Math 510 HW 18 Michael Snyder

18.1 consider the example

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1.0001 \\ 1 & 1.0001 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ 0.0001 \\ 4.0001 \end{bmatrix}$$

(a) What are the matrices A^+ and P for this example? Give exact answers.

Solution

$A^+ = (A^*A)^{-1}A^*$ and $P = AA^+$.

First we compute A^+.

$$A^*A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1.0001 & 1.0001 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1.0001 \\ 1 & 1.0001 \end{bmatrix} = \begin{bmatrix} 3 & 3.0002 \\ 3.0002 & 3.0004 \end{bmatrix}$$

$$(A^*A^{-1}) = \frac{1}{3(3.0004)-(3.0002)(3.0002)} \begin{bmatrix} 3.0004 & -3.0002 \\ -3.0002 & 3 \end{bmatrix}$$

$$= -4.0 \times 10^{-8} \begin{bmatrix} 3.0004 & -3.0002 \\ -3.0002 & 3 \end{bmatrix}$$

$$(A^*A)^{-1}A = -4.0 \times 10^{-8} \begin{bmatrix} 3.0004 & -3.0002 \\ -3.0002 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1.0001 & 1.0001 \end{bmatrix}$$

$$A^+ = -4.0 \times 10^{-8} \begin{bmatrix} 0.0002 & 0.00010002 & 0.00010002 \\ 0.0002 & -6.0005 & -6.0005 \end{bmatrix}$$

$$P = AA^+$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 1.0001 \\ 1 & 1.0001 \end{bmatrix} \begin{bmatrix} 0.0002 & 0.00010002 & 0.00010002 \\ 0.0002 & -6.0005 & -6.0005 \end{bmatrix}$$

$$= -4.0 \times 10^{-8} \begin{bmatrix} 0.0004 & -6.00039998 & -6.00039998 \\ -2 \times 10^{-8} & -6.0010003 & -6.0010003 \end{bmatrix}$$

(b) Find exact solutions x and $y = Ax$ to the least squares problem $Ax \approx b$.

Solution:

$$x = A^+ b$$

$$= -4.0 \times 10^{-8} \begin{bmatrix} 0.0002 & 0.00010002 & 0.00010002 \\ 0.0002 & -6.0005 & -6.0005 \end{bmatrix} \begin{bmatrix} 2 \\ 0.0001 \\ 4.0001 \end{bmatrix}$$

$$= -4.0 \times 10^{-8} \begin{bmatrix} 0.0008\,001\,00004 \\ -24.0028001 \end{bmatrix}$$

and

$$b \approx y = Ax = AA^+ b$$

$$= -4.0 \times 10^{-8} \begin{bmatrix} 1 & 1 \\ 1 & 1.0001 \\ 1 & 1.0001 \end{bmatrix} \begin{bmatrix} 0.0008\,001\,00004 \\ -24.0028001 \end{bmatrix}$$

$\downarrow$

$$= -4.0 \times 10^{-8} \begin{bmatrix} -24.0019999999996 \\ -24.0044002800006 \\ -24.0044002800006 \end{bmatrix}$$

(c) See Jupyter Notebook

(d) See Jupyter Notebook

(e) See Jupyter Notebook,

18.2 One might think that the more variables one included in such a model, the more information one would obtain, but this is not always true. Explain this phenomenon from the point of view of conditioning, make specific reference to the results of Theorem 18.1.

Solution Three of the four quantities in the table for Theorem 18.1 are proportional to $\kappa(A)$. This implies that poor conditioning on A will result in large perturbations in solutions x & y. Thus, including additional variables that cause A to be poorly conditioned may return undesirable results, particularly if data is noisy. For example, a minor mis-reporting in a parents IQ or years of education may result in a major change in predicted annual income, making the model untrustworthy.