SMCB Project 7.

Problem 17. Junction tree

DAG

Moral graph

Junction graph.

$$P(U) = \psi(U)$$
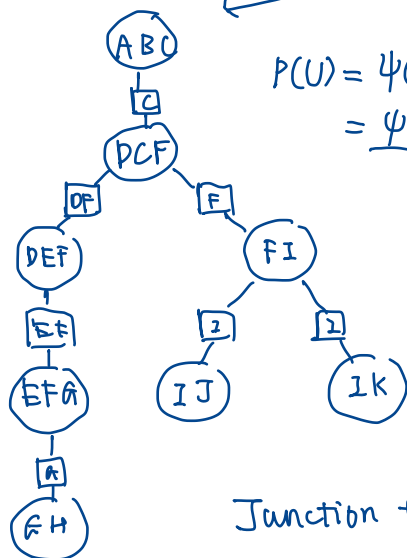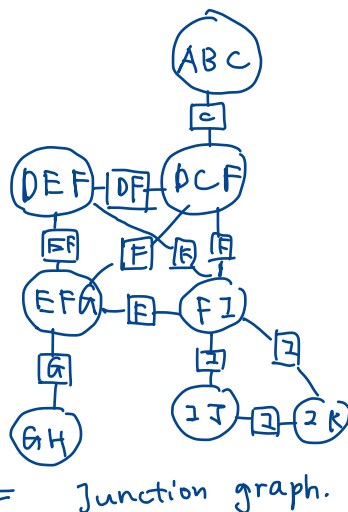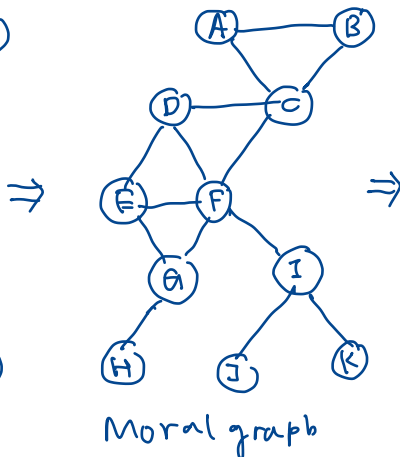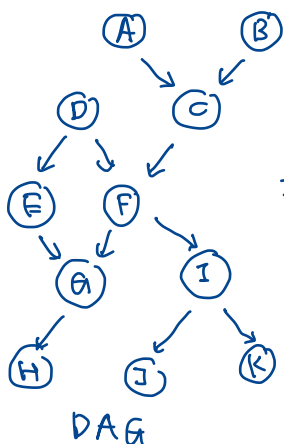
$$= \frac{\psi(ABC)\,\psi(DCF)\,\psi(DEF)\,\psi(EFG)\,\psi(AH)\,\psi(FI)\,\psi(IJ)\,\psi(IK)}{\psi(C)\,\psi(DF)\,\psi(F)\,\psi(EF)\,\psi(G)\,\psi(I)^2}$$

Junction tree

Problem 18

(a). forward message :

$$\mu_a(x_n) = \sum_{x_{n-1}} \psi_{n-1,n}(x_{n-1}, x_n) \mu_a(x_{n-1})$$

with initialization

$$\mu_a(x_1) = 1 \quad (\text{so that } \mu_a(x_2) = \sum_{x_1} \psi_{1,2}(x_1, x_2) \cdots)$$

backward message

$$\mu_\beta(x_n) = \sum_{x_{n+1}} \psi_{n,n+1}(x_n, x_{n+1}) \mu_\beta(x_{n+1})$$

with initialization

$$\mu_\beta(x_5) = 1 \quad (\text{so that } \mu_\beta(x_4) = \sum_{x_3} \psi_{x_4, x_5}(x_4, x_5) \cdot)$$

(b). $O(NK^2)$

Each node K values. Forward $(n-1)K^2$, backward $(N-n)K^2$.
Total: $(N-1)K^2 \rightarrow O(NK^2)$

(c). If values re stored, after doing one message passing in forward direction
and one message passing in backward direction, we can directly "read
off" all probabilities. The total calculation is $2(N-1)K^2$. or $O(NK^2)$

In the general case, it will be $N(N-1)K^2$ or $O(N^2K^2)$