Vortex Flow: Numerical Simulation of the Taylor–Green Vortex

Masud

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Outline

Vortex Flow

Vortex flows represent rotational motion of fluid elements, fundamental to turbulence, mixing, and aerodynamics.

The **Taylor–Green vortex** is a canonical test problem for CFD:

$$
\begin{aligned}
u(x, y, 0) &= U_0 \sin(kx) \cos(ky), \\
v(x, y, 0) &= -U_0 \cos(kx) \sin(ky), \\
p(x, y, 0) &= \frac{\rho U_0^2}{4} \left[\cos(2kx) + \cos(2ky) \right].
\end{aligned}
$$

- Incompressible, viscous, two-dimensional flow
- Initially periodic velocity field
- Decays over time due to viscosity

Navier–Stokes Equations

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \nu \nabla^2 \mathbf{u},$$
$$\nabla \cdot \mathbf{u} = 0.$$

- Viscosity ν controls vortex decay rate
- Periodic boundary conditions in both directions

Analytical Solution (Decay of Vortex)

The velocity field decays exponentially due to viscosity:

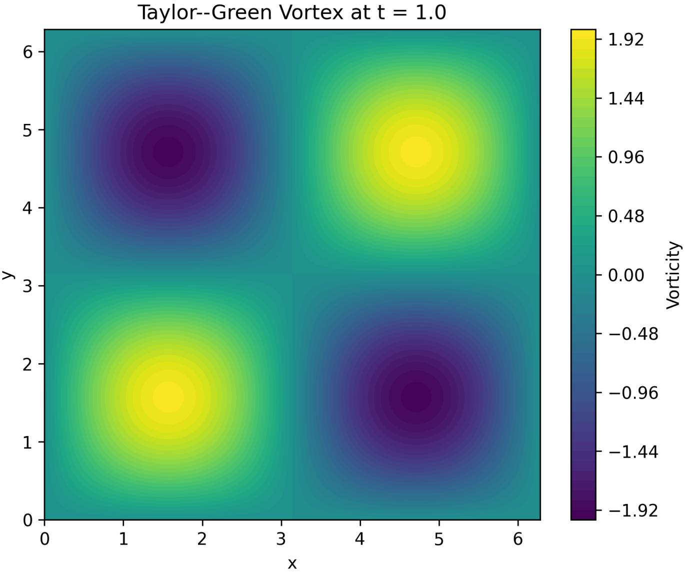
$$u(x, y, t) = U_0 \sin(kx) \cos(ky)\, e^{-2\nu k^2 t},$$
$$v(x, y, t) = -U_0 \cos(kx) \sin(ky)\, e^{-2\nu k^2 t}.$$

- The structure remains identical over time
- Amplitude decreases with $e^{-2\nu k^2 t}$

Discretization and Simulation

- Spatial discretization: central differences for derivatives
- Time integration: explicit schemes (RK2, RK4)
- Pressure projection step enforces $\nabla \cdot \mathbf{u} = 0$
- Periodic boundary conditions applied in both x and y

Velocity Field



Observations

- Energy decays exponentially with viscosity
- Flow remains symmetric and periodic
- Excellent benchmark for assessing numerical dissipation and accuracy

Conclusion

- The Taylor–Green vortex demonstrates fundamental vortex dynamics
- Used extensively for CFD solver validation
- Provides analytical reference for numerical error analysis