

Simple Linear Regression

Introduction

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Topics

- Use simple linear regression to describe the relationship between a quantitative predictor and quantitative response variable.

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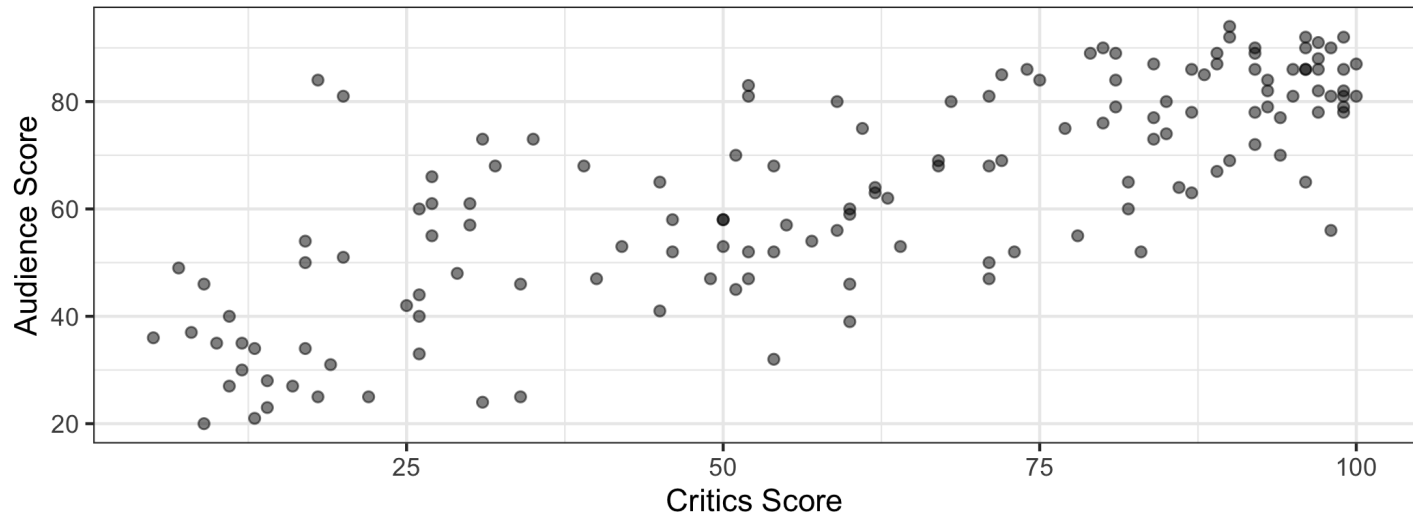
- Use simple linear regression to describe the relationship between a quantitative predictor and quantitative response variable.
- Estimate the slope and intercept of the regression line using the least squares method.

Topics

- Use simple linear regression to describe the relationship between a quantitative predictor and quantitative response variable.
- Estimate the slope and intercept of the regression line using the least squares method.
- Interpret the slope and intercept of the regression line.

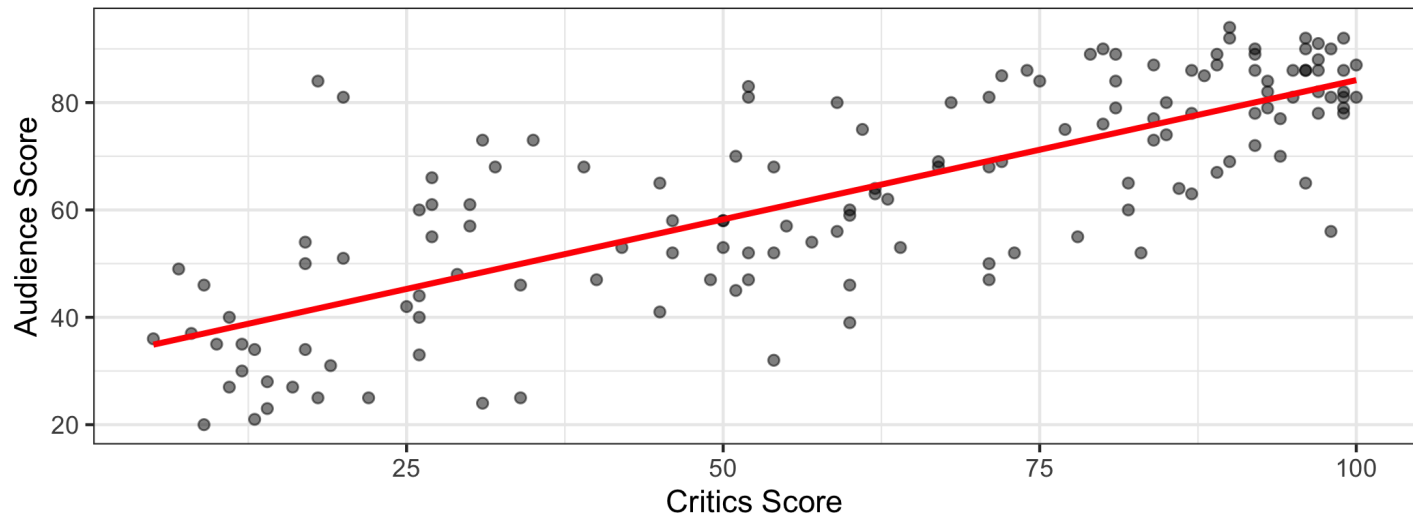
Movie ratings data

The data set contains the "Tomatometer" score (**critics**) and audience score (**audience**) for 146 movies rated on rottentomatoes.com.



Movie ratings data

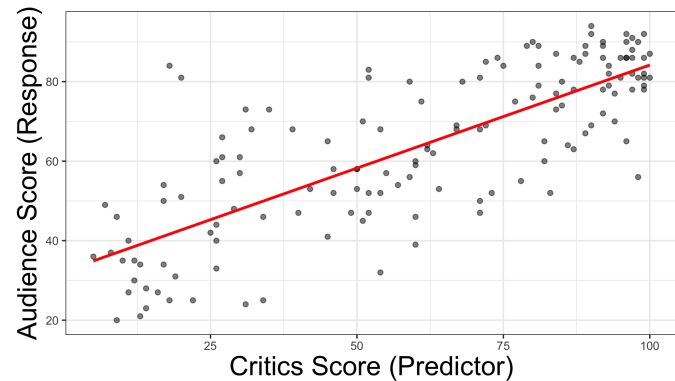
We want to fit a line to describe the relationship between the critics score and audience score.



Terminology

The **response, Y** , is the variable describing the outcome of interest.

The **predictor, X** , is the variable we use to help understand the variability in the response.



Regression model

A **regression model** is a function that describes the relationship between the response, Y , and the predictor, X .

$$Y = \mathbf{Model} + \text{Error}$$

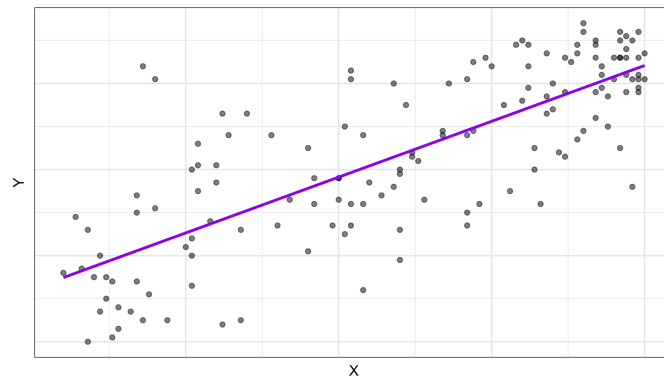
$$= \mathbf{f(X)} + \epsilon$$

$$= \mu_{Y|X} + \epsilon$$

$$Y = \text{Model} + \text{Error}$$

$$= \mathbf{f}(\mathbf{X}) + \epsilon$$

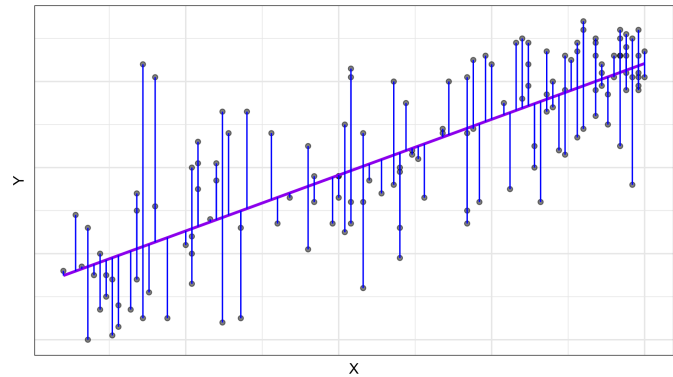
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$$Y = \text{Model} + \text{Error}$$

$$= \mathbf{f(X)} + \epsilon$$

$$= \mu_{Y|X} + \epsilon$$



Simple linear regression

When we have a quantitative response, Y , and a single quantitative predictor, X , we can use a **simple linear regression** model to describe the relationship between Y and X .

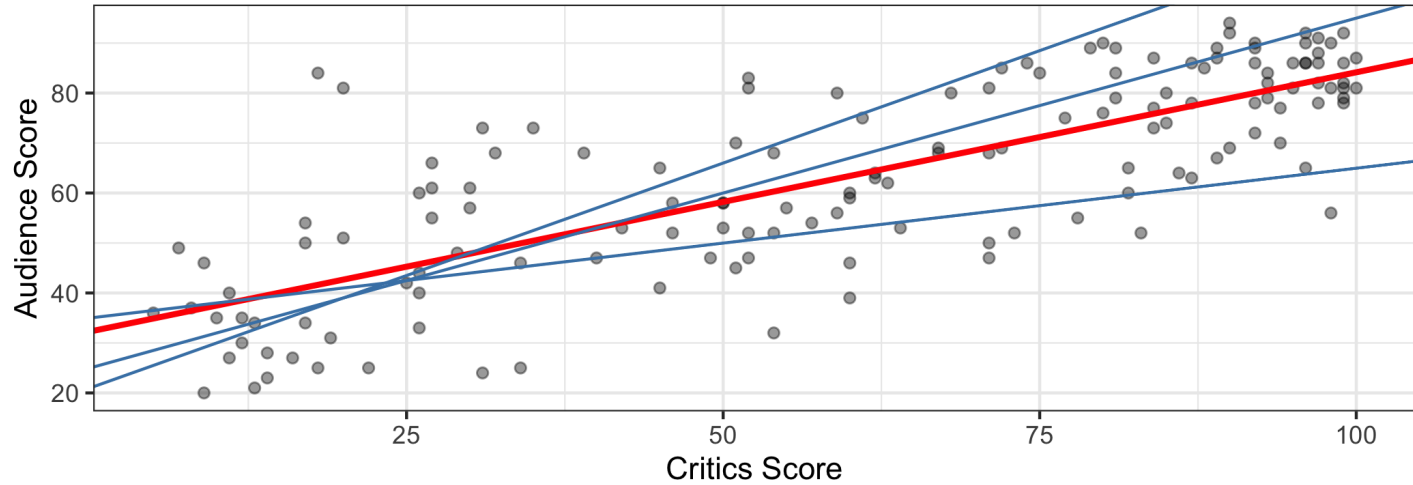
$$Y = \beta_0 + \beta_1 X + \epsilon$$

β_1 : Slope β_0 : Intercept

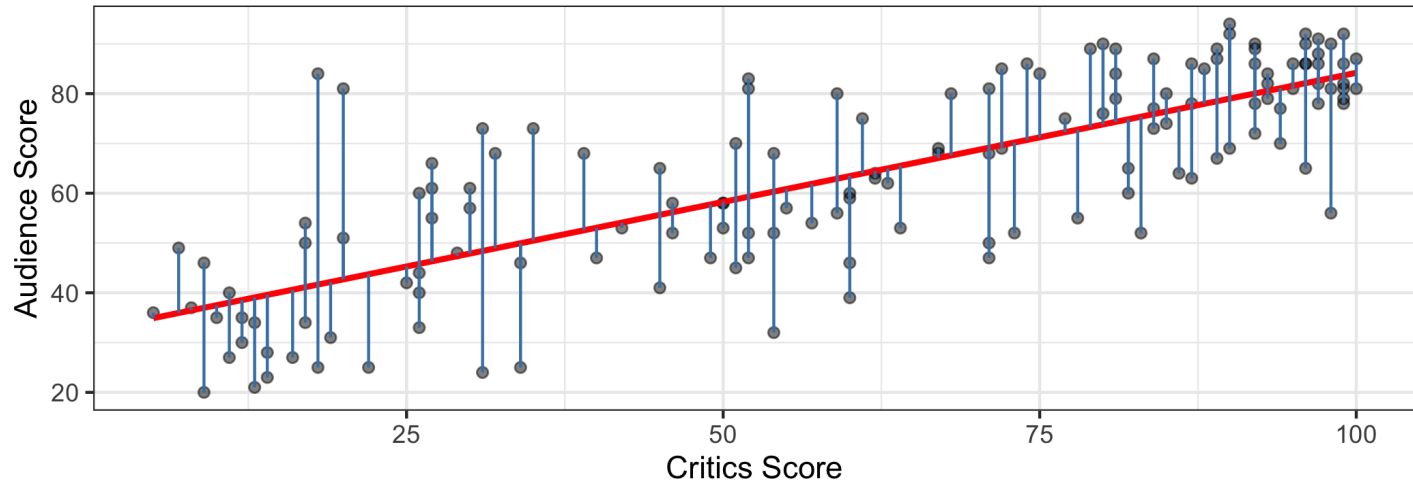
$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X$$

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How do we choose values for $\hat{\beta}_1$ and $\hat{\beta}_0$?



Residuals



$$\text{residual} = \text{observed} - \text{predicted} = y - \hat{y}$$

Least squares line

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- The residual for the i^{th} observation is

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- The **sum of squared residuals** is

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- The **least squares line** is the one that minimizes the sum of squared residuals

Estimating the slope

$$\hat{\beta}_1 = r \frac{s_Y}{s_X}$$

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$$r = 0.781$$

$$\begin{aligned}\hat{\beta}_1 &= 0.781 \times \frac{20.024}{30.169} \\ &= \mathbf{0.518}\end{aligned}$$

Estimating the intercept

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$$\bar{x} = 60.850$$

$$\bar{y} = 63.877$$

$$\hat{\beta}_1 = 0.518$$

Estimating the intercept

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$

$$\bar{x} = 60.850$$

$$\bar{y} = 63.877$$

$$\hat{\beta}_1 = 0.518$$

$$\begin{aligned}\hat{\beta}_0 &= 63.877 - 0.518 \times 60.850 \\ &= \mathbf{32.296}\end{aligned}$$

Interpreting slope & intercept

$$\hat{\text{audience}} = 32.296 + 0.518 \times \text{critics}$$

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Interpreting slope & intercept

$$\hat{\text{audience}} = 32.296 + 0.518 \times \text{critics}$$

Slope: For every one point increase in the critics score, we expect the audience score to increase by 0.518 points, on average.

Intercept: If the critics score is 0 points, we expect the audience score to be 32.296 points.

Does it make sense to interpret the intercept?

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✓ Interpret the intercept if

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❌ Otherwise, don't interpret the intercept!

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- Used the least squares method to estimate the slope and intercept.
- We interpreted the slope and intercept.
 - **Slope:** For every one unit increase in x , we expect y to change by $\hat{\beta}_1$ units, on average.
 - **Intercept:** If x is 0, then we expect y to be $\hat{\beta}_0$ units.