Rabi frequency dependence on two-mode squeezing generated by a SQUID

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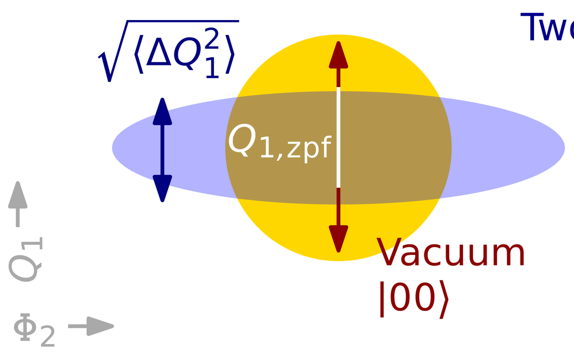
In a squeezed mode, fundamental quantum uncertainty in the magnetic field magnitude is less than the uncertainty in the electric field. The reason is the quantum correlations between them. Squeezing has been already used for sensing such as in LIGO gravitational wave detection.
In a multimode field, certain type of quantum correlations can arise between two different modes. That is dubbed two-mode squeezing and may be used for applications as well.
We explore how a driven dc SQUID can create two-mode squeezing and how this state influences Rabi oscillations with a transmon.

How to understand two-mode squeezing?



Two-mode squeezed vacuum:

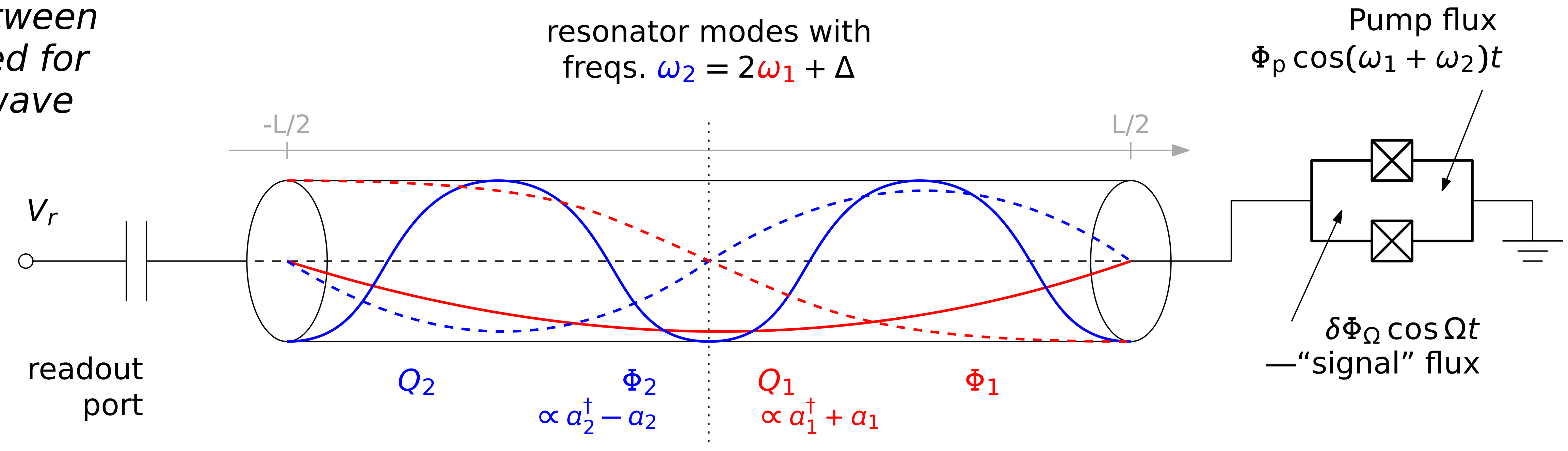
$$|\xi\rangle = e^{\xi(a_1 a_2 - a_1^\dagger a_2^\dagger)}\,|00\rangle$$

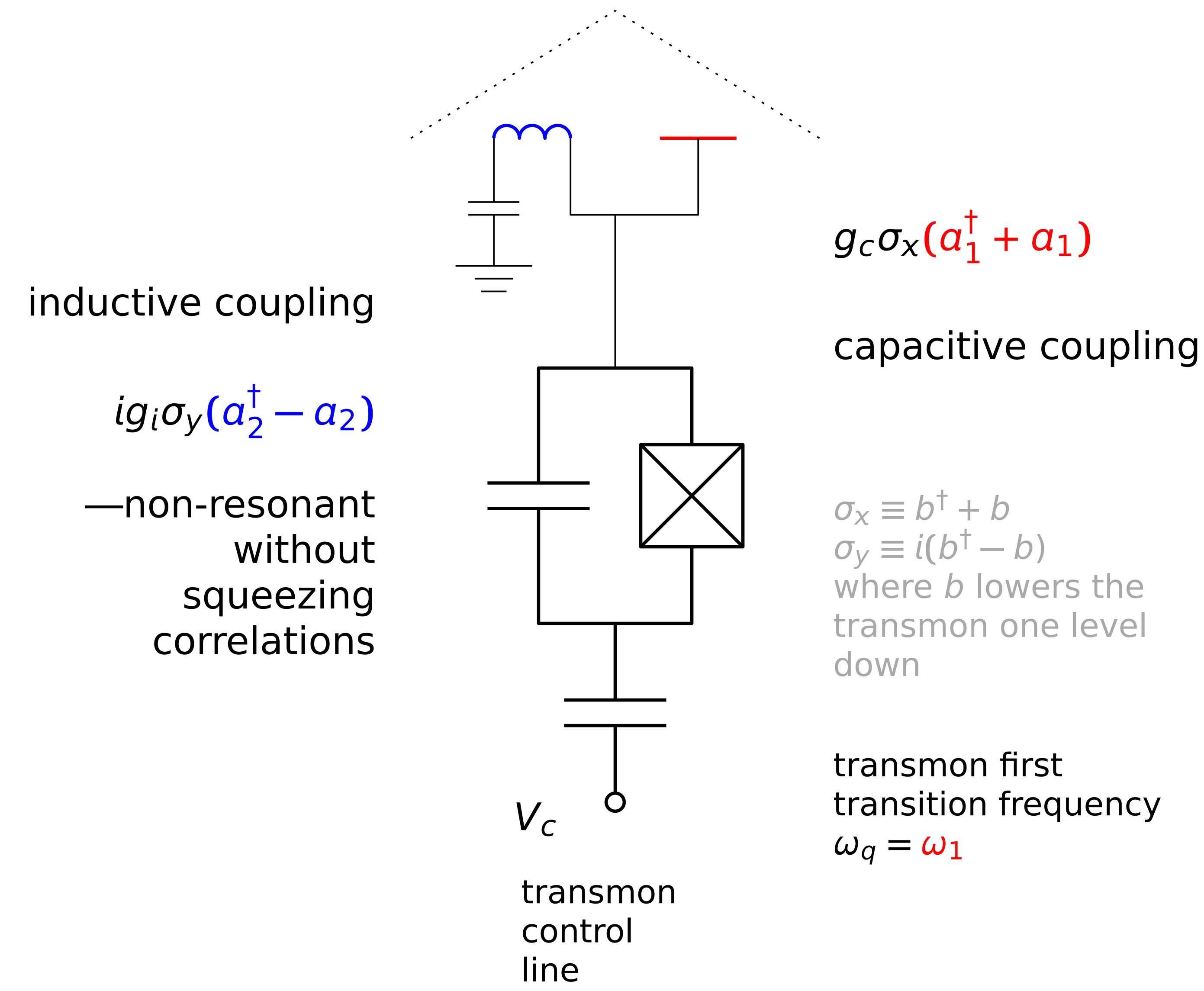
with squeezing

$$V = \frac{\langle \Delta Q_1^2 \rangle}{Q_{1,\mathrm{zpf}}^2} = e^{-2\xi}$$

$$\frac{1}{V} = \frac{\langle \Delta \Phi_2^2 \rangle}{\Phi_{2,\mathrm{zpf}}^2} = e^{2\xi}$$

I. Proposed setup





Protocol:

i. *At dc SQUID, signal $\delta\Phi$ component at frequency Ω multiplies with the drive Φ_e to produce two-mode squeezing*

ii. *Transmon qubit is initially prepared in its excited state $b^\dagger|0\rangle$. Its Rabi frequency depends on the two-mode squeezing. Hence the qubit population on waiting depends on the squeezing.*

iii. *Qubit is read out, for example with a dispersive readout via V_r and the second off-resonant mode. The probability to find the qubit in its excited state depends on the squeezing.*

II. Results

dc SQUID driven at $\omega_1+\omega_2$ and Ω squeezes the two resonator modes

The squeezing ratio ξ is the solution of $\sinh 2\xi = \frac{2\pi\delta\Phi_\Omega}{\Phi_0}\frac{G}{\Omega}$ where $G = 16\pi^4 E_J \frac{\Phi_1^\mathrm{zpf}\Phi_2^\mathrm{zpf}\Phi_\mathrm{p}}{\Phi_0^3}$ is proportional to the pump amplitude Φ_p and the Josephson energy of the SQUID junction E_J.

When the other drive is at $\Omega = \omega_2 - \omega_1$, Rabi frequency depends on the squeezing

Non-resonant mode picks up a resonant mode contribution:

$a_2 \to a_2\cosh\xi - a_1\sinh\xi$

In the squeezed-rotated frame:

$H_\mathrm{qr} \approx g_c b^\dagger a_1 \cosh\xi - g_i b^\dagger a_1 \sinh\xi + \mathrm{h.\,c.}$

b, a_1, and a_2 rotate at freq. $-\frac{\Omega}{2}\cosh 2\xi$

Exponential (de-)amplification of the Rabi frequency

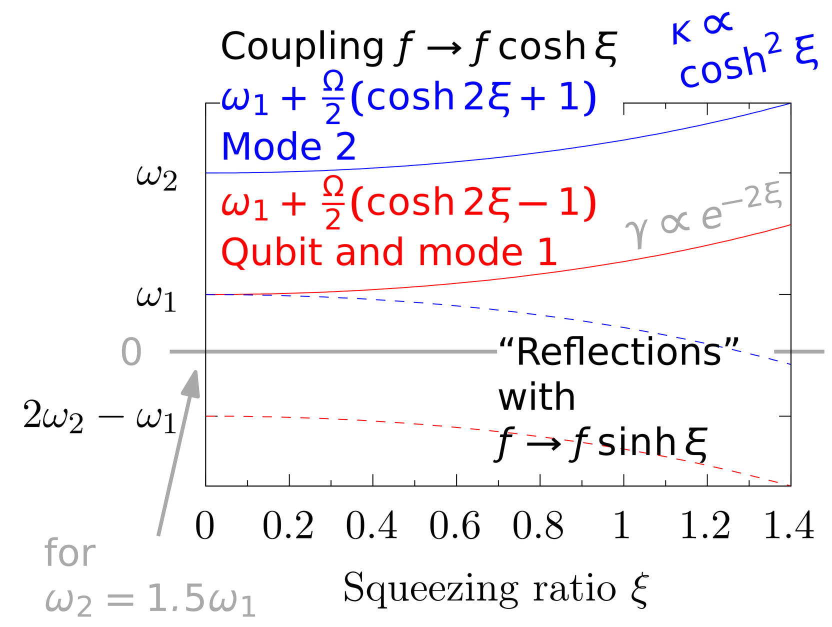
$g_c = g_i = g$:

$H_\mathrm{qr} = g e^{-\xi} b^\dagger a_1 + \mathrm{h.\,c.}$

$g_c = -g_i = g$:

$H_\mathrm{qr} = g e^{\xi} b^\dagger a_1 + \mathrm{h.\,c.}$

Spectrum and lifetimes



Amplification of qubit lifetime $1/\gamma$???

III. Calculations

Hamiltonian

$H = H_\mathrm{r} + H_\mathrm{q} + H_\mathrm{qr} + H_\mathrm{s}$

$H_\mathrm{r} = \omega_1 a_1^\dagger a_1 + \omega_2 a_2^\dagger a_2 \qquad H_\mathrm{q} = \omega_\mathrm{q} b^\dagger b + \frac{\Xi}{2} b^{\dagger 2} b^2$

$H_\mathrm{qr} = g_c(b^\dagger + b)(a_1^\dagger + a_1) + g_i(b^\dagger - b)(a_2^\dagger - a_2)$

$H_\mathrm{s} = -2E_J\cos\left[\pi\frac{\Phi_e+\delta\Phi}{\Phi_0}\right]\cos\left[2\pi\frac{\Phi(L/2)}{\Phi_0}\right]$

Origin of two-photon squeezing

$\Phi(L/2) \approx i\Phi_1^\mathrm{zpf}(a_1^\dagger - a_1) + i\Phi_2^\mathrm{zpf}(a_2^\dagger - a_2)$

Jacobi-Anger and Taylor-expand and pick the four-wave process: $H_\mathrm{s} = H_\mathrm{s}' + \ldots$

$H_\mathrm{s}' \approx G\frac{\delta\Phi_\Omega}{\Phi_0} a_1 a_2 e^{i(\omega_1+\omega_2)t}\cos\Omega t + \mathrm{h.\,c.}$

$G = 16\pi^4 E_J \frac{\Phi_1^\mathrm{zpf}\Phi_2^\mathrm{zpf}\Phi_\mathrm{p}}{\Phi_0^3}$

Remove H_s' explicit time dependence

$$R = \exp\Big[-i(\omega_1 - \Omega/2)a_1^\dagger a_1 t - i(\omega_2 - \Omega/2)a_2^\dagger a_2 t - i(\omega_1 - \Omega/2)b^\dagger b t\Big]$$

$H_\mathrm{s}' \to R^\dagger H_\mathrm{s}' R \approx G\frac{\delta\Phi_\Omega}{\Phi_0} a_1 a_2 + \mathrm{h.\,c.}$

$H_\mathrm{r} \to R^\dagger H_\mathrm{r} R - iR^\dagger \dot{R} = \frac{\Omega}{2}(a_1^\dagger a_1 + a_2^\dagger a_2)$

$H_\mathrm{q} \to \frac{\Omega}{2} b^\dagger b^\dagger + \frac{\Xi}{2} b^{\dagger 2} b^2$

$H_\mathrm{qr} \to g_c(\tilde{b}^\dagger + \tilde{b})(\tilde{a}_1^\dagger + \tilde{a}_1) + ig_i(\tilde{b}^\dagger - \tilde{b})(\tilde{a}_2^\dagger - \tilde{a}_2)$

$\tilde{a}_{1,2} = a_{1,2}e^{-i(\omega_{1,2}-\Omega/2)t} \qquad \tilde{b} = be^{-i(\omega_1-\Omega/2)t}$

Absorb squeezing into wavefunction

$S_{12} = \exp[\xi(a_1 a_2 - a_1^\dagger a_2^\dagger)], \ |\xi\rangle = e^{\xi(a_1 a_2 - a_1^\dagger a_2^\dagger)}|00\rangle$

$\sinh 2\xi = \frac{2\pi\delta\Phi_\Omega}{\Phi_0}\frac{G}{\Omega}$ —removes H_s' the squeezing term:

$H_\mathrm{r} + H_\mathrm{s}' \to \frac{\tilde{\Omega}}{2}(a_1^\dagger a_1 + a_2^\dagger a_2), \qquad \tilde{\Omega} = \Omega\cosh 2\xi$

$H_\mathrm{qr} \to g_c(\tilde{b}^\dagger + \tilde{b})[(a_1^\dagger\cosh\xi - a_2\sinh\xi)e^{i(\omega_1-\Omega/2)t} + \mathrm{h.\,c.}]$
$\quad + ig_i(\tilde{b}^\dagger - \tilde{b})[(a_2^\dagger\cosh\xi - a_1\sinh\xi)e^{i(\omega_2-\Omega/2)t} - \mathrm{h.\,c.})$

With $\Omega = \omega_2 - \omega_1$

$H_\mathrm{qr} \approx g_c b^\dagger a_1 \cosh\xi - g_i b^\dagger a_1 \sinh\xi + \mathrm{h.\,c.}$

Effective waveguide coupling

Bare coupling: $f\int dk\, w_k^\dagger(a_2 + a_1) + \mathrm{h.\,c.}$

—transforms similarly to H_qr

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