```
In [ ]: import numpy as np
```

Problem 18.1

Below are the computations for Parts (c), (d), and (e). These computation use the following information:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1.0001 \\ 1 & 1.0001 \end{bmatrix}$$

$$b = \begin{bmatrix} 2 \\ 0.0001 \\ 4.0001 \end{bmatrix}$$

and the computed

$$x = 4.0 \times 10^{-8} \begin{bmatrix} 0.000800100004 \\ -24.0028001 \end{bmatrix}$$

$$y = 4.0 \times 10^{-8} \begin{bmatrix} 24.001999999996 \\ 24.004400280006 \\ 24.004400280006 \end{bmatrix}$$

```
In [ ]: A = np.array([[1.0, 1.0], [1.0, 1.0001], [1.0, 1.0001]], dtype=np.float64)
        b = np.array([2, 0.0001, 4.0001], dtype=np.float64)
        x = np.array([0.000800100004,  -24.0028001], dtype=np.float64)
        y = 4.0e-8 * np.array([24.001999999996, 24.004400280006, 24.004400280006], dtype=np
```

(c) What $\kappa(A)$, θ, and η?

```
In [ ]: kappa_A = np.linalg.cond(A, p=2)
        theta = np.arccos(np.linalg.norm(y, ord=2) / np.linalg.norm(b, ord=2))
        eta = (np.linalg.norm(A, ord=2) * np.linalg.norm(x, ord=2)) / np.linalg.norm(y, ord
        print('kappa(A): ' + str(kappa_A))
        print('theta: ' +  str(theta))
        print('eta: ' + str(eta))
```

```
kappa(A): 42429.235416083044
theta: 1.5707959549401582
eta: 35355339.07897037
```

(d) What are the 4 condition numbers of Theorem 18.1?

$$\kappa_{y\mapsto b} = \frac{1}{\cos\theta}$$

$$\kappa_{y\mapsto A} = \frac{\kappa(A)}{\cos\theta}$$

$$\kappa_{x\mapsto b} = \frac{\kappa(A)}{\eta\cos\theta}$$

$$\kappa_{x\mapsto A} = \kappa(A) + \frac{\kappa(A)^2\tan\theta}{\eta}$$

In [ ]:

```
# Computations for each of the 4 condition numbers listed above.
kappa_y_b = 1/np.cos(theta)
kappa_y_A = np.linalg.cond(A, p=2)/np.cos(theta)
kappa_x_b = np.linalg.cond(A, p=2)/(eta * np.cos(theta))
kappa_x_A = ((np.linalg.cond(A, p=2) ** 2) * np.tan(theta))/(eta)

print('Sensitivity of y to perterbations in b is kappa_y_b: ' + str(kappa_y_b))
print('Sensitivity of y to perterbations in A is kappa_y_A: ' + str(kappa_y_A))
print('Sensitivity of x to perterbations in b is kappa_x_b: ' + str(kappa_x_b))
print('Sensitivity of x to perterbations in A is kappa_y_A: ' + str(kappa_x_A))
```

```
Sensitivity of y to perterbations in b is kappa_y_b: 2689222.1524829348
Sensitivity of y to perterbations in A is kappa_y_A: 114101639793.84401
Sensitivity of x to perterbations in b is kappa_x_b: 3227.281727916238
Sensitivity of x to perterbations in A is kappa_y_A: 136931096.18777186
```

(e) Give examples of perturbations δb and δA that approximately attain these four condition numbers.

In [ ]:

```
# Perturbations of b and A
delta_b = np.array([2 + 1e-14, 0.0001, 4.0001], dtype=np.float64)
delta_A = A = np.array([[1.0 + 1e-14, 1.0], [1.0, 1.0001], [1.0, 1.0001]], dtype=np
```

In [ ]:

```
# Computations for each of the 4 condition numbers listed above with delta_b and de
kappa_delta_A = np.linalg.cond(delta_A, p=2)
theta = np.arccos(np.linalg.norm(y, ord=2) / np.linalg.norm(b, ord=2))
eta = (np.linalg.norm(delta_A, ord=2) * np.linalg.norm(x, ord=2)) / np.linalg.norm(
print('kappa(delta_A): ' + str(kappa_delta_A))
print('theta: ' +  str(theta))
print('eta: ' + str(eta))



kappa_y_b = 1/np.cos(theta)
kappa_y_delta_A = np.linalg.cond(delta_A, p=2)/np.cos(theta)
kappa_x_b = np.linalg.cond(delta_A, p=2)/(eta * np.cos(theta))
kappa_x_delta_A = ((np.linalg.cond(delta_A, p=2) ** 2) * np.tan(theta))/(eta)

print('Sensitivity of y to perterbations in b is kappa_y_b: ' + str(kappa_y_b))
print('Sensitivity of y to perterbations in A is kappa_y_delta_A: ' + str(kappa_y_d
print('Sensitivity of x to perterbations in b is kappa_x_b: ' + str(kappa_x_b))
print('Sensitivity of x to perterbations in A is kappa_ydelta__A: ' + str(kappa_x_d
```

```
kappa(delta_A): 42429.2354118404
theta: 1.5707959549401582
eta: 35355339.07897044
Sensitivity of y to perterbations in b is kappa_y_b: 2689222.1524829348
Sensitivity of y to perterbations in A is kappa_y_delta_A: 114101639782.43459
Sensitivity of x to perterbations in b is kappa_x_b: 3227.281727593525
Sensitivity of x to perterbations in A is kappa_ydelta__A: 136931096.16038716
```