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Scheduling the Belgian Soccer League

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Every sports league needs a game schedule; such a schedule is important because it influences all parties involved and even the outcome of the sports competition. Interest in Belgian soccer has increased during recent years, as has interest in sports leagues in other countries. This paper describes our experiences in scheduling the highest division in the Belgian soccer league. We describe how we automated and improved the development of the 2006–2007 season schedule, and explain how we achieved additional improvement by dividing the scheduling problem into two problems. The resulting calendar has been accepted for the 2007–2008 season. We also compare the quality of the schedules generated using different scheduling methods.

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In Europe, soccer has become a major business involving many stakeholders, such as teams, police, fans, and broadcasting companies, and much money. In Belgium, Belgacom TV pays an unprecedented €12 million per year for the soccer broadcasting rights. Although this is a trifle compared to the amounts companies pay to broadcast soccer competition in other countries, such as the United Kingdom (£900 million), it illustrates a rising interest in Belgian soccer; hence, it shows the increasing importance of the league schedule. In addition to the obvious influence on the results of the sports competition, the schedule also affects the game attendance, public interest in the competition, and the event's profitability to broadcasters, sponsors, and advertisers. Each involved party has its (possibly conflicting) constraints and wishes; developing a schedule that all parties consider fair and acceptable is a challenge.

This paper is organized as follows. First, we review the literature on scheduling a national soccer league, explaining the different characteristics in various countries. Next, we describe the Belgian soccer league and our cooperation with the Royal Belgian Soccer Association (KBVB). We conclude by detailing our solution method and the impact of its implementation.

Soccer League Scheduling in Other Countries

Many papers in the literature describe the scheduling of national soccer leagues in European or South American countries. In some ways, all national soccer leagues are similar. For example, they consist mainly (if not fully) of a round-robin tournament, and each game is played in the home stadium of one of the opponents. However, because of some specific constraints that characterize each of these competitions, and different assessments of what each league considers as most important, the models that these papers present are diverse and not readily applicable to soccer league scheduling problems in other countries.

In most countries, each team plays against each other team twice, once at home and once away; this is termed a double round-robin tournament. One of the first papers on soccer league scheduling deals with the competition in The Netherlands (Schreuder 1992). The primary constraints for developing this schedule are as follows: (1) a pair of teams cannot be scheduled to play a home (away) game at the same time, (2) a minimal number of breaks (a break is defined as a series of consecutive home (away) games) is mandated, and (3) several constraints are necessary because of police and railway company attempts to prevent hooliganism. Schreuder (1992) describes a constructing and partitioning heuristic for this approach, which was used to create the schedule for the 1989–1990 season.

In Italy, two television companies have soccer league broadcasting rights because Italian law forbids monopolies. Each team is assigned to one of the television companies, which has the right to broadcast that team's home games. The main challenge is to find a schedule that is balanced with respect to television coverage. A television station would not want to broadcast home games for all its assigned teams in one round and have no games to broadcast in another round. This coverage constraint is so important in Italy that schedules with more breaks than necessary are acceptable. Della Croce and Oliveri (2006) use an integer programming approach to produce a schedule for the Italian competition; however, they mention only preliminary contacts with the Italian Football League for a possible application.

Germany has a constraint requiring that weak and strong opponents for each team must be distributed evenly throughout the season. Therefore, teams with roughly the same strength are grouped in so-called strength groups. The schedule should be developed such that two teams that belong to the same strength group are not scheduled to play against each other in two consecutive rounds (Briskorn 2008). Bartsch et al. (2006) present a semi-greedy algorithm; the German soccer association used it once and then abandoned it for reasons that the authors did make evident. However, Austria used a variant of their algorithm to schedule its Austrian competition (a quadruple round-robin tournament with 10 teams) for the 1997–1998 through 2002–2003 seasons. Brazil, although it has only one television company, limits broadcasting rights. Indeed, it allows a game to be broadcast everywhere in the country, except in the city in which it takes place. Furthermore, it permits only one game per round to be broadcast in each city. One of Brazil's scheduling goals is to maximize the number of relevant games that the television company can broadcast and minimize the number of breaks. Ribeiro and Urrutia (2007) describe an integer programming approach to improve several official schedules; however, they do not mention whether Brazil used their method in practice.

In some countries, the competition consists of a single round-robin tournament. In such a tournament, each team meets each other team only once; therefore, the number of home games and the strength of the home game opponents strongly determine the fairness of the schedule. It is more difficult to become the league champion if one plays away against the strong teams and at home against the weak teams, and the opposite is true if the goal is simply to stay in the league; therefore, if a team meets each opponent only once, it matters whether that team plays at home or away against that opponent. Chile organizes its 20 teams into four groups of five teams, although each plays once against each of the other 19 teams. After this single round-robin tournament, the best two teams of each group advance to a play-off stage, where the league champion is decided. In Chile, many constraints stem from its geographical shape. For example, the television-station constraints are based on geography; the stations do not want the venues of the four games that they broadcast per round to be located far apart. Given these and other hard constraints, one of Chile's objectives is to have as many "decisive games" as possible played near the end of the season. Duran et al. (2007) describe an integer programming method that the Chilean soccer association has used since 2005.

Denmark is one of the few countries that play a triple round-robin tournament (i.e., each pair of teams meets three times). As in Chile, this leads to an uneven distribution of home and away games, which has a great impact on the fairness of the schedule. For example, the schedule should ensure that each team plays two (high-revenue) home games against at least one top team. Rasmussen (2008) developed an algorithm, which was used to create the official Denmark 2006–2007 schedule, based on Benders' decomposition and column generation.

The above overview illustrates that we cannot generalize the soccer league scheduling problem. Indeed, because countries differ with respect to factors such as legislation, geography, and traditions, the requirements for their soccer league schedules vary considerably. The Belgian soccer league also has its own peculiarities.

Problem Description

The Jupiler League, the highest division in Belgian soccer, is organized as a double round-robin tournament with 18 teams. Based on their performances in the past 10 years, four of these teams are labeled *top teams;* a game between two top teams is called a *top game*. The Jupiler League mandates that its teams not play more than two consecutive home (away) matches, and that the total number of breaks is minimal. Furthermore, no team should start or end the season with a break. In Belgium, it is considered fair to schedule the second half of a competition identically to the first half, but to invert (i.e., mirror) the home advantage. A review of the current soccer schedules in other countries (e.g., France, Austria, the United Kingdom, and The Netherlands) shows that this view is not universally accepted. A calendar committee, which the Royal Belgian Football Association (KBVB) determines and which consists of representatives from six of the 18 clubs, determines the 34 weekends on which a round is played; weekday games generally do not occur. The calendar committee is responsible for producing a schedule six weeks prior to the start of competition.

The calendar committee must consider several requirements, which originate from four stakeholders. First, a mayor can forbid the playing of a game in his or her city on particular dates. This usually occurs if another event (e.g., a fair or cycle race) that requires the attention of local police has been scheduled in that city at that time. Many games (called "risk games") are recognized as having an increased risk of hooliganism. Because the additional police needed to guarantee the safety of citizens at a risk game might not be available on a specific date, a mayor might forbid a risk game but allow a regular game to take place on that date.

Second, clubs have a wide variety of wishes. For example, a team might prefer not to play at home when another specific team is also playing at home. This could be because the two teams share a stadium; it could also be because of a fear that some of its potential spectators would attend the other game. In a small country like Belgium, where most teams have half a dozen of competitors within a 50-km range, this fear is well founded. Teams also have a number of wishes related to the fairness of the schedule (e.g., no team wants to face all traditionally strong opponents in a row), to the European schedule (e.g., some clubs prefer to play a home game after having played a game in a European competition), or to the expected number of spectators (e.g., a top game should not be scheduled during the summer when many fans are abroad for holidays); moreover, a team might simply want to increase its chances of playing a good season (e.g., most clubs prefer to start the season with a home game and easy opponents). Some teams with cash flow problems want to play at home against the top teams during the first half of the season; thus, they would have the consequent revenues available early in the season. Other teams that fear relegation tend to want the opposite; they prefer to play (and win) their home games in the first half of the season, hoping that this will boost the team's morale and result in an easy second half of the season.

Third, the television station that acquired the broadcasting rights (Belgacom TV) wants to maximize the number of viewers. Belgium television stations are not hindered by any of the constraints one finds in Chile, Brazil, or Italy; Belgacom TV can broadcast all games throughout the country; travel distances are negligible. Although Belgium matches are normally played on Saturday, the broadcasting company has the right to shift one match to Friday and two matches to Sunday on a month's notice to increase the number of viewers. However, because for each team there should be at least two days between its consecutive games, this could be problematic for teams that also play a midweek match (e.g., the Champions League). Therefore, where possible, matches between those teams should be scheduled on weekends that are not preceded or followed by a midweek match. According to Belgacom TV, another way to increase the number of viewers is to develop a schedule where on each round, at least one (and preferably two) of the four top teams plays an away game. Its underlying motivation is that a top team's home games are less interesting because it usually wins these games without much effort. Belgacom TV also prefers that on each round, at least three of the six teams that finished at the top of the league during the previous season (which usually include the top teams) play an away game. Finally, it wishes to have only one top game per round and no top games in the first four rounds. Moreover, the top games should be spread over the season. This is the opposite of what countries such as Spain and The Netherlands do currently; they play top games on the same day.

Finally, the calendar committee also has several constraints for consideration. Because most teams prefer not to play against all top teams consecutively, the committee demands that no team play more than twice against a top team in four consecutive games.

Furthermore, it states that each team should receive a top team at home at least once in each half of the season. The calendar committee also considers the "carry-over effect." For example, if team A is very strong, its opponent, team B, will be exhausted after playing against team A. Because team B will play against another team, team C, in the next round, we can see that team A's game against team B has an impact on team B's game against team C. Thus, there is a carry-over effect from teams A to C. Clearly, a fair schedule should avoid the situation in which team C always plays against the team that played against a strong team (team A) in the previous match. Ideally, team A should carry over to team C at most twice during the season, such that the carry-over effects are maximally balanced over the teams (Russell 1980). As far as we know, Belgium's league is the first soccer league that considers the relevance of balancing carry-over effects. We believe that this is because a coach had suggested carry-over effects as a cause for his team's relegation in the Belgium media (Geril 2007).

The scheduling problem is to determine for each round which teams play against each other and which of each pair plays at home, while satisfying as many requirements as possible.

Scheduling the Soccer League Manually

In the traditional approach, which Belgian soccer used for decades until the 2005–2006 season, a schedule was constructed manually, under the supervision of the calendar committee's secretary. This manual approach begins with a "basic match schedule" (BMS). A basic match schedule consists of numbers, which determine the matches (home-away assignment and opponents) in each round. Table 1 shows the first seven rounds of the BMS.

The schedule shows that number 1 opens the season with a home game against number 3. On the second round, number 1 plays an away game against number 5; on the third round, number 1 plays number 7. This BMS is a canonical schedule (De Werra 1980), which satisfies the mirroring constraint and minimizes the number of breaks.

Clearly, a schedule is found by assigning each team to a number in the BMS. The calendar committee's secretary does this task manually, using an Excel sheet to

1	2	3	4	5	6	7
1-3	2-4	1-7	2-8	1-11	2-12	1-15
4-17	3-18	3-5	4-6	3-9	4-10	3-13
6-15	5-1	6-2	5-18	5-7	6-8	5-11
8-13	7-16	8-17	7-3	8-4	7-18	7-9
10-11	9-14	10-15	9-1	10-2	9-5	10-6
12-9	11-12	12-13	11-16	12-17	11-3	12-4
14-7	13-10	14-11	13-14	14-15	13-1	14-2
16-5	15-8	16-9	15-12	16-13	15-16	16-17
18-2	17-6	18-4	17-10	18-6	17-14	18-8

Table 1: This table shows the first seven rounds of the BMS.

visualize the assignments. First, he assigns a number from the BMS to the each of the four top clubs, bearing in mind a reasonable spread of their mutual games and the police constraints. Next, he assigns a number to each of the other teams, one by one, again trying to satisfy the police demands. Afterwards, he makes an attempt to improve the schedule by iteratively swapping (manually) the assignments of a pair of teams.

Not surprisingly, these calendars satisfy only a minority of the constraints, and often ignore the wishes of the clubs and Belgacom TV. Consequently, several teams thought that the league schedule was unbalanced and unfair (Lambaerts 2005). Moreover, even the teams that had a representative in the calendar committee were unclear about which wishes the committee had considered and how it constructed the schedule. This led to insinuations that the chairman of the calendar committee was favoring his own team (Reunes 2005). The schedule also failed to balance the benefits and the burdens of the carry-over effect. Pressure from Belgacom TV to deal with its wishes contributed to automating the scheduling procedure.

Automating the Scheduling Procedure

After Demasure (2006) illustrated in a master's thesis that there was ample room for schedule improvement for the then ongoing 2005–2006 season, the KBVB invited us to work on the schedule for the new season.

As we were gathering wishes from all involved parties, we realized that we could not satisfy all of them because they conflicted. After some deliberations, the calendar committee agreed to attach a priority level to each of the wishes. It established five levels and reserved the highest (i.e., level 1) for constraints

Priority	Level 1	Level 2	Level 3	Level 4	Level 5
Importance	15,000	500	20	4	1

Table 2: This table shows the importance of wishes from various priority levels in terms of level 5 wishes, as determined by the calendar committee.

arising from physical impossibilities (e.g., two teams sharing the same stadium, or a stadium not being available because of maintenance work). The second-highest level related to police and local government wishes; for example, a Belgium mayor has the right to forbid a game if he or she thinks that the police cannot guarantee safety. The calendar committee made an assessment of the club and Belgacom TV wishes, balancing the underlying financial or sporting motives with the fairness of the schedule, and assigned these wishes to one of the three remaining priority levels.

Next, we linked a penalty, incurred if the corresponding constraint was violated, to each priority level. To determine the values of these penalties, we asked the calendar committee to tell us the number of wishes of a lower priority level that it would like to see satisfied to give up a higher-priority wish. After some discussion, the committee agreed upon the relative importance of the wishes (Table 2); we were then able to establish the penalties.

Note that given this trade-off, it makes little sense to violate wishes coming from the police to satisfy lower priority wishes, such as several club constraints, whereas some bargaining among wishes of the clubs, the committee, and Belgacom TV is desirable.

We constructed a mixed-integer programming model, automating the assignment of numbers in the BMS to teams as the committee had previously done manually. Appendix A contains a detailed discussion of this model. We solved the model and developed a corresponding schedule with a minimal sum of incurred penalties within a few hours using ILOG CPLEX 8.1 on a Pentium IV 2 GHz computer. We held several meetings with the calendar committee and representatives of Belgacom TV to fine-tune the penalties and priority levels, and to arrive at the model's final specifications and the calendar, which we presented to the press as the official calendar for the 2006–2007 season.

Beyond the BMS

The KBVB has used the BMS for decades to schedule the first division, the seven other national divisions, and numerous regional leagues. The schedules for these leagues depend on the first-division schedule because many lower-division teams prefer not to play a home game when a neighboring higher-division team is also playing at home. Because the first division is the most constrained league, we scheduled it first. The KBVB then scheduled the second division, and so on, following the hierarchy among the divisions. The personnel involved in scheduling these leagues are familiar with the BMS and asked us to use it for the 2006–2007 season schedule.

The BMS has many interesting properties, such as a minimum number of breaks; however, it is also highly restrictive. We discovered that the wish of lower-division teams not to play at home at the same time as a neighboring higher-division team was the only relevant dependency between the leagues. Nevertheless, this concern does not require using the BMS. Indeed, by using the home-away patterns that follow from the BMS, we already establish compatibility with it. For each round, a home-away pattern states whether a team plays at home or away. Moreover, for each home-away pattern, the BMS includes another home-away pattern that is its opposite with respect to home and away games. Thus, if a lower-division team cannot play a home game when a particular first-division team plays at home, it suffices for the lower-division scheduler to assign the lower-division team to that number in the BMS using a home-away pattern that is opposite to that of the first-division team. Thus, compatibility with lower-division schedules does not necessitate using the matches that the BMS prescribes.

Using the home-away patterns as a starting point allows us to use a two-phased approach. In the first phase, each team is assigned a home-away pattern; in the second phase, the actual opponents are determined. Therefore, in phase 1, we can only consider constraints that relate to whether a team plays at home or away. These constraints are the complementarity constraints (i.e., two teams do not want to play home games at the same time), place constraints (i.e., a team does not want to play a home (away) game in a given round), and television constraints (i.e., for each round a number of top teams must play an away game). The first phase results in a home-away pattern for each team; this serves as an input to the second phase, which considers all other constraints—constraints that depend on actual opponents. For example, we can enforce a constraint that a team not be scheduled to play against a top team in a given round. The two-phased approach offers a wider range of solutions than the assignment model. Given a set of home-away patterns, matches can be fixed in many ways. The way in which this was done in the BMS is just one way, which need not be optimal. Conversely, the two-phased approach leaves the choice of the actual matches (to a large extent) open for optimization in phase 2, following the assignment of teams to home-away patterns in phase 1. ILOG CPLEX 8.1 allows us to solve phase 1 to optimality within a minute; solving phase 2 takes only a few seconds. It should be clear, however, that this does not guarantee an optimal schedule. In fact, the relatively short computation times of each of these phases allow us to employ a tabu search procedure. We consider a neighborhood in phase 1 by swapping the home-away patterns of two teams. For each of the solutions in this neighborhood, we compute the corresponding schedule by solving phase 2. We select the swap in this neighborhood that leads to the lowest total penalty (over both phases) as a starting point for the next iteration. To ensure that a swap is not reversed immediately as the algorithm proceeds, we maintain a tabu list of forbidden swaps. After several swaps that do not improve the best solution, we stop the algorithm, select the best schedule, and present it to the calendar committee. Appendix B includes the mixed-integer programs for both phases and more details on the tabu search algorithm.

One advantage of a phased approach is that the resulting schedule seems more able to withstand changes in the constraints or priorities than a BMS-based schedule. Indeed, using the BMS approach, even the smallest input change typically results in a very different schedule, whereas using the two-phased approach, an additional constraint does not necessarily change the previous schedule dramatically. This allows the members of the calendar committee to manage changes in the input (e.g., adding another wish or adjusting the priority level of a wish). Another advantage of using the two-phased approach is that it allows us to reduce the carry-over effect, which is not possible using the BMS-based schedule. The Royal Belgian Soccer Association used the schedule that the two-phased approach generated for the 2007–2008 season.

Results

A comparison of the three methods discussed is not straightforward because manual schedules for the 2006–2007 and 2007–2008 seasons are not available. Furthermore, because the wishes and constraints were never formalized and classified into priority categories until we became involved (beginning with the 2006–2007 season), it is impossible to use our approaches to find schedules that we can compare directly with a manual schedule. Nevertheless, the number of teams and the type of constraints did not change over the years; however, the number of constraints did increase because of our involvement. Table 3 summarizes the most important results.

According to the goal-function value, which is the total penalty of the unsatisfied wishes and is to be minimized, the quality of the schedules created using

	Manual approach	Assignment approach (BMS)		Two-phased approach	
Season	2005–2006	2006–2007	2007–2008	2006–2007	2007–2008
Goal-function value	>75,000	11,698	9,806	1,528	2,144
Police wishes (satisfied) (%)	70	95	96	95	100
Club wishes (satisfied) (%)	32	68	58	81	66
Television wishes (satisfied)					
Top teams (%)	29 (100)	76 (100)	59 (100)	70 (100)	82 (100)
Top 6 (%)	6 (65)	53 (100)	47 (88)	59 (100)	58 (100)
Teams with top games in both season halves	13	18	17	18	18
Unbalancedness of carry-over effects	4,386	4,386	4,386	992	1,006

Table 3: This table shows the results of the three solution methods for several seasons.

the assignment approach is approximately seven times better than those generated manually (although for the goal-function value of the manual schedule, we considered only police constraints and constraints similar to the ones that the calendar committee assigned priority 1 or 2 in subsequent years). The two-phased approach improves additionally on the assignment model by a factor of five, satisfying all nonconflicting wishes of priority 3 or higher. This improvement in quality is also evident when we look at the number of constraints that are satisfied. Whereas the manual schedule satisfied 70 percent of the police constraints for the 2005–2006 season, the assignment-approach schedule increased this to 95 percent for the 2006–2007 season. Using the two-phased approach, we reached 100 percent for the 2007–2008 season. The proportion of club wishes that could be satisfied doubled to over two-thirds when an automated approach replaced manual scheduling, despite an ever increasing number of club wishes; this explains why the proportion of satisfied club wishes dropped again for the 2007–2008 season. The television wishes that Table 3 mentions refer to the percentage of rounds that have as many top teams playing a home game as top teams playing away (and between brackets the percentage of rounds with at least one top team—or at least two teams in the top-six case—playing an away game). In the manual schedule, the consideration of television wishes was limited to a reasonable spread of the top games; consequently, it provided few rounds that showed an equal balance between top teams playing a home game and top teams playing away. Using the assignment approach, especially the two-phased approach, the concerns of Belgacom TV were met for the vast majority of the rounds. Furthermore, all 18 teams now play at least one top club at home in both halves of the season; using the manual schedule, only 13 teams did. Finally, the two-phased approach allowed us to reduce the unbalancedness of carry-over effects by a factor of four. Russell (1980) provides additional details.

Impact

The Jupiler League played its 2006–2007 season according to the schedule that the assignment model generated; we developed its 2007–2008 season schedule using the two-phased approach. The results show that all involved parties have benefited from our approach. Indeed, it met the wishes of the police, clubs, and Belgacom TV. Two factors were considered major improvements of our approach: (1) All teams play at least one home game against a top team in each half of the season, and (2) club wishes were satisfied to a far greater extent than previously. Therefore, the clubs gradually gave us more wishes, making the scheduling of Belgian soccer a challenging task year after year. In more general terms, generating a schedule has become more professional; this, in turn, has influenced the clubs and Belgacom TV to systematically list their wishes.

Furthermore, we note that either the assignment model, which we generate in approximately two hours, or the two-phased approach, which we generate in 10 minutes, allows us to obtain a schedule much faster than the manual approach, which takes one week, does. This is important because it gives the calendar committee extra time to suggest improvements; hence, it allows the committee to investigate alternative schedules. The computational efficiency of our approach also allows us to show stakeholders the negative consequences (with respect to the other constraints) of granting one of their unfulfilled wishes. Thus, we can show the committee members that we have considered all their wishes in selecting the final schedule. Until 2005, developing any schedule was difficult; it was virtually impossible to redo schedules to respond to all the concerns of the calendar-committee members. This left them with an uncomfortable feeling about the quality of the schedule, and led to debates and complaints in the media.

Finally, the media received our approach positively. Most newspapers noticed that clubs now have a better schedule and praised the calendar committee for the professional nature of the scheduling process (Cuvelier 2006); criticisms were mostly based on defects that any schedule would have (e.g., not all teams can start with a home game or avoid games against the top clubs in the beginning of the season). Although a careful selection of the priorities made it possible to grant at least one wish from each team, negative comments from the clubs seem unavoidable; some coaches think that it is in their best interest to hedge their team against a poor season start by pointing to a tough schedule. Guy Mangelschots, former coach of Sint-Truiden, summarized it as follows: "When after five games you look at the points you have, only then you can say whether the schedule was good or bad" (Martens 2007).

Conclusion

We argued that there is no single model that is readily applicable for all soccer-league scheduling problems. Nevertheless, the concept of linking a penalty to each wish and subsequently searching for the schedule that minimizes the total penalty of the violated constraints is applicable to any such problem. Of course, an all-comprising model will not be solvable within a reasonable amount of time. Therefore, dividing the problem into smaller subproblems is a reasonable approach. We used a variant of the "first-break-then-schedule" approach (Rasmussen and Trick 2008), which determines the home-away pattern of each team first, and then decides the actual opponents. For scheduling Belgian soccer, this method seems quite appropriate because it considers constraints related to breaks to be the most important, followed by those constraints that are related to assigning the home-away patterns (because they often originate from the police or from the unavailability of a stadium). Wishes related to the actual opponent are generally less important. Any sports-scheduling problem with the same structure could follow a similar approach.

Appendix A. The Assignment Model

In the BMS-based model, we used the following notation:

Variables

$x_{t,n} = 1$ if team t is assigned to number n, 0 otherwise.

$y_c = 1$ if constraint c is violated, 0 otherwise.

Parameters

$d_{t,n} =$ penalty associated with assigning number n to team t.

$q_c =$ penalty associated with violating constraint c.

$a_{t,n}, b_c =$ parameters to model wishes that we cannot express directly in the goal function.

We can derive a home-away pattern from each number n in the BMS. The following model results in an assignment of teams to numbers in the BMS, such that the total penalty of the violated constraints is minimized:

$$\begin{aligned} \min \quad & \sum_t \sum_n d_{t,n} x_{t,n} + \sum_c q_c y_c \\ \text{subject to} \quad & \sum_t x_{t,n} = 1 \quad \text{for all } n, \\ & \sum_n x_{t,n} = 1 \quad \text{for all } t, \\ & \sum a_{t,n} x_{t,n} \le b_c + y_c \quad \text{for all } c, \\ & x_{t,n} \in \{0,1\} \quad \text{for all } t, n, \\ & y_c \in \{0,1\} \quad \text{for all } c. \end{aligned}$$

The first set of constraints ensures that each number is assigned to exactly one team; the second set ensures that each team is assigned to exactly one number. Whereas we can model some wishes (e.g., a team does not want to play a home game in a given round) by plugging in the penalty with the corresponding x variables directly in the goal function, there are wishes that must be modeled using the third set of constraints. For example, when team A prefers not to play against team B in a given round, we need a constraint $x_{A,n1} + x_{B,n2} \le 1 + y$ for each pair of numbers $(n1, n2)$ in the BMS that have a game in that given round. The third set of constraints can be used to model all constraints relevant to scheduling Belgian soccer by selecting appropriate values for $a_{t,n}$ and b_c.

Appendix B. The Two-Phased Approach

In the first phase, we assign a home-away pattern to each team, using as variables:

$x_{t,p} = 1$ if team t is assigned to home-away pattern p, 0 otherwise.

$y_c = 1$ if constraint c is violated, 0 otherwise.

As parameters, we used:

$d_{t,p} =$ penalty associated with assigning home-away pattern p to team t.

$q_c =$ penalty associated with violating constraint c.

$a_{t,p}, b_c =$ parameters to model wishes that we cannot express directly in the goal function.

With the exception of the definition of the x variables, the resulting model looks very similar to the assignment model; however, the number of constraints that it considers is considerably smaller:

$$\begin{aligned}
\min\ & \sum_t \sum_p d_{t,p} x_{t,p} + \sum_c q_c y_c \\
\text{subject to}\ & \sum_t x_{t,p} = 1 \quad \text{for all } p, \\
& \sum_p x_{t,p} = 1 \quad \text{for all } t, \\
& \sum a_{t,n} x_{t,n} \le b_c + y_c \quad \text{for all } c, \\
& x_{t,p} \in \{0,1\} \quad \text{for all } t, p, \\
& y_c \in \{0,1\} \quad \text{for all } c.
\end{aligned}$$

The goal function minimizes the total penalty of the violated constraints. The first set of constraints ensures that each home-away pattern is assigned to exactly one team; the second set enforces the constraint that each team is assigned to precisely one home-away pattern. Many wishes that are relevant in this phase can be enforced directly in the goal function (e.g., a team does not want to play an away game in a given round). Other wishes require constraints of the third set to be modeled. For example, when two teams, A and B, want at most three rounds in which they simultaneously play a home game, we enforce constraint $x_{A,p1} + x_{B,p2} \le 1 + y$ for each pair of patterns $(p1, p2)$ with more than three rounds with simultaneous home games.

In the second phase, we determine the actual games, using the home-away pattern assignment from phase 1 as an input. We use the following variables:

$x_{i,j,k} = 1$ if team i plays a home game against team j on round k, 0 otherwise (provided that this game is compatible with the home-away patterns of team i and j).

$y_c = 1$ if constraint c is violated, 0 otherwise.

We used the following parameters:

$d_{i,j,k} =$ penalty associated with team i playing at home against team j on round k.

$q_c =$ penalty associated with violating constraint c.

$a_{i,j,k}, b_c =$ parameters to model wishes that we cannot express directly in the goal function.

The model for the second phase determines the actual opponents of each team in every round. We note that an $x_{i,j,k}$ variable exists only insofar as a home game of team i against team j on round k is allowed by the home-away patterns that were assigned to teams i and j. Note also that because of the mirroring assumption, we only need to schedule the first half of the season (we can model the constraints related to the second half of the season in terms of the first half of the season):

$$\begin{aligned}
\min\ & \sum_i \sum_j \sum_k d_{i,j,k} x_{i,j,k} + \sum_c q_c y_c \\
\text{subject to}\ & \sum_j (x_{i,j,k} + x_{j,i,k}) = 1 \quad \text{for all } i, k, \\
& \sum_k (x_{i,j,k} + x_{j,i,k}) = 1 \quad \text{for all } i, j\colon i \neq j, \\
& \sum a_{i,j,k} x_{i,j,k} \le b_c + y_c \quad \text{for all } c, \\
& x_{i,j,k} \in \{0,1\} \quad \text{for all } i, j, k \text{ allowed by the home-away patterns of team } i \text{ and } j, \\
& y_c \in \{0,1\} \quad \text{for all } c.
\end{aligned}$$

The first set of constraints ensures that each team plays exactly once on each round; the second set ensures that the teams in each pair meet each other precisely once in each half of the season. Again, we can directly model most relevant wishes in this phase using the goal function (e.g., do not schedule a top game in the first round). Other wishes must be modeled using the third set of constraints. For example, let us consider the television station's wish that there is at most one top game per round. Because a top game is a game between two of the four top teams, the league has 12 top games, and there can be at most two top games per round. When G is the set of top games, each round k requires a constraint c as follows: $\sum_{(i,j) \text{ in } G} x_{i,j,k} \le 1 + y_c$. This illustrates that by selecting appropriate values for $a_{i,j,k}$ and b_c, we can model any of the constraints relevant in this phase by using one or more constraints of the third set.

The tabu search algorithm uses as a neighborhood those phase 1 solutions that can be reached by swapping the home-away patterns of two teams, and continues with the best move within this neighborhood over the two phases. Note that computing the consequences of a swap in terms of phase 1 penalties is trivial; we do not have to solve phase 2 if the result of solving phase 1 is worse than the best solution thus far in this neighborhood. Furthermore, we maintain a tabu list containing the last five swaps. Unless one of the last two iterations shows improvement compared to its predecessor, the algorithm stops after 10 consecutive iterations without further improvement of the best solution thus far (which happens typically after less than 50 iterations). Typically, the tabu search procedure, which we implemented in C++ using Microsoft Visual Studio, improves the solution by simply solving phase 1, and phase 2 subsequently, by a factor of three.

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Robert Sterckx, Secretary-General of the League of Professional Football (LPF), Liga Beroepsvoetbal—K.B.V.B., Avenue Houba de Strooper, 145, 1020, Brussels, Belgium, writes: "I hereby state that the schedules found in a collaboration between the Royal Belgian Football Association and K.U. Leuven's team Dries Goossens and Frits Spieksma are being used in practice.

"Due to the involvement of the K.U. Leuven team, we have been able to come up with schedules that are much more satisfying for our partners (the police, Belgacom TV, and the clubs).

"In addition, the transparency of the process of agreeing upon a schedule has improved considerably as well.

"We look forward to extending this collaboration."