Assignment 3: Linear Programming

1TD184 Optimization

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Option 1: Staff Planning

Background

A hotel has the following estimated numbers of minimum required housekeeping staff for every week of the coming months.

Day	Staff
Monday	50
Tuesday	75
Wednesday	75
Thursday	70
Friday	65
Saturday	50
Sunday	40

Tasks

1. Suppose each employee works five consecutive days and then takes two days off. How many employees are needed in total?

2. The manager is considering dividing the employees into two groups. All employees of the first group work from Monday to Friday. An employee of the second group works on three of the five weekdays, plus Saturday and Sunday. Moreover, for each individual of the second group, the manager can freely choose which three weekdays are the working days. How many employees are needed in total with this two-group setting? Is the number lower than what you obtained above?

3. Suppose the required staff on Thursday is decreased by 15 and that on Sunday is increase by 15. Which of the above two schemes requires a higher number of employees? Can you provide some underlying reason for the observations you have made?

Option 2: Production Optimization

Background

A production manager has made the following forecast of a product for the coming six months. It is possible to put part of the production of a month in an inventory to meet the demand of later months, subject to an inventory cost.

Let us assume there is no initial inventory. Also, if necessary, make additional assumptions with justification.

Month	Market demand	Production unit cost	Inventory unit cost	Production capacity
1	500	12	2	600
2	600	12	2	700
3	650	15	1	750
4	650	18	1	750
5	800	18	1	900
6	550	20	2	550

Tasks

- Make an optimal production plan by formulating the above problem as an LP and solve it.
- Suppose having the inventory for the coming six months imposes an extra rental cost of c (which does not depend on the amount of product in the inventory). At what value of c should the manager decide not to use the inventory at all?
- Suppose it is possible to make an investment, with unit cost of v to increase the production capacity of one of the months by 100. That is, the total investment cost is $100v$. Such an investment potentially can reduce the total cost. What is the threshold of v such that it is worth to make this investment and in which month? (You should solve this part without considering any extra rental cost in the question above).