Adam-Bashforth Method: Explanation and Worked Example

Ibrahim Hossen Masud

October 1, 2025

Abstract

This note presents the Adam-Bashforth 2-step method for solving ordinary differential equations (ODEs). It includes derivation, a step-by-step worked example, and discussion on accuracy, advantages, and limitations.

1 Introduction

Adam-Bashforth methods are explicit linear multistep methods used to approximate solutions of ODEs:

$$\frac{dy}{dt} = f(t, y), \quad y(t_0) = y_0$$

The 2-step Adam-Bashforth method uses information from the previous two points to predict the next value.

2 Derivation of 2-Step Adam-Bashforth Method

Using polynomial interpolation for $f(t, y)$ over two previous points (t_n, f_n) and (t_{n-1}, f_{n-1}), the update formula is:

$$y_{n+1} = y_n + \frac{h}{2}\big(3f_n - f_{n-1}\big)$$

where:

- h is the step size
- $f_n = f(t_n, y_n)$
- $f_{n-1} = f(t_{n-1}, y_{n-1})$

This formula gives a second-order accurate approximation, improving on Euler's first-order method.

3 Worked Example

Consider the differential equation:

$$\frac{dy}{dt} = y - t^2 + 1, \quad y(0) = 0.5$$

We first need a starting value from another method (e.g., Euler) for y_1. Assume:

$$y_0 = 0.5, \quad y_1 = 0.806 \quad \text{(from Euler method with } h = 0.2)$$

Compute y_2 using Adam-Bashforth 2-step:

$$
\begin{aligned}
f_1 &= f(t_1, y_1) = y_1 - t_1^2 + 1 = 0.806 - 0.2^2 + 1 = 1.766 \\
f_0 &= f(t_0, y_0) = 0.5 - 0^2 + 1 = 1.5 \\
y_2 &= y_1 + \frac{0.2}{2}(3f_1 - f_0) = 0.806 + 0.1(3 \cdot 1.766 - 1.5) \\
&= 0.806 + 0.1(5.298 - 1.5) = 0.806 + 0.3798 \approx 1.1858
\end{aligned}
$$

Similarly, $y_3, y_4, \ldots$ can be computed using the previous two points.

4 Discussion on Accuracy and Advantages

- Adam-Bashforth 2-step method is second-order accurate, better than Euler's method.
- Being explicit, it is easy to implement, but stability depends on step size and problem type.
- Requires a starting procedure (like Euler or RK4) to generate the first point.
- Can be extended to 3-step, 4-step, etc., for higher-order accuracy.
- Particularly useful for problems where previous function evaluations are already available.

5 Conclusion

The Adam-Bashforth 2-step method is a simple yet effective multistep method for solving ODEs. It demonstrates how using information from multiple previous steps can improve accuracy and efficiency over single-step methods.