1081 Correct:(half-correct:)error %/ avg point

Part I

◎ **Part 1: 單選擇題**

(Multiple-Choice Questions, Each Problem with Single Correct Answer)

(單選十題, 每題五分, 共五十分。)

(10 questions, each question is worth 5 points, for 50 points in total.)

1. The limit $\lim_{x\to 0}(\cos x - \sin x)^{\frac{1}{\tan x}}$ is 59:41

   (A) 0. (B) 1. (C) e. **(D)** $\boxed{\boldsymbol{e^{-1}}.}$

Solution: Let $y = (\cos x - \sin x)^{\frac{1}{\tan x}}$, $\ln y = \dfrac{\ln(\cos x - \sin x)}{\tan x}$.

$\lim_{x\to 0} \ln y \overset{l'h}{=} \lim_{x\to 0} \dfrac{-\sin x - \cos x}{(\cos x - \sin x)\sec^2 x} = \dfrac{-0-1}{(1-0)1^2} = -1,$ $\quad$ **$(1^\infty \to \frac{0}{0})$**

$\lim_{x\to 0} y = \lim_{x\to 0} e^{\ln y} = e^{\lim_{x\to 0}\ln y} = e^{-1}$.

[Quick sol] $\cos x \approx 1$, $\sin x \approx x$, $\tan x \approx x$,

$\lim_{x\to 0}(\cos x - \sin x)^{\frac{1}{\tan x}} = \lim_{x\to 0}(1-x)^{\frac{1}{x}} = e^{-1}$.

3. Consider the following functions.

$$f(x) = \begin{cases} 0, & x \neq 0 \\ 1, & x = 0 \end{cases}, \qquad g(x) = \begin{cases} x\cos x, & x \neq 0 \\ 1, & x = 0 \end{cases}.$$

   Which of the following statements is **TRUE**? 68:32

   (A) $\lim_{x\to 0} f(x) = 1$. (B) $\lim_{x\to 0} g(x) = 1$.

   (C) $\lim_{x\to 0} g(f(x)) = \cos 1$. **(D)** $\boxed{\boldsymbol{\lim_{x\to 0} f(g(x)) = 0}.}$

Solution: $\lim_{x\to 0} f(x) = 0$, $\lim_{x\to 0} g(x) = 0\cos 0 = 0$,

$\lim_{x\to 0} g(f(x)) = g(0) = 1$, $\lim_{x\to 0} f(g(x)) = \lim_{x\to 0} f(x\cos x) = 0$.

6. Let f be a differentiable function defined on $(0, \infty)$. Which of the following must be **TRUE**? 29:71

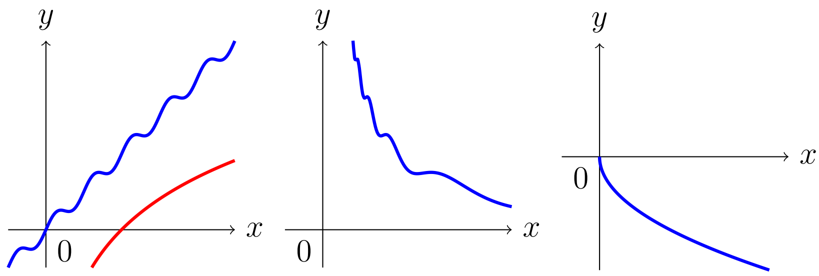
(A) If $\lim_{x\to\infty} f(x) = \infty$, then $\lim_{x\to\infty} f'(x) > 0$.

(B) $\boxed{\textbf{If } \lim_{x\to\infty} f'(x) > 0\textbf{, then } \lim_{x\to\infty} f(x) = \infty.}$

(C) If $\lim_{x\to 0^+} f(x) = \infty$, then $\lim_{x\to 0^+} f'(x) = -\infty$.

(D) If $\lim_{x\to 0^+} f'(x) = -\infty$, then $\lim_{x\to 0^+} f(x) = \infty$.

Solution: (A) $\lim_{x\to\infty} (x + \sin x) = \infty$,
but $\lim_{x\to\infty} (x + \sin x)' = \lim_{x\to\infty} (1 + \cos x)$ does not exist.
[Or] $\lim_{x\to\infty} \ln x = \infty$, but $\lim_{x\to\infty} (\ln x)' = \lim_{x\to\infty} 1/x = 0$.
(C) $\lim_{x\to 0^+} (\frac{1}{x} + \sin\frac{1}{x}) = \infty$,
but $\lim_{x\to 0^+} (\frac{1}{x} + \sin\frac{1}{x})' = \frac{-1}{x^2}(1 + \cos\frac{1}{x})$ does not exist.
(D) $\lim_{x\to 0^+} (-\sqrt{x})' = \lim_{x\to 0^+} \frac{-1}{2\sqrt{x}} = -\infty$, but $\lim_{x\to 0^+} (-\sqrt{x}) = 0$.



10. Consider the function $F(x) = (f \circ g')(x)$, where

$$g(x) = \begin{cases} x^2 - 3 & \text{if } x \le 1 \\ -\frac{2}{x} - \frac{x-1}{2} & \text{if } x > 1 \end{cases}, \quad f(x) = 4x^3 - 15x^2 + 12x.$$

Which of the following about the absolute maximum value y_M and absolute minimum value y_m of F on $\{x : 0 \le x \le 2 \text{ and } F(x) \text{ is defined}\}$ is true? 23:77

(A) $y_M = 1, y_m = 0$.

(B) $y_M = \frac{11}{4}, y_m = -4$.

(C) $\boxed{\boldsymbol{y_M = \frac{11}{4}}\textbf{, no absolute minimum value.}}$

(D) $y_M = 1, y_m = -4$.

Solution: $g' = \begin{cases} 2x & \text{if } x < 1 \\ \frac{2}{x^2} - \frac{1}{2} & \text{if } x > 1 \end{cases}$,

$\lim\limits_{x\to 1^-} g'(x) = \lim\limits_{x\to 1^-} 2x = 2$,

$\lim\limits_{x\to 1^+} g'(x) = \lim\limits_{x\to 1^+} (\frac{2}{x^2} - \frac{1}{2}) = \frac{3}{2}$,

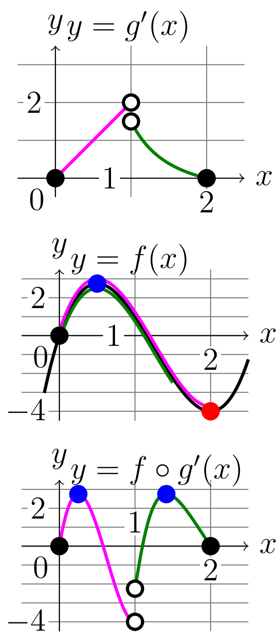
g' is not defined at 1.

$f'(x) = 12x^2 - 30x + 12 = 12(x - \frac{1}{2})(x - 2)$,

max is $f(\frac{1}{2}) = \frac{11}{4}$ and min is $f(2) = -4$.

For $g'([0,1)) = [0,2)$, $g'((1,2]) = [0, \frac{3}{2})$,

$y_M = F(\frac{1}{4}) = F(\sqrt{2}) = \frac{11}{4}$, but no abs min.



⊙ **Part 2: 多選擇題**

(Multiple-Choice Questions with More Than One Correct Answers)

(多選五題, 每題五分, 共二十五分。錯一個選項扣兩分, 錯兩個選項以上不給分, 分數不倒扣。)

(5 questions, each question is worth 5 points, for 25 points in total. The correct answer is worth 5 points. Answers at a distance 1 from the correct answer are worth 3 points, other answers are worth no points.)

11. Which of the following statements must be **TRUE**? 27:38:35

(A) If $\lim_{x\to 0} |f(x)| = |L|$, then $\lim_{x\to 0} f(x) = L$.

(B) $\boxed{\textbf{If } \lim_{x\to 0} f(x) = L\textbf{, then } \lim_{x\to 0} |f(x)| = |L|.}$

(C) If f is an odd function defined in $(-\infty, \infty)$, then $\lim_{x\to 0} f(x) = 0$.

(D) **Let f and g be defined in $(-\infty, \infty)$. If f is an even function and g is an odd function, then both $f \circ g$ and $g \circ f$ are even functions.**

Solution: (A) $f(x) = 1$ and $L = -1$.

(B) $\because |x|$ is continuous, $\lim_{x\to 0} |f(x)| = |\lim_{x\to 0} f(x)| = |L|$.

(C) $f(x) = x/|x|$ when $x \neq 0$ and $f(0) = 0$, $\lim_{x\to 0} f(x)$ does not exist.

(D) $f(g(-x)) = f(-g(x)) = f(g(x))$, $g(f(-x)) = g(f(x))$.

14. Consider the following function:

$$f(x) = \begin{cases} x^2 \sin\frac{1}{x} & \text{for } x \neq 0, \\ 0 & \text{for } x = 0. \end{cases}$$

Which of the following statements are **True**? 7:37:56

(A) $\boxed{f \textbf{ is differentiable at 0.}}$

(B) $\boxed{f \textbf{ has infinitely many local maxima.}}$

(C) $\boxed{\textbf{The curve } y = f(x) \textbf{ has infinitely many inflection points.}}$

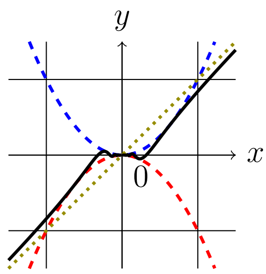
(D) $\boxed{\textbf{The line } y = x \textbf{ is a slant asymptote of } y = f(x).}$

Solution: (A) $f'(0) = \lim_{h\to 0} \dfrac{f(h) - f(0)}{h}$

$= \lim_{h\to 0} \dfrac{h^2 \sin\frac{1}{h} - 0}{h} = \lim_{h\to 0} h \sin\frac{1}{h} \overset{S.T.}{=} 0.$

(B)(C) trivial.

(D) as $x \to \pm\infty$, $x \sin\frac{1}{x} \to 1$, $f \approx x$.



End

Part II

◎ **Part 1: 單選擇題**

(Multiple-Choice Questions, Each Problem with Single Correct Answer)

(單選十題, 每題五分, 共五十分。)

(10 questions, each question is worth 5 points, for 50 points in total.)

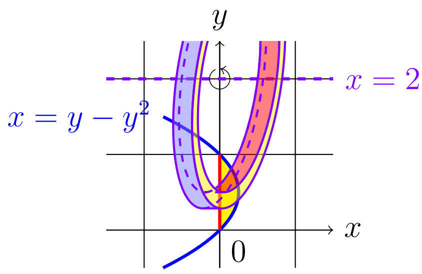
2. Let R be the region enclosed by $y^2 - y + x = 0$ and $x = 0$. The volume of the solid obtained by rotating R about the line $y = 2$ is 67:33

   (A) $\pi/4$. **(B)** $\boxed{\boldsymbol{\pi/2.}}$ (C) π. (D) $3\pi/2$.

Solution: $V = \int_0^1 2\pi(2 - y)(y - y^2)\, dy$

$= 2\pi \int_0^1 (2y - 3y^2 + y^3)\, dy$

$= 2\pi \left[y^2 - y^3 + \frac{y^4}{4}\right]_0^1 = \frac{\pi}{2}.$



4. The length of the curve $y = \ln(\cos x)$ from $(0, 0)$ to $(\pi/4, \ln(1/\sqrt{2}))$ is 74:26

   (A) $\boxed{\mathbf{ln}(\sqrt{\mathbf{2}} + \mathbf{1}).}$ (B) $\ln(2\sqrt{2} + 1)$. (C) $\ln(\frac{1}{\sqrt{2}} + 1)$. (D) 1.

Solution: $L = \int_0^{\pi/4} \sqrt{[(\ln \cos x)']^2 + 1}\, dx$

$= \int_0^{\pi/4} \sqrt{(-\tan)^2 + 1}\, dx = \int_0^{\pi/4} |\sec x|\, dx = \int_0^{\pi/4} \sec x\, dx$

$= \ln|\sec x + \tan x|\Big|_{x=0}^{\pi/4} = \ln(\sqrt{2} + 1).$

5. Let f be an odd function defined on $(-\infty,\infty)$. If f' is continuous and $f(1)=1$, then $\int_0^2 xf'(1-x)\,dx$ equals 56:44

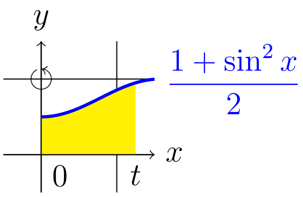
(A) 1. (B) -1. **(C)** $\boxed{\mathbf{2.}}$ (D) -2.

Solution: $\int_0^2 xf'(1-x)\,dx \overset{u=1-x}{=} \displaystyle\int_1^{-1} -(1-u)f'(u)\,du$
$$= \int_{-1}^{1} (1-u)f'(u)\,du = (1-u)f(u)\Big|_{-1}^{1} - \int_{-1}^{1} f(u)\cdot(-1)\,du$$
$$\overset{odd}{=} 0f(1) - 2f(-1) + 0 = 2f(1) = 2.$$

7. Let $V(t)$ be the volume obtained by rotating the region
$$A(T) := \{(x,y) : 0 \le x \le t, 0 \le y \le \frac{1+\sin^2(x)}{2}\}$$
about the y-axis. For what value of $-\infty < r < \infty$ and $0 < c < \infty$ does one have
$$\lim_{t\to 0^+} \frac{V(t)}{t^r} = c?$$
39:61

(A) $r=1$, $c=\pi$. (B) $r=1$, $c=\pi/2$.
(C) $r=2$, $c=\pi$. **(D)** $\boxed{\boldsymbol{r=2,\ c=\pi/2.}}$

Solution: $\displaystyle\lim_{t\to 0^+}\frac{V(t)}{t^r} = \lim_{t\to 0^+}\frac{\int_0^t 2\pi x\frac{1+\sin^2 x}{2}\,dx}{t^r}$
$$\overset{l'H}{=} \lim_{t\to 0^+}\frac{2\pi t\frac{1+\sin^2 t}{2}}{rt^{r-1}} = \lim_{t\to 0^+}\frac{\pi+\pi\sin^2 t}{rt^{r-2}},$$
exist when $r=2$, and $c = \dfrac{\pi+\pi\cdot 0}{2} = \dfrac{\pi}{2}$.



8. Assume that f is continuous and satisfies the following equation

$$\lim_{n\to\infty} \frac{f(\frac{x^2}{n}) + f(\frac{2x^2}{n}) + \cdots + f(\frac{(n-1)x^2}{n}) + f(\frac{nx^2}{n})}{n} = \frac{\sin(\pi x)}{x},$$

for any real number $x \neq 0$. Find $f(1)$. 20:80

(A) π. **(B)** $\boxed{\boldsymbol{\pi/2.}}$ (C) 0. (D) $-\pi/2$.

Solution: $I = \lim_{n\to\infty} \sum_{i=1}^{n} f(t_i x^2)\Delta t = \int_0^{x^2} \frac{f(t)}{x^2}\, dt = \frac{\sin \pi x}{x}$,
$\int_0^{x^2} f(t)\, dt = x \sin \pi x$, $(\frac{d}{dx}:)$ $f(x^2) \cdot 2x = \sin \pi x + \pi x \cos \pi x$,
$(x = \pm 1:)$ $f(1) = \frac{\sin \pm\pi \pm \pi \cos \pm\pi}{\pm 2} = \frac{\mp\pi}{\pm 2} = -\frac{\pi}{2}$.

9. Find the condition under which the value of the following integral must be positive

$$\int_0^{2\pi/\beta} e^{\alpha t}\Big(\alpha \cos(\beta t) - \beta \sin(\beta t)\Big)\, dt.$$

26:74

(A) $\alpha > 0, \beta < 0$. (B) $\beta > 2\pi\alpha$. **(C)** $\boxed{\boldsymbol{\alpha\beta > 0.}}$ (D) $\beta < 2\pi\alpha$.

Solution: $I = \int_0^{2\pi/\beta} e^{\alpha t}\Big(\alpha \cos(\beta t) - \beta \sin(\beta t)\Big)\, dt = \Big[e^{\alpha t} \cos(\beta t)\Big]_0^{2\pi/\beta}$
$= e^{2\pi\alpha/\beta} - 1$, $\because e^x$ is increasing,
$I > 0 \iff e^{2\pi\alpha/\beta} > 1 = e^0 \iff 2\pi\alpha/\beta > 0 \iff \alpha\beta > 0$.

1. (108-2) If the length of the polar curve $r = 3\theta^2$ with $0 \leq \theta \leq 2\pi$ is $a[(\pi^2 + 1)^{3/2} - 1]$, then a is equal to 71:29

(A) 2. (B) 4. **(C)** $\boxed{\textbf{8.}}$ (D) 16.

Solution: $L = \int_0^{2\pi} \sqrt{r^2 + (r')^2}\, d\theta = \int_0^{2\pi} \sqrt{(3\theta^2)^2 + (6\theta)^2}\, d\theta$
$= \int_0^{2\pi} 3\theta\sqrt{\theta^2 + 4}\, d\theta = \Big[(\theta^2 + 4)^{3/2}\Big]_0^{2\pi} = 8[(\pi^2 + 1)^{3/2} - 1]$.

⊙ **Part 2:** 多選擇題

(Multiple-Choice Questions with More Than One Correct Answers)

(多選五題, 每題五分, 共二十五分。錯一個選項扣兩分, 錯兩個選項以上不給分, 分數不倒扣。)

(5 questions, each question is worth 5 points, for 25 points in total. The correct answer is worth 5 points. Answers at a distance 1 from the correct answer are worth 3 points, other answers are worth no points.)

12. Consider the function $f(x) = \int_0^x |\sin(t)|\, dt$, where $-\infty < x < \infty$. Which of the following statements are **TRUE**? 40:30:30

    (A) $\boxed{\boldsymbol{f(x)} \textbf{ is an increasing function.}}$
    (B) $\boxed{\boldsymbol{f(x)} \textbf{ is a differentiable function.}}$
    (C) $\boxed{\boldsymbol{f'(x)} \textbf{ is a continuous function.}}$
    (D) $f'(x)$ is a differentiable function.

Solution: (A) $f'(x) = |\sin x| \geq 0$, f is increasing.
(B)(C) $\because |\sin x|$ is continuous, by TFTC.
(D) $|\sin x|$ is not differentiable at $x = n\pi$, $n \in \mathbb{Z}$.

13. Define $f(x) = \int_1^{x^2} \frac{t^2}{2(t^2+1)}\, dt$. Which of the following statements are **True**? 16:35:49

    (A) $\boxed{\boldsymbol{f} \textbf{ is continuous on } \boldsymbol{(-\infty, \infty)}.}$
    (B) $\boxed{\boldsymbol{f'} \textbf{ has a slant asymptote } \boldsymbol{y = x}.}$
    (C) $\boxed{\boldsymbol{f} \textbf{ is concave upward on } \boldsymbol{(-\infty, \infty)}.}$
    (D) $\boxed{\textbf{The graph of } \boldsymbol{f'} \textbf{ is symmetric about the origin.}}$

Solution: (A) $\because \frac{x^2}{2(x^2+1)}$ is continuous on $\mathbb{R}$.
(B) $f' = \frac{x^5}{x^4+1} \approx x$ as $x \to \pm\infty$.
(C) $f'' = \frac{x^4(x^4+5)}{(x^4+1)^2} > 0$.
(D) $f'(-x) = -f'(x)$, f' is odd.

15. Let f and g be functions which are continuous on $[a, b]$ and differentiable on (a, b) where $a < b$. Which of the following conditions can guarantee that f and g are equal on $[a, b]$? 18:57:24

(A) $\boxed{\lim_{y\to x} f(y) = \lim_{y\to x} g(y) \text{ for any } x \text{ on } (a, b).}$

(B) $f'(x) = g'(x)$ for any x on (a, b).

(C) $\boxed{\int_a^x f(t)\ dt = \int_a^x g(s)\ ds \text{ for any } x \text{ on } (a, b).}$

(D) $\boxed{f(x) + \int_a^x f(t)\ dt = g(x) + \int_a^x g(s)\ ds \text{ for any } x \text{ on } (a, b).}$

Solution: (A) $\because$ f, g are continuous on $[a, b]$, $f(x) = \lim_{y\to x} f(y) = \lim_{y\to x} g(y) = g(x)$ on (a, b), and so $f(a) = \lim_{x\to a^+} f(x) = \lim_{x\to a^+} g(x) = g(a)$, $f(b) = \lim_{x\to b^-} f(x) = \lim_{x\to b^-} g(x) = g(b)$.

(B) $f(x) = g(x) + C$ fails.

(C) By TFTC $f(x) = g(x)$ on (a, b) and by the continuity.

(D) Let $h = f - g$. $f(a) = \lim_{x\to a^+}(f(x) + \int_a^x f(t)\ dt) = \lim_{x\to a^+}(g(x) + \int_a^x g(s)\ ds) = g(a)$, then $h(a) = 0$; if $h(c) \gtrless 0$ at some smallest c then $f(x) + \int_a^x f(t)\ dt - g(x) - \int_a^x g(s)\ ds = h(c) + \int_a^c h(t)\ dt \gtrless 0$, contradiction.

11. (108-2) Consider the following polar equations.

$$\gamma_1 : r^2 = \sin(2\theta), \quad \gamma_2 : r^2 = \cos(2\theta), \quad \gamma_3 : r = 1 + \sin\theta, \quad \gamma_4 : r = -1 + \sin\theta.$$

Which of the following statements are **TRUE**? 14:31:55

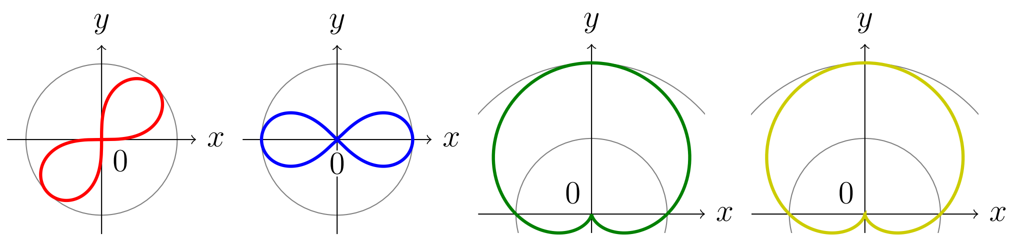
(A) $\boxed{\textbf{There are three horizontal tangent lines on the graph of } \boldsymbol{\gamma_1}.}$

(B) $\boxed{\textbf{The graph of } \boldsymbol{\gamma_2} \textbf{ is symmetric about the } \boldsymbol{x}\textbf{-axis and the } \boldsymbol{y}\textbf{-axis.}}$

(C) The graphs of γ_3 and γ_4 are different.

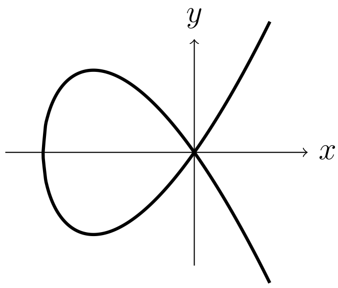
(D) $\boxed{\textbf{The area of the region enclosed by the graph of } \boldsymbol{\gamma_1} \textbf{ is the same as the area of the region enclosed by the graph of } \boldsymbol{\gamma_2}.}$

Solution:



⊚ **Part 3: 計算/證明題 (Questions of calculations and proofs)**
(答題時應將推理或解題過程說明清楚, 且得到正確答案, 方可得到滿分。如果計算錯誤, 則酌給部紛紛數。如果只有答案對, 但觀念錯誤, 或是過程不合理, 則無法得到分數。)
(Answer the problems as thoroughly as possible. Be sure to include all your work. Partial credit will be given even if the answer is not fully correct.)

1. A curve C is defined by the equation $y^2 = x^2(x+2)$. From Figure 1, you can see that part of the curve forms a loops.



(A) (5 points) At what points does the curve C have horizontal tangents? .64

(B) (5 points) Find the area of the region enclosed by the loop. .66

Solution: (A) $y = \pm x\sqrt{x+2}$, $y' = \pm(\sqrt{x+2} + \dfrac{x}{\sqrt{x+2}})$,
$y' = 0$ when $x = -\dfrac{4}{3}$, $y = \pm(-\dfrac{4}{3})\sqrt{-\dfrac{4}{3}+2} = \mp\dfrac{3}{4}\sqrt{\dfrac{2}{3}}$. .. $\Big(-\dfrac{4}{3}, \pm\dfrac{4}{3}\sqrt{\dfrac{2}{3}}\Big)$.
(B) $A = 2\displaystyle\int_{-2}^{0} -x\sqrt{x+2}\;dx \overset{u=x+2}{=} 2\int_{0}^{2} -(u-2)\sqrt{u}\;dx$
$= 2\Big[-\dfrac{2}{5}u^{5/2} + \dfrac{4}{3}u^{3/2}\Big]_0^2 = -\dfrac{16}{5}\sqrt{2} + \dfrac{16}{3}\sqrt{2} = \dfrac{32}{15}\sqrt{2}$. .... $\dfrac{32}{15}\sqrt{2}$.

2. Consider the parametric curve $\gamma(t) = (e^{-t}\sin t, e^{-t}\cos t)$.

   (A) (4 points) Compute the length of $\{\gamma(t) | 0 \le t < \infty\}$. .64

   (B) (4 points) Determine whether the area of the surface obtained by rotating $\{\gamma(t) | 0 \le t < \infty\}$ about $x = 1$ is finite of inginite. .64

   (C) (5 points) For $i = 1, 2, 3, 4$, let L_i be the tangent line to γ at $\gamma(\pi i/2)$. Find the area of the region enclosed by the L_1, L_2, L_3, L_4. .66

Solution: (A) $L = \displaystyle\int_0^\infty \sqrt{[(e^{-t}\sin t)']^2 + [(e^{-t}\cos t)']^2}\, dt$

$= \displaystyle\int_0^\infty \sqrt{[e^{-t}(-\sin t + \cos t)]^2 + [e^{-t}(-\cos t - \sin t)]^2}\, dt = \int_0^\infty \sqrt{2}e^{-t}\, dt$

$= \displaystyle\lim_{s\to\infty} \sqrt{2}\int_0^s e^{-t}\, dt = \sqrt{2}\lim_{s\to\infty}\Big[-e^{-t}\Big]_0^s = \sqrt{2}\lim_{s\to\infty}(1 - e^{-s}) = \sqrt{2}.\quad \ldots\sqrt{2}.$

(B) $S = \displaystyle\int 2\pi|1 - x|\, ds = \int_0^\infty 2\pi(1 - e^{-t}\sin t)\sqrt{2}e^{-t}\, dt$

$< 2\sqrt{2}\pi \displaystyle\int_0^\infty (1 + 1)e^{-t}\, dt = 4\sqrt{2}\pi < \infty.$ $\qquad (-e^{-t}\sin t < 1)$

[Or] $S = \displaystyle\int_0^\infty 2\pi(1 - e^{-t}\sin t)\sqrt{2}e^{-t}\, dt = 2\sqrt{2}\pi\int_0^\infty (e^{-t} - e^{-2t}\sin t)\, dt$

$= 2\sqrt{2}\pi \displaystyle\lim_{s\to\infty}\Big[-e^{-t} + e^{-2t}\frac{\cos t + 2\sin t}{5}\Big]_0^s$

$= 2\sqrt{2}\pi \displaystyle\lim_{s\to\infty}\Big(1 - e^{-s} + e^{-2s}\frac{\cos s + 2\sin s}{5} - \frac{1}{5}\Big) = \frac{8}{5}\sqrt{2}\pi < \infty.$

(C) $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{e^{-t}(-\cos t - \sin t)}{e^{-t}(-\sin t + \cos t)} = \begin{cases} 1 & \text{when } t = \pi/2, 3\pi/2, \\ -1 & \text{when } t = \pi, 2\pi. \end{cases}$

$L_i : y = (-1)^i(x - (x(\pi i/2)) + y(\pi i/2)$, rectangular region R.

$d(L_1, L_3) = \dfrac{|(x(\pi/2) - y(\pi/2) - x(3\pi/2) + y(3\pi/2)|}{\sqrt{2}} = \dfrac{e^{-\pi/2} + e^{-3\pi/2}}{\sqrt{2}},$

$d(L_2, L_4) = \dfrac{|(x(\pi) + y(\pi) - x(2\pi) - y(2\pi)|}{\sqrt{2}} = \dfrac{e^{-\pi} + e^{-2\pi}}{\sqrt{2}},$

$A(R) = \dfrac{e^{-\pi/2} + e^{-3\pi/2}}{\sqrt{2}} \times \dfrac{e^{-\pi} + e^{-2\pi}}{\sqrt{2}} = \dfrac{e^{-3\pi/2}(1 + e^{-\pi})^2}{2}.$

End