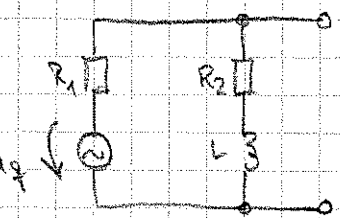
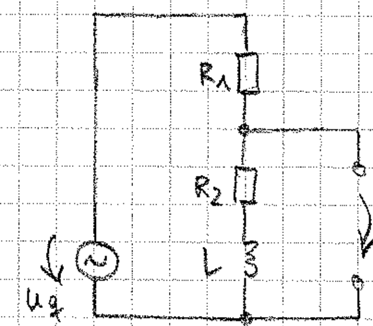
GET② Rechenübung ⑲ Aufgabe ①   Markus Symmank   30.04.2025

geg: $u_q = \hat{u}_q \sin(\omega t)$ ges: U_{qers} , I_{qers} , Z_{iers}



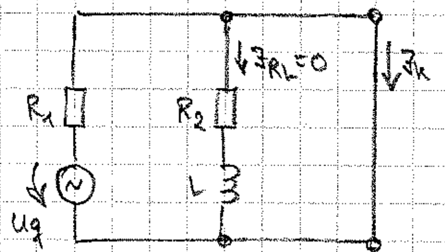
$U_{qers} = U_L$ (Leerlaufspannung)



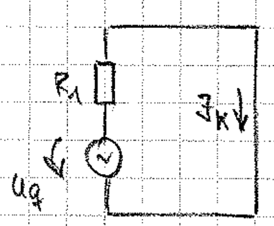
Anwendung Spannungsteiler

$$U_{qers} = U_L = \frac{R_2 + j\omega L}{R_1 + R_2 + j\omega L} \cdot U_q$$

$I_{qers} = I_K$ (Kurzschluss)

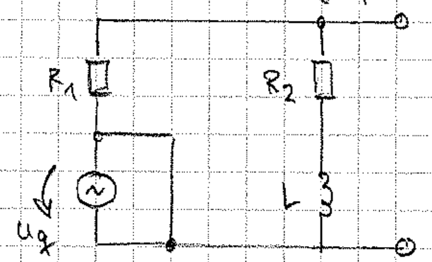


↓ kein Stromfluss durch den mittleren Pfad

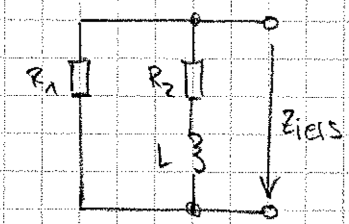


$$I_{qers} = I_K = \frac{U_q}{R_1}$$

Z_{iers} (Kurzschluss der Spannungsquelle)



Betrachtung als Widerstandsnetzwerk

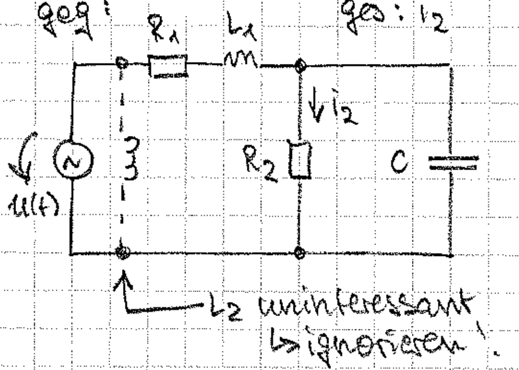


$$Z_{ier} = R_1 \parallel (R_2 + j\omega L)$$

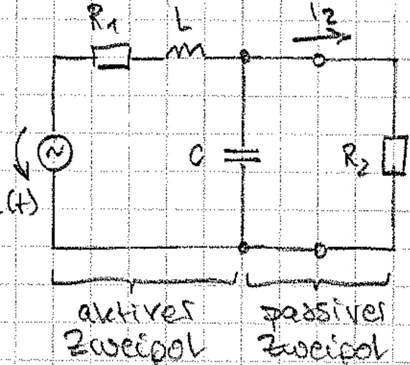
$$Z_{iers} = \frac{R_1 R_2 + R_1 j\omega L}{R_1 + R_2 + j\omega L}$$

GET ② Rechenübung ⑲ Aufgabe ② Markus Szymanke 30.04.2025

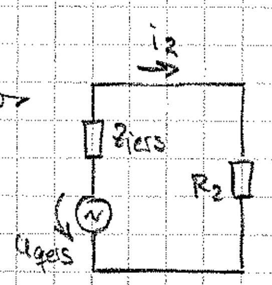
geg: ges: i_2



$\rightsquigarrow$

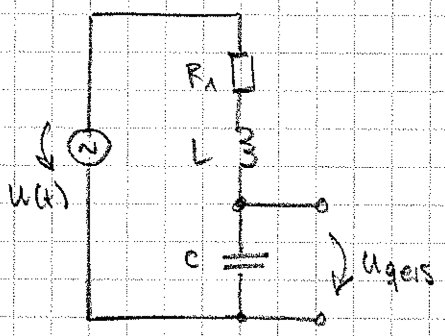


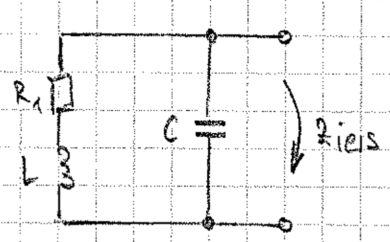
$\rightsquigarrow$



$u_{ers} = I_2 \cdot (Z_{ers} + R_2)$

$\rightsquigarrow I_2 = \frac{u_{ers}}{Z_{ers}+R_2}$





$$Z_{ers} = \frac{1}{j\omega C} \| (R_1 + j\omega L)$$

$$= \frac{\frac{1}{j\omega C}(R_1+j\omega L)}{\frac{1}{j\omega C}+j\omega L+R_1}$$

$$= \frac{R_1+j\omega L}{1-\omega^2 LC+jR_1\omega C}$$

$$u_{ers} = \frac{u X_C}{R_1+X_L+X_C} = \frac{\frac{u}{j\omega C}}{R_1+j\omega L+\frac{1}{j\omega C}}$$

$$u_{ers} = \frac{u}{1-\omega^2 LC+jR_1\omega C}$$

$$I_2 = \frac{u_{ers}}{Z_{ers}+R_2} \rightsquigarrow \frac{1}{I_2} = \frac{Z_{ers}+R_2}{u_{ers}} = \frac{\frac{R_1+j\omega L}{\cancel{1-\omega^2 LC+jR_1\omega C}}}{\frac{u}{\cancel{1-\omega^2 LC+jR_1\omega C}}} + \frac{R_2}{\frac{u}{1-\omega^2 LC+jR_1\omega C}}$$

$$\frac{1}{I_2} = \frac{R_1+j\omega L}{u} + \frac{R_2}{u}(1-\omega^2 LC+jR_1\omega C) \quad \Big| \cdot u$$

$$\frac{u}{I_2} = R_1+j\omega L + R_2(1-\omega^2 LC) + R_1R_2\omega C j = \underbrace{R_1+R_2(1-\omega^2 LC)}_{a} + j\omega\underbrace{(L+R_1R_2C)}_{b}$$

$$\frac{u}{I_2} = a+jb = \underbrace{\sqrt{(a^2+b^2)}}_{A}\exp(\underbrace{\mathrm{atan}(\tfrac{b}{a})}_{\varphi}j)$$

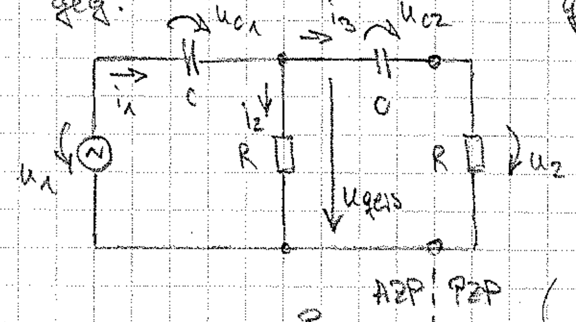
$$u = \hat{u}\sin(\omega t) = \hat{u}\exp(j\omega t)$$

$$\frac{u}{I_2} = A\exp(\varphi j) \rightsquigarrow \frac{I_2}{u} = \frac{1}{A}\exp(-\varphi j) \quad \Big|\cdot u \rightsquigarrow I_2 = \frac{\hat{u}}{A}\exp(j(\omega t-\varphi))$$

$$I_2 = \frac{\hat{u}}{\sqrt{((R_1+R_2(1-\omega^2 LC))^2+\omega^2(L+R_1R_2C)^2)}}\exp\left(j\left(\omega t - \mathrm{atan}\left(\frac{\omega(L+R_1R_2C)}{R_1+R_2(1-\omega^2 LC)}\right)\right)\right)$$

GET ② Rechenübung ⑲ Aufgabe ③ - I   Markus Sgummank   02.05.2025

geg:



ges: R u. C für $\varphi(u_2) = \varphi(u_1) + \frac{\pi}{2}$

⇝ Ansatz: $u_1 = \hat{u}_1 \exp(j\omega t)$

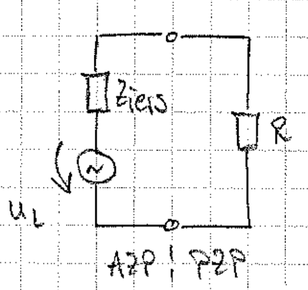
$u_2 = \hat{u}_2 \exp\left(j\left(\omega t + \frac{\pi}{2}\right)\right)$

$$\frac{u_1}{u_2} = \frac{\hat{u}_1}{\hat{u}_2} \exp\left(j\left(\omega t - \omega t - \frac{\pi}{2}\right)\right) = \frac{\hat{u}_1}{\hat{u}_2} \underbrace{\exp\left(-j\frac{\pi}{2}\right)}_{\text{rein imaginär}}$$

$$\frac{u_1}{u_2} = 0 - \frac{\hat{u}_1}{\hat{u}_2} j = -kj = \frac{k}{j}$$

$$u_{quers} = u_L = \frac{R}{R + \frac{1}{j\omega C}} u_1$$

$$= \frac{Rj\omega C}{Rj\omega C + 1} u_1$$



$$Z_{ers} = \frac{1}{j\omega C} + \frac{R \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{j\omega C} + \frac{R}{Rj\omega C + 1}$$

$$u_2 = \frac{R}{R + Z_{ers}} \cdot u_L = \frac{R}{R + \frac{1}{j\omega C} + \frac{R}{Rj\omega C + 1}} \cdot \frac{Rj\omega C}{Rj\omega C + 1} u_1$$

$$\frac{u_1}{u_2} = \frac{k}{j} = \frac{1}{R^2 j\omega C}\left(R + \frac{1}{j\omega C} + \frac{R}{Rj\omega C + 1}\right)(Rj\omega C + 1) \quad |\cdot j$$

$$k = \frac{1}{R^2\omega C}\left(R \cdot \frac{Rj\omega C + 1}{Rj\omega C + 1} + \frac{1}{j\omega C} + \frac{R}{Rj\omega C + 1}\right)(Rj\omega C + 1)$$

$$= \frac{1}{R^2\omega C}\left(\frac{R^2 j\omega C + R}{Rj\omega C + 1} + \frac{R}{Rj\omega C + 1} + \frac{1}{j\omega C}\right)(Rj\omega C + 1)$$

$$= \frac{1}{R^2\omega C}\left(\frac{R^2 j\omega C + 2R}{Rj\omega C + 1} + \frac{1}{j\omega C}\right)(Rj\omega C + 1) = \frac{1}{R^2\omega C}\left(2R + R^2 j\omega C + R + \frac{1}{j\omega C}\right)$$

$$= \frac{1}{R^2\omega C}\left(3R + j\left(R^2\omega C - \frac{1}{\omega C}\right)\right) \rightsquigarrow k = \frac{3R}{R^2\omega C} + \underbrace{j\frac{R^2\omega C - \frac{1}{\omega C}}{R^2\omega C}}$$

k ist rein reell, da das Verhältnis der Spannungen sonst aus dem 90° Phasenversatz „herausdreht". Daher muss der Imaginärteil Null sein.

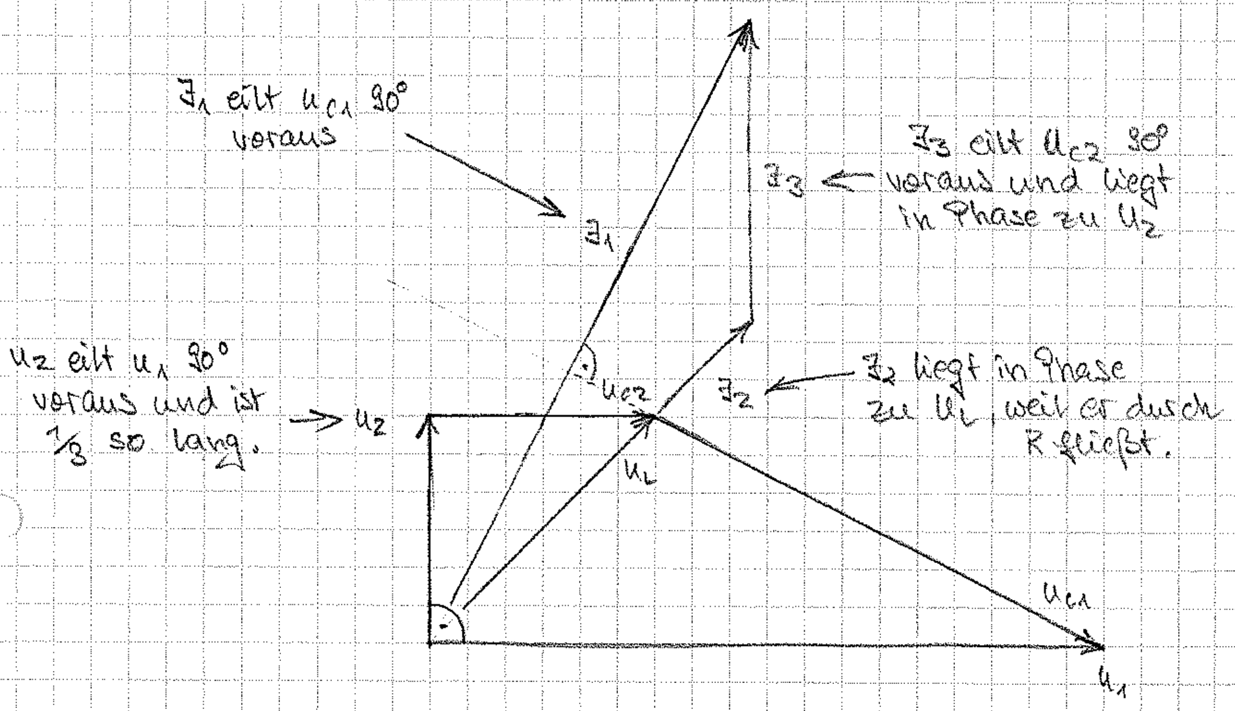
$$\mathrm{Im}(k) = 0 = \frac{R^2\omega C - \frac{1}{\omega C}}{R^2\omega C} = 0 = R^2\omega C - \frac{1}{\omega C} \Longleftarrow$$

$$\rightsquigarrow \frac{1}{\omega C} = R^2\omega C \rightsquigarrow 1 = R^2\omega^2 C^2$$

$$\rightsquigarrow R\omega C = 1 \rightsquigarrow \underline{\underline{R = \frac{1}{\omega C}}}$$

$$k = \frac{3}{R\omega C} = \frac{\hat{u}_1}{\hat{u}_2} = \frac{u_1}{u_2} \quad \text{mit } R\omega C = 1$$

$$\rightsquigarrow \frac{u_1}{u_2} = 3 \quad \text{oder} \quad \underline{\underline{u_2 = \frac{1}{3} u_1}}$$


I_1 eilt U_{C1} 90° voraus
I_3 eilt U_{C2} 90° voraus und liegt in Phase zu U_2
I_3
I_1
U_2 eilt U_1 90° voraus und ist 1/3 so lang.
U_2
U_{C2}
I_2
I_2 liegt in Phase zu U_L, weil er durch R fließt.
U_L
U_{C1}
U_1