a)

$$U_{ers} = U_1 + U_2$$

$$L_{ers} \frac{d\bar{i}}{dt} = L_1 \cdot \frac{d\bar{i}}{dt} + L_2 \frac{d\bar{i}}{dt}$$

$$\leadsto L_{ers} = L_1 + L_2$$

b)
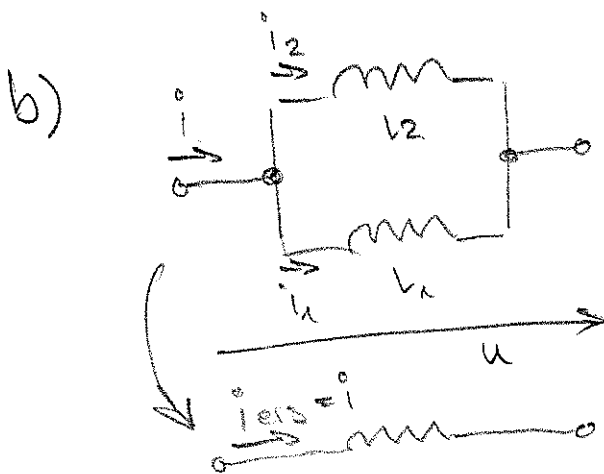
$$i_{ers} = i_1 + i_2 \quad \Big| \frac{d}{dt}$$

$$\frac{d i_{ers}}{dt} = \frac{d i_1}{dt} + \frac{d i_2}{dt}$$

$$\boxed{\frac{di}{dt} = \frac{u}{L}}$$

$$\leadsto \frac{1}{L_{ers}} = \frac{1}{L_1} + \frac{1}{L_2}$$

$$\frac{u}{L_{ers}} = \frac{u}{L_1} + \frac{u}{L_2}$$

$$1 = \frac{L_{ers}}{L_1} + \frac{L_{ers}}{L_2}$$

$$L_2 \cdot L_1 = L_{ers} L_1 + L_{ers} L_2$$

$$= L_{ers} (L_1 + L_2)$$

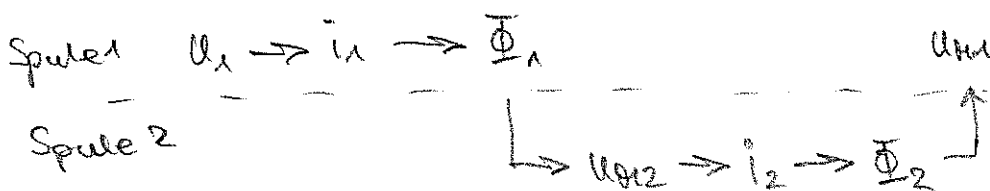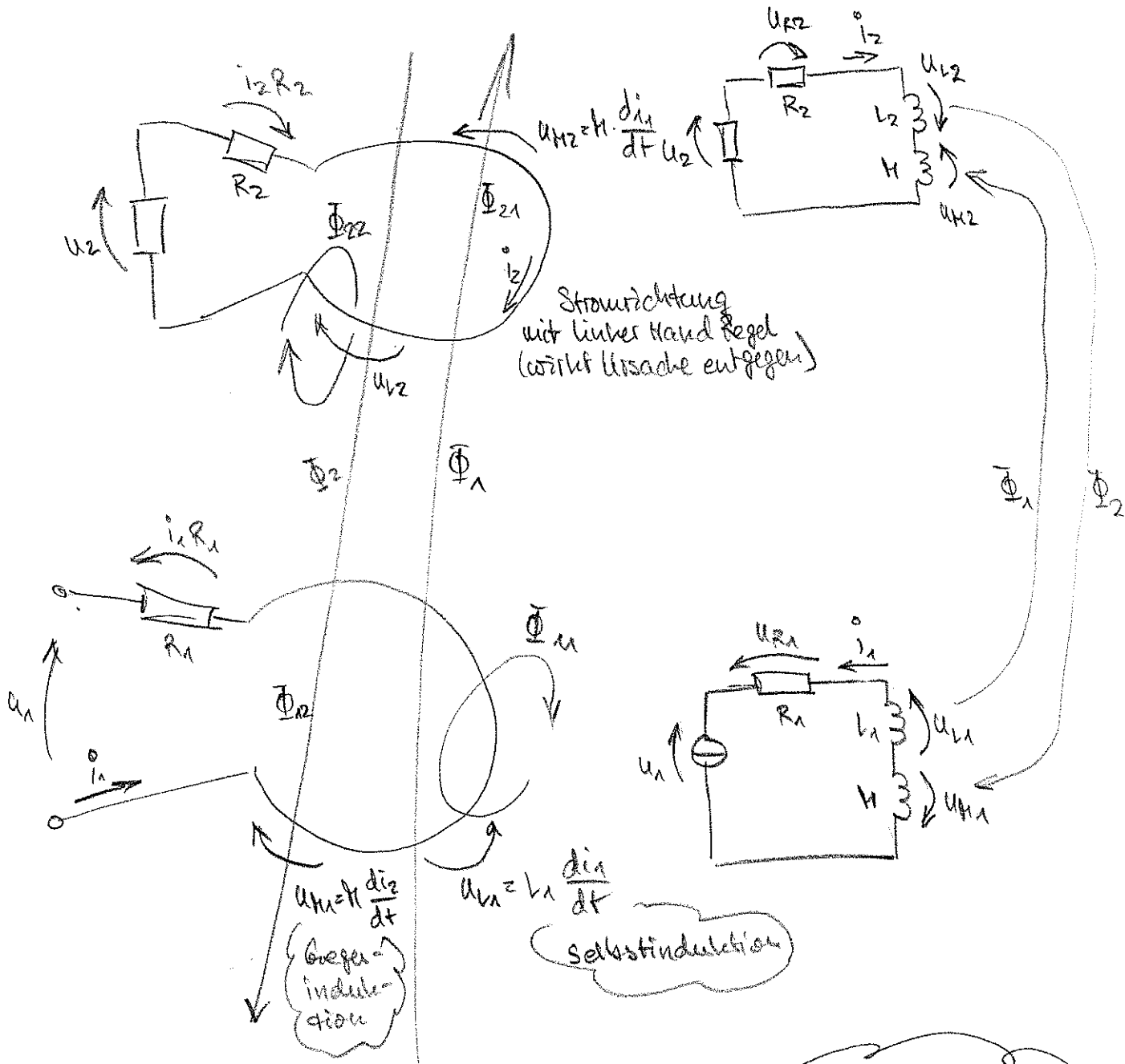$$\leadsto L_{ers} = \frac{L_1 \cdot L_2}{L_1 + L_2}$$

$$u_1 = u_{R1} + u_{L1} - u_{M1}$$

$$u_1 = i_1 R_1 + \frac{di_1}{dt} L_1 - \frac{di_2}{dt} M$$

$$u_2 + u_{R2} + u_{L2} = u_{M2}$$

$$u_2 = u_{M2} - u_{R2} - u_{L2}$$

$$u_2 = \frac{di_1}{dt} M - i_2 R_2 - \frac{di_2}{dt} L_2$$

$i_2 R_2$

R_2

u_2

Φ_{22} Φ_{21}

i_2

$u_{M2} = M \cdot \frac{di_1}{dt}$ u_2

u_{L2}

u_{R2} i_2

R_2 L_2

M

u_{M2}

Stromrichtung
mit linker Hand Regel
(wirkt Ursache entgegen)

$\overline{\Phi}_2$ $\overline{\Phi}_1$

Φ_1 Φ_2

$i_1 R_1$

R_1

Φ_{12}

Φ_{11}

u_1

i_1

u_{R1} i_1

R_1 L_1 u_{L1}

u_1

M u_{M1}

$u_{M1} = M \frac{di_2}{dt}$ $u_{L1} = L_1 \frac{di_1}{dt}$

(Gegen-
induk-
tion)

Selbstinduktion

Spule 1 $u_1 \rightarrow i_1 \rightarrow \overline{\Phi}_1$ u_{M1}

Spule 2 $\longmapsto u_{M2} \rightarrow i_2 \rightarrow \overline{\Phi}_2$

$\overline{\Phi}_{ij}$ = Fluss erzeugt durch
Strom j geht durch Spule i

$M = k \cdot \sqrt{(L_1 \cdot L_2)}$

$k = \sqrt{(1 - \delta)}$

Streufluss

0 1

kein Streu-
fluss   keine
induktive
Kopplung

$$W_m = \frac{1}{2} L J^2 \quad \text{mit } L = N^2 \mu \frac{A}{l} \quad \text{und} \quad J = \frac{U}{R_{cu}}$$

$$W_m = \frac{1}{2} N^2 \mu \frac{A}{l} \frac{U^2}{R_{cu}^2} = \frac{1}{2} N^2 \mu_r \mu_0 \frac{\pi d_2^2}{4 \pi d_1} \cdot \frac{U^2}{R_{cu}^2}$$

$$= \frac{1}{2} N^2 \mu_r 4\pi \cdot 10^{-7} \left[\frac{Vs}{Am}\right] \frac{d_2^2}{4 \cdot 2{,}5 \, d_2} \frac{U^2}{R_{cu}^2}$$
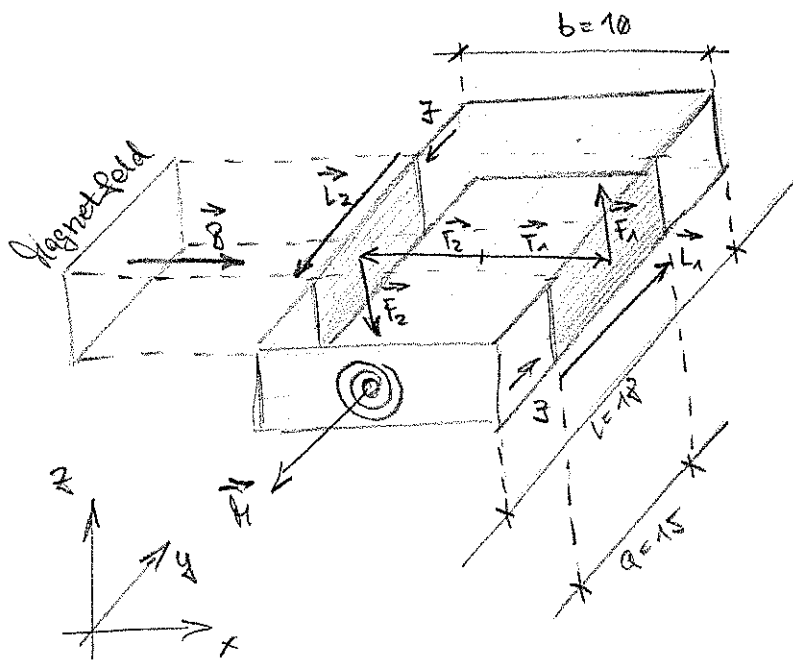
$$= 2\pi N^2 \mu_r \cdot 10^{-8} \left[\frac{Vs}{Am}\right] d_2 \frac{U^2}{R_{cu}^2}$$

$$= 2\pi \cdot \underbrace{2{,}25 \cdot 10^6}_{1500^2} \cdot 2{,}17 \cdot 10^{-8} \frac{Vs}{Am} \cdot 20 \cdot 10^{-3} \, m \cdot \frac{\overbrace{144 \, V^2 A^2}^{(12V)^2}}{\underbrace{2704 \, V^2}_{(52\Omega)^2}}$$

$$= 2\pi \cdot 2{,}25 \cdot 2{,}17 \cdot 10^{-5} \cdot 20 \cdot \frac{144}{2704} \, VAs$$

$$= 90\pi \cdot 2{,}17 \cdot \frac{144}{2704} \cdot 10^{-5} \, VAs = 32674 \cdot 10^{-6} \, Ws$$

$$= 32{,}7 \cdot 10^{-3} \, Ws$$

Magnetfeld $\vec{B}$ $b=10$

Ansatz: Lorentzkraft

$$\vec{F} = Q(\vec{v} \times \vec{B})$$

Geschwindig-keit der Ladungsträger ← → Flussrichtung des ext. Magnetf.

mit $Q = J \cdot t$ und $\vec{v} = \frac{\vec{s}}{t}$

$$\vec{F} = J \cancel{t} \cdot \frac{1}{\cancel{t}} (\vec{s} \times \vec{B})$$

$\downarrow J \cdot N$

$$\rightarrow \vec{F} = J \cdot N (\vec{s} \times \vec{B})$$

Weg des Stromes durch d. Magnetf.
→ Betrag $\hat{=} a$

$$\vec{F_1} = J_E (\vec{L_1} \times \vec{B}) = J_E \begin{pmatrix} 0 \\ 0 \\ -aB \end{pmatrix}$$

$$\begin{matrix} a & 0 \\ 0 & 0 \\ 0 & B \\ a & 0 \end{matrix}$$

$$\vec{F_2} = J_E (\vec{L_2} \times \vec{B}) = J_E \begin{pmatrix} 0 \\ 0 \\ aB \end{pmatrix}$$

$$\begin{matrix} -a & 0 \\ 0 & 0 \\ 0 & B \\ -a & 0 \end{matrix}$$

Wiederholung Kreuzprodukt

$$\vec{a} \times \vec{b} = \begin{Bmatrix} a_y & b_y \\ a_z & b_z \\ a_x & b_x \\ a_y & b_y \end{Bmatrix} = \begin{pmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{pmatrix}$$

$$\vec{L_1} = \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} \quad \vec{L_2} = \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix}$$

$$\vec{M} = \sum \vec{M_n}$$

$$\vec{M} = \vec{M_1} + \vec{M_2}$$

$$\vec{M} = \vec{r_1} \times \vec{F_1} + \vec{r_2} \times \vec{F_2}$$

mit $\vec{F_1} = J_E aB \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$, $\vec{r_1} = \frac{b}{2} \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$

$\vec{F_2} = J_E aB \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, $\vec{r_2} = \frac{b}{2} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$$\vec{M} = J_E aBb \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\vec{M_1} = \vec{r_1} \times \vec{F_1} = J_E aB \frac{b}{2} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\vec{M_2} = \vec{r_2} \times \vec{F_2} = J_E aB \frac{b}{2} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$M = J_E aBb = J \cdot N \cdot aB$$

$$= 2 \cdot 10^{-3} A \cdot 10^2 \cdot 15 \cdot 10^{-3} m \cdot 10 \cdot 10^{-3} m \cdot 0.8 \frac{Vs}{m^2}$$

$$= 300 \cdot 10^{-7} \cdot 0.8 \, VAs = 24 \cdot 10^{-6} \, Nm$$

$$\alpha = \frac{M}{c_F} = \frac{24 \cdot 10^{-6} \, Nm \cdot °}{0.49 \cdot 10^{-6} \, Nm} = \frac{24°}{0.49} = \underline{\underline{48,98°}}$$

z, y, x (coordinate axes)