Homework 1

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1 Question 1

1.1 Example

In this part, we give an DGP example in the table given below (as requested with binary covariate X and treatment T).

Index	**X**	**T**	$\mathbf{Y_0}$	$\mathbf{Y_1}$	**Y**
1	0	0	2	3	2
2	0	0	1	2	1
3	0	1	2	3	3
4	1	1	4	5	5
5	1	1	3	4	4
6	1	0	3	4	3

1.2 $E[Y_t]$ and ATE calculation

$$E[Y_0] = \frac{1}{6}(2+1+2+4+3+3) = 2.5$$
$$E[Y_1] = \frac{1}{6}(3+2+3+5+4+4) = 3.5$$
$$ATE = E[Y_1 - Y_0] = 1$$

1.3 DGP exmaple with different ATE based on ignorability

Notation: $Y = T \cdot Y_1 - (1-T) \cdot Y_0$

Recap from Tutorial 2: Conditional ignorability means that the covariate X affects both the treatment assignment and the outcome - $Y(0), Y(1) \perp T \,|X$, while the full ignorability lets us assume that treatment is unaffected by X (meaning it is random): $Y(0), Y(1) \perp T$

In our example in Section 1.1, X affects both the treatment assignment and the outcome. For X = 0, we have that only $\frac{1}{3}$ of the items received treatment and overall Y values are lower in both outcomes. And for X = 1, we have that $\frac{2}{3}$ of items recived the treatment, with higher outcome values Y.

1.4 ATE estiamtion

First let's assume that full ignorability assumption holds for this example, then we have:

$$A\hat{T}E = \hat{E}[Y|T=1] - \hat{E}[Y|T=0] = \frac{1}{3}(3+5+4-2-1-3) = 2$$

As we can see it overestimates an ATE by 1 unit.
Now let's assume that only conditional ignorability holds, meaning:

$$A\hat{T}E = E[E[Y|T=1,X] - E[Y|T=0,X]]$$

For that we need to calculate:

$$\hat{E}[Y|T=1,X=1] = \frac{1}{2}(5+4) = 4.5$$

$$\hat{E}[Y|T=1,X=0] = 3$$

$$\hat{E}[Y|T=0,X=1] = 3$$

$$\hat{E}[Y|T=0,X=0] = \frac{1}{2}(2+1) = 1.5$$

In the end we have: In the end we have:

$$A\hat{T}E = \frac{1}{2}(\hat{E}[Y|T=1,X=1] + \hat{E}[Y|T=1,X=0]) - \frac{1}{2}(\hat{E}[Y|T=0,X=1] + \hat{E}[Y|T=0,X=0])$$
$$= \tfrac{1}{2}(4.5+3-3-1.5) = 1.5$$

Clearly that the ATE that was adjusted to affect of X on both Y and T was closer to real ATE value. If we had a bigger data size (with similiar correlation between the variables) the result would have been even more accurate.