GET ② Rechenübung ⑱ Aufgabe ① Markus Symmank 28.04.2025

$$I_1 = (\sqrt{2} + \sqrt{2}\,j)\,A$$
$$I_2 = 4A \exp(-j\,30^\circ)$$
$$I = I_1 + I_2$$

$$I_1 = (\underbrace{\sqrt{2}}_{\text{Re}(I_1) = \sqrt{2}A} + \underbrace{\sqrt{2}}_{\text{Im}(I_1) = \sqrt{2}A}\,j)\,A$$

$\underline{\text{Re}(I_1) = \sqrt{2}A}$ $\underline{\text{Im}(I_1) = \sqrt{2}A}$

$$|I_1| = \sqrt{\text{Re}^2(I_1) + \text{Im}^2(I_1)}$$
$$= \sqrt{2A^2 + 2A^2} = \sqrt{4A^2} = \underline{2A}$$

$$\text{Im}(z_1) = \text{Re}(z_1)\tan(\varphi_1) \rightsquigarrow \varphi_1 = \text{atan}\left(\frac{\text{Im}(I_1)}{\text{Re}(I_1)}\right)$$

$$\varphi_1 = \text{atan}\left(\frac{\sqrt{2}A}{\sqrt{2}A}\right) = \text{atan}(1) = \frac{\pi}{4} = \underline{45^\circ}$$

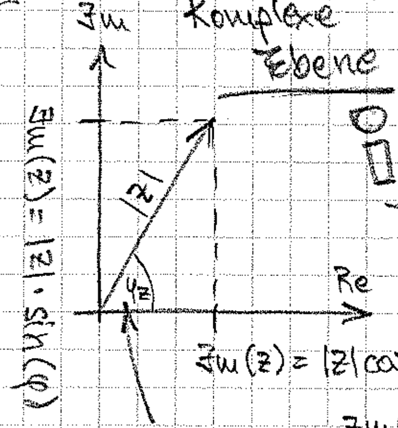
$$I_2 = 4A\exp(-j\,30^\circ) = \underbrace{4A}_{}\exp\Big(-j\underbrace{\frac{\pi}{6}}_{}\Big)$$

$$\underline{|I_2| = 4A} \qquad \underline{\varphi_2 = -\frac{\pi}{6}}$$

$$\text{Re}(I_2) = |I_2|\cos(\varphi) = 4A \cdot \cos\left(-\frac{\pi}{6}\right)$$
$$= 4A \cdot 0{,}866 = \underline{3{,}464\,A}$$

$$\text{Im}(I_2) = |I_2|\sin(\varphi) = 4A\sin\left(-\frac{\pi}{6}\right)$$
$$= 4A \cdot (-0{,}5) = \underline{-2A}$$

Erinnerung Komplexe Ebene



$$\text{Im}(z) = |z|\cos(\varphi)$$

$$\varphi_z = \text{atan}\left(\frac{\text{Im}(z)}{\text{Re}(z)}\right)$$

$$I = I_1 + I_2 = \text{Re}(I_1) + \text{Re}(I_2) + (\text{Im}(I_1) + \text{Im}(I_2))\,j$$

$$I = (\sqrt{2} + 3{,}464 + (\sqrt{2} - 2)\,j)\,A = \underline{\underbrace{4{,}878\,A}_{\text{Re}(I) = 4{,}878A} - \underbrace{0{,}586\,j\,A}_{\text{Im}(I) = -0{,}586A}}$$

$$|I| = \sqrt{4{,}878^2 + 0{,}586^2}\,A$$
$$= \underline{4{,}913\,A}$$

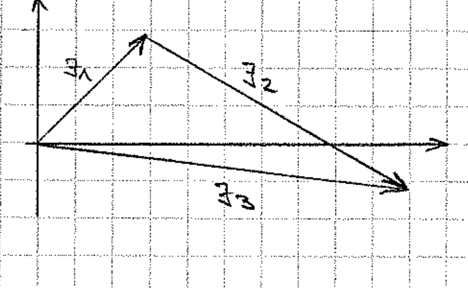
$$\varphi_I = \text{atan}\left(\frac{\text{Im}(I)}{\text{Re}(I)}\right) = \text{atan}\left(-\frac{0{,}586}{4{,}878}\right)$$

$$\underline{\varphi_I = -0{,}12 = -6{,}847^\circ}$$

$$I_1 = \sqrt{2}A + \sqrt{2}A\,j$$
$$I_1 = 2A\cos(45^\circ) + 2A\sin(45^\circ)\,j$$
$$I_1 = 2A\exp\left(\frac{\pi}{4}j\right)$$
$$I_2 = 3{,}464\,A - 2A\,j$$
$$I_2 = 4A\cos(-30^\circ) - 4A\sin(-30^\circ)\,j$$
$$I_2 = 4A\exp\left(-j\frac{\pi}{6}\right)$$
$$I_3 = 4{,}878\,A - 0{,}586\,A\,j$$
$$I_3 = 4{,}913\,A\cos(-6{,}847^\circ) - 4{,}913\sin(-6{,}847^\circ)\,j$$
$$I_3 = 4{,}913\,A\exp(-0{,}12\,j)$$

Lösung

Grafische Lösung



GET (2) Rechenübung (1) Aufgabe (2) Markus Symmank 28.04.2025

$Z_1 = (1+2j)\,k\Omega$ ges.: Z_2 mit doppeltem Scheinwiderstand durch Änderung der Induktivität

Scheinwiderstand
$S = |Z|$

$$|Z_2| = 2\,|Z_1| = 2\sqrt{(1^2+2^2)}\,k\Omega = 2\sqrt{5}\,k\Omega$$

$$|Z_2| = 2\sqrt{5}\,k\Omega = \sqrt{(R^2+\omega^2 L^2)} = \sqrt{(R^2+4\pi^2 f^2 L^2)} \quad \Big|\,(\cdot)^2$$

$$|Z_2|^2 = R^2 + 4\pi^2 f^2 L^2 \rightsquigarrow L^2 = \frac{|Z_2|^2 - R^2}{4\pi^2 f^2}$$

$$L = \sqrt{\left(\frac{|Z_2|^2 - R^2}{4\pi^2 f^2}\right)} \qquad R = 1\,k\Omega\,,\ f = 50\,Hz$$

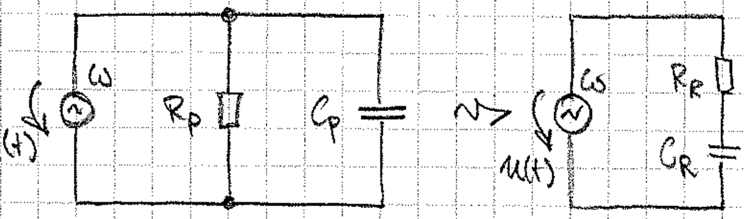
$$= \sqrt{\left(\frac{4\cdot 5\cdot 10^6\,\Omega^2 - 10^6\,\Omega^2}{4\cdot\pi^2\cdot 50^2\,1/s^2}\right)} = \sqrt{\left(\frac{(20-1)\cdot 10^6\,V^2s^2}{4\pi^2\cdot 50^2\,A^2}\right)}$$

$$= \sqrt{\left(\frac{19}{100^2\,\pi^2}\right)}\cdot 10^3\,\frac{Vs}{A} = \sqrt{\left(\frac{19}{\pi^2}\cdot 10^{-4}\right)}\cdot 10^3\,\frac{Vs}{A}$$

$$= \sqrt{\left(\frac{19}{\pi^2}\right)}\cdot 10\,\frac{Vs}{A} \rightsquigarrow \underline{\underline{L = \frac{\sqrt{19}}{\pi}\cdot 10\,\frac{Vs}{A} = 13{,}875\,\frac{Vs}{A}}}$$

GET ② Rechenübung ⑫ Aufgabe ②   Markus Szymanski   29.04.2025

geg: Z_p ges: $Z_R = Z_p$



Ansatz:

$$Z_R = Z_p = \underbrace{Re(Z_p)}_{} + \underbrace{Im(Z_p)}_{}\, j$$
$$= R_R - j\frac{1}{\omega C_R}$$

$$\leadsto \underline{\underline{R_R = Re(Z_p)}}$$

$$\leadsto \frac{1}{\omega C_R} = -Im(Z_p)$$

$$\leadsto \underline{\underline{C_R = -\frac{1}{\omega\, Im(Z_p)}}}$$

⇝ Finde Z_p in arithmetischer Form!

$$Z_p = R_p \parallel X_{C_p} = \frac{-R_p j \frac{1}{\omega C_p}}{R_p - j\frac{1}{\omega C_p}} = \frac{-R_p j}{R_p \omega C_p - j}$$

⇝ Konjugiert komplex erweitern, um j aus dem Nenner herauszubekommen

$$Z_p = \frac{-R_p j}{R_p \omega C_p - j} \cdot \frac{R_p \omega C_p + j}{R_p \omega C_p + j} = \frac{R_p - R_p^2 \omega C_p j}{R_p^2 \omega^2 C_p^2 + 1} = \underbrace{\frac{R_p}{R_p^2\omega^2 C_p^2 + 1}}_{Re(Z_p)} - \underbrace{\frac{R_p^2 \omega C_p}{R_p^2 \omega^2 C_p^2 + 1}}_{Im(Z_p)}\, j$$

$$\leadsto \underline{\underline{R_R = Re(Z_p) = \frac{R_p}{R_p^2 \omega^2 C_p^2 + 1}}}$$

$$C_R = -\frac{1}{\omega\, Im(Z_p)} = \frac{1}{\omega}\left(\frac{1}{R_p^2 \omega C_p} + \frac{\cancel{R_p^2}\omega^{\cancel{2}} C_p^{\cancel{2}}}{\cancel{R_p^2}\cancel{\omega C_p}}\right) = \frac{1}{\omega}\left(\frac{1}{R_p^2 \omega C_p} + \omega C_p\right)$$

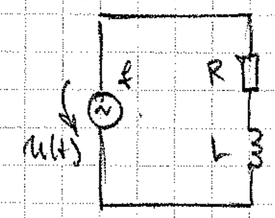
$$\underline{\underline{C_R = \frac{1}{R_p^2 \omega C_p} + C_p}}$$

GET (2) Rechenübung (18) Aufgabe (4) — Markus Symmank 28.04.2025

geg:

$$Z = \frac{20+12j}{8-10j}\,\Omega$$

ges: R, L für $Z(f = 50\,\text{Hz})$



$$Z = \mathrm{Re}(Z) + \mathrm{Im}(Z)\,j = R + j\omega L$$

$$\rightsquigarrow R = \mathrm{Re}(Z) \qquad \rightsquigarrow L = \frac{\mathrm{Im}(Z)}{\omega}$$

$$Z = \frac{20+12j}{8-10j}\,\Omega = \frac{10+6j}{4-5j}\,\Omega = \frac{10+6j}{4-5j}\cdot\frac{4+5j}{4+5j}\,\Omega = \frac{40+24j+50j-30}{16+25}\,\Omega$$

$$Z = \frac{10+74j}{41}\,\Omega = \frac{10}{41}\,\Omega + \frac{74j}{41}\,\Omega$$

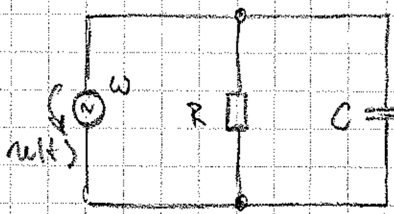
$$\underline{\underline{R = \frac{10}{41}\,\Omega = 0{,}2439\,\Omega}}$$

$$L = \frac{\mathrm{Im}(Z)}{\omega} = \frac{74}{2\pi 50\cdot 41}\,\frac{\text{Vs}}{\text{A}} = \frac{74}{4100\pi}\,\frac{\text{Vs}}{\text{A}} = \underline{\underline{5{,}745\,\text{mH}}}$$

GET (2) Rechenübung (17) Aufgabe (5) Markus Symmann 30.04.2025

$$Z = \frac{(10-5j)(10-2j)}{2+j}\,\Omega + 50\,\Omega \exp(-15^\circ j)$$

ges: R, C für $Z(f = 1\,\text{MHz})$



Ansatz: $\frac{1}{Z} = \frac{1}{R} + \frac{1}{X_C} = \frac{1}{R} + j\omega C$

$$\frac{1}{Z} = Z^{-1} = \text{Re}(Z^{-1}) + j\,\text{Im}(Z^{-1})$$

$$\leadsto \frac{1}{R} = \text{Re}(Z^{-1}) \leadsto \underline{\underline{R = \frac{1}{\text{Re}(Z^{-1})}}} \qquad \omega C = \text{Im}(Z^{-1}) \leadsto \underline{\underline{C = \frac{\text{Im}(Z^{-1})}{\omega}}}$$

$$Z = \underbrace{\frac{(10-5j)(10-2j)}{2+j}}_{Z_1}\,\Omega + \underbrace{50\exp(-15^\circ j)}_{Z_2}\,\Omega \qquad \varphi_{Rad} = \frac{-15^\circ}{180^\circ}\pi = -\frac{\pi}{12}$$

$$Z_1 = \frac{(10-5j)(10-2j)}{2+j}\cdot\frac{2-j}{2-j} = \frac{(100-50j-20j-10)(2-j)}{4+1} = \frac{(90-70j)(2-j)}{5}$$

$$= \frac{180-140j-90j-70}{5} = \frac{110-230j}{5} = \underline{\underline{22-46j}}$$

$$Z_2 = 50\cos\left(-\frac{\pi}{12}\right) + j\,50\sin\left(-\frac{\pi}{12}\right) = \underline{\underline{48{,}296 - 12{,}941j}}$$

$$Z = (Z_1 + Z_2)\,\Omega = (70{,}296 - 58{,}941j)\,\Omega$$

$$Z^{-1} = \frac{1}{Z} = \frac{A}{(70{,}296-58{,}941j)V}\cdot\frac{(70{,}296+58{,}941j)V}{(70{,}296+58{,}941j)V} = \frac{(70{,}296+58{,}941j)\,A}{(4941{,}6+3474)\,V}$$

$$= \frac{(70{,}296+58{,}941j)\,A}{} = \underline{\underline{(8{,}3531 + 7{,}0038)\cdot 10^{-3}\,\frac{A}{V}}}$$

$$R = \frac{1}{\text{Re}(Z^{-1})} = \frac{10^3\,V}{8{,}3531\,A} = \underline{\underline{119{,}72\,\Omega}}$$

$$C = \frac{\text{Im}(Z^{-1})}{\omega} = \frac{7{,}0038\cdot 10^{-3}\,As}{2\pi\cdot 10^6\,V} = \frac{7{,}0038}{2\pi}\cdot 10^{-9}\,\frac{As}{V} = \underline{\underline{1{,}1147\,nF}}$$