MiniEff の操作的意味論

2024 年 9 月 3 日

Computation c

Value v

Result r ::= Val v | Eff op v (x.c)

Environment E

Effect signature Σ

(int) $E, \Sigma \vdash i \Downarrow \text{Val } i$

(bool) $E, \Sigma \vdash b \Downarrow \text{Val } b$

(unit) $E, \Sigma \vdash () \Downarrow \text{Val } ()$

(val) $E, \Sigma \vdash \text{Val } v \Downarrow \text{Val } v$

(var) $E, \Sigma \vdash x \Downarrow \text{Val } v \ (x \in \text{dom}(E), E(x) = v)$

(opc) $E, \Sigma \vdash op \Downarrow \text{Val (Op } op) \ (op \in \Sigma)$

(fun) $E, \Sigma \vdash \text{fun } x \to c \Downarrow \text{Val (Cls } E \ x \ c)$

(handler) $E, \Sigma \vdash \text{handler } ocs \Downarrow \text{Val (Han } E \ ocs)$

$$\text{(pair-val)} \quad \frac{E, \Sigma \vdash c1 \Downarrow \text{Val } v1 \qquad E, \Sigma \vdash c2 \Downarrow \text{Val } v2}{E, \Sigma \vdash (c1, c2) \Downarrow \text{Val } (v1, v2)}$$

$$\text{(pair-eff-left)} \quad \frac{E, \Sigma \vdash c1 \Downarrow \text{Eff } op \ v \ (x.c)}{E, \Sigma \vdash (c1, c2) \Downarrow \text{Eff } op \ v \ (x.(c, c2))}$$

$$\text{(pair-eff-right)} \quad \frac{E, \Sigma \vdash c1 \Downarrow \text{Val } v \qquad E, \Sigma \vdash c2 \Downarrow \text{Eff } op \ v' \ (x.c)}{E, \Sigma \vdash (c1, c2) \Downarrow \text{Eff } op \ v' \ (x.(\text{Val } v, c))}$$

$$\text{(let-val)} \quad \frac{E, \Sigma \vdash c1 \Downarrow \text{Val } v \qquad E\{x{:}v\}, \Sigma \vdash c2 \Downarrow r}{E, \Sigma \vdash \text{let } x = c1 \text{ in } c2 \Downarrow r}$$

$$\text{(let-eff)} \quad \frac{E, \Sigma \vdash c1 \Downarrow \text{Eff } op \ v \ (y.c)}{E, \Sigma \vdash \text{let } x = c1 \text{ in } c2 \Downarrow \text{Eff } op \ v \ (y.\text{let } x = c \text{ in } c2)}$$

$$\text{(let rec)} \quad \frac{E\{f{:}\text{Rec } E \ f \ x \ c1)\}, \Sigma \vdash c2 \Downarrow r}{E, \Sigma \vdash \text{let rec } f \ x = c1 \text{ in } c2 \Downarrow r}$$

$$\text{(cond-true)} \quad \frac{E, \Sigma \vdash c1 \Downarrow \text{Val true} \qquad E, \Sigma \vdash c2 \Downarrow r2}{E, \Sigma \vdash \text{if } c1 \text{ then } c2 \text{ else } c3 \Downarrow r2}$$

$$\text{(cond-false)} \quad \frac{E, \Sigma \vdash c1 \Downarrow \text{Val false} \qquad E, \Sigma \vdash c3 \Downarrow r3}{E, \Sigma \vdash \text{if } c1 \text{ then } c2 \text{ else } c3 \Downarrow r3}$$

$$\text{(cond-eff)} \quad \frac{E, \Sigma \vdash c1 \Downarrow \text{Eff } op \ v \ (x.c)}{E, \Sigma \vdash \text{if } c1 \text{ then } c2 \text{ else } c3 \Downarrow \text{Eff } op \ v \ (x.\text{if } c \text{ then } c2 \text{ else } c3}$$

$$\text{(unary)} \quad \frac{E, \Sigma \vdash c \Downarrow \text{Val } v}{E, \Sigma \vdash + \ c \Downarrow \text{Val prim}(+, v)}$$

$$\text{(unary-eff)} \quad \frac{E, \Sigma \vdash c \Downarrow \text{Eff } op \ v \ (x.c')}{E, \Sigma \vdash + \ c \Downarrow \text{Eff } op \ v \ (x.+ \ c')}$$

(binary)
$$\frac{E, \Sigma \vdash c1 \Downarrow \text{Val } v1 \qquad E, \Sigma \vdash c2 \Downarrow \text{Val } v2}{E, \Sigma \vdash c1 + c2 \Downarrow \text{Val prim}(+, v1, v2)}$$

(binary-left)
$$\frac{E, \Sigma \vdash c1 \Downarrow \text{Eff op v } (x.c)}{E, \Sigma \vdash c1 + c2 \Downarrow \text{Eff op v } (x.c + c2)}$$

(binary-right)
$$\frac{E, \Sigma \vdash c1 \Downarrow \text{Val } v1 \qquad E, \Sigma \vdash c2 \Downarrow \text{Eff op v } (x.c)}{E, \Sigma \vdash c1 + c2 \Downarrow \text{Eff op v } (x.(\text{Val } v1) + c)}$$

(app-cls)
$$\frac{E1, \Sigma \vdash c1 \Downarrow \text{Val (Cls E2 x c)} \qquad E1, \Sigma \vdash c2 \Downarrow \text{Val } v2 \qquad E2\{x{:}v2\}, \Sigma \vdash c \Downarrow r}{E1, \Sigma \vdash c1\ c2 \Downarrow r}$$

(app-rec)
$$\frac{E1, \Sigma \vdash c1 \Downarrow \text{Val (Rec E2 f x c)} \qquad E1, \Sigma \vdash c2 \Downarrow \text{Val } v2 \qquad E2\{f{:}(\text{Rec E2 f x c}), x{:}v2\}, \Sigma \vdash c \Downarrow r}{E1, \Sigma \vdash c1\ c2 \Downarrow r}$$

(op-call)
$$\frac{E, \Sigma \vdash c1 \Downarrow \text{Val (Op op)} \qquad E, \Sigma \vdash c2 \Downarrow \text{Val } v}{E, \Sigma \vdash c1\ c2 \Downarrow \text{Eff op v } (x.x)}$$

(app-eff)
$$\frac{E, \Sigma \vdash c1 \Downarrow \text{Val v (v は Cls か Rec か Op)} \qquad E, \Sigma \vdash c2 \Downarrow \text{Eff op v' } (x.c)}{E, \Sigma \vdash c1\ c2 \Downarrow \text{Eff op v' } (x.(\text{Val v})\ c)}$$

(with-handle-val)
$$\frac{\begin{array}{c} \text{han} = \text{Han E' ocs} \\ E, \Sigma \vdash h \Downarrow \text{Val han} \end{array} \qquad \begin{array}{c} x \to c' = \text{valc}(ocs) \\ E, \Sigma \cup \text{sig}(ocs) \vdash c \Downarrow \text{Val } v \end{array} \qquad E'\{x{:}v\}, \Sigma \vdash c' \Downarrow r}{E, \Sigma \vdash \text{with h handle c} \Downarrow r}$$

(with-handle-eff)
$$\frac{\begin{array}{c} \text{han} = \text{Han E' ocs} \\ E, \Sigma \vdash h \Downarrow \text{Val han} \end{array} \qquad \begin{array}{c} \text{opc}(op, ocs) = op\ x\ k \to c' \qquad E, \Sigma \cup \text{sig}(ocs) \vdash c \Downarrow \text{Eff op v } (x.c'') \\ E'\{x{:}v, k{:}(\text{fun } x \to \text{with (Val han) handle } c'')\}, \Sigma \vdash c' \Downarrow r \end{array}}{E, \Sigma \vdash \text{with h handle c} \Downarrow r}$$

(with-handle-out)
$$\frac{\begin{array}{c} \text{han} = \text{Han E' ocs} \\ E, \Sigma \vdash h \Downarrow \text{Val han} \end{array} \qquad \begin{array}{c} \text{opc}(op, ocs) = \text{nil} \\ E, \Sigma \cup \text{sig}(ocs) \vdash c \Downarrow \text{Eff op v } (x.c'') \end{array}}{E, \Sigma \vdash \text{with h handle c} \Downarrow \text{Eff op v } (x.\text{with (Val han) handle } c'')}$$

$$\begin{aligned} &valc(nil) = nil \\ &valc(x \to c|ocs) = x \to c \\ &valc(\_|ocs) = valc(ocs) \end{aligned}$$

$$\begin{aligned} &opc(op, nil) = nil \\ &opc(op, op\ x\ k \to c|ocs) = op\ x\ k \to c \\ &opc(op, \_|ocs) = opc(op, ocs) \end{aligned}$$

$$\begin{aligned} &sig(nil) = \{\} \\ &sig(op\ x\ k \to c|ocs) = \{op\} \cup sig(ocs) \\ &sig(\_|ocs) = sig(ocs) \end{aligned}$$