# Q-Learning

Hung-yi Lee

#### Outline

Introduction of Q-Learning

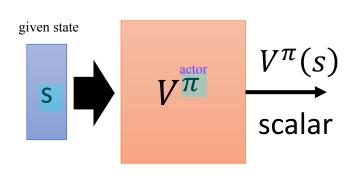
Tips of Q-Learning

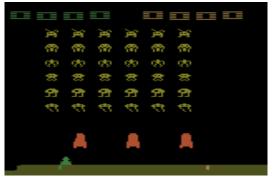
Q-Learning for Continuous Actions

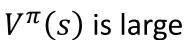
## Critic

The output values of a critic depend on the actor evaluated.

- A critic does not directly determine the action.
- Given an actor  $\pi$ , it evaluates how good the actor is
- State value function  $V^{\pi}(s)$ 
  - When using actor  $\pi$ , the *cumulated* reward expects to be obtained after visiting state s









 $V^{\pi}(s)$  is smaller

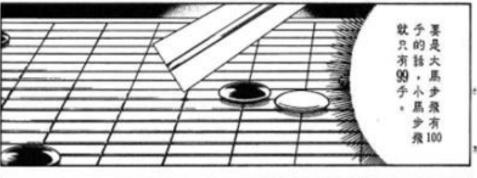
#### Critic

V以前的阿光(大馬步飛) = badV變強的阿光(大馬步飛) = good











# How to estimate $V^{\pi}(s)$

- Monte-Carlo (MC) based approach
  - The critic watches  $\pi$  playing the game

至少把遊戲玩到結束

After seeing  $s_a$ ,

Until the end of the episode, the cumulated reward is  $G_a$ 

 $S_a$   $\rightarrow$   $V^{\pi}$   $\rightarrow$   $V^{\pi}(S_a)$   $\leftarrow$   $G_a$ 

After seeing  $s_b$ ,

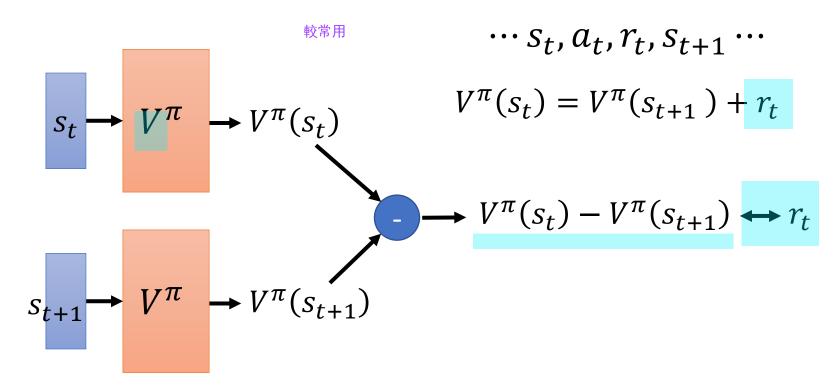
Until the end of the episode, the cumulated reward is  $G_h$ 

$$s_b \rightarrow V^{\pi} \rightarrow V^{\pi}(s_b) \rightarrow G_b$$

# How to estimate $V^{\pi}(s)$

不用將遊戲玩到最後

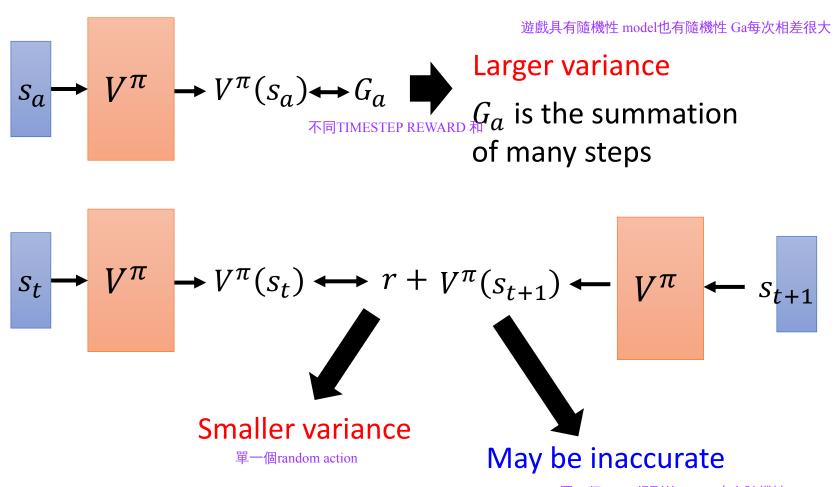
#### Temporal-difference (TD) approach



Some applications have very long episodes, so that delaying all learning until an episode's end is too slow.

$$Var[kX] = k^2 Var[X]$$

#### MC v.s. TD



同一個action得到的reward也有隨機性

#### MC v.s. TD

[Sutton, v2, Example 6.4]

- The critic has the following 8 episodes
  - $s_a, r = 0, s_b, r = 0$ , END
  - $s_b, r = 1$ , END
  - $s_h, r = 1$ , END
  - $s_b, r = 1$ , END
  - $s_h, r = 0$ , END

$$V^{\pi}(s_b) = 3/4$$

$$V^{\pi}(s_a) = ? 0? 3/4?$$

Monte-Carlo: 
$$V^{\pi}(s_a) = 0$$

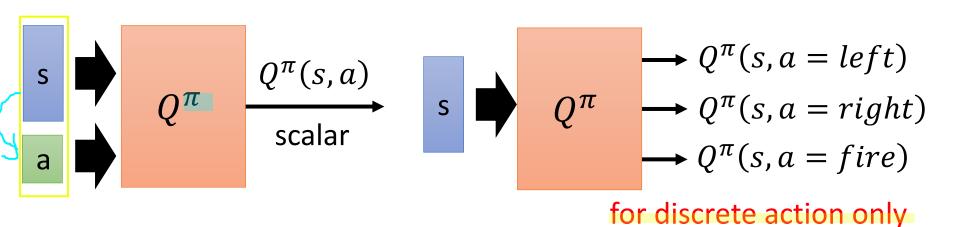
Temporal-difference:

$$V^{\pi}(s_a) = V^{\pi}(s_b) + r$$
  
3/4 3/4 0

#### **Another Critic**

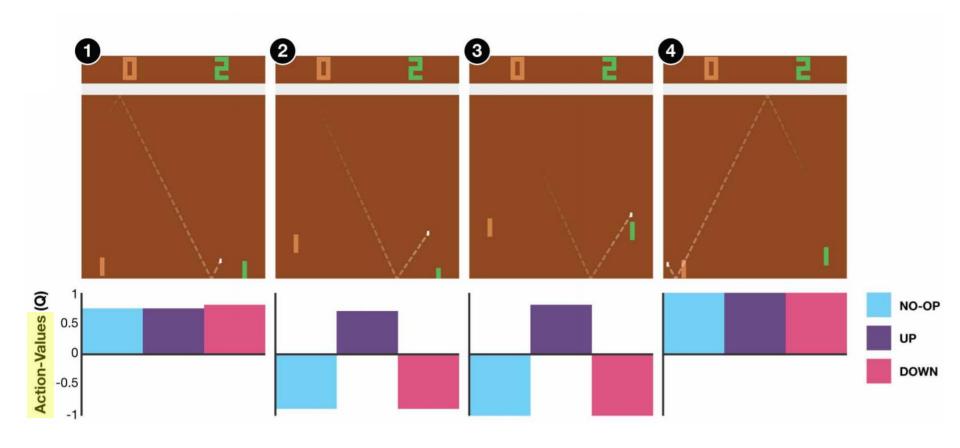
- State-action value function  $Q^{\pi}(s,a)$ 
  - When using actor  $\pi$ , the *cumulated* reward expects to be obtained after taking a at state s

強制在s採取a



可以窮舉的

## State-action value function



https://web.stanford.edu/class/psych209/Readings/MnihEtAlHassibis15NatureControlDeepRL.pdf

# Another Way to use Critic: Q-Learning

 $\pi$  interacts with the environment

有了Q value of a pi actor

保證可以找到一個比原本更好的 policy: pi prime

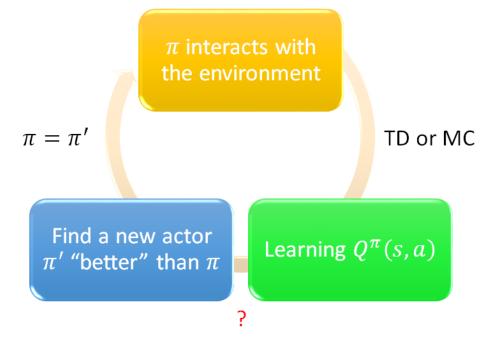
 $\pi = \pi'$ 

TD or MC

Find a new actor  $\pi'$  "better" than  $\pi$ 

Learning  $Q^{\pi}(s,a)$ 

# Q-Learning



- Given  $Q^{\pi}(s, a)$ , find a new actor  $\pi'$  "better" than  $\pi$ 
  - "Better":  $V^{\pi'}(s) \ge V^{\pi}(s)$ , for all state s

事實上s出來不一定是a只是這裡強迫

$$\pi'(s) = \arg\max_{a} Q^{\pi}(s, a)$$

所有可能的action哪個可以使得q function出來的reward(value)會最大

- $\succ \pi'$  does not have extra parameters. It depends on Q
- > Not suitable for continuous action a (solve it later)

#### **Q-Learning**

$$\pi'(s) = \arg\max_{a} Q^{\pi}(s, a)$$

$$V^{\pi'}(s) \geq V^{\pi}(s), \text{ for all state s}$$

$$V^{\pi}(s) = Q^{\pi}(s, \pi(s))$$

$$\leq \max_{a} Q^{\pi}(s, a) = Q^{\pi}(s, \pi'(s))$$

$$V^{\pi}(s) \leq Q^{\pi}(s, \pi'(s))$$

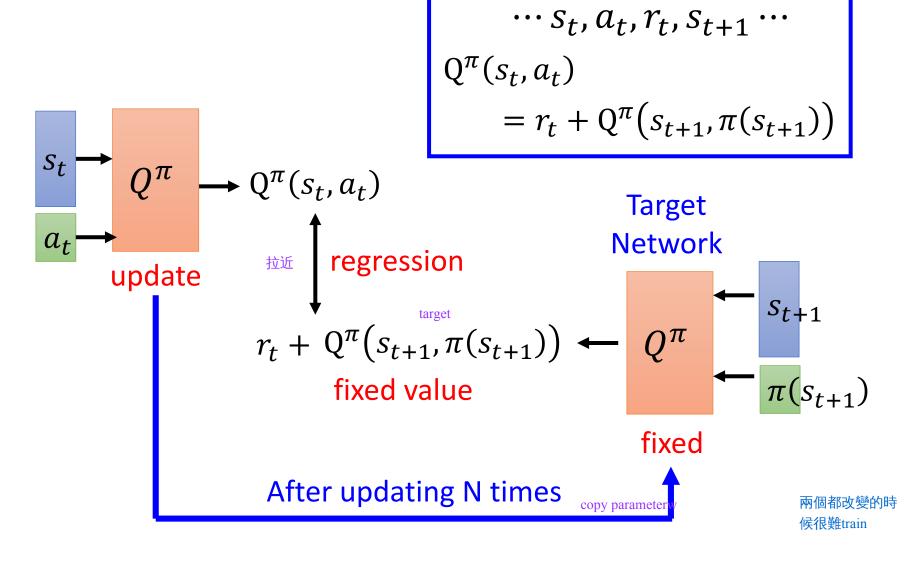
$$= E[r_{t+1} + V^{\pi}(s_{t+1})|s_{t} = s, a_{t} = \pi'(s_{t})]$$

$$\leq E[r_{t+1} + Q^{\pi}(s_{t+1}, \pi'(s_{t+1}))|s_{t} = s, a_{t} = \pi'(s_{t})]$$

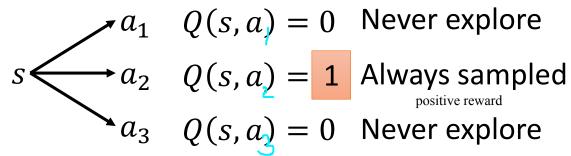
$$= E[r_{t+1} + r_{t+2} + V^{\pi}(s_{t+2})| \dots]$$

$$\leq E[r_{t+1} + r_{t+2} + Q^{\pi}(s_{t+2}, \pi'(s_{t+2}))| \dots] \dots \leq V^{\pi'}(s)$$

# Target Network



# Exploration



The policy is based on Q-function

$$a = arg \max_{a} Q(s, a)$$

This is not a good way for data collection.

#### **Epsilon Greedy**

 $\varepsilon$  would decay during learning

$$a = egin{cases} lpha g \max_{a} Q(s,a), \ a = a \end{pmatrix}$$
  $a = egin{cases} a max \ a \ random, \ random, \ randoms \end{pmatrix}$ 

with probability 1-arepsilon = 0.4 = 0.4 = 0.6 = 0.4 = 0.6 = 0.4 = 0.6 = 0.4 = 0.6 = 0.4 = 0.6 = 0.6 = 0.4 = 0.6 = 0.4 = 0.6 = 0.6

#### **Boltzmann Exploration**

Q越大 採取這個動作的機率就越大 取指數: 因為Q有正有負 但機率沒有負數

$$P(a|s) = rac{expig(Q(s,a)ig)}{\sum_{a} expig(Q(s,a)ig)$$
 Q越大讓他被選到

# Replay Buffer

Put the experience into buffer.

 $\pi$  interacts with the environment

$$\pi = \pi'$$

The experience in the buffer comes from different policies.

Drop the old experience if the buffer is full.

Find a new actor  $\pi'$  "better" than  $\pi$ 

Learning  $Q^{\pi}(s,a)$ 

# Replay Buffer

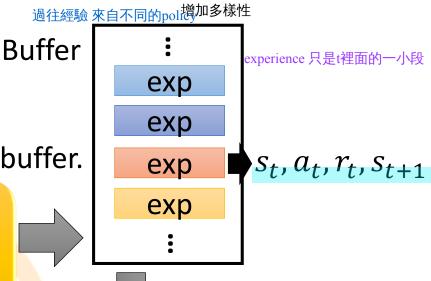
Put the experience into buffer.

 $\pi$  interacts with the environment

$$\pi = \pi'$$

Find a new actor  $\pi'$  "better" than  $\pi$ 

Learning  $Q^{\pi}(s, a)$ 



how to sample?

In each iteration:

sample的方法是要被定義的

從replay buffrer sampling experience來做

- 1. Sample a batch
- 2. Update Q-function

Off-policy

# Typical Q-Learning Algorithm

- Initialize Q-function Q, target Q-function  $\hat{Q} = Q$
- In each episode
  - For each time step t
    - Given state  $s_t$ , take action  $a_t$  based on Q (epsilon greedy)
    - Obtain reward  $r_t$ , and reach new state  $s_{t+1}$
    - Store  $(s_t, a_t, r_t, s_{t+1})$  into buffer
    - Sample  $(s_i, a_i, r_i, s_{i+1})$  from buffer (usually a batch)
    - Target  $y = r_i + \max_{a} \widehat{Q}(s_{i+1}, a)$
    - Update the parameters of Q to make  $Q(s_i, a_i)$  close to y (regression)
    - Every C steps reset  $\hat{Q} = Q$

#### Outline

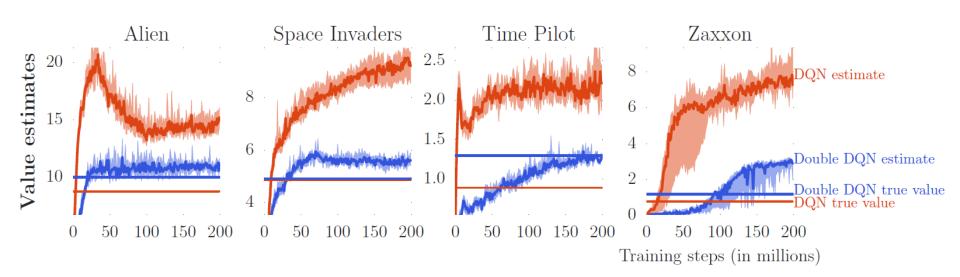
Introduction of Q-Learning

Tips of Q-Learning

Q-Learning for Continuous Actions

# Double DQN

Q value is usually over-estimated

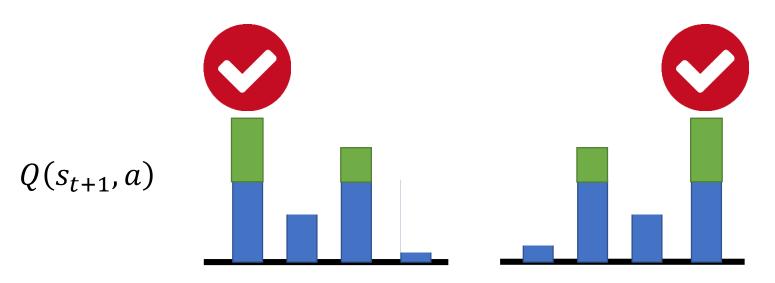


## Double DQN

Q value is usually over estimate



Tend to select the action that is over-estimated



## Double DQN

Q value is usually over estimate

$$Q(s_t, a_t) \longleftarrow r_t + \max_a Q(s_{t+1}, a)$$

• Double DQN: two functions Q and Q' Target Network

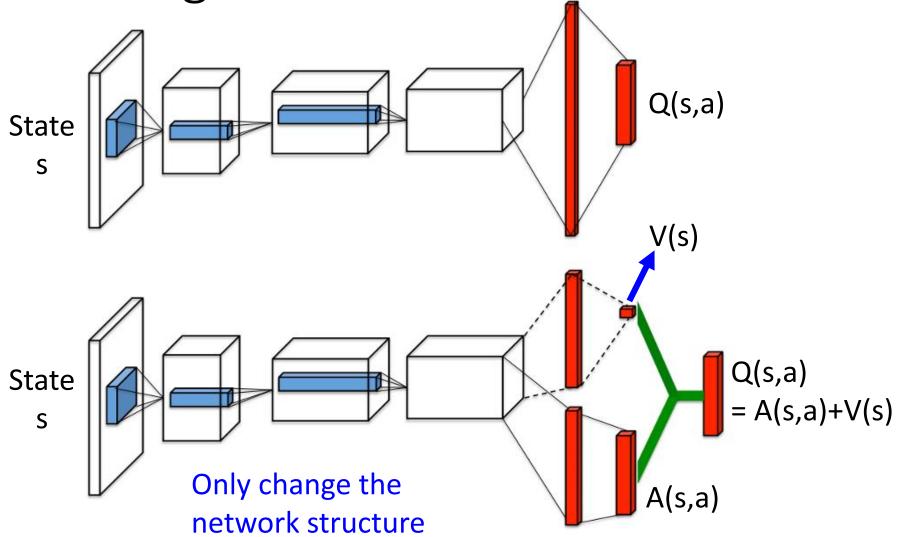
$$Q(s_t, a_t) \longleftrightarrow r_t + Q'\left(s_{t+1}, arg \max_a Q(s_{t+1}, a)\right)$$

If Q over-estimate a, so it is selected. Q' would give it proper value. How about Q' overestimate? The action will not be selected by Q.

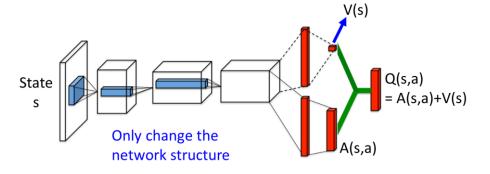
Hado V. Hasselt, "Double Q-learning", NIPS 2010 Hado van Hasselt, Arthur Guez, David Silver, "Deep Reinforcement Learning with Double Q-learning", AAAI 2016

# **Dueling DQN**

Ziyu Wang, Tom Schaul, Matteo Hessel, Hado van Hasselt, Marc Lanctot, Nando de Freitas, "Dueling Network Architectures for Deep Reinforcement Learning", arXiv preprint, 2015



# Dueling DQN



#### state

Q(s,a) action

П

3	3, 4	3	1
1	<u>-</u> Y 0	6	1
2	-2 -1	3	1

П

V(s) Average of column

A(s,a) sum of

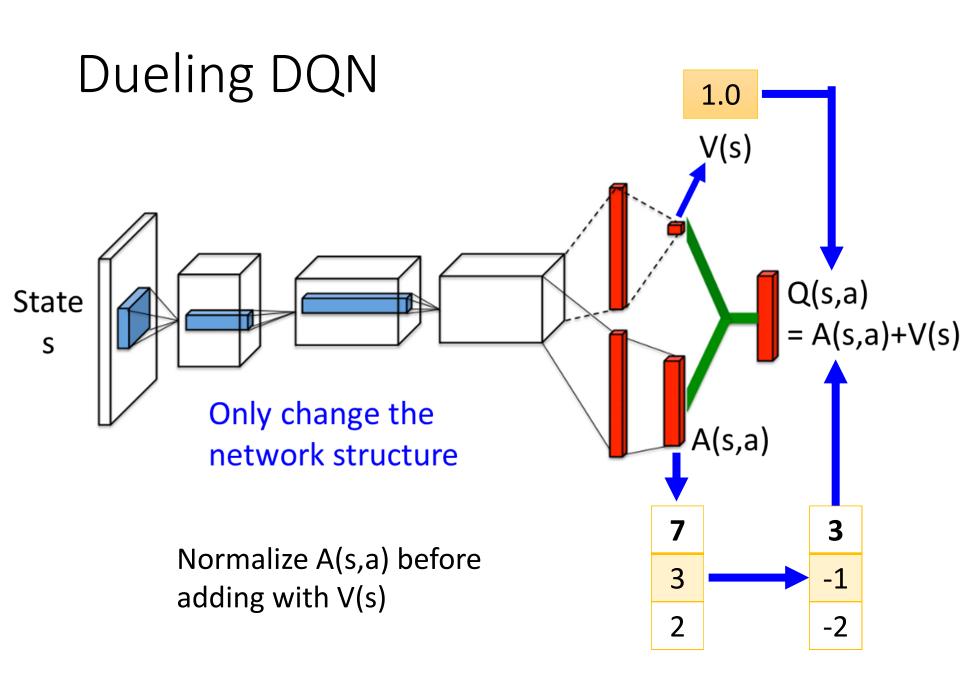
column = 0

2	8 1	4	1		
+					

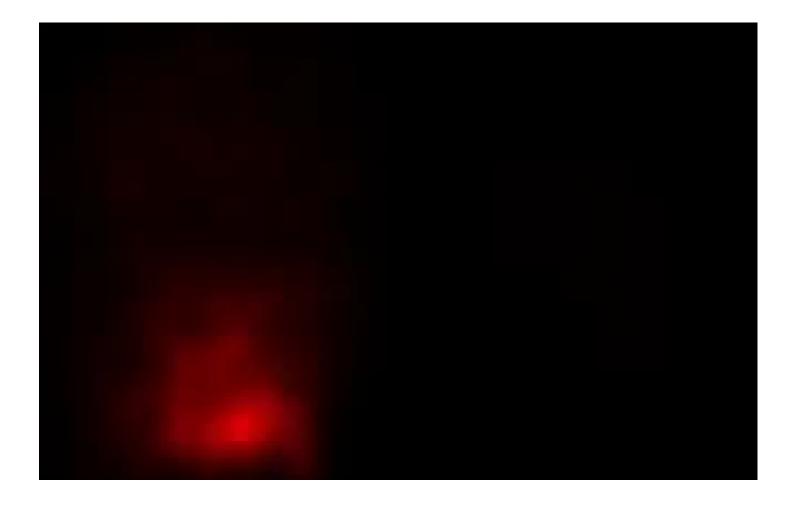
 1
 3
 -1
 0

 -1
 -1
 2
 0

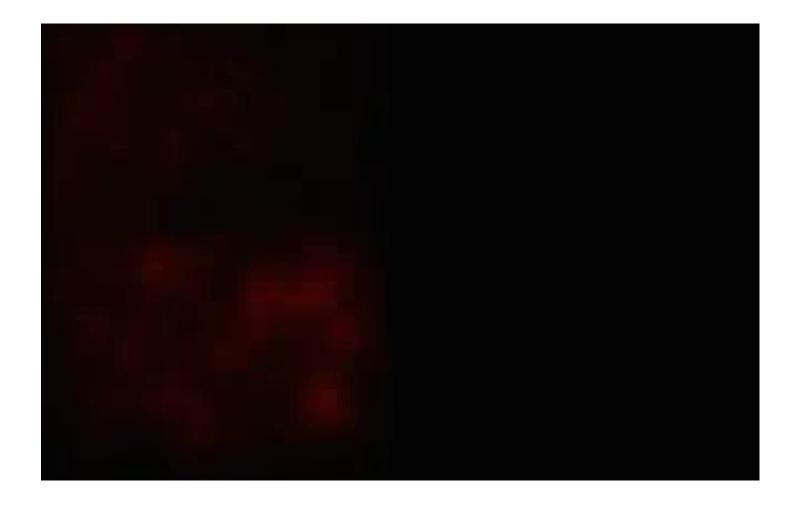
 0
 -2
 -1
 0



# Dueling DQN - Visualization

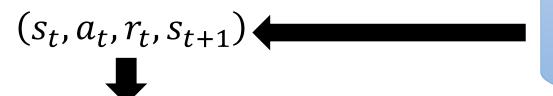


# Dueling DQN - Visualization

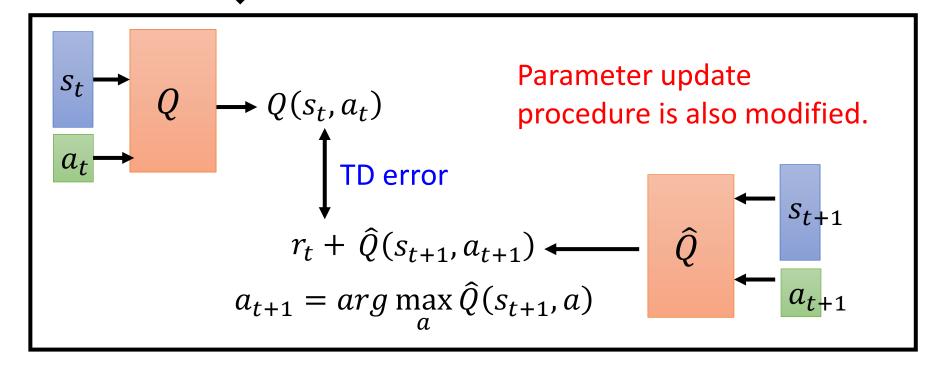


# Prioritized Reply

The data with larger TD error in previous training has higher probability to be sampled.

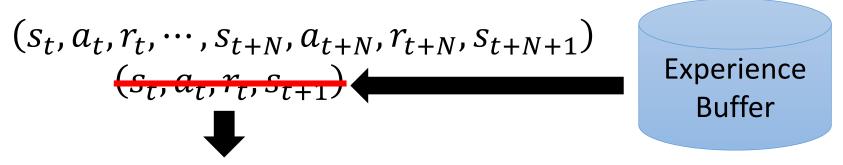


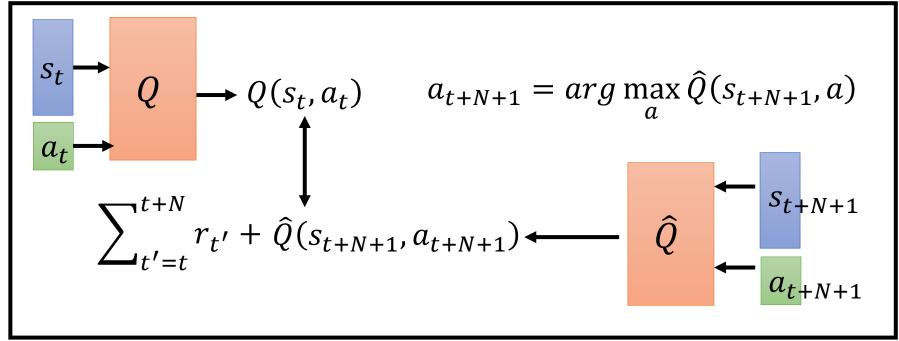
Experience Buffer



# Multi-step

#### Balance between MC and TD





# Noisy Net

https://arxiv.org/abs/1706.01905 https://arxiv.org/abs/1706.10295

Noise on Action (Epsilon Greedy)

$$a = \begin{cases} arg \max_{a} Q(s, a), & with probability 1 - \varepsilon \\ random, & otherwise \end{cases}$$

Noise on Parameters

 $a = arg \max_{a} \tilde{Q}(s, a)$ 

Inject noise into the parameters of Q-function at the beginning of each episode

$$Q(s,a) \longrightarrow \tilde{Q}(s,a)$$
Add noise

The noise would **NOT** change in an episode.

# Noisy Net

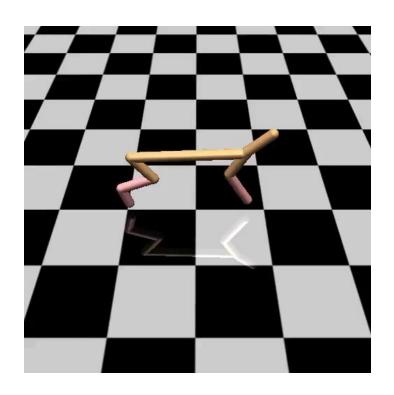
- Noise on Action
  - Given the same state, the agent may takes different actions.
  - No real policy works in this way
- Noise on Parameters
  - Given the same (similar) state, the agent takes the same action.
    - → State-dependent Exploration
  - Explore in a consistent way

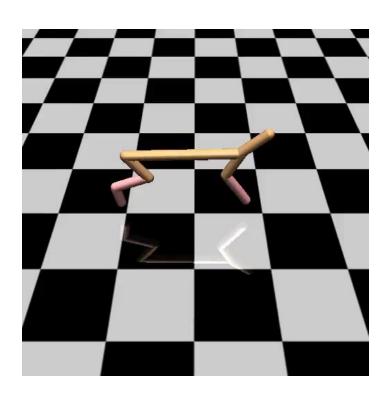
有系統地試

隨機亂試

## Demo

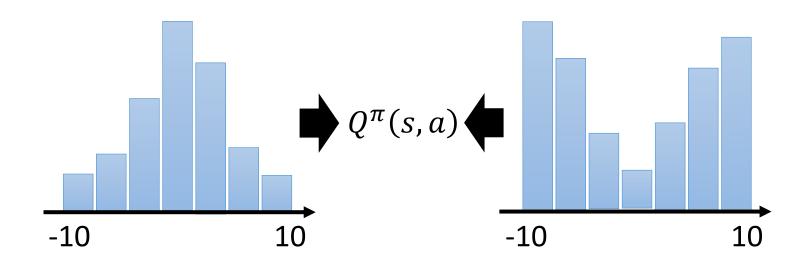
https://blog.openai.com/better-exploration-with-parameter-noise/





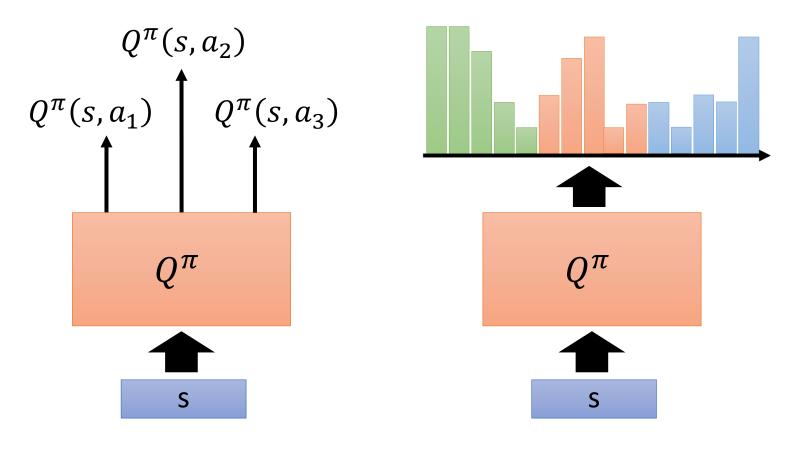
## Distributional Q-function

- State-action value function  $Q^{\pi}(s, a)$ 
  - When using actor  $\pi$ , the *cumulated* reward expects to be obtained after seeing observation s and taking a



Different distributions can have the same values.

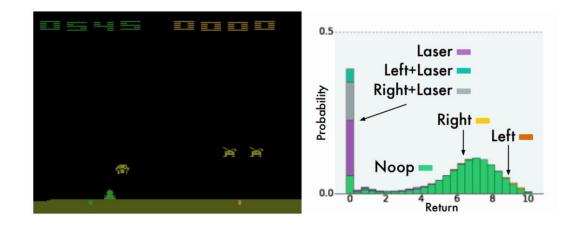
## Distributional Q-function

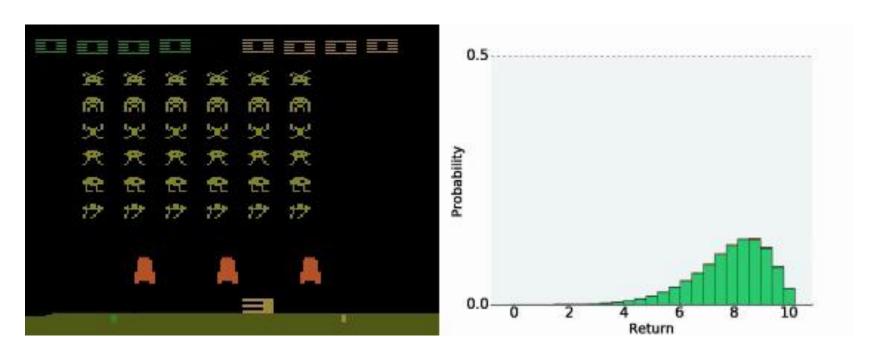


A network with 3 outputs

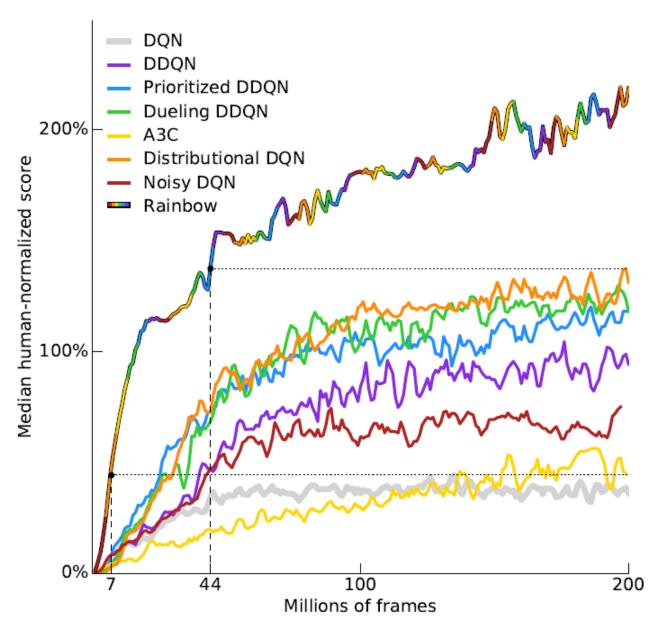
A network with 15 outputs (each action has 5 bins)

## Demo

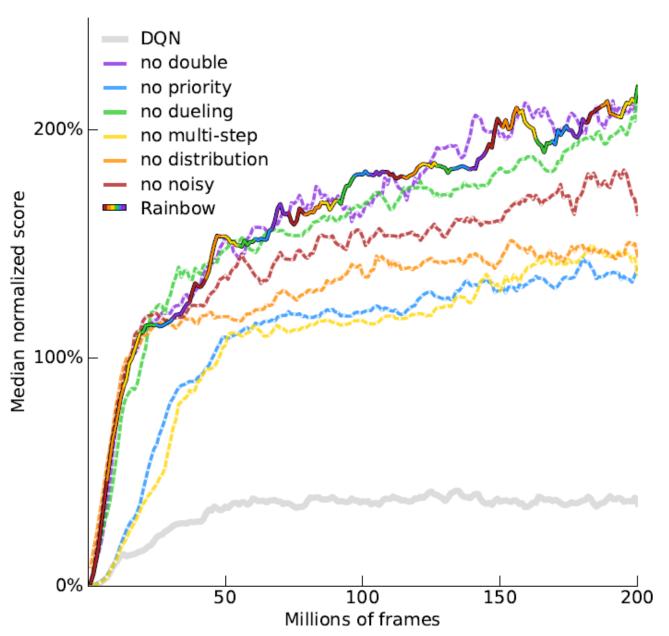




## Rainbow







#### Outline

Introduction of Q-Learning

Tips of Q-Learning

Q-Learning for Continuous Actions

#### Continuous Actions

Action a is a continuous vector

$$a = arg \max_{a} Q(s, a)$$

#### **Solution 1**

Sample a set of actions:  $\{a_1, a_2, \dots, a_N\}$ 

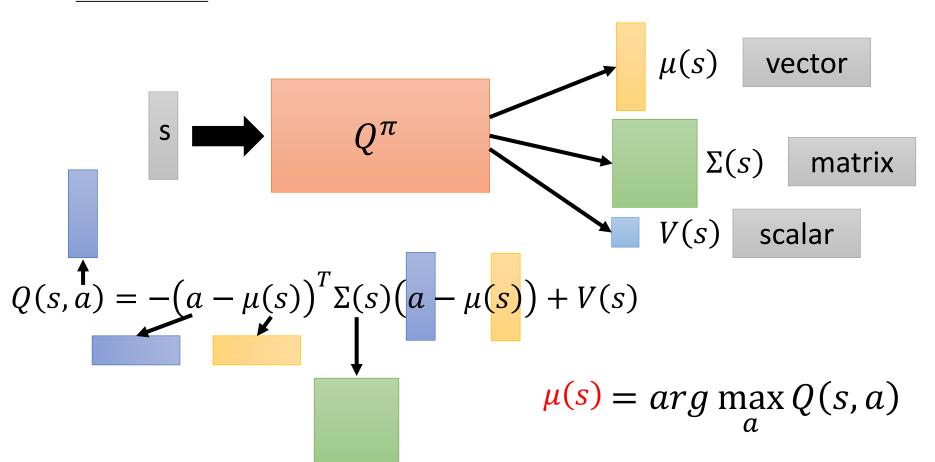
See which action can obtain the largest Q value

#### **Solution 2**

Using gradient ascent to solve the optimization problem.

#### Continuous Actions

**Solution 3** Design a network to make the optimization easy.

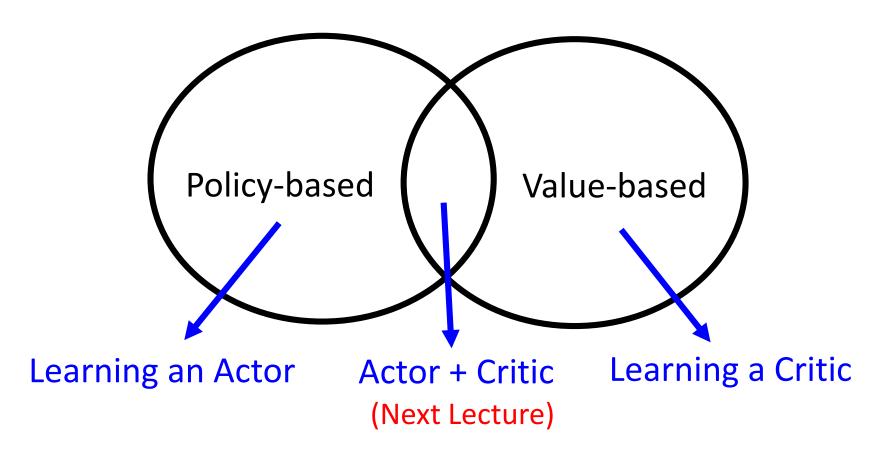




https://www.youtube.com/watch?v=ZhsEKTo7V04

#### Continuous Actions

**Solution 4** Don't use Q-learning



# Acknowledgement

• 感謝林雨新同學發現投影片上的錯字