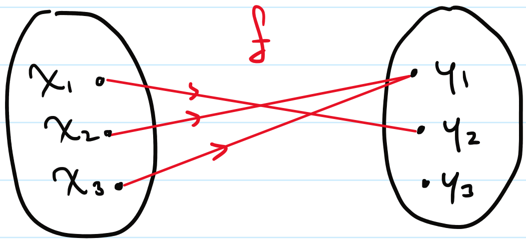
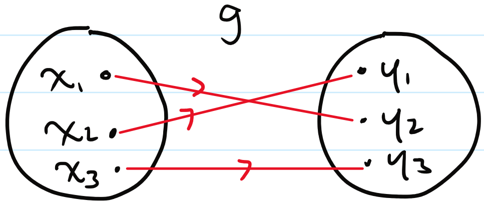
(Functions)

An <u>injective function</u>, also known as a one-to-one function is a function that maps distinct elements of its domain to distinct elements in the codomain



<u>not</u> an injective function because y_1 is the output of x_2 and x_3. In other words it isn't one-to-one

Remember: A function maps values of x in the domain to a <u>single</u> value in the codomain, so f here <u>is</u> still a function



g is an injective function because each output value is associated to an individual input.

An example of a non-injective function is

$f(x) = x^2$

because $f(-1) = (-1)^2 = 1$

$f(1) = 1^2 = 1$

$f(-1) = f(1) = 1$

Here we have the same output for two

different inputs.

An example of an injective function would be

$f(x) = x^3$

We can also write this as

$y = x^3$ if y is our output value.

If a y value is equal to 8

then

$8 = x^3$

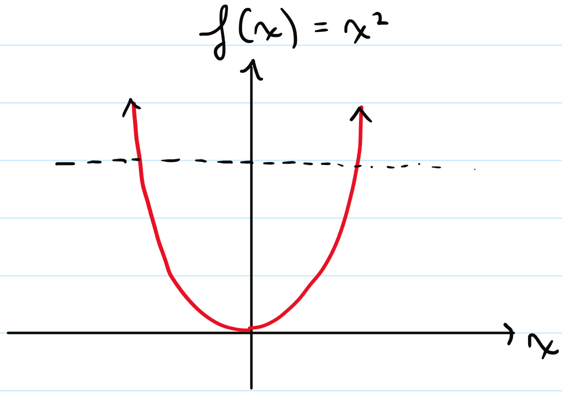
$x = \sqrt[3]{8}$

$x = 2$

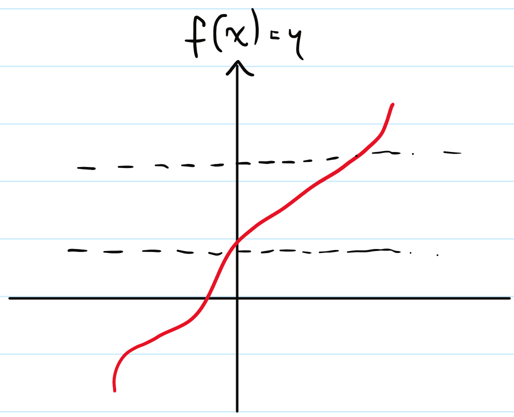
x can't also be equal to -2 or anything else

so $f(x) = x^3$ is one-to-one (injective).

The graph test for injectivity is a <u>horizontal line test</u>



If any horizontal line cuts the graph <u>more than once</u> the function cannot be one-to-one



There is no horizontal line that will cut our function at two points.
So it must be injective

☆ If <u>any</u> horizontal line cuts a given function at most once it is injective

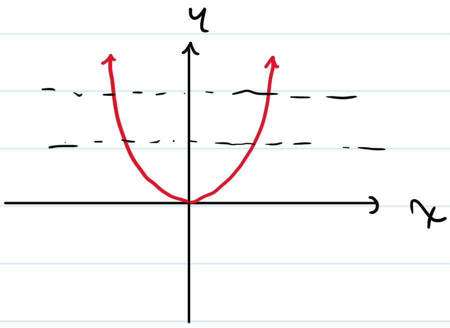
☆ If any horizontal line cuts a given function at most once it is injective

Surjective functions

A surjective function is a function such that for every y in the codomain there is at least one corresponding x in the domain

Graphical test for surjective functions

Any horizontal line in the codomain cuts the graph at least once



The values of y in the codomain for $f(x) = x^2$ are the positive real numbers, including 0.

And any horizontal line in this codomain will cut the graph at least once

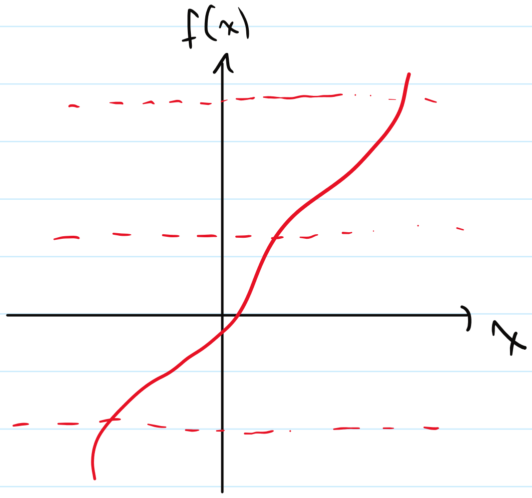
Bijective functions are functions that are both injective and surjective

Injective: distinct elements in the codomain correspond to distinct elements in the domain

Surjective: Every y in the codomain has at least one x corresponding to it.

Bijective: Every y in the codomain has one and only one corresponding x in the domain.

The graphical test for a bijective function is that every horizontal line in the codomain cuts the graph once and only once.



bijective.