Stat Models Moed Aleph 5775

1a. Suppose we have

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}$$

according to the weights $\{\pi_i\}$, $\{\tau_j\}$.

Define α_i^* to be the analogue of α_i under the weights $\{\pi_i^*\}$, $\{\tau_i^*\}$ (and similarly we use $*$ for other quantities under the new wts.)

We then have

$$\sum c_i \alpha_i^* = \sum_i c_i [\bar{\mu}_{i\cdot}^* - \bar{\mu}_{\cdot\cdot}^*]$$

$$= \sum c_i \bar{\mu}_{i\cdot}^* - \bar{\mu}_{\cdot\cdot} \sum_i c_i$$

$$= \sum_i c_i \bar{\mu}_{i\cdot}^*$$

$$= \sum_i c_i \sum_j \tau_j^* \mu_{ij}$$

$$= \sum_i c_i \sum_j \tau_j^* [\mu + \alpha_i + \beta_j]$$

$$= \sum_i \sum_j c_i \tau_j^* \mu + \sum_i \sum_j c_i \tau_j^* \alpha_i + \sum_i \sum_j c_i \tau_j^* \beta_j$$

$$= \mu \left(\sum_i c_i\right)\left(\sum_j \tau_j^*\right) + \left(\sum_i c_i \alpha_i\right)\left(\sum_j \tau_j^*\right) + \left(\sum_i c_i\right)\left(\sum_j \tau_j^* \beta_j\right)$$

$$= \sum_i c_i \alpha_i \quad \left(\text{since } \sum_i c_i = 0 \text{ and } \sum_j \tau_j^* = 1\right)$$

QED.

1b. Write the model in the form

$$Y = X\eta + \varepsilon$$

with

$$\eta = [\mu, \alpha_1, \dots, \alpha_{I-1}, \beta_1, \dots, \beta_{J-1}]^T$$

We have

$$\hat{\eta} = (X^T X)^{-1} X^T Y$$

$$\sim N(\eta, \sigma^2 (X^T X)^{-1})$$

$$\hat{\sigma}^2 = s^2 = \frac{1}{N-(I+J-1)} \sum_{ijk} (Y_{ijk} - \hat{Y}_{ijk})^2$$

$$s^2 \sim \frac{\sigma^2}{N-I-J+1} \chi^2_{N-I-J+1}$$

indept of $\hat{\eta}$

Let $\hat{\alpha} = [\hat{\alpha}_1, \dots, \hat{\alpha}_{I-1}]^T$

Let H be the $(I-1)\times(I-1)$ submatrix of $(X^T X)^{-1}$ corresponding to α

We then have $\hat{\alpha} \sim N(\alpha, \sigma^2 H)$

$$\hat{\psi}(c) \sim N(\psi(c), \sigma^2 c^T H c)$$

and the CI at level $1-\gamma$ is

$$\hat{\psi}(c) \pm t_{N-I-J+1}(1-\tfrac{\gamma}{2}) [c^T H c]^{1/2}$$

ג.

$\beta = [\mu]$

$x_i' = 1$

$\varepsilon_i = \begin{bmatrix} \varepsilon_{i11} \\ \varepsilon_{i12} \\ \varepsilon_{i13} \\ \varepsilon_{i21} \\ \varepsilon_{i22} \\ \varepsilon_{i23} \end{bmatrix}$

$b_i = \begin{bmatrix} a_i \\ b_{i1} \\ b_{i2} \end{bmatrix}$

$z_i' = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

ד. א.

$H_0: \sigma_1 = \sigma_2 = \dots = \sigma_I$

ב. נוכל לבצע מבחן פרמטרי בלי הנחת שוויון שונויות (מבחן וולש)
נוכל לבצע מבחן א-פרמטרי kruskal wallis
מבחן א-פרמטרי לבדיקת השונות במידה ויש אינדיקציה.

ה. א.

term amount age land

$x_1 = [1, 24, 10{,}000, 30, 0]$

נניח שהתפלגות תוחלת של הניבוי
של x לא נשנה את ההגדרה
לפי הנרמול

$$p(x_1) = \frac{e^{x_1'\hat{\beta}}}{1 + e^{x_1'\hat{\beta}}}$$

$$\hat{\beta} \sim N(\beta, \hat{V}^*(\hat{\beta}))$$

$$x_1'\hat{\beta} \mp z_{1-\frac{\alpha}{2}} \sqrt{x_1' \hat{V} x_1}$$

[illegible] $x_1'\hat{\beta} \sim$ [illegible]

$$x_1'\beta = [1 \quad \ln(24) \quad \ln(10{,}000) \quad \ln(30) \quad 0] \begin{bmatrix} -1.1179 \\ -0.9297 \\ 0.187 \\ 0.956 \\ -0.7341 \end{bmatrix} =$$

$$= 0.9013$$

$$z = 1.96$$

$$x_1'\hat{V} = [-0.086 \quad -0.012 \quad 0.0185 \quad -0.0045 \quad -0.0009]$$

$$x_1'\hat{V}x = 0.0304$$

$$l = x_1'\beta_l^* = 0.9013 - 1.96 \cdot \sqrt{0.0304} = 0.5597$$

$$h = x_1'\beta_h = 1.243$$

$$p(x_1)_l = e^l / 1+e^l = 0.6364$$

$$p(x_1)_h = 0.7761$$

$$\frac{p(x_2)}{1-p(x_2)} = \frac{e^{x_2'\beta}}{1+e^{x_2'\beta}} \Big/ 1 - \frac{e^{x_2'\beta}}{1+e^{x_2'\beta}} =$$

$$= \frac{e^{x_2'\beta}}{1+e^{x_2'\beta}} \Big/ \frac{1}{1+e^{x_2'\beta}} = e^{x_2'\beta}$$

ולכן צריך רו"ס עבור $e^{x_2'\beta - x_1'\beta} = e^{(x_2 - x_1)'\beta}$

$u = x_2 - x_1 = [0 \quad 12 \quad 5000 \quad 10 \quad 1]$ forgot to take logs here

נשתמש באומד β קודם נמצא רו"ס ל- $u'\beta$

$u'\beta = 0.7497$

$u'\hat{V} = [-1.0579 \quad -0.0398 \quad 0.09 \quad 0.1357 \quad -0.0296]$

$u'\hat{V}u = 0.9568$

$u'\beta_l = -1.1675$

$u'\beta_h = 2.6669$

$e^{u'\beta_l} = e^{u'\beta}{}_l = 0.3111$

$e^{u'\beta}{}_h = 14.3946$

ג. A השכלה → ומעל (AC, BC)

B מין $\pi_{ijk}^{ABC} = \exp(\bar\theta_{\cdot\cdot\cdot} + \lambda_A^i + \lambda_B^j + \lambda_C^k + \lambda_{AC}^{ik} + \lambda_{BC}^{jk})$

C גיל

$$\hat\pi_{ijk} = \frac{\frac{f_{i\cdot k}}{n}\cdot\frac{f_{\cdot jk}}{n}}{\frac{f_{\cdot\cdot k}}{n}} = \frac{f_{i\cdot k}\, f_{\cdot jk}}{n f_{\cdot\cdot k}} = \frac{f_{i\cdot k}\, f_{\cdot jk}}{n f_{\cdot\cdot k}}$$

נחשב את הf המתאימים

A - 0 - נמוך, 1 - בינוני, 2 - גבוהה

B - 0 - זכר, 1 - נקבה

C - 0 - צעיר, 1 - מבוגר

$n = 175$

$f_{0\cdot 0} = 9$ $\quad f_{\cdot 00} = 55$ $\quad f_{\cdot\cdot 0} = 95$

$f_{0\cdot 1} = 23$ $\quad f_{\cdot 01} = 50$ $\quad f_{\cdot\cdot 1} = 80$

$f_{1\cdot 0} = 36$ $\quad f_{\cdot 10} = 40$

$f_{1\cdot 1} = 35$ $\quad f_{\cdot 11} = 30$

$f_{2\cdot 0} = 50$

$f_{2\cdot 1} = 22$

π

A	B	C = 0	C = 1
0	0	$\frac{9\cdot 55}{175\cdot 95} = 0.03$	$\frac{23\cdot 50}{175\cdot 80} = 0.08$
	1	$\frac{9\cdot 40}{175\cdot 95} = 0.02$	$\frac{23\cdot 30}{175\cdot 80} = 0.05$
1	0	$\frac{36\cdot 55}{175\cdot 95} = 0.12$	$\frac{35\cdot 50}{175\cdot 80} = 0.1$[illegible]
	1	$\frac{36\cdot 40}{175\cdot 95} = 0.09$	$\frac{35\cdot 30}{175\cdot 80} = 0.0$[illegible]
2	0	$\frac{50\cdot 55}{175\cdot 95} = 0.17$	$\frac{22\cdot 50}{175\cdot 80} = 0.08$
	1	$\frac{50\cdot 40}{175\cdot 95} = 0.12$	$\frac{22\cdot 30}{175\cdot 80} = 0.05$

$\log \Pi$

A	B	C = 0	C = 1
0	0	−1.5	−2.5
0	1	−3.9	−3
1	0	−2.1	−2.1
1	1	−2.4	−2.7
2	0	−1.8	−2.5
2	1	−2.1	−3

$\overline{\theta_{\cdots}} = -2.63$

$\sum_{j,k} \log \Pi_{ijk} = JK\lambda_A^i + JK\overline{\theta_{\cdots}}$ מסכמים על j,k מקבלים

$-12.9 = 4\lambda_A^0 + 4\cdot(-2.63) \Rightarrow \lambda_A^0 = -0.595$

$-9.3 = 4\lambda_A^1 + 4(-2.63) \Rightarrow \lambda_A^1 = 0.305$

$\lambda_A^2 = -\lambda_A^1 - \lambda_A^0 = 0.29$

$\sum_{ik} \log \Pi_{ijk} = IK\overline{\theta_{\cdots}} + IK\lambda_B^j$ באופן דומה

$-14.5 = 6(-2.63) + 6\lambda_B^0 \Rightarrow \lambda_B^0 = 0.213 \Rightarrow \lambda_B^1 = -0.213$

$\sum_{ij} \log \Pi_{ijk} = IJ\overline{\theta_{\cdots}} + IJ\lambda_C^k$

$-15.8 = 6(-2.63) + 6\lambda_C^0 \Rightarrow \lambda_C^0 = -0.003 \quad \lambda_C^1 = 0.003$

$\sum_i \log \Pi_{ijk} = I\overline{\theta_{\cdots}} + I\lambda_B^j + I\lambda_C^k + I\lambda_{BC}^{jk}$

$-7.4 = 3(-2.63) + 3\cdot 0.213 + 3\cdot(-0.003) + 3\lambda_{BC}^{00} \Rightarrow \lambda_{BC}^{00} = -0.047 = \lambda$

$\lambda_{BC}^{10} = \lambda_{BC}^{01} = 0.047$

$\sum_j \log \Pi_{ijk} = J\overline{\theta_{\cdots}} + J\lambda_A^i + J\lambda_C^k + J\lambda_{AC}^{ik}$

$-7.4 = 2(-2.63) + 2(-0.595) + 2(-0.003) + 2\lambda_{AC}^{00} \Rightarrow \lambda_{AC}^{00} = -0.472$

$-4.5 = 2(-2.63) + 2\cdot 0.305 + 2(-0.003) + 2\lambda_{AC}^{10} \Rightarrow \lambda_{AC}^{10} = 0.078$

$i = 0,1,2 \quad \lambda_{AC}^{i1} = -\lambda_{AC}^{i0} \qquad \lambda_{AC}^{20} = 0.394$