Subset Sum (medium)

We'll cover the following

Problem Statement

Given a set of positive numbers, determine if a subset exists whose sum is equal to a given number ‘S’.

Example 1:

```
Input: {1, 2, 3, 7}, S=6
Output: True
The given set has a subset whose sum is '6': {1, 2, 3}
```

Example 2:

```
Input: {1, 2, 7, 1, 5}, S=10
Output: True
The given set has a subset whose sum is '10': {1, 2, 7}
```

Example 3:

```
Input: {1, 3, 4, 8}, S=6
Output: False
The given set does not have any subset whose sum is equal to '6'.
```

Try it yourself

Try solving this question here:

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```java
class SubsetSum {

  public boolean canPartition(int[] num, int sum) {
    // TODO: Write your code here
    return false;
  }

  public static void main(String[] args) {
    SubsetSum ss = new SubsetSum();
    int[] num = { 1, 2, 3, 7 };
    System.out.println(ss.canPartition(num, 6));
    num = new int[] { 1, 2, 7, 1, 5 };
    System.out.println(ss.canPartition(num, 10));
    num = new int[] { 1, 3, 4, 8 };
    System.out.println(ss.canPartition(num, 6));
  }
}
```

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Basic Solution

This problem follows the **0/1 Knapsack pattern** and is quite similar to Equal Subset Sum Partition. A basic brute-force solution could be to try all subsets of the given numbers to see if any set has a sum equal to ‘S’.

So our brute-force algorithm will look like:

```
for each number 'i'
  create a new set which INCLUDES number 'i' if it does not exceed 'S', and recursively
      process the remaining numbers
  create a new set WITHOUT number 'i', and recursively process the remaining numbers
return true if any of the above two sets has a sum equal to 'S', otherwise return false
```

Since this problem is quite similar to Equal Subset Sum Partition, let's jump directly to the bottom-up dynamic programming solution.

Bottom-up Dynamic Programming

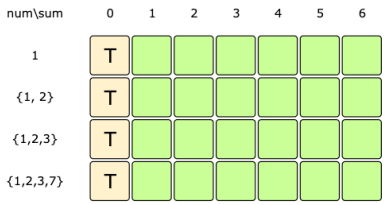
We'll try to find if we can make all possible sums with every subset to populate the array `dp[TotalNumbers][S+1]`.

For every possible sum 's' (where 0 <= s <= S), we have two options:

1. Exclude the number. In this case, we will see if we can get the sum 's' from the subset excluding this number => `dp[index-1][s]`
2. Include the number if its value is not more than 's'. In this case, we will see if we can find a subset to get the remaining sum => `dp[index-1][s-num[index]]`

If either of the above two scenarios returns true, we can find a subset with a sum equal to 's'.

Let's draw this visually, with the example input {1, 2, 3, 7}, and start with our base case of size zero:



'0' sum can always be found through an empty set

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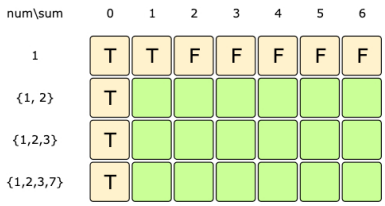

With only one number, we can form a subset only when the required sum is equal to that number

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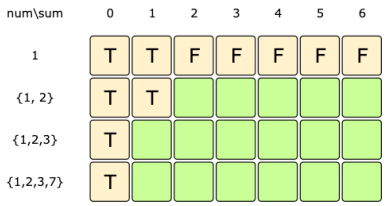

sum: 1, index:1=> (dp[index-1][sum] , as the 'sum' is less than the number at index '1' (i.e., 1 < 2)

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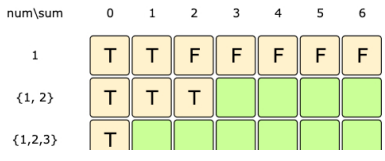

sum: 2, index:1=> (dp[index-1][sum] || dp[index-1][sum-2])

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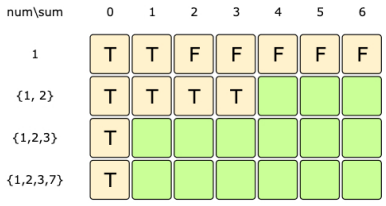

sum: 3, index:1=> (dp[index-1][sum] || dp[index-1][sum-2])

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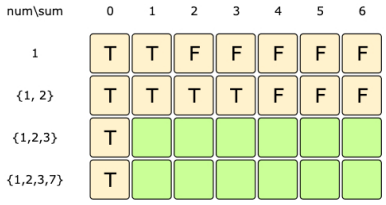

sum: 4-6 index:1=> (dp[index-1][sum] || dp[index-1][sum-2])

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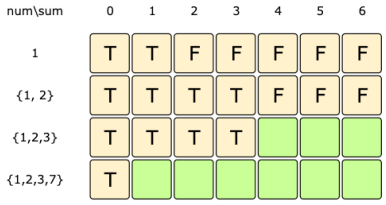

sum: 1,2,3, index:2=> (dp[index-1][sum] || dp[index-1][sum-3])

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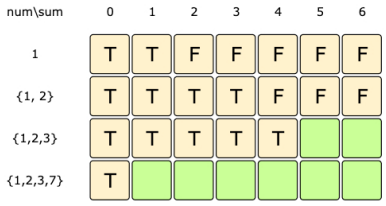

sum: 4, index:2=> (dp[index-1][sum] || dp[index-1][sum-3])

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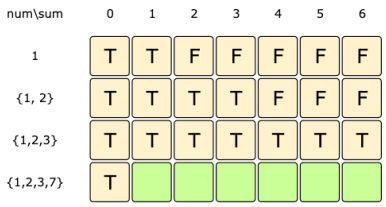

sum: 5, 6, index:2=> (dp[index-1][sum] || dp[index-1][sum-3])

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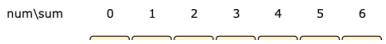

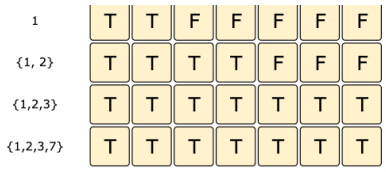

sum: 1-6, index:3=> (dp[index-1][sum] || dp[index-1][sum-7])

Code

Here is the code for our bottom-up dynamic programming approach:

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```java
class SubsetSum {

  public boolean canPartition(int[] num, int sum) {
    int n = num.length;
    boolean[][] dp = new boolean[n][sum + 1];

    // populate the sum=0 columns, as we can always form '0' sum with an empty set
    for (int i = 0; i < n; i++)
      dp[i][0] = true;

    // with only one number, we can form a subset only when the required sum is
    // equal to its value
    for (int s = 1; s <= sum; s++) {
      dp[0][s] = (num[0] == s ? true : false);
    }

    // process all subsets for all sums
    for (int i = 1; i < num.length; i++) {
      for (int s = 1; s <= sum; s++) {
        // if we can get the sum 's' without the number at index 'i'
        if (dp[i - 1][s]) {
          dp[i][s] = dp[i - 1][s];
        } else if (s >= num[i]) {
          // else include the number and see if we can find a subset to get the remaining
          // sum
          dp[i][s] = dp[i - 1][s - num[i]];
        }
      }
```

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Time and Space complexity

The above solution has the time and space complexity of $O(N * S)$, where 'N' represents total numbers and 'S' is the required sum.

Challenge

Can we improve our bottom-up DP solution even further? Can you find an algorithm that has $O(S)$ space complexity?

Hide Hint

Similar to the space optimized solution for 0/1 Knapsack

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```java
class SubsetSum {

  static boolean canPartition(int[] num, int sum) {
    //TODO: Write - Your - Code
    return false;
  }
}
```

Test | Hide Solution | Save | Reset

Solution

```java
class SubsetSum {
```

```
class SubsetSum {

  static boolean canPartition(int[] num, int sum) {
    int n = num.length;
    boolean[] dp = new boolean[sum + 1];

    // handle sum=0, as we can always have '0' sum with an empty set
    dp[0] = true;

    // with only one number, we can have a subset only when the required sum is equal to its value
    for (int s = 1; s <= sum; s++) {
      dp[s] = (num[0] == s ? true : false);
    }

    // process all subsets for all sums
    for (int i = 1; i < n; i++) {
      for (int s = sum; s >= 0; s--) {
        // if dp[s]==true, this means we can get the sum 's' without num[i], hence we can move on to
        // the next number else we can include num[i] and see if we can find a subset to get the
        // remaining sum
        if (!dp[s] && s >= num[i]) {
          dp[s] = dp[s - num[i]];
        }
      }
    }

    return dp[sum];
  }
```


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Equal Subset Sum Partition (medium)

Next

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