GET ② Rechenübung ⑱ Aufgabe ① — Markus Szymanski 28.04.2025

$$I_1 = (\sqrt{2} + \sqrt{2}\,j)A$$
$$I_2 = 4A \exp(-j\,30°)$$
$$I = I_1 + I_2$$

$$|I_1| = \sqrt{(Re^2(I_1) + Im^2(I_1))}$$
$$= \sqrt{(2A^2 + 2A^2)} = \sqrt{(4A^2)} = \underline{2A}$$

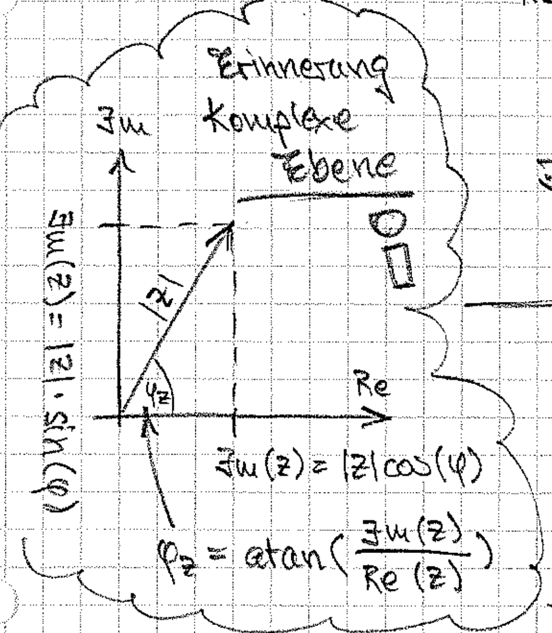
$$Im(I_1) = Re(I_1)\tan(\varphi_1) \rightsquigarrow \varphi_1 = \operatorname{atan}\left(\frac{Im(I_1)}{Re(I_1)}\right)$$
$$\varphi_1 = \operatorname{atan}\left(\frac{\sqrt{2}A}{\sqrt{2}A}\right) = \operatorname{atan}(1) = \frac{\pi}{4} = \underline{45°}$$

$$I_1 = (\sqrt{2} + \sqrt{2}\,j)A$$
$$\underline{Re(I_1) = \sqrt{2}A} \qquad \underline{Im(I_1) = \sqrt{2}A}$$

$$I_2 = 4A\exp(-j\,30°) = \underbrace{4A}\exp\Big(-j\underbrace{\frac{\pi}{6}}\Big)$$
$$\underline{|I_2| = 4A} \qquad \underline{\varphi_2 = -\frac{\pi}{6}}$$

$$Re(I_2) = |I_2|\cos(\varphi) = 4A \cdot \cos\left(-\frac{\pi}{6}\right)$$
$$= 4A \cdot 0{,}866 = \underline{3{,}464\,A}$$

$$Im(I_2) = |I_2|\sin(\varphi) = 4A\sin\left(-\frac{\pi}{6}\right)$$
$$= 4A \cdot (-0{,}5) = \underline{-2A}$$



$$I = I_1 + I_2 = Re(I_1) + Re(I_2) + (Im(I_1) + Im(I_2))\,j$$
$$I = (\sqrt{2} + 3{,}464 + (\sqrt{2} - 2)\,j)A = \underbrace{4{,}878A}_{Re(I)} - \underbrace{0{,}586\,j\,A}_{Im(I)}$$
$$\underline{Re(I) = 4{,}878A} \qquad \underline{Im(I) = -0{,}586A}$$

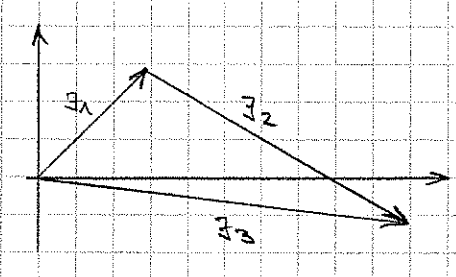
$$|I| = \sqrt{(4{,}878^2 + 0{,}586^2)}\,A$$
$$= \underline{4{,}913\,A}$$

$$\varphi_I = \operatorname{atan}\left(\frac{Im(I)}{Re(I)}\right) = \operatorname{atan}\left(-\frac{0{,}586}{4{,}878}\right)$$
$$\underline{\varphi_I = -0{,}12 = -6{,}847°}$$

$$I_1 = \sqrt{2}A + \sqrt{2}A\,j$$
$$I_1 = 2A\cos(45°) + 2A\sin(45°)\,j$$
$$I_1 = 2A\exp\left(\frac{\pi}{4}j\right)$$
$$I_2 = 3{,}464A - 2A\,j$$
$$I_2 = 4A\cos(-30°) - 4A\sin(-30°)\,j$$
$$I_2 = 4A\exp\left(-j\frac{\pi}{6}\right)$$
$$I_3 = 4{,}878A - 0{,}586A\,j$$
$$I_3 = 4{,}913A\cos(-6{,}847°) - 4{,}913\sin(-6{,}847°)\,j$$
$$I_3 = 4{,}913A\exp(-0{,}12\,j)$$

Lösung

Grafische Lösung



GET (2) Rechenübung (10) Aufgabe (2)   Markus Symmanne   28.04.2025

$Z_1 = (1+2j)\,k\Omega$ ges.: Z_2 mit doppeltem Scheinwiderstand durch Änderung der Induktivität

Scheinwiderstand
$S = |Z|$

$$|Z_2| = 2\,|Z_1| = 2\sqrt{(1^2+2^2)}\,k\Omega = 2\sqrt{5}\,k\Omega$$

$$|Z_2| = 2\sqrt{5}\,k\Omega = \sqrt{(R^2+\omega^2 L^2)} = \sqrt{(R^2+4\pi^2 f^2 L^2)} \quad \Big|\,(\cdot)^2$$

$$|Z_2|^2 = R^2 + 4\pi^2 f^2 L^2 \rightsquigarrow L^2 = \frac{|Z_2|^2 - R^2}{4\pi^2 f^2}$$

$$L = \sqrt{\left(\frac{|Z_2|^2 - R^2}{4\pi^2 f^2}\right)} \qquad R = 1\,k\Omega\ ,\ f = 50\,Hz$$

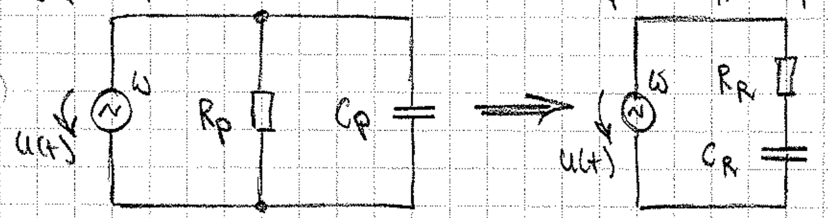
$$= \sqrt{\left(\frac{4\cdot 5\cdot 10^6\,\Omega^2 - 10^6\,\Omega^2}{4\cdot\pi^2\cdot 50^2\,1/s^2}\right)} = \sqrt{\left(\frac{(20-1)\cdot 10^6\,V^2 s^2}{4\pi^2\cdot 50^2\,A^2}\right)}$$

$$= \sqrt{\left(\frac{19}{100^2\,\pi^2}\right)}\cdot 10^3\,\frac{Vs}{A} = \sqrt{\left(\frac{19}{\pi^2}\cdot 10^{-4}\right)}\cdot 10^3\,\frac{Vs}{A}$$

$$= \sqrt{\left(\frac{19}{\pi^2}\right)}\cdot 10\,\frac{Vs}{A} \rightsquigarrow \underline{\underline{L = \frac{\sqrt{19}}{\pi}\cdot 10\,\frac{Vs}{A} = 13{,}875\,\frac{Vs}{A}}}$$

GET (2) Rechenübung (12) Aufgabe (3) Markus Szymanik 28.04.2025

geg: Z_P ges: $Z_R = Z_P$



Ansatz:

$$Z_R = Z_P = Re(Z_P) + Im(Z_P)\,j$$
$$= R_R - j\frac{1}{\omega C_R}$$

$\rightsquigarrow$ $\underline{R_R = Re(Z_P)}$

$\rightsquigarrow$ $\frac{1}{\omega C_R} = -Im(Z_P)$

$\rightsquigarrow$ $\underline{C_R = -\frac{1}{\omega\, Im(Z_P)}}$

$$Z_P = R_P \parallel X_{C_P} = \frac{-R_P j \frac{1}{\omega C_P}}{R_P - j\frac{1}{\omega C_P}} = \frac{-R_P j}{R_P\omega C_P - j} = \frac{Z_{P1}}{Z_{P2}}$$

$Z_{P1} = 0 - R_P j \rightsquigarrow |Z_{P1}| = R_P \rightsquigarrow \varphi_{P1} = -90° = -\frac{\pi}{2}$

$Z_{P1} = R_P \exp\left(-\frac{\pi}{2} j\right)$

$Z_{P2} = R_P\omega C_P - 1j \rightsquigarrow |Z_{P2}| = \sqrt{R_P^2\omega^2 C_P^2 + 1} \rightsquigarrow \varphi_{P2} = \operatorname{atan}\left(-\frac{1}{R_P\omega C_P}\right)$

$= -\operatorname{atan}\left(\frac{1}{R_P\omega C_P}\right)$

$$Z_P = \frac{Z_{P1}}{Z_{P2}} = \frac{|Z_{P1}|}{|Z_{P2}|}\exp\left((\varphi_{P1} - \varphi_{P2})j\right)$$

$$Z_P = \underbrace{\frac{R_P}{\sqrt{R_P^2\omega^2 C_P^2 + 1}}}_{|Z_P|}\exp\Big(\big(\underbrace{-\tfrac{\pi}{2} + \operatorname{atan}\left(\tfrac{1}{R_P\omega C_P}\right)}_{\varphi_{ZP}}\big)j\Big)$$

$\cos\left(x - \frac{\pi}{2}\right) = \sin(x)$

$$Re(Z_P) = |Z_P|\cos(\varphi_{ZP}) = \frac{R_P}{\sqrt{R_P^2\omega^2 C_P^2 + 1}}\cos\left(-\frac{\pi}{2} + \operatorname{atan}\left(\frac{1}{R_P\omega C_P}\right)\right)$$

$$\underline{\underline{Re(Z_P) = \frac{R_P}{\sqrt{R_P^2\omega^2 C_P^2 + 1}}\sin\left(-\operatorname{atan}\left(\frac{1}{R_P\omega C_P}\right)\right) = R_R}}$$

$\sin\left(x - \frac{\pi}{2}\right) = -\cos(x)$

$$Im(Z_P) = |Z_P|\sin(\varphi_{ZP}) = \frac{R_P}{\sqrt{R_P^2\omega^2 C_P + 1}}\sin\left(\frac{\pi}{2} + \operatorname{atan}\left(\frac{1}{R_P\omega C_P}\right)\right)$$

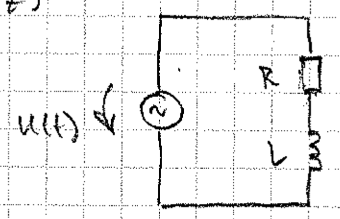
$$Im(Z_P) = \frac{R_P}{\sqrt{R_P^2\omega^2 C_P^2 + 1}}\cos\left(\operatorname{atan}\left(\frac{1}{R_P\omega C_P}\right)\right)$$

$$\underline{\underline{C_R = -\frac{1}{\omega\, Im(Z_P)} = -\frac{1}{\omega}\,\frac{\sqrt{R_P^2\omega^2 C_P^2 + 1}}{R_P\cos\left(\operatorname{atan}\left(1/R_P\omega C_P\right)\right)}}}$$

GET ② Rechenübung ⑱ Aufgabe ④ — Markus [illegible] — 28.04.2025

$$Z = \frac{20+12j}{8-10j}\,\Omega$$

ges: R, L für $Z(f=50\,Hz)$



$$Z = R + j\omega L = Re(Z) + Im(Z)j$$

$$\rightsquigarrow R = Re(Z) = |Z| \cos(\varphi_Z)$$

$$\rightsquigarrow L = \frac{Im(Z)}{\omega} = \frac{|Z|}{\omega}\sin(\varphi_Z)$$

$$Z_1 = 10+6j$$
$$|Z_1| = \sqrt{136}$$
$$\varphi_1 = atan\left(\frac{3}{5}\right)$$

$$Z = \frac{20+12j}{8-10j}\,\Omega = \frac{10+6j}{4-5j}\,\Omega = \frac{Z_1}{Z_2}\,\Omega$$

$$Z_2 = 4-5j$$
$$|Z_2| = \sqrt{41}$$
$$\varphi_2 = -atan\left(\frac{5}{4}\right)$$

$$Z = \frac{|Z_1|}{|Z_2|}\exp((\varphi_1-\varphi_2)j)$$

$$= \underbrace{\sqrt{\left(\frac{136}{41}\right)}}_{|Z|}\exp\Big(\underbrace{\left(atan\left(\tfrac{3}{5}\right)+atan\left(\tfrac{5}{4}\right)\right)}_{\varphi_Z}j\Big)\,\Omega$$

$$Re(Z) = |Z|\cos(\varphi_Z) = \sqrt{\left(\frac{136}{41}\right)}\cos\left(atan\left(\frac{3}{5}\right)+atan\left(\frac{5}{4}\right)\right)\Omega = R$$

$$\rightsquigarrow \underline{\underline{R = 0{,}2439\,\Omega}}$$

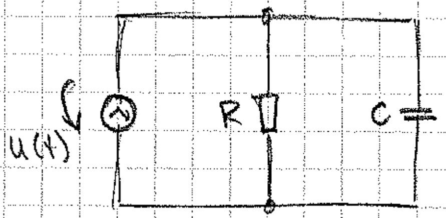
$$Im(Z) = |Z|\sin(\varphi_Z) = \sqrt{\left(\frac{136}{41}\right)}\sin\left(atan\left(\frac{3}{5}\right)+atan\left(\frac{5}{4}\right)\right)\Omega = 1{,}8049\,\Omega$$

$$\rightsquigarrow L = \frac{Im(Z)}{\omega} = \frac{1{,}8049\,V\,s}{2\pi\cdot 50\,A} = \underline{\underline{5{,}745\cdot 10^{-3}\,\frac{Vs}{A}}}$$

GET ② Rechenübung ⑱ Aufgabe ⑤ Markus Symmank 28.04.2025

$$Z = \frac{(10-5j)(10-2j)}{2+j}\,\Omega + 50\,\Omega \exp(-15^\circ j)$$

ges: R, C für $Z(f = 1\,\text{MHz})$



Ansatz: $\frac{1}{Z} = \frac{1}{R} + \frac{1}{X_C} = \frac{1}{R} + j\omega C$

$$\frac{1}{Z} = Z^{-1} = \mathrm{Re}(Z^{-1}) + \mathrm{Im}(Z^{-1})\,j = \frac{1}{R} + j\omega C$$

$$\rightsquigarrow \frac{1}{R} = \mathrm{Re}(Z^{-1}) \rightsquigarrow \underline{R = \frac{1}{\mathrm{Re}(Z^{-1})}} = \qquad \omega C = \mathrm{Im}(Z^{-1}) \rightsquigarrow \underline{C = \frac{\mathrm{Im}(Z^{-1})}{\omega}}$$

$$Z = \frac{Z_{11}\,Z_{12}}{Z_2} + Z_3 \qquad Z_{11} \cdot Z_{12} = Z_1 = (10-5j)(10-2j)$$

$$= 100 - 20j - 50j + 10j^2$$

$$Z = \frac{Z_1}{Z_2} + Z_3 \qquad = 100 - 70j - 10 = \underline{90 - 70j}$$

	$\lvert\cdot\rvert$	φ	Re(z)	Im(z)	(φ[°])
Z_1	114,018	-0,661	90	-70	-37,875
Z_2	2,236	0,464	2	1	26,565
Z_1/Z_2	50,99	-1,125	22	-46	-64,44
Z_3	50	-0,262	48,296	-12,941	-15
Z	91,737	-0,6978	70,296	-58,941	-39,979

$$\frac{1}{Z} = \frac{1}{|Z|} \exp(-\varphi_Z j) = \underbrace{\frac{1}{91{,}737}}_{|Z^{-1}|} \exp(\underbrace{0{,}6978}_{\varphi(Z^{-1})} j)$$

$$\mathrm{Re}(Z^{-1}) = |Z^{-1}| \cos(\varphi(Z^{-1})) = 8{,}3531 \cdot 10^{-3}$$

$$\mathrm{Im}(Z^{-1}) = |Z^{-1}| \sin(\varphi(Z^{-1})) = 7{,}0038 \cdot 10^{-3}$$

$$\underline{R = \frac{1}{\mathrm{Re}(Z^{-1})} = 119{,}72\ \Omega} \qquad \underline{C = \frac{\mathrm{Im}(Z^{-1})}{\omega} = \frac{7{,}0038 \cdot 10^{-3}\,\mathrm{AS}}{2\pi \cdot 10^6\ \mathrm{V}} = 1{,}1147\ \mathrm{nF}}$$