The Hunting of the zk-SNARK

CMI Student Seminars

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Outline

Introduction

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- This talk is about what these words mean

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- Poem 'The Hunting of the SNARK' published by Lewis Carroll in 1876
- Also the title of a paper by Goldwasser et. al. in 2014

Zero-Knowledge

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- Can do this using a zero-knowledge proof (ZKP)

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- We also want **protection against a dishonest Alice**

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Soundness

Alice can only convince Bob of the truth of the statement if it is in fact true.

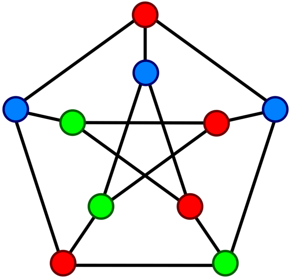
What is a Zero-Knowledge Proof?

Zero-Knowledge

Bob does not know anything other than the truth of the statement.

Formal example: He does not learn the witness or any information about it whatsoever.

A first Zero-Knowledge Proof



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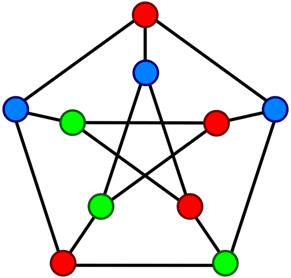

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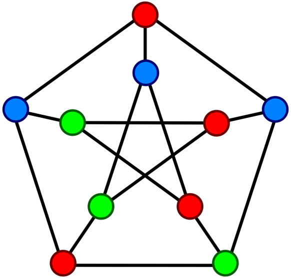

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- This problem is NP-hard
- We will show a zero-knowledge proof for it

The Setup

- Alice and Bob agree on a graph $G = (V, E)$ beforehand

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- Bob does not know whether G is 3-colorable
- Alice can figure it out with her unlimited computational power
- If $G \in 3Color$, Alice can cook up witnesses

The Algorithm

- Alice chooses a witness for the graph (this witness may or may not be correct) and colors it.
- Bob asks Alice for some edge e.
- Alice sends e along with the node colorings to Bob.
- If the colors are different, Bob accepts, otherwise he rejects.
- Alice permutes the colors of the graph, and they repeat the above until Bob either rejects or is ‘satisfied’.

Analysis - Completeness

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The proof is sound.

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The proof is zero-knowledge.

Natural Question

Most people after hearing this protocol ask the question:

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- A third party is not required, however. It can be done without it.

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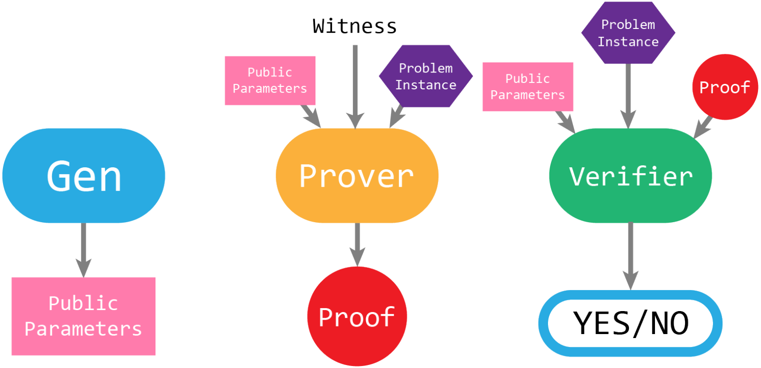
- Alice sends Bob only **one** message
- Can he be ‘satisfied’?

Introducing the NARG: Non-interactive Argument



A non-interactive argument.

Non-Interactive Arguments

A Non-Interactive argument is a tuple of 3 algorithms: (Gen, Prove, Verify).

- *Gen*$(1^\kappa) \to (\sigma, \tau)$: Takes input a *security parameter* κ which contains information about how 'satisfied' Bob needs to be, and generates two reference strings, σ and τ which are given to Alice and Bob respectively.

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- $\mathit{Prove}(\sigma, x, w) \to \pi$: Takes the reference string, a problem instance x, a witness w, and produces a proof π.
- $\mathit{Verify}(\tau, \pi, x) \to b$: Takes the reference string, the proof and the problem instance, and outputs a bit b corresponding to a success or a failure respectively.

What conditions do NARGs have?

Like ZKPs, NARGs also have some properties attached with them.

Completeness

If w is indeed a witness for x, then Bob will always be convinced.

This is standard.

What conditions do NARGs have?

Here's where things get fun - we now start thinking like cryptographers, and consider an *adversarial* model of soundness.

Soundness (Non-Adaptive)

Let our language be L and consider an adversary $\mathcal{A}$ to which we give $x \notin L$ and σ. Then $\mathcal{A}$ cannot produce a proof π which will be verified by Bob.

What conditions do NARGs have?

Soundness (Adaptive)

Let our language be L and consider an adversary $\mathcal{A}$ to which we give and σ. Then $\mathcal{A}$ cannot produce a proof π for some $x \notin L$ of his choice which will be verified by Bob.

These notions are important: they turn up again later.

Knowledge Soundness

Finally, we can add a final condition which turns our NARG into a NARK - A non-interactive argument **of knowledge**.

Knowledge Soundness

Suppose Alice produces a proof *pi* for some statement x. Then there exists an *extractor* $\mathcal{E}$ with equal power as Alice which can produce a witness w for x.
In other words, Alice can only produce a proof if she actually possesses a witness - regardless of whether $x \in L$.

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We have now defined our primitive, the **zk-NARK**. To reiterate:

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- Bob does not learn anything other than the obvious.

Succinctness

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- These proofs were first developed in the 1980s
- However, they have skyrocketed in popularity in the last decade
- The modern development of ZKPs and NARKs began after work by Alessandro Chiesa, professor at UC Berkeley
- These proofs carry an additional, ground-breaking property

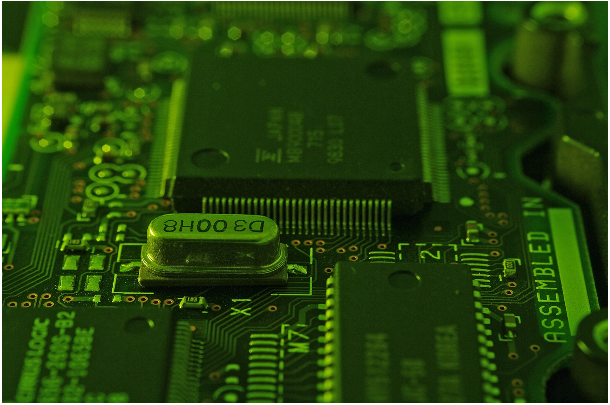
Succinctness

The proof is *short* and finding it is *efficient*. In particular, the size of the proof is usually a parameter like

$$\mathrm{poly}(|x| + |\kappa|)$$

where x is the size of the problem instance and κ is the security parameter. Furthermore, the algorithms are efficient, and work in time polynomial in κ.

What does this mean in practice?



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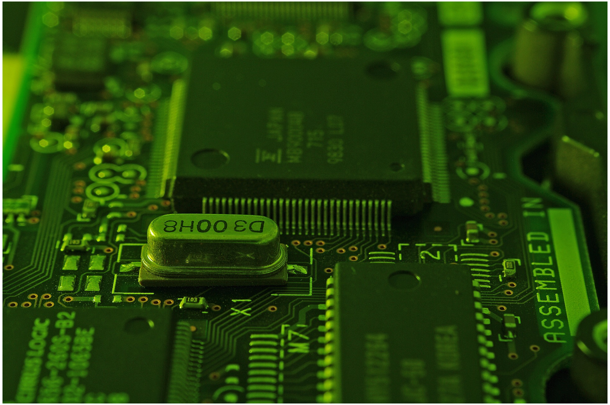

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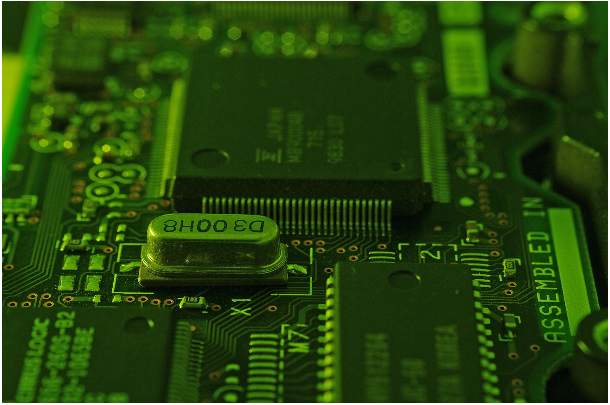

- In practice, this means that ZKPs and NARKs can be verified in **microseconds**
- The size of proofs is no longer than a few **hundred bits**
- These proofs are called SNARKs and (if they possess zero-knowledge) zk-SNARKs

What do we lose?

- Unfortunately, succinct arguments have one big problem

The Statement: **No succinct argument with an adaptive proof of soundness can base its security on a falsifiable assumption (as long as the reduction is black-box).**

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- Unfortunately, succinct arguments have one big problem
- Proved in 2013 by Craig Gentry and Daniel Wichs
- To understand this, let's look at falsifiable assumptions

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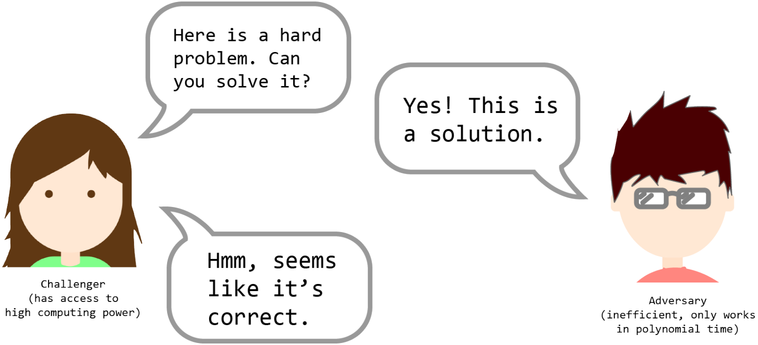
Falsifiable assumptions



A falsifiable assumption can be proved modelled as a game between an adversary and a challenger.

Falsifiable Assumptions

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- Prime factorization is hard
- Calculating discrete logarithms is hard
- Inverting shifted dot products is hard
- SAT is hard
- Separating very similar probability distributions is hard

Unfalsifiable Assumptions

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However, this doesn't mean that our constructions are *not* secure - it just means that we can't prove them assuming (a) a falsifiable assumption, and (b) that we possess no knowledge of the adversary.

New techniques are in development and it's possible we might be able to prove security using them.

Modern Work and Applications

Developments in the last few years

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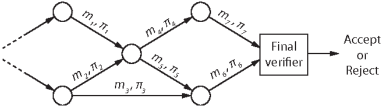
- All of these are under active development and new research

Distributed Computing



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- SNARKs are used to construct PCD, which is basically a SNARK on drugs

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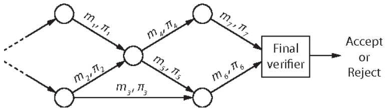

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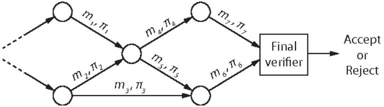

Proof-Carrying Data

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- Consists of a distributed system consisting of data flow and local inputs
- Can be used to prevent network fault, securely exchange messages, etc

Blockchain

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- Drawback: doesn't protect against NFT pfp

Gaming

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- Knowing the private ID could be used to attack the player
- Uses SNARK to verify whether hash corresponds to ID

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- Can be used in online gaming to prevent the system/company from knowing information about the players
- Used in Privacy-Preserving KYC where no information except existence of a human is known to the company

The Future

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- Also quantum-resilient
- Introduced only in 2018, so work is still at the basic level

Thank You

Questions?