Constraint Satisfaction Problem – Schedule generator

Mateusz Baryła,Piotr Janus

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1 Classes description

1.1 Record

Attributes:

- teacher : a teacher object
- room_id: unique id of a room
- subject_id: unique id of a subject which the teacher can teach
- time_id: randomly chosen time slot from available timeslots for the teacher

1.2 Teacher

Attributes:

- id: id of a teacher
- subjects: list of subject which teacher can teach
- notAvailHours: list of timeslots in which teacher is not able to teach
- limit: teaching hours limit

2 Subject

Attributes:

- id: id of a subject
- name: a name of the subject
- timesPerWeek: number of times the subject should appear in a timetable

3 Timetable

Attributes:

- subjectsList: list of subjects in a timetable
- roomN: number of rooms
- records: records in the timetable
- conflicts: number of conflicts
- teacherList: list of teachers

4 PG

Attributes:

- initN: popoluation size
- iterNum: iteration number
- mutPr: probability of mutation
- crossPr: probability of crossing
- pop: population of timetables
- descendents: list of descendent, initially empty
- parents:list of parents, initially empty
- pairs: list of pairs, initially empty
- conflicts: list of conflicts in whole population, initially empty

5 Genetic algorithm

- createPop
- classify
- i $\leftarrow$ 0
- while i $<$ iterNumber do
  - selection
  - makePairs
  - crossing
  - mutation
  - classify
- return bestTimetable

5.1 createPop

Creates list of *initN* randomly generated timetables.

5.2 classify

Classifies every timetable by setting number of conflicts in it. The less conflicts are in a timetable, the better the timetable is.
Constraints and weight on conflict in brackets:

- every teacher can work only in specified hours **[1]**
- every teacher can teach only specified subjects **[1]**
- every teacher can work teach only one subject in the same time **[1]**

- every teacher has limit of teaching hours in a week **[1]**
- every room can be used only by one class **[2]**
- every subject must be included in the timetable specified number of times **[3]**

5.3 selection

Algorithm firstly adds to parents list timetables with lowest number of conflicts, the uses tournament selection, in which it randomly **draws with replacement** two timetable from population. A winner of "tournament" is added to the parents population.

5.4 makePairs

Creates $\frac{initN}{2}$ randomly chosen pairs from parents population.

5.5 crossing

Every created pair has *crossPr* chance to be crossed. Algorithm:

- pr $\leftarrow$ rand(0,100)
- if pr $<$ *crossPr*
  - choose random record from the first and the second element of a pair and swap them
  - check both timetables conflicts
    - if swapping produces more conflicts in both timetables then do not cross them
    - if swapping produces less conflicts then change the pair to a swapping product
    - else assign each element of the pair to the better timetable generated during crossing
- add each element of the pair to the population of descendents
- return descendents

5.6 mutation

Every timetable in a descendents population has *mutPr* to be mutated. Algorithm:

- pr $\leq$ rand(0,100)
- if pr $<$ *mutPr*
  - choose randomly record from the timetable
  - if record mutation produces less conflicts then change the record in a timetable to the mutated one
  - return descendents

	A	B	C	D	E	F
1		**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
2	**7:30-9:00**	teacher_id 3 room_id 4 subject_id 5 teacher_id 2 room_id 1 subject_id 4	teacher_id 10 room_id 5 subject_id 3 teacher_id 12 room_id 3 subject_id 5	teacher_id 10 room_id 4 subject_id 3	teacher_id 2 room_id 1 subject_id 4 teacher_id 13 room_id 2 subject_id 6 teacher_id 12 room_id 4 subject_id 5 teacher_id 13 room_id 1 subject_id 6	teacher_id 13 room_id 3 subject_id 6
3	**9:15-10:45**		teacher_id 1 room_id 4 subject_id 2		teacher_id 6 room_id 1 subject_id 11 teacher_id 4 room_id 2 subject_id 7	
4	**13:15-14:45**	teacher_id 11 room_id 4 subject_id 4	teacher_id 3 room_id 4 subject_id 5		teacher_id 3 room_id 5 subject_id 5 teacher_id 10 room_id 4 subject_id 3	teacher_id 2 room_id 2 subject_id 4 teacher_id 6 room_id 1 subject_id 11
5	**15:15-17:00**	teacher_id 10 room_id 1 subject_id 3 teacher_id 5 room_id 5 subject_id 9 teacher_id 5 room_id 5 subject_id 9	teacher_id 13 room_id 2 subject_id 6	teacher_id 11 room_id 3 subject_id 4 teacher_id 2 room_id 5 subject_id 4	teacher_id 3 room_id 4 subject_id 5 teacher_id 8 room_id 2 subject_id 1 teacher_id 5 room_id 2 subject_id 9	teacher_id 9 room_id 3 subject_id 2 teacher_id 4 room_id 3 subject_id 7

Figure 1: Timetable before mutation

	A	B	C	D	E	F
1		**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
2	**7:30-9:00**	teacher_id 3 room_id 4 subject_id 5 teacher_id 2 room_id 1 subject_id 4	teacher_id 10 room_id 5 subject_id 3 teacher_id 12 room_id 3 subject_id 5	teacher_id 10 room_id 4 subject_id 3	teacher_id 2 room_id 1 subject_id 4 teacher_id 13 room_id 2 subject_id 6 teacher_id 12 room_id 4 subject_id 5 teacher_id 13 room_id 1 subject_id 6	teacher_id 13 room_id 3 subject_id 6
3	**9:15-10:45**		teacher_id 1 room_id 4 subject_id 2		teacher_id 6 room_id 1 subject_id 11 teacher_id 4 room_id 2 subject_id 7	
4	**13:15-14:45**	teacher_id 11 room_id 4 subject_id 4	teacher_id 3 room_id 4 subject_id 5		teacher_id 3 room_id 5 subject_id 5 teacher_id 10 room_id 4 subject_id 3	teacher_id 2 room_id 2 subject_id 4 teacher_id 6 room_id 1 subject_id 11
5	**15:15-17:00**	teacher_id 10 room_id 1 subject_id 3 teacher_id 5 room_id 5 subject_id 9 teacher_id 5 room_id 5 subject_id 9	teacher_id 13 room_id 2 subject_id 6	teacher_id 11 room_id 3 subject_id 4 teacher_id 2 room_id 5 subject_id 4	teacher_id 3 room_id 4 subject_id 5 teacher_id 8 room_id 2 subject_id 1 teacher_id 1 room_id 1 subject_id 2	teacher_id 9 room_id 3 subject_id 2 teacher_id 4 room_id 3 subject_id 7

Figure 2: Timetable after mutation

	A	B	C	D	E	F
1		**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
2	**7:30-9:00**	teacher_id 6 room_id 1 subject_id 12	teacher_id 3 room_id 4 subject_id 6 teacher_id 11 room_id 3 subject_id 4		teacher_id 11 room_id 4 subject_id 4	teacher_id 6 room_id 3 subject_id 11
3	**9:15-10:45**		teacher_id 11 room_id 4 subject_id 4	teacher_id 4 room_id 1 subject_id 8 teacher_id 1 room_id 2 subject_id 1 teacher_id 5 room_id 4 subject_id 9 teacher_id 10 room_id 3 subject_id 3 teacher_id 4 room_id 4 subject_id 8 teacher_id 3 room_id 3 subject_id 5		teacher_id 3 room_id 3 subject_id 6 teacher_id 11 room_id 4 subject_id 4
4	**13:15-14:45**		teacher_id 4 room_id 1 subject_id 7 teacher_id 11 room_id 4 subject_id 4	teacher_id 11 room_id 1 subject_id 4 teacher_id 7 room_id 4 subject_id 13	teacher_id 3 room_id 3 subject_id 5	teacher_id 4 room_id 3 subject_id 8 teacher_id 7 room_id 2 subject_id 13 teacher_id 1 room_id 5 subject_id 2
5	**15:15-17:00**		teacher_id 5 room_id 4 subject_id 9 teacher_id 4 room_id 1 subject_id 7 teacher_id 7 room_id 1 subject_id 13	teacher_id 2 room_id 3 subject_id 4 teacher_id 5 room_id 4 subject_id 10	teacher_id 10 room_id 1 subject_id 3 teacher_id 3 room_id 4 subject_id 6	teacher_id 13 room_id 1 subject_id 6

	A	B	C	D	E	F
1		**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
2	**7:30-9:00**	teacher_id 3 room_id 4 subject_id 5 teacher_id 2 room_id 1 subject_id 4	teacher_id 10 room_id 5 subject_id 3 teacher_id 12 room_id 3 subject_id 5	teacher_id 10 room_id 4 subject_id 3	teacher_id 2 room_id 1 subject_id 4 teacher_id 13 room_id 2 subject_id 6 teacher_id 12 room_id 4 subject_id 5 teacher_id 13 room_id 1 subject_id 6	teacher_id 13 room_id 3 subject_id 6
3	**9:15-10:45**		teacher_id 1 room_id 4 subject_id 2		teacher_id 6 room_id 1 subject_id 11 teacher_id 4 room_id 2 subject_id 7	
4	**13:15-14:45**	teacher_id 11 room_id 4 subject_id 4	teacher_id 3 room_id 4 subject_id 5		teacher_id 3 room_id 5 subject_id 5 teacher_id 10 room_id 4 subject_id 3	teacher_id 2 room_id 2 subject_id 4 teacher_id 6 room_id 1 subject_id 11
5	**15:15-17:00**	teacher_id 10 room_id 1 subject_id 3 teacher_id 5 room_id 5 subject_id 9 teacher_id 5 room_id 5 subject_id 9	teacher_id 13 room_id 2 subject_id 6	teacher_id 11 room_id 3 subject_id 4 teacher_id 2 room_id 5 subject_id 4	teacher_id 3 room_id 4 subject_id 5 teacher_id 8 room_id 2 subject_id 1 teacher_id 5 room_id 2 subject_id 9	teacher_id 9 room_id 3 subject_id 2 teacher_id 4 room_id 3 subject_id 7

Figure 3: Two timetables chosen to crossing. Two randomly chosen records from both timetables are red marked.

	A	B	C	D	E	F
1		**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
2	**7:30-9:00**	teacher_id 6 room_id 1 subject_id 12	teacher_id 3 room_id 4 subject_id 6 teacher_id 11 room_id 3 subject_id 4		teacher_id 11 room_id 4 subject_id 4	teacher_id 6 room_id 3 subject_id 11
3	**9:15-10:45**		teacher_id 11 room_id 4 subject_id 4	teacher_id 4 room_id 1 subject_id 8 teacher_id 1 room_id 2 subject_id 1 teacher_id 5 room_id 4 subject_id 9 teacher_id 10 room_id 3 subject_id 3 teacher_id 4 room_id 4 subject_id 8 teacher_id 3 room_id 3 subject_id 5		teacher_id 3 room_id 3 subject_id 6 teacher_id 11 room_id 4 subject_id 4
4	**13:15-14:45**		teacher_id 4 room_id 1 subject_id 7 teacher_id 11 room_id 4 subject_id 4	teacher_id 11 room_id 1 subject_id 4 teacher_id 7 room_id 4 subject_id 13	teacher_id 3 room_id 3 subject_id 5	teacher_id 4 room_id 3 subject_id 8 teacher_id 7 room_id 2 subject_id 13 teacher_id 1 room_id 5 subject_id 2
5	**15:15-17:00**		teacher_id 5 room_id 4 subject_id 9 teacher_id 4 room_id 1 subject_id 7 teacher_id 9 room_id 3 subject_id 2	teacher_id 2 room_id 3 subject_id 4 teacher_id 5 room_id 4 subject_id 10	teacher_id 10 room_id 1 subject_id 3 teacher_id 3 room_id 4 subject_id 6	teacher_id 13 room_id 1 subject_id 6

	A	B	C	D	E	F
1		**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
2	**7:30-9:00**	teacher_id 3 room_id 4 subject_id 5 teacher_id 2 room_id 1 subject_id 4	teacher_id 10 room_id 5 subject_id 3 teacher_id 12 room_id 3 subject_id 5	teacher_id 10 room_id 4 subject_id 3	teacher_id 2 room_id 1 subject_id 4 teacher_id 13 room_id 2 subject_id 6 teacher_id 12 room_id 4 subject_id 5 teacher_id 13 room_id 1 subject_id 6	teacher_id 13 room_id 3 subject_id 6
3	**9:15-10:45**		teacher_id 1 room_id 4 subject_id 2		teacher_id 6 room_id 1 subject_id 11 teacher_id 4 room_id 2 subject_id 7	
4	**13:15-14:45**	teacher_id 11 room_id 4 subject_id 4	teacher_id 3 room_id 4 subject_id 5		teacher_id 3 room_id 5 subject_id 5 teacher_id 10 room_id 4 subject_id 3	teacher_id 2 room_id 2 subject_id 4 teacher_id 6 room_id 1 subject_id 11
5	**15:15-17:00**	teacher_id 10 room_id 1 subject_id 3 teacher_id 5 room_id 5 subject_id 9 teacher_id 5 room_id 5 subject_id 9	teacher_id 13 room_id 2 subject_id 6	teacher_id 11 room_id 3 subject_id 4 teacher_id 2 room_id 5 subject_id 4	teacher_id 3 room_id 4 subject_id 5 teacher_id 8 room_id 2 subject_id 1 teacher_id 1 room_id 1 subject_id 2	teacher_id 7 room_id 1 subject_id 13 teacher_id 4 room_id 3 subject_id 7

Figure 4: Two timetables after crossing with swapped records.

	A	B	C
1	ID	Name	Times per week
2	1	AI	2
3	2	Database	2
4	3	Advanced Numerical Analysis	1
5	4	Risk Managment	1
6	5	Calculus	2
7	6	Algebra	3
8	7	Programming	2
9	8	Database2	4
10	9	Data Structures	4
11	10	Probability	3
12	11	Partial differental eq	2
13	12	IT	2
14	13	Physics	2

Figure 5: Initial data – subjects.

	A	B	C	D
1	Teacher id	Subjects	Not available during timeslots	Limit
2	1	1;2	1;2;10	4
3	2	3;4	12;13;14	2
4	3	5;6	15;20	5
5	4	7;8	18;19	6
6	5	9;10	15;2;4;5	7
7	6	11;12	8;7	4
8	7	13	1;2	2
9	8	1	3;1	1
10	9	2	11;2	1
11	10	3	10;4	1
12	11	4	5;9	1
13	12	5	6;12	1
14	13	6	7;18	1

Figure 6: Initial data – teachers.

	A	B	C	D	E	F	G
1		**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**	
2	**7:30-9:00**	teacher_id 6 room_id 1 subject_id 12	teacher_id 3 room_id 4 subject_id 6 teacher_id 11 room_id 3 subject_id 4		teacher_id 11 room_id 4 subject_id 4	teacher_id 6 room_id 3 subject_id 11	
3	**9:15-10:45**		teacher_id 11 room_id 4 subject_id 4	teacher_id 4 room_id 1 subject_id 8 teacher_id 1 room_id 2 subject_id 1 teacher_id 5 room_id 4 subject_id 9 teacher_id 10 room_id 3 subject_id 3 teacher_id 4 room_id 4 subject_id 8 teacher_id 3 room_id 3 subject_id 5		teacher_id 3 room_id 3 subject_id 6 teacher_id 11 room_id 4 subject_id 4	timesPerWeek conflict room conflict teacher conflict
4	**13:15-14:45**		teacher_id 4 room_id 1 subject_id 7 teacher_id 11 room_id 4 subject_id 4	teacher_id 11 room_id 1 subject_id 4 teacher_id 7 room_id 4 subject_id 13	teacher_id 3 room_id 3 subject_id 5	teacher_id 4 room_id 3 subject_id 8 teacher_id 7 room_id 2 subject_id 13 teacher_id 1 room_id 5 subject_id 2	
5	**15:15-17:00**		teacher_id 5 room_id 4 subject_id 9 teacher_id 4 room_id 1 subject_id 7 teacher_id 7 room_id 1 subject_id 13	teacher_id 2 room_id 3 subject_id 4 teacher_id 5 room_id 4 subject_id 10	teacher_id 10 room_id 1 subject_id 3 teacher_id 3 room_id 4 subject_id 6	teacher_id 13 room_id 1 subject_id 6	

Figure 7: First generated timetable. Total number of conflicts - 68

	A	B	C	D	E	F
1		**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
2	**7:30-9:00**	teacher_id 12 room_id 3 subject_id 5	teacher_id 4 room_id 4 subject_id 7 teacher_id 11 room_id 2 subject_id 4	teacher_id 6 room_id 3 subject_id 12	teacher_id 7 room_id 2 subject_id 13	teacher_id 6 room_id 3 subject_id 11
3	**9:15-10:45**	teacher_id 4 room_id 3 subject_id 8	teacher_id 5 room_id 1 subject_id 10	teacher_id 5 room_id 5 subject_id 9	teacher_id 13 room_id 3 subject_id 6	teacher_id 3 room_id 4 subject_id 6 teacher_id 7 room_id 1 subject_id 13
4	**13:15-14:45**	teacher_id 4 room_id 3 subject_id 8 teacher_id 5 room_id 1 subject_id 10	teacher_id 1 room_id 4 subject_id 1	teacher_id 5 room_id 5 subject_id 9	teacher_id 9 room_id 1 subject_id 2 teacher_id 4 room_id 3 subject_id 7	teacher_id 4 room_id 4 subject_id 8
5	**15:15-17:00**	teacher_id 6 room_id 4 subject_id 12 teacher_id 10 room_id 3 subject_id 3 teacher_id 1 room_id 1 subject_id 2	teacher_id 5 room_id 5 subject_id 10 teacher_id 3 room_id 2 subject_id 5	teacher_id 5 room_id 1 subject_id 9 teacher_id 3 room_id 4 subject_id 6	teacher_id 5 room_id 4 subject_id 9	teacher_id 8 room_id 4 subject_id 1 teacher_id 4 room_id 3 subject_id 8 teacher_id 6 room_id 5 subject_id 11

Figure 8: Best generated timetable, 0 conflicts.

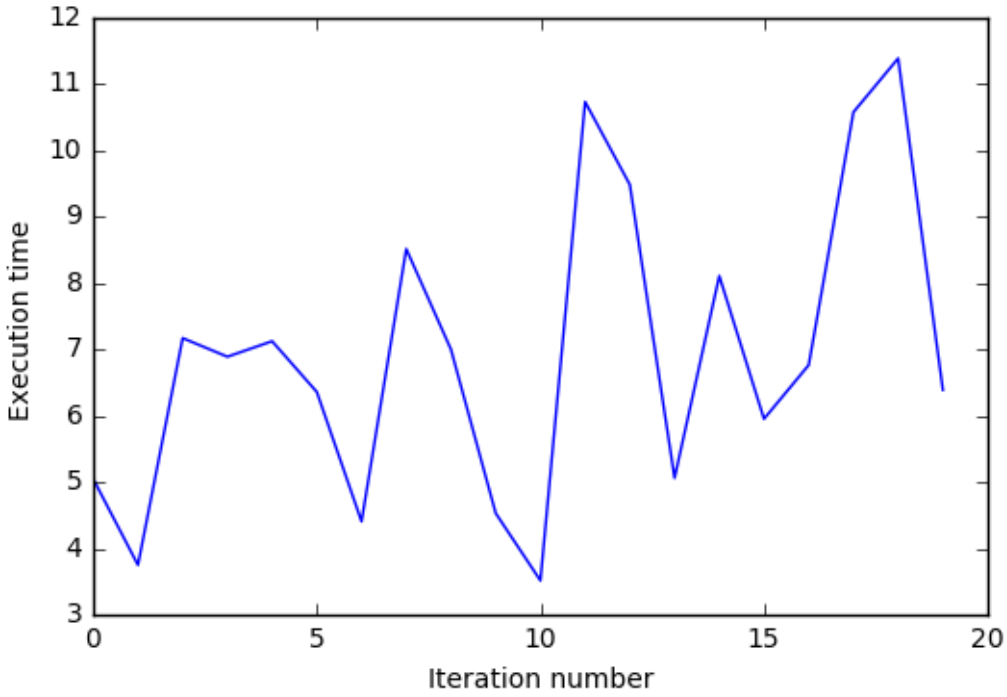

Figure 9: Population 40, 56 iterations, 50 % mutation probability, 50 % crossing probability, estimated execution time 4.5 seconds

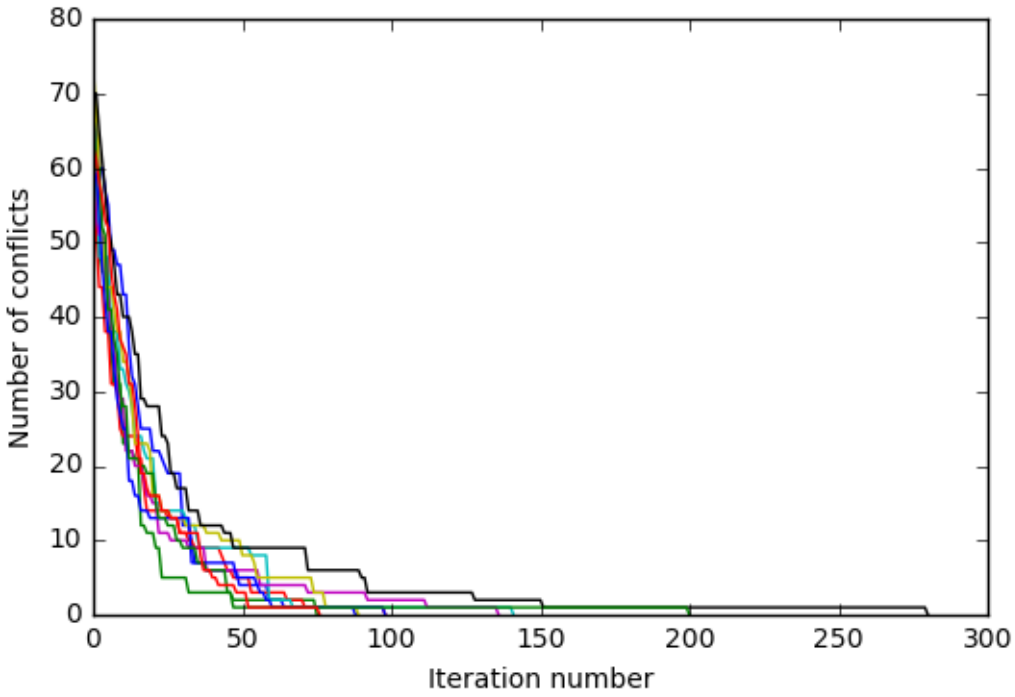

Figure 10: Population 40, max 500 iterations, 50 % mutation probability, 50 % crossing probability, estimated execution time 7.5 seconds per timetable

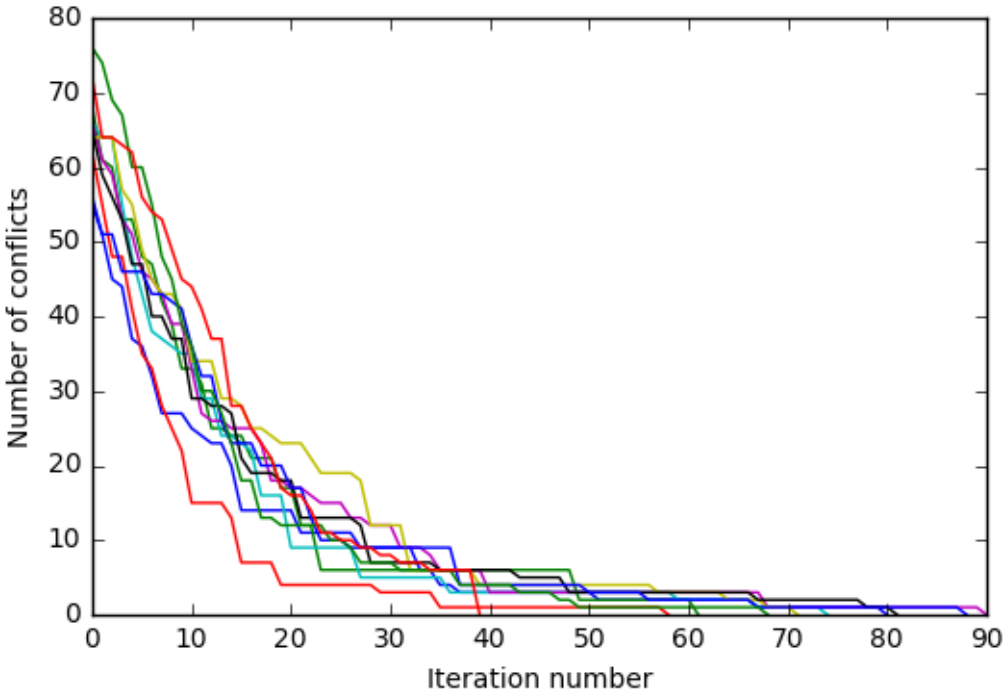

Figure 11: Population 40, max 500 iterations, 100 % mutation probability, 0 % crossing probability, estimated execution time 5.89 seconds per timetable

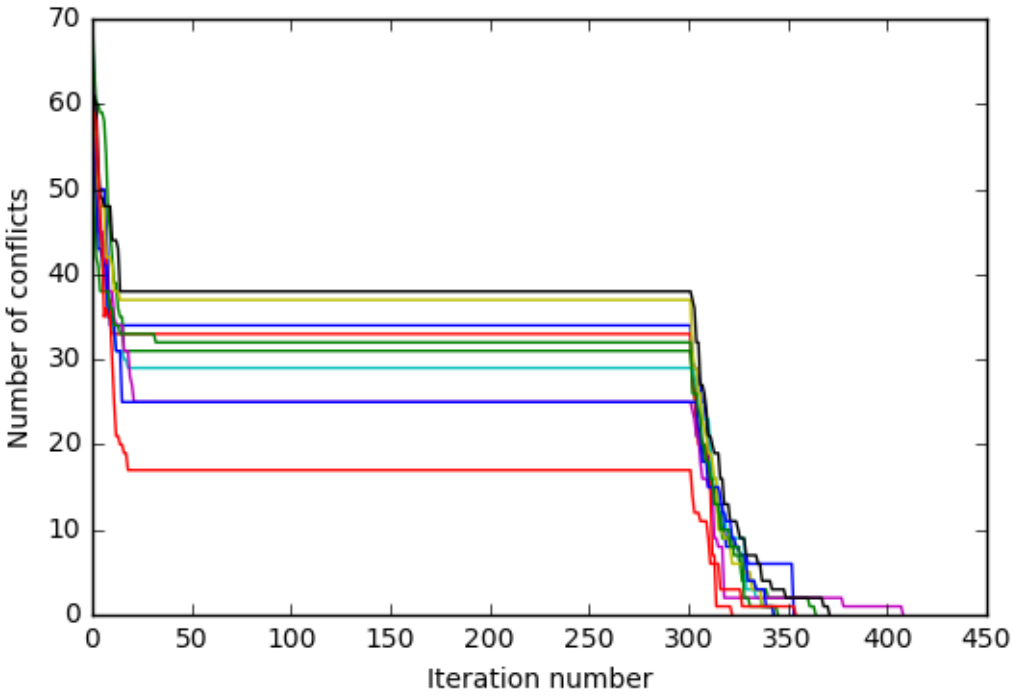

Figure 12: Population 40, max 500 iterations, 0 % mutation probability, 100 % crossing probability, estimated execution time 20.72 seconds per timetable

As we can see crossing struggle with stuck in local minimum problem. We decided to change $mutPr = 100$ $crossPr = 0$ after 60 % of iteration. As we can see in figure 12 after 300 iteration the change in reducing conflicts is noticeable.

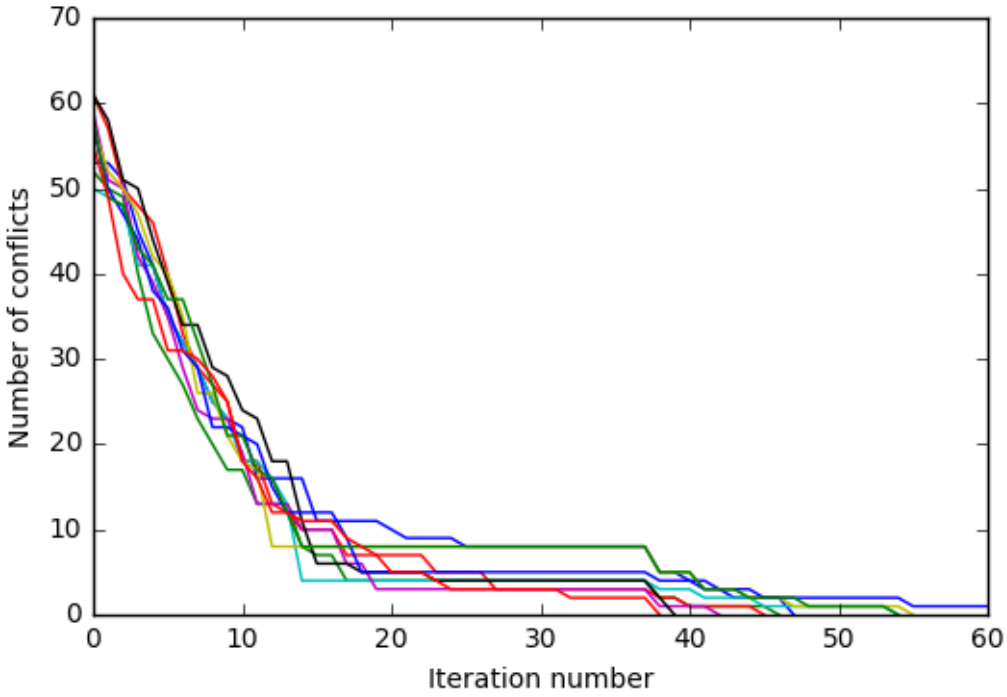

Figure 13: Population 300, max 60 iterations, 0 % mutation probability, 100 % crossing probability, estimated execution time 25.42 seconds per timetable

After increasing size of population algorithm performs much better. Of course changing *mutPr* and *crossPr* also helps a lot.

We have estimated best parameters for our genetic algorithm taking average time of 20 executions, with *mutPr = 100 %* and *crossPr = 15 %*.

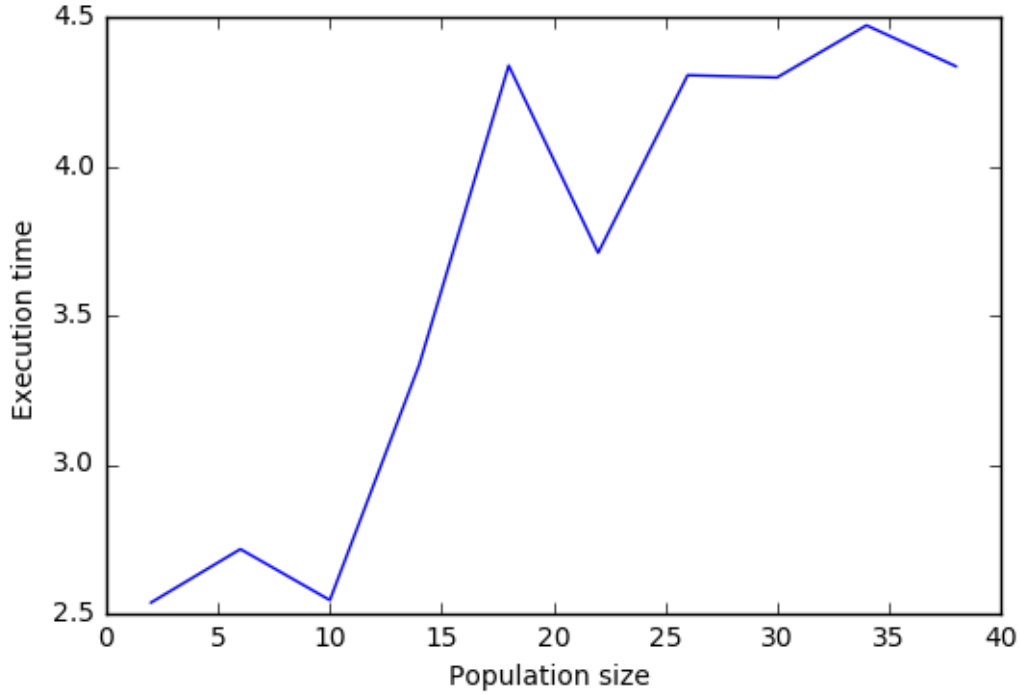

Figure 14: Execution time increases with increasing population.

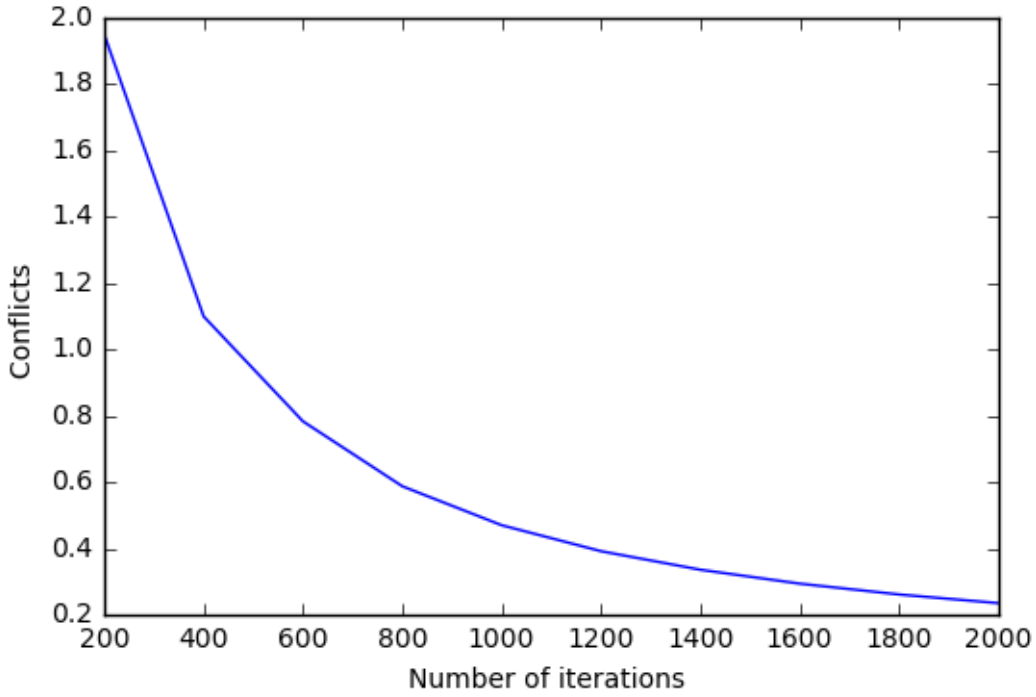

Figure 15: Average conflicts number decreases with increasing number of iterations

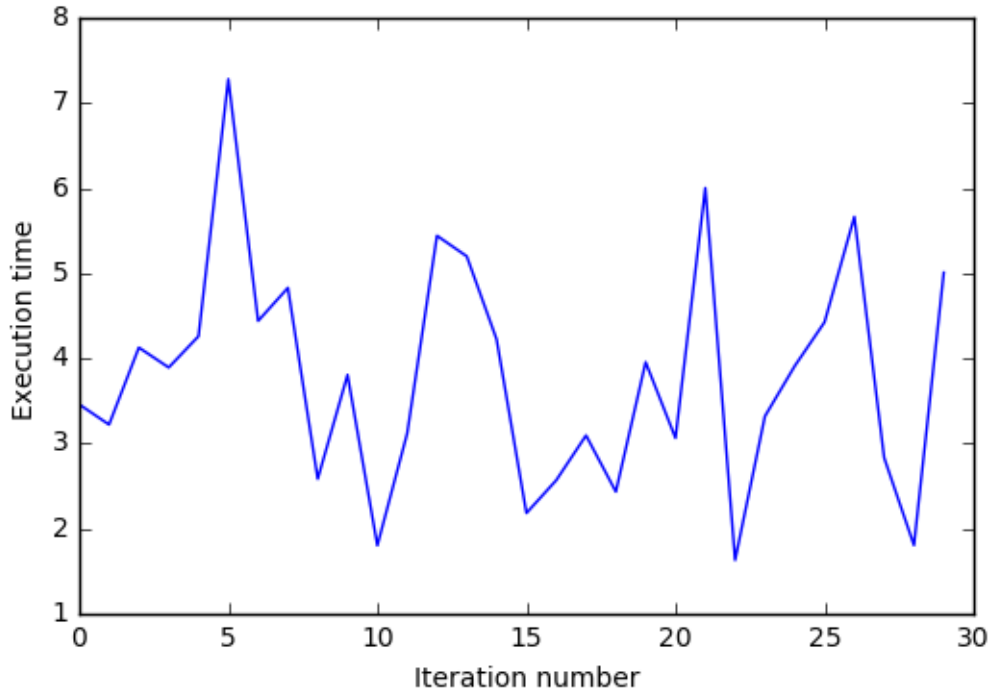

Figure 16: Average time: 3.78 seconds. Population: 8, 100 % mutation probability, 15 % crossing probability

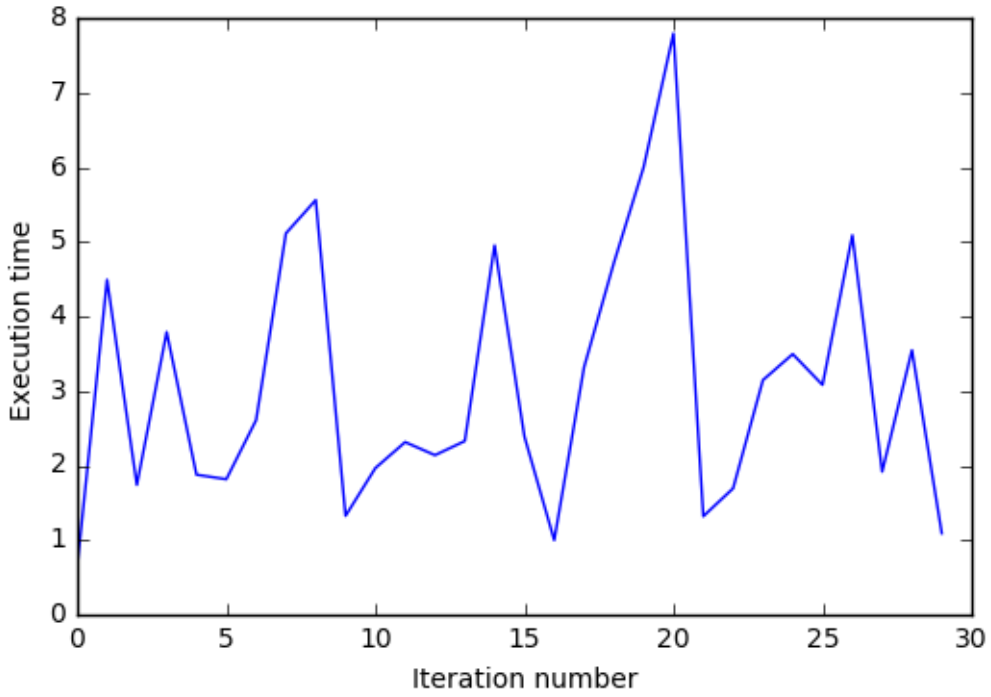

Figure 17: Average time: 3.07 seconds. Population: 8, 100 % mutation probability, 0 % crossing probability

Now estimated execution time is equal to 3.07 seconds so by choosing best parameters the time decreased to 45% of its initial (6.90 seconds) time!

6 Combining genetic algorithm and beam search

When we take a data set which leaves no room for many variations, like every teacher is able to teach at most two subjects out of all thirteen and what is more we only have two rooms to teach in, then it becomes very difficult to solve the problem without conflicts. Such timetable is shown on Figure 18 and an example data set below.

	Monday	Tuesday	Wednesday	Thursday	Friday
7:30-9:00	teacher_id 5 room_id 1 subject_id 9 teacher_id 4 room_id 2 subject_id 8	teacher_id 3 room_id 1 subject_id 6 teacher_id 6 room_id 2 subject_id 12	teacher_id 3 room_id 1 subject_id 5 teacher_id 5 room_id 2 subject_id 9	teacher_id 2 room_id 1 subject_id 4 teacher_id 6 room_id 2 subject_id 11	teacher_id 4 room_id 2 subject_id 8 teacher_id 1 room_id 1 subject_id 1
9:15-10:45	teacher_id 8 room_id 2 subject_id 1 teacher_id 7 room_id 1 subject_id 13	teacher_id 1 room_id 2 subject_id 2 teacher_id 4 room_id 1 subject_id 8	teacher_id 4 room_id 2 subject_id 7 teacher_id 1 room_id 1 subject_id 2	teacher_id 2 room_id 1 subject_id 3 teacher_id 7 room_id 2 subject_id 13	teacher_id 2 room_id 1 subject_id 3 teacher_id 6 room_id 2 subject_id 11
13:15-14:45	teacher_id 4 room_id 1 subject_id 8 teacher_id 6 room_id 2 subject_id 11	teacher_id 3 room_id 2 subject_id 6 teacher_id 8 room_id 1 subject_id 1	teacher_id 6 room_id 1 subject_id 12 teacher_id 9 room_id 2 subject_id 2	teacher_id 5 room_id 2 subject_id 10 teacher_id 3 room_id 1 subject_id 6	teacher_id 4 room_id 1 subject_id 7 teacher_id 6 room_id 2 subject_id 12
15:15-17:00	teacher_id 7 room_id 2 subject_id 13 teacher_id 5 room_id 1 subject_id 9	teacher_id 2 room_id 2 subject_id 3 teacher_id 5 room_id 1 subject_id 10	teacher_id 11 room_id 1 subject_id 4 teacher_id 5 room_id 2 subject_id 9	teacher_id 2 room_id 2 subject_id 4 teacher_id 5 room_id 1 subject_id 10	teacher_id 6 room_id 1 subject_id 11 teacher_id 12 room_id 2 subject_id 5

Figure 18: Timetable with 40 classes during one week and only 2 available rooms, created with 0 conflicts

Although it is possible, our algorithm needs at least 2000 iterations, what takes in the best case $\sim$ 30 seconds. That brings up a question if we are able to manage such situation to evaluate it faster. We tried to combine beam search with k best descendants.

6.1 Beam search algorithm

After some fixed x iterations without any progress (decreasing number of conflicts) in generic algorithm we execute our **beamSearch(*pop*,*k*)** function.

Teacher id	Subjects	Not available during timeslots	Limit
1	1;2	1;2;10	7
2	3;4	12;13;14	7
3	5;6	15;20	6
4	7;8	18;19	7
5	9;10	15;2;4;5	8
6	11;12	8;7	8
7	13	1;2	4
8	1	3;1	2
9	2	11;2	1
10	3	10;4	2
11	4	5;9	2
12	5	6;12	1
13	6	7;18	2

ID	Name	Times per week
1	AI	3
2	Database	3
3	Advanced Numerical Analysis	3
4	Risk Managment	3
5	Calculus	2
6	Algebra	3
7	Programming	2
8	Database2	4
9	Data Structures	4
10	Probability	3
11	Partial differental eq	4
12	IT	3
13	Physics	3

- Take $k = 2$ best timetables from *pop*
- For each timetable:
  - Create $len(timetable.records)$ new timetables by replacing only one record in every new timetable
  - Count conflicts for every timetable
- Choose best k timetables
- Come back to point no. 2 until algorithm finds better timetables that the best out of k timetables initially given.

Unfortunately within 2000 iterations algorithm with beam search required $\sim$ 35 sec with average 2.1 conflicts (4 times without any conflicts), while within the same period of time the algorithm with beam search turned off, managed to archive only 1.6 conflicts on average (3 times without conflicts).