Exponential & Logarithmic Functions

An exponential function is a relation between two variables x, y which can be written in the form

$$y = ka^{bx} \quad \text{where } k, a, b \text{ are constants}$$
$$k, a, b \in \mathbb{R}$$
$$a > 0$$

Example

Show that $3^{-\frac{1}{2}x+5}$ can be written in the form ka^{bx}

The usual rules of powers apply

$$3^{-\frac{1}{2}x+5} = 3^{-\frac{1}{2}x} \cdot 3^5 = 3^5 \cdot 3^{-\frac{1}{2}x}$$
$$= 243 \cdot 3^{-\frac{1}{2}x}$$

Properties of exponential functions

1. They never cross the x-axis
2. They always cross the y-axis $(x = 0)$

   $ka^{b(0)} = ka^0 = k(1) = k$

   $(0, k)$ is the point where the function will cross the y-axis
3. If $k > 0$ the function curve will be above the x-axis

   If $k < 0$ the curve will be below the x-axis
4. The curves are always increasing or decreasing

For $y = ka^{bx} + c$ the curve is just shifted vertically by a factor of c and so can cross the x-axis if $c < 0$

Log functions

Logarithmic functions are of the form

$f(x) = k \log_a x$ where $k, a \in \mathbb{R}$ constants
and $a > 0$
$x, y = f(x)$ variables

Exponential functions and log functions are inverses of each other

$f(x) = y = 2^x$
$\log_2 y = x$
$f^{-1}(x) = \log_2 x$

In general $\log_a x$ is equal to the power you must raise a to to get x

$2^4 = 8 \iff \log_2 8 = 4$

Properties of logs

1. $\log_a 1 = 0$ for all $a \neq 0$

$\log_a$ (negative number) does not exist where a is any base

3. $\log_a b =$ negative number if b is between 0 and 1
$0 < b < 1$
and $a > 1$ $a, b \in \mathbb{R}$

4. $\log_a a = 1$ because $a^1 = a$

Rules for logs

- $\log_a x + \log_a y = \log_a (xy)$
- $\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$

must have the same base

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- $k\log_a x = \log_a (x)^k$ multiplication by a number

- $\log_a x = \dfrac{\log_b x}{\log_b a}$ change of base

- $\log_a b = \dfrac{1}{\log_b a}$

Back to log functions

Properties

$y = k\log_a x \qquad k, a \in \mathbb{R},\ a > 0$

1. A log curve always crosses the x-axis at $x=1$
2. A log curve always increases or decreases
   If $k>0$ it is increasing
   If $k<0$ it is decreasing