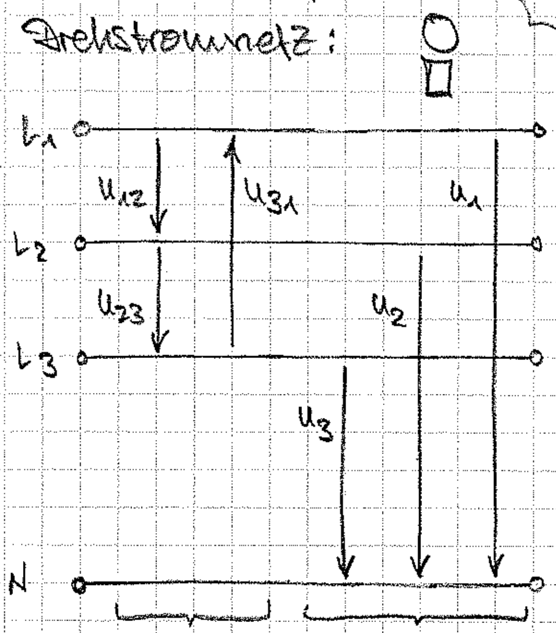
GET (2) Rechenübung (22) Aufgabe (1) Markus Symmank 12.05.2025

Drehstromnetz:



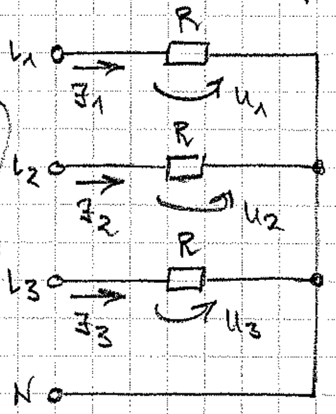
Außen-Leiterspannungen

Strangspannungen

$U_{St} = U_1 = U_2 = U_3 = 230\,V$

$U_L = U_{12} = U_{23} = U_{31} = \sqrt{3}\,U_{St} = 400\,V$

Sternschaltung



geg: $U_L = 400\,V$

$R = 20\,\Omega$

ges: I, P für λ u. Δ

$$P = 3 \cdot U \cdot I$$

$$= 3 \cdot U_{St} \cdot \frac{U_{St}}{R}$$

$$= 3 \cdot \frac{U_{St}^2}{R}$$

$$= 3 \cdot \frac{U_L^2}{\sqrt{3}^2 R}$$

$$P = \frac{U_L^2}{R} = \frac{(400\,V)^2}{20\,\Omega}$$

$$= \frac{160000\,V^2 A}{20\,\cancel{V}} = 8000\,W$$

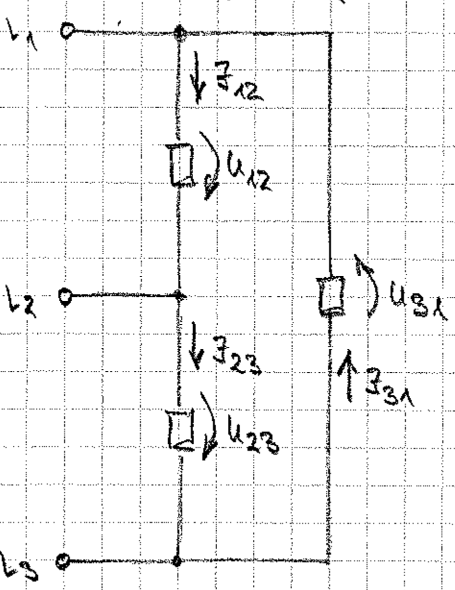
$$\underline{\underline{P_\lambda = 8\,kW}}$$

$$I = I_1 = I_2 = I_3 = \frac{U_{St}}{R} = \frac{U_L}{\sqrt{3}R} = \frac{400\,VA}{\sqrt{3}\,20\,V} = \frac{20}{\sqrt{3}}\,A$$

$$\underline{\underline{I_\lambda = 11{,}55\,A}}$$

Dreieckschaltung

$I = I_{12} = I_{23} = I_{31}$



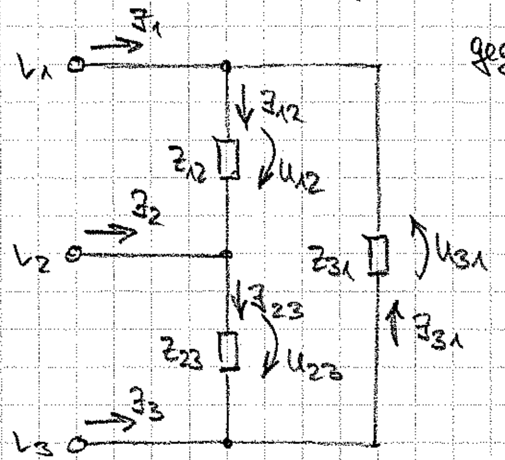
$$P = 3 \cdot U \cdot I = 3 \cdot U_L \cdot I = 3 \cdot U_L \cdot \frac{U_L}{R}$$

$$P = 3\,\frac{U_L^2}{R} = 3 \cdot \frac{(400\,V)^2}{R}$$

$$P = \frac{3 \cdot 160000\,V^2 A}{20\,V} = 24000\,W$$

$$\underline{\underline{P_\Delta = 24\,kW}}$$

$$I = \frac{U_L}{R} = \frac{400\,V\,A}{20\,V} \rightsquigarrow \underline{\underline{I_\Delta = 20\,A}}$$



geg: $Z_{12} = -j\,|X_C|$ $R = |X_C| = X_L = 100\,\Omega$

$Z_{23} = R$ $U_L = 400\,V$

$Z_{31} = R + jX_L$

ges: $I_L = \{I_1, I_2, I_3\}$ mit $\varphi_{12} = 0$

\+ Zeigerdiagramm

$$I_{12} = \frac{U_{12}}{Z_{12}} = -\frac{U_{12}}{j|X_C|} = -\frac{400V\exp(0)}{100\Omega\exp(j\frac{\pi}{2})} = -4A\exp(-j\frac{\pi}{2}) = \underline{\underline{4A\exp(j\frac{\pi}{2}) = 4Aj}}$$

$$I_{23} = \frac{U_{23}}{Z_{23}} = \frac{U_{23}}{R} = \frac{400V\exp(-j\frac{2\pi}{3})}{100\Omega} = \underline{\underline{4A\exp(-j\frac{2\pi}{3})}}$$

$$= 4A\left(\cos(-\frac{2\pi}{3}) + j\sin(-\frac{2\pi}{3})\right)$$

$$= \underline{\underline{-2A - 3{,}464\,A\,j}}$$

$$I_{31} = \frac{U_{31}}{Z_{31}} = \frac{U_{31}}{R + jX_L}$$

$$= \frac{400V\exp(j\frac{2\pi}{3})}{100\Omega\underbrace{(1+j)}_{45°}} = \frac{400V\exp(j\frac{2\pi}{3})}{100\cdot\sqrt{2}\,\Omega\exp(j\frac{\pi}{4})} = \frac{4}{\sqrt{2}}A\exp\left(j\left(\frac{2}{3}\pi - \frac{\pi}{4}\right)\right)$$

$$= \frac{4}{\sqrt{2}}A\exp\left(j\frac{5}{12}\pi\right) = \overset{2{,}83}{\frac{4}{\sqrt{2}}}A\left(\cos(\frac{5}{12}\pi) + j\sin(\overset{75°}{\frac{5}{12}\pi})\right) = \underline{\underline{0{,}732A + 2{,}732A\,j}}$$

$$I_1 = I_{12} - I_{31} = -0{,}732A + j(4 - 2{,}732)A = \underline{\underline{(-0{,}732 + j\,1{,}268)A}} = \underline{\underline{1{,}464A\cdot\exp(j\frac{2}{3}\pi)}}$$

$$I_2 = I_{23} - I_{12} = -2A + j(-3{,}464 - 4)A = \underline{\underline{(-2 - j\,7{,}464)A}}$$

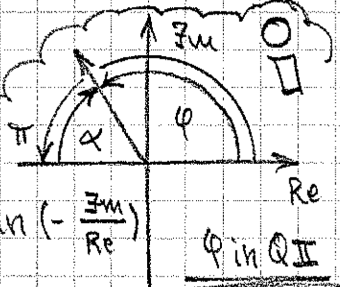
$$= \underline{\underline{7{,}73A\exp(j - \frac{7}{12}\pi)}} = 7{,}73A\exp(-j\,105°)$$

$$I_3 = I_{31} - I_{23} = 0{,}732A + 2A + j(2{,}732 + 3{,}464)A$$

$$= \underline{\underline{(2{,}732 + j\,6{,}196)A}}$$

$$= \underline{\underline{6{,}772A\exp(j\,1{,}156)}}$$

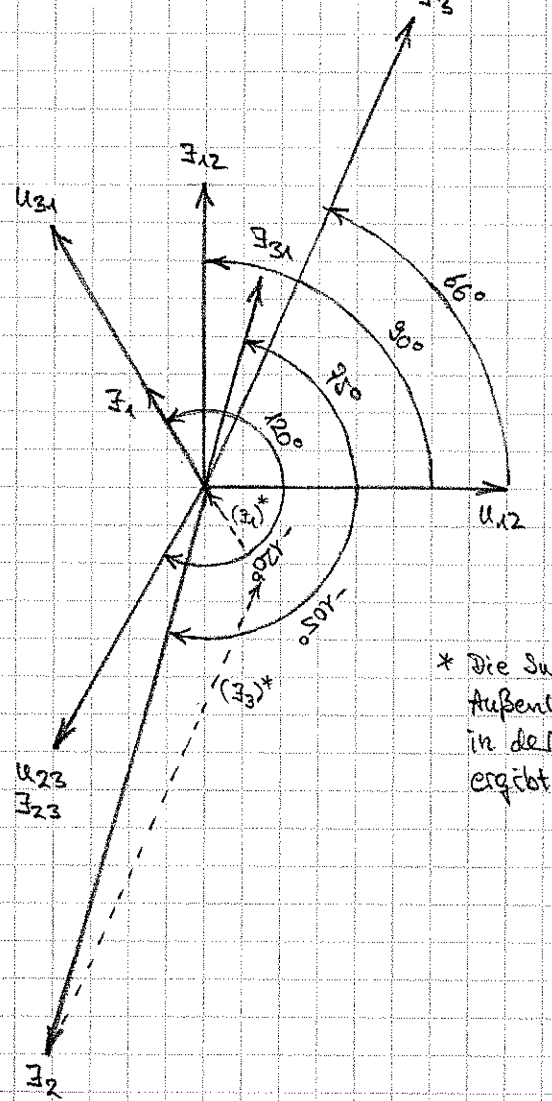
$$= 6{,}772A\exp(j\,66{,}2°)$$



$\alpha = \operatorname{atan}(-\frac{Jm}{Re})$ $\underline{\varphi \text{ in } Q\,II}$

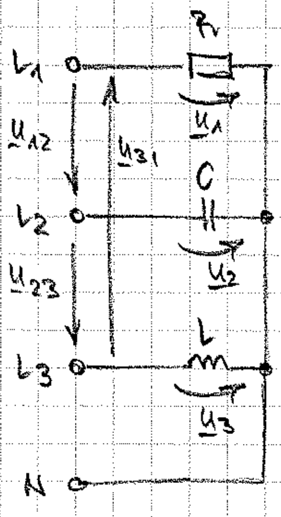
$\varphi = \pi - \alpha$

$$\underline{\underline{\varphi = \pi - \operatorname{atan}(-\frac{Jm}{Re})}}$$



* Die Summe der Außenleiterströme in der Dreieckschaltung ergibt Null.

GET 2 Rechenübung 22 Aufgabe 3 — Markus Symmanke 13.05.2025



$$\left.\begin{aligned} U_L &= U_{12} = U_{23} = U_{31} = 400\,\mathrm{V} \\ U_{St} &= U_1 = U_2 = U_3 = \frac{400}{\sqrt{3}}\,\mathrm{V} \end{aligned}\right\}\ \text{Effektivwerte}$$

Komplexe, stehende Effektivwertzeiger (mit $\underline{U}_1$ als Referenz $\rightsquigarrow \varphi_1 = 0$)

$\underline{U}_1 = U_{St}\exp(0) = U_{St}$

$\underline{U}_2 = U_{St}\exp\left(-j\frac{2}{3}\pi\right)$ (eilt $\underline{U}_1$ 120° nach)

$\underline{U}_3 = U_{St}\exp\left(-j\frac{4}{3}\pi\right)$ (eilt $\underline{U}_1$ 240° nach)

ges: P $\rightsquigarrow$ Wirkleistungsumsatz geschieht nur in Strang 1 über R

$$P = S_1 = \frac{U_1^2}{R} = \underline{\frac{U_{St}^2}{R}} = \frac{\left(\frac{400\,\mathrm{V}}{\sqrt{3}}\right)^2}{R} = \frac{16\cdot 10^4\,\mathrm{V^2 A}}{3\cdot 100\,\mathrm{V}} = \frac{1600}{3}\,\mathrm{W} = \underline{\underline{533{,}3\,\mathrm{W}}}$$

ges: $|Q_{XC}|$, $|Q_{XL}|$ $\rightsquigarrow$ $|Q_{XC}| = |S_2|$ und $|Q_{XL}| = |S_3|$ weil die Stränge 2 und 3 keine Wirkwiderstände enthalten

$$|Q_{XC}| = |S_2| = \left|\frac{\underline{U}_2^2}{\underline{Z}_2}\right| \text{ mit}$$

$$\underline{Z}_2 = \frac{1}{j\omega C} = \frac{|X_C|}{j} = -|X_C|j = |X_C|\exp\left(-j\frac{\pi}{2}\right)$$

$$S_2 = \underbrace{\frac{U_{St}^2\exp\left(-j\frac{4}{3}\pi\right)}{|X_C|\exp\left(-j\frac{\pi}{2}\right)}}_{} = \frac{U_{St}^2}{|X_C|}\exp\left(j\left(\frac{\pi}{2} - \frac{4}{3}\pi\right)\right) = \underbrace{\frac{U_{St}^2}{|X_C|}}_{= \frac{U_{St}^2}{R} = 533{,}3\,\mathrm{W}}\exp\left(-j\frac{5}{6}\pi\right)$$

$$= \underbrace{533{,}3\,\mathrm{W}}_{}\ \underbrace{\left(-0{,}866 - 0{,}5j\right)}_{}$$

Bezogen auf $\underline{U}_2$ ist S_2 rein imaginär $\rightsquigarrow$ „wirklich" eine reine Blindleistung.

$$\underline{\underline{|Q_{XC}| = 533{,}3\,\mathrm{W}}}$$

Bezogen auf $\underline{U}_1$ ist S_2 nicht rein imaginär $\rightsquigarrow$ keine reine Blindleistung

$$|Q_{XL}| = |S_3| = \left|\frac{\underline{U}_3^2}{\underline{Z}_3}\right| \text{ mit } \underline{Z}_3 = j\omega L = |X_L|j = |X_L|\exp\left(j\frac{\pi}{2}\right)$$

$$S_3 = \frac{U_{St}^2\exp\left(-j\frac{8}{3}\pi\right)}{|X_L|\exp\left(j\frac{\pi}{2}\right)} = \frac{U_{St}^2}{|X_L|}\exp\left(j\left(\frac{\pi}{2} - \underbrace{\frac{8}{3}\pi}_{\hat{=}\,\frac{2}{3}\pi}\right)\right) = \underbrace{\frac{U_{St}^2}{|X_L|}}_{= \frac{U_{St}^2}{R} = 533{,}3\,\mathrm{W}}\exp\left(-j\frac{\pi}{6}\right)$$

$$S_3 = 533{,}3\,\mathrm{W}\left(0{,}866 - 0{,}5j\right)$$

$$\rightsquigarrow \underline{\underline{|Q_{XL}| = 533{,}3\,\mathrm{W}}}$$

ges: $|Q_\Sigma| = |Q_{XL} + Q_{XC}|$

$$Q_{XL} + Q_{XC} = 533{,}3\,\mathrm{W}\left(0{,}866 - 0{,}866 + j(-0{,}5 - 0{,}5)\right)$$

$$Q_{XL} + Q_{XC} = 533{,}3\,\mathrm{W}\,(-j) = -533{,}3\,\mathrm{W}j \qquad \underline{\underline{|Q_{XL} + Q_{XC}| = |Q_\Sigma| = 533{,}3\,\mathrm{W}}}$$