Equal Subset Sum Partition (medium)

We'll cover the following

Problem Statement

Given a set of positive numbers, find if we can partition it into two subsets such that the sum of elements in both subsets is equal.

Example 1:

```
Input: {1, 2, 3, 4}
Output: True
Explanation: The given set can be partitioned into two subsets with equal sum: {1, 4} & {2, 3}
```

Example 2:

```
Input: {1, 1, 3, 4, 7}
Output: True
Explanation: The given set can be partitioned into two subsets with equal su
m: {1, 3, 4} & {1, 7}
```

Example 3:

```
Input: {2, 3, 4, 6}
Output: False
Explanation: The given set cannot be partitioned into two subsets with equal sum.
```

Try it yourself

This problem looks similar to the 0/1 Knapsack problem. Try solving it before moving on to see the solution:

Java | Python3 | JS | C++

```python
def can_partition(num):
  # TODO: Write your code here
  return False

def main():
  print("Can partition: " + str(can_partition([1, 2, 3, 4])))
  print("Can partition: " + str(can_partition([1, 1, 3, 4, 7])))
  print("Can partition: " + str(can_partition([2, 3, 4, 6])))


main()

```

Run Save Reset

Basic Solution

This problem follows the **0/1 Knapsack pattern**. A basic brute-force solution could be to try all combinations of partitioning the given numbers into two sets to see if any pair of sets has an equal sum.

Assume that `S` represents the total sum of all the given numbers. Then the two equal subsets must have a sum equal to `S/2`. This essentially transforms our problem to: "Find a subset of the given numbers that has a total sum of `S/2`".

So our brute-force algorithm will look like:

```
for each number 'i'
  create a new set which INCLUDES number 'i' if it does not exceed 'S/2', and recursively
      process the remaining numbers
  create a new set WITHOUT number 'i', and recursively process the remaining items
return true if any of the above sets has a sum equal to 'S/2', otherwise return false
```

Code

Here is the code for the brute-force solution:

Java | Python3 | C++ | JS

```python
def can_partition(num):
  s = sum(num)
  # if 's' is a an odd number, we can't have two subsets with equal sum
  if s % 2 != 0:
    return False

  return can_partition_recursive(num, s / 2, 0)


def can_partition_recursive(num, sum, currentIndex):
  # base check
  if sum == 0:
    return True

  n = len(num)
  if n == 0 or currentIndex >= n:
    return False

  # recursive call after choosing the number at the `currentIndex`
  # if the number at `currentIndex` exceeds the sum, we shouldn't process this
  if num[currentIndex] <= sum:
    if(can_partition_recursive(num, sum - num[currentIndex], currentIndex + 1)):
      return True

  # recursive call after excluding the number at the 'currentIndex'
  return can_partition_recursive(num, sum, currentIndex + 1)


def main():
  print("Can partition: " + str(can_partition([1, 2, 3, 4])))
  print("Can partition: " + str(can_partition([1, 1, 3, 4, 7])))
  print("Can partition: " + str(can_partition([2, 3, 4, 6])))


main()
```

Run Save Reset

Time and Space complexity

The time complexity of the above algorithm is exponential $O(2^n)$, where 'n' represents the total number. The space complexity is $O(n)$, which will be used to store the recursion stack.

Top-down Dynamic Programming with Memoization

We can use memoization to overcome the overlapping sub-problems. As stated in previous lessons, memoization is when we store the results of all the previously solved sub-problems so we can return the results from memory if we encounter a problem that has already been solved.

Since we need to store the results for every subset and for every possible sum, therefore we will be using a two-dimensional array to store the results of the solved sub-problems. The first dimension of the array will represent different subsets and the second dimension will represent different 'sums' that we can calculate from each subset. These two dimensions of the array can also be inferred from the two changing values (sum and currentIndex) in our recursive function `canPartitionRecursive()`.

Code

Here is the code for Top-down DP with memoization:

Java | Python3 | C++ | JS

```python
def can_partition(num):
  s = sum(num)

  # if 's' is a an odd number, we can't have two subsets with equal sum
  if s % 2 != 0:
    return False

  # initialize the 'dp' array, -1 for default, 1 for true and 0 for false
  dp = [[-1 for x in range(int(s/2)+1)] for y in range(len(num))]
  return True if can_partition_recursive(dp, num, int(s / 2), 0) == 1 else False


def can_partition_recursive(dp, num, sum, currentIndex):
  # base check
  if sum == 0:
    return 1

  n = len(num)
  if n == 0 or currentIndex >= n:
    return 0

  # if we have not already processed a similar problem
  if dp[currentIndex][sum] == -1:
    # recursive call after choosing the number at the currentIndex
    # if the number at currentIndex exceeds the sum, we shouldn't process this
    if num[currentIndex] <= sum:
      if can_partition_recursive(dp, num, sum - num[currentIndex], currentIndex + 1) == 1:
        dp[currentIndex][sum] = 1
        return 1

    # recursive call after excluding the number at the currentIndex
    dp[currentIndex][sum] = can_partition_recursive(
      dp, num, sum, currentIndex + 1)

  return dp[currentIndex][sum]


def main():
  print("Can partition: " + str(can_partition([1, 2, 3, 4])))
  print("Can partition: " + str(can_partition([1, 1, 3, 4, 7])))
  print("Can partition: " + str(can_partition([2, 3, 4, 6])))


main()
```

Run | Save | Reset

Time and Space complexity

The above algorithm has the time and space complexity of $O(N * S)$, where 'N' represents total numbers and 'S' is the total sum of all the numbers.

Bottom-up Dynamic Programming

Let's try to populate our `dp[][]` array from the above solution by working in a bottom-up fashion. Essentially, we want to find if we can make all possible sums with every subset. **This means, `dp[i][s]` will**

be ‘true’ if we can make the sum ‘s’ from the first ‘i’ numbers.

So, for each number at index ‘i’ (0 <= i < num.length) and sum ‘s’ (0 <= s <= S/2), we have two options:

1. Exclude the number. In this case, we will see if we can get ‘s’ from the subset excluding this number: `dp[i-1][s]`
2. Include the number if its value is not more than ‘s’. In this case, we will see if we can find a subset to get the remaining sum: `dp[i-1][s-num[i]]`

If either of the two above scenarios is true, we can find a subset of numbers with a sum equal to ‘s’.

Let’s start with our base case of zero capacity:

num\sum	0	1	2	3	4	5
1	T					
{1, 2}	T					
{1,2,3}	T					
{1,2,3,4}	T					

'0' sum can always be found through an empty set

1 of 10

num\sum	0	1	2	3	4	5
1	T	T	F	F	F	F
{1, 2}	T					
{1,2,3}	T					
{1,2,3,4}	T					

With only one number, we can form a subset only when the required sum is equal to its value

2 of 10

num\sum	0	1	2	3	4	5
1	T	T	F	F	F	F
{1, 2}	T	T				
{1,2,3}	T					
{1,2,3,4}	T					

sum: 1, index:1=> (dp[index-1][sum] , as the 'sum' is less than the number at index '1'

3 of 10

num\sum	0	1	2	3	4	5
1	T	T	F	F	F	F
{1, 2}	T	T	T			
{1,2,3}	T					
{1,2,3,4}	T					

sum: 2, index:1=> (dp[index-1][sum] || dp[index-1][sum-2])

4 of 10

num\sum	0	1	2	3	4	5
1	T	T	F	F	F	F
{1, 2}	T	T	T	T		
{1,2,3}	T					
{1,2,3,4}	T					

sum: 3, index:1=> (dp[index-1][sum] || dp[index-1][sum-2])

5 of 10

num\sum	0	1	2	3	4	5
1	T	T	F	F	F	F
{1, 2}	T	T	T	T	F	F
{1,2,3}	T					
{1,2,3,4}	T					

sum: 4,5, index:1=> (dp[index-1][sum] || dp[index-1][sum-2])

6 of 10

num\sum	0	1	2	3	4	5
1	T	T	F	F	F	F
{1, 2}	T	T	T	T	F	F
{1,2,3}	T	T	T	T		
{1,2,3,4}	T					

sum: 1,2,3, index:2=> (dp[index-1][sum] || dp[index-1][sum-3])

7 of 10

num\sum	0	1	2	3	4	5
1	T	T	F	F	F	F
{1, 2}	T	T	T	T	F	F
{1,2,3}	T	T	T	T	T	
{1,2,3,4}	T					

sum: 4, index:2=> (dp[index-1][sum] || dp[index-1][sum-3])

8 of 10

num\sum	0	1	2	3	4	5
1	T	T	F	F	F	F
{1, 2}	T	T	T	T	F	F
{1,2,3}	T	T	T	T	T	T

{1,2,3,4}	T					

sum: 5, index:2=> (dp[index-1][sum] || dp[index-1][sum-3])

9 of 10

num\sum	0	1	2	3	4	5
1	T	T	F	F	F	F
{1, 2}	T	T	T	T	F	F
{1,2,3}	T	T	T	T	T	T
{1,2,3,4}	T	T	T	T	T	T

sum: 1-5, index:3=> (dp[index-1][sum] || dp[index-1][sum-4])

10 of 10

From the above visualization, we can clearly see that it is possible to partition the given set into two subsets with equal sums, as shown by bottom-right cell: `dp[3][5] => T`

Code

Here is the code for our bottom-up dynamic programming approach:

Java | Python3 | C++ | JS

```python
def can_partition(num):
  s = sum(num)

  # if 's' is a an odd number, we can't have two subsets with same total
  if s % 2 != 0:
    return False

  # we are trying to find a subset of given numbers that has a total sum of 's/2'.
  s = int(s / 2)

  n = len(num)
  dp = [[False for x in range(s+1)] for y in range(n)]

  # populate the s=0 columns, as we can always for '0' sum with an empty set
  for i in range(0, n):
    dp[i][0] = True

  # with only one number, we can form a subset only when the required sum is
  # equal to its value
  for j in range(1, s+1):
    dp[0][j] = num[0] == j

  # process all subsets for all sums
  for i in range(1, n):
    for j in range(1, s+1):
      # if we can get the sum 'j' without the number at index 'i'
      if dp[i - 1][j]:
        dp[i][j] = dp[i - 1][j]
      elif j >= num[i]:  # else if we can find a subset to get the remaining sum
        dp[i][j] = dp[i - 1][j - num[i]]

  # the bottom-right corner will have our answer.
  return dp[n - 1][s]


def main():
  print("Can partition: " + str(can_partition([1, 2, 3, 4])))
  print("Can partition: " + str(can_partition([1, 1, 3, 4, 7])))
  print("Can partition: " + str(can_partition([2, 3, 4, 6])))


main()
```

Run Save Reset

Time and Space complexity

The above solution the has time and space complexity of $O(N * S)$, where 'N' represents total numbers and 'S' is the total sum of all the numbers.