(Functions)

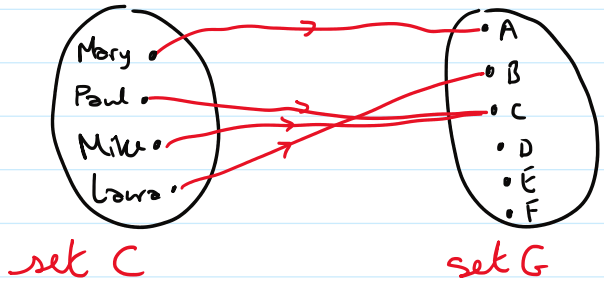
Relations

A relation is a correspondence between sets

Let set C be the students in a class of 4 students. And let G be the set of possible grades that can be obtained in an exam.

C = { Mary, Paul, Mike, Laura }
G = { A, B, C, D, E, F }

We can model exam results with a digram. After an exam, each student will have a corresponding grade



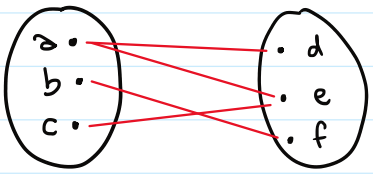
set C is called the domain
set G is called the codomain

A set S of ordered pairs can be used to represent this relation

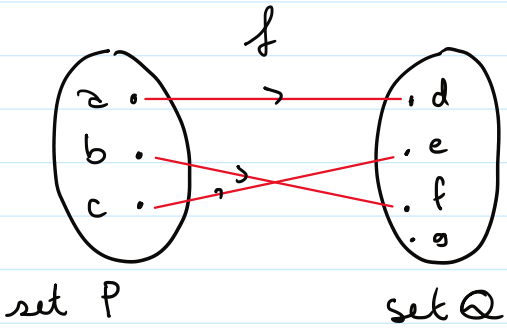
S = { (Mary, A), (Paul, C), (Mike, C), (Laura, B) }

Functions

A function is a relation that to each element in the domain one and only one value in the codomain



is not a function because an element of the domain is mapped to two elements.
(a, d) & (a, e) are in our set. So a is mapped to both d & e.



is a function because each element in the domain P, is mapped to one and only one element in the set Q

A function can be thought of as a relation between a set of inputs in the domain to a set of outputs in the codomain. The subset of the codomain that has elements that are output values is called the range.
Here .g is not in the range because it doesn't have a corresponding input value.
The range can be thought of as the set of elements that end at arrows.
So of our function above,

domain = {a, b, c}
codomain = {d, e, f, g}
range = {d, e, f}

A function can be defined by a rule
Our input can be notated by x, and our output by f(x)

$f(x) = x + 1 \qquad x \in \mathbb{R}$

This is the function that takes elements from the real numbers (our domain) and maps to each x one and only one output number.
This output number is defined by the rule in plain English "take your input number and add 1 to it".

There are different ways of representing this.
Using a table:

*

input x	-2	-1	0	1	2	3	4
output $f(x)$	-1	0	1	2	3	4	5

We can check our output for any value of x such that x is a real number ($x \in \mathbb{R}$).

$f(0) = 1$ from the table

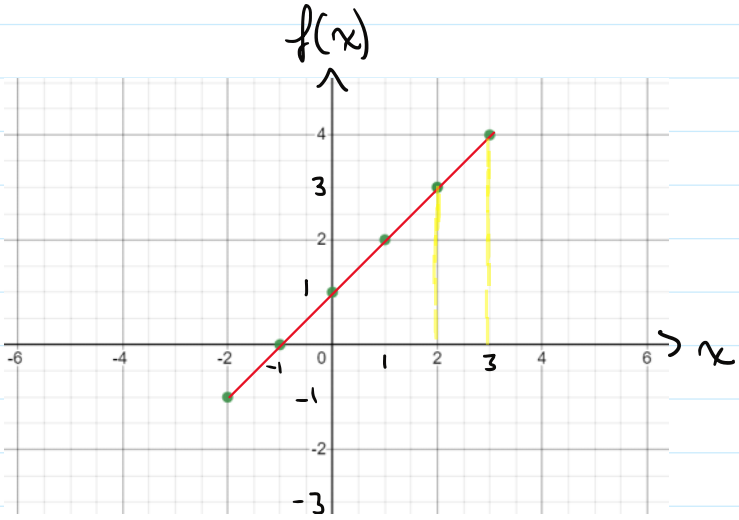
$f(2.5) = 2.5 + 1 = 3.5$

$f(\frac{1}{2}) = \frac{1}{2} + 1 = \frac{3}{2}$

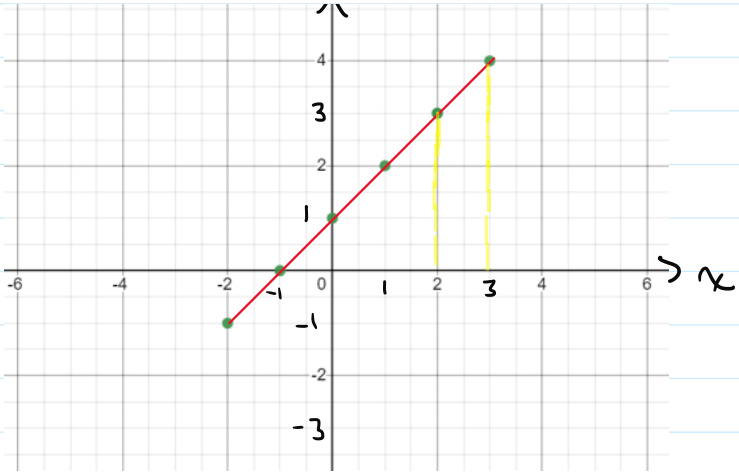
$f(\pi) = \pi + 1 = 4.14\ldots$

Another natural way of representing a functions inputs and outputs is with a graph

Plot the points from table *

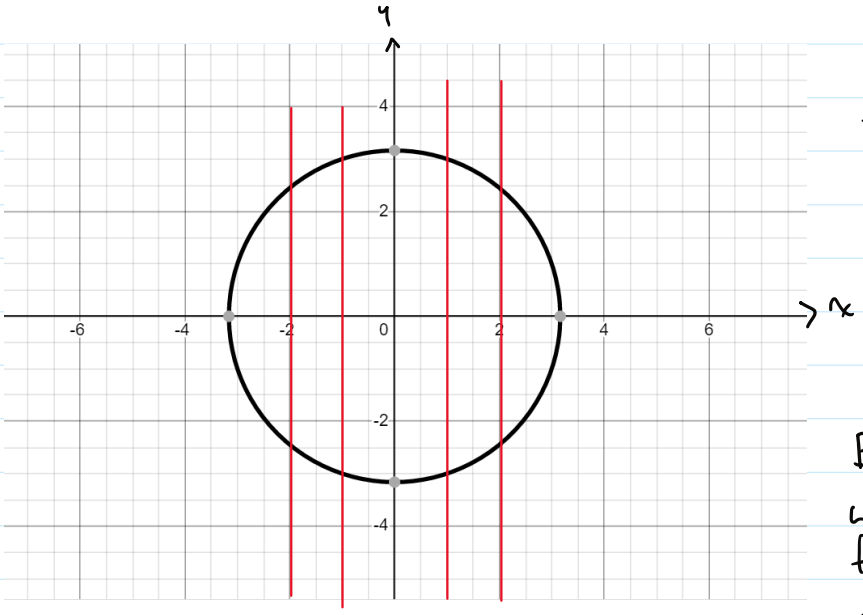


We see that when we plot the points we get a line



We see that when we plot the points we get a line

We can check that is a function with the vertical line test.
If vertical lines pass through only one point on the function graph, then we have a function



The equation of this circle is
$x^2 + y^2 = 10$
(unimportant for now)

But we can see that we don't have a function because some x values along the x-axis have two y values.

Substitute test.

You can also check if a given equation is a function using the substitute test
Consider the relation between x and y such that

$y^2 = x$

$y = \pm\sqrt{x}$

so for any output value y you can have two input values.