Numerical Linear Algebra: Homework #8

Due on September 19, 2022 at 10:00PM

Instructor: Professor Blake Barker
Section 1

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Problem 8.1

Let A be an $m \times n$ matrix. Determine the exact number of floating point operations (i.e., additions, subtractions, multiplications, and divisions) involved in computing the factorization $A = \hat{Q}\hat{R}$ by Algorithm 8.1.

Solution: First we note that because we are considering the reduced QR factorization, Q is $m \times n$ and R is $n \times n$. With this in mind, and beginning from the inner loop, we find that

Inner Loop

$r_{ij}q_i$ results m multiplications and $v_j - r_{ij}q_i$ results in m subtractions. Thus, $v_j = v_j - r_{ij}q_i$ results in $2m$ floating point operations.

$q_i^* v_j$ results in m multiplications and $m-1$ additions. Thus, $r_{ij} = q_i^* v_j$ results in $m + m - 1 = 2m - 1$ floating point operations.

Adding these two assignments within the inner loop together, we have $4m - 1$ floating point operations.

Since these assignments happen for $i = 1$ to n and for $j = i + 1$ to n, we are summing up the $4m - 1$ floating point operations n times starting at $j = 1 + 1 = 2$ to n which means we have $\frac{n}{2}(n - (i+1) + 1) = \frac{n}{2}(n - 2 + 1) = \frac{n(n-1)}{2}$ terms. Thus we have

$$\sum_{i=1}^{n} \sum_{j=i+1}^{n} (4m - 1) = \frac{1}{2} \cdot n(n-1) \cdot (4m-1) = \frac{n(n-1)}{2}(4m-1)$$

floating point operations.

Outer Loop

The outer loop consists of m divisions in v_i / r_{ii} and m multiplications and $m - 1$ additions in $\|v_i\|$.
Then summing those up for $i = 1$ to n we have $n(3m - 1)$ floating point operations.

We note here prior to summing all the floating point operations that the top loop of Algorithm 8.1 (i.e., with $v_i = a_i$) does not result in any floating point operations since it is only reassignment.

Now, adding up the inner and outer loops, we have

$$\text{inner loop } + \text{ outer loop } = \frac{n(n-1)}{2}(4m-1) + n(3m-1)$$

floating point operations.