Signals & systems #7

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Part 1 :

الف)

$$V_R(t) + V_L(t) + V_C(t) = V_{in}(t) \rightarrow \quad \text{(الف}$$

$$R\,i(t) + L\frac{d(i)}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau = V_{in}(t) \rightarrow$$

$$\frac{L\,d^2 i(t)}{dt^2} + R\frac{di(t)}{dt} + \frac{1}{C}\,i(t) = \frac{dV_{in}(t)}{dt}$$

ب)

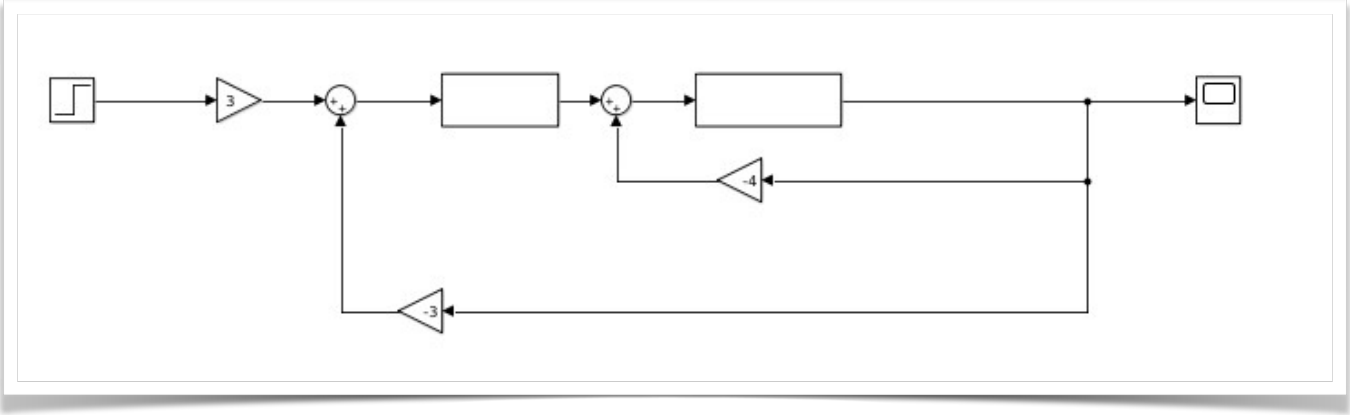
$$S^2 L I(S) + sR\,I(S) + \frac{I(S)}{C} = s\,V_{in}(S) \quad \text{(ب}$$

ج)

$$y(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau \rightarrow i(t) = C\frac{dy(t)}{dt} \rightarrow \quad \text{(ج}$$

$$\rightarrow I(S) = Cs\,Y(s)$$

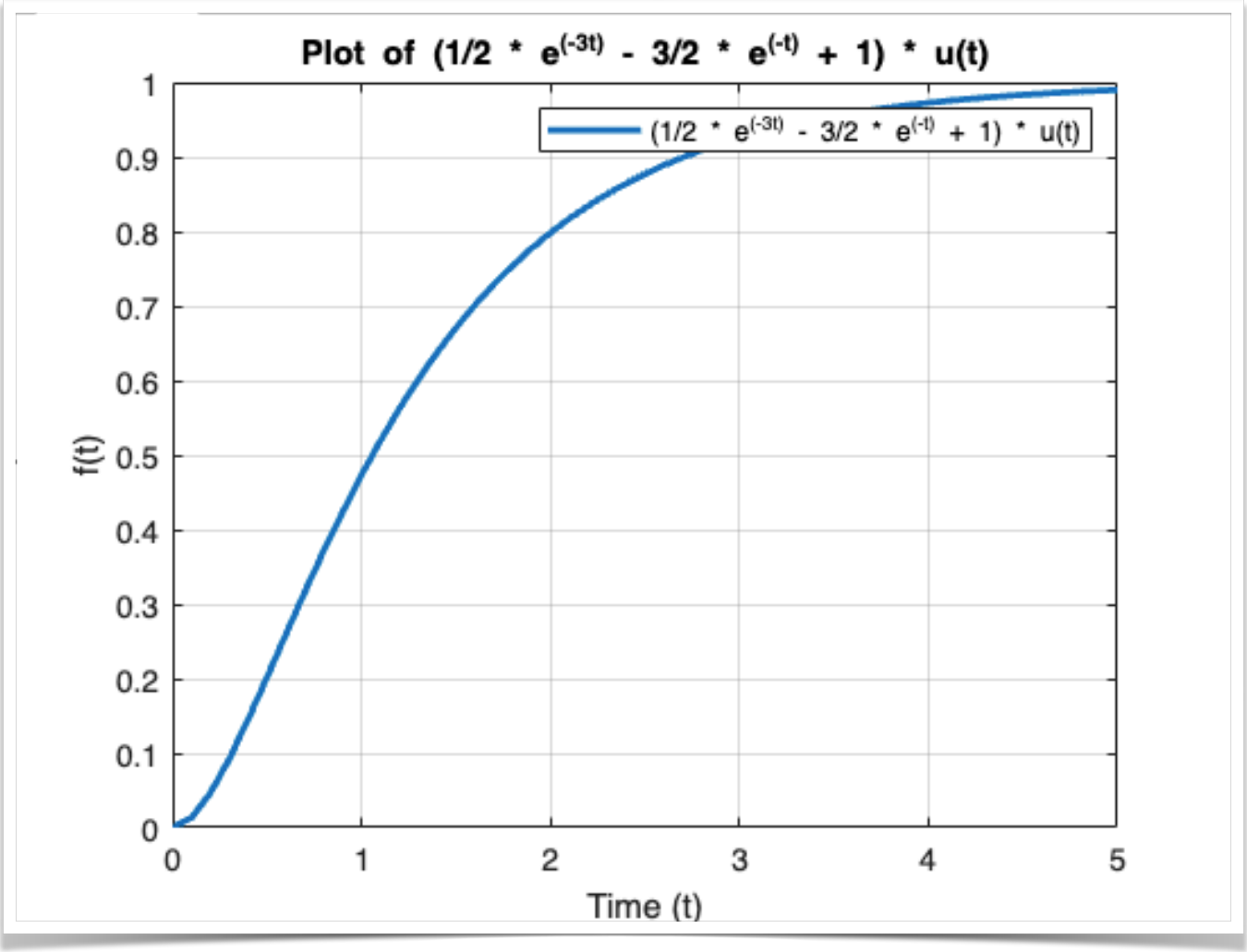
د) هر دو transfer func ها 1/s هستند :



$$\frac{1}{3}s^2Y(s) + \frac{4}{3}sY(s) + Y(s) = X(s) \rightarrow$$

$$Y(s) = \frac{1}{s^2}\left(3X(s) - 3Y(s)\right) - \frac{4}{s}Y(s)$$

و)


Plot of (1/2 * e^(-3t) - 3/2 * e^(-t) + 1) * u(t)
(1/2 * e^(-3t) - 3/2 * e^(-t) + 1) * u(t)
f(t)
Time (t)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
1
2
3
4
5

```
1
2
3   % Define the time range
4   t = 0:0.1:5; % Adjust the step size as needed
5
6   % Define the function
7   f_t = (1/2 * exp(-3*t) - 3/2 * exp(-t) + 1) .* (t >= 0);
8
9   % Plot the function
10  plot(t, f_t, 'LineWidth', 2);
11
12  % Label the axes
13  xlabel('Time (t)');
14  ylabel('f(t)');
15
16  % Title for the plot
17  title('Plot of (1/2 * e^(-3t) - 3/2 * e^(-t) + 1) * u(t)');
18
19  % Show the grid
20  grid on;
21
22  % Show the plot
23  legend('(1/2 * e^(-3t) - 3/2 * e^(-t) + 1) * u(t)');
24
```

$$H(s) = \frac{3}{s^2+4s+3} = \frac{3}{2}\left(\frac{-1}{s+3}\right) + \frac{3}{2}\left(\frac{1}{s+1}\right) \quad (7$$

$$\rightarrow \mathcal{L}\{e^{-at}u(t)\} = \frac{1}{a+s} \rightarrow \mathcal{L}^{-1}(H) = \frac{3}{2}\left(e^{-t} - e^{-3t}\right)u(t)$$

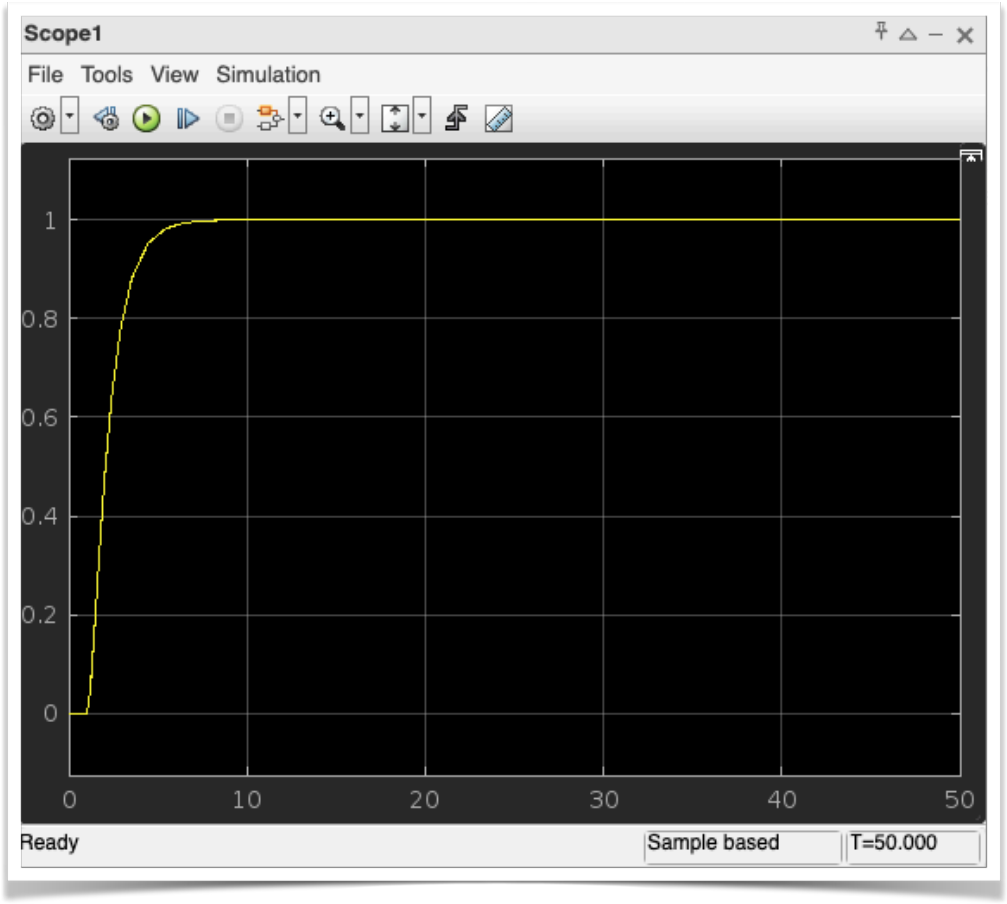
$$\rightarrow h(t) = u(t) \times \frac{3}{2}\left(e^{-t} - e^{-3t}\right)$$

$$s(t) = \int_{-\infty}^{t} h(t)\,dt \rightarrow s(t) = \int_{-\infty}^{t} u(\tau)\,\frac{3}{2}\left(e^{-\tau} - e^{-3\tau}\right)d\tau$$

$$= \frac{3}{2}\left(\frac{1}{3}e^{-3t} - e^{-t} - \left(\frac{1}{3} - 1\right)\right) \times u(t) =$$

$$= \left(\frac{1}{2}e^{-3t} - \frac{3}{2}e^{-t} + 1\right) \times u(t)$$

ه) بله خروجی کاملا مشابه مقدار تئوری است .



Part 2 :

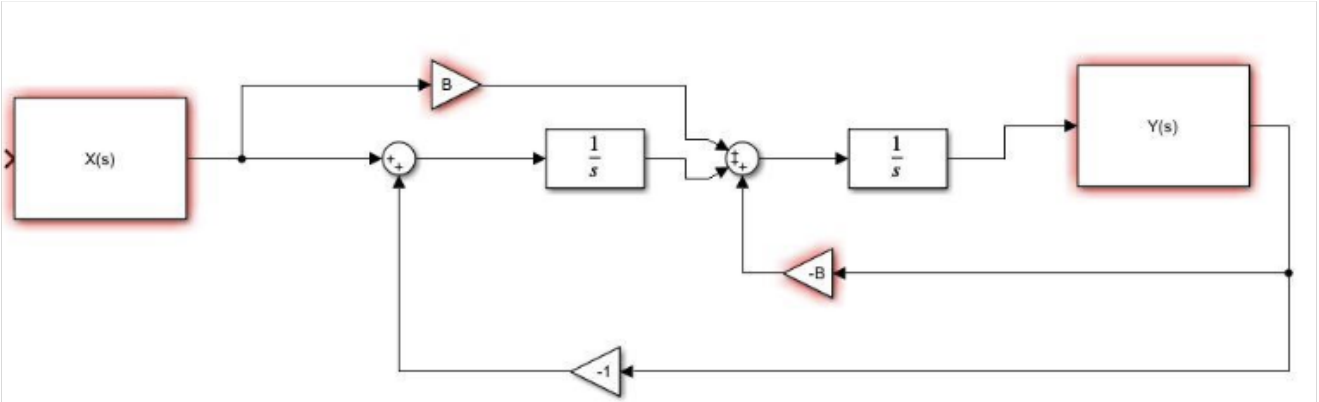
الف)

$$\frac{d^2y(t)}{dt^2} + B\frac{dy(t)}{dt} + y(t) = B\frac{dx_i(t)}{dt} + x_i(t) \qquad (الف$$

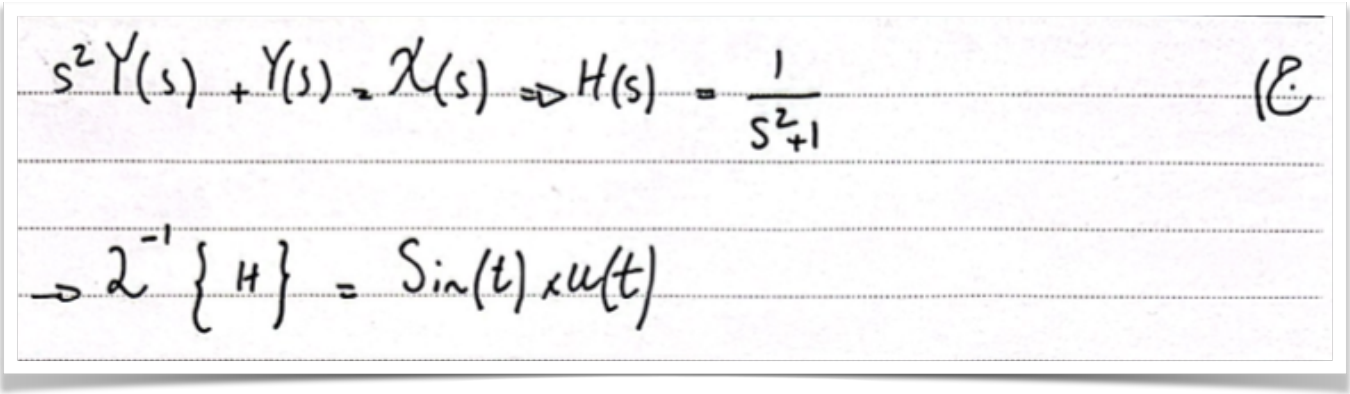
ب)

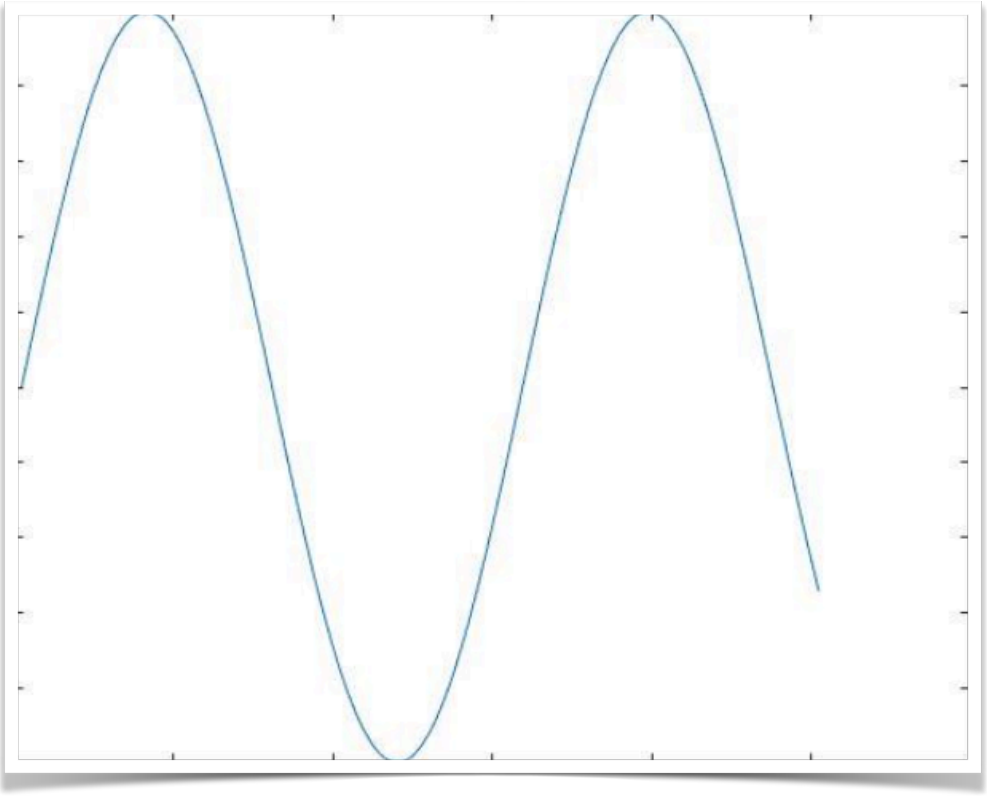
$$s^2Y(s) + BsY(s) = BsX_i(s) + X_i(s) - Y(s) \Rightarrow \qquad (ب$$

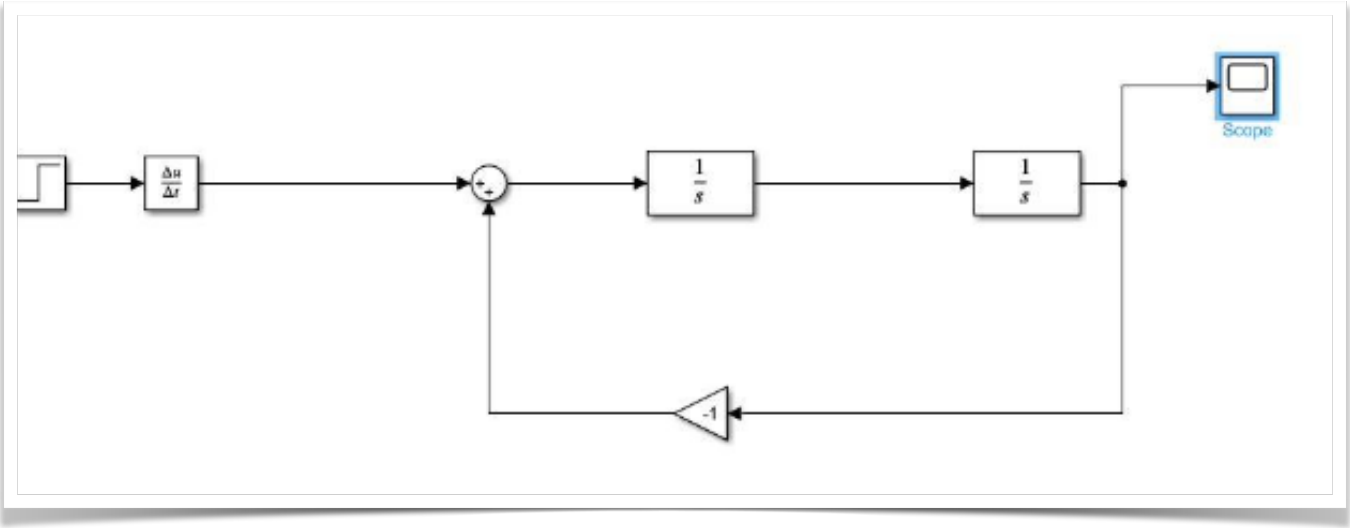
$$Y(s) = \frac{1}{s}\left(BX_i(s) - BY(s)\right) + \frac{1}{s^2}\left(X_i(s) - Y(s)\right)$$

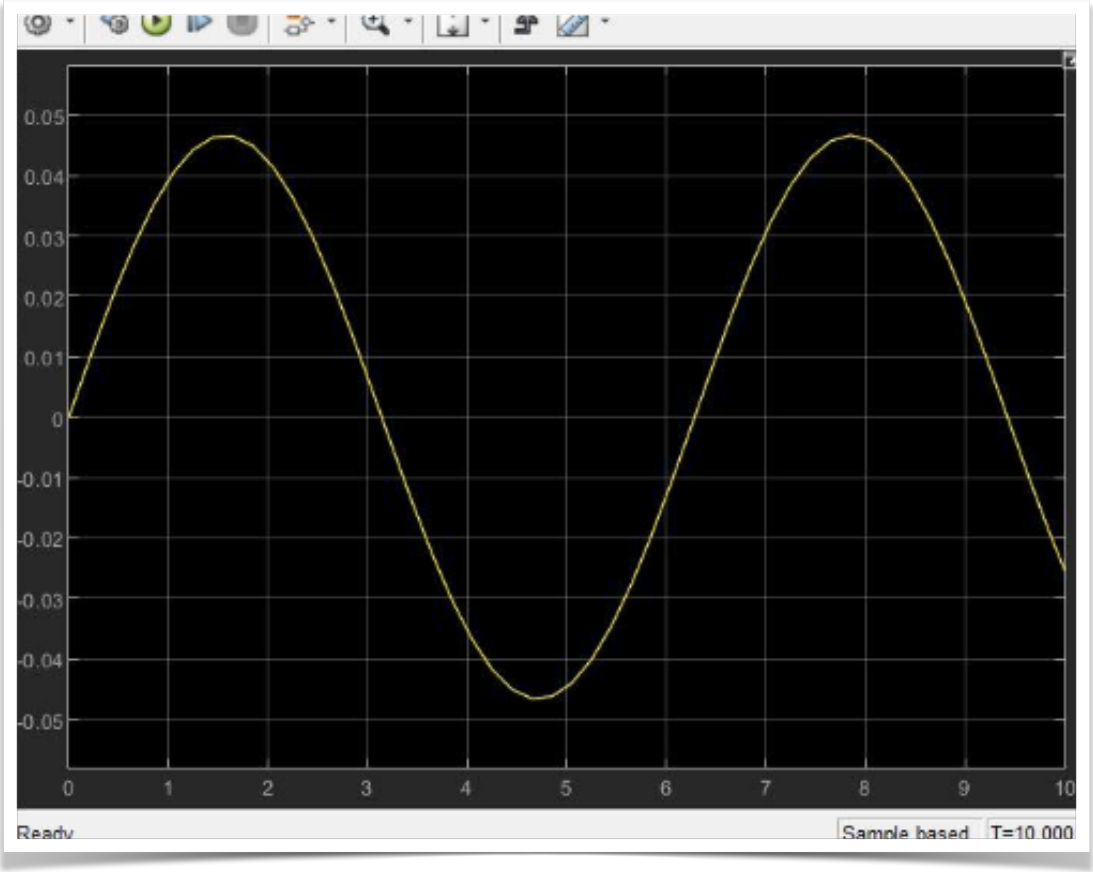


ج)مشاهده میشود حرکت کابین خودرو به صورت سینوسی خواهد بود و چون میرا نیست، نوسانات زیادی دارد .






Scope
-1


0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
0
1
2
3
4
5
6
7
8
9
10
Ready
Sample based
T=10.000

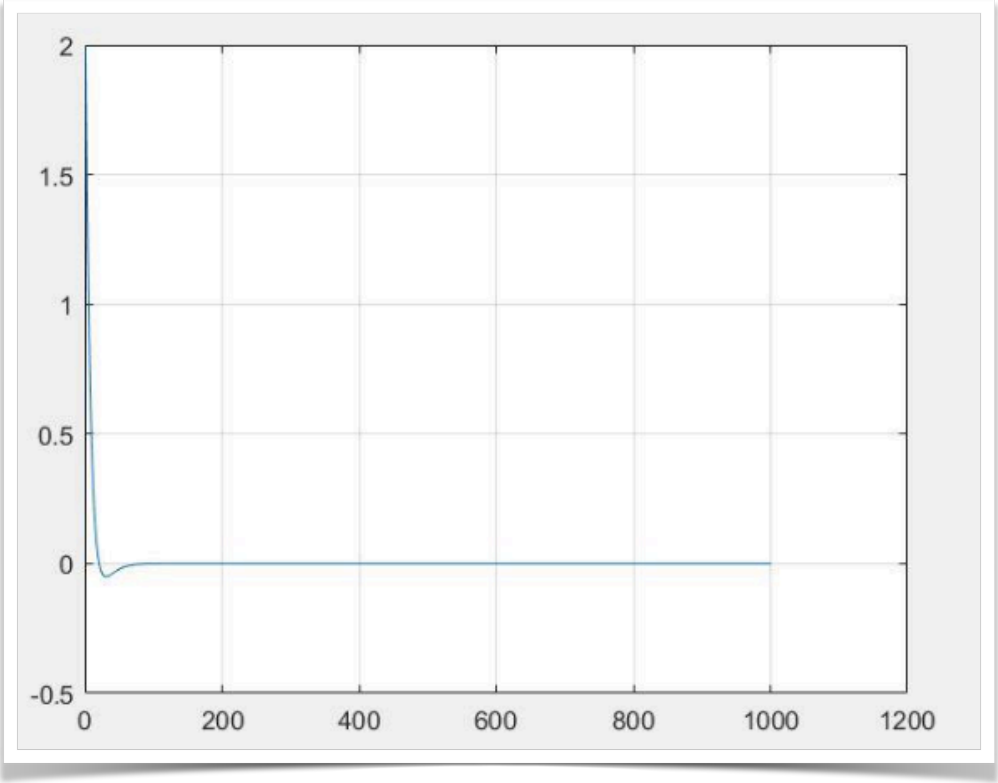
د)نوسانات کابین کمتر از حالت قبل است و میرا میشود !!!

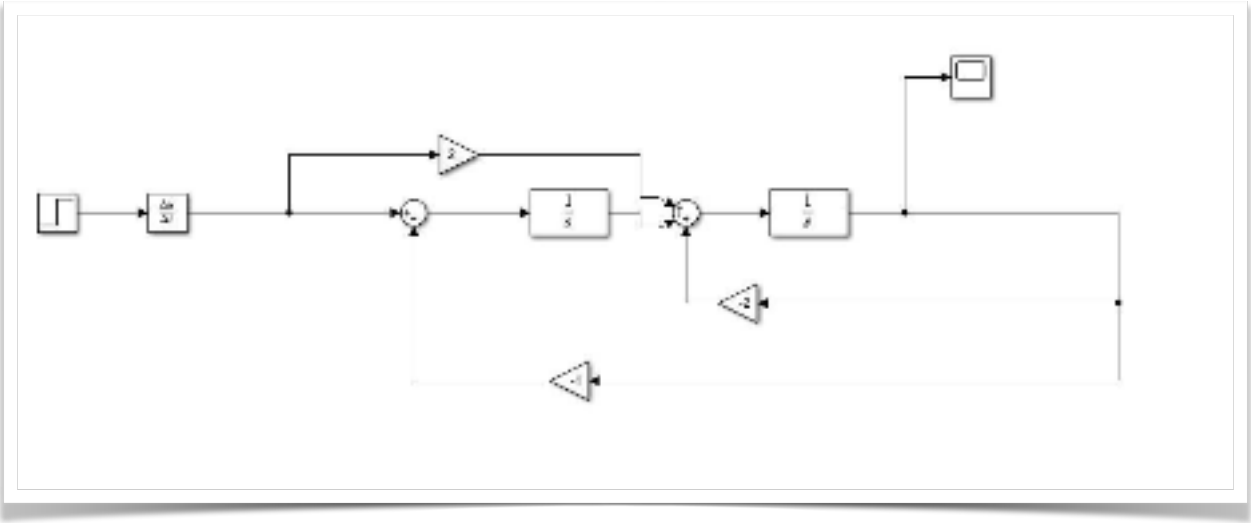
$$H(s) = \frac{Bs+1}{s^2+Bs+1} \rightarrow s^2+Bs+1=0 \rightarrow B^2-4>0 \rightarrow$$

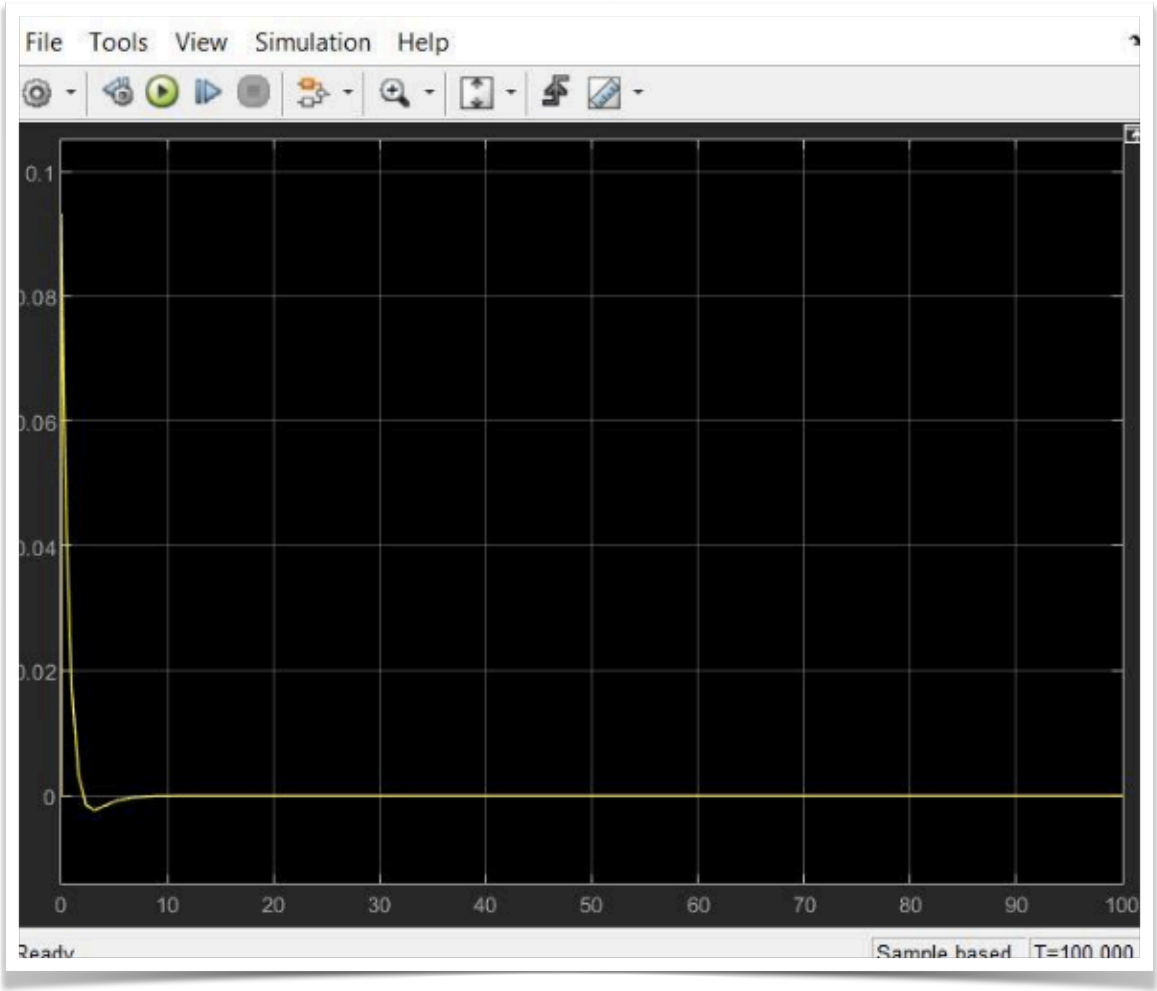
$$B_{min} = 2 \rightarrow H(s) = \frac{2s+1}{(s+1)^2}$$

$$\rightarrow \mathcal{L}^{-1}\left\{\frac{2s+1}{s^2+2s+1}\right\} = \mathcal{L}^{-1}\left\{\frac{2s}{(s+1)^2}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s^2+2s+1}\right\} =$$

$$\Rightarrow x(t) = \left(\frac{d}{dt}(2te^{-t}) + te^{-t}\right)u(t) = (2e^{-t} - te^{-t})u(t)$$



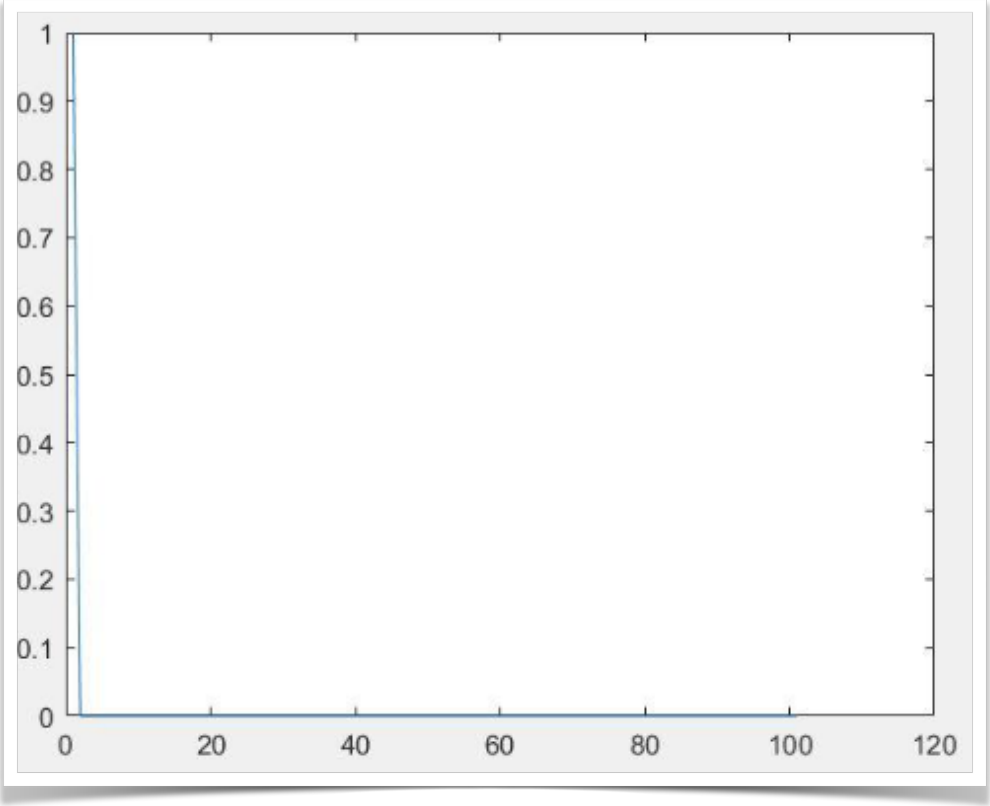


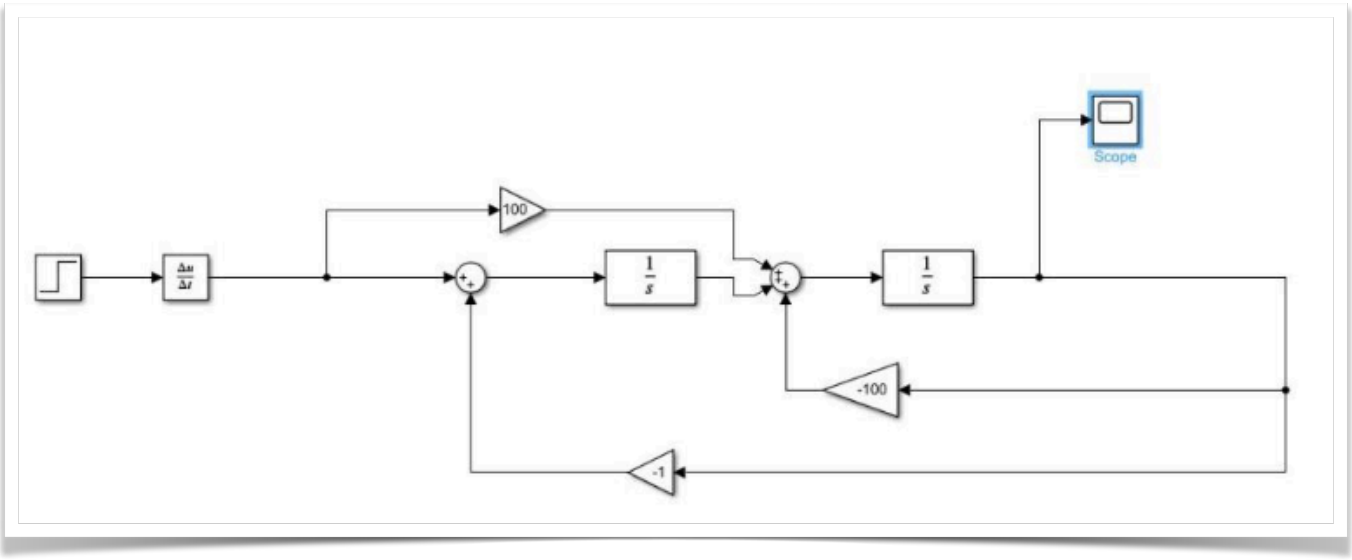

File Tools View Simulation Help
0.1
0.08
0.06
0.04
0.02
0
0 10 20 30 40 50 60 70 80 90 100
Ready
Sample based
T=100.000

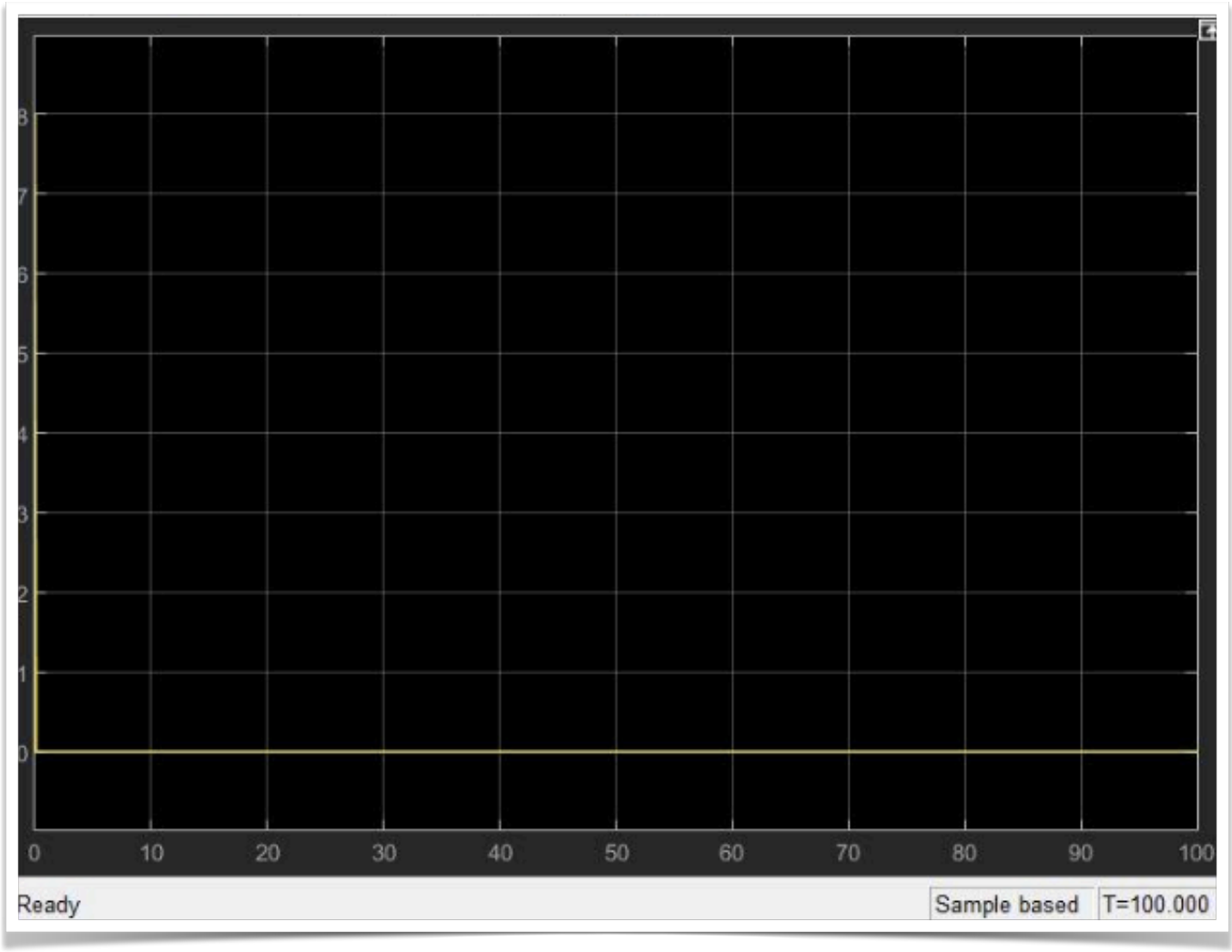
و) به شدت میرا است.

$$H(s) = \frac{100s+1}{(s+100)\left(s+\frac{1}{100}\right)} = \frac{100}{100+s} \rightarrow (9$$

$$\mathcal{L}^{-1}\{H(s)\} = e^{-100t}u(t)$$




Scope
100
-100
-1


0
10
20
30
40
50
60
70
80
90
100
Ready
Sample based
T=100.000

ه)در حالت "ج" نوسانات میرا نیستند که این برای ما مطلوب نیست.

در حالت "و" نوسانات میرایی زیادی دارند .

اما دامنه آن بسیار زیاد است.پس حالت "د" حالت مطلوب ما است.

Part 3:

الف)

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 5u(t) \rightarrow$$ (الف

$$s^2Y(s) - s\times y(0) - y'(0) + 3sY(s) - 3y(0) + 2Y(s) = 0 \rightarrow$$

$$\rightarrow (s^2+3s+2)Y(s) = s+4 \rightarrow Y(s) = \frac{s+4}{s^2+3s+2} = \frac{s+4}{(s+2)(s+1)}$$

$$\rightarrow y_1(t) = -2e^{-2t} + 3e^{-t} \rightsquigarrow$$ پاسخ ناشی از شرایط اولیه

$$(s^2+3s+2)Y(s) = \frac{5}{s} \rightarrow Y(s) = \frac{5}{(s+2)(s+1)\times s} \rightarrow$$

$$Y(s) = \frac{5/2}{s+2} - \frac{5}{s+1} + \frac{5/2}{s} \xrightarrow{\mathcal{L}^{-1}}$$

$$y_2(t) = \left(\frac{5}{2}e^{-2t} - 5e^{-t} + \frac{5}{2}\right)u(t) \rightsquigarrow$$ پاسخ حاصل از ورودی

ب)بله . با تئوری تطبیق دارد.

```
syms y(t)
Dy= diff(y);
ode= diff(y,t,2)+3*diff(y,t,1)+2*y==5*heaviside(t);
cond1=y(0)==1;
cond2=Dy(0)==1;
conds=[cond1 cond2];

ySol(t)=dsolve(ode,conds);
ySol=simplify(ySol)

tstart=0;tend=10;ts=1/100;
t1=tstart:ts:tend-ts;

y=(exp(-2.*t1).*(5*exp(2.*t1) + 2.*exp(t1) + 5.*sign(t1) ...
    - 10.*exp(t1).*sign(t1) + 5.*exp(2.*t1).*sign(t1) - 3))/4;


plot(t1,y,Linewidth=2);
```

```
>> part3

ySol(t) =

(exp(-2*t)*(5*exp(2*t) + 2*exp(t) + 5*sign(t) - 10*exp(t)*sign(t) + 5*exp(2*t)*sign(t) - 3))/4

>>
```

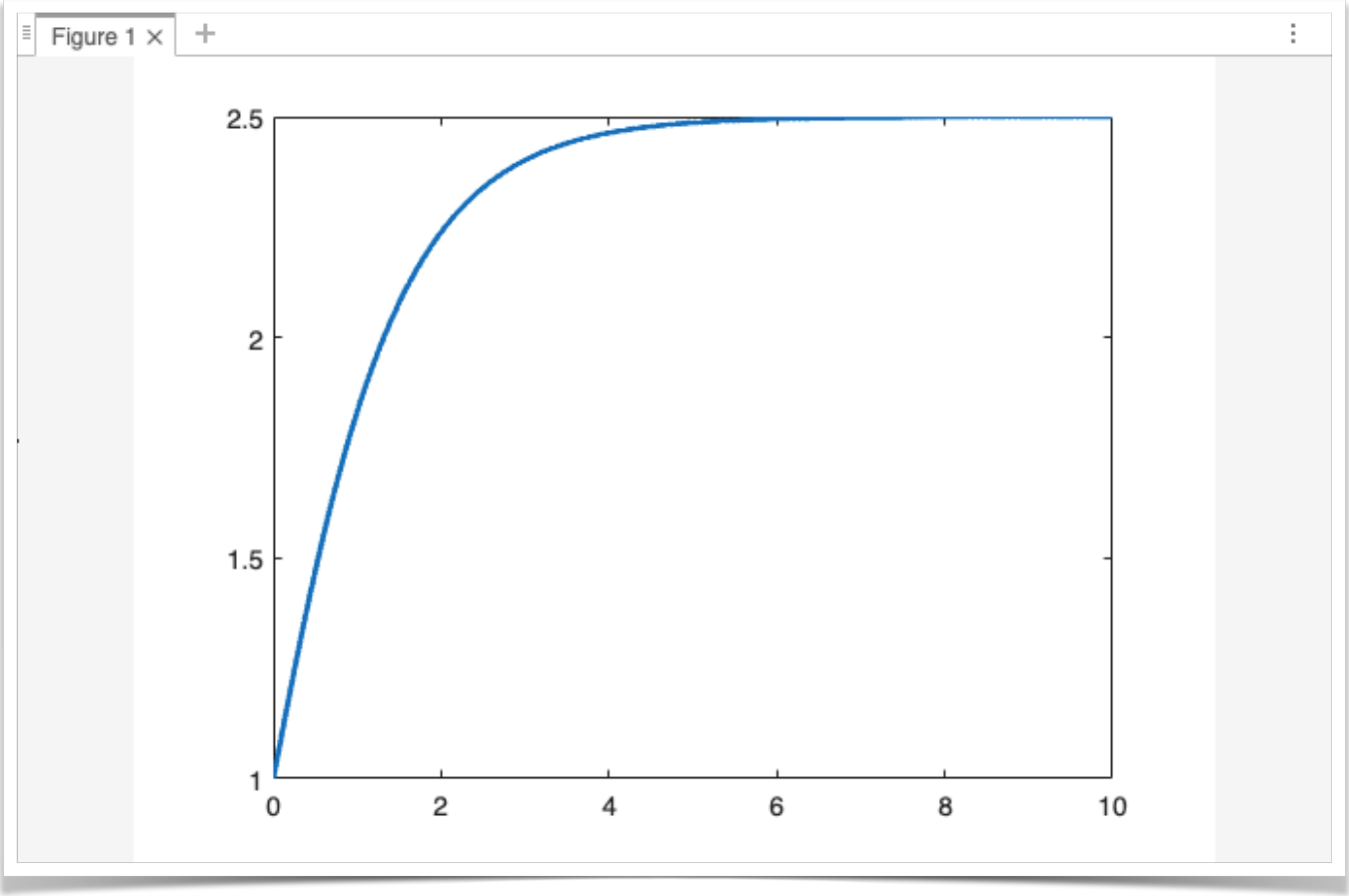


Figure 1
2.5
2
1.5
1
0
2
4
6
8
10