

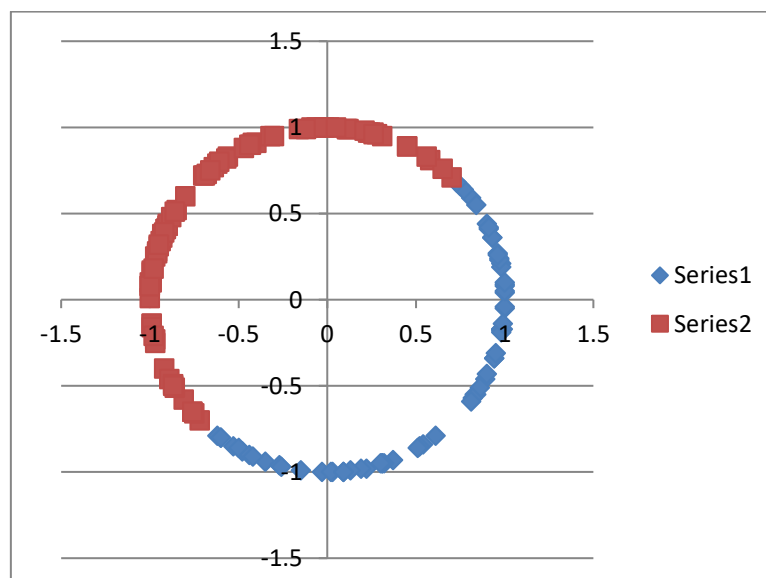
**Problem 1.** What is the VC-dimension of the infinite set of uni-directional balls on three dimensional points? Prove your result.

**Problem 2.**

- A. Given a set of rules  $H$  with VC-dimension  $h$ , and a different set of rules  $J$  with VC-dimension  $j$ , show that the joint set  $H \cup J$  has VC-dimension at most  $h+j+1$ .
- B. Give an upper-bound on the VC-dimension of bidirectional balls in the plane.
- C. Prove an upper-bound for the VC-dimension of the infinite phases of the moon. Each phase is the intersection of two circles, one with white inside, and the other with either white inside or with white outside.



**Problem 3.** The “two\_circle” data set contains 150 points on the circle:



Implement Perceptron on this set (code the version we learned in class). What is the final direction vector? How many mistakes were made by the algorithm? What margin was achieved by the final direction vector?

Now compute the optimal margin using brute force – how does it compare to the margin that Perceptron discovered?

Hand in your python code. Also hand the answers in the same file as the rest of the answers (not together with the code).