GET (2) Rechenübung (16) Aufg (1) Markus Szymanek 11.04.2025

Ansatz: Effektivwert $U_{eff} = \sqrt{\frac{1}{T}\int_0^T u^2(t)\,dt}$

engl. RMS – Square
 └ Mean
 └ Root

Warum quadratisch? Der Effektivwert einer Wechselspannung entspricht dem Wert einer Gleichspannung, die an einem ohmschen Widerstand die gleiche Leistung umsetzt.
→ Leistung ist die Referenzgröße.
→ Spannung geht quadratisch in die Leistung ein.

$i(t) = I_0 + \hat{I}\sin(\omega t)$

$\hookrightarrow\ i^2(t) = I_0^2 + 2I_0\hat{I}\sin(\omega t) + \hat{I}^2\sin^2(\omega t)$

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt} = \sqrt{\frac{1}{T}\int_0^T\left(I_0^2 + 2I_0\hat{I}\sin(\omega t) + \hat{I}^2\sin(\omega t)\right)dt}$$

$$= \sqrt{\frac{1}{T}\left(I_0^2\int_0^T dt + 2I_0\hat{I}\int_0^T\sin(\omega t)\,dt + \hat{I}^2\int_0^T\sin^2(\omega t)\,dt\right)}$$

$\int \sin^2(ax)\,dx$

Substitution:
$ax = u \rightarrow a\,dx = du \rightarrow dx = \frac{1}{a}du$

Additionstheorem: $\sin^2(x) = \frac{1}{2}(1-\cos(2x))$

$$\int\sin^2(ax)\,dx = \frac{1}{a}\int\sin^2(u)\,du = \frac{1}{a}\int\frac{1}{2}(1-\cos(2u))\,du$$

Integrationskonst. fällt bei bestimmter Integr. weg

$$= \frac{1}{2a}\left(\int du - \int\cos(2u)\,du\right) = \frac{1}{2a}\left(u - \frac{1}{2}\sin(2u) + C\right) = \frac{u}{2a} - \frac{1}{4a}\sin(2u) + C$$

⇝ Rücksubst.: $= \frac{ax}{2a} - \frac{1}{4a}\sin(2ax) \Rightarrow \boxed{\int\sin^2(ax)\,dx = \frac{x}{2} + \frac{1}{4a}\sin(2ax) + C}$

$$= \sqrt{\frac{1}{T}\left(I_0^2\,[t]_0^T - 2I_0\hat{I}\,[\cos(\omega t)]_0^T + \hat{I}^2\left[\frac{t}{2} - \frac{1}{4\omega}\sin(2\omega t)\right]_0^T\right)}$$

Wie groß ist T? → $\omega = 2\pi f = \frac{2\pi}{T} \rightarrow T = \frac{2\pi}{\omega}$

$$= \sqrt{\frac{\omega}{2\pi}\left(I_0^2\frac{2\pi}{\omega} - 2I_0\hat{I}\left(\overset{=1}{\cancel{\cos(2\pi)}} - \overset{=1}{\cancel{\cos(0)}}\right) + \hat{I}^2\left(\frac{\pi}{\omega} - \overset{=0}{\cancel{\frac{\sin(4\pi)}{4\omega}}} - 0 + \overset{=0}{\cancel{\frac{\sin(0)}{4\omega}}}\right)\right)}$$

$$= \sqrt{\frac{\omega}{2\pi}\left(I_0^2\frac{2\pi}{\omega} - 0 + \hat{I}^2\frac{\pi}{\omega}\right)} = \sqrt{I_0^2 + \frac{1}{2}\hat{I}^2}$$

$$\Longrightarrow\ I_{eff} = \sqrt{I_0^2 + \frac{1}{2}\hat{I}^2}$$

In Zahlen mit $I_0 = \hat{I} = 3A = I$

$I_{eff} = \sqrt{I^2 + \frac{1}{2}I^2} = I\sqrt{\frac{3}{2}} = 3\cdot\sqrt{\frac{3}{2}}\,A = 3{,}6742\,A$

GET ② Rechenübung ⑯ Aufg. ② Markus Szymanek 11.04.2025

Ansatz: Effektivwert $U_{eff} = \sqrt{\frac{1}{T}\int_0^T u^2(t)\,dt}$

Spannung: $u(t) = \frac{\hat{u}}{2} + \frac{\hat{u}}{2T}t = \frac{\hat{u}^2}{2}\left(1 + \frac{t}{T}\right) \rightarrow \underline{u^2(t) = \frac{\hat{u}^2}{4}\left(1 + 2\frac{t}{T} + \frac{t^2}{T^2}\right)}$

$$U_{eff} = \sqrt{\frac{1}{T}\int_0^T \frac{\hat{u}^2}{4}\left(1 + 2\frac{t}{T} + \frac{t^2}{T^2}\right)dt}$$

$$= \sqrt{\frac{\hat{u}^2}{4T}\left(\int_0^T dt + \frac{2}{T}\int_0^T t\,dt + \frac{1}{T^2}\int_0^T t^2\,dt\right)}$$

$$= \sqrt{\frac{\hat{u}^2}{4T}\left(\left[t\right]_0^T + \frac{2}{T}\left[\frac{t^2}{2}\right]_0^T + \frac{1}{T^2}\left[\frac{t^3}{3}\right]_0^T\right)}$$

$$= \sqrt{\frac{\hat{u}^2}{4T}\left(T + \frac{\cancel{2}}{\cancel{T}}\frac{T^{\cancel{2}}}{\cancel{2}} + \frac{1}{\cancel{T^2}}\frac{T^{\cancel{3}}}{3}\right)}$$

$$= \sqrt{\frac{\hat{u}^2}{4\cancel{T}}\left(\cancel{T} + \cancel{T} + \frac{\cancel{T}}{3}\right)} = \sqrt{\frac{\hat{u}}{4}\left(1 + 1 + \frac{1}{3}\right)}$$

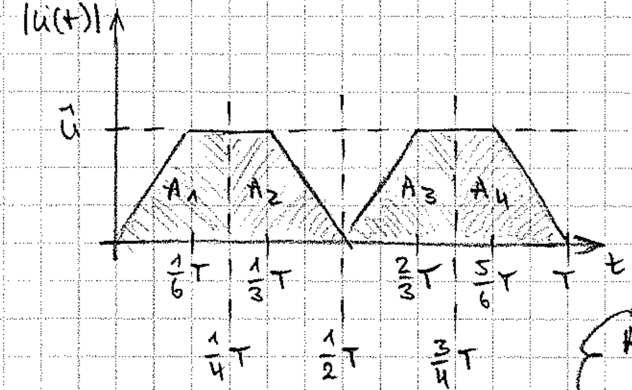
$$= \sqrt{\frac{\hat{u}}{4}\left(\frac{3}{3} + \frac{3}{3} + \frac{1}{3}\right)} = \sqrt{\frac{\hat{u}^2}{4}\frac{7}{3}}$$

$$\underline{\underline{U_{eff} = \frac{\hat{u}}{2}\sqrt{\frac{7}{3}}}}$$

GET (2) Rechenübung (16) Aufgabe (3) — Markus Szymanek 10.04.2025

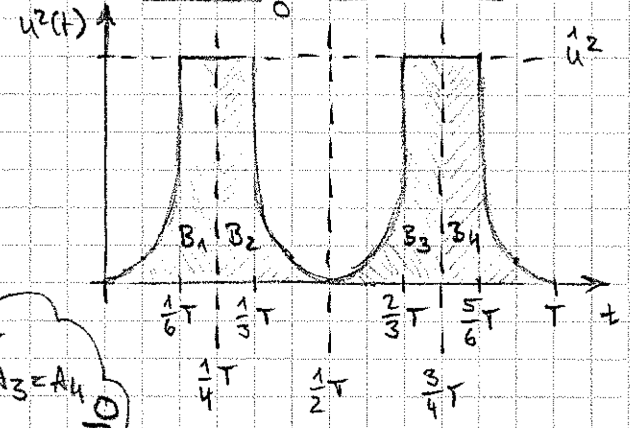
a) Gleichrichtwert

$$\bar{u} = \frac{1}{T}\int_0^T |u(t)|\,dt$$



b) Effektivwert

$$u_{eff} = \sqrt{\left(\frac{1}{T}\int_0^T u^2(t)\,dt\right)}$$



Weil $A_1 = A_2 = A_3 = A_4$ bzw. $B_1 = B_2 = B_3 = B_4$ gilt

$$\bar{u} = \frac{1}{\frac{T}{4}}\int_0^{T/4} |u(t)|\,dt \qquad u_{eff} = \sqrt{\left(\frac{1}{\frac{T}{4}}\int_0^{T/4} u^2(t)\,dt\right)}$$

$$u\left(t = 0 \ldots \frac{T}{6}\right) = \hat{u}\,\frac{t}{T/6} = 6\,\hat{u}\,\frac{t}{T} \qquad u^2\left(t = 0 \ldots \frac{T}{6}\right) = 36\,\hat{u}^2\,\frac{t^2}{T^2}$$

$$u\left(t = \frac{T}{6} \ldots \frac{T}{4}\right) = \hat{u} \qquad u^2\left(t = \frac{T}{6} \ldots \frac{T}{4}\right) = \hat{u}^2$$

$$\bar{u} = \frac{4}{T}\left(\int_0^{T/6} 6\,\hat{u}\,\frac{t}{T}\,dt + \int_{T/6}^{T/4} \hat{u}\,dt\right)$$

$$\bar{u} = \frac{4\hat{u}}{T}\left(\frac{6}{T}\int_0^{T/6} t\,dt + \int_{T/6}^{T/4} dt\right)$$

$$\bar{u} = \frac{4\hat{u}}{T}\left(\frac{6}{T}\left[\frac{1}{2}t^2\right]_0^{T/6} + \left[t\right]_{T/6}^{T/4}\right)$$

$$\bar{u} = \frac{4\hat{u}}{\cancel{T}}\left(\frac{\cancel{6}}{\cancel{T}}\,\frac{1}{2}\,\frac{\cancel{T^2}}{6^{\cancel{2}}} + \frac{\cancel{T}}{4} - \frac{\cancel{T}}{6}\right)$$

$$\bar{u} = 4\hat{u}\left(\frac{1}{12} + \frac{3}{12} - \frac{2}{12}\right)$$

$$\bar{u} = \frac{4 \cdot 2\,\hat{u}}{12}$$

$$\bar{u} = \frac{2}{3}\,\hat{u}$$

$$u_{eff} = \sqrt{\left(\frac{4}{T}\left(\int_0^{T/6} 36\,\hat{u}^2\,\frac{t^2}{T^2}\,dt + \int_{T/6}^{T/4} \hat{u}^2\,dt\right)\right)}$$

$$u_{eff} = \sqrt{\left(\frac{4\hat{u}^2}{T}\left(\frac{36}{T^2}\int_0^{T/6} t^2\,dt + \int_{T/6}^{T/4} dt\right)\right)}$$

$$u_{eff} = \sqrt{\left(\frac{4\hat{u}^2}{T}\left(\frac{36}{T^2}\left[\frac{1}{3}t^3\right]_0^{T/6} + \left[t\right]_{T/6}^{T/4}\right)\right)}$$

$$u_{eff} = \sqrt{\left(\frac{4\hat{u}^2}{\cancel{T}}\left(\frac{\cancel{36}}{\cancel{T^2}}\,\frac{1}{3}\,\frac{\cancel{T^3}}{6^{\cancel{3}}} + \frac{\cancel{T}}{4} - \frac{\cancel{T}}{6}\right)\right)}$$

$$u_{eff} = 2\hat{u}\sqrt{\left(\frac{2}{36} + \frac{9}{36} - \frac{6}{36}\right)}$$

$$u_{eff} = 2\hat{u}\sqrt{\left(\frac{5}{36}\right)}$$

$$u_{eff} = \frac{\sqrt{5}}{3}\,\hat{u}$$