HW 11

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11.1

Suppose the $m \times n$ matrix A has the form

$$A = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$$

where A_1 is a nonsingular matrix of dimension $n \times n$ and A_2 is an arbitrary matrix of dimension $(m-n) \times n$. Prove that $\|A^+\|_2 \leq \|A_1^{-1}\|_2$.

Proof: By definition, $A^+ = (A^*A)^{-1}A^*$. The matrix A has SVD $A = U\Sigma V^*$. Thus we may write

$$A^+ = [(U\Sigma V^*)^*(U\Sigma V^*)]^{-1} A^*$$

$$= [V\Sigma\Sigma V^*]^{-1}(V\Sigma U^*)$$

$$= V^{-*}\Sigma^{-1}\Sigma^{-1}V^{-1}V\Sigma U^* \leftarrow \text{since } V \text{ \& } \Sigma \text{ are invertible and the product } V\Sigma\Sigma V^* \text{ is invertible.}$$

$$= V^{-*}\Sigma^{-1}U^*$$

$$= V\Sigma^{-1}U^{-1} \longleftarrow \text{since } V \text{ \& } U \text{ are unitary}$$

$$= (U\Sigma V^*)^{-1}$$

$$\leq A_1^{-1}$$

Thus, we have $\|A^+\|_2 \leq \|A^{-1}\|_2$. Since A is $m \times n$ with A_1 full rank, $(U\Sigma V^*)^{-1}$ must only construct A_1^{-1}, otherwise it would not be invertible.