Simple Linear Regression

Prediction

Dr. Maria Tackett

Topics

Topics

- Predict the response given a value of the predictor variable

Topics

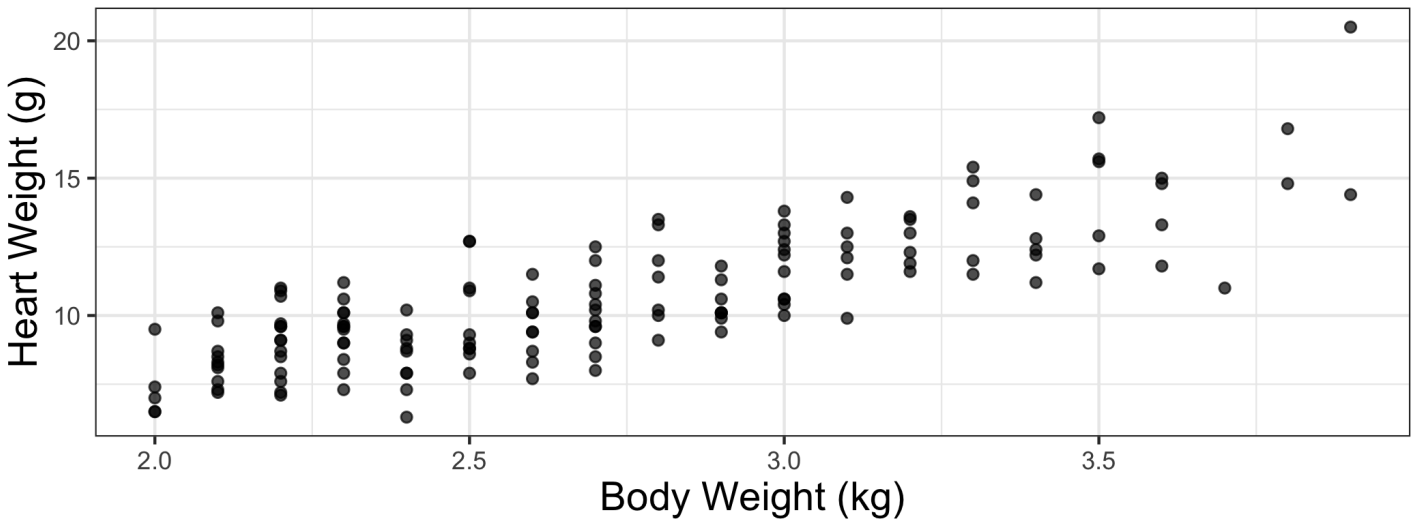
- Predict the response given a value of the predictor variable
- Use intervals to quantify the uncertainty in the predicted values

Topics

- Predict the response given a value of the predictor variable
- Use intervals to quantify the uncertainty in the predicted values
- Define *extrapolation* and why we should avoid it

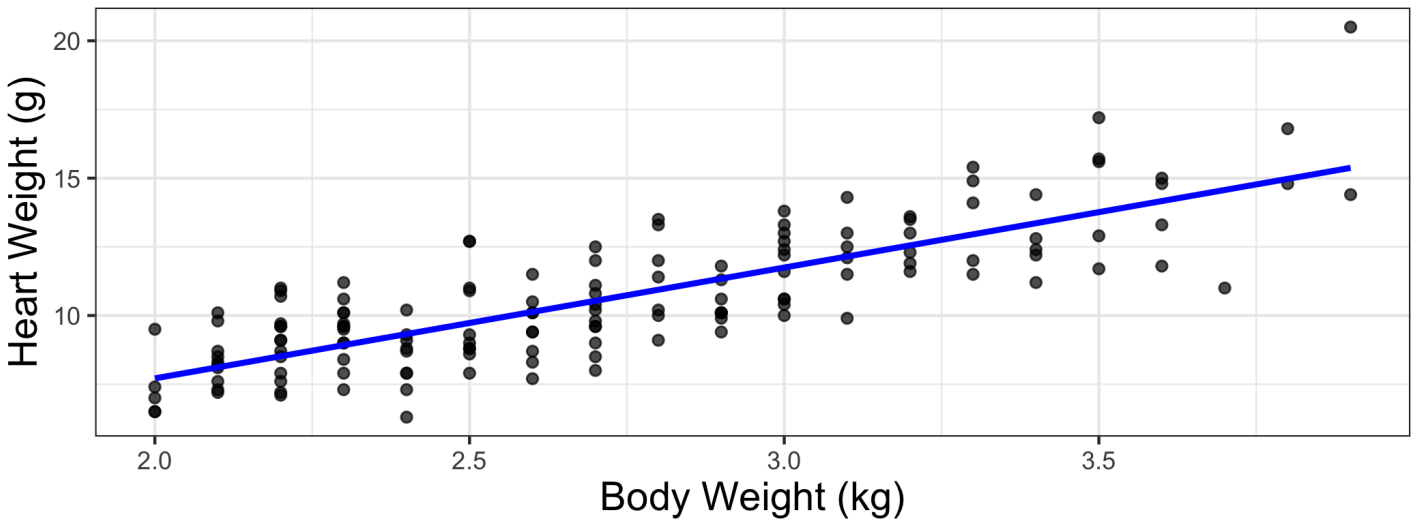
Cats data

The data set contains the **heart weight** (**`Hwt`**) and **body weight** (**`Bwt`**) for 144 domestic cats.



Cats data

The weight of a cat's heart is used to determine the appropriate dosage for heart medicine.



We want to fit a model so we can use a cat's body weight to predict how much its heart weighs.

The model

$$\hat{\text{Hwt}} = -0.357 + 4.034 \times \text{Bwt}$$

term	estimate	std.error	statistic	p.value
(Intercept)	-0.357	0.692	-0.515	0.607
Bwt	4.034	0.250	16.119	0.000

Prediction

We can use the regression model to

Prediction

We can use the regression model to

Estimate the **mean** response when the predictor variable is equal to a value x_0

Prediction

We can use the regression model to

Estimate the **mean** response when the predictor variable is equal to a value x_0

Predict the response for an **individual** observation with a value of the predictor equal to x_0

Calculating a predicted value

My cat Mindy weighs about 3.18 kg (7 lbs).

Based on this model, about how much does her heart weigh?

Calculating a predicted value

My cat Mindy weighs about 3.18 kg (7 lbs).

Based on this model, about how much does her heart weigh?

$$\hat{Hwt} = -0.357 + 4.034 \times \mathbf{3.18}$$
$$= \mathbf{12.471} \text{ grams}$$

Uncertainty in predictions

Uncertainty in predictions

Confidence interval for the mean response

$$\hat{y} \pm t^*_{n-2} \times \mathbf{SE}_{\hat{\boldsymbol{\mu}}}$$

Uncertainty in predictions

Confidence interval for the mean response

$$\hat{y} \pm t^*_{n-2} \times \mathbf{SE}_{\hat{\boldsymbol{\mu}}}$$

Prediction interval for an individual observation

$$\hat{y} \pm t^*_{n-2} \times \mathbf{SE}_{\hat{\mathbf{y}}}$$

Standard errors

Standard errors

$$SE_{\hat{\mu}} = \hat{\sigma} \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2}}$$

Standard errors

$$SE_{\hat{\mu}} = \hat{\sigma} \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2}}$$

$$SE_{\hat{y}} = \hat{\sigma} \sqrt{1 + \frac{1}{n} + \frac{(x - \bar{x})^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2}}$$

Standard errors

$$SE_{\hat{\mu}} = \hat{\sigma} \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2}}$$

$$SE_{\hat{y}} = \hat{\sigma} \sqrt{\mathbf{1} + \frac{1}{n} + \frac{(x - \bar{x})^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2}}$$

Confidence interval

Confidence interval

The 95% **confidence interval** for the ***mean*** heart weight of cats that weigh 3.18 kg is

Confidence interval

The 95% **confidence interval** for the ***mean*** heart weight of cats that weigh 3.18 kg is

fit	lwr	upr
12.472	12.143	12.801

Confidence interval

The 95% **confidence interval** for the ***mean*** heart weight of cats that weigh 3.18 kg is

fit	lwr	upr
12.472	12.143	12.801

We are 95% confident that mean heart weight for the subset of cats that weigh 3.18 kg is between 12.143 g and 12.801 g.

Prediction interval

Prediction interval

The 95% **prediction interval** for an ***individual*** cat that weighs 3.18 kg is

Prediction interval

The 95% **prediction interval** for an ***individual*** cat that weighs 3.18 kg is

fit	lwr	upr
12.472	9.582	15.362

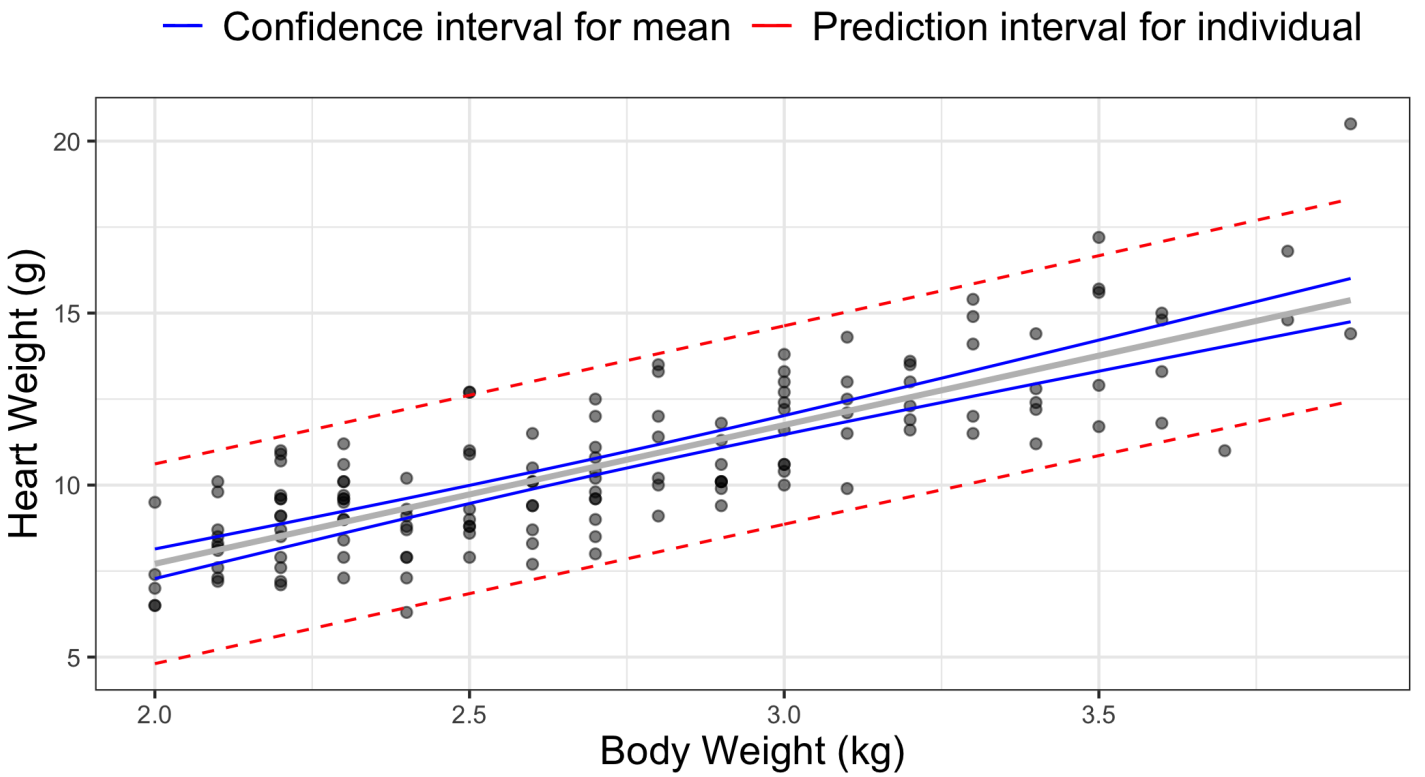
Prediction interval

The 95% **prediction interval** for an ***individual*** cat that weighs 3.18 kg is

fit	lwr	upr
12.472	9.582	15.362

We are 95% confident the heart weight for an individual cat that weighs 3.18 kg is between 9.582 g and 15.362 g.

Comparing intervals



Caution! Extrapolation

We should **not** use the model to predict for values of X far outside the range of values used to fit the model.

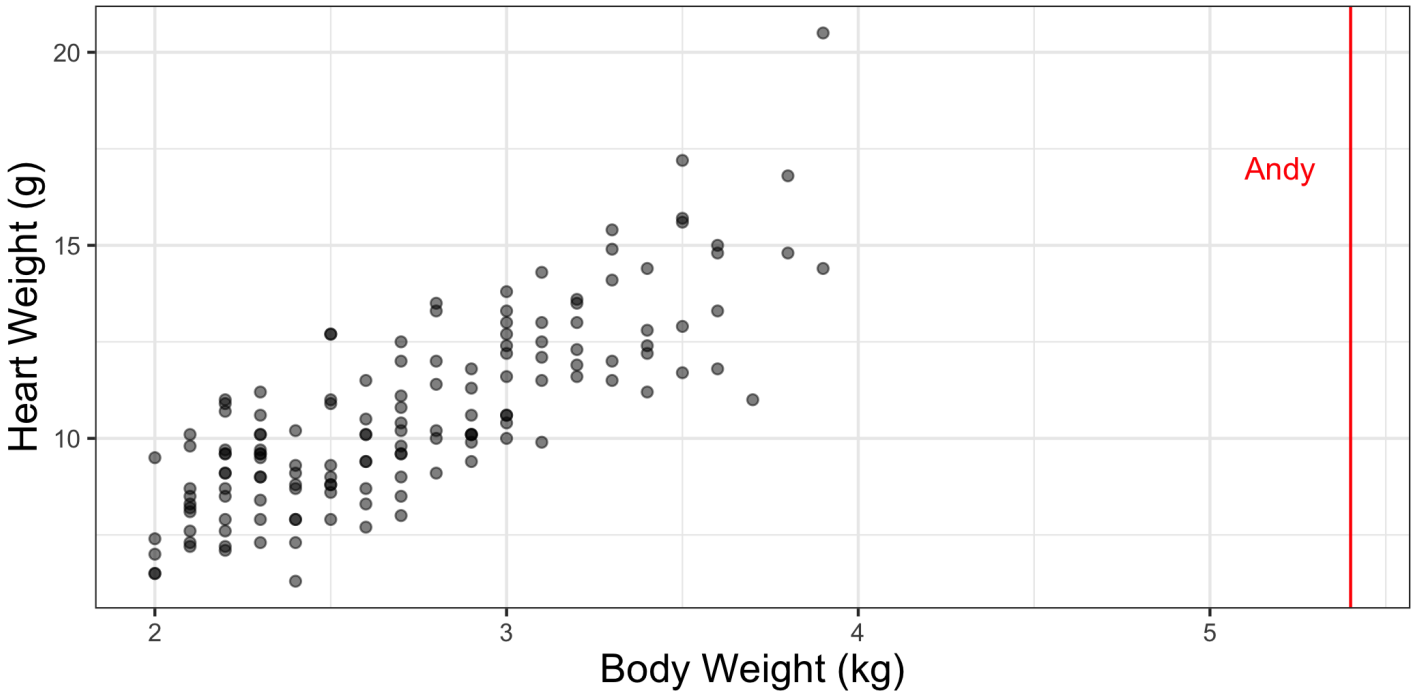
This is called **extrapolation**.

Predict Andy's heart weight?

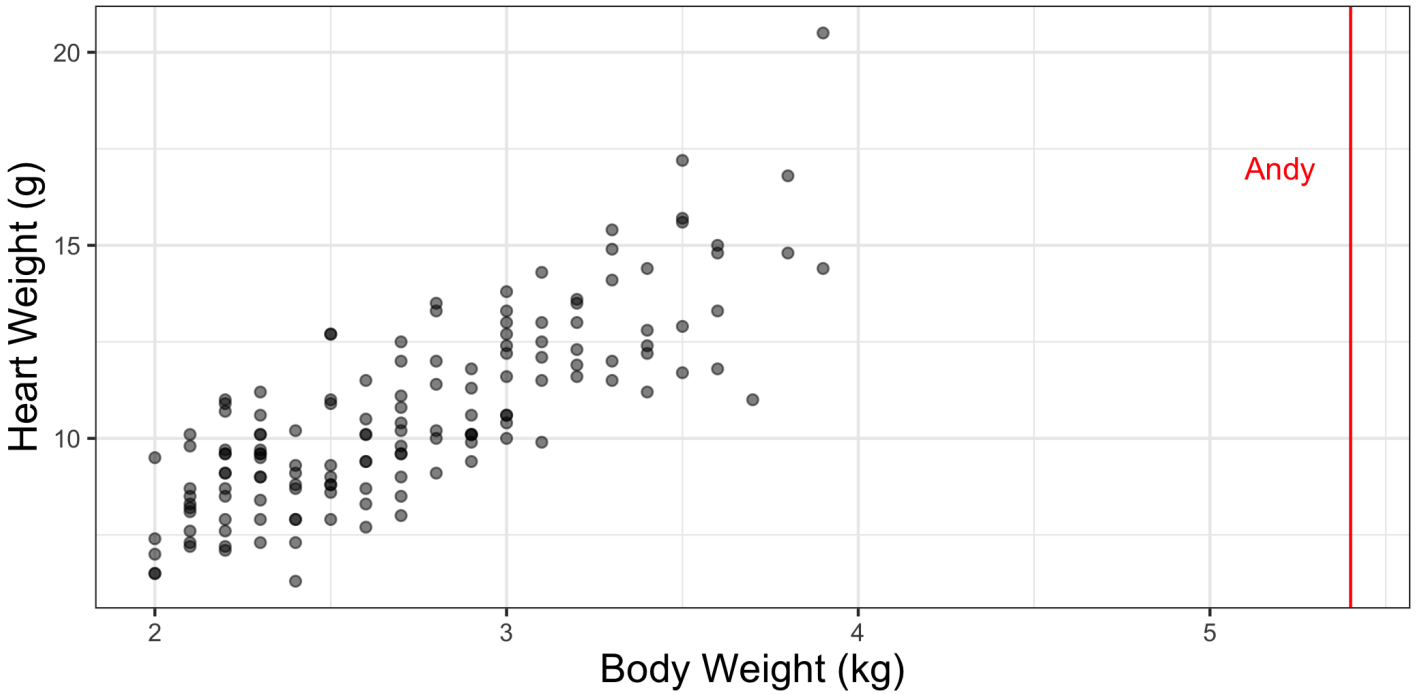
My cat Andy weighs about 5.44 kg (12 lbs).

Should we use this regression model to predict his heart weight?

Predict Andy's heart weight?



Predict Andy's heart weight?



We should **not** use this model to predict Andy's heart weight, since that would be **extrapolation**.

Recap

Recap

- Predicted the response given a value of the predictor variable

Recap

- Predicted the response given a value of the predictor variable
- Used intervals to quantify the uncertainty in the predicted values
  - Confidence interval for the mean response
  - Prediction interval for individual response

Recap

- Predicted the response given a value of the predictor variable
- Used intervals to quantify the uncertainty in the predicted values
  - Confidence interval for the mean response
  - Prediction interval for individual response
- Defined **extrapolation** and why we should avoid it