

Simple Linear Regression

Prediction

Dr. Maria Tackett



Topics

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- Predict the response given a value of the predictor variable

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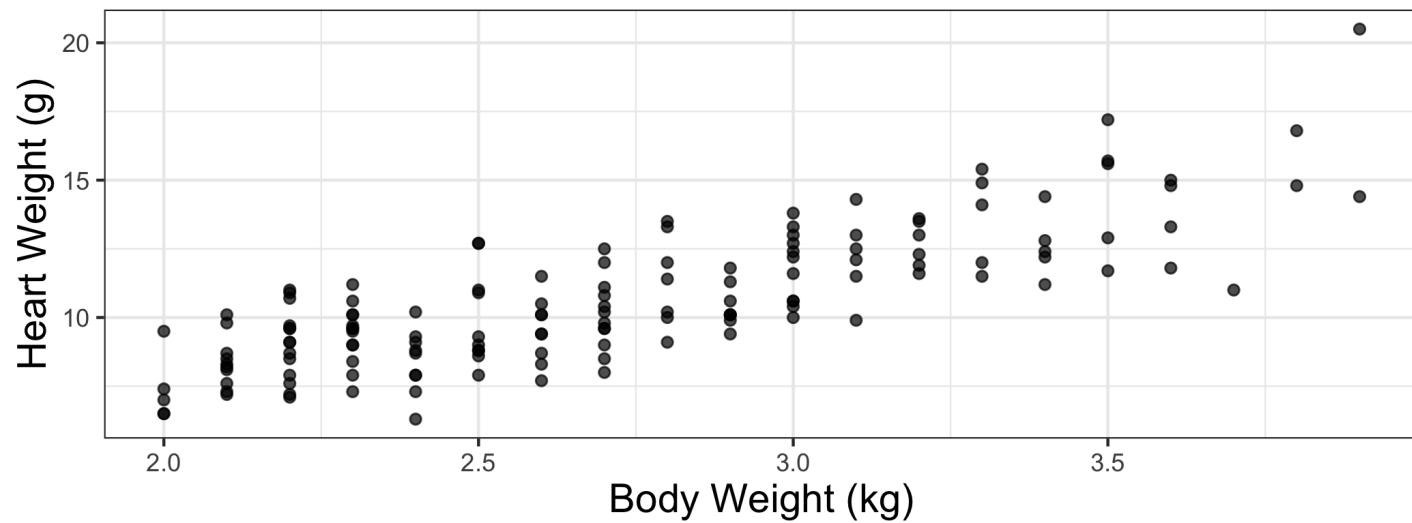
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- Use intervals to quantify the uncertainty in the predicted values

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- Predict the response given a value of the predictor variable
- Use intervals to quantify the uncertainty in the predicted values
- Define *extrapolation* and why we should avoid it

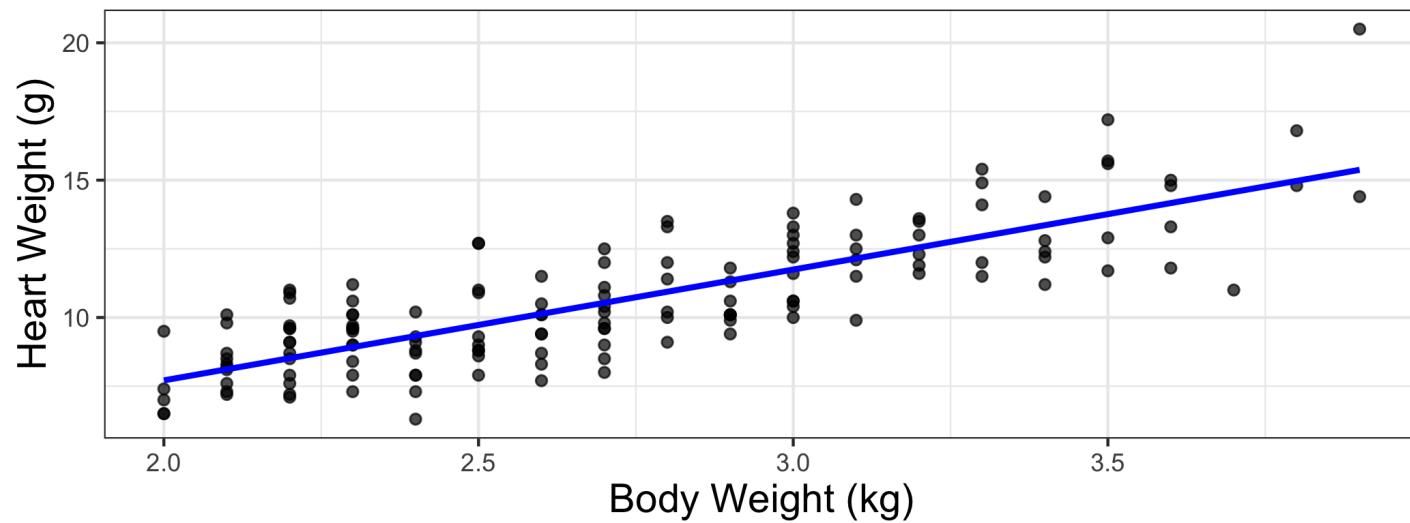
Cats data

The data set contains the heart weight (**Hwt**) and body weight (**Bwt**) for 144 domestic cats.



Cats data

We want to fit a model so we can use a cat's body weight to predict how much its heart weighs.



The model

$$\hat{Hwt} = -0.357 + 4.034 \times Bwt$$

term	estimate	std.error	statistic	p.value
(Intercept)	-0.357	0.692	-0.515	0.607
Bwt	4.034	0.250	16.119	0.000

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Estimate the **mean** response when the predictor variable is equal to a value x_0

Predict the response for an **individual** observation with a value of the predictor equal to x_0

Calculating a predicted value

My cat Mindy weighs about 3.18 kg (7 lbs).

Based on this model,
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$$\begin{aligned}\hat{H}_{wt} &= -0.357 + 4.034 \times 3.18 \\ &= 12.471 \text{ g}\end{aligned}$$

Uncertainty in predictions

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Confidence interval for the mean response

$$\hat{y} \pm t_{n-2}^* \times \mathbf{SE}_{\hat{\mu}}$$

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Prediction interval for an individual observation

$$\hat{y} \pm t_{n-2}^* \times \mathbf{SE}_{\hat{y}}$$

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We are 95% confident that mean heart weight for the subset of cats that weigh 3.18 kg is between 12.143 g and 12.801 g.

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12.472	9.582	15.362

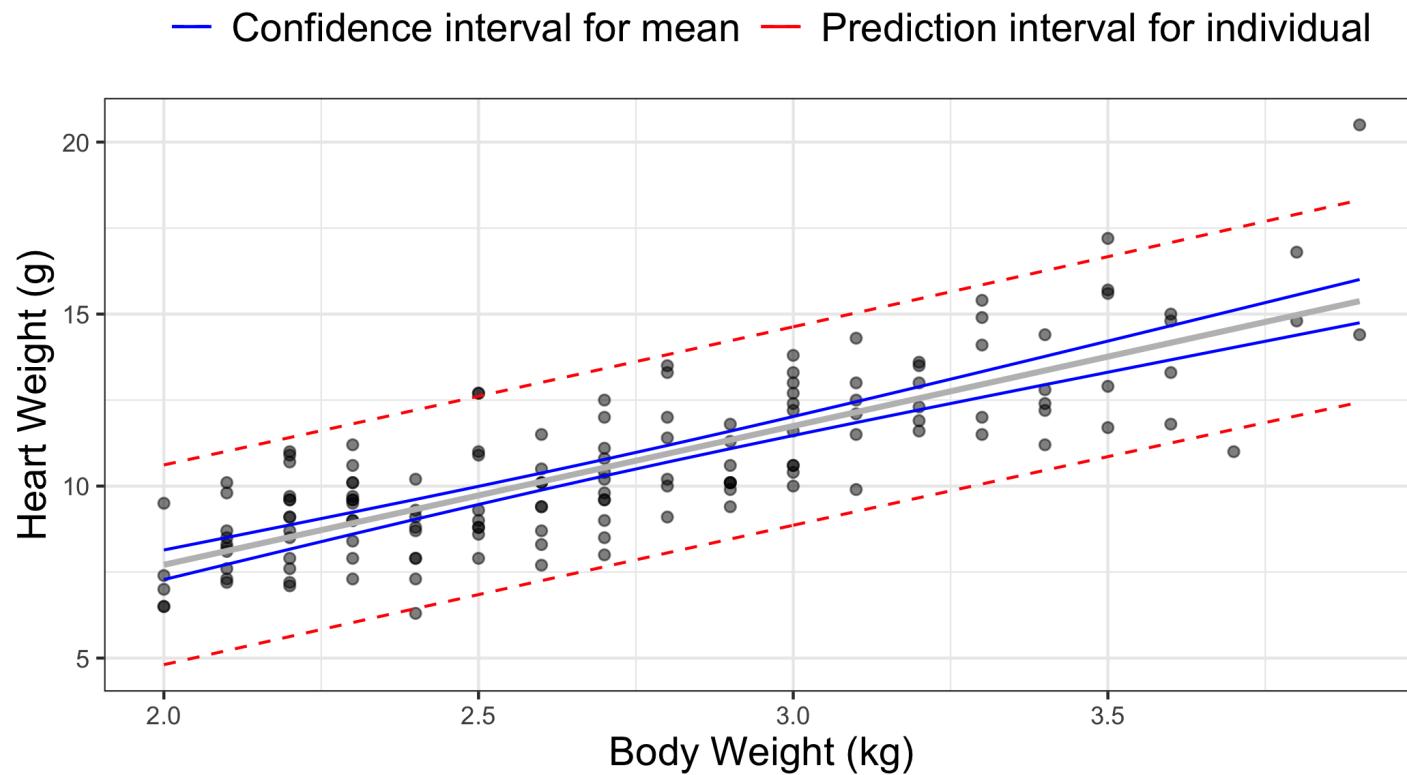
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fit	lwr	upr
12.472	9.582	15.362

We can predict with 95% confidence that Mindy's heart weighs between 9.582 g and 15.362 g.

Comparing intervals



Caution! Extrapolation

We should not use the model to predict for values of X far outside the range of values used to fit the model.

This is called **extrapolation**.

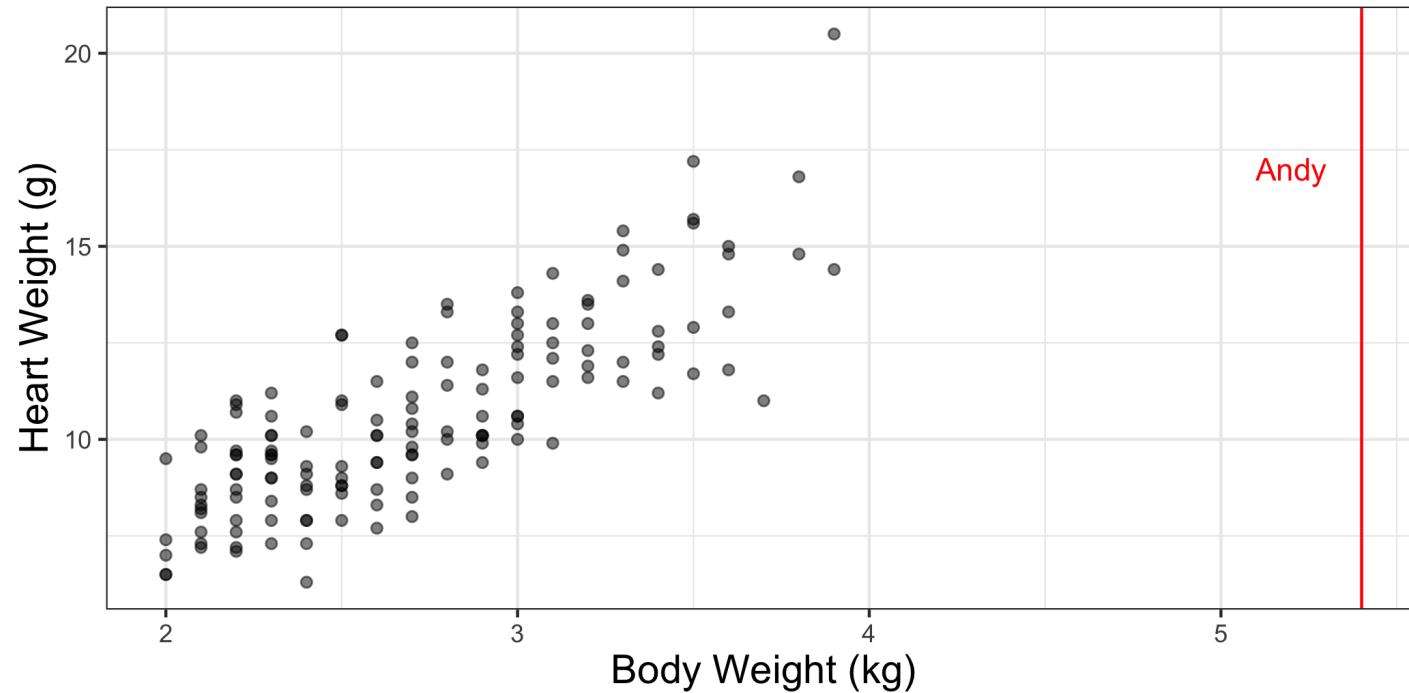
Predict Andy's heart weight?

My cat Andy weighs about 5.44 kg (12 lbs).

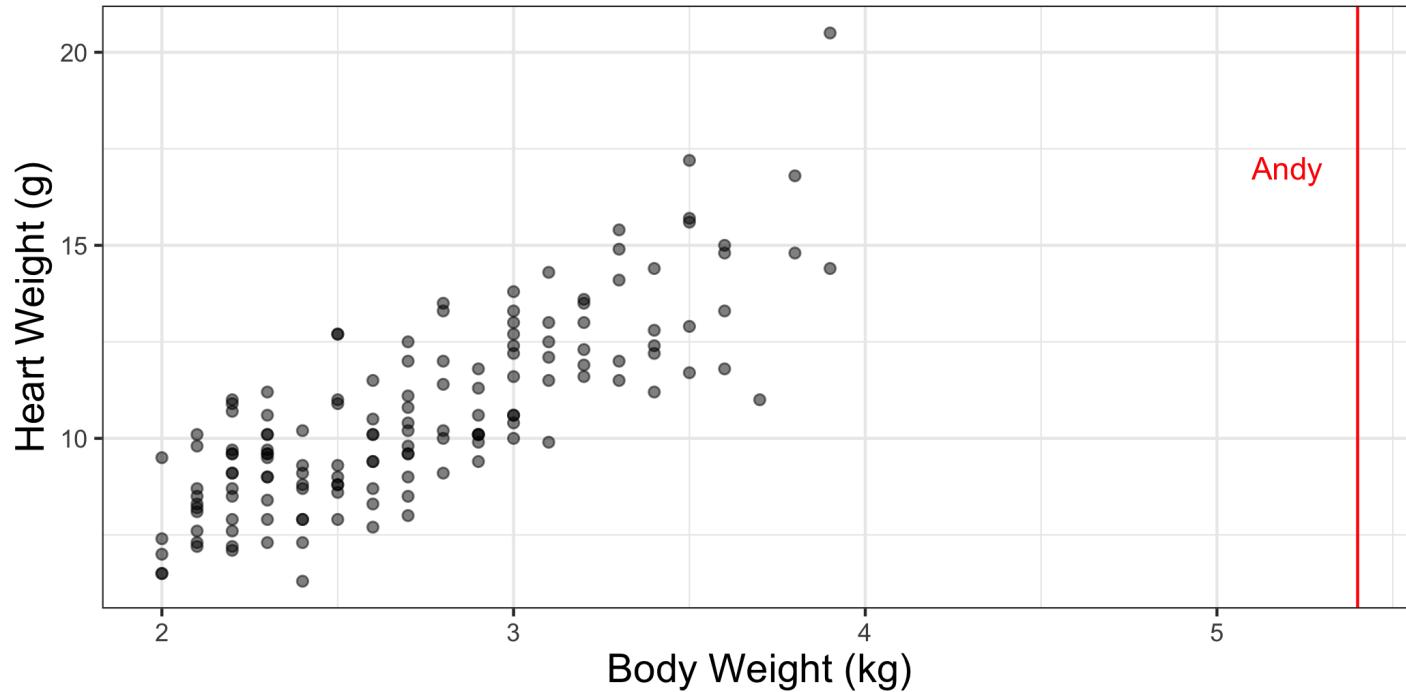
Should we use this regression model to predict how much his heart weighs?



Predict Andy's heart weight?



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We should not use this model to predict Andy's heart weight, since that would be **extrapolation**.

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