Part I

◎ 單選擇題 (單選十題, 每題五分, 共五十分, 答錯不倒扣。)

1. How many horizontal, vertical and slant **asymptotes** does the function $f(x) = \dfrac{x^3-1}{x(x+1)}$ have? 65:35

   (A) 4; **(B)** $\boxed{\textbf{3;}}$ (C) 2; (D) 1.

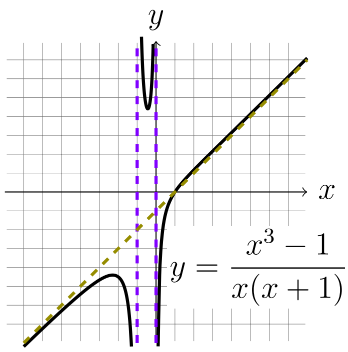
Solution:

$\lim\limits_{x\to\pm\infty} f(x) = \pm\infty$. .... No H.A..

$\lim\limits_{x\to 0^\pm} f(x) = \mp\infty$, ... V.A.: $x = 0$.

$\lim\limits_{x\to -1^\pm} f(x) = \pm\infty$, ... V.A.: $x = -1$.

$f(x) = x - 1 + \dfrac{x-1}{x(x+1)}$, S.A.: $y = x - 1$.



2. Given that $g(3) = 3$, $g'(3) = 7$, $h(6) = 3$, and $h'(6) = -2$, and let $f(x) = \dfrac{g(h(x))}{h(x)}$. Then $f'(6) =$ 85:15

   (A) $-\dfrac{8}{3}$; (B) $-\dfrac{16}{3}$; (C) -2; **(D)** $\boxed{\mathbf{-4.}}$

Solution: $f'(x) = \dfrac{g'(h(x))h'(x)h(x) - g(h(x))h'(x)}{[h(x)]^2}$,

$f'(6) = \dfrac{g'(h(6))h'(6)h(6) - g(h(6))h'(6)}{[h(6)]^2}$

$= \dfrac{g'(3)\cdot(-2)\cdot 3 - g(3)\cdot(-2)}{3^2} = \dfrac{7\cdot(-2)\cdot 3 - 3\cdot(-2)}{3^2} = -4.$

3. Find the **derivative** of $f(x) = \ln|x^3 - 4x + 1|$ when $x^3 - 4x + 1 \neq 0$. 67:33

(A) $\boxed{\boldsymbol{f'(x) = \frac{3x^2 - 4}{x^3 - 4x + 1};}}$ (B) $\frac{3x^2 - 4}{|x^3 - 4x + 1|}$;

(C) $-\frac{3x^2 - 4}{|x^3 - 4x + 1|}$; (D) $-\frac{3x^2 - 4}{x^3 - 4x + 1}$.

Solution: Let $u = x^3 - 4x + 1$, $\frac{d}{dx}f(x) = \frac{df(u)}{du}\frac{du}{dx}$
$= (\ln|u|)'(x^3 - 4x + 1)' = \frac{1}{u}(3x^2 - 4) = \frac{3x^2 - 4}{x^3 - 4x + 1}$.
[Quick sol] $(\ln|u|)' = \frac{u'}{u} = \frac{(x^3 - 4x + 1)'}{x^3 - 4x + 1} = \frac{3x^2 - 4}{x^3 - 4x + 1}$.

4. **The limit** $\lim_{x\to\infty}\left(1 - \frac{1}{x} - \frac{2}{x^2}\right)^x =$ 45:55

(A) 1; **(B)** $\boxed{\boldsymbol{e^{-1};}}$ (C) e^{-2}; (D) e^{-3}.

Solution: Let $y = \left(1 - \frac{1}{x} - \frac{2}{x^2}\right)^x$, $\ln y = x\ln\left(1 - \frac{1}{x} - \frac{2}{x^2}\right)$,

$\lim_{x\to\infty}\ln y = \lim_{x\to\infty}\frac{\ln(1 - 1/x - 2/x^2)}{1/x}$ $\qquad (\mathbf{1^\infty \to \infty \cdot 0 \to \frac{0}{0}})$

$= \lim_{t\to 0^+}\frac{\ln(1 - t - 2t^2)}{t}$ $\qquad (t = \frac{1}{x} \to 0^+ \text{ as } x \to \infty, (\mathbf{\frac{0}{0} \to \frac{\infty}{\infty}}))$

$\overset{l'H}{=} \lim_{t\to 0^+}\frac{\frac{-1-4t}{1-t-2t^2}}{1} = -1$, $\lim_{x\to\infty} y = \lim_{x\to\infty} e^{\ln y} = e^{\lim_{x\to\infty}\ln y} = e^{-1}$.

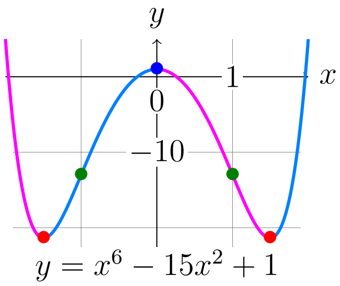
[Quick sol] $\lim_{x\to\infty}\left(1 - \frac{1}{x} - \frac{2}{x^2}\right)^x = \lim_{x\to\infty}\left[\left(1 + \frac{1}{x}\right)^x\left(1 - \frac{2}{x}\right)^x\right]$
$= \lim_{x\to\infty}\left(1 + \frac{1}{x}\right)^x \lim_{x\to\infty}\left(1 + \frac{-2}{x}\right)^x = e^1e^{-2} = e^{-1}$.

[Quicker sol] When $x \to \infty$, $1 - \frac{1}{x} - \frac{2}{x^2} \approx 1 - \frac{1}{x}$,
$\lim_{x\to\infty}\left(1 - \frac{1}{x} - \frac{2}{x^2}\right)^x = \lim_{x\to\infty}\left(1 + \frac{-1}{x}\right)^x = e^{-1}$.

5. How **many points of inflection** does the function $f(x) = x^6 - 15x^2 + 1$ have? 83:17

(A) 0; (B) 1; **(C)** $\boxed{\mathbf{2;}}$ (D) 4.

Solution: $f'(x) = 6x^5 - 30x$, $f''(x) = 30(x^4 - 1) = 0$ when $x = \pm 1$. $f''(x) > 0$ when $x < -1$ or $x > 1$, and $f''(x) < 0$ when $-1 < x < 1$, and $f(x)$ is continues at $x = \pm 1 \implies$ two inflection points.



◎ 多選擇題 (多選五題, 每題五分, 共二十五分。答錯一個選項扣兩分, 錯兩個選項以上不給分, 分數不倒扣。)

11. If $f(x) = \dfrac{ax}{x^2 + b^2}$ has a local minimum at $x = -2$ and $f'(0) = 1$, then a and b could be 92:3:4

(A) $\boxed{\boldsymbol{a = 4,\ b = 2;}}$ **(B)** $\boxed{\boldsymbol{a = 4,\ b = -2;}}$
(C) $a = 2$, $b = 4$; (D) $a = -2$, $b = 4$.

Solution: $f'(x) = \dfrac{a(x^2 + b^2) - ax(2x)}{(x^2 + b^2)^2} = \dfrac{a(b^2 - x^2)}{(x^2 + b^2)^2} = 0$ when $x = \pm b = -2$, $b = \pm 2$. $f'(0) = \dfrac{a}{b^2} = 1$, $a = b^2 = 4$.

12. Which of the following statements are **True** for $f(x) = x^{2/3}(9-x)^{1/3}$? 14:39:47

(A) $\boxed{\boldsymbol{f} \textbf{ is increasing on } \boldsymbol{(0,4)}.}$

(B) f has a local minimum at $x = 9$.

(C) $\boxed{\boldsymbol{f} \textbf{ has a local minimum at } \boldsymbol{x = 0}.}$

(D) $\boxed{\boldsymbol{f} \textbf{ has a local maximum at } \boldsymbol{x = 6}.}$

Solution: $f'(x) = \dfrac{2}{3}x^{-1/3}(9-x)^{1/3} - \dfrac{1}{3}x^{2/3}(9-x)^{-2/3}$

$= \dfrac{2(9-x)-x}{x^{1/3}(9-x)^{2/3}} = \dfrac{3(6-x)}{\sqrt[3]{x(9-x)^2}}$,

$f'(x) = 0$ when $x = 6$ and

does not exist when $x = 0, 9$.

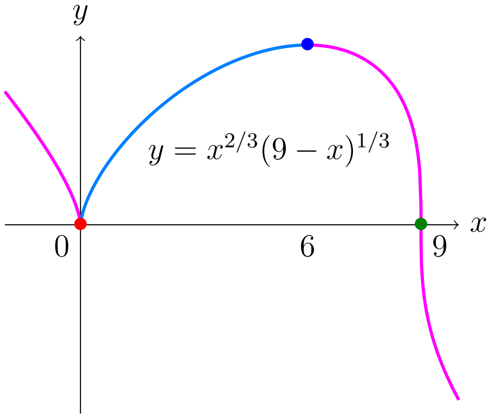
$f'(x) < 0$ when $x < 0$ or $x > 6$,

$f'(x) > 0$ when $0 < x < 6$.

f has a local min at $x = 0$ and

f has a local max at $x = 6$,

but no extreme at $x = 9$.



◎ 填空題 (填空五題, 每題五分, 共二十五分, 答錯不倒扣。)

16. The limit $\lim\limits_{x\to\infty} \dfrac{x^{2017}}{2^x} =$ 78:19

Solution: 0.

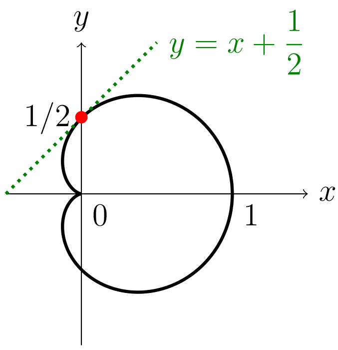
$\lim\limits_{x\to\infty} \dfrac{x^{2017}}{2^x} \overset{l'H^{2017}}{=} \lim\limits_{x\to\infty} \dfrac{2017!}{2^x(\ln 2)^{2017}} = 0.$ (ℓ'Hospital rule 2017 times.)

17. The **tangent line** to the curve $x^2 + y^2 = (2x^2 + 2y^2 - x)^2$ at the point $\left(0, \frac{1}{2}\right)$. 55:37

Solution: $y = x + \frac{1}{2}$.

....

$x^2 + y^2 = (2x^2 + 2y^2 - x)^2$,
$\frac{d}{dx} : 2x + 2yy'$
$= 2(2x^2 + 2y^2 - x)(4x + 4yy' - 1)$,
take $\left(0, \frac{1}{2}\right) : 2(0) + 2\frac{1}{2}y'$
$= 2(2(0)^2 + 2\frac{1}{2}^2 - 0)(4(0) + 4\frac{1}{2}y' - 1)$,
$y' = 2y' - 1$, $y' = 1$.
tangent line: $y = 1(x - 0) + \frac{1}{2} = x + \frac{1}{2}$.

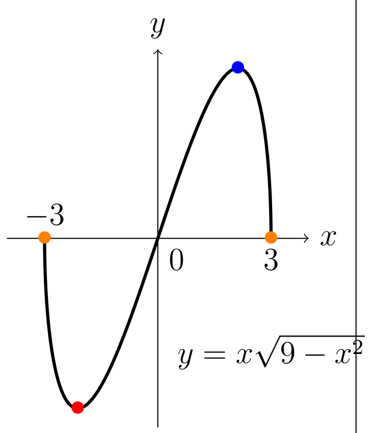


18. The **absolute maximum value** of the function $f(x) = x\sqrt{9 - x^2}$, $-3 \le x \le 3$ is 58:38

Solution: $f(\frac{3\sqrt{2}}{2}) = \frac{9}{2}$.

....

$f'(x) = \frac{9 - 2x^2}{2\sqrt{9 - x^2}} = 0$ when $x = \pm\frac{3}{\sqrt{2}}$,
does not exist when $x = \pm 3$.
$f(\pm\frac{3}{\sqrt{2}}) = \pm\frac{9}{2}$, $f(\pm 3) = 0$.
The abs. max $f(\frac{3}{\sqrt{2}}) = \frac{9}{2}$.
[Quick sol]
$x = 3\sin t$, $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$,
$f(x(t)) = \frac{9}{2}\sin 2t \le \frac{9}{2}$.



End

Part II

◎ 單選擇題 (單選十題, 每題五分, 共五十分, 答錯不倒扣。)

6. $\displaystyle\int_0^{\frac{1}{2}\ln 3} e^x\sqrt{1+e^{2x}}\,dx=$ 57:43

(A) $\boxed{\mathbf{\frac{1}{2}\left(2\sqrt{3}-\sqrt{2}+\ln\frac{2+\sqrt{3}}{1+\sqrt{2}}\right);}}$ (B) $2\sqrt{3}-\sqrt{2}+\ln\frac{2+\sqrt{3}}{1+\sqrt{2}}$;

(C) $\frac{1}{2}\left(2\sqrt{2}-\sqrt{3}+\ln\frac{2+\sqrt{2}}{1+\sqrt{3}}\right)$; (D) $2\sqrt{2}-\sqrt{3}+\ln\frac{2+\sqrt{2}}{1+\sqrt{3}}$.

Solution: Let $u=e^x=\tan t$, $du=e^x\,dx=\sec^2 t\,dt$,
$0\le x\le\frac{1}{2}\ln 3$, $1\le u\le\sqrt{3}$, $\pi/4\le t\le\pi/3$.

$$\int_0^{\frac{1}{2}\ln 3} e^x\sqrt{1+e^{2x}}\,dx=\int_1^{\sqrt{3}}\sqrt{1+u^2}\,du$$

$$=\int_1^{\sqrt{3}}\sqrt{1+\tan^2 t}\,d(\tan t)=\int_{\pi/4}^{\pi/3}\sec^3 t\,dt$$

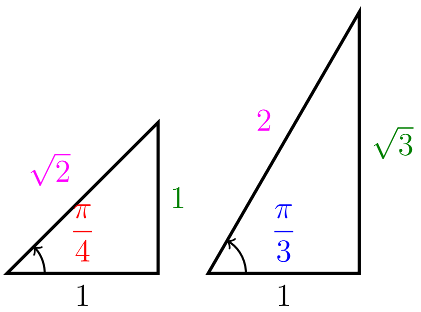
$$=\frac{1}{2}\Big[\sec t\tan t+\ln|\sec t+\tan t|\Big]_{\pi/4}^{\pi/3}$$

$$\left(=\frac{1}{2}\Big[u\sqrt{1+u^2}+\ln|\sqrt{1+u^2}+u|\Big]_1^{\sqrt{3}}\right)$$

$$\left(=\frac{1}{2}\Big[e^x\sqrt{1+e^{2x}}+\ln|\sqrt{1+e^{2x}}+e^x|\Big]_0^{\frac{1}{2}\ln 3}\right)$$

$$=\frac{1}{2}\Big[(2\sqrt{3}+\ln|2+\sqrt{3}|)-(\sqrt{2}(\times 1)+\ln|\sqrt{2}+1|)\Big]$$

$$=\frac{1}{2}\left(2\sqrt{3}-\sqrt{2}+\ln\frac{2+\sqrt{3}}{1+\sqrt{2}}\right).$$



7. If the infinite region $\Omega = \left\{(x,y)|x \geq 1, 0 \leq y \leq \frac{1}{x}\right\}$ is rotated about the **x-axis**, how about the **volume** of the resulting solid and its **surface area**? 49:51

(A) Volume is finite. Surface area is finite.

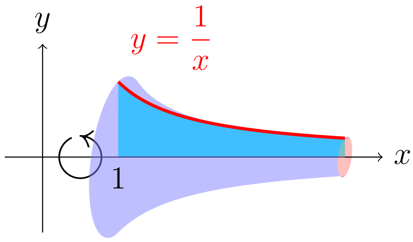
(B) Volume is infinite. Surface area is finite.

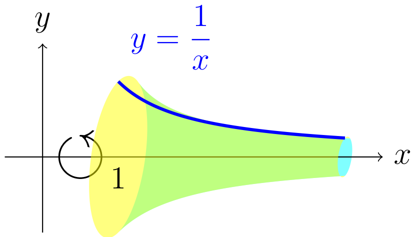
(C) $\boxed{\textbf{Volume is finite. Surface area is infinite.}}$

(D) Volume is infinite. Surface area is infinite.

Solution: Gabriel's horn:

$$V = \int_1^\infty \frac{\pi}{x^2}\, dx = \lim_{t\to\infty} \int_1^t \frac{\pi}{x^2}\, dx$$
$$= \lim_{t\to\infty} -\frac{\pi}{x}\Big|_1^t = \pi - \lim_{t\to\infty} \frac{\pi}{t}$$
$$= \pi < \infty,$$
$$S = \int_1^\infty 2\pi\frac{1}{x}\sqrt{1+\frac{1}{x^4}}\, dx$$
$$\geq \int_1^\infty 2\pi\frac{1}{x}\, dx = \infty.$$



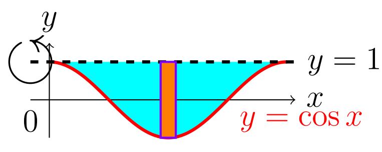


8. Let R be the region bounded by $y = \cos x$ and the line $y = 1$ with $x \in [0, 2\pi]$. The **volume** of the solid obtained by rotating the region R about the line $\boldsymbol{y = 1}$ is 59:41

(A) π^2; (B) $2\pi^2$; **(C)** $\boxed{\boldsymbol{3\pi^2};}$ (D) $4\pi^2$.

Solution: Disk:

$$V = \int_0^{2\pi} \pi(1-\cos x)^2\, dx$$
$$= \pi\int_0^{2\pi} (1 - 2\cos x + \cos^2 x)\, dx$$
$$= \pi\int_0^{2\pi} \left(\frac{3}{2} - 2\cos x + \frac{\cos 2x}{2}\right)\, dx$$



$$= \pi \left[\frac{3}{2}x - 2\sin x + \frac{\sin 2x}{4} \right]_0^{2\pi} = \pi \cdot \frac{3}{2} \cdot 2\pi = 3\pi^2.$$

9. The **length** of the curve $x = 3\cos t - \cos 3t$ and $y = 3\sin t - \sin 3t$, $0 \le t \le \pi$ is 77:23

(A) 10; (B) 11; **(C)** $\boxed{\textbf{12;}}$ (D) 13.

Solution: $ds = \sqrt{(x')^2 + (y')^2}\, dt$, $\sqrt{(x')^2 + (y')^2}$

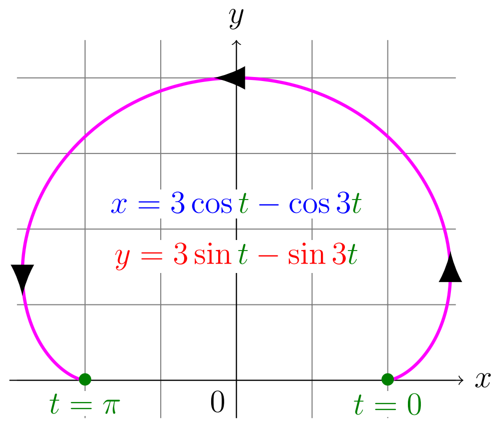
$= \sqrt{(-3\sin t + 3\sin 3t)^2 + (3\cos t - 3\cos 3t)^2}$

$= 3\sqrt{\sin^2 t - 2\sin t \sin 3t + \sin^2 3t + \cos^2 t - 2\cos t \cos 3t + \cos^2 3t}$

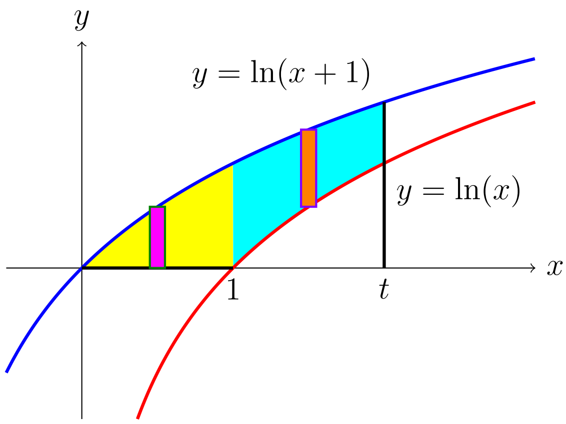
$= 3\sqrt{2 - 2(\cos 3t \cos t + \sin 3t \sin t)} = 3\sqrt{2 - 2\cos(3t - t)}$

$= 3\sqrt{2 - 2\cos 2t} = 3\sqrt{4\sin^2 t} = 3|2\sin t| = 6\sin t.$ $(0 \le t \le \pi)$

$L = \displaystyle\int ds = \int_0^{\pi} 6\sin t\, dt = 6\Big[-\cos t\Big]_0^{\pi} = 6 \cdot 2 = 12.$



10. Let R be the region enclosed by $y = \ln x$, $y = \ln(x+1)$, $y = 0$, and $x = t$ $(t > 1)$. If $V(t)$ is the volume of the solid obtained by rotating R about the ***y*-axis**, then the limit $\lim_{t\to\infty}\left(\frac{d}{dt}V(t)\right) =$ 61:39



(A) π;

(B) $\boxed{\mathbf{2\pi;}}$

(C) 3π;

(D) 4π.

Solution: Cylindrical shell:

$$V(t) = \int_0^1 2\pi x \ln(x+1)\, dx + \int_1^t 2\pi x[\ln(x+1) - \ln x]\, dx,$$

$$V'(t) = 0 + 2\pi t[\ln(t+1) - \ln t] = 2\pi t \ln(1 + \frac{1}{t}) = 2\pi \ln\left(1 + \frac{1}{t}\right)^t.$$

$$\lim_{t\to\infty} V'(t) = 2\pi \lim_{t\to\infty} \frac{\ln(1+1/t)}{1/t} = 2\pi \lim_{s\to 0^+} \frac{\ln(1+s)}{s} \qquad \left(\boldsymbol{\infty \cdot 0 \to \frac{0}{0}}\right)$$

$$\overset{l'H}{=} 2\pi \lim_{s\to 0^+} \frac{1/(1+s)}{1} = 2\pi \cdot 1 = 2\pi.$$

[Quick sol]

$$\lim_{t\to\infty} V'(t) = 2\pi \lim_{t\to\infty} \ln\left(1 + \frac{1}{t}\right)^t = 2\pi \ln \lim_{t\to\infty}\left(1 + \frac{1}{t}\right)^t$$

$$= 2\pi \ln e = 2\pi.$$

◎ 多選擇題 (多選五題, 每題五分, 共二十五分。答錯一個選項扣兩分, 錯兩個選項以上不給分, 分數不倒扣。)

13. Consider $x \in [0,1]$ and $f(x) = \dfrac{\sin x}{x}$ if $x \neq 0$, $f(x) = 0$ if $x = 0$. Which of the following statements are **True**? 15:40:44

(A) $f(x)$ is continuous.

(B) $\boxed{\boldsymbol{f(x)} \textbf{ is differentiable on } \boldsymbol{(0,1)}.}$

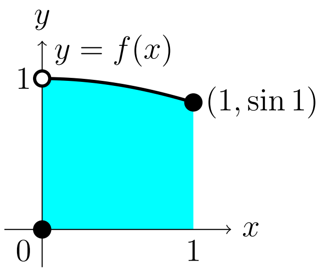
(C) $\boxed{\boldsymbol{f(x)} \textbf{ is integrable.}}$

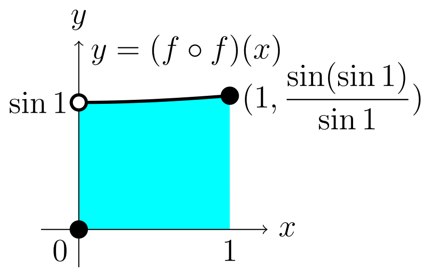
(D) $\boxed{\boldsymbol{(f \circ f)(x)} \textbf{ is integrable.}}$

Solution: (A) $\lim\limits_{x \to 0^+} f(x) = 1 \neq 0 = f(0)$.

(B) $\sin x$ and $\dfrac{1}{x}$ differentiable for $x \neq 0$.

(C)(D) f and $f \circ f$ are dis-continuous at finite points in $[0,1]$.





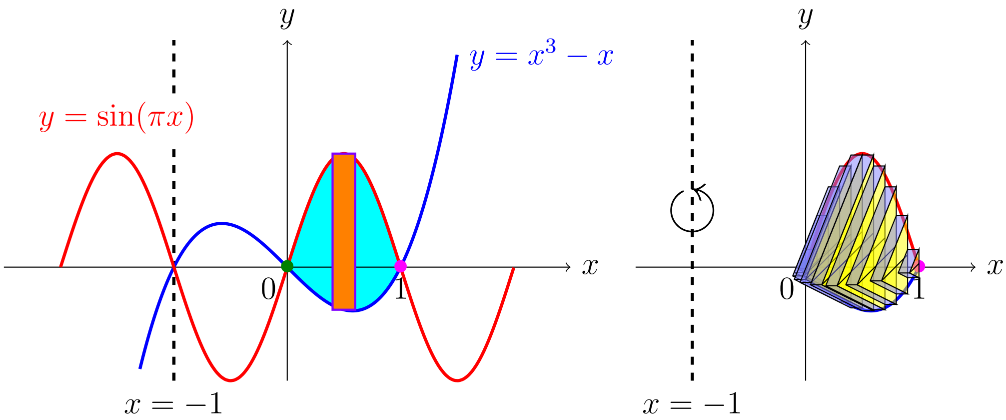
14. Let R be the region bounded **below** by the graph of $y = x^3 - x$ and bounded **above** by the graph $y = \sin(\pi x)$. Which of the following statements are **True**? 34:48:18

(A) $(0, 0)$ and $(\pi, 0)$ are on the boundary of the region R.

(B) $\boxed{\text{The area of } \boldsymbol{R = \int_0^1 (\sin(\pi x) - x^3 + x)\ dx}.}$

(C) **Let $\boldsymbol{S}$ be a solid with the base $\boldsymbol{R}$ and each cross-section perpendicular to the base $\boldsymbol{R}$ is an equilateral triangle. Then the volume of this solid is equal to $\boldsymbol{\frac{\sqrt{3}}{4}\int_0^1 (x^3 - x - \sin(\pi x))^2\ dx}$.**

(D) **The volume of the solid obtained by rotating the region $\boldsymbol{R}$ about the line $\boldsymbol{x = -1}$ can be evaluated as $\boldsymbol{2\pi\int_0^1 (x+1)(\sin(\pi x) - x^3 + x)\ dx}$.**



Solution: $A = \int_0^1 [\sin(\pi x) - (x^3 - x)]\ dx$.

$A(x) = \frac{\sqrt{3}}{4}[\underbrace{\sin(\pi x) - (x^3 - x)}_{\text{邊長}}]^2$, $V = \int_0^1 A(x)\ dx$.

$V = \int_0^1 \underbrace{2\pi(x+1)}_{\text{均周長}}\underbrace{[\sin(\pi x) - (x^3 - x)]}_{\text{高度}}\underbrace{dx}_{\text{厚度}}$.

15. Let f be the function given by $f(x) = \int_1^x (t^2 - 4t + 3)e^{-t}\,dt$. Which of the following statements about f must be **True**? 62:20:17

(A) f is increasing on the interval $(1, 3)$.

(B) $\boxed{\boldsymbol{f} \textbf{ is increasing on the interval } \mathbf{(3,4).}}$

(C) $f(3) > 0$.

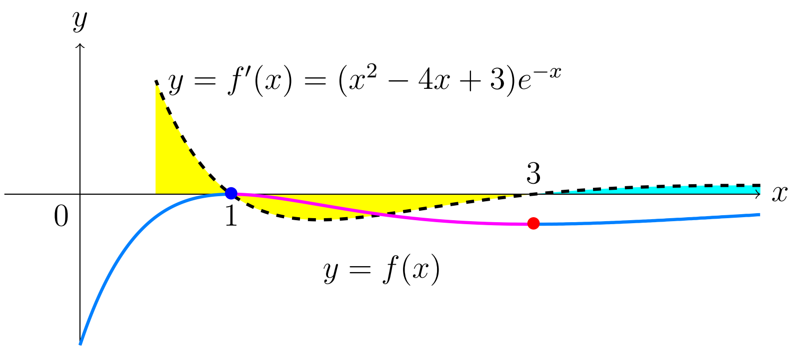
(D) $\boxed{\boldsymbol{f(1) = 0.}}$

Solution: $f'(x) = (x^2 - 4x + 3)e^{-x} = 0$ when $x = 1, 3$.

$f'(x) > 0$ when $x < 1$ or $x > 3$, $f'(x) < 0$ when $1 < x < 3$.

$f(x) = \int_1^x (t^2 - 4t + 3)e^{-t}\,dt = [-(x^2 - 4x + 3) - (2x - 4) - (2)]e^{-x}$
$= -(x - 1)^2 e^{-x}$, $f(3) = -4e^{-3} < 0$, $f(1) = 0$.

[Quick sol] f is decreasing on $(1, 3)$, $f(3) < f(1) = \int_1^1 \cdots\,dt = 0$.



◎ 填空題 (填空五題, 每題五分, 共二十五分, 答錯不倒扣。)

19. Let $f(x) = \int_0^x e^{t^2}\,dt$. Then $f''(x) =$ 71:25

Solution: $2xe^{x^2}$.

....

By TFTC, $f'(x) = e^{x^2}$, by Chain Rule $f''(x) = e^{x^2}(x^2)' = 2xe^{x^2}$.

20. Determine **all values** of p such that $\int_1^\infty x^p|\sin x|\,dx$ **converges**. 34:51

Solution: $p < -1$.

. . . .

$\because 0 \le |\sin x| \le 1$, $0 \le x^p|\sin x| \le x^p$, for $x > 0$.

$\int_1^\infty x^p\,dx = \int_1^\infty \frac{1}{x^{-p}}\,dx$ converges $\iff -p > 1 \iff p < -1$ by the Comparison Theorem, $\int_1^\infty x^p|\sin x|\,dx$ converges when $p < -1$.

For $p \ge -1$, $\int_1^\infty x^p|\sin x|\,dx \ge \int_1^\infty \frac{|\sin x|}{x}\,dx \qquad (x^p > x^{-1} = \frac{1}{x})$

$\ge \frac{1}{2}\Big(\int_\pi^\infty \frac{|\sin x|}{x}\,dx + \int_{\pi/2}^\infty \frac{|\sin x|}{x}\,dx\Big) \qquad (\pi > \pi/2 > 1)$

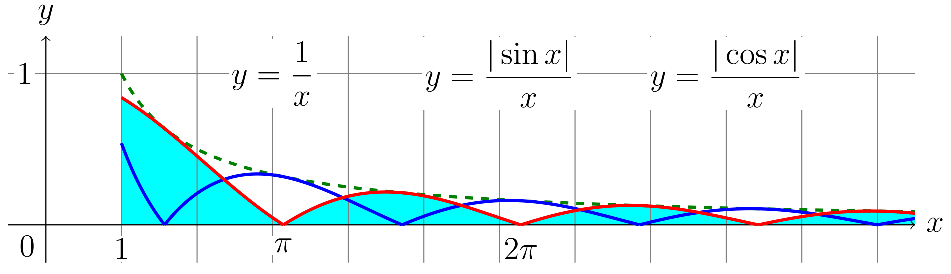
$\ge \frac{1}{2}\Big(\int_\pi^\infty \frac{|\sin x|}{x}\,dx + \int_\pi^\infty \frac{|\cos x|}{x-\pi/2}\,dx\Big) \qquad (\sin(x-\pi/2) = \cos x)$

$\ge \frac{1}{2}\int_\pi^\infty \Big(\frac{|\sin x|}{x} + \frac{|\cos x|}{x}\Big)\,dx \qquad (\frac{1}{x-\pi/2} \ge \frac{1}{x})$

$\ge \frac{1}{2}\int_\pi^\infty \frac{\sin^2 x + \cos^2 x}{x}\,dx \qquad (|\sin x| \ge \sin^2 x,\ |\cos x| \ge \cos^2 x)$

$= \frac{1}{2}\int_\pi^\infty \frac{1}{x}\,dx (= \infty)$ diverges, by the Comparison Theorem,

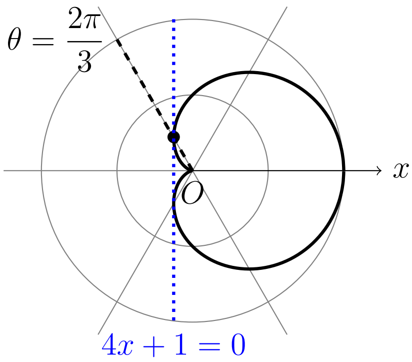
$\int_1^\infty x^p|\sin x|\,dx$ diverges when $p \ge -1$.



16. (105-2) Suppose that the equation of the **tangent line** to the polar curve $r = 1 + \cos\theta$ at the point $(r, \theta) = (\frac{1}{2}, \frac{2\pi}{3})$ is $ax + by + 1 = 0$, then the pair (a, b) is 14:71

Solution: $(4, 0)$.

. . . .



$x = r\cos\theta = (1 + \cos\theta)\cos\theta = -\frac{1}{4}$ when $\theta = \frac{2\pi}{3}$,

$\frac{dx}{d\theta} = -\sin\theta - \sin 2\theta = 0$ when $\theta = \frac{2\pi}{3}$,

$y = r\sin\theta = (1 + \cos\theta)\sin\theta = \frac{\sqrt{3}}{4}$ when $\theta = \frac{2\pi}{3}$,

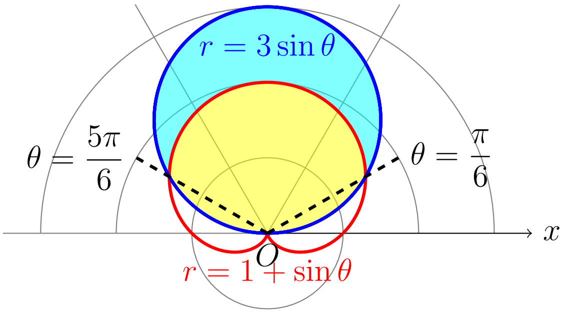
$\frac{dy}{d\theta} = \cos\theta + \cos 2\theta = -1$ when $\theta = \frac{2\pi}{3}$,

$\lim_{\theta \to 2\pi/3^-} \frac{dy}{dx} = \infty$ and $\lim_{\theta \to 2\pi/3^+} \frac{dy}{dx} = -\infty$,

vertical tangent line: $x = -\frac{1}{4}$, $4x + 0y + 1 = 0$.

17. (105-2) The **area** of the region that lies inside both curves $r = 3\sin\theta$ and $r = 1 + \sin\theta$ is 0+1:86-1

Solution: $\frac{5\pi}{4}$.



$3\sin\theta = r = 1 + \sin\theta$,
$\sin\theta = \frac{1}{2}, \theta = \frac{\pi}{6}, \frac{5\pi}{6}$.

$$A = \int_{\pi/6}^{5\pi/6} \frac{1}{2}(1+\sin\theta)^2\, d\theta + \int_0^{\pi/6} \frac{1}{2}(3\sin\theta)^2\, d\theta + \int_{5\pi/6}^{\pi} \frac{1}{2}(3\sin\theta)^2\, d\theta$$

$$= 2\int_{\pi/6}^{\pi/2} \left(\frac{1}{2} + \sin\theta + \frac{1}{2}\sin^2\theta\right)\, d\theta + 2\int_0^{\pi/6} \frac{9}{2}\sin^2\theta\, d\theta$$

$$= \int_{\pi/6}^{\pi/2} \left(\frac{3}{2} + 2\sin\theta - \frac{1}{2}\cos 2\theta\right)\, d\theta + \int_0^{\pi/6} \frac{9}{2}(1-\cos 2\theta)\, d\theta$$

$$= \left[\frac{3\theta}{2} - 2\cos\theta - \frac{1}{4}\sin 2\theta\right]_{\pi/6}^{\pi/2} + \left[\frac{9\theta}{2} - \frac{9}{4}\sin 2\theta\right]_0^{\pi/6}$$

$$= \left[\left(\frac{3\pi}{4} - 0 - 0\right) - \left(\frac{\pi}{4} - \sqrt{3} - \frac{\sqrt{3}}{8}\right)\right] + \left[\left(\frac{3\pi}{4} - \frac{9\sqrt{3}}{8}\right) - (0-0)\right] = \frac{5}{4}\pi.$$

[Sol 2] $$B = \int_{\pi/6}^{5\pi/6} \frac{1}{2}[(3\sin\theta)^2 - (1+\sin\theta)^2]\, d\theta$$

$$= 2\int_{\pi/6}^{\pi/2} \left(\frac{9}{2}\sin^2\theta - \frac{1}{2} - \sin\theta - \frac{1}{2}\sin^2\theta\right)\, d\theta$$

$$= \int_{\pi/6}^{\pi/2} (3 - 2\sin\theta - 4\cos 2\theta)\, d\theta = \Big[3\theta + 2\cos\theta - 2\sin 2\theta\Big]_{\pi/6}^{\pi/2}$$

$$= \left[\left(\frac{3\pi}{2} + 0 - 0\right) - \left(\frac{\pi}{2} + \sqrt{3} - \sqrt{3}\right)\right] = \pi. \qquad \text{(10.4 example)}$$

$$A = \pi\left(\frac{3}{2}\right)^2 - B = \frac{5\pi}{4}.$$

End