

# Optimal Manpower Assignment Using a Weighted Hungarian Algorithm for Multiple Criteria

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## 1 Introduction: The Need for Multi-Criteria Optimization in Manpower Assignment

Effective manpower assignment is a critical factor in organizational success, directly influencing productivity and operational costs. While the standard Hungarian algorithm provides an efficient solution for optimal one-to-one assignments based on a single cost or benefit criterion, real-world scenarios often involve balancing multiple, potentially conflicting objectives. Factors such as worker availability, skill levels, workhours, and task complexity all play a significant role in determining the most suitable assignment. Ignoring these multiple criteria can lead to suboptimal decisions, resulting in inefficiencies and increased costs.

To address this challenge, the Hungarian algorithm can be extended to incorporate multiple criteria through methods like weighted sums. This approach allows decision-makers to assign relative importance to different factors, ensuring that the optimization process aligns with organizational priorities. This report will delve into how the Hungarian algorithm can be adapted to handle multiple criteria using a weighted approach. It will discuss the core principles of the algorithm, the mathematical formulation of multi-criteria manpower assignment problems, the process of quantifying input parameters and assigning weights, real-world applications, computational considerations, limitations, and potential alternatives.

## 2 Detailed Explanation of the Hungarian Algorithm

### 2.1 Core Principles and Steps

The Hungarian algorithm is a combinatorial optimization technique used to solve the assignment problem, aiming to find a minimum-cost (or maximum-profit) perfect matching in a bipartite graph. It operates in polynomial time, making it efficient for finding optimal assignments. The fundamental goal is to assign each worker to a unique task (or vice versa) in a way that optimizes a single objective, such as minimizing total cost or maximizing total profit.

The algorithm involves the following key steps:

1. **Cost Matrix Creation:** Construct a square cost matrix where each element  $(i, j)$  represents the cost of assigning worker  $i$  to task  $j$ . For unbalanced problems, dummy rows or columns with zero costs are added to create a square matrix.
2. **Row Reduction:** Subtract the minimum element of each row from all elements in that row.
3. **Column Reduction:** Subtract the minimum element of each column from all elements in that column.
4. **Covering Zeros:** Cover all zeros in the matrix using the minimum number of horizontal and vertical lines.
5. **Optimality Test:** If the number of covering lines equals the size of the matrix ( $n$ ), an optimal assignment is possible.
6. **Improving the Matrix:** If the number of lines is less than  $n$ , find the smallest uncovered element, subtract it from every uncovered element, and add it to every element at the intersection of two covering lines. Repeat from Step 4.

7. **Assignment:** Once the optimality test is met, select a set of  $n$  zeros such that no two are in the same row or column. Each selected zero represents an optimal assignment.

## 2.2 Mathematical Formulation (Matrix and Bipartite Graph)

The assignment problem can be mathematically represented using a cost matrix  $\mathbf{C}$  where  $C_{ij}$  is the cost of assigning task  $j$  to worker  $i$ . The objective is to find a permutation that minimizes the trace of the resulting matrix:

$$\text{Minimize}_{\mathbf{P}} \text{Tr}(\mathbf{PC}) \quad (1)$$

where  $\text{Tr}$  denotes the trace of the matrix and  $\mathbf{P}$  is a permutation matrix.

Alternatively, the problem can be modeled as a complete bipartite graph with worker vertices  $S$  and job vertices  $T$ . Each edge  $(i, j)$  has a cost  $c(i, j)$ , and the goal is to find a perfect matching with minimum total cost.

## 2.3 Variations for Minimization and Maximization

The Hungarian algorithm is typically used for minimization problems. Maximization problems can be converted by negating the cost matrix or by subtracting all elements from the maximum value in the matrix.

# 3 Extending the Hungarian Algorithm for Multiple Criteria

## 3.1 Limitations of the Standard Algorithm

The standard Hungarian algorithm is designed to optimize a single objective function. In manpower assignment, however, we often need to consider multiple factors simultaneously. For example, we might want to minimize cost while also ensuring a good match between worker skills and task complexity. The basic Hungarian algorithm does not inherently support such multi-criteria optimization.

## 3.2 Weighted Sum Approach

One common method to extend the Hungarian algorithm for multiple criteria is the **weighted sum approach**. This involves assigning a weight to each criterion, reflecting its relative importance. For manpower assignment, we can define a cost matrix where the cost of assigning worker  $i$  to task  $j$  is a weighted sum of several factors:

$$\text{Cost}_{ij} = (w_{\text{availability}} \times \text{Cost}_{\text{availability}_{ij}}) + (w_{\text{skill}} \times \text{Cost}_{\text{skill}_{ij}}) + (w_{\text{workhours}} \times \text{Cost}_{\text{workhours}_{ij}}) + (w_{\text{complexity}} \times \text{Cost}_{\text{complexity}_{ij}}) \quad (2)$$

where  $w$  represents the weight for each criterion, and  $\text{Cost}$  represents the cost associated with that criterion for the specific worker-task assignment. The weights should sum to 1 or can be used as relative importance factors.

## 3.3 Quantifying Input Parameters for Weighted Approach

To use the weighted sum approach, we need to quantify each input parameter and determine its associated cost for each worker-task pair.

- **Worker Availability:** Can be represented as a binary cost (high if unavailable, low if available) or a cost based on the degree of availability (e.g., penalty for assigning more hours than available).
- **Worker Skill Levels:** The cost can be based on the difference between the worker's skill level and the task's required skill level. A higher cost for a larger skill gap encourages better matching.
- **Workhours:** The cost can be related to the worker's wage rate and the time required for the task. Penalties can be added for exceeding a worker's available hours or preferred workload.
- **Task Complexity:** The cost can be lower for workers with higher skill levels assigned to complex tasks, reflecting efficiency. Conversely, a high cost can be assigned if a worker lacks the skills for a complex task.

### 3.4 Assigning Weights

The weights assigned to each criterion should reflect the organization's priorities. For example, if meeting deadlines is critical, the 'workhours' criterion might receive a higher weight. If quality is paramount, 'skill level' and 'task complexity' might be weighted more heavily. The weights can be determined based on expert opinions, organizational goals, or through analytical methods.

### 3.5 Other Multi-Objective Optimization Techniques

While the weighted sum method is straightforward, other multi-objective optimization techniques can also be used, such as Pareto optimization or goal programming, to find a set of non-dominated solutions that offer different trade-offs between the criteria. Fuzzy Hungarian algorithms can handle uncertainty in the parameters.

## 4 Mathematical Modeling of Manpower Assignment Problems with Weighted Criteria

### 4.1 Formulating the Objective Function and Constraints

With the weighted sum approach, the objective function for a minimization problem becomes:

$$\text{Minimize } Z = \sum_{i=1}^n \sum_{j=1}^n [(w_{\text{availability}} \times c_{\text{availability\_ij}}) + (w_{\text{skill}} \times c_{\text{skill\_ij}}) + (w_{\text{workhours}} \times c_{\text{workhours\_ij}}) + (w_{\text{complexity}} \times c_{\text{complexity\_ij}})] \times x_{ij} \quad (3)$$

where  $c$  represents the cost associated with each criterion for worker  $i$  and task  $j$ , and  $x_{ij}$  is a binary variable indicating the assignment.

The constraints remain the same:

- $\sum_{j=1}^n x_{ij} \leq 1$  for all workers  $i$  (Each worker is assigned to at most one task).
- $\sum_{i=1}^n x_{ij} = 1$  for all tasks  $j$  (Each task is assigned to exactly one worker).
- $x_{ij} \in \{0, 1\}$  for all workers  $i$  and tasks  $j$  (Binary constraint).

### 4.2 Representing Workers and Tasks with Weighted Criteria

Workers and tasks are still represented by the rows and columns of the cost matrix. However, the values within the matrix now reflect the combined weighted cost of assigning a particular worker to a specific task, considering all the defined criteria and their respective weights.

## 5 Real-World Applications and Case Studies

The weighted Hungarian algorithm can be applied in various real-world scenarios where multiple factors need to be considered in manpower assignment.

- **Project Staffing:** Assigning project team members to tasks based on their skills, availability, and the priority of the task. Weights can be adjusted based on project deadlines and critical skill requirements.
- **Healthcare Scheduling:** Allocating nurses to shifts considering their skills, preferences, and patient needs. Weights can prioritize critical care skills or minimize overtime costs.
- **Manufacturing Job Allocation:** Assigning workers to machines based on their expertise, machine availability, and production targets. Weights can balance production speed and worker skill utilization.
- **Service Industry Task Assignment:** Matching customer service agents to inquiries based on their skills, language proficiency, and the urgency of the issue. Weights can prioritize customer satisfaction or response time.

Successful implementations often lead to improved efficiency, reduced costs, and better alignment of resources with organizational goals.

## 6 Computational Complexity and Scalability of the Weighted Hungarian Algorithm

The computational complexity of the Hungarian algorithm remains  $O(n^3)$  even when using a weighted cost matrix, as the core steps of the algorithm are the same. The addition of weights only affects the calculation of the cost matrix elements. For large-scale problems, parallelization or GPU acceleration techniques can still be employed to improve performance.

## 7 Limitations and Challenges in Applying the Weighted Hungarian Algorithm

While the weighted Hungarian algorithm extends the applicability of the standard algorithm, it still has limitations.

- **Determining Appropriate Weights:** Assigning accurate and meaningful weights to different criteria can be subjective and challenging. It requires a clear understanding of organizational priorities and may involve input from various stakeholders.
- **Handling Complex Constraints:** Incorporating complex constraints beyond the basic one-to-one assignment, such as task dependencies or worker preferences, might still require additional techniques or alternative optimization methods.
- **Dynamic Environments:** The weighted approach, like the standard algorithm, might need to be re-run or adapted for dynamic environments where worker availability or task requirements change frequently.
- **Incorporating Qualitative Factors:** While weights can reflect the importance of certain qualitative aspects indirectly, directly incorporating non-quantifiable factors like worker morale can be difficult.

## 8 Potential Solutions and Alternatives to the Weighted Hungarian Algorithm

To address the limitations of the weighted Hungarian algorithm in practical manpower assignment problems, several alternatives and extensions can be considered.

- **Multi-Objective Optimization Algorithms:** Techniques like NSGA-II or MOEA/D can find a set of Pareto-optimal solutions for problems with multiple objectives without relying on a single weighted sum.
- **Goal Programming:** This approach allows setting target levels for each objective and aims to minimize the deviation from these targets.
- **Constraint Programming:** This paradigm can handle complex constraints and preferences in assignment problems.
- **Metaheuristic Algorithms:** For very large-scale problems or problems with highly complex constraints where finding an exact optimal solution is computationally infeasible, algorithms like genetic algorithms or simulated annealing can provide good approximate solutions.
- **Min-Cost Flow:** The assignment problem can be modeled as a min-cost flow problem, which offers flexibility in handling capacities and multiple assignments.

## 9 Conclusion and Recommendations for Implementing Optimal Manpower Assignment with Weighted Criteria

Extending the Hungarian algorithm with a weighted sum approach provides a valuable framework for solving manpower assignment problems involving multiple criteria. By carefully quantifying input parameters and assigning weights that reflect organizational priorities, decision-makers can achieve more effective and efficient resource allocation.

When implementing this approach, it is crucial to:

1. **Clearly define the criteria** relevant to the manpower assignment problem.
2. **Develop a robust method for quantifying** each criterion for every worker-task pair.
3. **Engage stakeholders to determine appropriate weights** for each criterion based on organizational goals.
4. **Consider the limitations** of the weighted sum approach and explore alternative multi-objective optimization techniques for complex scenarios.
5. **Regularly review and adjust weights** as organizational priorities evolve.