Part I

◎ 單選擇題 (單選十題, 每題五分, 共五十分, 答錯不倒扣。)

1. The **slope** of the tangent line to the curve $x^3+y^3-9xy=0$ at $(2,4)$ is 86:14

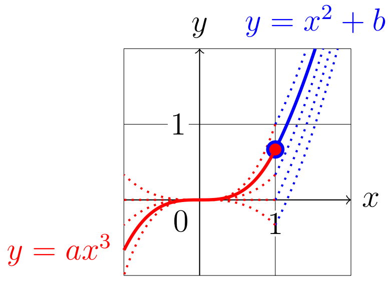
(A) $\frac{5}{4}$; **(B)** $\boxed{\mathbf{\frac{4}{5}};}$ (C) $\frac{3}{2}$; (D) $\frac{2}{3}$.

Solution: $(\frac{d}{dx}:)\ 3x^2+3y^2y'-9y-9xy'=0$,
$(2,4): 3(2)^2+3(4)^2y'-9(4)-9(2)y'=0,\ y'=\frac{4}{5}$.

5. Let $f(x)=\begin{cases} ax^3 & \text{if } x\le 1, \\ x^2+b & \text{if } x>1. \end{cases}$ If f is differentiable on $\mathbb{R}$, then the ordered pair (a,b) is 93:7

(A) $\boxed{\mathbf{\left(\frac{2}{3},\frac{-1}{3}\right)};}$ (B) $\left(\frac{-1}{3},\frac{2}{3}\right)$; (C) $\left(\frac{2}{3},\frac{1}{3}\right)$; (D) $\left(\frac{1}{3},\frac{2}{3}\right)$.

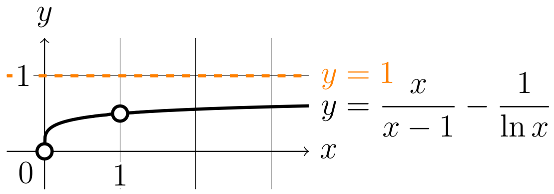
Solution: f is continuous at 1, $f'(x)=\begin{cases} 3ax^2 & \text{if } x<1, \\ 2x & \text{if } x>1. \end{cases}$
differentiable: $3a(1)^2=\lim_{x\to 1^-}f'(x)=\lim_{x\to 1^+}f'(x)=2(1),\ a=\frac{2}{3}$.
continuous: $a(1)^3=\lim_{x\to 1^-}f(x)=\lim_{x\to 1^+}f(x)=(1)^2+b,\ b=\frac{-1}{3}$.



8. The limit $\lim\limits_{x\to 1}\left(\dfrac{x}{x-1}-\dfrac{1}{\ln x}\right)$ is 58:42

(A) 1; **(B)** $\boxed{\mathbf{\frac{1}{2}};}$ (C) 0; (D) ∞.

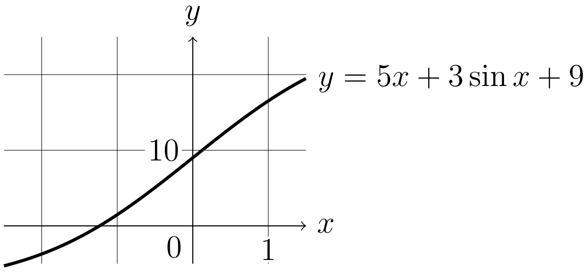
Solution: $\lim\limits_{x\to 1}\left(\dfrac{x}{x-1}-\dfrac{1}{\ln x}\right)=\lim\limits_{x\to 1}\dfrac{x\ln x-x+1}{(x-1)\ln x}$ $\left(\boldsymbol{\infty-\infty\to\frac{0}{0}}\right)$
$\overset{l'H}{=}\lim\limits_{x\to 1}\dfrac{\ln x}{\ln x+1-1/x}\overset{l'H}{=}\lim\limits_{x\to 1}\dfrac{1/x}{1/x+1/x^2}=\dfrac{1/1}{1/1+1/1^2}=\dfrac{1}{2}.$



10. How many **roots** does the equation $5x+3\sin x+9$ have? 81:19

(A) 0; **(B)** $\boxed{\mathbf{1;}}$ (C) 2; (D) 3.

Solution: Let $f(x)=5x+3\sin x+9$. $f(0)f(-\pi)=9(9-5\pi)<0$, $f'(x)=5+3\cos x>0$, $f(x)$ is increasing on $\mathbb{R}$. By the Locating Root Theorem, $f(x)$ has one, and hence exactly one, root on $[-\pi,0]$.



◎ 多選擇題 (多選五題, 每題五分, 共二十五分。答錯一個選項扣兩分, 錯兩個選項以上不給分, 分數不倒扣。)

13. Suppose that $\lim_{x\to 0} f(x) = L$ and $\lim_{x\to 0} g(x) = M$. Which of the following statements are **TRUE**? 13:40:47

(A) If $f(x) < g(x)$ for all $x \neq 0$, then $L < M$.

(B) $\boxed{\lim_{x\to 0} |f(x)| = |L|.}$

(C) If $L = 0$, then $\lim_{x\to 0} g(f(x)) = M$.

(D) $\boxed{\text{If } f(x) = g(x) \text{ for all } x \neq 0\text{, then } L = M.}$

Solution: $f(x) = -x^2 < x^2 = g(x)$ but $L = 0 = M$. .... (A)

$\because |x|$ is continuous on $\mathbb{R}$, $\lim_{x\to 0} |f(x)| = |\lim_{x\to 0} f(x)| = |L|$. .... (B)

$g(x) = \dfrac{\sin x}{x}$, $f(x) = 0$ ($\notin g$'s domain), $\lim_{x\to 0} g(f(x)) \nexists$. .... (C)

$L = \lim_{x\to 0} f(x) = \lim_{x\to 0} g(x) = M$. .... (D)

15. Let f be a continuous function on $\mathbb{R}$. Which of the following are **TRUE**? 52:18:30

(A) If $f'(x) > 0$ for all x, then $\lim_{x\to\infty} f(x) = \infty$.

(B) $\boxed{\text{If } f'(x) > 0 \text{ and } f''(x) > 0 \text{ for all } x\text{, then } \lim_{x\to\infty} f(x) = \infty.}$

(C) If $f'(x) > 0$ and $f''(x) < 0$ for all x, then $\lim_{x\to\infty} f(x) = \infty$.

(D) **If $f'(x) > 0$ and $f''(x) < 0$ for all x, then there is x such that $f(x) < -20180110$.**

Solution: Let $f(x) = -e^{-x}$, $f'(x) = e^{-x} > 0$, $f''(x) = -e^{-x} < 0$,

but $\lim_{x\to\infty} f(x) = 0 < \infty$. .... (A)(C)

$f''(x) > 0$, f CU $\iff f(x) \geq f'(a)(x-a) + f(a)$,

$f'(x) > 0 \implies \lim_{x\to\infty} f(x) \geq \lim_{x\to\infty} [f'(a)(x-a) + f(a)] = \infty$. ..... (B)

$f''(x) < 0$, f CD $\iff f(x) \leq f'(a)(x-a) + f(a)$,

$f'(x) > 0 \implies \lim_{x\to-\infty} f(x) \leq \lim_{x\to-\infty} [f'(a)(x-a) + f(a)] = -\infty$.

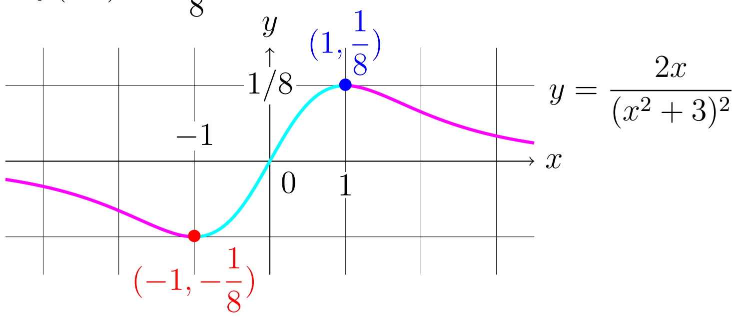
$\therefore \exists M < 0 \ni x < M \implies f(x) < -20180110$. .... (D)

◎ 填空題 (填空五題, 每題五分, 共二十五分, 答錯不倒扣。)

17. The **slant** asymptote of $f(x) = x + x\sin\frac{1}{x}$ is 34:56

> **Solution:** $y = x + 1$.
>
> ....
>
> $\lim\limits_{x\to\pm\infty} x\sin\frac{1}{x} \overset{t=1/x}{=} \lim\limits_{t\to 0^{\pm}} \frac{\sin t}{t} = 1$, $\lim\limits_{x\to\pm\infty} |f(x) - (x+1)| = 0$.

19. Let $f(x) = \dfrac{2x}{(x^2+3)^2}$ for $x \in \mathbb{R}$. Then, the **absolute minimum** of f is 57:40

> **Solution:** $f(-1) = -\frac{1}{8}$.
>
> ....
>
> $f'(x) = \dfrac{6(1-x^2)}{(x^2+3)^3} = 0$ when $x = \pm 1$, $f'(x)$ $- \to +$ at $x = -1$,
>
> f has a local minimum $f(-1) = -\frac{1}{8}$. (相對極小不一定是絕對最小。)
>
> For $x > 1$, $\because \lim\limits_{x\to\infty} f(x) = 0$ and $f'(x) < 0$, so $f(x) > 0$. (沒人更小。)
>
> $\therefore f(-1) = -\frac{1}{8}$ is also an absolute minimum.
>
> 
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End

Part II

◎ 單選擇題 (單選十題, 每題五分, 共五十分, 答錯不倒扣。)

2. Consider the curve defined by the parametric equations, $x = \theta - \sin\theta$ and $y = 1 - \cos\theta$ with $\theta \in [0, 2\pi]$. Find the **volume** of the solid obtained by rotating the region bounded by this curve and the x-axis about the x-axis. 送分

(A) $3\pi^2$; (B) $4\pi^2$; **(C)** $\boxed{\mathbf{5\pi^2;}}$ (D) $6\pi^2$.

Solution:

One arch of the cycloid: $0 \le \theta \le 2\pi$, $dx = (1 - \cos\theta)\ d\theta$.

$V = \int \pi y^2\ dx = \int_0^{2\pi} \pi(1-\cos\theta)^2 \cdot (1-\cos\theta)\ d\theta$

$= \pi \int_0^{2\pi} (1-\cos\theta)^3\ d\theta = \pi \int_0^{2\pi} (1 - 3\cos\theta + 3\cos^2\theta - \cos^3\theta)\ d\theta$

$= \pi \int_0^{2\pi} (1 + \frac{3}{2}(1 + \underbrace{\cos 2\theta}_{u=2\theta}) - \underbrace{(3 + 1 - \sin^2\theta)\cos\theta}_{v=\sin\theta})\ d\theta$

$\left(= \pi\left[\int_0^{2\pi} \frac{5}{2}\ d\theta + \int_0^{2\pi} \frac{3}{4}\cos 2\theta\ d(2\theta) + \int_0^{2\pi} (\sin^2\theta - 4)\ d(\sin\theta)\right]\right)$

$= \pi\left[\frac{5}{2}\theta + \frac{3}{4}\sin 2\theta + \frac{1}{3}\sin^3\theta - 4\sin\theta\right]_0^{2\pi} = 5\pi^2.$

★ (§10.2 Example: Extra) cycloid $V = 5\pi^2 r^3$.

3. The **length** of the curve $y = \int_0^x \sqrt{\cos 2t}\ dt$ from $x = 0$ to $x = \frac{\pi}{4}$ equals 70:30

(A) $\frac{1}{2}$; **(B)** $\boxed{\mathbf{1;}}$ (C) $\frac{3}{2}$; (D) 2.

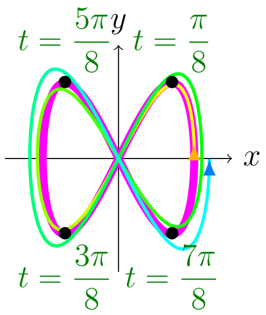
Solution: $y' = \sqrt{\cos 2x}$, $1 + (y')^2 = 1 + \cos 2x$, $\sqrt{1 + (y')^2}$
$= \sqrt{1 + \cos 2x} \overset{(半角)}{=} \sqrt{2\cos^2 x} = \sqrt{2}|\cos x| \overset{(負不合)}{=} \sqrt{2}\cos x$.

$L = \int\ ds = \int_0^{\pi/4} \sqrt{2}\cos x\ dx = \sqrt{2}\sin x\Big|_0^{\pi/4} = \sqrt{2}(\frac{1}{\sqrt{2}} - 0) = 1.$

4. Consider the parametric curve, $x = \cos 2t$ and $y = \sin 4t$ for $0 \leq t \leq 2\pi$.
How many points on this curve at which **tangent** lines are **horizontal**? 21:79

(A) 2; **(B)** $\boxed{\mathbf{4;}}$ (C) 6; (D) 8.

Solution: 繞兩次 (週期 π), 只看 $0 \leq t \leq \pi$.
$\frac{dy}{dt} = 4\cos 4t = 0$, $4t = (n + \frac{1}{2})\pi$,
$t = \frac{2n+1}{8}\pi = \frac{\pi}{8}, \frac{3}{8}\pi, \frac{5}{8}\pi, \frac{7}{8}\pi$.
$\frac{dx}{dt} = -2\sin 2t \neq 0$ for these t's,
$(x, y) = (\frac{\sqrt{2}}{2}, \pm 1), (-\frac{\sqrt{2}}{2}, \pm 1)$.



6. The limit $\lim_{x \to 0^+} x^{-\frac{3}{2}} \int_0^{\sqrt{x}} \sin(t^2)\cos(t)\; dt$ equals 40:60

(A) ∞; (B) $\frac{1}{2}$; **(C)** $\boxed{\mathbf{\frac{1}{3};}}$ (D) 0.

Solution: $\lim_{x \to 0^+} x^{-\frac{3}{2}} \int_0^{\sqrt{x}} \sin(t^2)\cos(t)\; dt$

$$= \lim_{x \to 0^+} \frac{\int_0^{\sqrt{x}} \sin t^2 \cos t\; dt}{x^{3/2}} \overset{l'H}{=} \lim_{x \to 0^+} \frac{\frac{d}{dx}\int_0^{\sqrt{x}} \sin t^2 \cos t\; dt}{3\sqrt{x}/2} \qquad (\frac{\mathbf{0}}{\mathbf{0}})$$

$$\overset{u=\sqrt{x}}{=} \lim_{x \to 0^+} \frac{\frac{d}{du}\int_0^{u} \sin t^2 \cos t\; dt \frac{d\sqrt{x}}{dx}}{3\sqrt{x}/2} = \lim_{x \to 0^+} \frac{\sin u^2 \cos u \frac{1}{2\sqrt{x}}}{3\sqrt{x}/2}$$

$$= \lim_{x \to 0^+} \frac{\sin x \cos \sqrt{x}}{3x} = \frac{1}{3} \lim_{x \to 0^+} \frac{\sin x}{x} \lim_{x \to 0^+} \cos \sqrt{x} = \frac{1}{3} \cdot 1 \cdot 1 = \frac{1}{3}.$$

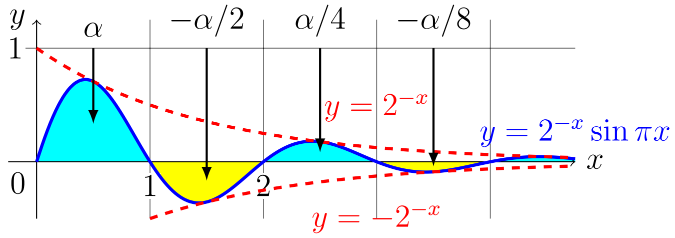
7. Let $\alpha = \int_0^1 2^{-x} \sin \pi x \, dx$. Then, the **improper** integral $\int_0^\infty 2^{-x} \sin \pi x \, dx$ equals 38:62

(A) $\frac{\alpha}{3}$; (B) $\frac{\alpha}{2}$; **(C)** $\boxed{\boldsymbol{\frac{2\alpha}{3}};}$ (D) α.

Solution: $\int_n^{n+1} 2^{-x} \sin \pi x \; dx \overset{u=x-n}{=} \int_0^1 2^{-u-n} \sin \pi(u+n) \; du$

$= \int_0^1 2^{-u} 2^{-n} \sin \pi u \cos n\pi \; du = \frac{(-1)^n}{2^n} \int_0^1 2^{-u} \sin \pi u \; du = (\frac{-1}{2})^n \alpha.$

$\int_0^\infty 2^{-x} \sin \pi x \; dx = \sum_{n=0}^\infty \int_n^{n+1} 2^{-x} \sin \pi x \; dx = \sum_{n=0}^\infty (\frac{-1}{2})^n \alpha = \frac{2\alpha}{3}.$



♦: [IxP] $\int 2^{-x} \sin \pi x \; dx \overset{\text{(omit)}}{=} \frac{-2^{-x}(\pi \cos \pi x + \ln 2 \sin \pi x)}{\pi^2 + (\ln 2)^2} + C,$

$\alpha = \int_0^1 2^{-x} \sin \pi x \; dx = \frac{\pi}{\pi^2 + (\ln 2)^2} - \frac{-1}{2} \frac{\pi}{\pi^2 + (\ln 2)^2} = \frac{3}{2} \frac{\pi}{\pi^2 + (\ln 2)^2},$

$\int_0^\infty 2^{-x} \sin \pi x \; dx = \lim_{t\to\infty} \int_0^t 2^{-x} \sin \pi x \; dx$

$= \frac{\pi}{\pi^2 + (\ln 2)^2} - \lim_{t\to\infty} \frac{2^{-t}(\pi \cos \pi t + \ln 2 \sin \pi t)}{\pi^2 + (\ln 2)^2} = \frac{\pi}{\pi^2 + (\ln 2)^2} = \frac{2}{3}\alpha.$

9. The limits $\lim_{n\to\infty} \sum_{i=1}^n \frac{1}{n + \sqrt{(i-1)i}}$ equals 48:51

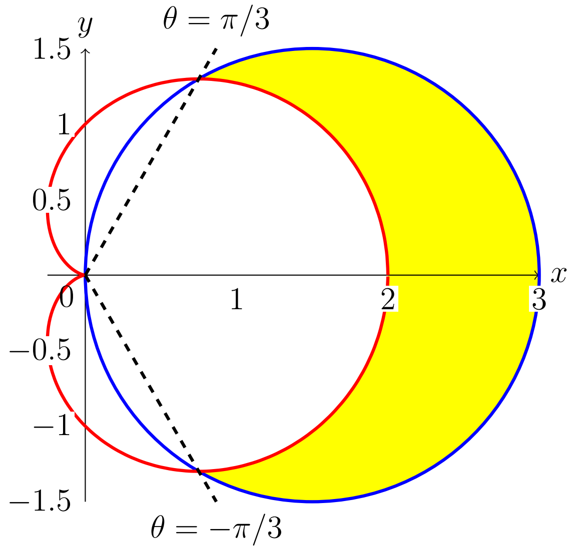
(A) 2; **(B)** $\boxed{\textbf{ln 2;}}$ (C) e^2; (D) $\tan^{-1} 2$.

Solution: $\Delta x = \frac{1}{n}$, $x_{i-1} = \frac{i-1}{n}$, $x_i = \frac{i}{n}$, $x_i^* = \sqrt{x_{i-1} x_i} = \frac{\sqrt{(i-1)i}}{n}$.

$\lim_{n\to\infty} \sum_{i=1}^n \frac{1}{n + \sqrt{(i-1)i}} = \lim_{n\to\infty} \sum_{i=1}^n \frac{1/n}{1 + \sqrt{(i-1)i}/n} = \lim_{n\to\infty} \sum_{i=1}^n \frac{\Delta x}{1 + x_i^*}$

$= \int_0^1 \frac{dx}{1+x} = \ln|1+x| \Big|_0^1 = \ln 2 - \ln 1 = \ln 2.$

1. (106-2) The area of the region that lies inside the polar curve $r = 3\cos\theta$ and outside the polar curve $1 + \cos\theta$ is 59:41



(A) $\pi + \sqrt{3}$; **(B)** $\boxed{\boldsymbol{\pi};}$ (C) $\pi - \sqrt{3}$; (D) $\frac{1}{2}\pi$.

Solution: $3\cos\theta = r = 1 + \cos\theta$, $\theta = \pm\pi/3$.

$A = \int_{-\pi/3}^{\pi/3} \frac{1}{2}[(3\cos\theta)^2 - (1 + \cos\theta)^2]\ d\theta$

$= \int_0^{\pi/3} [9\cos^2\theta - (1 + 2\cos\theta + \cos^2\theta)]\ d\theta$

$= \int_0^{\pi/3} (-1 - 2\cos\theta + 8\cos^2\theta)\ d\theta = \int_0^{\pi/3} (3 - 2\cos\theta + 4\cos 2\theta)\ d\theta$

$= \Big[3\theta - 2\sin\theta + 2\sin 2\theta\Big]_0^{\pi/3} = (\pi - \sqrt{3} + \sqrt{3}) = \pi.$

◎ 多選擇題 (多選五題, 每題五分, 共二十五分。答錯一個選項扣兩分, 錯兩個選項以上不給分, 分數不倒扣。)

11. Let f be a differentiable **odd** funtcion on $\mathbb{R}$ satisfying $\int_0^1 f(x)\,dx = 0$.

Then, $\int_0^1 xf'(1-x)\,dx$ **MUST** equal. 20:21:59

(A) $f(1)-f(0)$; (B) $f(1)$; **(C)** $\boxed{-f(0);}$ **(D)** $\boxed{0.}$

Solution: $\int_0^1 xf'(1-x)\,dx \overset{u=1-x}{=} \int_1^0 (1-u)f'(u)(-\,du)$

$= \int_0^1 (1-u)f'(u)\,du = (1-u)f(u)\Big|_0^1 - \int_0^1 -f(u)\,du$

$= (1-1)f(1) - (1-0)f(0) + 0 = -f(0) = 0.$ ($\because$ odd)

12. Consider the function $f(x) = \int_0^{x^2} \frac{dt}{1+t^4}$. Which of the following statements are **TRUE**? 34:28:38

(A) $\boxed{f(x) \text{ is continuous on } \mathbb{R}.}$

(B) $f(x)$ has neither local maxima nor local minima.

(C) $\boxed{f(x) \text{ is concave upward on some interval and concave downward on some interval.}}$

(D) $f(x)$ has exactly one inflection point.

Solution: $f'(x) = \frac{d}{dx}\int_0^{x^2} \frac{dt}{1+t^4} \overset{u=x^2}{=} \frac{d}{du}\int_0^{u} \frac{dt}{1+t^4}\frac{dx^2}{dx} = \frac{2x}{1+u^4}$

$= \frac{2x}{1+x^8}$ exists on $\mathbb{R}$, f is differentiable $\Longrightarrow$ continuous. ..... (A)

$f'(x) = 0$ when $x = 0$, $f'(x)$ $- \to +$ at 0 (or $f''(0) = 2 > 0$),

$f(x)$ has one local(absolute) minimum value $f(0) = 0$. .... (B)

$f''(x) = \frac{2-14x^8}{(1+x^8)^2}$, $f''(x) = 0$ when $x = \pm\sqrt[8]{1/7}$,

$f''(x) > 0$ and hence CU when $-\sqrt[8]{1/7} < x < \sqrt[8]{1/7}$,

$f''(x) < 0$ and hence CD when $x < -\sqrt[8]{1/7}$ or $x > \sqrt[8]{1/7}$. ..... (C)

$f(x)$ has two infection point at $x = \pm\sqrt[8]{1/7}$. .... (D)

14. Which of the following represents the **surface area** of revolution obtained by rotating the curve, $y = \sin x$ with $x \in [0, \pi]$, about the x-axis? 36:42:22

(A) $\boxed{\mathbf{2\pi \int_0^\pi \sin x \sqrt{1+\cos^2 x}\, dx.}}$ (B) $2\pi \int_0^\pi \cos x \sqrt{1+\sin^2 x}\, dx.$

(C) $\boxed{\mathbf{2\pi(\sqrt{2} + \ln(\sqrt{2}+1)).}}$ (D) $2\pi(\sqrt{2} + 2\ln(\sqrt{2}+1)).$

Solution: $y' = \cos x$, $\sqrt{1+(y')^2} = \sqrt{1+\cos^2 x}$.

$$S = \int 2\pi y\, ds = 2\pi \int_0^\pi \sin x \sqrt{1+\cos^2 x}\, dx \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(A)}$$

$$\overset{u=-\cos x}{=} 2\pi \int_{-1}^1 \sqrt{1+u^2}\, du = 4\pi \int_0^1 \sqrt{1+u^2}\, du$$

$$\overset{u=\tan t}{=} 4\pi \int_0^{\pi/4} \sec^3 t\, dt = 4\pi \left[\frac{1}{2}\sec t \tan t + \frac{1}{2}\ln|\sec t + \tan t|\right]_0^{\pi/4}$$

$$= 2\pi\left[u\sqrt{1+u^2} + \ln\left|\sqrt{1+u^2}+u\right|\right]_0^1 = 2\pi(\sqrt{2} + \ln(\sqrt{2}+1)). \text{ (C)(D)}$$

$$2\pi \int_0^\pi \cos x \sqrt{1+\sin^2 x}\, dx \overset{v=\sin x}{=} 2\pi \int_0^\pi \cos x \sqrt{1+(\sin x)^2}\, dx$$

$$= 2\pi \int_0^0 \sqrt{1+v^2}\, dv = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(B)}$$

◎ 填空題 (填空五題, 每題五分, 共二十五分, 答錯不倒扣。)

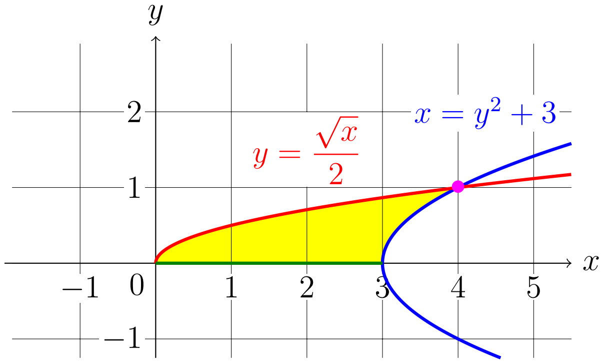
16. Let $f(x) = \sqrt{x} \int_0^{\sqrt{x}} e^{xt^2} dt$. Then $f'(x)$ is 3:82

Solution: e^{x^2}.

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$f'(x) = \frac{d}{dx}\left(\sqrt{x} \int_0^{\sqrt{x}} e^{(\sqrt{x}t)^2}\, dt\right) \overset{u=\sqrt{x}t}{=} \frac{d}{dx} \int_0^x e^{u^2}\, du = e^{x^2}.$$

18. The **area** of the region in the first quadrant enclosed by the three curves, $y = 0$, $y = \dfrac{\sqrt{x}}{2}$ and $x = y^2 + 3$, is 73:26



Solution: 2.

$$y = \frac{\sqrt{x}}{2} \iff x = 4y^2;\ y^2 + 3 = x = 4y^2,\ y^2 = 1,\ y = 1.$$

$$A = \int_0^1 (y^2 + 3 - 4y^2)\,dy = \Big[3y - y^3\Big]_0^1 = 2.$$

20. If the integral $\displaystyle\int_0^\infty \left(\frac{x}{x^2+1} - \frac{a}{2x+1}\right) dx$ is **convergent**, then $a =$ 55:37

Solution: 2.

. . . .

$$\frac{x}{x^2+1} - \frac{a}{2x+1} \approx \frac{1}{x} - \frac{a}{2x} = \frac{2-a}{2}\frac{1}{x} \text{ as } x \to \infty,$$

$\displaystyle\int_0^\infty \frac{2-a}{2}\frac{1}{x}\, dx$ diverges when $a \neq 2$. Check convergence of $a = 2$:

[Comparison Theorem] $\displaystyle\int_0^1 \left(\frac{x}{x^2+1} - \frac{2}{2x+1}\right) dx$ is proper,

$$\int_1^\infty \left(\frac{x}{x^2+1} - \frac{2}{2x+1}\right) dx = \int_1^\infty \frac{x-2}{2x^3+x^2+2x+1}\, dx$$

$$< \int_1^\infty \frac{x-2}{2x^3}\, dx = \frac{1}{2}\int_1^\infty \frac{dx}{x^2} - \int_1^\infty \frac{dx}{x^3} (= 0) \text{ converges.}$$

[Improper integral] $\displaystyle\int_0^\infty \left(\frac{x}{x^2+1} - \frac{2}{2x+1}\right) dx$

$$= \lim_{t\to\infty} \int_0^t \left(\frac{x}{x^2+1} - \frac{2}{2x+1}\right) dx = \lim_{t\to\infty} \left[\frac{1}{2}\ln(x^2+1) - \ln|2x+1|\right]_0^t$$

$$= \frac{1}{2}\lim_{t\to\infty} \ln\left|\frac{t^2+1}{(2t+1)^2}\right| \overset{\div t^2}{=} \frac{1}{2}\ln\left|\lim_{t\to\infty}\frac{1+1/t^2}{(2+1/t)^2}\right| = \frac{1}{2}\ln\frac{1}{2^2} = -\ln 2.$$

16. (106-2) The **slope** of the tangent line to the polar curve $r = \dfrac{1}{\theta}$ at $\theta = \pi$ is 51:45

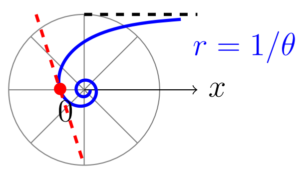
Solution: $-\pi$.

. . . .

$$x = r\cos\theta = \frac{\cos\theta}{\theta},\ y = r\sin\theta = \frac{\sin\theta}{\theta},$$

$$\frac{dy}{dx} = \frac{\dfrac{dy}{d\theta}}{\dfrac{dx}{d\theta}} = \frac{\dfrac{\theta\cos\theta - \sin\theta}{\cancel{\theta^2}}}{\dfrac{-\theta\sin\theta - \cos\theta}{\cancel{\theta^2}}}$$

$$= \frac{\pi\cdot(-1) - 0}{-\pi\cdot 0 - (-1)} = -\pi.$$



End