

Simple Linear Regression

Foundation

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General form of model

$$Y = f(X) + \epsilon$$

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ϵ : random error

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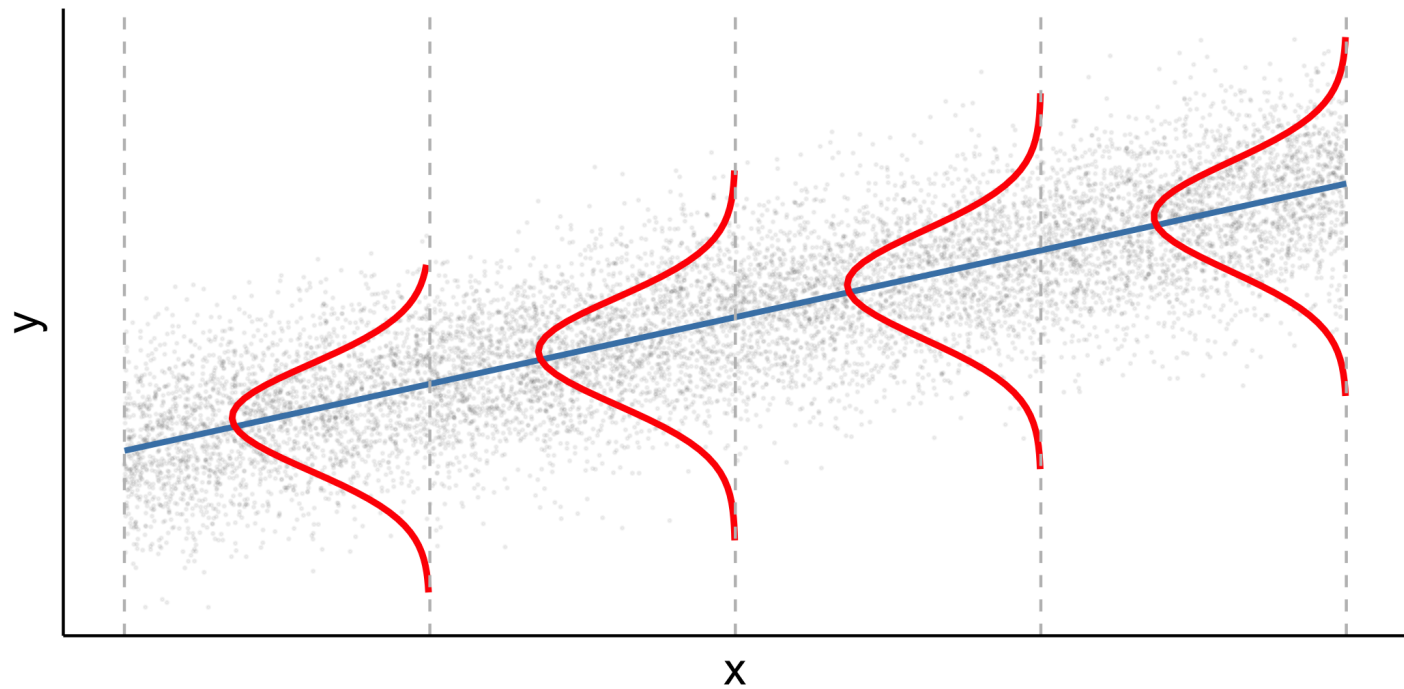
$$= \mu_{Y|X} + \epsilon$$

$$= \beta_0 + \beta_1 X + \epsilon$$

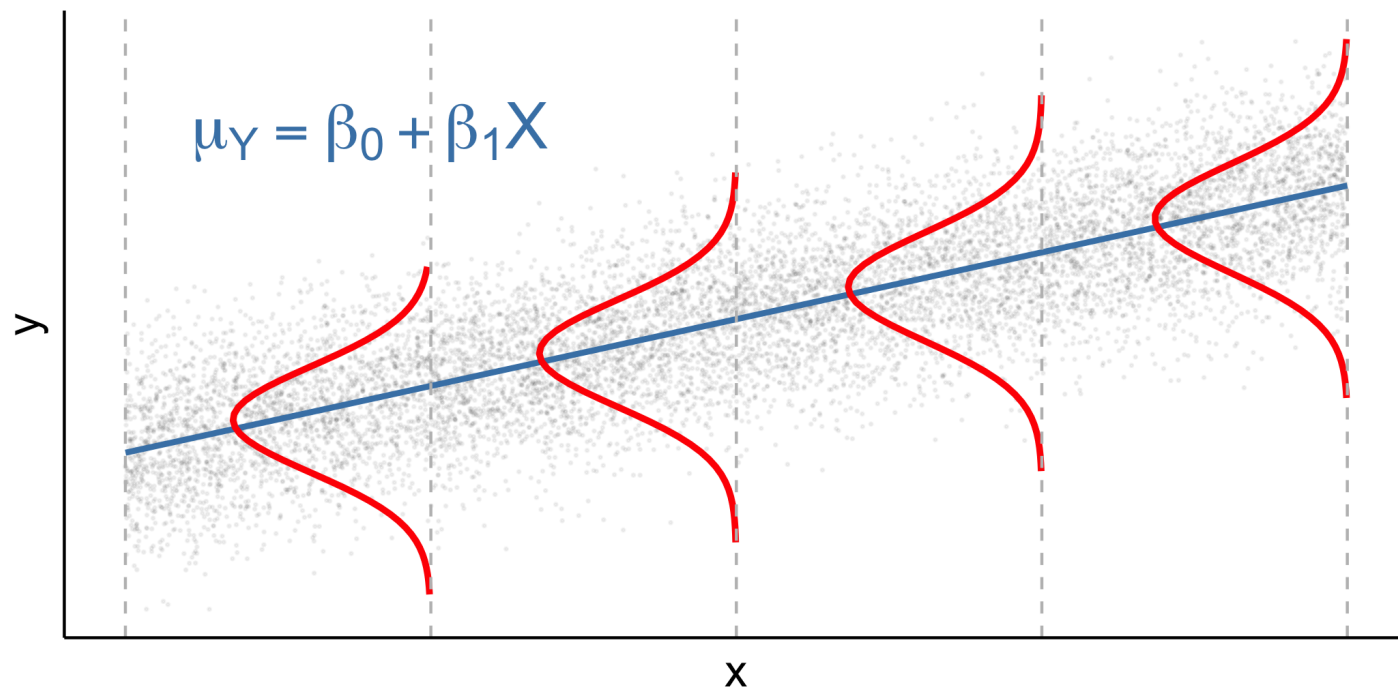
$$Y = \beta_0 + \beta_1 X + \epsilon$$

where the errors are independent and normally distribution, $\epsilon \sim N(0, \sigma_\epsilon)$

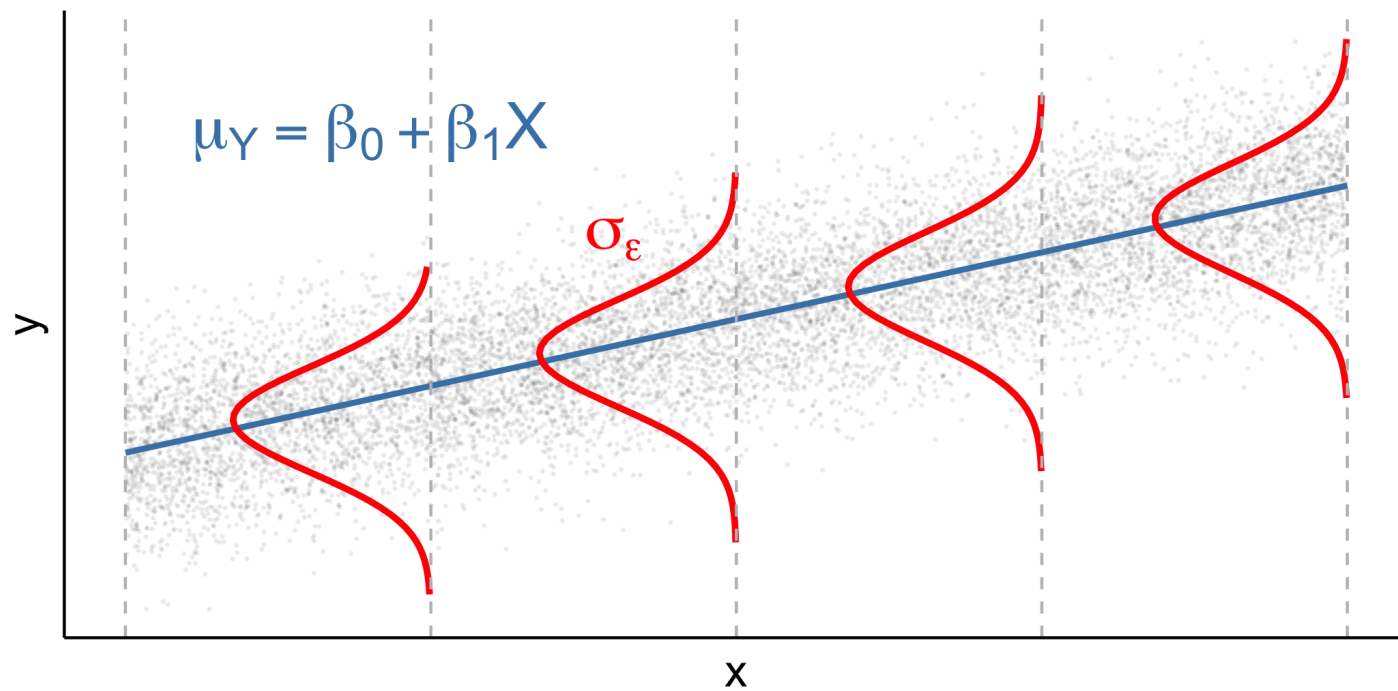
$$Y|X \sim N(\beta_0 + \beta_1 X, \sigma_\epsilon)$$



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Regression standard error

Once we fit the model, we can use the residuals to calculate the vocab[regression standard error, $\hat{\sigma}_\epsilon$]

$$\hat{\sigma}_\epsilon = \sqrt{\frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n - 2}} = \sqrt{\frac{\sum_{i=1}^n e_i^2}{n - 2}}$$

Standard error of $\hat{\beta}_1$

$$SE_{\hat{\beta}_1} = \hat{\sigma}_\epsilon \sqrt{\frac{1}{(n-1)s_X^2}}$$

