1.

a. $(x^2+7x+9)(2x^3+9x^2+5)$

$= (2x^5+9x^4+3x^4+8x^3+7x^3+5x^2+4x^2+2x+1)$

$= 2x^5+x^4+4x^3+9x^2+2x+1$ #

b. $5x^3+4=0 \Rightarrow 5x^3=3 \Rightarrow x^3=3\cdot5^{-1}=9=2$

$2x^5+3x+2 = 2(x^3)x^2+3x+2$

$= 4x^2+3x+2$ #

C. $2x^3+9x^2+10x+1=0 \Rightarrow 2x^3=2x^2+x+10 \Rightarrow x^3=x^2+6x+5$

$x^4+8x^3+7x+8 = x(x^2+6x+5)+8(x^2+6x+5)+7x+8$

$= x^3+6x+5+6x^2+5x+8x^2+4x+7+7x+8$

$= 4x^2+9$

$4x^2+9=0 \Rightarrow 4x^2=2 \Rightarrow x^2=6$

$2x^3+9x^2+10x+1 = 12x+10+10x+1$

$= 0$

$\Rightarrow 4x^2+9$ #

d.

r	q	a	b
x^4+x+1	x	0	1
x^3+x+1	x	1	0
x^2+1	x	$-x$	1
1	x	x^2+1	$-x$

$\Rightarrow x^2+1$ #

2. a. Assume that $x^3+x+1 = f(x)\cdot g(x)$, where $\deg(f(x)) = 1$, $\deg(g(x)) = 2$

$x=0 \Rightarrow 0^3+0+1 = 1$

$x=1 \Rightarrow 1^3+1+1 = 3 = 1$

$\Rightarrow x^3+x+1$ is irreducible #

b. $x=1 \Rightarrow 1+1+1+1 = 4 = 0$

$\Rightarrow x^4+x^2+x+1 = (x+1)(x^3+x^2+1)$

$\Rightarrow x^4+x^2+x+1$ is reducible #

3. $a(y)\cdot b(y) = [(x+1)y^3+y^2+y+x][(x^3+x+1)y^3+(x^3+x^2+1)y^2+(x^3+1)y+(x^3+x^2+x)]$

$= (x^4+x^3+x^2+2x+1)(y^4)y^2+[(x^3+x+1)+(x^4+2x^3+x^2+x+1)](y^4)y$

$+[(x^4+x^3+x+1)+(x^3+x^2+1)+(x^3+x+1)](y^4)$

$+[(x^4+2x^3+2x^2+x)+(x^3+1)+(x^3+x^2+1)+(x^4+x^2+x)]y^3$

$+[(x^3+x^2+x)+(x^3+1)+(x^4+x^3+x)]y^2$

$+[(x^3+x^2+x)+(x^4+x)]y$

$+ x^4+x^3+x^2$ $\quad (y^4+1=0 \Rightarrow y^4=1$

$= (2x^4+4x^3+4x^2+2x+2)y^3+(2x^4+4x^3+2x^2+4x+2)y^2$

$+(2x^4+4x^3+2x^2+4x+2)y+(2x^4+4x^3+2x^2+2x+3)$

$= 1$ (due to GF(2)'s modulo)

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