Algebra 2 Formulas

krista king MATH

First order equations

Powers of negative bases

$$-b^2 = -1(b^2)$$

$$(-b)^2 = (-b)(-b)$$

Powers of fractions

$$\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$$

Zero as an exponent

Any nonzero real number raised to the power 0 is equal to 1, which means anything that looks like a^0 is equal to 1 if a is not equal to 0.

$$\frac{x^n}{x^n} = x^{n-n} = x^0$$

Negative exponents

$$a^{-b} = \frac{1}{a^b}$$

$$a^b = \frac{1}{a^{-b}}$$

Negative exponents and product rule

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n} = \frac{b^n}{a^n}$$

Fractional exponents

$$x^{\frac{a}{b}} = \sqrt[b]{x^a}$$

$$x^{\frac{1}{2}} = \sqrt[2]{x} = \sqrt{x}$$

$$x^{\frac{1}{3}} = \sqrt[3]{x}$$

$$x^{\frac{4}{3}} = \sqrt[3]{x^4}$$

$$x^{a/b} = [(x)^a]^{\frac{1}{b}}$$

$$x^a \cdot x^{\frac{c}{d}} = x^{a+\frac{c}{d}} = x^{\frac{ad}{d}+\frac{c}{d}} = x^{\frac{ad+c}{d}}$$

Rationalizing the denominator

Rules with radicals:

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

$$a \cdot \sqrt{b} = a\sqrt{b}$$

$$\sqrt{a} \cdot \sqrt{a} = a$$

Rationalizing the denominator:

$$\frac{\sqrt{a}}{\sqrt{b}} \to \frac{\sqrt{a}}{\sqrt{b}}\left(\frac{\sqrt{b}}{\sqrt{b}}\right) \to \frac{\sqrt{a}\sqrt{b}}{\sqrt{b}\sqrt{b}} \to \frac{\sqrt{ab}}{b}$$

Rationalizing with conjugate method

Conjugate: The conjugate of the binomial is the same terms with the opposite sign in between. The conjugate of of $5-\sqrt{3}$ is $5+\sqrt{3}$.

Rationalizing the denominator with conjugate method:

$$\frac{3}{5-\sqrt{3}} \to \frac{3}{5-\sqrt{3}}\left(\frac{5+\sqrt{3}}{5+\sqrt{3}}\right) \to \frac{3(5+\sqrt{3})}{(5-\sqrt{3})(5+\sqrt{3})} \to \frac{3(5+\sqrt{3})}{25+5\sqrt{3}-5\sqrt{3}-3} \to \frac{3(5+\sqrt{3})}{22}$$

Ratios and proportions

Ratios and proportions

Ratio: A comparison of two numbers (or other mathematical expressions), often written as a fraction

Proportion: An equality between two ratios

Cross multiplication:

$$\frac{a}{b} = \frac{c}{d} \quad \to \quad a \cdot d = c \cdot b$$

Chemical compounds

Molar mass: The sum of products of the atomic weight of each element that make up the compound and the numbers of atoms of the corresponding element in one molecule of that compound. Measured in grams per mole, g/mol.

Fractions to decimals to percents

Percents to decimals: Divide by 100.

Decimals to percents: Multiply by 100.

Fractions to percents: Change the fraction to a decimal, then change the decimal to a percent.

Percent markup

Percent markup formulas, where “new price,” or “selling price,” is the price the end customer pays the store for the item, “original price,” “manufacturer’s price,” or “original purchase price,” is the price the store paid the manufacturer for the item, and “percent markup” is the percent of the original price by which the store marked up the item in order to get to the new price.

$$\text{New Price} = \text{Original Price} + \text{Markup Amount}$$

$$\text{New Price} = \text{Original Price}\left(1 + \frac{\text{Percent Markup}}{100}\right)$$

$$\text{Markup Amount} = \text{Original Price}\left(\frac{\text{Percent Markup}}{100}\right)$$

Percent markdown

Percent markdown formulas, where "percent markdown" is the percentage off the original price of the item, "original price" is the price the person or store was selling the item for before they discounted it, the "discount amount" is the amount in dollars that is taken off the original price, and "sale price" is the price of the item after the discount is applied.

$$\text{Discount Price} = \text{Original Price} - \text{Discount Amount}$$

$$\text{Discount Price} = \text{Original Price}\left(1 - \frac{\text{Percent Markdown}}{100}\right)$$

$$\text{Discount Amount} = \text{Original Price}\left(\frac{\text{Percent Markdown}}{100}\right)$$

$$\frac{\text{Discount Amount}}{\text{Original Price}} = \frac{\text{Percent Markdown}}{100}$$

Calculating commission

Commission: The amount of money a salesperson earns on the sale of an item.

Commission = Selling Price · Percent Commission

Calculating simple interest

Simple interest: The amount we earn on an investment each year, and the interest doesn't compound, such that we earn the same interest each year. If I is interest earned, P is principal, r is interest rate, and t is time, then

$$I = Prt$$

Total account balance: If A is account balance, P is principal, r is interest rate, and t is time, then

$$A = P(1 + rt)$$

Complex fractions

Complex fractions: An algebraic expression with fraction(s) in either the numerator, the denominator, or both.

Fraction division: To divide by a fraction, we can multiply by its reciprocal.

Imaginary and complex numbers

Imaginary number: $i = \sqrt{-1}$ and $i^2 = -1$

Complex number: The sum of a real number and an imaginary number, $a + bi$

Addition and subtraction complex numbers:

$$z_1 + z_2 = (a + c) + i(b + d)$$

$$z_1 - z_2 = (a - c) + i(b - d)$$

Rationalizing complex denominators

Multiplication of complex numbers:

$$(a + bi)(c + di) = (ac - bd) + (ad + bc)i$$

Complex conjugate: Formed by changing the sign of the imaginary part and leaving the real part unchanged.

Conjugate method: To rationalize a fraction that has a complex number in the denominator, we multiply it by the fraction in which both the numerator and the denominator are the complex conjugate of that complex number.

Factoring

Difference of cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Sum of cubes

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

Rational functions

Adding and subtracting rational functions

When we add and subtract fractions, we need a common denominator. The lowest common denominator is the least common multiple of the denominators in the individual fractions.

Factoring to find a common denominator

Rational expression: A fraction in which the numerator and denominator are polynomials.

Dividing rational functions

When we divide one rational function by another, the first fraction is the dividend, the second fraction is the divisor, and the result of doing the division gives us the quotient.

$$\frac{\text{dividend}}{\text{divisor}} = \text{quotient}$$

Values that make a quotient undefined:

1. Any values(s) where the dividend's denominator is 0

2. Any value(s) where the divisor's denominator is 0

3. Any values(s) where the divisor's numerator is 0

Advanced equations

Direct variation

Direct variation relationship where, as one variable increases, the other variable also increases, and where k is a constant:

$$y = kx$$

Inverse variation

Inverse variation relationship where, as one variable increases, the other variable decreases, and where k is a constant:

$$y = \frac{k}{x}$$

Fractional equations

Solving fractional equations:

1. Find the lowest common denominator (LCD).

2. Multiply both sides of the equation by the LCD to remove the fractions.

3. Solve the equation and check the solution.

Rational equations

Rational equations: Fractional equations where we have variables in the numerator and/or denominator of the fractions

Solving rational equations:

1. Find any value of the variable that would make any denominator equal to zero.

2. Find the least common denominator (LCD).

3. Multiply both sides of the equation to clear the fractions.

4. Solve the resulting equation and check the solutions. Don't forget to discard any values from step 1 if they are algebraic solutions.

Extraneous solution: Values that look like solutions but actually aren't

Radical equations

Solving radical equations:

1. Isolate the radical expression on one side of the equation.

2. Square both sides of the equation.

3. Rearrange and solve the equation.

Eliminating a radical with index n by raising it to the n power:

$$(\sqrt[n]{a})^n = a$$

Multivariable rational equations

Abstract fractional equation: An equation that has at least one fraction with a variable in its denominator

Systems of equations

Uniform motion problems

Distance, rate, and time:

Distance = Rate · Time

$$D = RT$$

Number word problems

Consecutive integers:

- Consecutive integers are integers that are in ascending order and have no integer between them, such as 4 and 5.

- Consecutive even numbers are even numbers that are in ascending order and have no even number between them, such as 4 and 6.

- Consecutive odd numbers are odd numbers that are ascending order and have no odd number between them, such as 5 and 7.

Graphing

Parallel and perpendicular lines

Parallel lines:

$$\begin{cases} y = mx + b_1 \\ y = mx + b_2 \end{cases} \qquad \text{with } b_1 \neq b_2$$

Perpendicular lines:

$$\begin{cases} y = mx + b_1 \\ y = -\frac{1}{m}x + b_2 \end{cases}$$

Graphing parabolas

Standard form of a parabola:

$y = ax^2 + bx + c$

Axis of symmetry $x = -b/2a$

Vertex $(-b/2a, f(-b/2a))$

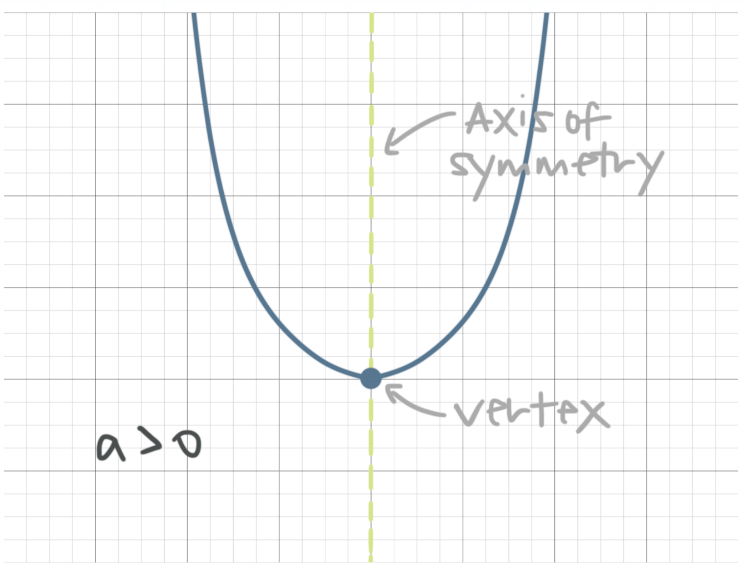
Vertex form of a parabola:

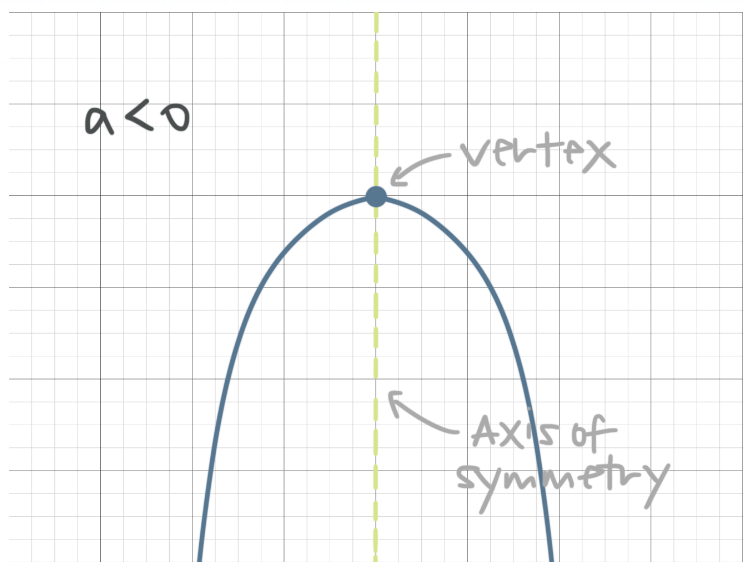
$y = a(x - h)^2 + k$

Axis of symmetry $x = h$

Vertex (h, k)

Parabola direction: If $a > 0$ the parabola opens upwards and the vertex is the point at the bottom of the parabola, and if $a < 0$ the parabola opens downwards and the vertex is the point at the top of the parabola.





Sketching parabolas:

1. Find the coordinates of the vertex.

2. Find the value of the y-intercept.

3. Find the x-coordinates of the x-intercepts by solving the equation $f(x) = 0$.

4. Make sure we've found at least one point to either side of the vertex. This helps make a better sketch. If we already have two x-intercepts from the previous step, we can use those. Otherwise, if we have zero or just one x-intercept, we can then use another x value or use the axis of symmetry and the y-intercept to get the second point.

5. Sketch the graph.

Center and radius of a circle

Standard form of the equation of a circle, where r is the radius and (h, k) are the coordinates of the center:

$$(x - h)^2 + (y - k)^2 = r^2$$

Distance between two points

Distance between two points:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint between two points:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

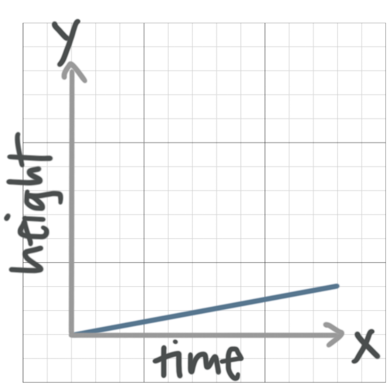
Modeling a piecewise-defined function

Piecewise-defined function: A function that's made up of different "pieces," each of which has its own "sub-function" (its own algebraic expression) and its own "sub-domain" (its own part of the domain of the entire piecewise function).

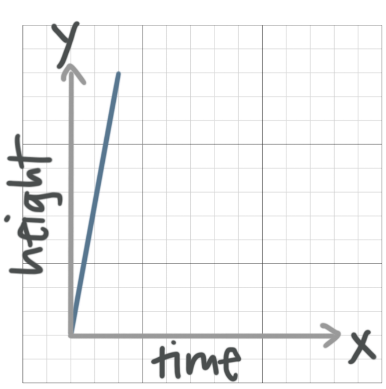
$$f(x) = \begin{cases} \text{Function}_1 & \text{if} \quad \text{Domain}_1 \\ \text{Function}_2 & \text{if} \quad \text{Domain}_2 \\ \text{Function}_3 & \text{if} \quad \text{Domain}_3 \\ \text{Function}_4 & \text{if} \quad \text{Domain}_4 \end{cases}$$

Sketching graphs from story problems

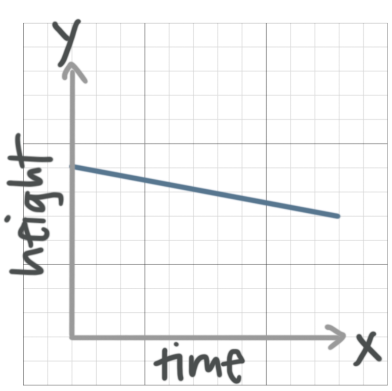
Slow increase:



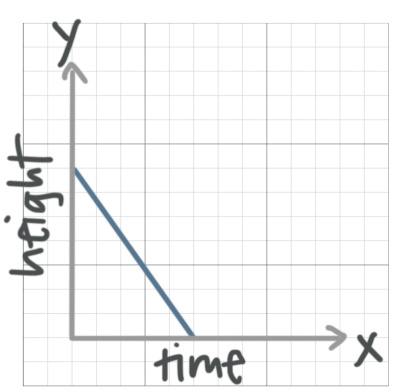
Fast increase:



Slow decrease:



Fast decrease:



Quadratic inequalities

Quadratic inequality: A quadratic equation where the equal sign has been replaced by an inequality sign

Solving quadratic inequalities algebraically:

1. Put the inequality into standard form.
2. Find the critical points, which are the solutions to the related quadratic equation.
3. Using the critical points, divide the number line into intervals.
4. Choose a point from each interval and substitute it into the quadratic expression to find its sign, either positive or negative.
5. Finally, choose the intervals where the inequality is true and write the solution using interval notation.

Solving quadratic inequalities graphically:

1. Write the inequality in standard form.
2. Determine the x-intercepts by looking at the graph, or by solving $ax^2 + bx + c = 0$.
3. Graph the parabolic function given by $f(x) = ax^2 + bx + c$.
4. Depending on the sign of the inequality, determine whether it asks for the value(s) of x that make the parabola negative (below the horizontal axis) or positive (above the horizontal axis).

Zeros of the quadratic from the discriminant:

$b^2 - 4ac > 0$	Two zeros
$b^2 - 4ac = 0$	One zero
$b^2 - 4ac < 0$	No zeros

Manipulating functions

Combinations of functions

Sum	$(f + g)(x) = f(x) + g(x)$
Difference	$(f - g)(x) = f(x) - g(x)$
Product	$(f \cdot g)(x) = f(x) \cdot g(x)$
Quotient	$(f \div g)(x) = \dfrac{f(x)}{g(x)}$

Composite functions

Composite of $f(x)$ and $g(x)$: $(f \circ g)(x)$ or $f(g(x))$

Domains of composite functions

Domain of a function: The set of x-values where the function is defined

Domain of a composite: Given the composite $f(g(x))$, the domain will exclude all values of x where $g(x)$ is undefined, and all values of x where $g(x)$ is defined but $f(g(x))$ is undefined.

One-to-one functions and the Horizontal Line Test

Horizontal Line Test: A graph passes the Horizontal Line Test when no horizontal line intersects it at more than one point.

One-to-one function: A graph represents a one-to-one function if and only if it passes the Vertical Line Test and the Horizontal Line Test. Passing the Vertical Line Test ensures that the graph represents a function, and then also passing the Horizontal Line Test ensures that the function it represents is one-to-one.

One-to-one functions algebraically: If $f(a)=f(b)$ implies $a=b$, then f is a one-to-one function.

Inverse functions

Invertible functions: A function is invertible (has an inverse) if and only if it's one-to-one. The inverse of a function $f(x)$ is $f^{-1}(x)$.

Exponential and logarithmic function

What is a logarithm?

Logarithms: Logarithms tell us how many times we multiply one number by itself in order to get a different number.

Base and argument: In the equation $\log_2(8) = 3$, the 2 is the base and the 8 is the argument of the log function.

General log rule: Given the exponential equation $a^x = y$, the associated logarithmic equation is $\log_a(y) = x$, and vice versa.

Common bases and restricted values

Common logarithm: When there's no subscript on the log, it means that we're dealing with the **common logarithm**, which always has a base of 10.

Euler's number: $e \approx 2.718281828459045235360287471352 7....$

Natural logarithms: Logarithms to base e, written as $\ln$

$$\log_e(x) = \ln(x)$$

Restricted values: In a logarithmic expression $\log_a(y)$, the base a must be positive (and not equal to 1), and the argument y must also be positive.

The general log rule

Equivalent log and exponential functions:

$\log_a(y) = x$ and $a^x = y$ are equivalent

$\log_a(x) = y$ and $a^y = x$ are equivalent

Inverse functions:

Both $\log_a(x) = y$ and $a^y = x$ are inverses of $\log_a(y) = x$

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Laws of logarithms

Product rule: $\log_a(xy) = \log_a x + \log_a y$

Quotient rule: $\log_a(x/y) = \log_a x - \log_a y$

Power rule: $\log_a(x^n) = n \log_a x$

Change of base

Change of base:

$$\log_a b = \frac{\log_c b}{\log_c a}$$

Laws of natural logs

General log rule with natural logs: Given the exponential equation $e^x = y$, the associated logarithmic equation is $\ln(y) = x$, and vice versa.

Rules of natural logs:

Product rule:	$\ln(xy) = \ln x + \ln y$
Quotient rule:	$\ln(x/y) = \ln x - \ln y$
Power rule:	$\ln(x^n) = n \ln x$

Features of the natural log:

$\ln x$ is undefined at all $x \le 0$

$\ln 1 = 0$

$\ln x \to \infty$ as $x \to \infty$

Graphing exponential functions

Graphing exponential functions:

1. Plug in $x = 100$ and $x = -100$, and use the values $f(100)$ and $f(-100)$ to determine the "end behavior" of the function, that is, what happens to the value of the function as $x \to \infty$ and as $x \to -\infty$.

2. One of these will result in an infinite value, the other will give a real-number value. The real-number value is the horizontal asymptote of the exponential function.

3. Plug in a few easy-to-calculate values of x, like $x = -1, 0, 1$, in order to get a couple of points that we can plot.

4. Connect the points with an exponential curve, and draw the horizontal asymptote.

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