In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt
from sympy import *
```

13.3

Consider the polynomial $p(x) = (x - 2)^9$. Plot this polynomial in it's expanded form and in the factored form.

In [ ]:
```
def p1(x):
    return x ** 9 - 18 * x ** 8 + 144 * x ** 7 - 672 * x ** 6 + 2016 * x ** 5 - 403
```

In [ ]:
```
x = np.round(np.linspace(-1.920, 2.080, 4000), decimals=3)
```

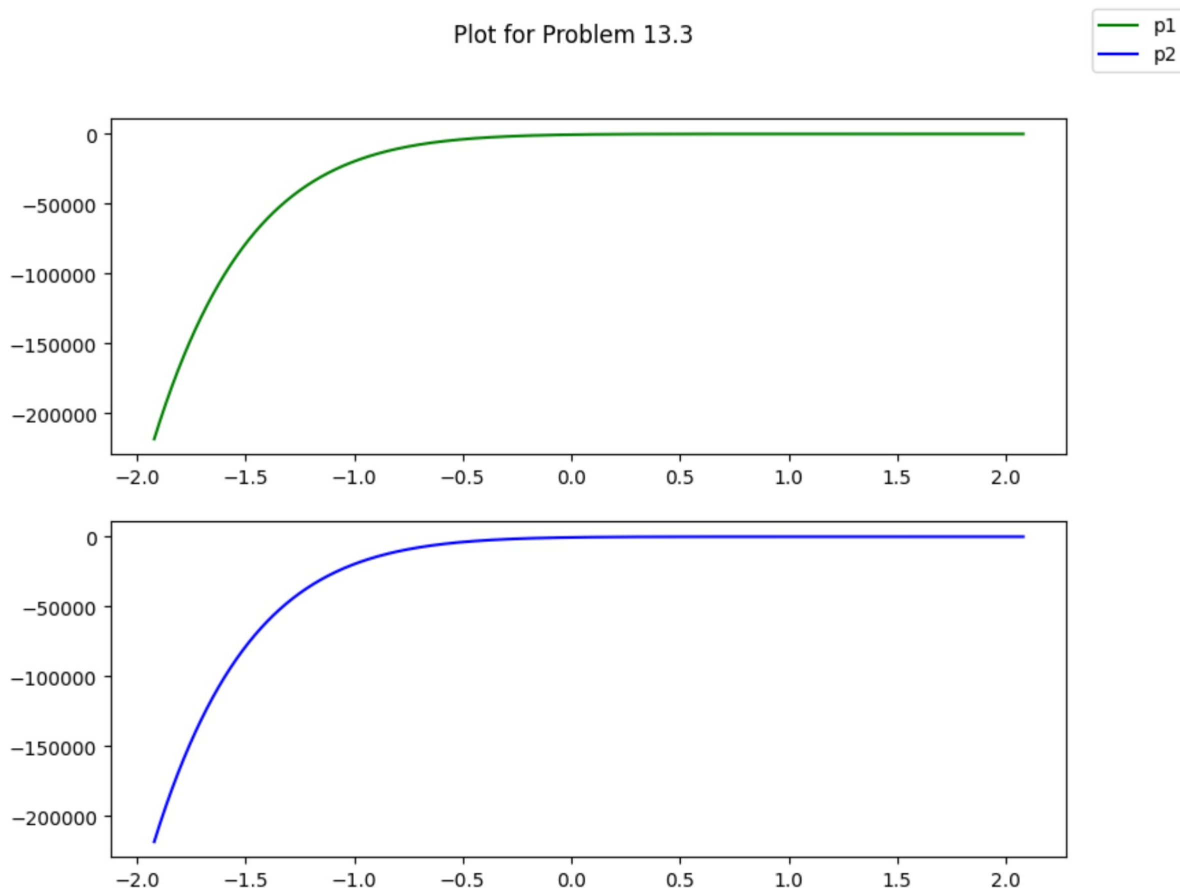
In [ ]:
```
x
```

Out[ ]:
```
array([-1.92 , -1.919, -1.918, ...,  2.078,  2.079,  2.08 ])
```

In [ ]:
```
def p2(x):
    return (x - 2) ** 9
```

In [ ]:
```
fig, axs = plt.subplots(2, figsize=(9, 7))
axs[0].plot(x, p1(x), color='green', label='p1')
axs[1].plot(x, p2(x), color='blue', label='p2')
fig.suptitle('Plot for Problem 13.3')
fig.legend()
```

Out[ ]:
```
<matplotlib.legend.Legend at 0x7fb18b919a60>
```



13.4

The polynomial $p(x) = x^5 - 2x^4 - 3x^3 + 3x^2 - 2x - 1$ has three real zeros. Applying Newton's method to p with initial guess $x_0 = 0$ produces a series of estimates $x_1, x_2, x_3, \ldots$ that converge rapidly to a zero $x_* \approx -0.315$.

(a) Compute $x_1, \ldots, x_6$ in floating point arithmetic with $\epsilon_{machine} \approx 10^{-16}$. How many digits do you estimate are correct in each of these numbers?

In [ ]:
```
# Machine epsilon for Numpy's float64 is approximately 10^{-16}
np.finfo(np.float64).eps
```

Out[ ]: $2.22044604925031 \cdot 10^{-16}$

In [ ]:
```
def newton(f, dfdx, x0, epsilon=1e-10, max_iters=500):
    # Quick iterative implementation of Newton's method
    iters = 0
    estimates = [x0]
    x_previous = x0
    while np.abs(f(x_previous)) > epsilon and iters < max_iters:
        x_next = x_previous - f(x_previous)/dfdx(x_previous)
        estimates.append(x_next)
        x_previous = x_next
        iters += 1
    return x_next, estimates
```

In [ ]:
```
f = lambda x: (x ** 5) - (2 * x ** 4) - (3 * x ** 3) + 3 * x ** 2 - 2 * x - 1
dfdx = lambda x: (5 * x ** 4) - (8 * x ** 3) - (9 * x ** 2) + (6 * x) - 2
x0 = np.float64(0.0)
```

In [ ]:
```
x_final, floating_point_xs = newton(f, dfdx, x0)
```

Below are the sequence of estimates produced by Newton's method. I would estimate that no more than 15 digits after the decimal are correct (but probably fewer due to the division in the algorithm).

In [ ]:
```
floating_point_xs
```

Out[ ]: $[0.0, \ -0.5, \ -0.336842105263158, \ -0.315728448396289, \ -0.315301162703277, \ -($

In [ ]:

(b) Compute $x_1, \ldots, x_6$ again *exactly* with the aid of a symbolic algebra system such as Maple or Mathematica. Each x_j is a rational number. How many digits are there in the numerator and the denominator for each j?

In [ ]:
```
# Using Sympy
x= symbols('x')
init_printing(use_unicode=True)
```

In [ ]:
```
f = (x ** 5) - (2 * x ** 4) - (3 * x ** 3) + 3 * x ** 2 - 2 * x - 1
dfdx = diff(f, x)
```

In [ ]:
```
dfdx
```

Out[ ]: $5x^4 - 8x^3 - 9x^2 + 6x - 2$

In [ ]:
```
def sym_newton(f, x0, epsilon=1e-10, max_iters=500):
    # symx_previous, symx_next = symbols('symx_previous symxnext dfdx')
    dfdx = diff(f)
    iters = 0
    estimates = [x0]
    fs = list()
    dfdxs = list()
    x_previous = x0
    symx_previous = x
    while np.abs(f.subs(x, x_previous)) > epsilon and iters < max_iters:
        x_next = x_previous - f.subs(x, x_previous)/dfdx.subs(x, x_previous)
        fs.append(f.subs(x, x_previous))
        dfdxs.append(dfdx.subs(x, x_previous))
        estimates.append(x_next)
        x_previous = x_next
        iters += 1
    return x_next, estimates, fs, dfdxs
```

In [ ]:
```
result, symbolic_xs, fs, dfdxs = sym_newton(f, x0=0.0)
```

In [ ]:
```
print('Floating point estimates: \n' + str(floating_point_xs))
print('Symbolic estimates \n' + str(symbolic_xs))
```

```
Floating point estimates: 
[0.0, -0.5, -0.3368421052631579, -0.31572844839628944, -0.31530116270327685, -0.315
30098645936266]
Symbolic estimates 
[0.0, -0.500000000000000, -0.336842105263158, -0.315728448396289, -0.31530116270327
7, -0.315300986459363]
```

In [ ]:
```
fs
```

Out[ ]: $\left[-1.0,\ 0.96875,\ 0.0986450190238758,\ 0.00191853843116641,\ 7.90693465868575 \cdot 10^{-7}\right]$

In [ ]:
```
dfdxs
```

Out[ ]: $\left[-2,\ -5.9375,\ -4.67209539521643,\ -4.49006007582294,\ -4.4863589749122\right]$

Taking a look at the values computed for f and df/dx at each step j, it appears that there are at least 15 digits after the second iteration.