Exercise 12

1. Consider the loglinear model (AB,C) with $I = J = K = 3$. In class we showed that setting the log-likelihood derivatives to zero leads to the following equations, where r, s, and t each range from 1 to 2:

$$(f_{r\cdot\cdot} - n\pi_{r\cdot\cdot}) - (f_{3\cdot\cdot} - n\pi_{3\cdot\cdot}) = 0 \tag{1}$$

$$(f_{\cdot s\cdot} - n\pi_{\cdot s\cdot}) - (f_{\cdot 3\cdot} - n\pi_{\cdot 3\cdot}) = 0 \tag{2}$$

$$f_{\cdot\cdot t} - n\pi_{\cdot\cdot t}) - (f_{\cdot\cdot t} - n\pi_{\cdot\cdot t}) = 0 \tag{3}$$

$$(f_{rs\cdot} - n\pi_{rs\cdot}) - (f_{r3\cdot} - n\pi_{r3\cdot}) - (f_{3s\cdot} - n\pi_{3s\cdot}) + (f_{33\cdot} - n\pi_{33\cdot}) \tag{4}$$

We showed further that (1)-(3) imply that for r, s, and t ranging from 1 to 3 we have

$$\hat{\pi}_{r\cdot\cdot} = f_{r\cdot\cdot}/n, \quad \hat{\pi}_{\cdot s\cdot} = f_{\cdot s\cdot}/n, \quad \hat{\pi}_{\cdot\cdot t} = f_{\cdot\cdot t}/n \tag{5}$$

We claimed that (1)-(4) together imply that for r, s, and t ranging from 1 to 3 we have

$$\hat{\pi}_{rs\cdot} = f_{rs\cdot}/n$$

Prove this claim. Hint: The proof is similar, although somewhat more complicated, to the proof of (4); it involves introducing some trivial equations and summing over equations.

2. The data file `ex12dat.txt` presents results from a study on satisfaction with a certain brand of beer. The first column is satisfaction (1=low, 2=medium, 3=high), the second is sex (1=male, 2=female), the third is age group (1=young, 2=old), and the fourth is the cell count. Denote A=satisfaction, B=Sex, C=Age. Attached is R code to fit the loglinear model (AB,C) using an R function for Poisson regression. Run the code and verify that the following relationships are satisfied:

$$\hat{\pi}_{ij\cdot} = f_{ij\cdot}/n, \quad \hat{\pi}_{\cdot\cdot k} = f_{\cdot\cdot k}/n$$