

# Thriving Through Drought: The Power of Plant Diversity

## Summary

*"Biodiversity increases the resistance of ecosystem productivity to climate extremes." [1]*

—Isbell et al., **Nature**, 2015

To analyze and simulate the interaction between different plant species encountering drought, we begin with building an Ordinary Differential Equation model for those complex dynamics. Instead of directly defining the competition between different plant species, we resort to a more authentic way by defining their demand for water in the soil as well as their water and soil protection capacity. Accordingly, a **Tree-Sapling-Grass-Soil** model is proposed to reflect the **real-world dynamics**.

In terms of parameter estimation, we collect credible parameters value from an array of established articles on environment modelling. For those crucial and unknown parameters, such as the colonization rate for species, we apply the **Monte Carlo approach** to determine those values by finding the Tree-Grass coexistence interval to determine the value above.

With the establishment of the dynamic model, a simulation for eleven years is implemented on irregular weather conditions from the real-world database. Despite the data being insufficient for the simulation of the whole process, we applied the **Bartlett-Lewis model** to extend the data length while keeping its **statistical features**. The simulation results based on **Euler method** clearly demonstrate a remarkable balance between the tree-grass community regardless of the drought.

To delve deeper into the impact of the number of species, we first define the metrics of evaluating the bio-community, that is, **plant spatial occupancy** and **species richness**. Then the grass and tree are subdivided into various types by randomly selecting their colonization and mortality rate based on a normal distribution. To eliminate the effects of randomness, we also applied the Monte Carlo approach to calculate the metrics of different grass-tree pairs. It turns out that **6 types of grass and 2 types of trees** can reach maximum mutual benefit in areas with irregular precipitation of 400-600 mm per year.

When modifying the model to make it more suitable for the real environment, we applied sensitivity analysis to our model regarding the rainfall changes from depletion to abundance. It shows that the plant community can fully benefit from the abundant precipitation with species richness of **23% higher** than usual, and **19% lower** than the average level when exposed to extremely low precipitation.

Furthermore, the sensitivity analysis regarding pollution and habitat reduction is conducted, followed by a **Poisson process** of a total 10% loss on habitat and vegetation for the whole simulation time series. It turned out that the effect of pollution and habitat reduction, though relatively milder than precipitation, still has a big impact on biodiversity, i.e. **15%** loss of species richness.

Overall, our mathematical model shows that despite the joint requirement of precipitation, their interaction incredibly plays a crucial role in their adaption to drought. To preserve these natural wonders, we suggest that planting many tree saplings is imperative to benefit a larger environment.

**Keywords:** Nonlinear dynamics, Bartlett-Lewis model, Monte Carlo simulation, Stochastic process

## Contents

|          |   |           |
|----------|---|-----------|
| <b>1</b> | <b>Introduction</b>   | <b>3</b>  |
| 1.1      | Background . . . . .  | 3         |
| 1.2      | Restatement . . . . .   | 4         |
| 1.3      | Organization of the Paper . . . . .                                     | 4         |
| <b>2</b> | <b>Notations and Parameters</b>   | <b>5</b>  |
| <b>3</b> | <b>The Tree-Sapling-Grass-Soil Model</b>                                | <b>6</b>  |
| 3.1      | Model Description . . . . .   | 6         |
| 3.1.1    | The Colonization Rate Function . . . . .                                | 8         |
| 3.1.2    | The Evaporation Rate and Evapotranspiration Rate Function . . . . .     | 9         |
| 3.2      | Data Pre-processing . . . . .   | 10        |
| 3.3      | Parameter Estimation . . . . .  | 10        |
| 3.4      | Model Implementation in Irregular Weather Conditions . . . . .          | 12        |
| <b>4</b> | <b>Analyze the Impact of Species Number</b>                             | <b>13</b> |
| 4.1      | Assessment Criteria . . . . .   | 13        |
| 4.2      | Evaluation Process of Species Number . . . . .                          | 14        |
| 4.3      | Application of the Evaluation Process . . . . .                         | 14        |
| <b>5</b> | <b>Sensitivity Analysis</b>   | <b>16</b> |
| 5.1      | The Impact of Climate Change . . . . .                                  | 16        |
| 5.2      | The Impact of Pollution and Habitat Reduction . . . . .                 | 17        |
| 5.2.1    | Establishment of Disruption by Possion Distribution . . . . .           | 17        |
| 5.2.2    | Simulation of Conditions under Disruptions . . . . .                    | 18        |
| <b>6</b> | <b>Possible Strategies for Long-term Viability</b>                      | <b>19</b> |
| 6.1      | Suggestion for Ensuring the Long-term Viability of Bio-system . . . . . | 20        |
| 6.2      | Possible Impact on Larger Environment . . . . .                         | 20        |

|  |           |
|--|-----------|
| <b>7 Model Evaluation and Further Discussion</b> | <b>21</b> |
| 7.1 Strengths of our Model . . . . .             | 21        |
| 7.2 Weakness of our Model . . . . .              | 22        |
| 7.3 Possible Improvement of the Model . . . . .  | 22        |
| <b>References</b>                                | <b>24</b> |

# 1 Introduction

## 1.1 Background

The natural world is a wonder to behold, with an incredible variety of plants that have evolved to develop unique and specific abilities to survive in a diverse range of environments. One of the most crucial factors in plant survival is water, and different types of plants are renowned for their ability to compete for this precious resource. In the complex and interconnected ecosystem of the natural world, these plants have evolved to develop an array of strategies to gain an advantage in the race for water.

However, a plethora of observations suggest that the number of different plant species present plays a critical role in how a plant community adapts to cycles of drought over successive generations. For instance, communities with only one species of plant do not adapt as well to drought conditions as those with four or more species.

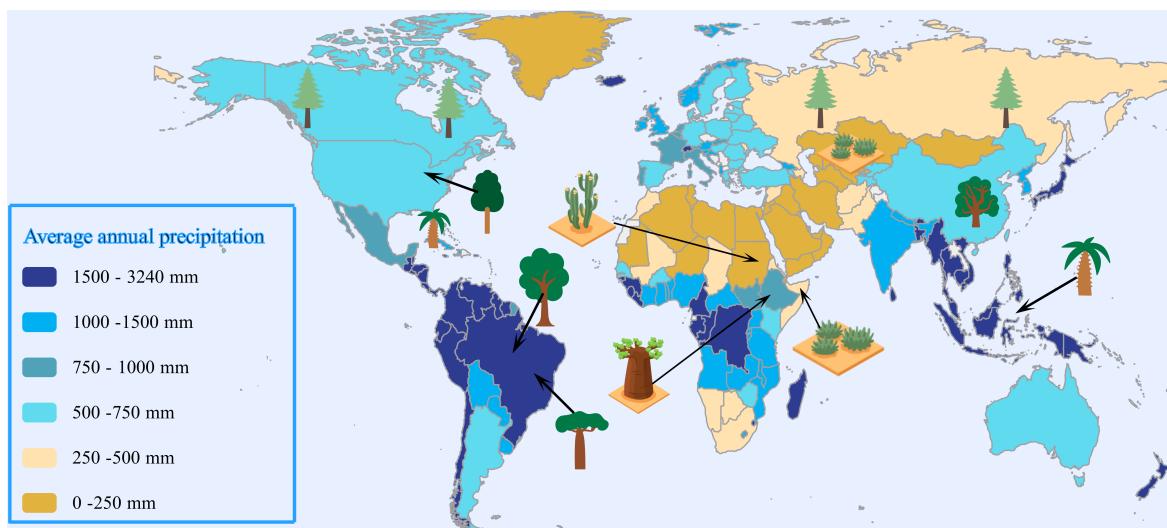


Figure 1: Average annual precipitation and related vegetation, data from FAO(UN) [2].

As can be seen in Figure 1, drought-prone areas are mainly concentrated in Asia and Africa, Corresponding to continental climate and savannah climate and desert. Given that the model's background is within a variable climate encompassing both dry and rainy seasons, our primary reference point for this analysis will be the rainfall records of tropical savannah climates. In the arid and semi-arid regions of the world, where precipitation is a precious resource, the coexistence of trees and grasses is a fascinating phenomenon that requires in-depth study.

Understanding the intricate interplay between these two vegetation types, as well as the role of rainfall temporal intermittency in shaping their competition, is crucial and highly relevant in these regions. By exploring the complex dynamics of this interplay, researchers can gain valuable insights into the short-term and long-term outcomes of tree-grass competition and coexistence, and how it impacts the local ecosystem [3]. In addition, the increasing frequency of droughts and extreme weather events due to global warming highlights the need for a better understanding of how ecosystems respond to changes in rainfall patterns. This is of great practical importance, as ecosystems provide essential goods and services to human societies, such as food, water, and air quality [4].

## 1.2 Restatement

This paper aims to delve into the intricate relationship between drought adaptability and the number of species in a plant community.

- **Establish a mathematical model** that can predict how a plant community will evolve over time when subjected to unpredictable weather cycles, including times of drought when precipitation is irregular.
- **Draw insightful conclusions** which will shed light on the long-term interactions.
  - **Investigate the number of plant species** required for the community.
  - **Explore the implications of** having a greater variety of species within the community and how the types of species impact the results.
  - **Consider the impact of** a greater or less frequency and wider variation of the occurrence of droughts.
  - **Sensitivity analysis for** pollution and habitat reduction like fire and man-made sabotage.
  - **Extend our model for** the long-term viability of the plant community and impact on the larger environment.

## 1.3 Organization of the Paper

The remaining sections of this paper are organized as follows. Section 2 provides a detailed list of the relevant notations and parameters used throughout the paper, as presented in Tables 1 and 2. In Section 3, we introduce the model we developed to study the plant community, as well as the solution methods used to obtain the results.

To evaluate the potential impacts of the number of species on the plant community, we establish a set of assessment criteria in Section 4. These criteria serve as a framework for evaluating the effects of various factors on the plant community, and provide a basis for our subsequent analysis.

In Section 5, we conduct a sensitivity analysis to examine the effects of climate change and environmental disruption on the plant community. This analysis sheds light on the potential vulnerabilities of the plant community to external factors, and highlights the need for effective management strategies.

Finally, in Section 6, we provide some long-term insights into the plant community, drawing on the results of our analysis and considering future trends and developments. In Section 7, we offer an overall evaluation of the model and its usefulness in studying the plant community and provide some suggestions for future research.

Additionally, we have provided a flowchart below, which outlines the various sections and their interrelationships in Figure 2.

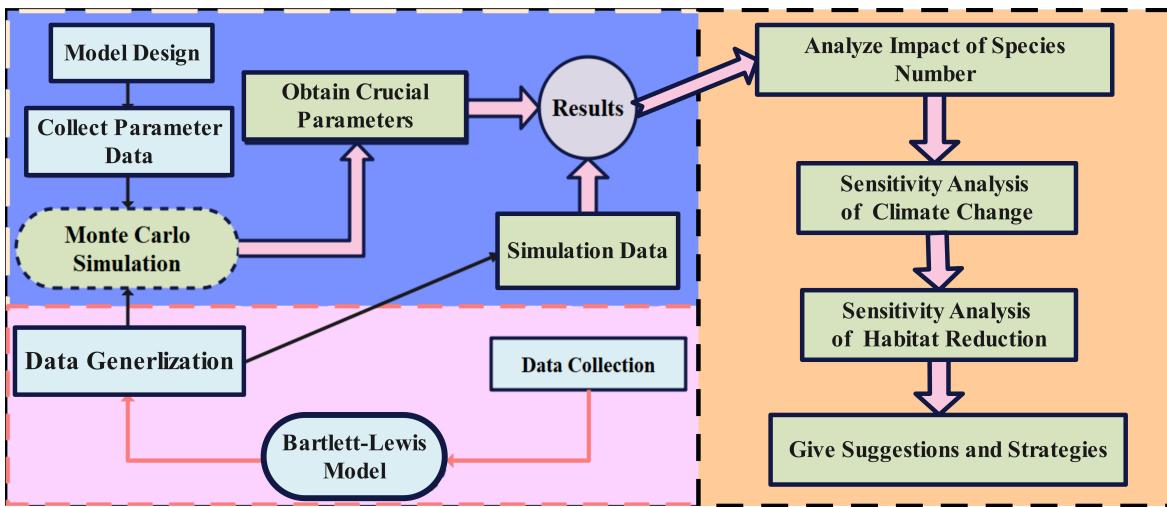


Figure 2: Flowchart of the Paper

## 2 Notations and Parameters

Some essential mathematical notations in our paper are listed in Table 1. Other crucial parameters are listed in Table 2.

Table 1: Notation

| Notation  | Definition                                  | Dimension        |
|-----------|---|------------------|
| $G$       | Grass cover                                 |                  |
| $S$       | Tree seedling cover                         |                  |
| $T$       | Adult tree cover                            |                  |
| $s$       | Soil moisture rate                          |                  |
| $t$       | Time  | $T$              |
| $R$       | Precipitation                               | $mm$             |
| $c_G(s)$  | Colonization rate for grassed               | $T^{-1}$         |
| $c_T(s)$  | Colonization rate for trees                 | $T^{-1}$         |
| $r(t)$    | Daily rainfall rate                         | $L \cdot T^{-1}$ |
| $I(s, r)$ | Infiltration rate of rainfall               | $L \cdot T^{-1}$ |
| $E_0(s)$  | Evaporation rate from bare soil             | $L \cdot T^{-1}$ |
| $E_G(s)$  | Evapotranspiration rate from grasses        | $L \cdot T^{-1}$ |
| $E_S(s)$  | Evapotranspiration rate from tree seedlings | $L \cdot T^{-1}$ |
| $E_T(s)$  | Evapotranspiration rate from adult trees    | $L \cdot T^{-1}$ |
| $L(s)$    | Leakage                                     | $L \cdot T^{-1}$ |

Table 2: Parameters

| Parameters  | Meaning                                   | Value | Unit              |
|-------------|---|-------|-------------------|
| $\beta$     | Water retention parameter                 | 8.5   |                   |
| $S_{mc,T}$  | Maximum colonization point for trees      | 0.185 |                   |
| $S_{mcG}$   | Maximum colonization point for grasses    | 0.175 |                   |
| $S_h$       | Hygroscopic point                         | 0.48  |                   |
| $S_{fc}$    | Soil field capacity                       | 0.29  |                   |
| $S_{wG}$    | Wilting point for grasses                 | 0.056 |                   |
| $S_{wT}$    | Wilting point for adult trees             | 0.085 |                   |
| $S^*$       | Fully open-stomata point                  | 0.175 |                   |
| $\tau_G$    | Grass mortality rate                      | 0.5   | $yr^{-1}$         |
| $\tau_S$    | Tree seedling mortality rate              | 5     | $yr^{-1}$         |
| $\tau_T$    | Adult tree mortality rate                 | 0.02  | $yr^{-1}$         |
| $g$         | Tree growth rate                          | 0.2   | $yr^{-1}$         |
| $C_{max,G}$ | Grass maximum colonization rate           | 2     | $yr^{-1}$         |
| $C_{max,T}$ | Tree maximum colonization                 | 3     | $yr^{-1}$         |
| $E_{max}$   | Maximum evapotranspiration rate           | 0.38  | $cm \cdot d^{-1}$ |
| $E_{fc}$    | Soil evaporation rate at $S_{fc}$         | 0.049 | $cm \cdot d^{-1}$ |
| $nZ_t$      | Soil porosity multiplied by rooting depth | 42    | $cm$              |
| $K_s$       | Saturated hydraulic conductivity          | 20    | $cm \cdot d^{-1}$ |
| $N_{wet}$   | Wet season length                         | 200   | $d$               |

### 3 The Tree-Sapling-Grass-Soil Model

The delicate balance of an ecosystem is maintained through the complex interactions between a wide range of plant and animal species, but none are quite so prominent or iconic as the grass and trees that form the backbone of so many important ecological systems. Therefore, we take grass and trees, as well as their respective sub-species, as the focus of our study.

We introduced a mathematical model to describe the tree-grass dynamics as well as their competition for water resources. In this way, the average dynamics of soil and vegetation can be approached according to an implicit method.

#### 3.1 Model Description

Our model could be divided into the following compartments according to individuals' properties as tree seedlings, adult trees, grass as well as soil.  $G, S, T$  denote grass, tree seedlings and adult trees respectively, which vary between 0 to 1 to describe the fraction of space they occupied in a certain region but never overlap each other.

Throughout the definition of our model, the relationship can be given by:

$$0 \leq G + S + T \leq 1.$$

Coupled with an equation for soil moisture (1d), the vegetation equation can be expressed as (1a)-(1c) :

$$\left\{ \begin{array}{l} \frac{dG}{dt} = c_G(s)(1 - T - G) - \tau_G G, \\ \frac{dS}{dt} = c_T(s)T(1 - T - G - S) - gS - c_G(s)GS - \tau_S S, \\ \frac{dT}{dt} = gS - \tau_T T, \end{array} \right. \quad (1a)$$

$$\left\{ \begin{array}{l} \frac{dS}{dt} = c_T(s)T(1 - T - G - S) - gS - c_G(s)GS - \tau_S S, \\ \frac{dT}{dt} = gS - \tau_T T, \end{array} \right. \quad (1b)$$

$$\left\{ \begin{array}{l} \frac{dT}{dt} = gS - \tau_T T, \\ \frac{ds}{dt} = \frac{1}{nZ_r}[I(s, r) - (1 - T - S - G)E_0(s) - TE_T(s) - SE_S(s) - GE_G(s) - L(s)]. \end{array} \right. \quad (1c)$$

$$\left\{ \begin{array}{l} \frac{ds}{dt} = \frac{1}{nZ_r}[I(s, r) - (1 - T - S - G)E_0(s) - TE_T(s) - SE_S(s) - GE_G(s) - L(s)]. \end{array} \right. \quad (1d)$$

where the model follows Assumptions 1-5 below. The meaning and dimension of the variable and function symbols, as well as the parameter symbols, are listed in Table 1 and Table 2 respectively.

- **Assumption 1:** In the early stages of growth, trees often face stiff competition from grasses. Nevertheless, with maturity, trees emerge as resilient and dominant occupants of their habitat, able to withstand and overcome various external pressures.
- **Assumption 2:** Adult trees cannot substitute for grass actively, which means they are not strictly the best competitor, and neither can grass. Hence, their colonization limits is constituted.
- **Assumption 3:** All of the three life forms have extinction rates. We assume that the mortality is constant, moreover,  $\tau_T < \tau_G < \tau_S$ .
- **Assumption 4:** The colonization rates increase linearly up to a constant value:  $C_{max,G}, C_{max,T}$ .
- **Assumption 5:** The growth rate of seedlings into adult trees remains a constant. On this basis, seedlings eventually reach a certain age at which they transition into adult trees

Central to this model is a set of equations that capture the key processes at work within the plant community. In equation (1a), for example, we see that grass is able to settle into new areas that have not yet been occupied by adult trees and grasses ( $1 - T - G$ ), in proportion to their practical coverage  $G$ . This process allows for the replacement of tree seedlings by grass, reflecting the delicate interplay between these two key species.

However, tree seedlings can only grow in bare soil ( $1 - T - G - S$ ), and their establishment is linearly dependent on adult tree coverage  $T$  according to equation (1b). This reflects the crucial importance of adult trees in providing the necessary conditions for the growth and development of new seedlings.

As seedlings grow and mature, they eventually reach adulthood, competing with grass and other seedlings for resources and space, as captured by equation (1c). This competitive process is a critical factor in shaping the overall behavior of the community, and underscores the importance of understanding the complex dynamics that govern these systems.

Finally, equation (1d) represents the mean water balance in the area, reflecting the crucial role that precipitation and other environmental factors play in shaping the plant community over time.

To conduct further works regarding parameter estimation and model simulation, the Tree-Sapling-Grass-Soil model can be transformed into the monthly difference form to calculate the number of each compartment in the model as ODEs (2a)-(2d):

$$\left\{ \begin{array}{l} G(t+1) = G(t) + c_G(s)(1 - T - G) - \tau_G G, \\ S(t+1) = S(t) + c_T(s)T(1 - T - G - S) - gS - c_G(s)GS - \tau_S S, \\ T(t+1) = T(t) + gS - \tau_T T, \\ s(t+1) = s(t) + (1/nZ_r) \cdot [I(s, r) - (1 - T - S - G)E_0(s) - TE_T(s) - SE_S(s) - GE_G(s) - L(s)]. \end{array} \right. \quad (2a)$$

$$\left\{ \begin{array}{l} G(t+1) = G(t) + c_G(s)(1 - T - G) - \tau_G G, \\ S(t+1) = S(t) + c_T(s)T(1 - T - G - S) - gS - c_G(s)GS - \tau_S S, \\ T(t+1) = T(t) + gS - \tau_T T, \\ s(t+1) = s(t) + (1/nZ_r) \cdot [I(s, r) - (1 - T - S - G)E_0(s) - TE_T(s) - SE_S(s) - GE_G(s) - L(s)]. \end{array} \right. \quad (2b)$$

$$\left\{ \begin{array}{l} G(t+1) = G(t) + c_G(s)(1 - T - G) - \tau_G G, \\ S(t+1) = S(t) + c_T(s)T(1 - T - G - S) - gS - c_G(s)GS - \tau_S S, \\ T(t+1) = T(t) + gS - \tau_T T, \\ s(t+1) = s(t) + (1/nZ_r) \cdot [I(s, r) - (1 - T - S - G)E_0(s) - TE_T(s) - SE_S(s) - GE_G(s) - L(s)]. \end{array} \right. \quad (2c)$$

$$\left\{ \begin{array}{l} G(t+1) = G(t) + c_G(s)(1 - T - G) - \tau_G G, \\ S(t+1) = S(t) + c_T(s)T(1 - T - G - S) - gS - c_G(s)GS - \tau_S S, \\ T(t+1) = T(t) + gS - \tau_T T, \\ s(t+1) = s(t) + (1/nZ_r) \cdot [I(s, r) - (1 - T - S - G)E_0(s) - TE_T(s) - SE_S(s) - GE_G(s) - L(s)]. \end{array} \right. \quad (2d)$$

To help visualize these complex relationships and dynamics, we have created a detailed flowchart that captures the key elements of our model.

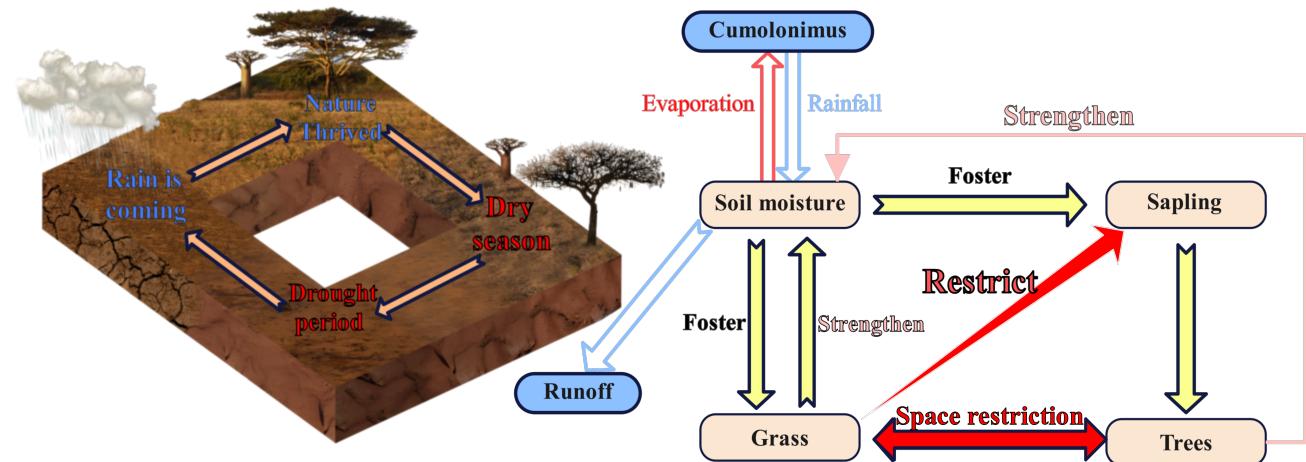


Figure 3: Flowchart of the Tree-Sapling-Grass-Soil Model

### 3.1.1 The Colonization Rate Function

In equation 1, colonization rates represent the capacity of a plant to successfully establish itself in new environments. This can be influenced by a variety of factors, such as the ability to disperse its seeds over long distances, the resilience of its seeds to environmental stresses, or the adaptability of its seedlings to varying soil and climate conditions. In our model, we define the colonization rate for grass and tree by soil moisture and wilting points. According to assumption 4, the function can be written as equation 3 and shown in Figure 4.

$$c_i(s) = \begin{cases} 0, & \text{if } s \leq s_{w,i}, \\ C_{\max,j} \frac{(s-s_{w,j})}{(s_{mc,i}-s_{w,i})}, & \text{if } s_{w,j} < s \leq s_{mc,i}, \quad i = T, G, \\ C_{\max,j}, & \text{if } s_{mc,i} < s \leq 1, \end{cases} \quad (3)$$

### 3.1.2 The Evaporation Rate and Evapotranspiration Rate Function

The output terms of the soil water balance are driven by three distinct mechanisms, each of which plays a critical role in regulating the movement and availability of water within the soil system. The first mechanism is the evaporation of water from the bare soil surface, which occurs when the energy from the sun heats the soil and causes the water to vaporize and escape into the atmosphere.

$$E_0 = \begin{cases} 0, & \text{if } s \leq s_h, \\ E_{fc} \cdot \frac{s-s_h}{s_{fc}-s_h}, & \text{if } s_h < s \leq s_{fc}, \\ E_{fc}, & \text{if } s_{fc} < s \leq 1. \end{cases} \quad (4)$$

When soil moisture  $s$  is below  $s_h$ ,  $E_0$  is defined to be 0, then increasing linearly up to a maximum value  $E_{fc}$  at which soil moisture equals to the capacity  $s_{fc}$ .

The secondary mechanism is evapotranspiration, which occurs when water is taken up by plants and transpired into the atmosphere through their leaves. To simplify the model, we assume that  $E_{max}$  does not vary according to the types of plants.

$$E_j(s) = \begin{cases} 0, & \text{if } s \leq s_h, \\ E_{fc} \cdot \frac{s-s_h}{s_{fc}-s_h}, & \text{if } s_h < s \leq s_{w,i}, \\ E_{w,i} + (E_{max,j} - E_{w,i}) \cdot \frac{s-s_{w,i}}{s_i^*-s_{w,i}}, & \text{if } s_{w,i} < s \leq s_i^*, \\ E_{max,j}, & \text{if } s_i^* < s \leq 1, \end{cases} \quad (5)$$

where  $j = T, G, S$  represents trees, grass and seedlings respectively. The maximum evapotranspiration of plants is defined as  $E_{max}$  when the soil moisture is abundant. As plants begin to close their stomata, evapotranspiration decreases linearly with soil moisture, reflecting the fact that water is becoming scarcer and more difficult to obtain.

Once the soil moisture level drops to the wilting point, plants fully close their stomata, and only soil evaporation can take place. This critical transition is captured by a linear decrease in the evapotranspiration function, which declines steadily from the value of  $E_{fc} \cdot \frac{s-s_h}{s_{fc}-s_h}$  down to zero at the hygroscopic point  $s_h$ . Figure 4 demonstrates a vague illustration.

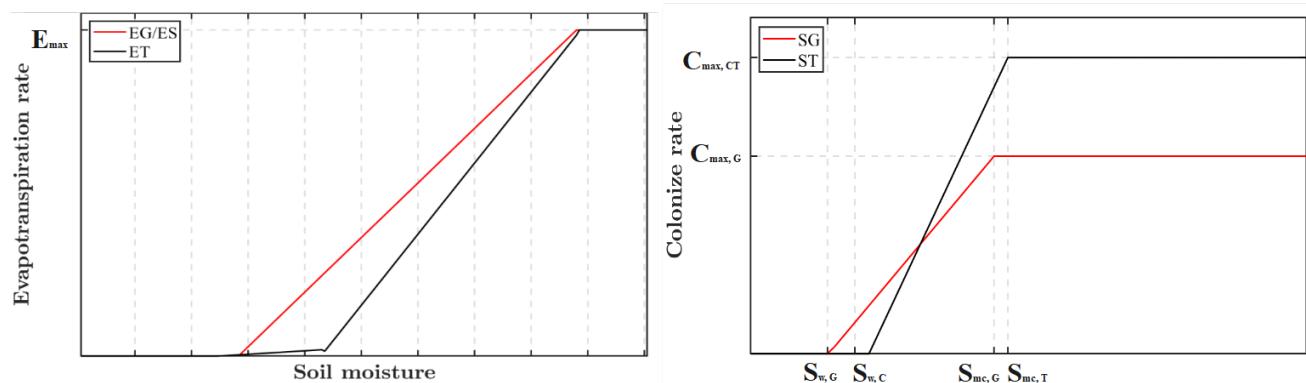


Figure 4:  $C_i$  and  $E_j$  with respect to soil moisture

Additionally, the third mechanism is leakage at the lowest boundary of the soil layer, which occurs when excess water drains out of the soil and enters groundwater or other water storage systems.

$$L(s) = K_s \cdot \frac{e^{\beta(s-s_{fc})} - 1}{e^{\beta(1-s_{fc})} - 1}, \quad \text{if } s_{fc} < s \leq 1. \quad (6)$$

Leakage term, which is defined as an increasing exponential function whose codomain is  $[s_{fc}, K_s]$ , where  $K_s$  is saturated hydraulic conductivity.

### 3.2 Data Pre-processing

In accordance with the requirements, Kenya is chosen as a typical example of a diverse climate area that encompasses both arid and semi-arid regions. The African Open Data repository provides open access to rainfall data [5], but the data is currently only updated to 2016, which is not sufficient.

To overcome this limitation, we employed the Bartlett-Lewis model to simulate precipitate, [6] which is designed to operate under the assumption that rainfall follows a Markov process, where the intensity of rainfall at any given time is dependent on the intensity at the previous time step. Despite the complexity of this model, we can easily resort to PyHSPF package in Python to generate synthetic monthly rainfall data that closely matches the actual weather patterns of the region, even for years leading up to 2020.

### 3.3 Parameter Estimation

The majority of the parameters incorporated in our model have been sourced from established scientific research, referred from Porporato (2004) [7], Baudena et al. (2010) [8], Jakoby (2014) [9]. These parameters boast a high level of accuracy and reliability and have demonstrated exceptional versatility, making them well-suited for a wide range of practical applications.

The colonization rate for grass, tree seedlings and adult trees is a crucial parameter, while there is no exact daily data available in the literature. To address this issue, we assumed that the colonization rate of grass is  $C_{max,G} = 2\text{yr}^{-1}$ . Furthermore, to determine the colonization rate of trees and their relationship with precipitation, we use the Monte Carlo method [10].

This approach involves generating 100,000 random groups of data that contain different combinations of colonization rates  $c_i(s)$  and precipitation  $R$ , allowing us to perform a comprehensive simulation that accurately reflects the real-world conditions of the system. Using this data, we can obtain the colonization rate of trees and the conditions under which they can coexist with grass.

To begin with, we establish a Type function to demonstrate the pattern of area (grass or trees).

$$\text{Type}(C_{max,T}, r) = \begin{cases} 1, & G = 0, \\ 0.5, & \text{otherwise,} \\ 0, & T = 0. \end{cases} \quad (7)$$

After that, we assign a set of initial values to our model, that is, grass cover  $G$ , tree seedling cover  $S$ , adult tree covers  $T$  and soil moisture rate  $s$  to be 0.2, 0, 3, 0.1, 0.1 respectively.

As is widely recognized, the delicate balance of a plant community can take years, even decades, to establish and stabilize. In order to explore this intricate process, we generate a series of monthly precipitation data for one year. By concatenating 30 copies of this time series data, that is, data of 30 years of precipitation, we are able to ensure that the grass and tree coverage in a particular area will remain stable. To put it in details: we set  $C_{max,T} \in [3, 7]$  and  $R \in [40, 1200]$ . Overall, the whole estimation process via our model could be conducted in Algorithm 1.

---

**Algorithm 1** Monte Carlo Simulation to Select the Crucial Parameters  $C_{max,T}$ 


---

**Input:** Model parameters listed in Table 22, Sample size ( $L=100,000$ ), Rain data within a year ( $R_B$ )

**Output:** Type  $(C_{max,T}, R)$  for  $R \in [40, 1200]$ ,  $C_{max,T} \in [3, 7]$

- 1: Use the rainfall data to run the  $B - L$  model.
  - 2: Generate monthly Rain data
  - 3: **for**  $i = 1$  to  $L$  **do**
  - 4:     Sample the value of  $C_{max,T}$ , from  $[3, 7]$ .
  - 5:     Sample the value of  $R$  from  $[40, 1200]$ .
  - 6:     Scaling monthly rainfall data to be suitable for the total amount of  $R_B$ , and extend the data sample  $L$  to a span of 30 years
  - 7:     Run the simulation for 30 years
  - 8:     Calculate Type  $(C_{max,T}, r)$
  - 9: **end for**
- 

The results are clearly shown in Figure 5 below, the boundary line divides the whole picture into three biome types, which are tree-only, grass-only and coexistence. Through simulation, it can be determined that there is a positive correlation between the level of precipitation and the success of trees in achieving dominance within a given ecosystem. Additionally, it has been observed that once annual precipitation levels exceed 600 mm, the symbiotic ratio between trees and grasses approaches a state of equilibrium.

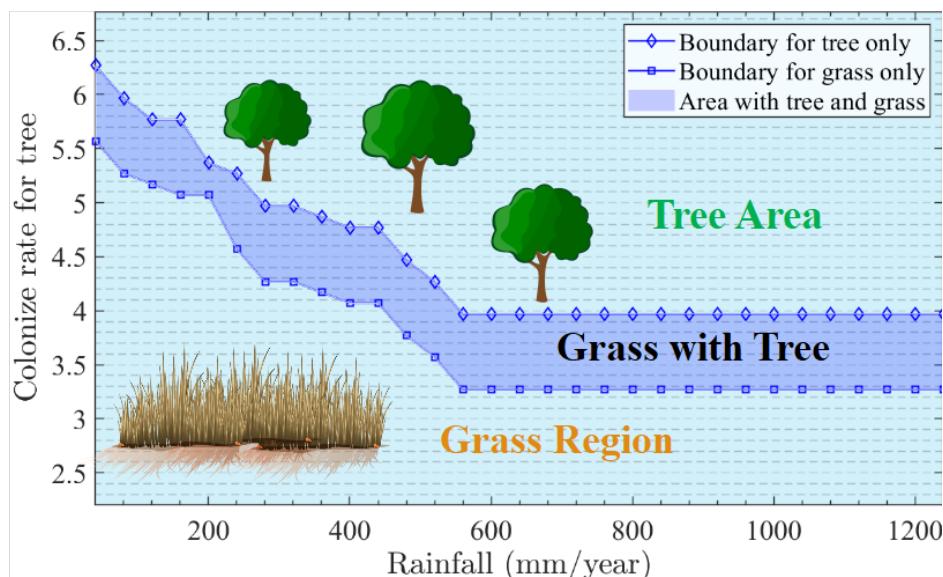


Figure 5: Monte Carlo simulation result of area type with different  $(C_{max,T}, r)$

One of the most notable strengths of our model is its excellent generalization ability. This means that, even with a wide range of different initial values, the model is able to produce highly consistent and reliable results that accurately reflect the underlying dynamics of the system.

Based on our analysis of the available data, we have determined that the average precipitation in the selected area is around  $400 - 600\text{ml}$ . Using this information, we set the value of  $C_{max,T}$  at  $5 \cdot yr^{-1}$  in future analysis.

### 3.4 Model Implementation in Irregular Weather Conditions

To simulate irregular weather conditions, we first utilized the left bar chart to show the historical precipitation data in Nairobi, with a typical tropical savannah climate. Based on this, we randomly decreased the precipitation in some of the months in wet seasons to represent drier conditions.

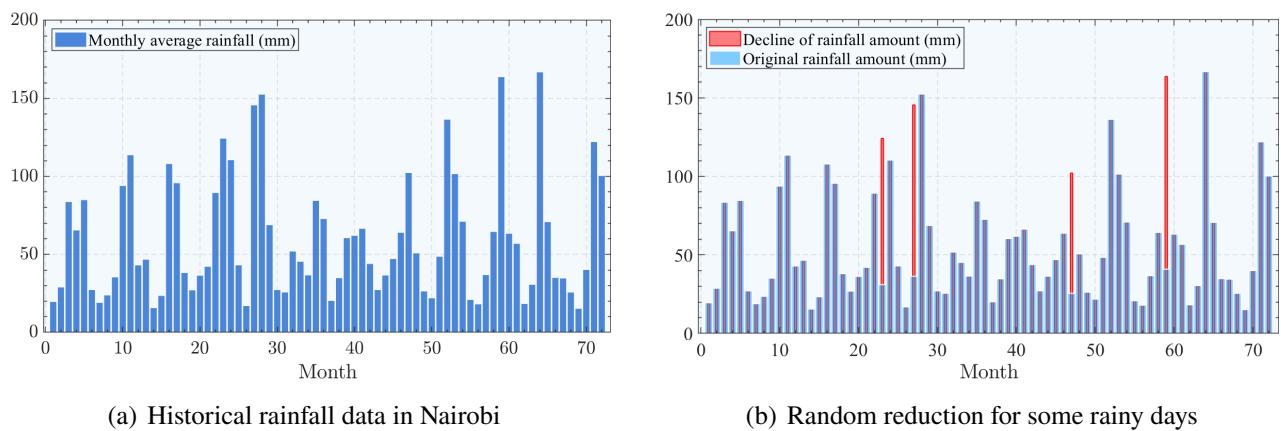


Figure 6: Precipitation data before and after disturbance, data from World Bank

By using B-L model to generate data for another four years again — a total of 10 years. In this case, we assume that the ecosystem contains only one species of grass and trees. After that, we use the Euler method to solve the model numerically. The results of the simulation through the ODEs 1, where the value of parameters are listed in Table 2 are shown in Figure 6:

As we can see, the coverage of grass increases significantly over time, as this hardy and adaptable plant species are able to take advantage of the available resources and establish a foothold in the ecosystem. At the same time, we observe a general increase in the number of trees present in the community, reflecting the important role that these towering plants play in shaping the landscape and supporting a wide range of other species. Besides, the number of seedlings remains relatively low, reflecting the delicate interplay between different species and the various factors that govern their growth and development.

Overall, despite facing extreme disturbance and a shortage of water, the grass-tree community has demonstrated a remarkable tendency towards balance, which serves as a powerful testament to the crucial importance of inter-species correlation. Through our rigorous analysis and Monte Carlo simulation done in Figure 7, we draw a similar conclusion: regardless of the amount of rainfall in a particular area, trees and grass can coexist in a delicate balance.

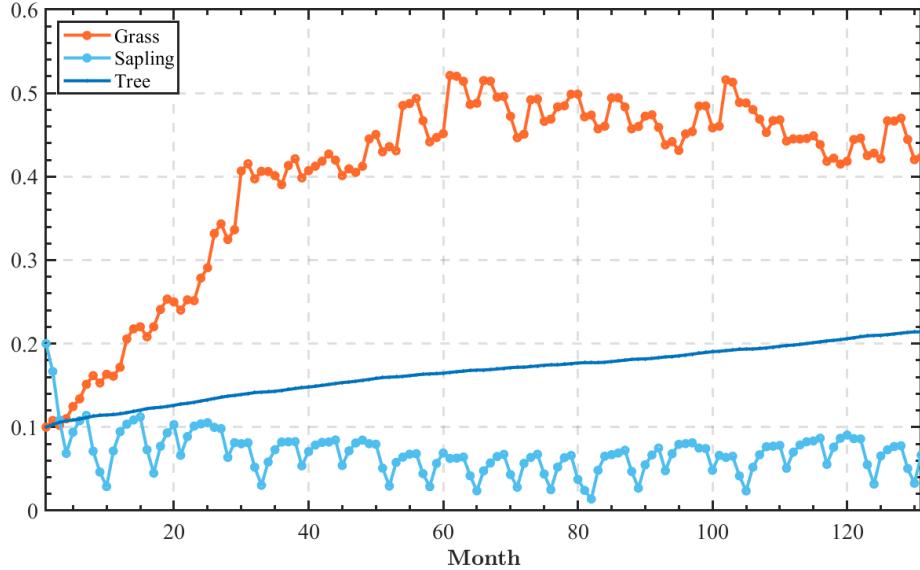


Figure 7: Simulation result of the one tree and one grass model

## 4 Analyze the Impact of Species Number

In this section, we focus on the development of several key indicators that enable us to gain valuable insights into how different species influence the plant community over time. The indicators include measures of species diversity and abundance, as well as resource utilization rates that capture the complex interplay between different plant species and their environment.

### 4.1 Assessment Criteria

In order to assess the biodiversity and space utilization of plants in a given area, we propose the following indicators which allow us to understand the various plant life present in the ecosystem, as well as how different species are distributed and interact with each other within the ecosystem:

- **Plant spatial occupancy** is to clarify the proportional relationship among trees, grass and saplings, here we define:

$$\sum_{i=1}^n T_i + \sum_{i=1}^n G_i + \sum_{i=1}^n S_i = S_p, \quad (8)$$

- **Species richness** is Shannon Entropy which is introduced to assess the species diversity of an area, which can be expressed mathematically as:

$$\widehat{H}_{total} = - \sum_{i=1}^S P_i \ln(P_i), \quad (9)$$

which is the Shannon diversity index as a measure of species diversity that takes into account both the number of species present in an ecosystem and the evenness of their abundance [11], where  $H$  is the Shannon diversity index,  $P_i$  is the proportion of the certain species.

## 4.2 Evaluation Process of Species Number

In the previous section, we developed a simplified model that assumes the ecosystem contains only one species of grass and trees, enabling us to focus on the interplay between these two critical components of the plant community. However, to gain a more comprehensive understanding of the dynamics at work within the system, we now turn our attention to a more nuanced analysis that sub-divides the different species of grass and trees in greater detail.

Through a careful review of related articles and research, we have identified a range of subtle but important differences between different plant species, including variations in reproductive characteristics and mortality rates. To capture these differences in our model, we have assumed that the colonization rate and mortality rate of trees and grass follow standard normal distributions [12].

The colonization rate of trees  $c_T(s)$  and grass  $c_G(s)$  follow a standard normal distribution, with the mean values 5 and 2, variance 1 and 0.4 separately. In addition, their mortality rate follows the standard normal distribution as well, the mean value of  $\tau_T$  and  $\tau_G$  are 0.02 and 0.5, and variances of 0.02 and 0.0008. By randomly selecting colonization and mortality rates from these distributions, followed by a Monte Carlo simulation, we are able to ensure the generalization ability and statistical significance of the model. Overall, this process is carefully outlined in Algorithm 2.

---

### Algorithm 2 Monte Carlo Simulation for $S_p, H_{total}$ calculation

---

**Input:** Model parameters listed in Table 2, Sample size L=100,000, tree species m, grass species n.

**Output:**  $\frac{\sum S_{p,i}}{L}, \frac{\sum H_{total,i}}{L}$

- 1: **for**  $i = 1$  to  $L$  **do**
  - 2:     Sample from  $[N(C_{max,G_j}, 0.4), N(u_{G_j}, 0.02)] \rightarrow G_j, j = 1, 2, \dots, m$ .
  - 3:     Sample from  $[N(C_{max,T_k}, 1), N(u_{T_k}, 0.0008)] \rightarrow T_k, k = 1, 2, \dots, n$ .
  - 4:     Run the simulation model based on the precipitation data shown in Section 3.4.
  - 5:     Calculate  $S_{p,i}, H_{total,i}$ .
  - 6: **end for**
  - 7: Calculate  $\frac{\sum S_{p,i}}{L}, \frac{\sum H_{total,i}}{L}$ .
- 

## 4.3 Application of the Evaluation Process

We define integer pair  $(Grass, Tree)$ , which represents types of grass and trees combination within the ecosystem. For each pair of grass-tree groups, we conduct Monte Carlo simulation shown in Algorithm 2 and calculate the  $S_p, H_{total}$  of those groups respectively. Using this approach, we have simulated a wide range of hypothetical plant communities, exploring the behavior of different combinations of grass and tree species over time.

The results of these Monte Carlo simulations are documented in Table 3. Particularly, one of the simulations among 10000 times that a community containing three types of grass and one type of tree is shown in Figure 8.

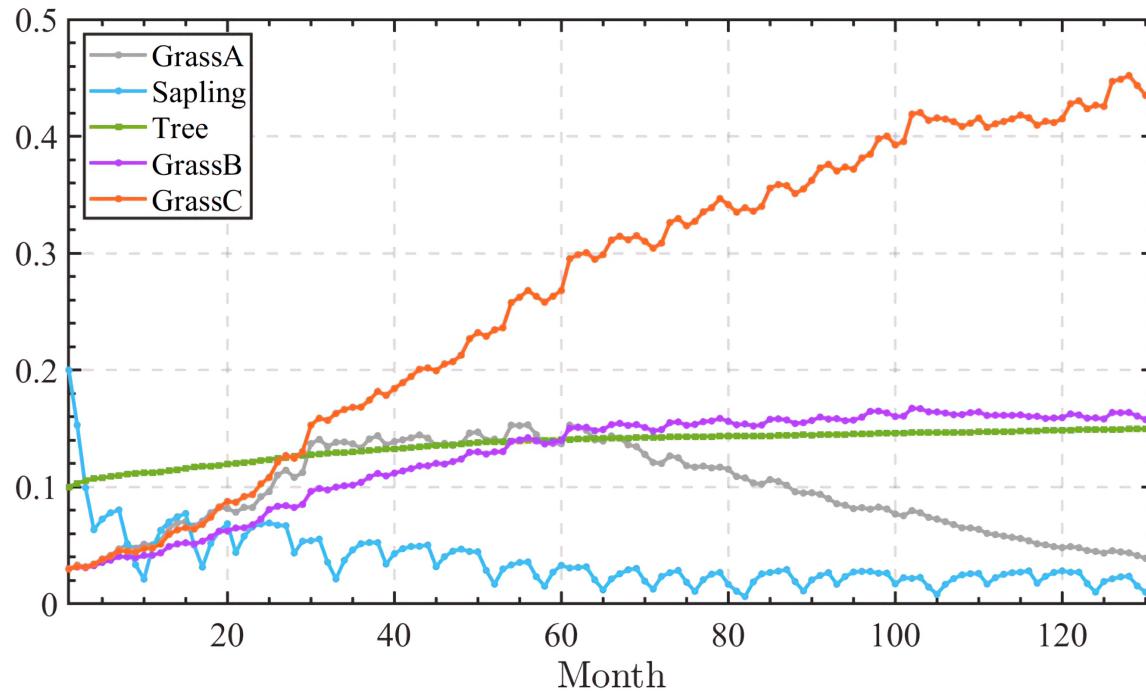


Figure 8: One of the situations among the Monte Carlo simulation with grass-tree pair (3,1)

Table 3: The experiment results of  $S_p$  and  $H_{total}$  in simulation

| $(Grass, Tree)$ | $S_p$          | $H_{total}$   |
|-----------------|----------------|---------------|
| (3, 1)          | 0.73557        | 1.1690        |
| (4, 1)          | 0.76473        | 1.1947        |
| (5, 1)          | 0.77862        | 1.2529        |
| (6, 2)          | <b>0.78109</b> | 1.2847        |
| (9, 3)          | 0.77134        | <b>1.2932</b> |

One key finding is that the value of  $S_p$  increases in direct proportion to the number of plant species present, up to a maximum of 0.78109 when the integer pair (Grass, Tree) equals (6,2) since the plant community is limited in terms of its ability to occupy space, due to constraints imposed by precipitation.

Moreover, we observe that the value of  $H_{total}$  generally increases as biodiversity increases, indicating that a greater diversity of plant species can have a positive impact on the health and resilience of the ecosystem. However, we also note that there are subtle differences between the communities represented by the integer pairs (6,2) and (9,3), suggesting that as the number of species increases, they may begin to compete more fiercely with one another for resources and space within the ecosystem.

Taking all of these factors into account, we conclude that the most beneficial community configuration in the context of precipitation levels between 400-600 ml is the integer pair (6,2), which is able to maximize the biodiversity and space utilization of the plant community, while still operating within the constraints imposed by the available resources and environmental conditions.

## 5 Sensitivity Analysis

In this section, we focus on the robustness and reliability of our model under the circumstance including worse or better weather conditions as well as other factors ranging from pollution to habitat reduction.

### 5.1 The Impact of Climate Change

To explore the behavior of our model under conditions of abundant precipitation, we select Ghana as a case study, where the yearly rainfall reaches a staggering 1000mm. By using data from this region, we are able to simulate a hypothetical plant community with an integer pair  $(Grass, Tree) = (6, 2)$ , using a process similar to the Monte Carlo method described earlier to discover the change in the  $S_p, H_{total}$  value from the original weather condition shown in Section 3 and 4.1.

Through careful analysis of the resulting data, we are able to draw several key conclusions, with one of the situations among the Monte Carlo simulation regarding the abundant precipitation illustrated in Figure 9 and Table 4.

Table 4: Results of  $S_p$  and  $H_{total}$  under Climate Change

| $(Grass, Tree) = (6, 2)$ | $S_p$   | $H_{total}$ |
|--------------------------|---------|-------------|
| Abundant precipitation   | 0.89953 | 1.2733      |
| Deficient precipitation  | 0.59387 | 1.0408      |
| Previous data            | 0.78109 | 1.2847      |

Specifically, we find that the values of  $S_p$  are significantly higher in this context, as compared to our previous simulations with more limited precipitation levels. This suggests that when conditions are favorable, the plant community is able to thrive and grow in a way that is highly beneficial for the environment and the broader ecosystem.

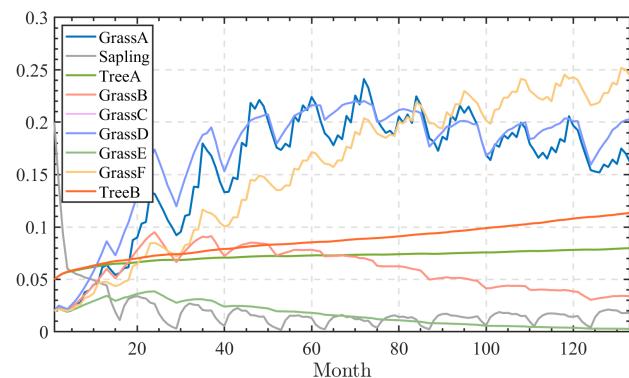
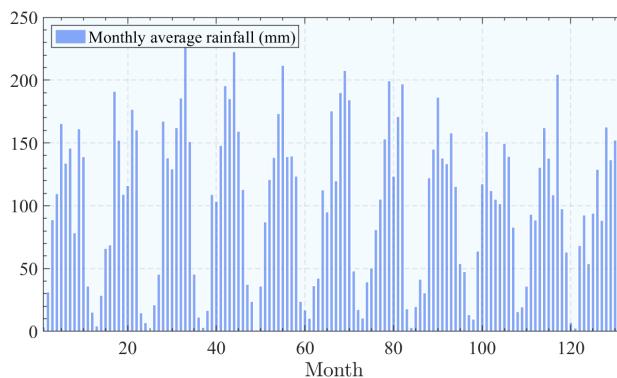


Figure 9: The abundant precipitation situation among the Monte Carlo simulation with grass-tree pair

In contrast to our previous simulations, we find that when precipitation levels are abundant, the competition between different types of plants becomes much more intense, leading to a greater level of overall instability and variability. Specifically, we observe that certain types of plants, which have

lower colonization rates or higher mortality rates, are unable to compete effectively with their more robust counterparts, and as a result, they are gradually replaced or die off over time.

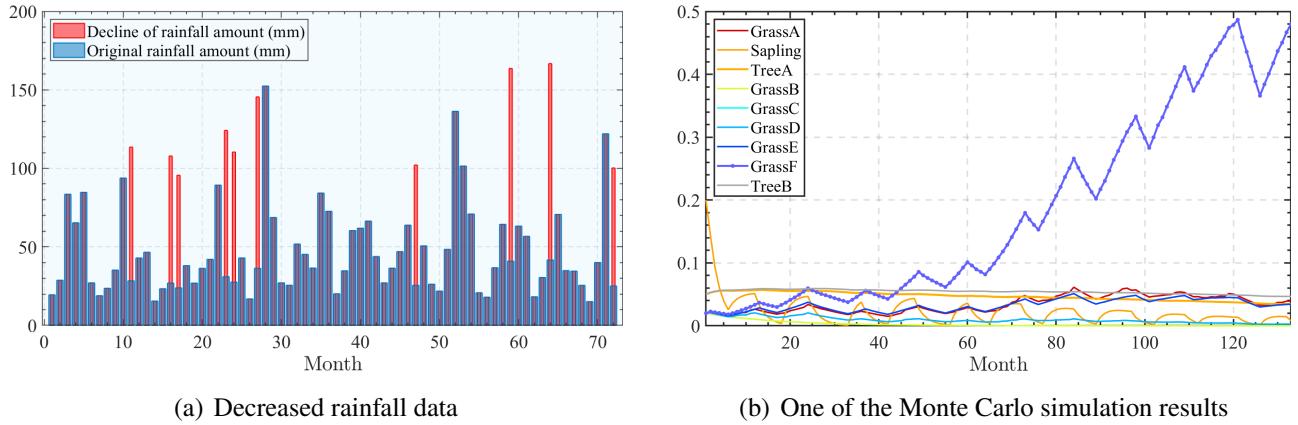


Figure 10: The decreased precipitation situation among the Monte Carlo simulation with grass-tree pair

To this end, we have simulated a hypothetical scenario in which precipitation levels are significantly lower, based on the data presented in Section 3.4. We cut down the precipitation to a greater extent to achieve the desired effect. Then we repeat the work above, the conclusion is shown in Figure 10:

We find that both  $S_p$  are significantly lower in this context, indicating that the plant community is struggling to survive and adapt in the face of scarce water resources. Moreover, we observe that the competition between different types of plants is even more intense under these conditions, with certain species struggling to survive and ultimately dying off. Under this circumstance, the advantage of growth higher is more obvious: in the early stage of the simulation, those grass species with a high colonization rate can quickly account for nearly half of the place, leaving nearly no space for other species to survive.

## 5.2 The Impact of Pollution and Habitat Reduction

Additionally, it is also important to take into account a wide range of other factors that can affect the health and stability of these ecosystems over time. Some of the most significant of these include environmental disruption, pollution, deforestation, wildfires, and other forms of human activity that can have a significant impact on local plant communities and their long-term health. We call these factors disruption in the following section.

We look up a range of relevant data and research findings, including reports on the declining forest coverage rates in Latin America and the Caribbean over the past decade, which is up to 10% of the total forest coverage, regarded as a staggering number. Hence, we introduce this circumstance to our model to discover the reaction of the plant community.

### 5.2.1 Establishment of Disruption by Poisson Distribution

To study the impact of environmental disruption on plant communities, we first make a fundamental assumption that the occurrences of disruption events follow a Poisson distribution [13]. The Poisson distribution is commonly used in probability theory to model the random occurrence of rare events

over time. To further investigate the effects of environmental disruption, we divide the total number of disruptions into several events, for instance, 100 times per year, each time will reduce 0.01% of the forest coverage, and distribute them across each month followed by a Poisson process.

Using the Poisson distribution whose  $\lambda = \frac{100}{12}$ , we can then create a probability distribution where the rate of disruption events in a certain month is proportional to the corresponding value of the Poisson distribution. After that implementation, we are able to discover the real-world simulation of a plant community encountering various disruptions. After that, we use Monte Carlo simulation to calculate the value of  $S_p$  and  $H_{total}$  after the disruptions. The whole process can be conducted in Algorithm 3.

---

**Algorithm 3** Sensitivity analysis among habitat disruptions
 

---

**Input:** Model parameters listed in Table 2, Sample size  $L=100,000$ ,  $\lambda = \frac{100}{12}$ .

**Output:**  $\frac{\sum S_{p,i}}{L}, \frac{\sum H_{total,i}}{L}$

- 1: **for**  $i = 1$  to  $L$  **do**
  - 2:   **for**  $t = 1$  to simulation length **do**
  - 3:     Generate a poison distribution  $N_{\lambda,t}$  with  $\lambda$ , size = 1 year.
  - 4:     Concatenating the above time series into one integral segment.
  - 5:   **end for**
  - 6:   Run the simulation model, for each month  $j$  in simulation,
  - 7:    $G \cdot [1 - 0.001 \cdot N_{\lambda}(j)] \rightarrow G$ ,
  - 8:    $T \cdot [1 - 0.0005 \cdot N_{\lambda}(j)] \rightarrow T$ ,
  - 9:   Calculate  $S_{p,i}, H_{total,i}$ .
  - 10: **end for**
  - 11: Calculate  $\frac{\sum S_{p,i}}{L}, \frac{\sum H_{total,i}}{L}$ .
- 

## 5.2.2 Simulation of Conditions under Disruptions

In addition to assuming that environmental disruption events follow a Poisson distribution, we also investigate the differential impact of disruption on grass and trees. Specifically, we assume that grass is more vulnerable to disruption compared to trees, and assign a disruption rate for grass that is twice that of trees. This assumption is based on empirical observations and prior knowledge about the relative resilience of these two types of plants. After the assumption, we conduct the Monte Carlo simulation based on Algorithm 3 and the results of our simulation as well as one of the situations in the Monte Carlo simulation are presented in Table 5 and Figure 11.

Table 5: Results of  $S_p$  and  $H_{total}$  under Disruptions

| $(Grass, Tree) = (6, 2)$ | $S_p$   | $H_{total}$ |
|--------------------------|---------|-------------|
| Disrupted                | 0.65669 | 1.0877      |
| Previous data            | 0.78109 | 1.2847      |

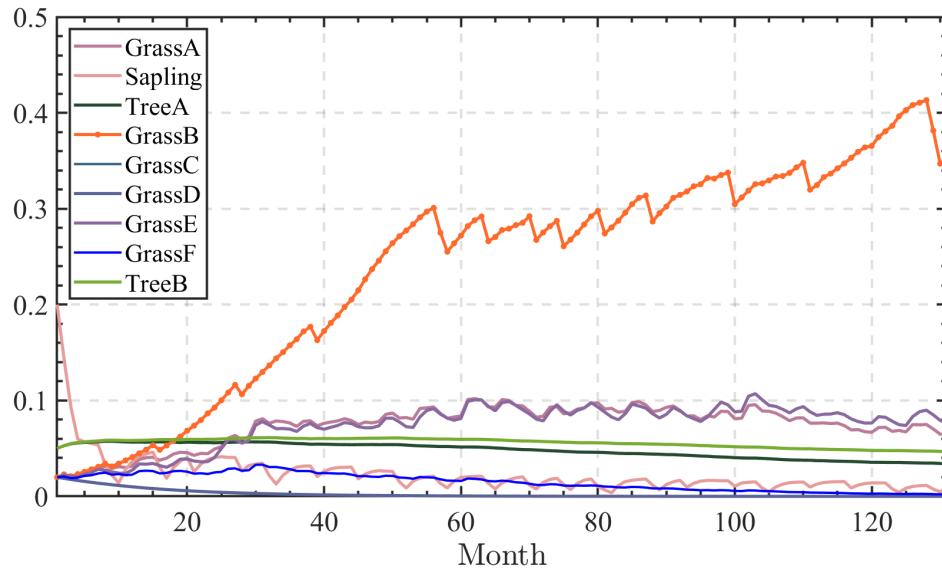


Figure 11: One of the situations from the Monte Carlo simulation regarding habitat disruption

Based on the analysis conducted in Section 5.2 and related results, we draw the conclusion that the impacts of environmental disruption on biodiversity can be significant. Our investigation reveals that while the effects on vegetation coverage were relatively minor, the disruption can have far-reaching consequences for the overall biodiversity of the ecosystem. Due to the disruption of habitat, most types of grass and trees can not thrive under that circumstance because their relatively low colonization rate is unable to support their colonization. Consequently, those types with relatively high colonization rates can easily take up considerable space, leading to a huge decline in biodiversity.

Given the potential impact of environmental disruption on biodiversity, it is essential to implement measures to protect and preserve habitats, and limit pollution. These efforts can help to mitigate the negative effects of disruption and maintain a healthy and diverse ecosystem. In addition to protecting the habitats, it is crucial to promote public awareness and education about the importance of biodiversity and the threats that it faces. This can help to build a strong and informed community that is committed to safeguarding the natural environment for future generations.

## 6 Possible Strategies for Long-term Viability

Our Tree-Sapling-Grass-Soil model provides a valuable window into the complex and dynamic interactions between soil, trees, and grasses in the real world. Through simulation, we have revealed critical insights into the factors that restrict biodiversity and threaten the long-term viability of plant communities. Specifically, our model demonstrates that habitat disruption, such as pollution and deforestation, can drive a considerable biodiversity loss.

Remarkably, our model has also shown that many plant species are capable of thriving in areas with relatively low precipitation, provided their habitats remain intact. This highlights the essential role of habitat conservation in maintaining plant community viability.

Therefore, our findings strongly emphasize the need to prioritize habitat conservation efforts since the extreme irregular weather may not occur in real situations. In particular, urgent action is needed to reduce the harmful effects of pollution and deforestation, which pose significant threats to plant communities and the biodiversity they support. By protecting and preserving habitats, we can help safeguard the future of these natural wonders for generations to come. Our detailed suggestions are listed below.

## **6.1 Suggestion for Ensuring the Long-term Viability of Bio-system**

Ensuring the long-term stability of tree populations is crucial to promoting healthy ecosystems and maintaining biodiversity. However, simply planting a few tree saplings after deforestation may not be sufficient to ensure long-term stability.

In fact, our understanding of the complex dynamics of forest ecosystems has shown that planting a much higher number of saplings is often necessary to account for the vulnerability of these young trees to environmental changes. A relatively low number of saplings may be unable to overcome the challenges of harsh environmental conditions or competition for resources, leading to a potential decline in the overall number of trees.

Therefore, it is imperative to recognize the importance of planting a substantial number of tree saplings after each instance of deforestation. By doing so, we can help ensure the long-term stability of tree populations and promote a healthy and sustainable environment. This approach not only benefits local ecosystems but also contributes to global efforts to combat climate change by increasing carbon sequestration and mitigating the impacts of deforestation.

## **6.2 Possible Impact on Larger Environment**

Our suggested approach to enhancing tree populations can have a significant impact on the larger environment. By promoting the planting and growth of trees, we can achieve a range of benefits that will have positive ripple effects throughout the ecosystem.

First and foremost, our approach will increase the overall water contain ability of the environment. Through our model, we have shown that trees could provide greater benefits than grass in terms of strengthening the soil's ability to capture water and reducing evaporation on bare soil. Although the average precipitation may not change significantly due to local geographical conditions, the enhanced ability of the soil to capture water can have a tremendous impact on water availability and sustainability. For example, the figure below vividly demonstrates the ability of trees to protect the stability of the whole environment.

Moreover, our approach can help promote overall environmental stability. As demonstrated in our model simulations, trees are less sensitive to habitat disruption, which means that as the number of trees in the environment increases, the overall resistance of the ecosystem will be greatly enhanced. This will, in turn, promote greater biodiversity and contribute to the overall health and sustainability of the environment.

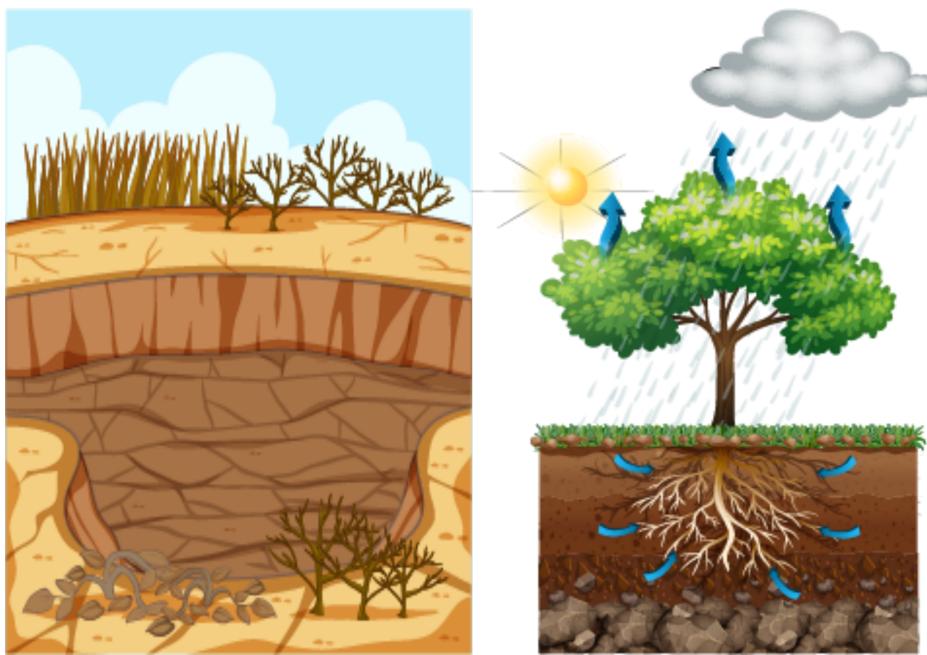


Figure 12: Trees have a strong capacity to protect water and soil resources in natural ecosystems.

In conclusion, our suggested approach to enhancing tree populations can have far-reaching positive effects on the larger environment. By promoting the growth of trees, we can increase water availability and sustainability, promote environmental stability, and contribute to the preservation of biodiversity. These benefits highlight the critical importance of prioritizing tree planting and growth efforts, both locally and globally, in our efforts to promote a healthy and sustainable environment for all.

## 7 Model Evaluation and Further Discussion

In this section, we take a critical reflection on our work and explore the potential for growth and improvement.

### 7.1 Strengths of our Model

- **Universal:** The model displays a high degree of generalization, as it is capable of producing convergent results even if rainfall data input with significant differences, which indicates that the model's predictive capacity is robust and can adapt to a wide range of input variables, resulting in a reliable and consistent output.
- **Highly Plausible:** We utilize an approach that does not explicitly define the competitive relationships between plants. Instead, we assume that the conflicts between them derived from infinite needs for limited water resources, and their protective effects on the soil can achieve mutual benefits. The results of our model are consistent with the harmonious coexistence of various plant species in natural ecosystems.
- **Rigorous and Scientific:** The accuracy of the results can be attributed to the use of Monte Carlo simulation and authoritative sources of information, which allows for a more robust and

reasonable selection of model parameters and provides a solid foundation for our methodology and enhances its reliability and validity.

## 7.2 Weakness of our Model

---

**Algorithm 4** SFF (Square-Free Factorization)

---

**Input:** A monic polynomial  $f$  in  $\mathbb{F}_q[x]$  where  $q = p^m$

**Output:** Square-free factorization of  $f$

```

1:  $R \leftarrow 1$ 
2:  $c \leftarrow \text{gcd}(f, f')$ 
3:  $w \leftarrow f/c$ 
4:  $i \leftarrow 1$ 
5: while  $w \neq 1$  do
6:    $y \leftarrow \text{gcd}(w, c)$ 
7:    $fac \leftarrow w/y$ 
8:    $R \leftarrow R \cdot fac^i$ 
9:    $w \leftarrow y$ 
10:   $c \leftarrow c/y$ 
11:   $i \leftarrow i + 1$ 
12: end while
13: if  $c \neq 1$  then
14:    $c \leftarrow c^{1/p}$ 
15:    $R \leftarrow R \cdot \text{SFF}(c)^p$ 
16: end if
17: return  $R$ 

```

---

- In the real world, climatic conditions are often more complex, and plant survival are also sensitive to temperature changes. This paper assumes that local temperature changes are mild, at least will not affect the survival of local vegetation.

## 7.3 Possible Improvement of the Model

Plants are an essential component of our ecosystem and play a critical role in maintaining the balance of nature. Over the long span of time, plants have evolved to adapt to changing environmental conditions, and some have developed the ability to resist drought.

Given the potential importance of these plants, it is worthwhile to consider whether they can help protect the entire plant community from environmental stressors.

Additionally, the plant communities studied in our paper provide important habitats for a wide range of animal species. These animal species, in turn, can play a key role in supporting the growth and development of plant communities. For example, herbivores can help disperse seeds and promote the colonization of new plant species, while predators can help control the population of herbivores and prevent overgrazing.

Future studies may wish to further explore the interrelationships between plants and animals in these communities, and consider the potential impacts of animal migration on the plant community.

## References

- [1] F. Isbell, D. Craven, J. Connolly, M. Loreau, B. Schmid, C. Beierkuhnlein, T. M. Bezemer, C. Bonin, H. Bruelheide, E. De Luca, et al., Biodiversity increases the resistance of ecosystem productivity to climate extremes, *Nature* 526 (7574) (2015) 574–577. doi:[10.1038/nature15374](https://doi.org/10.1038/nature15374).
- [2] Food and Agriculture Organization of the United Nations, Average annual precipitation, <https://www.who.int/emergencies/diseases-outbreak-news/item/2022-DON385> (2022).
- [3] D. d'Onofrio, M. Baudena, F. d'Andrea, M. Rietkerk, A. Provenzale, Tree-grass competition for soil water in arid and semiarid savannas: The role of rainfall intermittency, *Water Resources Research* 51 (1) (2015) 169–181. doi:[10.1002/2014WR015515](https://doi.org/10.1002/2014WR015515).
- [4] P. Huang, S.-P. Xie, K. Hu, G. Huang, R. Huang, Patterns of the seasonal response of tropical rainfall to global warming, *Nature Geoscience* 6 (5) (2013) 357–361. doi:[10.1038/ngeo1792](https://doi.org/10.1038/ngeo1792).
- [5] World Bank, Kenya climate data | 1991-2016, <https://climateknowledgeportal.worldbank.org/download-data> (2020).
- [6] C. Onof, H. S. Wheater, Modelling of british rainfall using a random parameter bartlett-lewis rectangular pulse model, *Journal of Hydrology* 149 (1) (1993) 67–95. doi:[10.1016/0022-1694\(93\)90100-N](https://doi.org/10.1016/0022-1694(93)90100-N).
- [7] I. R. Iturbe, A. Porporato, Ecohydrology of water-controlled ecosystems: soil moisture and plant dynamics (2004).
- [8] M. Baudena, F. D'Andrea, A. Provenzale, An idealized model for tree–grass coexistence in savannas: the role of life stage structure and fire disturbances, *Journal of Ecology* 98 (1) (2010) 74–80. doi:[10.1111/j.1365-2745.2009.01588.x](https://doi.org/10.1111/j.1365-2745.2009.01588.x).
- [9] O. Jakoby, V. Grimm, K. Frank, Pattern-oriented parameterization of general models for ecological application: Towards realistic evaluations of management approaches, *Ecological Modelling* 275 (2014) 78–88. doi:<https://doi.org/10.1016/j.ecolmodel.2013.12.009>.
- [10] I. Buvat, D. Lazaro, Monte Carlo simulations in emission tomography and GATE: An overview, *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* 569 (2) (2006) 323–329. doi:[10.1016/j.nima.2006.08.039](https://doi.org/10.1016/j.nima.2006.08.039).
- [11] M. Hejda, P. Pyšek, V. Jarošík, Impact of invasive plants on the species richness, diversity and composition of invaded communities, *Journal of ecology* 97 (3) (2009) 393–403. doi:[10.1111/j.1365-2745.2009.01480.x](https://doi.org/10.1111/j.1365-2745.2009.01480.x).
- [12] A. Kletter, J. von Hardenberg, Patterned vegetation and rainfall intermittency, *Journal of Theoretical Biology* 256 (4) (2009) 574–583. doi:[10.1016/j.jtbi.2008.10.020](https://doi.org/10.1016/j.jtbi.2008.10.020).
- [13] S. M. Edie, P. D. Smits, D. Jablonski, Probabilistic models of species discovery and biodiversity comparisons, *Proceedings of the National Academy of Sciences* 114 (14) (2017) 3666–3671. doi:[10.1073/pnas.1616355114](https://doi.org/10.1073/pnas.1616355114).