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Point values are assigned for each question. Points earned: ____ / 100

1. Consider the algorithm on page 148 in the textbook for binary reflected Gray codes. What change(s) would you make so that it generates the binary numbers **in order** for a given length `n`? Your algorithm must be recursive and keep the same structure as the one in the textbook. Describe only the change(s). (10 points)

   The L2 should be reversed before it is appended to L1 to get *L* in order for the binary numbers to appear in order for a given length n.

2. Show the steps to multiply 72 x 93 with Russian peasant multiplication, as seen in Figure 4.11b on page 154 in the textbook. (10 points)

N	M
72	93
36	186
18	32
9	**744**
4	1488
2	2976
1	**5952**

   744 + 5952 = 6696

3. Suppose you use the LomutoPartition() function on page 159 in the textbook in your implementation of quicksort. (10 points, 5 points each)
   a. Describe the types of input that cause quicksort to perform its worst-case running time

      Inputs that would cause Lomuto Partition to perform its worst-case running time would be arrays that are already sorted in either increasing order looking for the biggest value or decreasing order and looking for the smallest value, with the pivot selected as the first element, every other element will either be all larger or all smaller than the pivot, making little progress in partitioning the array .

   b. What is that running time?
      This running time is $\Theta(n^2)$

4. Compute 2205 x 1132 by applying the divide-and-conquer algorithm outlined in the text. Repeat the process until the numbers being multiplied are each 1 digit. For each multiplication, show the values of c_2, c_1, and c_0. Do not skip steps. (10 points)

2205 = 22*100+5
1132 = 11*100+32
2205*1132 = (22*100+5)*(11*100+32)
= (22*11)*10000 + (22 * 32 + 5*11) * 100 + (5*34)

(22*32+5*11) = ((22+5) * (11+32) – 22*11 – 5*34)

= (22*11)*10000 + ((22+5) * (11+32) – 22*11 – 5*34)
* 100 + (5*34)
= (22*11)*10000 + (27*43 – 22*11 – 5*34)*100 + 5*34

= (22*11)*10000 + ((22+5) * (11+32) – 22*11 – 5*34))* 100 + (5*34)
22*11 = (2*10 +2) * (1*10 + 1)
= (2*1) * 100 + (2*1 + 2*1) * 10 + 2*1
= (2*1)*100 + ((2+2)*(1+1) – 2*1 – 2*1)*10 + 2*1
= 2*1 *100 + (4*2 – 2*1 – 2*1)*10 + 2*1
= 200 + 40+2

05*34 = (0*10+5)*(3*10+4)
= (0*3) *100 + (0*4+5*3)*10+ 4*5
= 0*3*100 + (0+5)*(3+4)-0*3-5*4)*10 + 4*5
= 0 + (5*7 -0-20)*10 + 20
= 150+20
= 170

27*43 = (2*10+7) * (4*10+3)
= (2*4) *100 + (2*3+7*4)*10 + 3*7
= 8*100 + ((2+7)*(3+4) – 2*4 – 7*3)*10 + 3*7
= 800 + 340+21
= 1161

= (22*11)*10000 + (27*43 – 22*11 – 5*34)*100 + 5*34
= 242*10000+ (1161-242 – 170)*100 + 170
= 2496060

5. Draw the binary search tree after inserting the following keys: 24 18 67 68 69 25 19 20 11 93 (10 points)

24

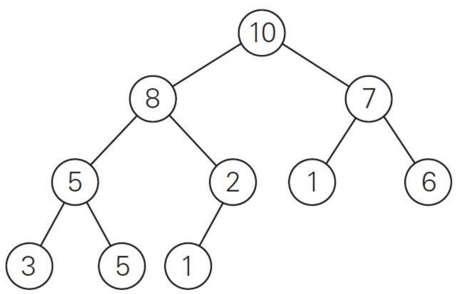
18 67

11 19 25 68

20 69

93

6. Consider the following binary tree. (16 points, 2 points each)



a. Traverse the tree preorder.
   10,8,5,3,5,2,1,7,1,6
b. Traverse the tree inorder.
   3,5,5,8,1,2,10,1,7,6
c. Traverse the tree postorder.
   3,5,5,1,2,8,1,6,7,10
d. How many internal nodes are there?

5 internal nodes (non-leaves)

e. How many leaves are there?
   5 Leaves
f. What is the maximum width of the tree?
   Max width is 4
g. What is the height of the tree?

Height is 3

h. What is the diameter of the tree?

Diameter is 5

7. Use the Master Theorem to give tight asymptotic bounds for the following recurrences. (25 points, 5 points each)

a. $T(n) = 2T(n/4) + 1$

A = 2; B = 4; D = 0

$2 > 4^0$

Θ($n^{\log4(2)}$) = **Θ(sqrt(n))**

b. $T(n) = 2T(n/4) + \sqrt{n}$

A = 2; B = 4; D = 1/2

2 = sqrt(4)

Θ(sqrt(n)*log4(2))= **Θ (sqrt(n)/2)**

c. $T(n) = 2T(n/4) + n$

A = 2; B = 4; D = 1

$2 < 4^1$

Θ(n)

d. $T(n) = 2T(n/4) + n^2$

A = 2; B = 4; D = 2

$2 < 4^2$

Θ(n^2)

e. $T(n) = 2T(n/4) + n^3$

A = 2; B = 4; D = 3

$2 < 4^3$

Θ(n^3)

8. Consider the following function. (9 points)

```
int function(int n) {
if (n <= 1) {
return 0;
```

```
    }     int temp = 0;     for
(int i = 1; i <= 6; ++i) {
temp += function(n / 3);
    }     for (int i = 1; i <= n; ++i) {
   for (int j = 1; j * j <= n; ++j) {
            ++temp;
         }
    }
    return temp;
}
```

a) Write an expression for the runtime $T(n)$ for the function. (4 points)

T(n) = 6T(n/3)+ (n*sqrt(n))

$T(n) = 6T(n/3) + (n^{3/2})$

b) Use the Master Theorem to give a tight asymptotic bound. Simplify your answer as much as possible. (5 points)

A = 6; B = 3; D = 3/2

$6 > 3^{3/2}$

$\Theta(n^{\log 3(6)})$