New Theory Frontiers 2024-Feb-14

Had a conversation with Jiayu in the morning, in which we discussed that we would not be working on things for a couple weeks, which gives me some time to think and retread on some of the problems I'm interested in solving. What I need to do is thoroughly go through some basic algorithmic and probabilistic problem-solving techniques from a mathematical standpoint. I've found a few resources towards this goal:

- The book 'Extremal Combinatorics.' I will try to go through the first section of this book with good writing.
- Ryan O'Donnell's Probability and Computing lecture notes from 2009, which are probably worth going through as well.
- Salil Vadhan's course 'A Theorists Toolkit' from 2005, which consists of 12 lectures. Will probably go through them at some point.
- Finally, Babai's notes on 'Linear Algebra methods in Combinatorics.' Will refer to them if I ever need them.

Anyway, I then read chapter 1 of the book on some basic counting. Did the binomial theorem and some definitions of factorials. This is slightly interesting:

Lemma. $\sum_{i=0}^{k}\binom{n}{i} \leq \left(\frac{en}{k}\right)^{k}$.

Proof uses the identity $1+t<e^t$. Factorial approximations may be useful; the stirling formula is supposedly important.

$$n! = \left(\frac{n}{e}\right)^{n}\sqrt{2\pi n}e^{\alpha_n}$$

where $1/(12n+1) < \alpha_n < 1/12n$.

Did the stars and bars counting method and some stuff related to partitions.

Turan's number is quite interesting here, because I remember it being defined only in the context of graphs. An intuitive explanation for this is as follows: the number $T(n,k,l)$ with $(l \leq k \leq n)$ is the smallest number of l-element subsets of an n element set X in a way such that every k-element subset of X contains at least one of those sets. Think of it this way: we have some set X of size n. Then we can divide it into l-element subsets. Suppose we pick a k-element subset; assuming that we have enough l-element subsets, k will necessarily contain one of them. Clearly, if l is simply $\binom{n}{l}$, this is true, but we can choose less of them as well. The smallest number we can choose is $T(l,k,n)$.