



# Algebra 2 Worksheet Solutions

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# Exponents and radicals

## worksheet



1. Circle each expression that has the same value as  $(-3/5)^2$ .

$$(-1)\left(\frac{3}{5}\right)^2$$

$$\frac{-9}{25}$$

$$\frac{3^2}{(-5)^2}$$

$$\frac{(-3)^2}{5^2}$$

$$\frac{-3^2}{5^2}$$

$$\frac{9}{25}$$

2. Choose the simplified expression.

$$\frac{x^{\frac{1}{5}}y^{-3}z^0}{x^{\frac{3}{5}}y^2z^{-2}}$$

$$\frac{z^2}{x^{\frac{2}{5}}y^5}$$

$$\frac{z^2}{x^2y^5}$$

$$\frac{1+z^2}{x^{\frac{2}{5}}y^5}$$

$$\frac{z^2}{x^{\frac{2}{5}}y^{-1}}$$

3. Match each radical with a rational exponent.

$$x^{\frac{2}{3}}$$

$$x^{\frac{3}{2}}$$

$$x^{\frac{1}{2}}$$

$$x^{\frac{1}{3}}$$

$$\sqrt[3]{x}$$

$$\sqrt{x^3}$$

$$\sqrt[3]{x^2}$$

$$\sqrt{x}$$

4. Circle the simplified expression.

$$\sqrt{\frac{1}{16}} + \sqrt{\frac{2}{3}} + \sqrt{\frac{1}{6}}$$

$$\frac{1}{4} + \sqrt{3}$$

$$\frac{3\sqrt{6}}{4}$$

$$\frac{1 + 2\sqrt{3}}{4}$$

$$\frac{1 + 2\sqrt{6}}{4}$$

5. Circle the first step at which a mistake is made in simplifying the expression.

$$\frac{1 + \sqrt{3}}{1 - \sqrt{3}}$$

Step 1:  $\frac{(1 + \sqrt{3})}{(1 - \sqrt{3})} \cdot \frac{(1 + \sqrt{3})}{(1 + \sqrt{3})}$

Step 2:  $\frac{(1 + \sqrt{3})^2}{1 - \sqrt{3} + \sqrt{3} - 3}$

Step 3:  $\frac{1 + 3}{1 - 3}$

Step 4:  $-2$

# Exponents and radicals

## KEY POINTS

Powers with negative bases

Powers of fractions

Zero exponent

Negative exponents

Fractional exponents

Rationalizing the denominator

## NOTES

Case 1:  $-b^a = -1 \cdot b^a$ ; Case 2:  $(-b)^a$ , multiply  $-b$  times itself  $a$  times. Ex:

$$(-2)^4 = (-2)(-2)(-2) = -8$$

Raise both the numerator and denominator

the the power. Ex:  $\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$

Any nonzero real number raised to the

power 0 is equal to 1. Ex:  $\frac{x^n}{x^n} = x^{n-n} = x^0 = 1$

The rule of thumb is to write any negative exponents as positive exponents following

the rules  $a^{-b} = \frac{1}{a^b}$  and  $\frac{1}{a^{-b}} = a^b$ . Ex:

$$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

When you have a fractional exponent, the numerator (of the exponent) is the power

and the denominator is the root. Ex:  $x^{\frac{a}{b}} = \sqrt[b]{x^a}$

Taking all of the radicals out of the denominator of a fraction and moving them

to the numerator. Ex:  $\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

# Exponents and radicals

## KEY POINTS

Rules about radicals

## NOTES

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}, \sqrt{ab} = \sqrt{a}\sqrt{b}, a \cdot \sqrt{b} = a\sqrt{b},$$

$$\sqrt{a} \cdot \sqrt{a} = a$$

# Ratios and proportions

## worksheet



1. There are 12 cats for every 8 dogs living in a city. If there are a total of 3,240 cats and dogs in the city, how many dogs are there?

1,080

1,296

1,944

2,160

2. Solve the proportion for the unknown value.  $[x = 4]$

$$\frac{3,136}{56} = \frac{224}{x}$$

3. Complete the table.

Fraction	Decimal	Percent
$\frac{5}{8}$	0.625	62.5%
$\frac{3}{4}$	0.75	75%
$\frac{1}{100}$	0.01	1%

4. A store purchased an item from the manufacturer for \$42 and then sold it to a customer for \$49.98. What was the percent markup that the store applied to the item?

0.798 %

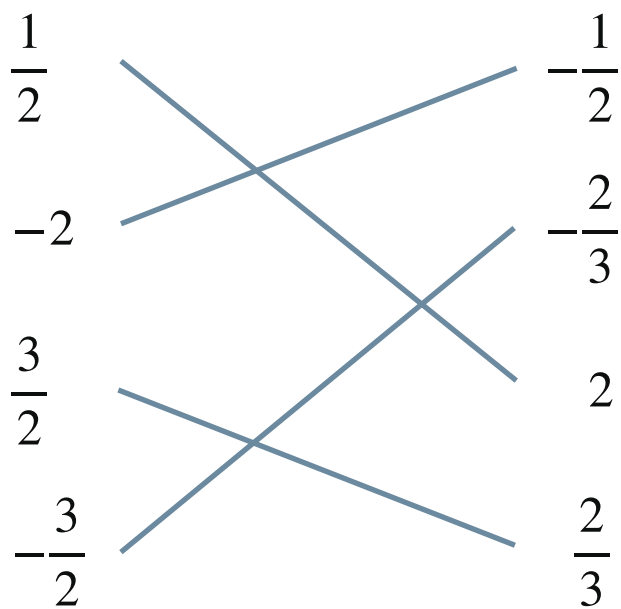
1.9 %

7.98 %

19 %

5. Find the amount of interest earned in one year if the annual interest rate is 0.3 % and the principal balance is \$2,000.  $[\$6]$

6. Match each fraction on the left with its reciprocal on the right.



7. Circle any complex fractions.

$$\frac{2}{3}$$

$$\frac{\frac{2}{3}}{4}$$

$$-\frac{5}{\frac{4}{x}}$$

$$\frac{\frac{x+2}{5}}{\frac{8}{x-1}}$$

8. True or false? Dividing a number by  $\frac{4}{5}$  or multiplying the number by  $\frac{5}{4}$  will give the same result. [True]

9. Solve the equation for  $x$ . [ $x = 22$ ]

$$\frac{\frac{x+2}{3}}{7} = \frac{2}{\frac{7}{4}}$$

10. Circle any expressions that are equivalent to the complex fraction.

$$\frac{\frac{2}{x-3}}{\frac{4}{x+5}}$$

$$\frac{2}{x-3} \cdot \frac{4}{x+5}$$

$$\frac{2x+10}{4x-12}$$

$$\frac{8}{(x-3)(x+5)}$$

$$\frac{x+5}{2(x-3)}$$

11. Circle the simplified version of  $\sqrt{-64}$ .

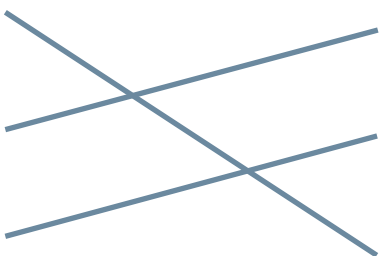



$-8$

$\pm 8i$

$8i$

$8i^2$

12. Match each power of  $i$  with its simplified form.

$i^{102}$		$i$
$i^{73}$		$-i$
$i^{211}$		$-1$
$i^{1000}$		$1$

13. Simplify the expression  $5i^3 - 2i^2 + i^4 - i + 8 - \sqrt{-4}$ .  $[11 - 8i]$

14. True or false? The conjugate of  $3 + 2i$  is  $-3 - 2i$ .

[False. The conjugate of  $3 + 2i$  is  $3 - 2i$ .]

15. Circle the simplified expression.

$$\frac{i+1}{i-1}$$



1

0

-1

$-i$

# Ratios and proportions

## KEY POINTS

### Ratio

A comparison of two numbers (or other mathematical expressions), and it is often written as a fraction. Ex: The ratio of cats to dogs can be written as  $\frac{\text{number of cats}}{\text{number of dogs}}$ .

### Proportion

A proportion is an equality between two ratios. Use cross multiplication to solve proportions. Ex: A proportion is written as  $\frac{a}{b} = \frac{c}{d}$  and we solve by using cross multiplication:  $a \cdot d = c \cdot b$ .

### Percent

An amount or part out of 100. Ex:  $4\% = \frac{4}{100}$

### Convert percent to decimal

Divide a percent by 100 to get a decimal number. Ex:  $49\% = \frac{49}{100} = 0.49$

### Convert decimal to percent

Multiply a decimal number by 100 to get a percent. Ex:  $0.05 \cdot 100 = 5\%$

### Convert fraction to percent

Change the fraction to a decimal, then change the decimal to a percent. Ex:  $\frac{1}{4} = 0.25$  and  $0.25 \cdot 100 = 25\%$

# Ratios and proportions

## KEY POINTS

Percent of a number

Percent markup

Percent markdown

Commission

Simple interest

## NOTES

To find a percent of a number, multiply the number by the decimal form or the fractional form of the percent. Ex: 10 % of 50 is  
 $10\% \cdot 50 = 0.1 \cdot 50 = 5$

Markup Amount = Original Price  $\left( \frac{\text{Percent Markup}}{100} \right)$

Discount Amount = Original Price  $\left( \frac{\text{Percent Markdown}}{100} \right)$

$$\frac{\text{Discount Amount}}{\text{Original Price}} = \frac{\text{Percent Markdown}}{100}$$

The amount a salesperson makes on the sale of an item. Ex:

Commission = Selling Price  $\cdot$  Percent Commission

The amount you earn on an investment each year. Ex:  $I = Prt$

# Ratios and proportions

## KEY POINTS

Complex fractions

Reciprocal

Solving complex fractions

Imaginary number,  $i$

Square of  $i$

Complex number

## NOTES

A fraction that includes fractions either in the numerator, denominator or both. Ex:

$$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)}.$$

The reciprocal of a fraction is just that fraction “flipped upside down.” Ex: The

reciprocal of  $\frac{a}{b}$  is  $\frac{b}{a}$ .

When a complex fraction in a proportion has a variable, you can solve for that variable in one of two ways: cross multiplication or multiply by reciprocals. Remember that

cross multiplication of  $\frac{a}{b} = \frac{c}{d}$  results in

$$ad = bc.$$

$$i = \sqrt{-1}$$

$$i^2 = -1$$

The sum of a real number and an imaginary number is written in the form  $a + bi$  where  $a$  and  $b$  are real numbers. Ex:  $8 - 6i$  where  $a = 8$  and  $b = -6$ .

# Ratios and proportions

## KEY POINTS

Complex conjugate

## NOTES

$a + bi$  and  $a - bi$  are conjugates

# Factoring

## worksheet



1. Circle the binomial factors of the quadratic  $6x^2 + 13x + 6$ .

$2x + 3$

$2x + 2$

$3x + 3$

$3x + 2$

2. What should  $A$  be to allow us to factor the polynomial  $5x^3 + 20x^2 + Ax + 8$  by grouping?

$4$

$-4$

$2$

$-2$

3. True or false? The binomial  $64x^6 - 1$  could be factored both as a difference of cubes or as a difference of squares. [\[True\]](#)

4. Match the binomial in standard form with its factored form.

$x^3 + 1$

$x^3 - 1$

$8x^3 - 27$

$8x^3 + 27$

$(x - 1)(x^2 + x + 1)$

$(2x + 3)(4x^2 - 6x + 9)$

$(x + 1)(x^2 - x + 1)$

$(2x - 3)(4x^2 + 6x + 9)$

5. Which quadratic would have roots  $x = -1$  and  $x = 6$ ?

$2x^2 - 10x - 12$

$2x^2 - 10x + 12$

$x^2 - 5x + 6$

$2x^2 - 5x - 6$

# Factoring

## KEY POINTS

Factoring quadratics when  
 $a \neq -1, 1$

Factor by grouping

Difference of two cubes

Sum of two cubes

## NOTES

To factor  $ax^2 + bx + c$ , find factors of the product  $ac$  whose sum is  $b$ . Rewrite the quadratic with 4 terms and factor by grouping.

When factoring a polynomial with 4 terms, group two terms and factor out a common factor, then group the other two terms and factor out a common factor. If the result of each groups is the same, factor out that common factor. Ex:  $4x^2 + 2x + 6x + 3$ , Group the first two terms and the last two terms, then factor out common factors.

$$(4x^2 + 2x) + (6x + 3) = 2x(2x + 1) + 3(2x + 1),$$

Factor out the common factor of  $(2x + 1)$ ,

$$2x(2x + 1) + 3(2x + 1) = (2x + 1)(2x + 3)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$\text{Ex: } x^3 - 8 = (x - 2)(x^2 + 2x + 4)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$\text{Ex: } x^3 + 27 = (x + 3)(x^2 - 3x + 9)$$

# Rational functions

worksheet



1. Circle the remainder after dividing  $3x^3 - 2x^2 - 5x + 7$  by  $3x + 4$ ?

$$3x + 4$$

$$\frac{11}{3x + 4}$$

$$x^2 - 2x + 1$$

$$\frac{3}{3x + 4}$$

2. What is the greatest common factor by which all terms can be reduced?  $[2xy^2]$

$$\frac{2xy^3 - 4x^2y^5}{8xy^2}$$

3. What is the least common denominator for the sum?  $[18x^4y^3]$

$$\frac{8}{9x^4y} + \frac{8}{6x^2y^3}$$

4. Circle the simplified expression.

$$\frac{4x}{x^2 - 16} \cdot \frac{x^2 - 5x + 4}{2x^2 + 2x - 4}$$

$$\frac{1}{x + 4}$$

$$\frac{2x}{(x + 4)(x + 2)}$$

$$\frac{1}{2x + 4}$$

$$\frac{4x}{(x + 4)(x + 2)}$$



5. Circle all of the values that should be excluded from the domain.

$$\frac{x+4}{x-3} \div \frac{x^2-1}{x}$$

$$x \neq 3$$

$$x \neq 2$$

$$x \neq -4$$

$$x \neq -3$$

$$x \neq 1$$

$$x \neq 4$$

$$x \neq -1$$

$$x \neq 0$$

# Rational functions

## KEY POINTS

Long division of polynomials

Simplifying rational expressions

Adding or subtracting rational expressions

Multiplying rational expressions

Dividing rational expressions

## NOTES

Follow the same steps for long division of real numbers. Make sure to align like terms.

Factor polynomials in the numerator and denominator completely and cancel factors that are shared by all terms of the numerator and denominator.

Common denominators are needed to add or subtract rational expressions, just like adding or subtracting fractions.

Cancel factors from the denominator of a product of rational expressions first, when possible, then multiply remaining polynomials. You may have to factor and cancel factors again. Don't forget to explicitly state the "hidden" values of the variable that are excluded from the domain.

Factor and simplify by canceling common factors in the numerator and denominator when possible. Remember to change fraction division to multiplication by the reciprocal of the second fraction.

# Advanced equations

## worksheet



1. Label each scenario as direct variation, inverse variation, or neither.

The number of songs downloaded on your phone, and the amount of memory you have left. inverse

The number of shoes you buy, and the cost of your bill.  
direct

The number of pets you have, and your grade in English.  
neither

2. Solve the equation for  $x$ .

$$0.01x - 0.5x + 0.078 = -1$$

$$x = -1.88$$

$$x = 1.88$$

$$x = 2.2$$

$$x = 2.695$$

3. Circle the solution(s) of the equation.

$$\sqrt{x-4} = \frac{1}{5}x$$

$$x = 4$$

$$x = 5$$

$$x = 20$$

no solution

4. True or false? If  $2a + 3b - 5c = 12$ , then the values of  $a$ ,  $b$ , and  $c$  are

$$a = \frac{-3b + 5c + 12}{2}$$

$$b = \frac{-2a + 5c + 12}{3}$$

$$c = \frac{2a + 3b - 12}{5} \text{ [True]}$$

5. Gwen can bike 30 miles in 2 hours. How far can she travel at this same pace if she bikes for 5.5 hours?

15 miles

52.5 miles

82.5 miles

165 miles

# Advanced equations

## KEY POINTS

Direct variation

Inverse variation

Solving decimal equations

Solving fraction equations

Extraneous solutions

## NOTES

Relationships in the form  $kx = y$ , where  $k$  is a constant and  $x$  and  $y$  are variables. As one variable increases, the other variable will also increase.

Relationships in the form  $y = \frac{k}{x}$ , where  $k$  is a constant and  $x$  and  $y$  are variables. As one variable increases the other variable decreases.

Multiply by an appropriate power of 10 to clear decimals and work with integers. Ex:  $0.2x = 10$  Multiply by 10 on both sides to clear the decimal term.  $0.2x(10) = 10(10)$ , Simplify.  $2x = 100$ , Divide by 2 on both sides.  $x = 50$ .

Clear fractions by multiplying both sides of the equation by the LCD of all fractions in the equation, then solve using traditional methods of solving equations.

Values that are not actually solutions of the original equation. When solving rational or radical equations, evaluate the original equations with the potential solutions you have found. If the statement is true, then the

# Advanced equations

## KEY POINTS

Solving multivariable equations

## NOTES

value is indeed a solution; otherwise, it is extraneous.

Use inverse operations to isolate the variable you're solving for.

# Systems of equations

## worksheet



1. Solve the system of equations for  $x_1$  using any method.

$$2x_1 - x_2 = 12$$

$$\frac{1}{5}x_1 + \frac{1}{2}x_2 = 0$$

$$x_1 = -2$$

$$x_1 = 0$$

$$x_1 = 5$$

$$x_1 = 10$$

2. True or false? There can be either 0, 1, or 2 solutions between the graph of a circle and a quadratic.

[False. While it's possible to have 0, 1, or 2 solutions, it's also possible that there are more than two solutions.]

3. Two even consecutive integers have a sum of 66. Circle the equation that must be true, with  $n$  representing the lesser of the two numbers.

$$2n + 2 = 66$$

$$2n + 1 = 66$$

$$n + 2 = 66$$

$$n^2 + 2n = 66$$

4. Four years ago, Bethany was 15 years older than Mackenzie, and Mackenzie was 3 years younger than twice Jared's age. Now, Jared is 10 years younger than Mackenzie. How old is Bethany now?

$$36$$

$$38$$

$$42$$

$$40$$

5. Solve the system of equations using any method.  $[(-5, 3, 13)]$

$$x + y + z = 11$$

$$-2x + 2y - z = 3$$

$$2x - y + 2z = 13$$



# Systems of equations

## KEY POINTS

Variables with subscripts

Uniform motion

Solution to systems of equations

Consecutive integers

Consecutive even integers

Consecutive odd integers

Systems of three equations

## NOTES

Variables with subscripts should be treated uniquely. Ex:  $t_1$ ,  $t_2$ ,  $t_3$  could represent three time measurements read as, “time 1,” “time 2,” and “time 3.”

Distance = Rate · Time

The solution to a system of equations in two variables is an ordered pair  $(x, y)$ , or multiple ordered pairs, in which the values satisfy both equations.

Integers that are in ascending order and have no integer between them. Ex: 4 and 5.

Even integers that are in ascending order and have no even integer between them. Ex: 4 and 6.

Odd integers that are in ascending order that have no odd integer between them. Ex: 3 and 5.

Three equations with three variables where the solution consists of one value for each variable that satisfies every equation. The

# Systems of equations

## KEY POINTS

## NOTES

solution can be written as an ordered triplet  $(x, y, z)$ .

# Graphing

## worksheet



1. Circle each line that's perpendicular to  $6x - 2y = 4$ .

$$y = 6x - 2$$

$$y = -\frac{1}{3}x + 3$$

$$-3y = x - 1$$

$$y = 3x + \frac{1}{2}$$

2. What will be the value of  $k$  after converting  $y = 2x^2 - 4x + 1$  to vertex form,  $y = a(x - h)^2 + k$ ?  $[-1]$

3. What's the distance from the center in Circle A,  $(x - 3)^2 + (y + 1)^2 = 4$ , to Circle B,  $(x + 5)^2 + (y + 3)^2 = 9$ ?

$$\sqrt{5}$$

$$5$$

$$2\sqrt{15}$$

$$2\sqrt{17}$$

4. True or false? If a circle has one end of its diameter at  $(-5, 3)$ , and the other end at  $(1, 3)$ , then the equation of the circle would be  $(x + 2)^2 + (y - 3)^2 = 9$ .

[True, the length of the diameter would be 6, so the circle has a radius of 3 and the midpoint of the diameter would be  $(-2, 3)$ .]

5. Use the function  $f(x)$  to calculate each value.

$$f(x) = \begin{cases} -x + 1 & x < -4 \\ 5 & -4 < x \leq 3 \\ -2x + 11 & x > 3 \end{cases}$$

$$f(-10) = \underline{11}$$

$$f(3) = \underline{5}$$

$$f(4) = \underline{3}$$

# Graphing

## KEY POINTS

Parallel lines

Perpendicular lines

Standard form of a quadratic

quadratic  
Axis of symmetry of a

Vertex form of a quadratic

Standard form of a circle

Distance formula

## NOTES

Lines whose slopes are equal but their  $y$ -intercepts are different. The graphs of parallel lines never cross or touch. Ex:

$$\begin{cases} y = 3x + 5 \\ y = 3x - 4 \end{cases}$$

Lines whose slopes are opposite reciprocal. The graphs of perpendicular lines intersect at a right angle.

$$\text{Ex: } \begin{cases} y = 3x + 5 \\ y = -\frac{1}{3}x - 4 \end{cases}$$

$$y = ax^2 + bx + c$$

$$x = -\frac{b}{2a}$$

$$y = a(x - h)^2 + k, (h, k)$$

$(x - h)^2 + (y - k)^2 = r^2$ , where  $r$  is the radius and  $(h, k)$  is the center. Ex:

$(x - 1)^2 + (y - 2)^2 = 16$  has a center at  $(1, 2)$  and a radius of 4.

The distance between two points  $(x_1, y_1)$  and

$$(x_2, y_2) \text{ is } d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

# Graphing

## KEY POINTS

Piecewise function

Story problems and horizontal lines

Story problems and positive slope

Story problems and negative slope

## NOTES

A function that is made up of different “pieces,” each of which has its own “sub-function” and its own “sub-domain.” Ex:

$$f(x) = \begin{cases} -3 & x \leq -2 \\ \frac{5}{4}x - 2 & -2 < x < 2 \\ 2 & x \geq 2 \end{cases}$$

They represent a value that doesn’t change. Ex: As time goes on, or increases, the height of an object stays constant.

Positive slope shows a relationship between two variables is increasing. Ex: As time increases, the height of an object increases.

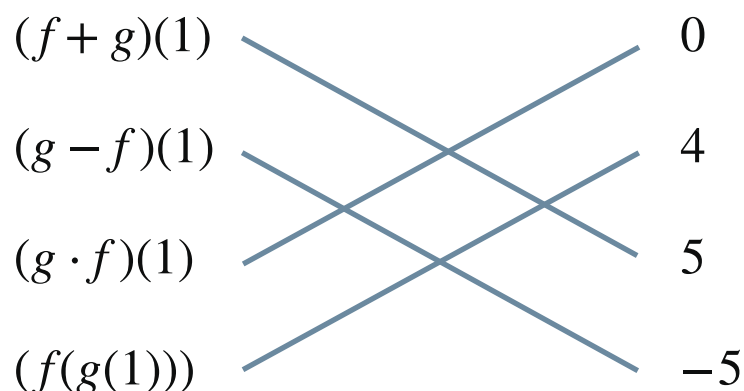
Negative slope shows that as one variable increases, the other variable decreases. Ex: As time increases, the height of an object decreases.

# Manipulating functions

worksheet



1. For  $f(x) = x^2 + 4$  and  $g(x) = \sqrt{x} - 1$ , match each function to its value.



2. Circle the statement that's always true.

$$f(g(x)) = g(f(x))$$

$$(f \cdot g)(x) = (g \cdot f)(x)$$

$$(f - g)(x) = (g - f)(x)$$

3. Fill every cell in the table with “Yes” or “No.”

	Passes the VLT	Passes the HLT	Is One-to-One
$y^2=x$	No	Yes	No
$y=x^3$	Yes	Yes	Yes
$y=x^2$	Yes	No	No

4. Is the inverse of  $f(x) = (x - 2)^3 + 4$  always  $f^{-1}(x) = \sqrt[3]{x + 2} - 4$ ? [No.]

5. The vertex of  $f(x)$  is at  $(3, 2)$  and  $f(1) = 6$ . Circle all outputs of  $f^{-1}(11)$ .

-3

0

3

6

# Manipulating functions

## KEY POINTS

Sum function

Difference function

Product function

Quotient function

Composite function

Domain of a composite function

One-to-one function

Horizontal line test

Inverse function

## NOTES

$$(f + g)(x) = f(x) + g(x)$$

$$(f - g)(x) = f(x) - g(x)$$

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

$$(f \div g)(x) = \frac{f(x)}{g(x)}$$

$(f \circ g)(x) = f(g(x))$  It means to treat the function  $g(x)$  as the variable in  $f(x)$ .

The domain will exclude all values of  $x$  where  $g(x)$  is undefined, and all values of  $x$  where  $g(x)$  is defined but  $f(g(x))$  is undefined.

A function is one-to-one, if it has exactly one  $x$ -value in its domain for each  $y$ -value in its range.

A graph passes the Horizontal Line Test if no two points of the graph have the same  $y$ -coordinate (if no horizontal line intersects the graph at more than one point).

A function that reverses or “undoes” another function. The inverse of a function  $f(x)$  is written as  $f^{-1}(x)$ .

# Exponential and logarithmic functions

## worksheet



1. If  $\log_b a = x$ , circle the equation that must be true.

$b^a = x$

$x^a = b$

$b^x = a$

$a^x = b$

2. Match each log expression on the left to its value on the right.

$\log 1,000$

$\ln e^4$

$\log_5 1$

$\log_2 32$

$\ln e$

$0$

$5$

$1$

$3$

$4$

3. Circle the inverse  $f^{-1}(x)$  of the function  $f(x) = \log_b(x + 4)$ .

$f^{-1}(x) = b^x - 4$

$f^{-1}(x) = x^b - 4$

$f^{-1}(x) = b^{x-4}$

$f^{-1}(x) = b^x + 4$

4. If  $\log_8 2 + \log_8(4x) = 1$ , circle all statements that must be true.

$\log_8(2 + 4x) = 1$

$x = 1$

$\log_8(8x) = 1$

$\log_8(4x) = \frac{2}{3}$

5. If  $\log_a b = 5$  and  $\log a = 2$ , then circle the value of  $\log b$ .

$0.2$

$3$

$7$

$10$



# Exponential and logarithmic functions

## KEY POINTS

Exponents

Logarithms

Base of a logarithm

Argument of a logarithm

Common logarithm

Natural logarithm

Product rule

Quotient rule

Power rule

## NOTES

An exponent tells us how many times to multiply the base by itself. Ex:  $2^3 = 2 \cdot 2 \cdot 2 = 8$

Tell us how many times we multiply one number by itself in order to get a different number. Logarithms (or “logs,” for short) tell us what the value of the exponent needs to be in order to make the equation true. Ex:  $\log_2(8) = 3$  means  $2^3 = 8$ .

In  $\log_b(a) = x$ ,  $b$  is the base.

In  $\log_b(a) = x$ ,  $a$  is the argument.

A logarithm that has a base of 10.

A logarithm with base of  $e$ . Ex:  $\log_e(x) = \ln(x)$

$\log_a(xy) = \log_a x + \log_a y$  and  $\ln(xy) = \ln x + \ln y$

$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$  and

$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$

$\log_a(x^n) = n \log_a x$  and  $\ln(x^n) = n \ln x$

# Exponential and logarithmic functions

## KEY POINTS

Change of base

## NOTES

$$\log_a b = \frac{\log_c b}{\log_c a} \quad \text{Ex: } \log_5 4 = \frac{\log_{10} 4}{\log_{10} 5} \approx 0.8614$$

