♦ 15.4 Application of double integrals

0.1 Classical mechanics 古典力學

A lamina [ˈlæmənə, “勒麼那”] 薄片 (不計厚度)

- ***density*** 密度: $\rho(x,y) = \lim_{\Delta A \to 0} \dfrac{\Delta m}{\Delta A}$ (質量與面積比值的極限),
  Δm 與 ΔA 是包含 (x,y) 矩形的質量與面積。

- ***mass*** 質量: $m = \displaystyle\iint_D \rho(x,y)\ dA$.

- ***moment*** (**of mass**) (力) 矩或動差 about x- and y-axis:

$$M_x = \iint_D y\rho(x,y)\ dA, \quad M_y = \iint_D x\rho(x,y)\ dA.$$

  (力矩 = 施力 × 距離; 考慮地心引力 $F = mg$, 重力加速度 $g = 9.8\ \mathrm{m/s^2}$, 可以簡化爲: 質量 × 距離。)

- ***center of mass*** 質心: $(\bar{x}, \bar{y})$, $\ni m\bar{x} = M_y$, $m\bar{y} = M_x$.

$$\bar{x} = \frac{M_y}{m} = \frac{\iint_D x\rho(x,y)\ dA}{\iint_D \rho(x,y)\ dA}, \quad \bar{y} = \frac{M_x}{m} = \frac{\iint_D y\rho(x,y)\ dA}{\iint_D \rho(x,y)\ dA}.$$

  (看成以密度爲權重的加權平均座標。)

- ***moment of inertia*** [ɪnˈɝʃə, “引|呢下”] or ***second moment*** 轉動慣量或慣性矩 (=質量×距離^2) about x-axis, y-axis, and the origin:

$$I_x = \iint_D y^2\rho(x,y)\ dA, \quad I_y = \iint_D x^2\rho(x,y)\ dA,$$
$$I_O = \iint_D (x^2+y^2)\rho(x,y)\ dA = I_x + I_y.$$

  (轉動慣量描述物體對於其旋轉運動的慣性 (改變的對抗)。)

  ♦ 動能 (kinetic [kɪˈnɛtɪk] energy) $E_k = \frac{1}{2}mv^2 = \frac{1}{2}I\omega^2$,
  m: 質量, v: 速度, I: 轉動慣量, ω: 角速度。

- ***radius of gyration*** [dʒɑɪˈreʃən, “宅略遜”] 迴轉半徑 $\bar{\bar{y}}$ with respect to x-axis and $\bar{\bar{x}}$ with respect to y-axis, $\ni m\bar{\bar{x}}^2 = I_y$, $m\bar{\bar{y}}^2 = I_x$.

$$\bar{\bar{x}} = \sqrt{\frac{I_y}{m}} = \sqrt{\frac{\iint_D x^2\rho(x,y)\ dA}{\iint_D \rho(x,y)\ dA}} \quad \bar{\bar{y}} = \sqrt{\frac{I_x}{m}} = \sqrt{\frac{\iint_D y^2\rho(x,y)\ dA}{\iint_D \rho(x,y)\ dA}}.$$

0.2 Statics 統計

- ***probability density function*** 機率密度函數 $f(x)$ of a continuous ***random variable*** 隨機變數 X:

$$f(x) \geq 0, \int_{-\infty}^{\infty} f(x)\ dx = 1,$$

and the ***probability*** 機率 that X lies between a and b is

$$P(a \leq X \leq b) = \int_a^b f(x)\ dx.$$

- ***joint (probability) density function*** 聯合機率密度函數 $f(x, y)$ of a pair continuous random variables X, Y:

$$f(x,y) \geq 0, \iint\limits_{\mathbb{R}^2} f(x,y)\ dA = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\ dx\ dy = 1,$$

and the probability that (X, Y) lies in a region D is

$$P((X,Y) \in D) = \iint\limits_D f(x,y)\ dA.$$

- X and Y are ***independent random variables*** 獨立隨機變數 if

$$f(x,y) = f_1(x)f_2(y).$$

- ***expected value*** 期望值 or **mean** 平均值 $\mu = \displaystyle\int_{-\infty}^{\infty} xf(x)\ dx.$

- ***standard deviation*** [diʊɪˋeʃən] 標準差 $\sigma = \sqrt{\displaystyle\int_{-\infty}^{\infty}(x-\mu)^2 f(x)\ dx}.$

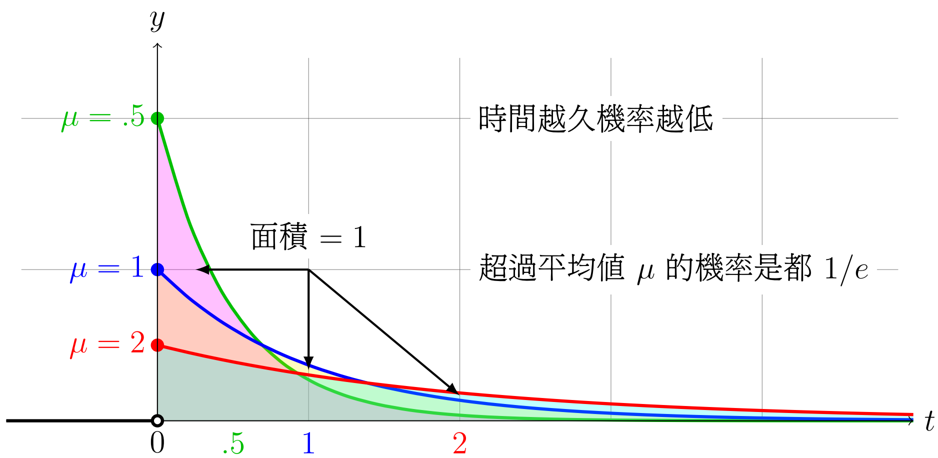
- **X-, Y-mean** X-, Y-平均值

$$\mu_1 = \iint\limits_{\mathbb{R}^2} xf(x,y)\ dA,\ \mu_2 = \iint\limits_{\mathbb{R}^2} yf(x,y)\ dA.$$

- waiting time is modeled by using ***exponential density function***:

$$f(t) = \begin{cases} \dfrac{1}{\mu} e^{-t/\mu} & \text{if } t \geq 0 \\ 0 & \text{if } t < 0 \end{cases}$$

where μ is the mean waiting time.
(這種模型稱爲指數分配 (exponential distribution), 常用於時間。)



- a single variable is ***normally distributed*** 常態分佈或高斯分佈 if its probability density function is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$$

where μ is the mean and σ is the standard deviation.
(統計學大多的模型都是建立在常態分佈的假設之下。)