Summary of MSc

Resummation of enhanced logarithms in QCD is a broad topic. In particular, threshold resummation refers to the all-order resummation of logarithms enhanced in the threshold limit, that is to say when the center-of-mass energy of the process is just enough for the process to occur. Threshold resummation typically relies on renormalization group equations. Identifying the soft scale on which the observables depend, enables writing renormalization group equations whose solutions actually perform resummation of logarithms in Mellin space. All logarithmic-order threshold resummation formulae for partonic total cross sections are the easiest to derive since they are one-scale dependent. The topic is revised in [1], where, considering the kinematically archetypical processes of Deep Inelastic Scattering and Drell-Yan production the following two soft scales are derived from the phase space:

- $\Lambda^2(Q^2, x) = Q^2(1-x)$ for DIS;
- $\Lambda^2(Q^2, x) = Q^2(1-x)^2$ for DY.

Q^2 is the hard scale of the process and x is the threshold variable ($x \to 1$ in the threshold limit). From distributional identities arising from factors $[Q^2(1-x)^a]^{-1+\epsilon}$ appearing in dimensional regularization, plus distributions of the soft scale appear in the final regularized result. These are the logarithms that are resummed in Mellin space.

Now, considering differential cross sections, more scales may arise. Consider the Higgs boson production process. The first differential observables that are usually computed are the transverse momentum distribution ($d\sigma/dp_T^2$) and the rapidity distribution ($d\sigma/dy$), where y is the longitudinal rapidity of the Higgs boson. Threshold resummation of the p_T-distribution up to any logarithmic accuracy has been achieved in 2021 [2]. In an unpublished thesis [3] threshold resummation of the rapidity distribution has been performed by identifying two soft scales (called here *soft* and *collinear*):

- $\Lambda^2_{coll} = m_H^2(1-x_1)(1-x_2)$
- $\Lambda^2_{soft} = m_H^2(1-x_1)^2$.

Using a technique analogous to the one adopted in the case of the transverse momentum distribution, the two scales were identified by decomposing the phase space into three subspaces. The first phase space singles out extra emissions that are soft in the threshold limit, the second the Higgs boson emission, and the third singles out emissions that are collinear in the threshold limit. The integrals of the first and the last phase space can be rewritten in dimensional regularization to make the dimensional scales on which they depend explicit. This gives rise to the two scales above.

This is the first analytical threshold resummation formula for the rapidity distribution. Other results in the literature (such as [4]) are limited to the resummation of logarithmic contributions enhanced only in a stronger threshold limit, that is when the Higgs is produced at rest (that is when both x_1 and x_2 goes to 1, while here we consider x_2 fixed). However, another numerical result exists obtained in Soft-Collinear-Effective Theory (SCET) [5] that also performs resummation in the weaker threshold limit. This motivates my thesis project: verifying the obtained resummation formula by identifying in a fixed-order result (particularly NNLO) the scales predicted by the 2022 argument. I have done this in my thesis by first rewriting the long NNLO result contained in [6] and identifying the two soft scales by rewriting them in terms of variables more relevant to amplitude calculations. Finally, I also have verified that the origin of those logarithms is exactly that predicted by the phase space argument by retracting their origin in the calculation starting from the color-ordered amplitudes in [7].

What I have been doing since I completed my thesis project is checking whether the SCET result is compatible with the analytical result through perturbative solutions of the SCET kernel that, computed numerically, performs resummation. If the formula passes this check, it can be published.

References

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