PROBLEM SET 2

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ECONOMICS 326

Problem 1 (Based on Angrist, Berry, and Kocatulum (2007) PSI, Q2)**:** This problem asks you to use R (or Stata if you prefer) to conduct a series of sampling experiments. This question can be answered by making some small changes to the R code at `https://bitbucket.org/paulschrimpf/econ326/src/master/notes/02/clt.R?at=master`, or the Stata code at `https://bitbucket.org/paulschrimpf/econ326/src/master/problemSets/02/clt.do?at=master`.

(i) Draw 5000 random samples of size 8 from a random number generator for a standard normal distribution. Then increase the sample size to 32. Finally, increase the sample size to 128. Plot histograms of the sampling distributions of (i) the sample mean and (ii) the sample variance, for each these three sample sizes.

(ii) Repeat your experiments from (i) for three samples drawn from another parametric distribution of your choice (e.g., a uniform distribution). Discuss the results of your experiments in light of the central limit theorem.

(iii) Repeat your experiments from (i) for three samples drawn from a Cauchy distribution. Discuss the results of your experiments in light of the central limit theorem.

(iv) Your experiments produce samples of sample means. Compute the mean and variance of the sample means generated by each experiment and compare them to the mean and variance predicted by statistical theory. Does the variance of the sample means (i.e., the sampling variance) decrease with sample size at the rate predicted by the theory? Does Normality matter for this?

Problem 2 (Wooldridge C.2)**:** Let $y_1, y_2, ..., y_n$ be n pairwise uncorrelated random variables with common mean μ and variance σ^2. Let $\bar{y}$ denote the sample average.

(i) Define the class of linear estimators of μ by

$$W_a = a_1 y_1 + ... + a_n y_n$$

where the a_i are constants. What restriction on a_i is needed for W_a to be an unbiased estimator of μ?

(ii) Find $\text{Var}(W_a)$.

(iii) For any numbers, $a_1, ..., a_n$, the following inequality holds:

$$(a_1 + ... + a_n)^2 / n \leq a_1^2 + ... + a_n^2$$

Use this, along with (i) and (ii), to show that $\text{Var}(W_a) \geq \text{Var}(\bar{y})$ whenever W_a is unbiased, so that $\bar{y}$ is the best linear unbiased estimator. [Hint: What does the inequality become when a_i satisfy the restriction from part (i)?]

Problem 3 (Based on Wooldridge C.4)**:** For positive random variables X and Y, suppose that $\text{E}[Y|X] = \theta X$ for some unknown parameter θ.

(i) Define the random variable $Z = Y/X$. Show that $\text{E}[Z] = \theta$. [Hint: First show that $\text{E}[Z|X] = \theta$ and then use the law of iterated expectations.]

(ii) Use part (i) to prove that the estimator $W_1 = \frac{1}{n}\sum_{i=1}^{n} \frac{Y_i}{X_i}$ is unbiased for θ, where $\{(X_i, Y_i) : i = 1, 2, ..., n\}$ is a random sample.

(iii) Explain why $W_2 = \frac{\bar{Y}}{\bar{X}}$ is not the same as W_1.
(iv) Show that W_2 is a consistent estimator of θ.
(v) Show that W_2 is also an unbiased estimator of θ.

Problem 4 (Angrist, Berry, and Kocatulum (2007) PSI, Q3)**:** Table 3 in Woodbury and Spiegelman (1987) reports the results of two social experiments examining the relationship between Unemployment Insurance (UI) and the length of time unemployed. In the Employer Experiment, any UI recipient finding employment for at least 4 months received a voucher worth $500 to his or her employer. In the Claimant Experiment, any UI recipient finding employment for at least 4 months received $500 directly.

(i) For each experiment, test the hypothesis that bonuses increased the proportion of UI claimants returning to work (i.e., ending benefits) within 11 weeks.
(ii) For each experiment, pick a significance level and test the hypothesis that the experiment reduced weeks of insured unemployment in the benefit year using a one-tailed and two-tailed test. Which test makes more sense in this case?

REFERENCES

Angrist, Joshua, James Berry, and Emre Kocatulum. 2007. "14.32 Econometrics." Massachusetts Institute of Technology: MIT OpenCourseWare. URL `http://ocw.mit.edu/courses/economics/14-32-econometrics-spring-2007/`.

Woodbury, Stephen A. and Robert G. Spiegelman. 1987. "Bonuses to Workers and Employers to Reduce Unemployment: Randomized Trials in Illinois." *The American Economic Review* 77 (4):pp. 513–530. URL `http://www.jstor.org/stable/1814528`.