Property: If $E[X^2]E[Y^2] = (E[XY])^2$,

then there must exist some $t \in \mathbb{R}$ such that

$$P(\{\omega : t \cdot X(\omega) = Y(\omega)\}) = 1.$$

Pf: Prove this by contradiction

Suppose for all $t \in \mathbb{R}$, we have $P(\{\omega : t \cdot X(\omega) = Y(\omega)\}) < 1$

Then, we have $\underbrace{E[(tX - Y)^2] > 0, \text{ for all } t \in \mathbb{R}}_{(*)}$

Recall that

$$E[(tX-Y)^2] = (\underbrace{E[X^2]}_{\geq 0})t^2 + (-2 \cdot E[XY])t + (E[Y^2])$$

$(*)$ implies that $(-2 \cdot E[XY])^2 - 4 \cdot E[X^2] \cdot E[Y^2] < 0$

Hence, $E[X^2] \cdot E[Y^2] > (E[XY])^2$

which leads to a contradiction

$\square$