = np.dot(m1-m2, (m1-m2).T)
```

```
assert sb.shape == (2,2)
print(f"Between-class scatter matrix SB: {sb}")
```

```
Between-class scatter matrix SB: [[ 0.41895314 -0.68052227]
 [-0.68052227  1.10539942]]
```

Q4:

4. Compute the Fisher's linear discriminant

```
[ ] # w is porportional to Sw-1 * (m2-m1)
    w = np.dot(np.linalg.inv(sw), (m2-m1))

    # w is restricted to 1 unit
    w /= np.sqrt(w[0]**2 + w[1]**2)
```

```
[ ] assert w.shape == (2,1)
    print(f"Fisher's linear discriminant: {w}")
```

```
Fisher's linear discriminant: [[-0.50266214]
 [ 0.86448295]]
```

Q5:

5. Project the test data by linear discriminant and get the class prediction by nearest-neighbor rule. Calculate the accuracy score

you can use `accuracy_score` function from `sklearn.metric.accuracy_score`

```
y_pred = np.zeros(y_test.size)
# compute train value
trained_class1_mean = np.mean(np.dot(w.T, class1.T))
trained_class2_mean = np.mean(np.dot(w.T, class2.T))
# compute tested value
for i, x in enumerate(x_test):
    tmp = np.dot(w.T, x.T)
# use mean value to estimate to reduce
# the influence of outliers
    if np.abs(trained_class1_mean-tmp)<np.abs(trained_class2_mean-tmp):
        y_pred[i] = 0
    else:
        y_pred[i] = 1
acc = accuracy_score(y_test, y_pred)
```

```
[ ] print(f"Accuracy of test-set {acc}")
```

```
Accuracy of test-set 0.908
```

Q6:

6. Plot the 1) best projection line on the training data and show the slope and intercept on the title (you can choose any value of intercept for better visualization) 2) colorize the data with each class 3) project all data points on your projection line. Your result should look like [this image]()

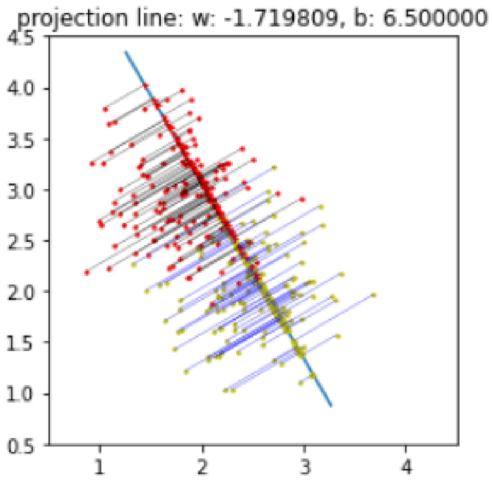
```
[ ] weight = w[1][0]/w[0][0]
    intercept = 6.5
    plt.title("projection line: w: %f, b: %f" %(weight, intercept))
    plt.axis('square')
    plt.xlim(0.5,4.5)
    plt.ylim(0.5,4.5)
    # plot the projection line
    plt.plot(np.array([-6.5*w[0][0], -2.5*w[0][0]]),
             np.array([intercept-6.5*w[1][0], intercept-2.5*w[1][0]]))

    # classify test data into two classes
    pred_class1 = np.empty((0,2))
    pred_class2 = np.empty((0,2))
    # give initial zeroes
    for i in range(y_test.size):
        # class1
        if y_test[i]==0:
            pred_class1 = np.append(pred_class1,
                                    np.array([x_test[i]]), axis=0)
        # class2
        else:
            pred_class2 = np.append(pred_class2,
                                    np.array([x_test[i]]), axis=0)

    # plot the value of each class
    plt.plot(pred_class1[:,0], pred_class1[:,1], 'y.',
             pred_class2[:,0], pred_class2[:,1], 'r.', markersize=3)
```

```
# calculate projection point of each point
proj_x1 = ((weight*(pred_class1[:,1]-intercept)+pred_class1[:,0])
          / (weight**2+1))
proj_y1 = weight*proj_x1 + intercept
proj_x2 = ((weight*(pred_class2[:,1]-intercept)+pred_class2[:,0])
          / (weight**2+1))
proj_y2 = weight*proj_x2 + intercept

proj_class1 = np.concatenate((proj_x1.reshape(1,-1),
                              proj_y1.reshape(1,-1)),
                              axis=0
                            ).T
proj_class2 = np.concatenate((proj_x2.reshape(1,-1),
                              proj_y2.reshape(1,-1)),
                              axis=0
                            ).T
plt.plot(proj_class1[:,0], proj_class1[:,1], 'y>',
         proj_class2[:,0], proj_class2[:,1], 'r+', markersize=3)
for i in range(len(pred_class1)):
    plt.plot(np.array([pred_class1[i][0], proj_class1[i][0]]),
             np.array([pred_class1[i][1], proj_class1[i][1]]),
             color='b', linewidth=0.3)
for i in range(len(pred_class2)):
    plt.plot(np.array([pred_class2[i][0], proj_class2[i][0]]),
             np.array([pred_class2[i][1], proj_class2[i][1]]),
             color='k', linewidth=0.3)
```



Part 2, Writing

1. To maximize the class separation criterion : $\nabla L(\lambda, w) = 0$

$$\Rightarrow \frac{\partial L(\lambda,w)}{\partial \lambda} = w^T w - 1 = 0 \Rightarrow w^T w = 1$$

$$\Rightarrow L(\lambda, w) = \frac{1}{w}(m_2 - m_1) + 0$$

$$\Rightarrow w \cdot L(\lambda, w) = m_2 - m_1$$

$$\Rightarrow w \propto (m_2 - m_1)$$

#

2.
$$\frac{(m_2-m_1)^2}{S_1^2+S_2^2} = \frac{[w^T(m_2-m_1)]^2}{\sum_{n\in C_1}(w^T x_1 - w^T m_1)^2 + \sum_{n\in C_2}(w^T x_2 - w^T m_2)^2}$$

$$= \frac{w^T(m_2-m_1)\cdot \overbrace{\boxed{w^T(m_2-m_1)}}^{\text{constant}}}{\sum_{n\in C_1}\left[w^T(x_1-m_1)\cdot \underbrace{\boxed{w^T(x_1-m_1)}}_{\text{constant}}\right] + \sum_{n\in C_2}\left[w^T(x_2-m_2)\cdot \underbrace{\boxed{w^T(x_2-m_2)}}_{\text{constant}}\right]}$$

$$= \frac{w^T(m_2-m_1)\cdot(m_2-m_1)^T\cdot w}{w^T\left(\sum_{n\in C_1}(x_1-m_1)\cdot(x_1-m_1)^T + \sum_{n\in C_2}(x_2-m_2)\cdot(x_2-m_2)^T\right)\cdot w}$$

$$= \frac{w^T \cdot S_B \cdot w}{w^T \cdot S_W \cdot w}$$

#

3.

$$\nabla E(w) = \frac{\partial\left(-\sum_{n=1}^{N}\{t_n \ln \Delta(a_n) + (1-t_n)\ln(1-\Delta(a_n))\}\right)}{\partial w}$$

$$= -\sum_{n=1}^{N}\left\{\frac{t_n}{\cancel{\Delta(a_n)}}\cdot\cancel{\Delta(a_n)}\cdot(1-\Delta(a_n))\cdot a_n' + \frac{1-t_n}{\cancel{1-\Delta(a_n)}}\cdot(-\Delta(a_n))\cdot\cancel{(1-\Delta(a_n))}\cdot a_n'\right\}$$

$$= -\sum_{n=1}^{N}\{(t_n - t_n y_n)\cdot\phi_n - (y_n - y_n t_n)\cdot\phi_n\}$$

$$= -\sum_{n=1}^{N}(t_n\phi_n - y_n\phi_n)$$

$$= \sum_{n=1}^{N}(y_n - t_n)\cdot\phi_n$$

$\#$