基礎方程式:

$$
\begin{gathered}
\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla P - \frac{1}{\mathrm{Re}_0}(\boldsymbol{u} - U_y \hat{\boldsymbol{y}}) + \frac{1}{\mathrm{Re}} \nabla^2 \boldsymbol{u} + \sin(ny) \hat{\boldsymbol{x}}, \\
\nabla \cdot \boldsymbol{u} = 0, \\
\boldsymbol{U} = \frac{a}{4\pi^2} \int dx dy \boldsymbol{u} = (0, U_y) \quad : \mathrm{const.}
\end{gathered}
$$

新しい変数:

$$
\boldsymbol{v} = \boldsymbol{u} - \boldsymbol{U} = \nabla \times (\psi \hat{\boldsymbol{z}}) = (\partial_y \psi) \hat{\boldsymbol{x}} + (-\partial_x \psi) \hat{\boldsymbol{y}}
$$

変換した基礎方程式:

$$
\begin{gathered}
\partial_t \boldsymbol{v} + \boldsymbol{v} \cdot \nabla \boldsymbol{v} + \boldsymbol{U} \cdot \nabla \boldsymbol{v} = -\nabla P - \frac{1}{\mathrm{Re}_0} \boldsymbol{v} + \frac{1}{\mathrm{Re}} \nabla^2 \boldsymbol{v} + \sin(ny) \hat{\boldsymbol{x}}, \\
\nabla \cdot \boldsymbol{v} = 0, \\
\frac{a}{4\pi^2} \int dx dy \boldsymbol{v} = 0
\end{gathered}
$$

$\nabla \times$ を掛けたい:

$$
\begin{gathered}
\nabla \times \boldsymbol{v} = \nabla \times \nabla \times (\psi \hat{\boldsymbol{z}}) = \nabla(\nabla \cdot \psi \hat{\boldsymbol{z}}) - \nabla^2 (\psi \hat{\boldsymbol{z}}) = (-\nabla^2 \psi) \hat{\boldsymbol{z}} \\
\nabla \times (\boldsymbol{v} \cdot \nabla \boldsymbol{v}) = J(\psi, \nabla^2 \psi) \hat{\boldsymbol{z}}, \\
\nabla \times (\boldsymbol{U} \cdot \nabla \boldsymbol{v}) = \boldsymbol{U} \cdot \nabla (\nabla \times \boldsymbol{v}) = (-U_y \partial_y \nabla^2 \psi) \hat{\boldsymbol{z}}
\end{gathered}
$$

ただし

$$
J(a, b) = (\partial_x a)(\partial_y b) - (\partial_y a)(\partial_x b)
$$

基礎方程式に $\nabla \times$ を掛ける:

$$
-\partial_t \nabla^2 \psi + J(\psi, \nabla^2 \psi) - U_y \partial_y \nabla^2 \psi = \frac{1}{\mathrm{Re}_0} \nabla^2 \psi - \frac{1}{\mathrm{Re}} \nabla^2 \nabla^2 \psi - n \cos(ny)
$$

整理:

$$
\begin{aligned}
\partial_t \nabla^2 \psi = n \cos(ny) - \frac{1}{\mathrm{Re}_0} \nabla^2 \psi - U_y \partial_y \nabla^2 \psi + \frac{1}{\mathrm{Re}} \nabla^2 \nabla^2 \psi \\
+ J(\psi, \nabla^2 \psi)
\end{aligned}
$$

線形部分:

$$
\partial_t \nabla^2 \psi = n \cos(ny) - \frac{1}{\mathrm{Re}_0} \nabla^2 \psi - U_y \partial_y \nabla^2 \psi + \frac{1}{\mathrm{Re}} \nabla^2 \nabla^2 \psi
$$

$\bar{\omega} = \nabla^2 \psi$:

$$
\partial_t \bar{\omega} = n \cos(ny) - \frac{1}{\mathrm{Re}_0} \bar{\omega} - U_y \partial_y \bar{\omega} + \frac{1}{\mathrm{Re}} \nabla^2 \bar{\omega}
$$

Fourier 変換, $F[\bar{\omega}](k,\ell)$:

$$\partial_t F[\bar{\omega}] = F[n\cos(ny)] - \frac{1}{\mathrm{Re}_0} F[\bar{\omega}] - i\ell U_y F[\bar{\omega}] - \frac{1}{\mathrm{Re}}(k^2+\ell^2) F[\bar{\omega}]$$

線形項を積分:

$$\begin{gathered}
\partial_t \{F[\bar{\omega}] e(t)\} = F[n\cos(ny)] e(t), \\
e(t) = \exp(t\alpha), \\
\alpha = \frac{1}{\mathrm{Re}_0} + i\ell U_y + \frac{1}{\mathrm{Re}}(k^2+\ell^2)
\end{gathered}$$

積分!!:

$$\begin{aligned}
F[\bar{\omega}](t) &= F[\bar{\omega}](t_0) e(-t+t_0) + e(-t) \int_{t_0}^{t} F[n\cos(ny)] e(s)\, ds \\
&= F[\bar{\omega}](t_0) e(-t+t_0) + F[n\cos(ny)] \cdot \frac{1 - e(-t+t_0)}{\alpha} \\
&= \left(F[\bar{\omega}](t_0) - \frac{F[n\cos(ny)]}{\alpha} \right) e(-t+t_0) + \frac{F[n\cos(ny)]}{\alpha}
\end{aligned}$$