Simple Linear Regression

Partitioning variability

Dr. Maria Tackett

Topics

- Use analysis of variance to partition variability in the response variable
- Define and calculate R^2
- Use ANOVA to test the hypothesis $H_0 : \beta_1 = 0$

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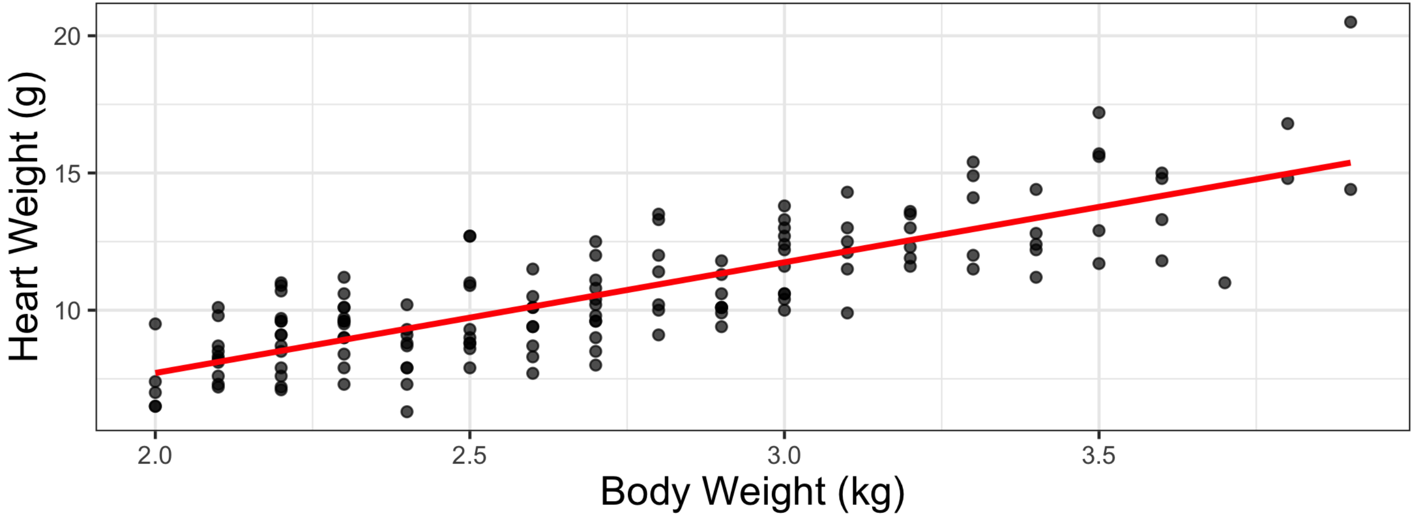
Cats data

The data set contains the **heart weight** (`Hwt`) and **body weight** (`Bwt`) for 144 domestic cats.

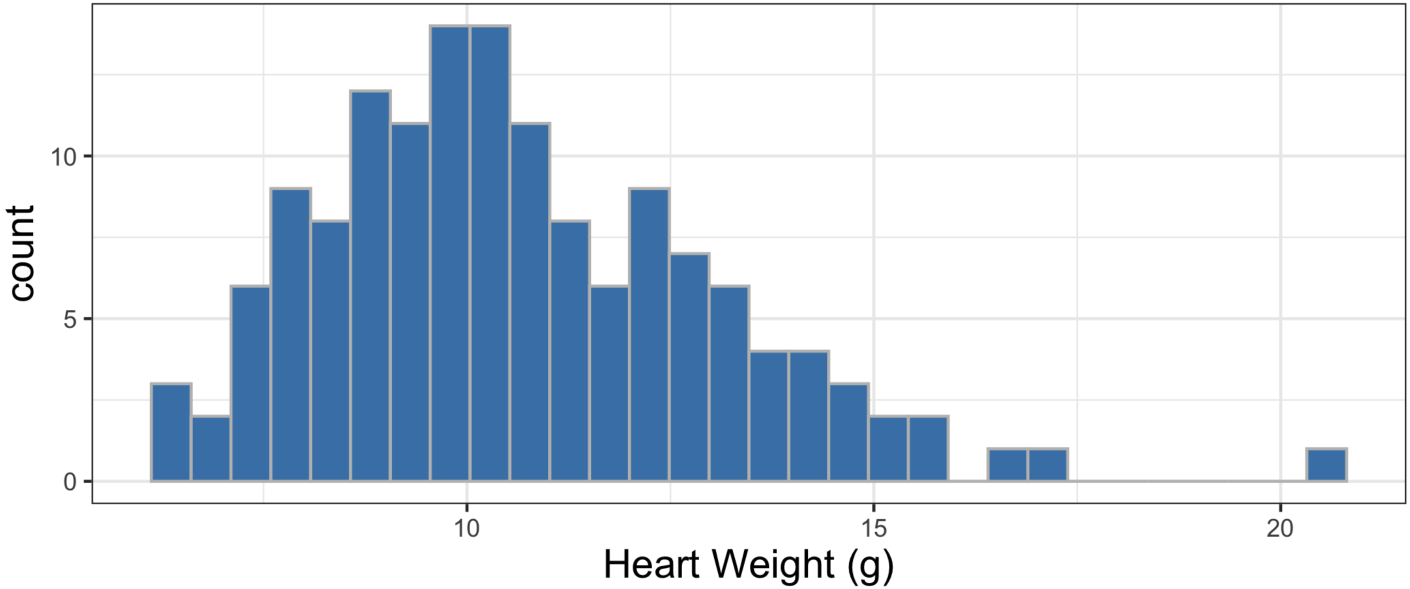


The model

$$\widehat{\text{Hwt}} = -0.357 + 4.034 \times \text{Bwt}$$

term	estimate	std.error	statistic	p.value
(Intercept)	-0.357	0.692	-0.515	0.607
Bwt	4.034	0.250	16.119	0.000

Distribution of response



Mean	Std. Dev.	IQR
10.631	2.435	3.175

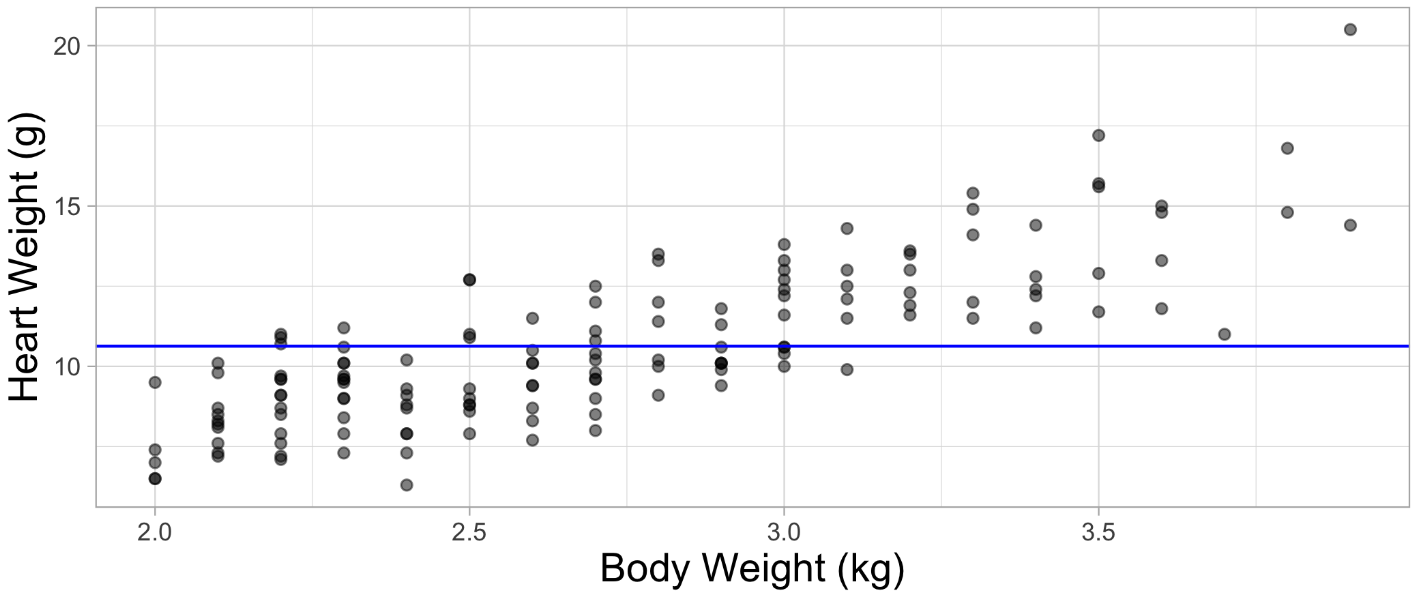
How much of the variability in cats' heart weights can be explained by knowing their body weights?

ANOVA

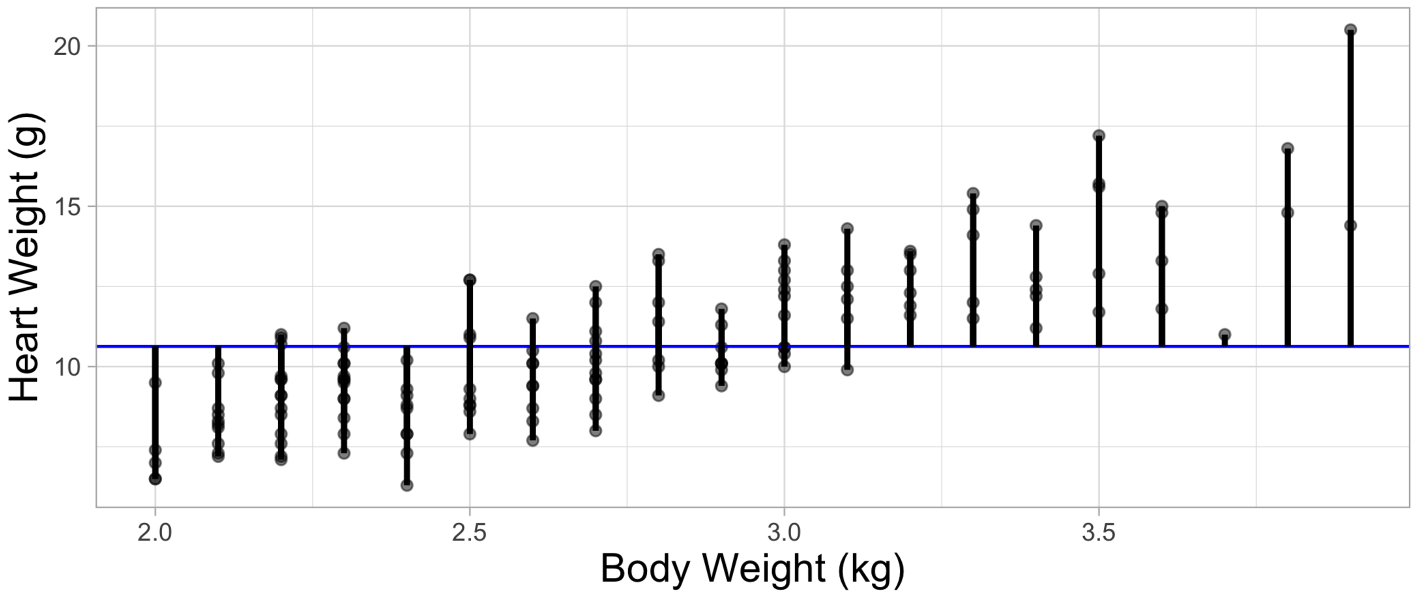
We will use **Analysis of Variance (ANOVA)** to partition the variation in the response variable Y.

Total variation in Y = Explained variation (Model) + Unexplained variation (Residuals)

Response variable, Y,

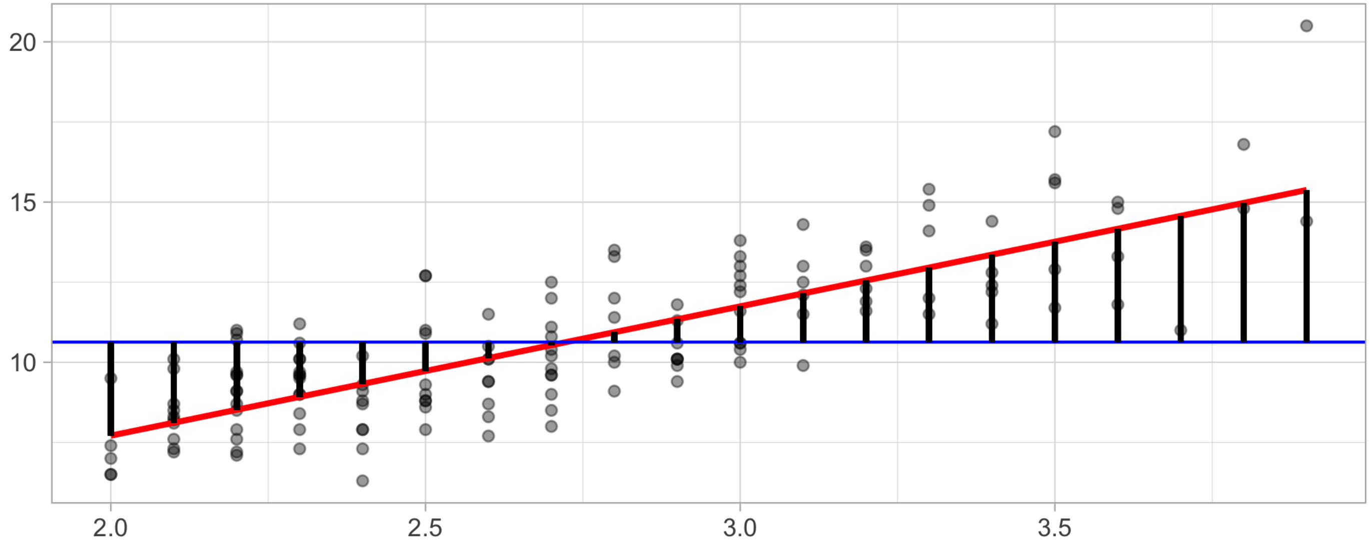


Total variation



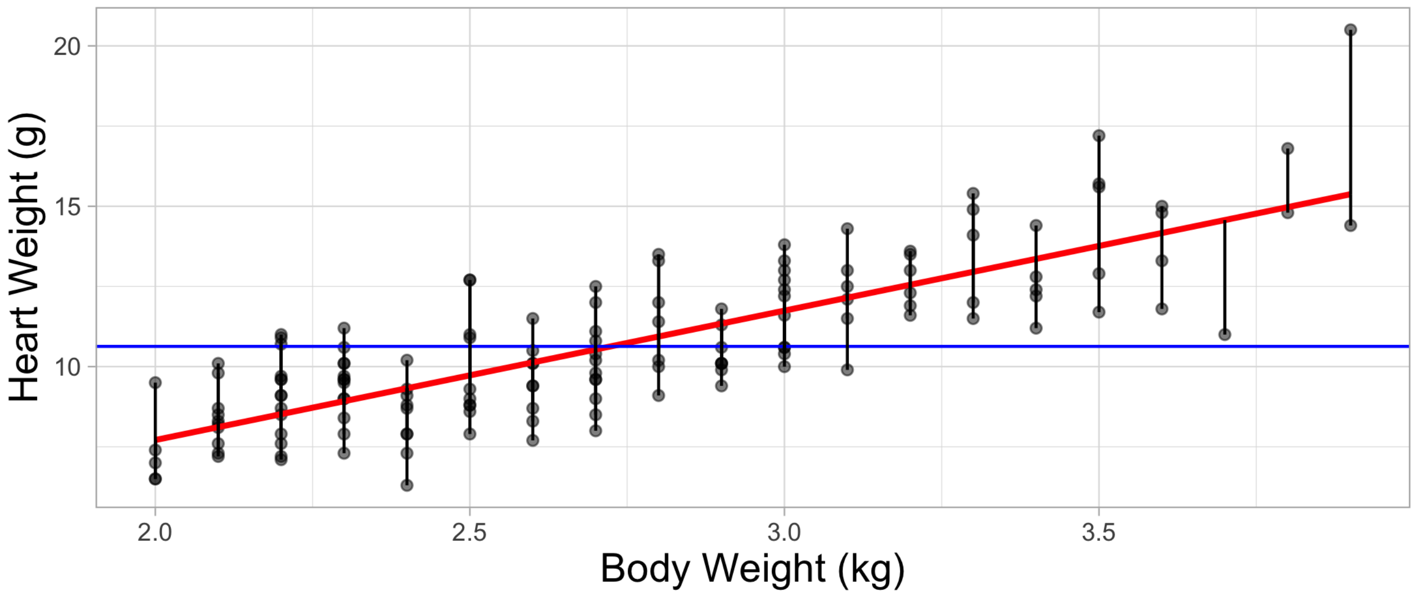
$$SS_{Total} = \sum_{i=1}^{n}(y_i - \bar{y})^2 = (n-1)s_y^2$$

Explained variation (Model)



$$SS_{Model} = \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2$$

Unexplained variation (Residuals)



$$SS_{Error} = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$\sum_{i=1}^{n}(y_i - \bar{y})^2 = \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2 + \sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

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R^2

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$$R^2 = \frac{SS_{Model}}{SS_{Total}} = 1 - \frac{SS_{Error}}{SS_{Total}}$$

R^2 for our model

$$SS_{Model} = 548.092$$

$$SS_{Error} = 299.533$$

$$SS_{Total} = 847.625$$

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$$= \mathbf{0.647}$$

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$$= \mathbf{0.647}$$

About 64.7% of the variation in the heart weight of cats can be explained by variation in body weight.

ANOVA

Source	Df	Sum Sq	Mean Sq	F Stat	Pr(> F)
Model	1	548.092	548.092	259.835	0
Residuals	142	299.533	2.109		
Total	143	847.625			

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Sum of squares

$$SS_{Total} = 847.625$$

$$SS_{Model} = 548.092$$

$$SS_{Error} = 847.625 - 548.092 = 299.533$$

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$$H_0 : \beta_1 = 0$$
$$H_a : \beta_1 \neq 0$$

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Degrees of freedom

$$df_{Total} = 144 - 1 = 143$$

$$df_{Model} = 1$$

$$df_{Error} = 143 - 1 = 142$$

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Mean squares

$$MS_{Model} = \frac{548.092}{1} = 548.092$$

$$MS_{Error} = \frac{299.533}{142} = 2.109$$

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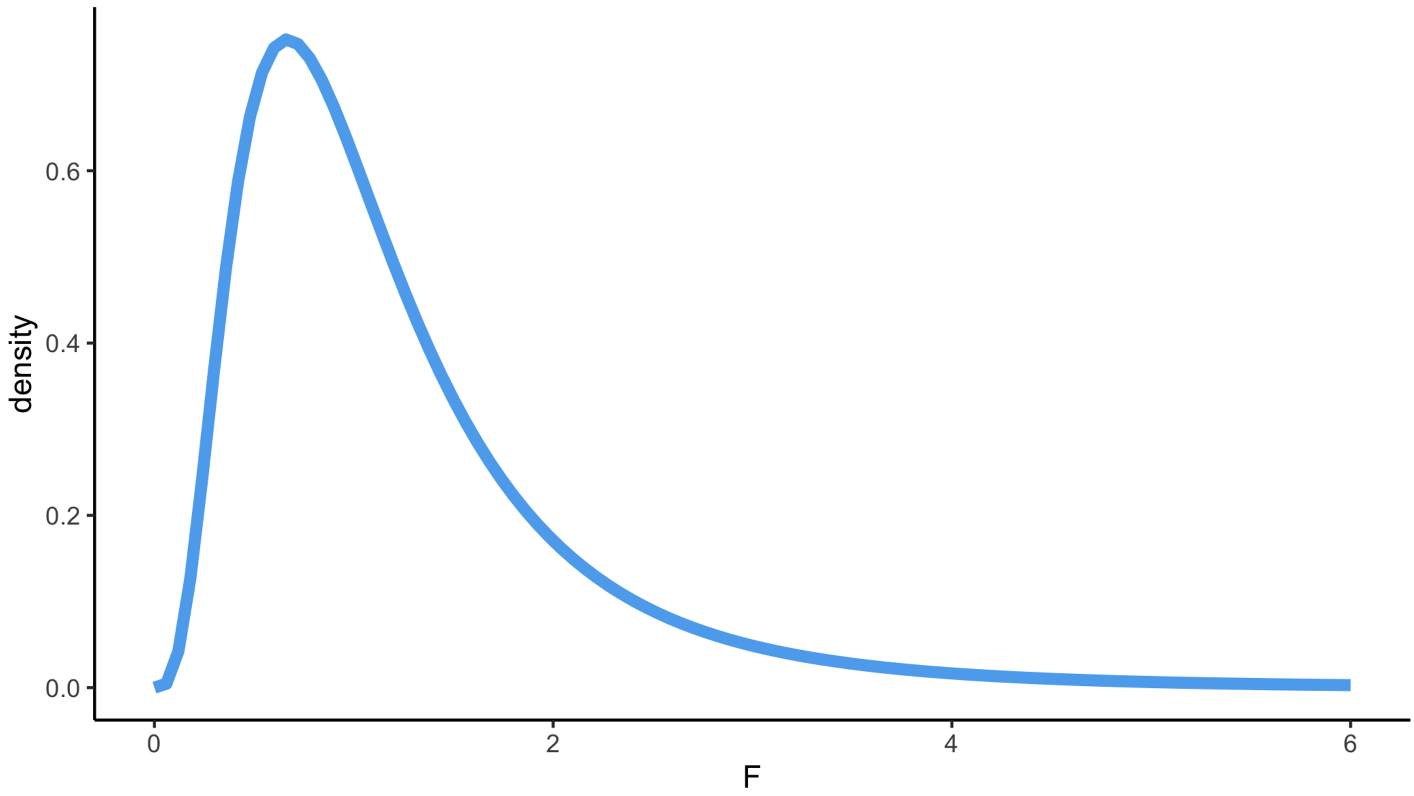
F test statistic: ratio of explained to unexplained variability

$$F_{(1,142)} = \frac{MS_{Model}}{MS_{Residuals}} = \frac{548.092}{2.109} = 259.835$$

F distribution



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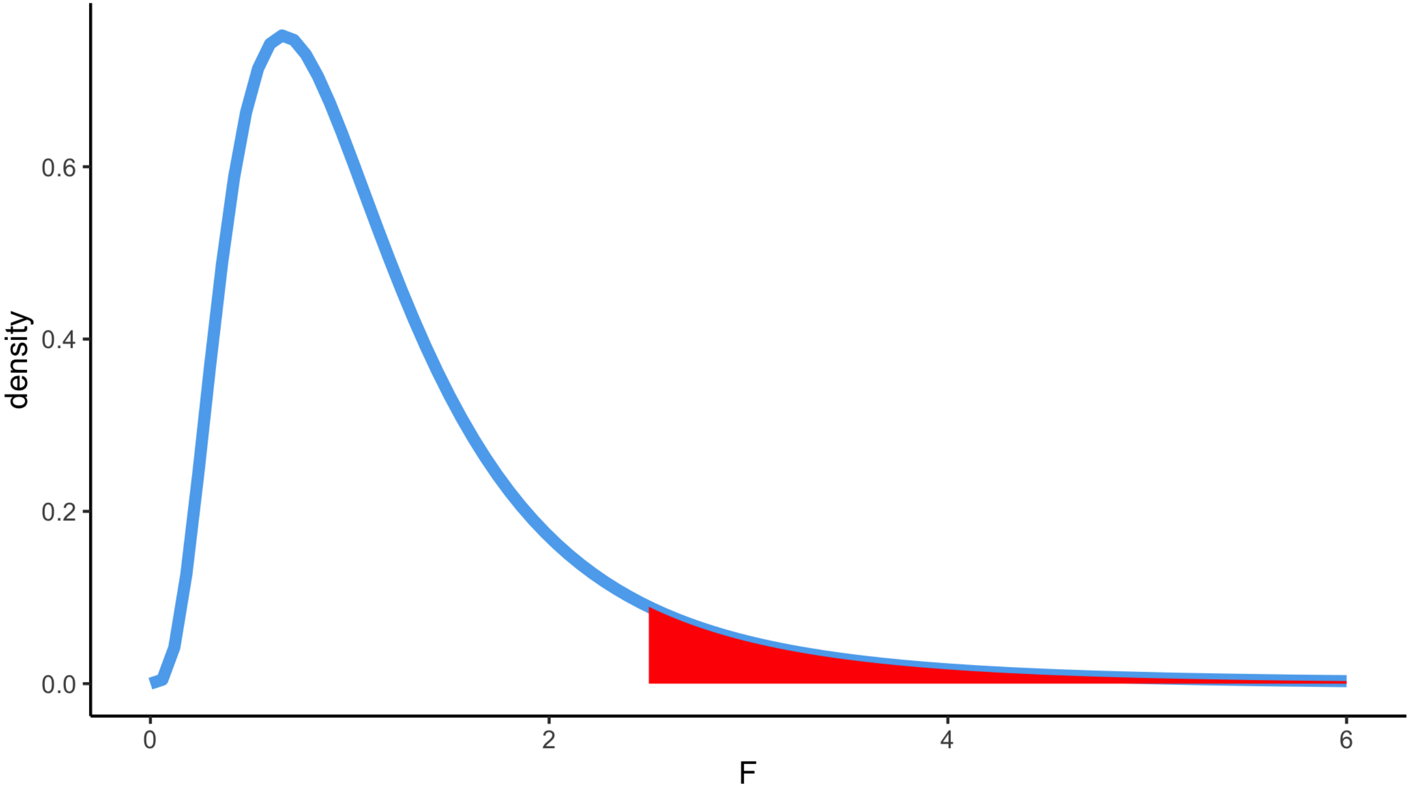

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P-value: Probability of observing a test statistic at least as extreme as *F Stat* given the true slope parameter is 0, i.e. $H_0 : \beta_1 = 0$

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The p-value is very small (≈ 0), so we reject H_0.

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There is evidence of a linear relationship between a cat's heart weight and body weight.

Recap

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