Simple Linear Regression

Foundation

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General form of model

$$Y = f(X) + \epsilon$$

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f: fixed but unknown function

ϵ: random error

Simple linear regression

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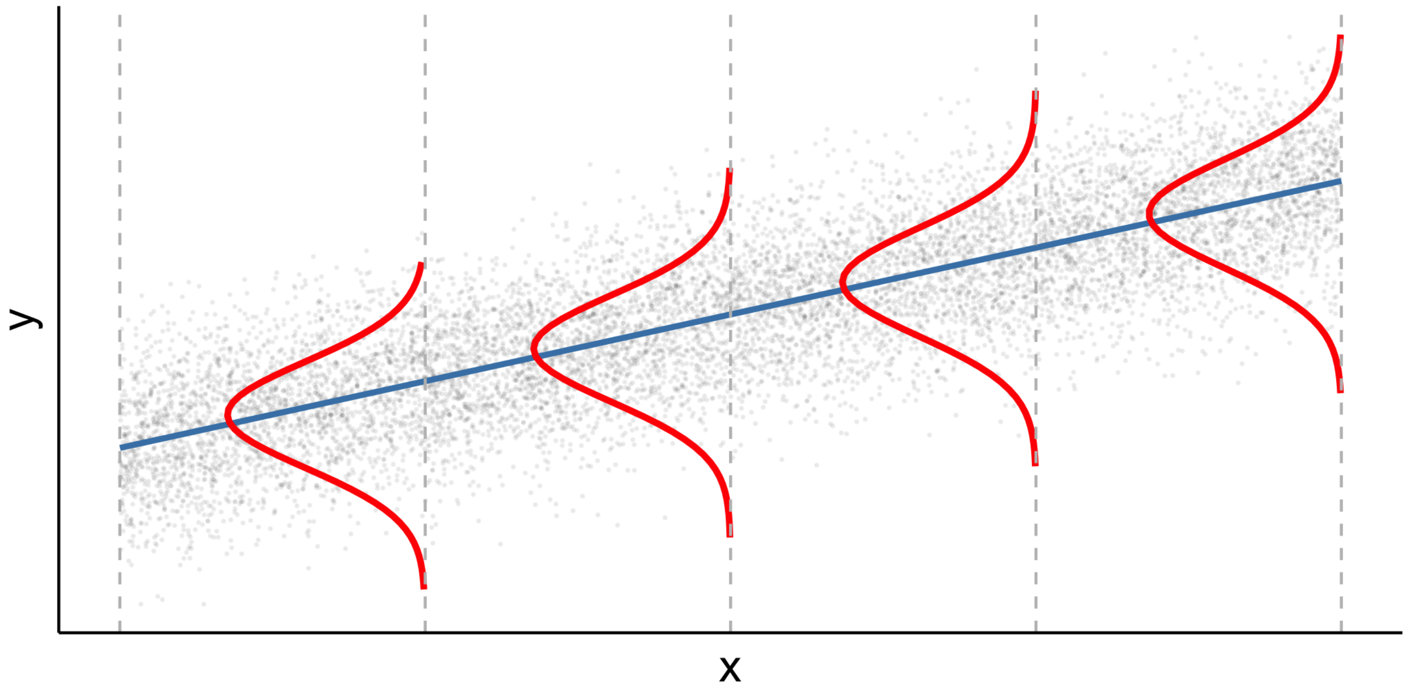
$$
\begin{aligned}
Y &= f(X) + \epsilon \\
&= \mu_{Y|X} + \epsilon \\
&= \beta_0 + \beta_1 X + \epsilon
\end{aligned}
$$

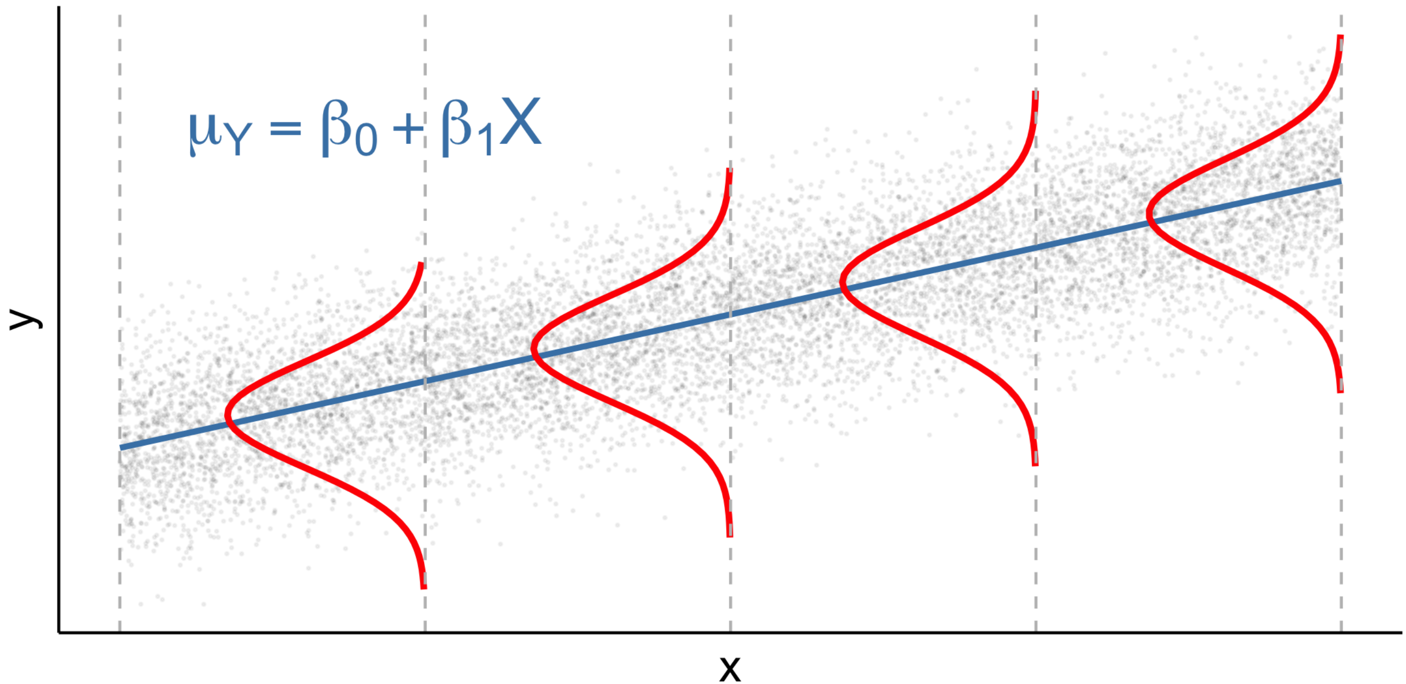
$$Y = \beta_0 + \beta_1 X + \epsilon$$

where the errors are independent and normally distribution, $\epsilon \sim N(0, \sigma_\epsilon)$

$$Y|X \sim N(\beta_0 + \beta_1 X, \sigma_\epsilon)$$



$$Y|X \sim N(\beta_0 + \beta_1 X, \sigma_\epsilon^2)$$



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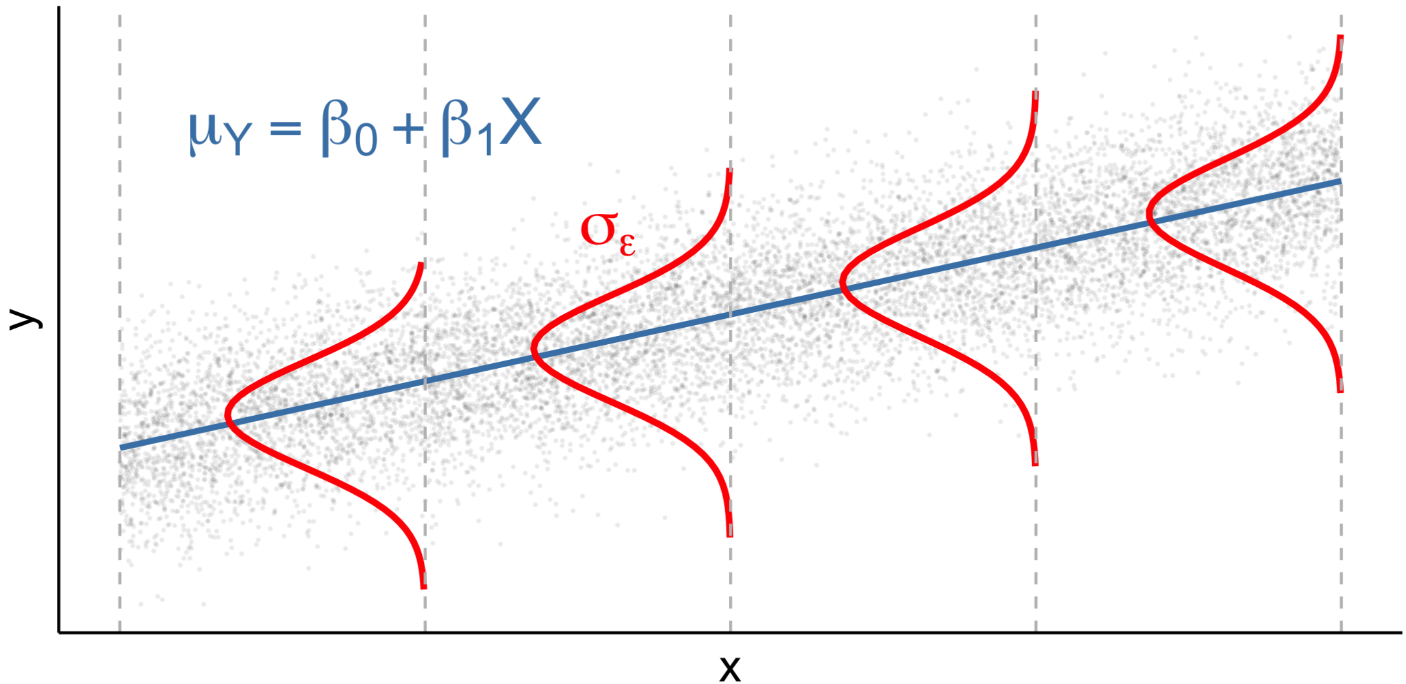


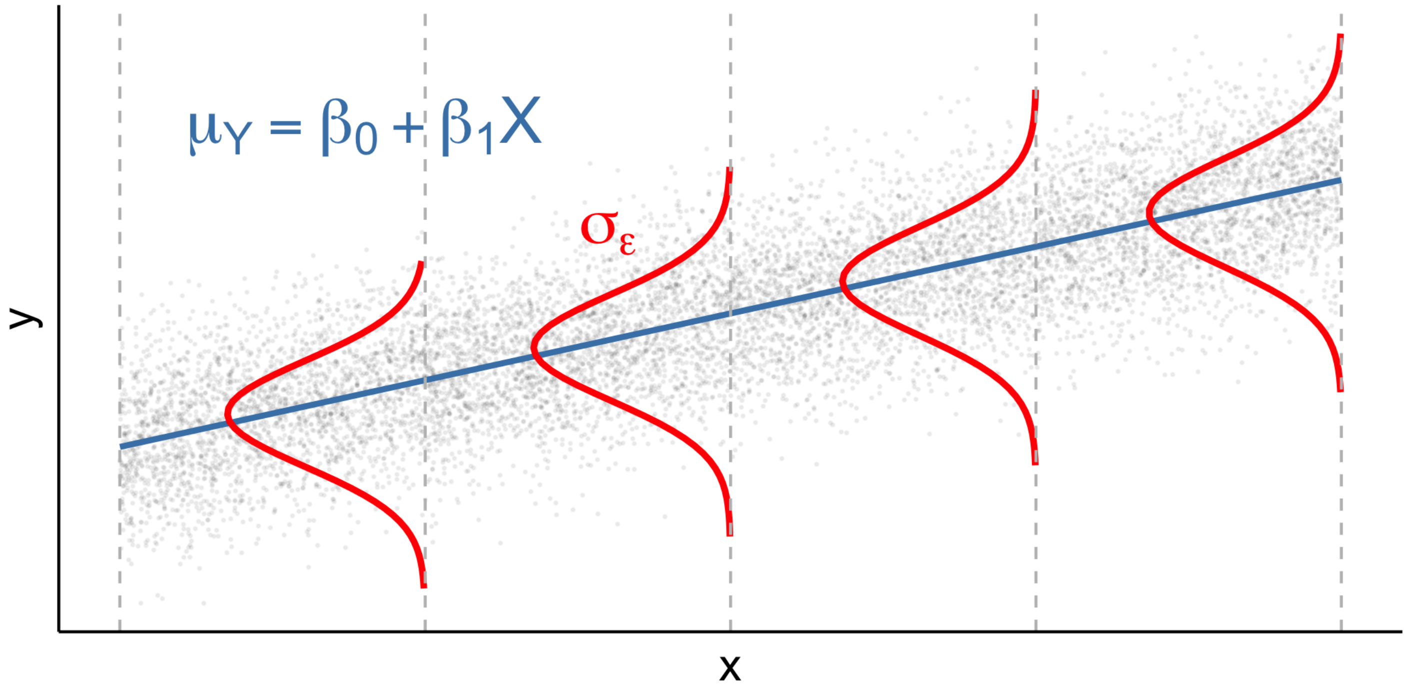
Regression standard error

Once we fit the model, we can use the residuals to calculate the regression standard error

$$\hat{\sigma}_\epsilon = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n-2}} = \sqrt{\frac{\sum_{i=1}^{n} e_i^2}{n-2}}$$

Standard error of $\hat{\beta}_1$

$$SE_{\hat{\beta}_1} = \hat{\sigma}_\epsilon \sqrt{\frac{1}{(n-1)s_X^2}}$$



$\mu_Y = \beta_0 + \beta_1 X$
σ_ε
y
x