CPSC 221 Assignment 1

1. Rank the following functions by order of growth. Partition your list into classes such that functions f(n) and g(n) are in the same class if and only if f(n) ∈ Θ(g(n)).

lg(lg(n))

lg(n)

ln(n)

lgn

2lg(n)

lg n! and nlg(n) -> Θ(nlgn)

n1.13

n2 and 4lgn  -> Θ(n2)

()n

2n

2n+1

n!

22n

2. Simplifying to Asymptotic Bounds.

(a) Example: T(n) = 47, answer Θ(1).

(b) T(n) = 8(n + 1) + 3 = 8n+11 -> get rid of lower order and coefficients

Answer is Θ(n).

(c) T(n) = (n − 1)(8(n + 1) + 3) = 8n2 +3n -11 -> get rid of lower order and coefficients

Answer is Θ(n2).

(d) T(n) = =(3n-1) = n2 – n -> get rid of lower order and coefficients

Answer is Θ(n2).

(e) T(n) == (n+3 + 2)

Answer is Θ(n).

(f) T(n) = = = (n-1) = n2 -

Answer is Θ(n2).

(g) T(0) = 1 and

T(n) = T(n − 1) + 8 ->

T(n) = T(n - 2) + 8\*2 ->

T(n) = T(n - 3) + 8\*3 ->

T(n) = T(n - k) + 8k

For k= n -> T(0) + 8n -> 1 + 8n -> 8n

Answer is Θ(n).

(h) T(0) = 1 and T(n) = 2T(n − 1) + 8

T(n) = 2T(n-1) + 8

T(n-1)= 2T(n-2) + 8

T(n-2) = 2T(n-3) + 8

T(n) = 2(2T(n-2) + 8) + 8 = 2(2T(n-2))+ 2(8) + 8

T(n) = 2(2(2T(n-3) + 8)) + 8 = 8T(n-3) + 4(8) + 2(8) + 8 = 23 T(n-3) + 23-2(8) + 8 + 23-2(8) + 8

For k = n

T(n) = 2nT(0) + 2n-1(8) + 2n-2(8) + 8

* 2n + 2n-1(8) + 2n-2(8) + 8
* 2n

Answer is Θ(n2).

(i) T(0) = 1 and T(n) = T() + 8

T() + 8 = T() + 8

T() + 8 = T() + 8 ->

T(n) = T() + 2(8)

= T(n) = T() + 3(8) = T() + k(8)

K = lg2n

T(n)= T() + lg2n(8)

Flooring…

T(n) = T() + 16lgn = T(0) + 16lgn = 1 + 16lgn -> lgn

Answer is Θ(lgn).

(j) T(0) = 1 and T(n) = T() + 8n

T() = T() + 8()

T() = T() + 8()

T(n) = T() + 8() + 8n

K = lg2n

T(n) = T() + 8() + 8n

Flooring…

T(n) = T() + + 8n = T(0) + + 8n = 1 + + 8n

Answer is Θ().