Noise in the Brumley-Carrara model of chemotaxis

$$\sigma = \sqrt{\frac{3C}{\pi a D_c T^3}}$$

The quantities have units $[D_c] = \mu m^2/s$, $[T] = s$, $[a] = \mu m$, $[C] = \mu M$; and we must have $[\sigma] = \mu M/s$ since it is noise on a temporal gradient measurement

$[aD_cT^3] = \mu m^3 s^2$ hence $[\sigma] = \frac{1}{s}\sqrt{\mu M/\mu m^3}$
The quantity under the square root has obviously the dimensions of a concentration but requires a prefactor arising from the conversion between μM and $1/\mu m^3$.

$$1\mu M = 10^{-6} mol/L = 10^{-21} mol/\mu m^3 = 10^{-21} N_A \, \mu m^{-3} \approx 602 \mu m^{-3}$$

hence

$$1\mu m^{-3} \approx 1.66 \times 10^{-3} \mu M$$

So we have

$$\sigma = \sqrt{\frac{3C \; [\mu M]}{\pi a D_c T^3 \; [\mu m^3 s^2]}} = \sqrt{\frac{3C}{\pi a D_c T^3}} \sqrt{1.66 \times 10^{-3}} \; [\mu M/s] \approx 0.041 \sqrt{\frac{3C}{\pi a D_c T^3}} \; [\mu M/s]$$

In other words, the value from the "naive" calculation has to be multiplied by a factor 0.041.
This is a **factor ~24** reduction in noise amplitude which might explain (at least in part) the *extreme* dependency to background signal that I observed in previous simulations. With the "uncorrected" σ, a chemotactic precision $\Pi = 1$ (the theoretical minimum) was equivalent to a $\Pi \approx 24$ for the corrected σ.