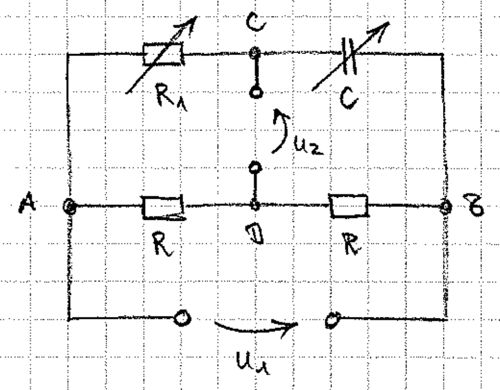
GET (2) Rechenübung (20) Aufgabe (1)   Markus Szymanski   05.05.2025

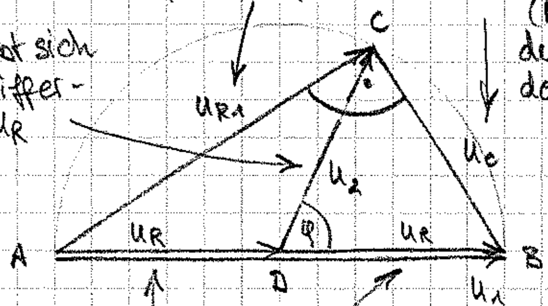
Hausrathbrücke



Qualitatives Zeigerdiagramm

(3) u_1 teilt sich in zwei 90° phasenversetzte Teilspannungen u_{R1} und u_c auf (Anwendung Satz des Thales)

(4) u_2 ergibt sich aus der Differenz $u_{R1} - u_R$



(2) u_1 teilt sich auf zwei gleiche Teilspannungen u_R auf

(1) u_1 als Bezugsgröße

Suche $|u_2| = f(|u_1|)$

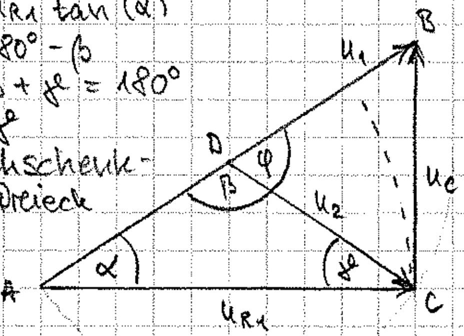
⇝ Aus dem Zeigerdiagramm ergibt sich, dass u_2 immer vom Mittelpunkt zum Rand des Halbkreises führt. Betragsmäßig entspricht u_2 somit immer dem Radius.

$$|u_2| = f(|u_1|) = |u_R| = \underline{\underline{\frac{1}{2}|u_1|}}$$

Suche $\varphi(R, \omega, C)$

Geometrische Betrachtung zeigt

- $u_c = u_{R1} \tan(\alpha)$
- $\varphi = 180° - \beta$
- $\alpha + \beta + \gamma = 180°$
- $\alpha = \gamma$ (gleichschenkliges Dreieck ACD)



$$\alpha + \beta + \gamma = 2\alpha + \beta = 180° = \pi$$

$$\rightsquigarrow 2\alpha = \pi - \beta$$

$$\varphi = 180° - \beta = \pi - \beta = 2\alpha$$

$$\rightsquigarrow \varphi = 2\alpha$$

$$|u_c| = |u_{R1}| \tan(\alpha) \quad \rightsquigarrow \tan(\alpha) = \frac{|u_c|}{|u_{R1}|}$$

$$\rightsquigarrow \alpha = \operatorname{atan}\left(\frac{|u_c|}{|u_{R1}|}\right) \quad \rightsquigarrow \varphi = 2 \cdot \operatorname{atan}\left(\frac{|u_c|}{|u_{R1}|}\right)$$

$$\frac{u_{R1}}{u_1} = \frac{R_1}{R_1 + \frac{1}{j\omega C}} \qquad \frac{u_c}{u_1} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \quad \rightsquigarrow \quad \frac{u_c}{u_{R1}} = \frac{1}{Rj\omega C} = -\underbrace{\frac{1}{R\omega C}}_{\frac{|u_c|}{|u_{R1}|}} j$$

$$\underline{\underline{\varphi = 2 \cdot \operatorname{atan}\left(\frac{1}{R\omega C}\right)}}$$

$$\varphi = 90°, \text{ wenn } \beta = 90° \rightsquigarrow \varphi = \beta \rightsquigarrow |u_{R1}| = |u_c| \rightsquigarrow \underline{\underline{\frac{|u_c|}{|u_{R1}|} = 1 = \frac{1}{R\omega C}}}$$

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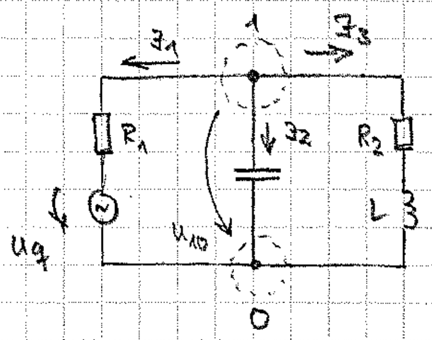


$$\underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0 \qquad G_1 = \frac{1}{R_1}$$

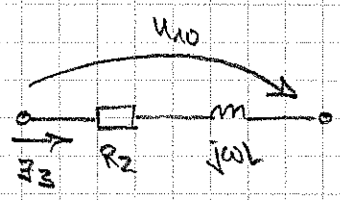
$$G_1(U_{10} - U_q) + \frac{U_{10}}{\frac{1}{j\omega C}} + \frac{U_{10}}{R_2 + j\omega L}$$

$$\frac{U_{10}}{R_1} - \frac{U_q}{R_1} + U_{10}\, j\omega C + \frac{U_{10}}{R_2 + j\omega L}$$

$$U_{10}\left(\frac{1}{R_1} + j\omega C + \frac{1}{R_2 + j\omega L}\right) = \frac{U_q}{R_1} \rightsquigarrow U_{10} = \frac{U_q}{1 + R_1 j\omega C + \frac{R_1}{R_2 + j\omega L}}$$

gesucht: $\underline{I}_3 \hat{=} i_2$

$$\underline{I}_3 = \frac{U_{10}}{R_2 + j\omega L} = \frac{U_q}{R_2 + j\omega L + R_1 R_2 j\omega C - R_1 \omega^2 CL + R_1}$$



$$\underline{I}_3 = \frac{U_q}{\underbrace{R_1 + R_2 - R_1\omega^2 CL}_{a} + j\underbrace{(\omega L + R_1 R_2 \omega C)}_{b}}$$

$$\underline{I}_3 = \frac{U_q}{\sqrt{(a^2+b^2)}} \exp\Big(\underbrace{-\operatorname{atan}\left(\frac{b}{a}\right)}_{\varphi_3} j\Big)$$

$$u_q(t) = \hat{u}_q \sin(\omega t + \varphi_{uq}) \rightsquigarrow U_q = \hat{u}_q\Big(\cos(\omega t - \underbrace{\varphi_{uq} - \frac{\pi}{2}}_{\varphi_q}) + j \sin(\omega t - \varphi_{uq} - \frac{\pi}{2})\Big)$$

$$U_q = \hat{u}_q \exp(\omega t - \varphi_q)$$

$$\underline{I}_3 = \frac{\hat{u}_q}{\sqrt{(a^2+b^2)}} \exp(\omega t - \varphi_q - \varphi_3)$$

$$\underline{I}_3 = \underline{I}_2 = \frac{\hat{u}_q}{\sqrt{((R_1+R_2-R_1\omega^2 CL)^2 + (\omega L + R_1 R_2 \omega C)^2)}} \exp\left(\left(\omega t - \varphi_{uq} - \frac{\pi}{2} - \operatorname{atan}\left(\frac{\omega L + R_1 R_2 \omega C}{R_1 + R_2 - R_1 \omega^2 CL}\right)\right) j\right)$$

Zeitfunktion (Realanteil des komplexen Stroms)

$$i_2(t) = \frac{\hat{u}_q}{\sqrt{((R_1+R_2-R_1\omega^2 CL)^2 + (\omega L + R_1 R_2 \omega C)^2)}} \sin\left(\omega t - \varphi_{uq} - \operatorname{atan}\left(\frac{\omega L + R_1 R_2 \omega C}{R_1 + R_2 - R_1 \omega^2 CL}\right)\right)$$

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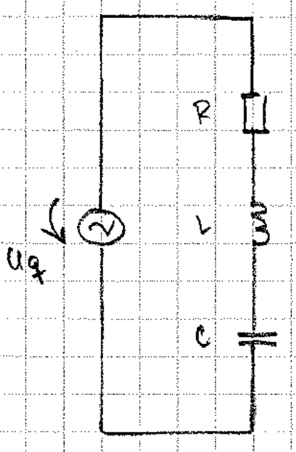


geg: $U_q = 230V$, $U_{cr} = 3{,}5kV$ (Spannung über Kondensator im Resonanzfall)

$L = 80H$, $f_r = 60Hz$

a) ges: C $\quad \omega_0 = \frac{1}{\sqrt{LC}} \rightsquigarrow \omega_0^2 = \frac{1}{LC} \rightsquigarrow C = \frac{1}{\omega_0^2 L}$

$$C = \frac{s^2 \quad A}{60^2 4\pi^2 \, 80 Vs} = 87{,}952416 \cdot 10^{-9} \frac{As}{V} = 88\,nF$$

b) ges: Q $\quad Q = \frac{U_{max}}{U_{ges}} = \frac{3500}{230} = 15{,}2173913$

c) Im Resonanzfall fällt die Quellspannung nur noch über dem ohm'schen Teil der Impedanz ab.

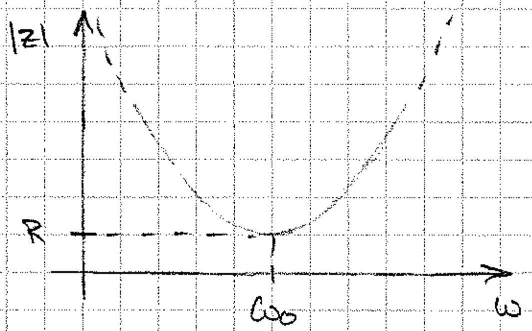
$$\rightsquigarrow I_{r,eff} = \frac{U_q}{R} \qquad Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$R = \frac{1}{Q}\sqrt{\frac{L}{C}} = \frac{1}{Q}\sqrt{\omega_0^2 L^2} = \frac{\omega_0 L}{Q}$$

$$= \frac{U_{ges}}{U_{max}} \omega_0 L = \frac{230}{3500} 2\pi \, 60 \frac{1}{s} \, 80 \frac{Vs}{A}$$

$$= 1981{,}896\,\Omega$$

$$I_{r,eff} = \frac{230V}{1982\Omega} = 116{,}05\,mA$$



$\omega \to 0$: $|Z| \to \infty$

ω_0 : $|Z| = R$

$\omega \to \infty$: $|Z| \to \infty$

d) $f(I_{eff} = \frac{1}{10} I_{reff})$

$$\frac{I_{reff}}{I_{eff}} = 10 = \frac{|Z|}{R} = \frac{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}{R} \quad |\cdot R$$

$$10R = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} \quad |(\cdot)^2 \rightsquigarrow 100R^2 = R^2 + (\omega L - \frac{1}{\omega C})^2 \quad |-R^2$$

$$\boxed{*1:\ 99R^2 = (\omega L - \frac{1}{\omega C})^2} \quad |\sqrt{\cdot} \rightsquigarrow \sqrt{99}R = \omega L - \frac{1}{\omega C} \quad |\cdot \omega C$$

$$\sqrt{99}R\omega C = \omega^2 LC - 1 \quad |-\sqrt{99}R\omega C \rightsquigarrow 0 = \omega^2 LC - \sqrt{99}R\omega C - 1 \quad |\cdot \frac{1}{LC}$$

$$0 = \omega^2 - \sqrt{99}\frac{R}{L}\omega - \frac{1}{LC} \quad \text{mit } R = \frac{\omega_0 L}{Q},\ \frac{R}{L} = \frac{\omega_0}{Q} \text{ und } \frac{1}{LC} = \omega_0^2$$

$$\rightsquigarrow 0 = \omega^2 \underbrace{- \sqrt{99}\frac{\omega_0}{Q}}_{p}\omega \underbrace{- \omega_0^2}_{q}$$

*2: Wenn man das neg. VZ rechnerisch vermeiden will, muss man bei *1 die Quadrierung ausführen, ω^4 zu ω'^2 substituieren und von $\omega'_{1/2}$ die Wurzel ziehen.

$$\omega_{1/2} = -\frac{p}{2} \pm \sqrt{(\frac{p}{2})^2 - q} = \frac{\sqrt{99}\,\omega_0}{2Q} \pm \sqrt{\frac{99\omega_0^2}{4Q^2} + \omega_0^2} = \omega_0\left(\frac{\sqrt{99}}{2Q} \pm \sqrt{\frac{99}{4Q^2} + 1}\right)$$

$$f_{1/2} = \frac{\omega_0}{2\pi}\left(\frac{\sqrt{99}}{2Q} \pm \sqrt{\frac{99}{4Q^2} + 1}\right) = \{82{,}74\,Hz;\ \overset{*2}{-43{,}51\,Hz}\} \overset{!}{=} \{82{,}74\,Hz;\ 43{,}51\,Hz\}$$

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Alternative (vorzeichenrichtige) Frequenzberechnung zu (d)

$$\frac{|Z|}{R} = 10 = \frac{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}{R} \quad \Big|\cdot R \quad \Big| (\cdot)^2$$

$$100 R^2 = R^2 + (\omega L - \frac{1}{\omega C})^2 \quad \Big| - R^2 \rightsquigarrow 99 R^2 = (\omega L - \frac{1}{\omega C})^2 \quad \Big| - 99 R^2$$

$$0 = -99R^2 + (\omega L - \frac{1}{\omega C})^2 = -99R^2 + \omega^2 L^2 - 2\frac{L}{C} + \frac{1}{\omega^2 C^2} \quad \Big| \cdot \omega^2 C^2$$

$$0 = -99R^2\omega^2C^2 + \omega^4 L^2 C^2 - 2LC\omega^2 + 1 = \omega^4 L^2 C^2 - (99R^2C^2 + 2LC)\omega^2 + 1 \quad \Big| \cdot \frac{1}{L^2C^2}$$

$$0 = \omega^4 - \frac{1}{L^2C^2}(99R^2C^2 + 2LC)\omega^2 + \frac{1}{L^2C^2}$$

$$R = \frac{\omega_0 L}{Q}, \quad \frac{R}{L} = \frac{\omega_0}{Q}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$0 = \omega^4 - (99\frac{R^2}{L^2} + \frac{2}{LC})\omega^2 + \frac{1}{L^2C^2}$$

$$\underbrace{0 = \omega^4 - (99\frac{\omega_0^2}{Q^2} + 2\omega_0^2)\omega^2 + \omega_0^4}_{} = \omega^4 - \omega_0^2(\frac{99}{Q^2} + 2)\omega^2 + \omega_0^4$$

$$x_{1/2} = -\frac{p}{2} \pm \sqrt{(\frac{p}{2})^2 - q}$$

$$= \omega'^2 \underbrace{- \omega_0^2(\frac{99}{Q^2} + 2)}_{p}\omega' + \underbrace{\omega_0^4}_{q}$$

$$\omega'_{1/2} = \frac{\omega_0^2}{2}(\frac{99}{Q^2} + 2) \pm \sqrt{\frac{\omega_0^4}{4}(\frac{99}{Q^2} + 2)^2 - \omega_0^4}$$

$$= \omega_0^2\left(\frac{1}{2}(\frac{99}{Q^2} + 2) \pm \sqrt{\frac{1}{4}(\frac{99}{Q^2} + 2)^2 - 1}\right)$$

$$= \left\{270268\ \frac{1}{s^2}\ ;\ 74735\ \frac{1}{s^2}\right\}$$

$$\omega_1 = \sqrt{\omega'_1} = 519{,}9\ Hz \rightsquigarrow \underline{\underline{f_1 = \frac{\omega_1}{2\pi} = 82{,}74\ Hz}}$$

$$\omega_2 = \sqrt{\omega'_2} = 273{,}4\ Hz \rightsquigarrow \underline{\underline{f_2 = \frac{\omega_2}{2\pi} = 43{,}51\ Hz}}$$