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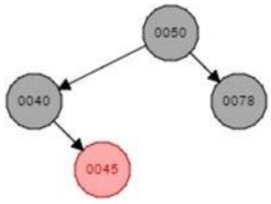
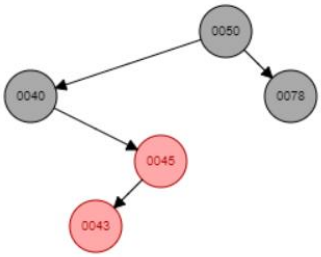
Points earned: ____ / 74 = ____ %

1. Show the red-black tree after inserting a node with the key 0043. Use the document on Canvas that explains the insertion process succinctly. List the case you applied (i.e. 1, 2a, 3b), and write the steps you took to fix the tree (also listed in the document).



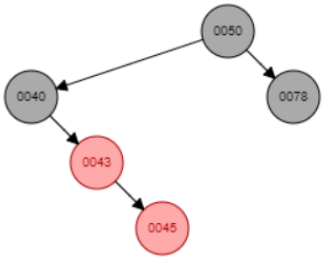
a) Draw the tree after doing a regular binary search tree insertion of 0043. (3 points)



b) Which RBT property is violated? (3 points) **Property 4 is violated "If a node is red then (both) its children are black".**
Case seen after the regular binary search tree insertion: (3 points**) Case 2b**

Steps taken to fix the tree: (3 points) **Node 43 becomes the parent of 45 and a right rotation is performed**

Draw the tree after taking the steps you just described. (3 points)



c) Which property is violated now? (3 points) **Property 4 is violated "If a node is red then (both) its children are black".**

d) Case seen after first fixup: (3 points) **Case 3b**

Steps taken to fix the tree: (3 points) **Node 43 becomes black, node 40 becomes red, and a left rotation is performed around node 43.**

Draw the tree after taking the steps you just described. (3 points)

2. Draw the 2-3 tree after inserting each of the following keys. Redraw the whole tree for each part.

a) 50 (1 point)

b) 76 (1 point)

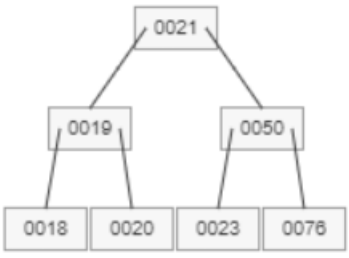
c) 23 (3 points)

d) 21 (3 points)

e) 20 (3 points)

f) 19 (3 points)

g) 18 (3 points)



3. Read pages 241-242 in the textbook. Using that information, write pseudocode for computing the LCM of an array A[1..n] of integers. You may assume there is a gcd() function available. (6 points)

```
ALGORITHM LCM(A[1..n]):
    // Computes the least common multiple of all the integer in array A
        Product = 1                          // will store A[0]*…*A[n-1]
        For i in array of integers (0 to n-1) {
                Multiply A[i] to product
        }
        Solution = product / gcd(A)          // Assuming GCD also takes an array of integers
        Return Solution
```

4. Horner's method: $p(x) = 4x^4 + 5x^3 - 2x^2 - 4x + 7$

   a) Repeatedly factor out x in the polynomial above so that you can apply Horner's method. Write your final expression for $p(x)$. (5 points)

   $$\boldsymbol{p(x) = 7 + x((-4) + x((-2) + x(5 + x(4))))}$$

   b) Show values of the array P[0..n] as needed to apply Horner's method. (3 points)
   [7, -4, -2, 5, 4]

   c) Apply Horner's method to evaluate the polynomial at x 2. Make a table as we did in class showing the values x, p, n, and i, and then state your final answer for $p(2)$. (5 points)

x	p	n	i
2	4	4	
	13		3
	24		2
	44		1
	95		0

$p(2) = 95$

d) Use **synthetic** (not long) **division** to divide $p(x)$ by x - 2 to check your work. Be sure to show your work. (5 points)

x_0 | x^4 x^3 x^2 x^1 x^0

---

2 | 4 5 -2 -4 7

| 0 8 26 48 88

---

0 | 4 13 24 44 **95**

5. Rewrite the *LeftRightBinaryExponentiation* algorithm on page 237 in the textbook to work for $n = 0$ as well as any positive integer. *No credit will be given for answers that simply start with an if statement for n = 0.* (6 points)

```
ALGORITHM LeftRightBinaryExponentiation(a, b(n)):
    // Computes a^n
    product ← a
    for i ← l − 1 downto 0 do:
            sum += b_i
    if (sum == 0):
            return 1
    for i ← l − 1 downto 0 do
            product ← product ∗ product
            if b_i = 1 product ← product ∗ a
    return product
```

I added a new variable called sum, which will sum the literal digits (1's or 0's) that make up the binary representation of b(n). If n is 0, then all digits in its binary representation will be zeros and sum will remain 0, which the conditional will recognize. Since anything to the 0 is 1, the algorithm will return 1.

Alternatively:

```
            …
    for i ← l − 1 downto 0 do {
            sum += b_i
            if (sum != 0):
                    break;
            return 1;
            }
            …
```

This will also work, as any presence of 1 in the representation will indicate it is not zero, else return 1.