Reflection and Refraction

Q. What do you mean by Reflection and Refraction?
→ Reflection and Refraction are two important optical phenomena that occur when light travels through different media.

Reflection: Reflection refers to the bouncing back of light when it strikes a surface such as mirror. When light strikes a smooth surface, it is reflected in a specific direction and the angle of incidence (the angle ~~bee~~ between the incoming light and the surface) is equal to the angle of reflection. (The angle between the reflected light and surface)

Refraction: Refraction occurs when light travels from one medium to another, such as from air to water. When light travels through a medium with a different refractive index it's speed and direction change.

Both reflection and refraction are important in various fields, including optics, physics and engineering. They have many practical applications such as in the design of mirrors, lenses etc.

Q. Electromagnetic wave: An electromagnetic wave is a type of wave that is created by the oscillation of electric and magnetic fields. These waves are ~~as~~ made up of oscillating ~~electron~~ electric and magnetic ~~field~~ fields that are ~~perf~~ perpendicular to each other. Electromagnetic ~~field~~ waves can travel through a vaccum.

Snell's law: Snell's law is a formula used to describe the relationship between the angle of ~~p~~ incidence and refraction. It states that the ratio of the sine of the angle of incidance to the sine of the angle of refraction is equal to a constant known as refractive index of the medium.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1} = \mu$$

Here, θ_1 = The angle of incidence

θ_2 = The angle of refraction

n_1 = Incident index

n_2 = Reflected index.

Critical angle: From the snell's law, it states that the ratio of the sine of ~~an~~ the angle of incidence to the sine of the angle of refraction is equal to the constant known as the refractive index of the medium.

$$n_1 \sin\theta_i = n_2 \sin\theta_r$$

$$\therefore \frac{\sin\theta_i}{\sin\theta_r} = \frac{n_2}{n_1} = \mu$$

Here, θ_i = The angle of incidance

θ_r = The angle of refraction

n_1 = Incident index

n_2 = Refracted index.

The critical angle is the angle of incidence at whi a light passing through a boundary between two different media is refracted at an angle of 90° d

The critical angle is determined by the refractive indices of the two media and it can be calculated using snell's law:

$$n_1 \sin\theta_i = n_2 \sin\theta_r$$
$$\Rightarrow n_1 \sin\theta_i = n_2 \sin 90^\circ \quad [\theta_r = 90^\circ]$$
$$\Rightarrow \sin\theta_i = \frac{n_2}{n_1}$$
$$\Rightarrow \sin\theta_{ic} = \frac{n_2}{n_1}$$
$$\therefore \theta_{ic} = \sin^{-1}(n_2/n_1)$$

when θ_{ic} the critical angle.

<u>Brewster angle:</u> Brewster angle is an angle of incidence at which light with a particular polarization is perfectly transmitted through a surface with no reflection, so the special angle of incidence that produces a 90° angle between the reflected and refracted ray, known as brewster angle.

$$\theta_{iB} = \tan^{-1}(n_2/n_1)$$
$$= \cot^{-1}(n_1/n_2)$$

<u>Depth of penetration:</u> The depth of penetration also known as the skin depth is a measure of the distance that an electromagnetic wave can travel into a material before its amplitude is reduced to 1/e of its original value, it is denoted as δ.

Now, $E_x = E_0 e^{-\alpha z}$

if ~~$\alpha z = 1$~~ if $\alpha z = 1$

then, $\frac{E_x}{E_0} = \frac{1}{e}$

on traversing distance, $z = \frac{1}{\alpha}$

The amplitude of the wave fall to $\frac{1}{e}$ times its value at $z = 0$.

By definition, such a distance is equal to ~~the~~ depth of penetration δ, so that for good conductor,

$$\cancel{\delta = \frac{1}{\alpha}} \quad \delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\frac{\omega\mu\sigma}{2}}}$$

$$= \sqrt{\frac{2}{\omega\mu\sigma}}$$

<u>Skin effect:</u> Skin ~~is~~ effect is a tendancy for alternating current (AC) to flow mostly near the outer surface of an electrical conductor such as metal wire, Skin effect increases with the increases ~~with~~ in frequency. At low frequency, there is a small increases in the current density near the surface of the conductor, But a high frequencies, Such a radio frequency, the whole of the currents flows on ~~a~~ the surface of the conductor.

Phase velocity and group velocity: The velocity of individual wave is termed as phase velocity.

Phase velocity, $V_p = \frac{\omega}{k}$

where, $k = |k|$ is the magnetude of propagation vector wave number.

ω = Angular frequency

When a region consists of two or more wave trains then the physical velocity of propagation of waves is termed as group velocity. Group velocity: $V_g = \frac{\partial \omega}{\partial k}$

Relation between phase velocity and group velocity:

The phase velocity of a wave, $V_p = \frac{\omega}{k}$ ——— (i)

The group velocity, $V_g = \frac{d\omega}{dk}$

where, ω = Angular frequency

k = Wave number

From equation (i), $V_p = \frac{\omega}{k}$

$\therefore \omega = kV_p$ ——— (ii)

Differentiating (ii) w.r.t. k,

$$\frac{d\omega}{dk} = \frac{d}{dk}(kV_p)$$

$$= V_p + k\frac{dV_p}{dk}$$

$$\therefore V_g = V_p + k\frac{dV_p}{dk} \quad \left[\because V_g = \frac{d\omega}{dk}\right]$$

Show that the phase velocity and group velocity are equal.

We know, $V_p = \frac{V}{\sin\theta_i}$

Again, $V = \frac{1}{\sqrt{\mu\epsilon}}$ and $\theta_i = 90°$

Then, $V_p = \frac{1}{\sqrt{\mu\epsilon}}$

$\frac{\omega}{k} = \frac{1}{\sqrt{\mu\epsilon}}$, $\therefore k = \omega\sqrt{\mu\epsilon}$

$\Rightarrow dk = d\omega\sqrt{\mu\epsilon}$

$\Rightarrow \frac{1}{\sqrt{\mu\epsilon}} = \frac{d\omega}{dk}$

$\therefore V_p = V_g$

<u>Fresnel's Equations</u>: The fresnel equations describe reflection and transmission of light when it is incident an interface between two different mediums. The fresn equation also known as fresnel co-efficients are defin as the ratio of the electric field of a reflected an transmitted wave to the electric field of the incident

We know that E and H vectors in plane electro- wave are always ~~penpet~~ perpendicular to the dire of propagation and to each other. The vector E of ~~indi~~ incident wave can be oriented in any directi perpendicular to vector n_i.

It is convenient to consider two cases:

(i) In which the incident wave is polarized such that its vector E is normal to the plane of incidence.

(ii) In which the vector E is parallel to the plane of incidence.

(A) Incident wave polarized with its vector E normal to the plane of incidence:

In this case the electronic field ~~circuit~~ and magnetic field vectors E and H of the incident wave are perpendicular to the direction of propagation k_i. The pictorial diagram of this case shown in figure. Since the media are isotropic, the electric field vectors of the reflected and transmitted wave will also be perpendicular to the plane of incidence.

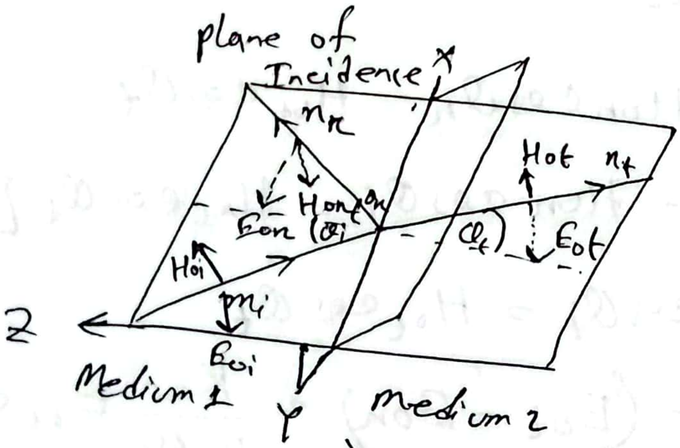

Figure: Showing incident, reflected and transmitted waves when the incident wave is polarized with the E vector normal to the plane of incidence.

The continuity of the tangential components of E & H at the interface requires that,

We know,

$$E_{0i}\cos\theta_i + E_{0r}\cos\theta_r = E_{0t}\cos\theta_t \quad \text{---} (i)$$

$$H_{0i}\cos\theta_i + H_{0r}\cos\theta_r = H_{0t}\cos\theta_t \quad \text{---} (ii)$$

From equation (i), we can write,

$$E_{0i} + E_{0r} = E_{0t} \quad \text{---} (iii)$$

Now, $\frac{E}{H} = \frac{\omega\mu}{k}$

$$\Rightarrow H = \frac{k}{\omega\mu}E$$

$$\left.\begin{aligned} &\Rightarrow H_{0i} = \frac{k_1}{\omega\mu_1}E_{0i} \\ &\Rightarrow H_{0r} = \frac{k_1}{\omega\mu_1}E_{0r} \\ &\Rightarrow H_{0t} = \frac{k_2}{\omega\mu_2}E_{0t} \end{aligned}\right\} \quad \text{---} (iv)$$

Again,

$$H_{0i}\cos\theta_i - H_{0r}\cos\theta_r = H_{0t}\cos\theta_t$$

$$\Rightarrow H_{0i}\cos\theta_i - H_{0r}\cos\theta_i = H_{0t}\cos\theta_t \quad [\because \theta_i = \theta_r]$$

$$\Rightarrow (H_{0i} - H_{0r})\cos\theta_i = H_{0t}\cos\theta_t$$

$$\Rightarrow \cos\theta_i \frac{k_1}{\omega\mu_1}(E_{0i} - E_{0r}) = \frac{k_2}{\omega\mu_2}E_{0t}\cos\theta_t \quad \text{---} (v)$$

From equation (iii)

$$E_{0i} + E_{0r} = E_{0t}$$

$$\therefore E_{0r} = E_{0t} - E_{0i}$$

Putting $E_{on} = E_{ot} - E_{oi}$ in equation (v),

$$\cos\theta_i \frac{k_1}{\omega\mu_1}(E_{oi} - E_{ot} + E_{oi}) = \frac{k_2}{\mu_2\omega} E_{ot}\cos\theta_t$$

$$\Rightarrow \cos\theta_i \frac{k_1}{\mu_1\omega}(2E_{oi} - E_{ot}) = \frac{k_2}{\omega\mu_2} E_{ot}\cos\theta_t$$

$$\Rightarrow 2E_{oi}\cos\theta_i \frac{k_1}{\mu_1\omega} - E_{ot}\cos\theta_i \frac{k_1}{\mu_1\omega} = \frac{k_2}{\omega\mu_2} E_{ot}\cos\theta_t$$

$$\Rightarrow 2E_{oi}\frac{k_1}{\mu_1\omega}\cos\theta_i = \frac{k_2}{\omega\mu_2} E_{ot}\cos\theta_t + \frac{k_1}{\omega\mu_1} E_{ot}\cos\theta_t$$

$$\Rightarrow 2E_{oi}\cos\theta_i \frac{k_1}{\mu_1\omega} = E_{ot}\left(\cos\theta_t \frac{k_2}{\omega\mu_2} + \cos\theta_t \frac{k_1}{\omega\mu_1}\right)$$

$$\Rightarrow \left(\frac{E_{oi}}{E_{ot}}\right)_N = \frac{\cos\theta_t \frac{k_2}{\omega\mu_2} + \cos\theta_t \frac{k_1}{\omega\mu_1}}{\frac{2k_1}{\mu_1\omega}\cos\theta_i}$$

$$\Rightarrow \left(\frac{E_{ot}}{E_{oi}}\right)_N = \frac{\frac{2k_1}{\omega\mu_1}\cos\theta_i}{\frac{k_2}{\omega\mu_2}\cos\theta_t + \frac{k_1}{\omega\mu_1}\cos\theta_i}$$

$$= \frac{\frac{2n_1}{\lambda_0 k m_1 \mu_0 \omega}\cos\theta_i}{\frac{n_2\cos\theta_t}{\lambda_0 k m_2 \mu_0 \omega} + \frac{n_1\cos\theta_i}{\lambda_0 k m_1 \mu_0 \omega}}$$

$$\therefore \left(\frac{E_{ot}}{E_{oi}}\right)_N = \frac{\frac{2n_1}{km_1}\cos\theta_i}{\frac{n_2}{km_2}\cos\theta_t + \frac{n_1}{km_1}\cos\theta_i}$$

where, $k_1 = \frac{n_1}{\lambda_0}$, $k_2 = \frac{n_2}{\lambda_0}$, $\mu_1 = km_1\mu_0$, $\mu_2 = km_2\mu_0$

From equation (v),

$$\cos\theta_i \frac{k_1}{\omega\mu_1}(E_{oi} - E_{or}) = \frac{k_2}{\omega\mu_2}(E_{oi} + E_{or})\cos\theta_t$$

$$\Rightarrow E_{oi}\frac{k_1}{\omega\mu_1}\cos\theta_i - E_{or}\frac{k_1}{\omega\mu_1}\cos\theta_i = E_{oi}\cos\theta_t\frac{k_2}{\omega\mu_2} + E_{or}\cos\theta_t\frac{k_2}{\omega\mu_2}$$

$$\Rightarrow E_{oi}\frac{k_1}{\omega\mu_1}\cos\theta_i - E_{oi}\frac{k_2}{\omega\mu_2}\cos\theta_t = E_{or}\frac{k_1}{\omega\mu_1}\cos\theta_i + E_{or}\frac{k_2}{\omega\mu_2}\cos\theta_t$$

$$\Rightarrow E_{oi}\left(\frac{k_1}{\omega\mu_1}\cos\theta_i - \frac{k_2}{\omega\mu_2}\cos\theta_t\right) = E_{or}\left(\frac{k_1}{\omega\mu_1}\cos\theta_i + \frac{k_2}{\omega\mu_2}\cos\theta_t\right)$$

$$\Rightarrow \left(\frac{E_{oi}}{E_{or}}\right)_N = \frac{\frac{k_1}{\omega\mu_1}\cos\theta_i + \frac{k_2}{\omega\mu_2}\cos\theta_t}{\frac{k_1}{\omega\mu_1}\cos\theta_i \cancel{+} - \frac{k_2}{\omega\mu_2}\cos\theta_t}$$

$$\therefore \left(\frac{E_{or}}{E_{oi}}\right)_N = \frac{\frac{n_1}{km_1}\cos\theta_i \ast \frac{n_2}{km_2}\cos\theta_t}{\frac{n_1}{km_1}\cos\theta_i + \frac{n_2}{km_2}\cos\theta_t}$$

Where,

(ii) When the incident wave is polarised with its vector E parallel to the plane of incidence. In this case the E vector of all three waves must be in the plane of incidence as shown in figure:

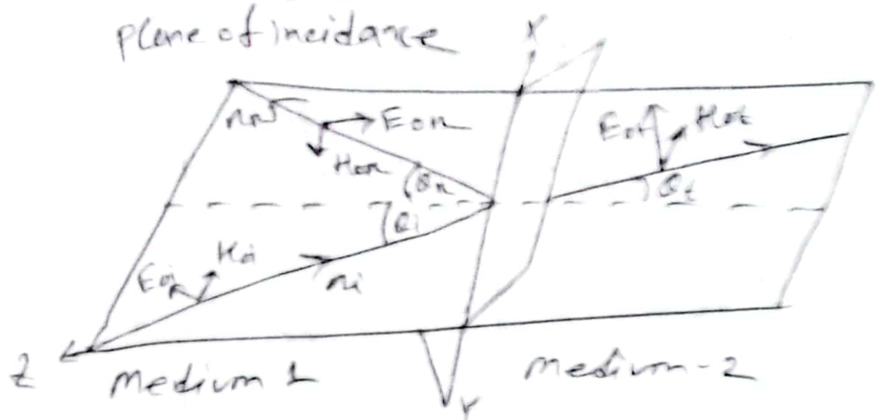

Figure: Showing the incidence, reflected and transmitted wave when the incident wave is ~~pla~~ polarised with its E vector parallel to the plane of incidence

The continuity of the tangenial components of E at the interface requires that,

$$E_{oi}\cos\theta_i + E_{or}\cos\theta_r = E_{ot}\cos\theta_t \quad \text{—— (i)}$$

similarly, the continuity of the tangenial components of H, $H_{oi}\cos\theta_i - H_{or}\cos\theta_r = H_{ot}\cos\theta_t$ —— (ii)

Now, $H_{oi} - H_{or} = H_{ot}$ —— (iii)

we know that, the ratio between E and H is,

$$\frac{\overline{E}}{\overline{H}} = \frac{\omega\mu}{K} \quad \therefore \overline{H} = \frac{K\overline{E}}{\omega\mu}$$

So that, $H_{or} = \frac{k_1}{\omega\mu_1} E_{or}$

$$H_{ot} = \frac{k_2}{\omega\mu_2} E_{ot}$$

$$H_{oi} = \frac{k_1}{\omega\mu_1} E_{oi}$$

From equation (iii) we get,

$$\frac{k_1}{\omega\mu_1} E_{oi} - \frac{k_1}{\omega\mu_1} E_{or} = \frac{k_2}{\omega\mu_2} E_{ot}$$

$$\Rightarrow \frac{k_1}{\omega\mu_1}(E_{oi} - E_{or}) = \frac{k_2}{\omega\mu_2} E_{ot}$$

$$\Rightarrow \frac{n_1}{\lambda_0 \omega K_{m_1} \mu_0}(E_{oi} - E_{or}) = \frac{n_2}{\lambda_0 \omega K_{m_2} \mu_0} E_{ot}$$

where, $k_1 = \frac{n_1}{\lambda_0}$, $k_2 = \frac{n_2}{\lambda_0}$, $\mu_1 = K_{m_1}\mu_0$, $\mu_2 = K_{m_2}\mu_0$

$$\Rightarrow \frac{n_1}{K_{m_1}}(E_{oi} - E_{or}) = \frac{n_2}{K_{m_2}} E_{ot}$$

$$\Rightarrow E_{oi} - E_{or} = \frac{n_2}{K_{m_2}} \frac{K_{m_1}}{n_1} E_{ot}$$

$$\therefore E_{or} = E_{oi} - \frac{n_2 K_{m_1}}{n_1 K_{m_2}} E_{ot} \quad \text{—— (IV)}$$

and, $E_{ot} = \frac{n_1 K_{m_2}}{n_2 K_{m_1}}(E_{oi} - E_{or})$ —— (V)

From

From equation (i) & (v) we get,

$$E_{oi}\cos\theta_i + \left(E_{oi} - \frac{n_2 k_{m_1}}{k_{m_2} n_1} E_{ot}\right)\cos\theta_R = E_{ot}\cos\theta_t$$

$$\Rightarrow E_{oi}\cos\theta_i + E_{oi}\cos\theta_R - \frac{n_2 k_{m_1}}{n_1 k_{m_2}} E_{ot}\cos\theta_R = E_{ot}\cos\theta_t$$

$$\Rightarrow E_{oi}\cos\theta_i + E_{oi}\cos\theta_R = E_{ot}\cos\theta_t + \frac{n_2 k_{m_1}}{n_1 k_{m_2}} E_{ot}\cos\theta_R$$

$$\Rightarrow 2E_{oi}\cos\theta_i = E_{ot}\left(\cos\theta_t + \frac{n_2 k_{m_1}}{n_1 k_{m_2}}\cos\theta_i\right) \quad [\because \theta_i = \theta_R]$$

$$\Rightarrow \left(\frac{E_{oi}}{E_{ot}}\right)_P = \frac{\cos\theta_t + \frac{n_2 k_{m_1}}{n_1 k_{m_2}}\cos\theta_i}{2\cos\theta_i}$$

$$\Rightarrow \left(\frac{E_{ot}}{E_{oi}}\right)_P = \frac{2\cos\theta_i}{\cos\theta_t + \frac{n_2 k_{m_1}}{n_1 k_{m_2}}\cos\theta_i}$$

Again, from the equation (i) and (v) we get,

$$\therefore \left(\frac{E_{ot}}{E_{oi}}\right)_P = \frac{2\frac{n_1}{k_{m_1}}\cos\theta_i}{\frac{n_1}{k_{m_1}}\cos\theta_t + \frac{n_2}{k_{m_2}}\cos\theta_i}$$

$$E_{oi}\cos\theta_i + E_{oR}\cos\theta_R = \frac{n_1 k_{m_2}}{n_2 k_{m_1}}\left(E_{oi} - E_{oR}\right)\cos\theta_t$$

$$\Rightarrow E_{oi}\cos\theta_i + E_{oR}\cos\theta_i = E_{oi}\cos\theta_t\frac{n_1 k_{m_2}}{n_2 k_{m_1}} - E_{oR}\cos\theta_t\frac{n_1 k_{m_2}}{n_2 k_{m_1}}$$

$$\Rightarrow E_{oR}\cos\theta_i + E_{oR}\cos\theta_t\frac{n_1 k_{m_2}}{n_2 k_{m_1}} = E_{oi}\cos\theta_t\frac{n_1 k_{m_2}}{n_2 k_{m_1}} - E_{oi}\cos\theta_i$$

$$\Rightarrow E_{or}\left(\cos\theta_i + \frac{n_1 k_{m2}}{n_2 k_{m1}}\cos\theta_t\right) = E_{oi}\left(\frac{n_1 k_{m2}}{n_2 k_{m1}}\cos\theta_t - \cos\right.$$

$$\therefore \left(\frac{E_{or}}{E_{oi}}\right)_P = \frac{-\cos\theta_i + \frac{n_1 k_{m2}}{n_2 k_{m1}}\cos\theta_t}{\cos\theta_i + \frac{n_1 k_{m2}}{n_2 k_{m1}}\cos\theta_t}$$

Q. Why Brewster angle is called polarizing angle?

→ We know from parallel incident,

$$\left(\frac{E_{or}}{E_{oi}}\right)_P = \frac{-\cos\theta_i + \frac{n_1 k_{m2}}{n_2 k_{m1}}\cos\theta_t}{\cos\theta_i + \frac{n_1 k_{m2}}{n_2 k_{m1}}\cos\theta_t}$$

For less dielectric media,

$$\left(\frac{E_{or}}{E_{oi}}\right)_P = \frac{-\cos\theta_i + \frac{n_1}{n_2}\cos\theta_t}{\cos\theta_i + \frac{n_1}{n_2}\cos\theta_t}$$

When, $\left(\frac{E_{or}}{E_{oi}}\right)_P = 0$

$$\therefore -\cos\theta_i + \frac{n_1}{n_2}\cos\theta_t = 0 \quad \text{——— (1)}$$

According to Snell's law,

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

$$\Rightarrow \frac{n_1}{n_2} = \frac{\sin\theta_t}{\sin\theta_i}$$

From eq. (1) we get,

$$-\cos\theta_i + \frac{\sin\theta_t}{\sin\theta_i}\cos\theta_t = 0$$

$$\Rightarrow -\sin\theta_i\cos\theta_i + \sin\theta_t\cos\theta_t = 0$$

$$\Rightarrow \sin 2\theta_t - \sin 2\theta_i = 0$$

$$\Rightarrow 2\cos\left(\frac{2\theta_t + \cancel{2\theta_t}\, 2\theta_i}{2}\right)\sin\left(\frac{2\theta_t - 2\theta_i}{2}\right) = 0$$

$$\Rightarrow \cos\left(\theta_t + \theta_i\right) \sin(\theta_t - \theta_i) = 0$$

Now, $\sin(\theta_t - \theta_i) = 0$, $\theta_t = \theta_i$ | $\cos(\theta_t - \theta_i) = 0$ $\Rightarrow \theta_t + \theta_i = \frac{\pi}{2}$ $\therefore \theta_t = \frac{\pi}{2} - \theta_i$

The brewster angle,

$$\frac{n_1}{n_2} = \frac{\sin\theta_t}{\sin\theta_i} = \frac{\sin(\pi/2 - \theta_i)}{\sin\theta_i}$$

$$= \frac{\sin(\pi/2 - \theta_{iB})}{\sin\theta_{iB}}$$

$$= \frac{\cos\theta_{iB}}{\sin\theta_{iB}} = \cot\theta_{iB}$$

$$\therefore \theta_{iB} = \cot^{-1}\left(\frac{n_1}{n_2}\right)$$

If an unpolarized wave is incident on an interference at the brewster angle then the only portion of the wave with E vector normal to the plane of incidence will be reflected.

Q. Show that the Brewster angle between two dielectric there is no reflected wave if the incident wave is polarized with the E vector parallel to the plane of incidence. / $\left(\frac{E_{or}}{E_{oi}}\right)_p = 0$

→ We know that, from fresnel's equation,

$$\left(\frac{E_{or}}{E_{oi}}\right)_p = \frac{-\cos\theta_i + \frac{n_1}{n_2}\cos\theta_t}{\cos\theta_i + \frac{n_1}{n_2}\cos\theta_t}$$

$$= \frac{\frac{n_1}{n_2}\cos\theta_t - \cos\theta_i}{\frac{n_1}{n_2}\cos\theta_t + \cos\theta_i} \quad \text{—— (1)}$$

According to the brewster angle,

$$\theta_i + \theta_t = \frac{\pi}{2}$$

$$\Rightarrow \theta_i = \frac{\pi}{2} - \theta_t \ , \ \theta_t = \frac{\pi}{2} - \theta_i$$

From equation (i),

$$\left(\frac{E_{or}}{E_{oi}}\right)_p = \frac{\frac{n_1}{n_2}\cos\left(\frac{\pi}{2} - \theta_i\right) - \cos\left(\frac{\pi}{2} - \cos\theta_t\right)}{\frac{n_1}{n_2}\cos\left(\frac{\pi}{2} - \theta_i\right) + \cos\left(\frac{\pi}{2} - \cancel{\theta}\cos\theta_t\right)}$$

$$= \frac{\frac{n_1}{n_2}\sin\theta_i - \sin\theta_t}{\frac{n_1}{n_2}\cancel{\cos}\sin\theta_i + \sin\theta_t}$$

$$= \frac{\sin\theta_i\left(\frac{n_1}{n_2} - \frac{\sin\theta_t}{\sin\theta_i}\right)}{\sin\theta_i\left(\frac{n_1}{n_2} + \frac{\sin\theta_t}{\sin\theta_i}\right)}$$

$$= \frac{\frac{n_1}{n_2} - \frac{n_1}{n_2}}{\frac{n_1}{n_2} + \frac{n_1}{n_2}} = 0$$

$$\therefore \left(\frac{E_{or}}{E_{oi}}\right)_p = 0$$