Subject: Chinese Remainder Theorem Date:

a)

system: $x \equiv 1 \pmod 3$

$x \equiv 2 \pmod 5$

$x \equiv 3 \pmod 7$

Total modulus $N = 3 \cdot 5 \cdot 7 = 105$

For each congruence $x = a_i \pmod{n_i}$

compute $N_i = N/n_i$ and inverse y_i of N_i mod

$\Rightarrow n_1 = 3, \quad a_1 = 1 : N_1 = 105/3 = 35$

$35 \equiv 2 \pmod 3$.

solve, $2y_1 = 1 \pmod 3 \Rightarrow y_1 = 2$

contribution: $a_1 N_1 y_1 = 1 \cdot 35 \cdot 2 = 70$

$\Rightarrow n_2 = 5, \; a_2 = 2 : N_2 = 105/5 = 21$

$\Rightarrow 21 \equiv 1 \pmod 5$ so, $y_2 = 1$

contribution: $2 \cdot 21 \cdot 1 = 42$

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$\Rightarrow n_3 = 7,\ a_3 = 3 :\ N_3 = 105/7 = 15$

$\Rightarrow 15 \equiv 1 \pmod 7.$ so, $y_3 = \underline{1}$

contribution $= 3 \cdot 15 \cdot 1 = 45$

sum $= 70 + 42 + 45 = ~~147~~\ 157$

Reduce modulo $105 : 157 \equiv 157 - 105 = 52$

$x \equiv 52 \pmod{105}$

since, $52 \bmod 3 = 1,$

$52 \bmod 5 = 2.$

$52 \bmod 7 = 3$

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b)

Eqn: $x \equiv 5 \pmod{11}$, $x \equiv 14 \pmod{29}$,

$x \equiv 15 \pmod{31}$

$\Rightarrow N = 11 \cdot 29 \cdot 31 = 9889$

$n_1 = 11, a_1 = 5 : N_1 = 9889/11 = 899$

$\Rightarrow 899 \equiv 8 \pmod{11}$. solve. $8y_1 \equiv 1 \pmod{11}$

$\Rightarrow y_1 \equiv 7$ (since $8 \cdot 7 = 56 \equiv 1$)

contribution: $5 \cdot 899 \cdot 7 = 5 \cdot 6293 = 31465$

$\Rightarrow n_2 = 29, a_2 = 14 : N_2 = 9889/29 = 341$

$\Rightarrow 341 \equiv 22 \pmod{29}$. solve $22y_2 \equiv 1 \pmod{29}$

$\Rightarrow y_2 \equiv 4$ (since $22 \cdot 4 = 88 \equiv 1$)

contribution; $14 \cdot 341 \cdot 4 = 14 \cdot 1364 = 19096$

$\Rightarrow n_3 = 31, a_3 = 15 : N_3 = 9889/31 = 319$

$\Rightarrow 319 \equiv 9 \pmod{31}$. solve $9y_3 \equiv 1 \pmod{31}$

$\Rightarrow y_3 \equiv 7$ (since $9 \cdot 7 = 63 \equiv 1$)

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contribution: $15 \cdot 3197 = 15 \cdot 2233 = 33495$

sum: $31465 + 19096 + 33495 = 84056$

Reduce modulo 9889.

$9889 \cdot 8 = 79112_0$

$84056 - 79112 = 4944$

$\boxed{x = 4944 \pmod{9889}}$

$4944 \mod 11 = 5$

$4944 \mod 29 = 14$

$4944 \mod 31 = 15$

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c)

system: $x \equiv 5 \pmod 6$, $x \equiv 4 \pmod{11}$

$x \equiv 3 \pmod{17}$

$N = 6 \cdot 11 \cdot 17 = 1122$

$n_1 = 6$, $a_1 = 5$: $N_1 = 1122/6 = 187$

$187 \equiv 1 \pmod 6 \Rightarrow y_1 = 1$

contribution: $5 \cdot 187 \cdot 1 = 935$

$n_2 = 11$, $a_2 = 4$, : $N_2 = 1122/11 = 102$

$102 \equiv 3 \pmod 4$, solve $3y_2 \equiv 1 \pmod{11}$ $3y_2 \equiv 4$

contribution: $4 \cdot 102 \cdot 4 = 4 \cdot 408 = 1632$

$\Rightarrow n_3 = 17$, $a_3 = 3$: $N_3 = 1122/17 = 66$

$66 \equiv 15 \pmod{17}$ solve $15y_3 \equiv 1 \pmod{17}$

Note $15 \equiv -2$, so $-2y_3 \equiv 1$

$\Rightarrow 2y_3 \equiv 16 \Rightarrow y_3 = 8$

contribution: $3 \cdot 66 \cdot 8 = 1584$

sum: $935 + 1632 + 1584 = 4151$

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Reduce modulo -

~~1122:~~

1122: 1122·3 = 3366,

4151 - 3366 = 785

~~4151 - 3366 = 7~~

$x = 785 \pmod{1122}$

$785 \bmod 6 = 5$

$785 \bmod 4 = 4$

$785 \bmod 17 = 3$