Project 9 Model Selection

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```r
set.seed(42)
```

Problem 23: d-separation

(i) Write down all the variables that are d-separated from A given C, D

Just G (because given $A \not\perp B|D$, hence B is not d-separated from A and consequently B's descendents F and E are also not d-separated).

(ii) Indicate whether each statement is true or false and expalin your choice.

We have the Theorem (Verma & Pearl): A is d-separated from B by C *if and only if* the join distribution over all variables satisfied $A \perp B|C$.

1. $B \perp C|D$: **False**.

   C and B are not d-separated because in the path $B - D - A - C$ the arrows meet head-to-head at D.

2. $G \perp E|D$: **False**.

   G and E are not d-separated because in the path $E - F - B - D - A - C - G$ the arrows meet head-to-head at D.

3. $C \perp F|A$: **True**.

   C and F are d-separated because the arrows meet tail-to-tail at A.

4. $C \perp E|MB(C)$: **True**.

   Given $MB(C) = A, D, G$, C and E are d-separated because in every path from E to C, it is either blockec by A (tail-to-tail) or D head-to-tail.

Load data for Problem 24 - 27.

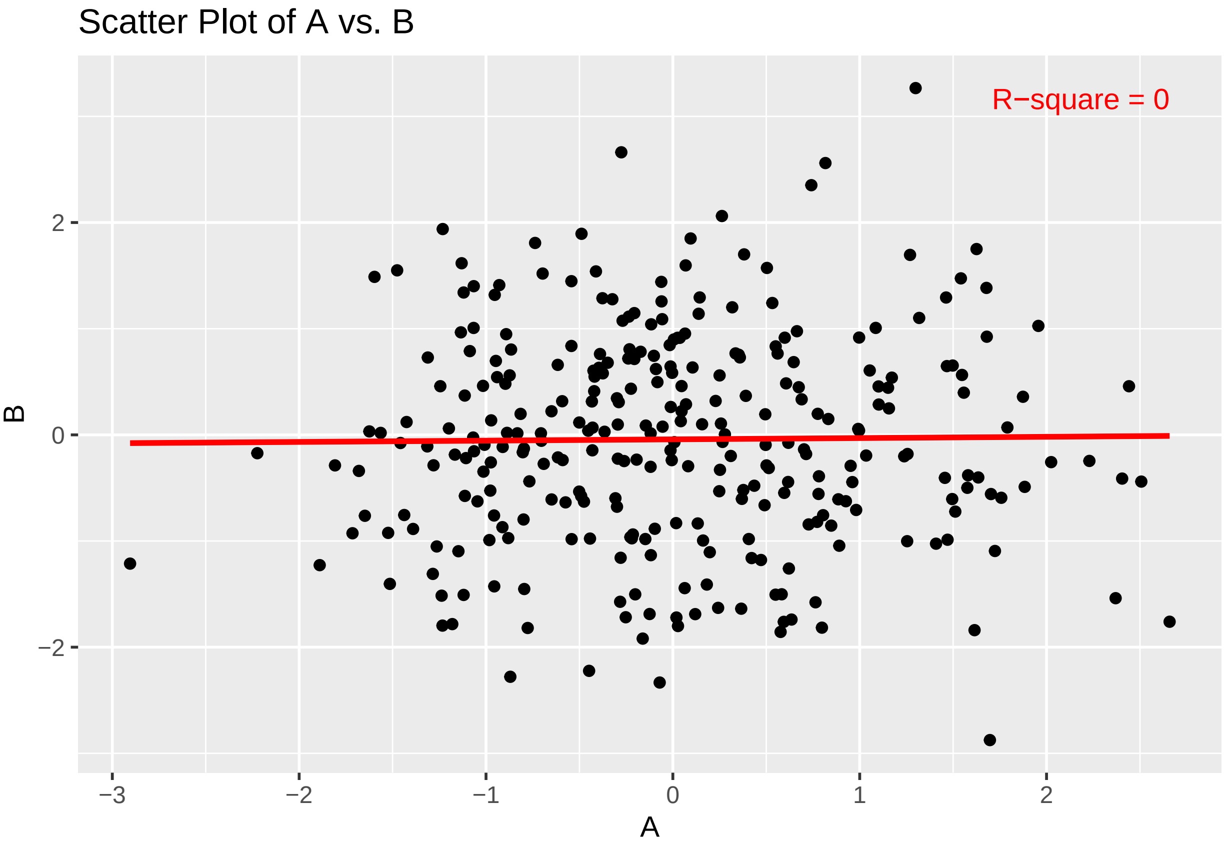
```r
mvn.dag <- readRDS("MVN_DAG.rds")
head(mvn.dag)
```

```
##              A           B          C           D           E          F
## 1 -0.294720447  0.09739665 -0.6693398 -1.83015111  0.41091561  0.8663284
## 2 -0.005767173 -0.23838695  1.3541249 -0.63615467 -0.62030667  0.5347883
## 3  2.404653389 -0.41182796 -0.6750873  2.40311737  0.25173361 -1.3942278
## 4  0.763593461 -1.57721805 -2.9092631 -0.52703829 -0.81604685  1.4960640
## 5 -0.799009249 -0.79727610 -2.5411415  0.18356375  0.07764537 -1.2424275
## 6 -1.147657009 -1.09623678 -0.8384432 -0.03142255 -0.23194797 -1.0266878
```

Problem 24: Testing for marginal correlation

```r
ggplot(mvn.dag, aes(x = A, y = B)) +
  geom_point() +
  xlab("A") +
  ylab("B") +
  ggtitle("Scatter Plot of A vs. B") +
  # Add linear regression line and R-squared value to better show that there's no correlation
  geom_smooth(method = "lm",
              se = FALSE,
              color = "red") +
  annotate(
    "text",
    x = max(mvn.dag$A),
    y = max(mvn.dag$B),
    label = paste("R-square =", round(summary(
      lm(B ~ A, data = mvn.dag)
    )$r.squared, 2)),
    hjust = 1,
    vjust = 1,
    color = "red"
  )
```

```
## `geom_smooth()` using formula = 'y ~ x'
```



The (very scattered) plot suggests that A and B are marginally independent (no correlation, $R^2 = 0$). And it confirms with Figure 2 because it also suggests that A and B are marginally independent.

```
cor <- cor.test(mvn.dag$A, mvn.dag$B)
cor
```

```
##
##  Pearson's product-moment correlation
##
## data:  mvn.dag$A and mvn.dag$B
## t = 0.20194, df = 298, p-value = 0.8401
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
##  -0.1016784  0.1247727
## sample estimates:
##        cor
## 0.01169715
```

The result of `cor.test()` on A and B also confirms that there's no correlation between A and B (the p-value is 0.840103 > 0.05, so we cannot reject the null hypothesis that there's no correlation between A and B).

Problem 25: Testing for partial correlation

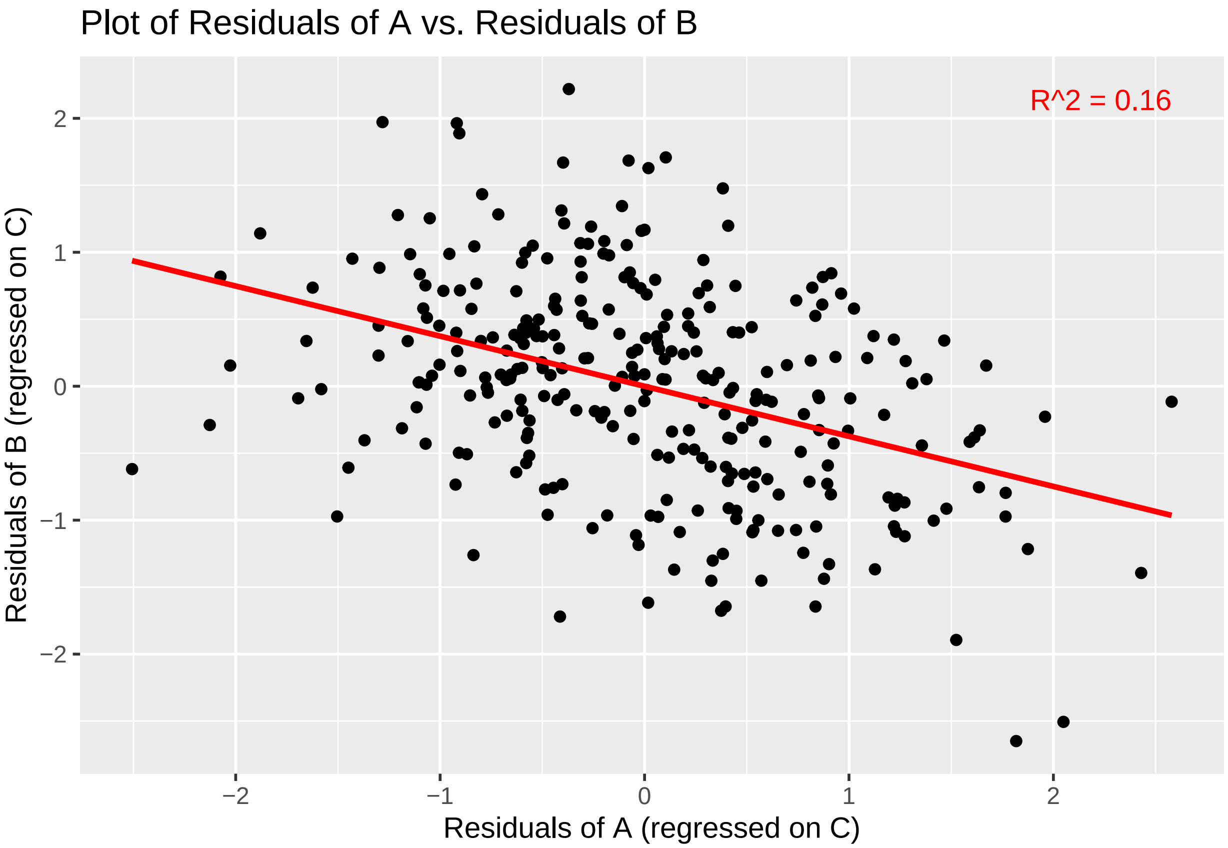
```
# Linear regression of A on C
model_A <- lm(A ~ C, data = mvn.dag)
resid_A <- residuals(model_A)

# Linear regression of B on C
model_B <- lm(B ~ C, data = mvn.dag)
resid_B <- residuals(model_B)

# Linear regression of residuals of A vs. residuals of B
model_resid <- lm(resid_B ~ resid_A)
resid_AB <- residuals(model_resid)

# Plot residuals of A vs. residuals of B, with linear regression and R-squared value
ggplot(
  data = data.frame(
    resid_A = resid_A,
    resid_B = resid_B,
    resid_AB = resid_AB
  ),
  aes(x = resid_A, y = resid_B)
) +
  geom_point() +
  geom_smooth(method = "lm",
              se = FALSE,
              color = "red") +
  xlab("Residuals of A (regressed on C)") +
  ylab("Residuals of B (regressed on C)") +
  ggtitle("Plot of Residuals of A vs. Residuals of B") +
  annotate(
    "text",
    x = max(resid_A),
    y = max(resid_B),
    label = paste("R^2 =", round(summary(model_resid)$r.squared, 2)),
    hjust = 1, vjust = 1, color = "red")
```

```
## `geom_smooth()` using formula = 'y ~ x'
```



There seems to be a correlation between $A|C$ and $B|C$.

```
cor2 <- cor.test(resid_A, resid_B)
cor2
```

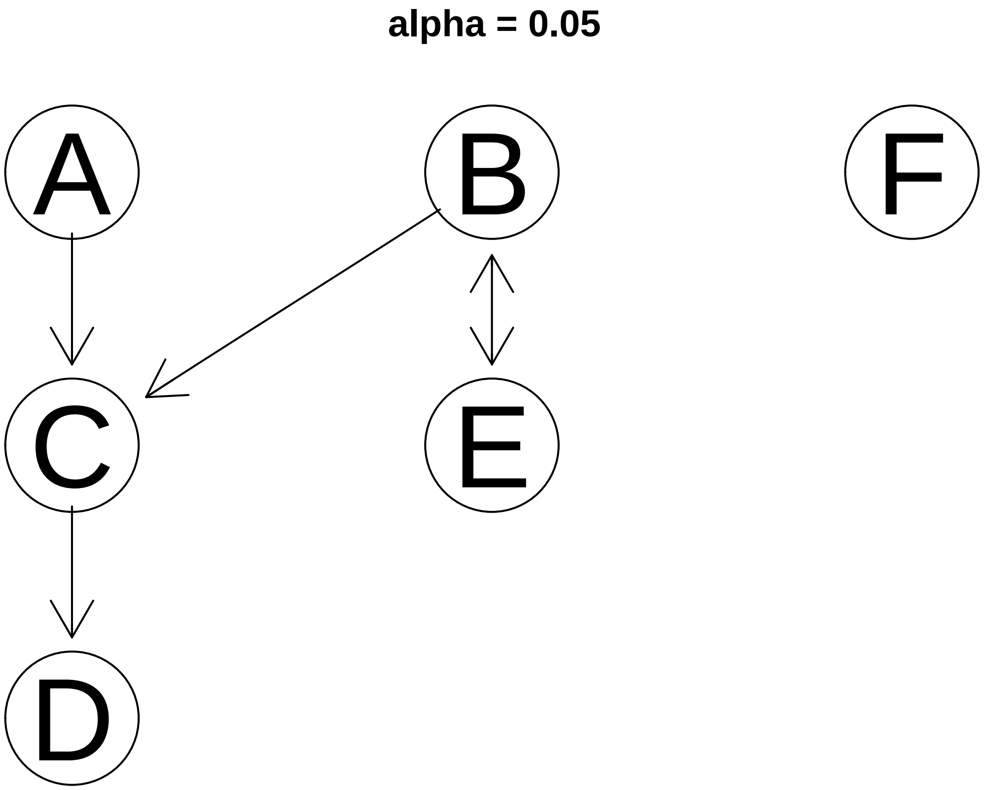
```
##
##  Pearson's product-moment correlation
##
## data:  resid_A and resid_B
## t = -7.5173, df = 298, p-value = 6.6e-13
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
##  -0.4903245 -0.2995546
## sample estimates:
##        cor
## -0.3992521
```

With a p-value of $6.5999545 \times 10^{-13} < 005$, we can reject the null hypothesis that $A|C$ and $B|C$ are not correlated, hence A and B are not conditionally independent given C. This also confirms with the uderlying DAG in Figure 2 because A and B explain away each other given C.

Problem 26: Running the PC algorithm

```
res <- pc(
  suffStat = list(C = cor(mvn.dag), n = nrow(mvn.dag)),
  indepTest = gaussCItest,
  alpha = 0.05,
  labels = colnames(mvn.dag)
)
```

```
plot(res, main="alpha = 0.05")
```



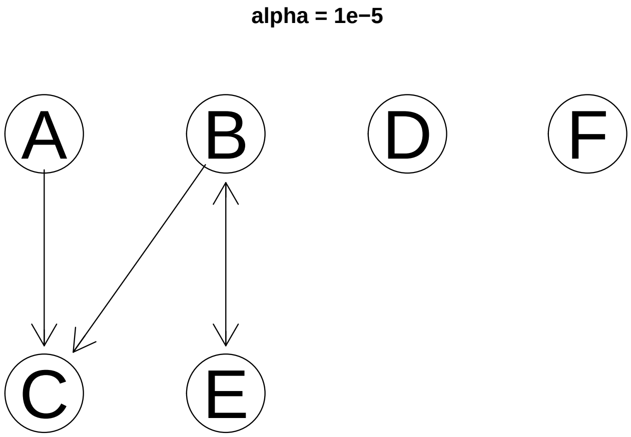
The PC algorithm has successfully learned the correct graph. The colliders are also successfully identified.

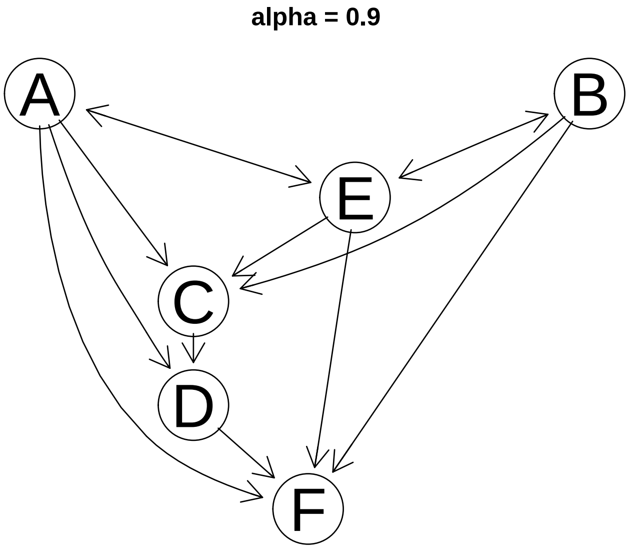
If the significance level α increase, more edges can get accepted, making the graph become more dense. (The code block below showed a very unrealistic setting)

```
res.very_small_alpha <- pc(
  suffStat = list(C = cor(mvn.dag), n = nrow(mvn.dag)),
  indepTest = gaussCItest,
  alpha = 1e-5,
  labels = colnames(mvn.dag)
)
plot(res.very_small_alpha,  main="alpha = 1e-5")

res.very_large_alpha <- pc(
  suffStat = list(C = cor(mvn.dag), n = nrow(mvn.dag)),
  indepTest = gaussCItest,
  alpha = 0.9,
  labels = colnames(mvn.dag)
)
```

```r
plot(res.very_large_alpha,  main="alpha = 0.9")
```

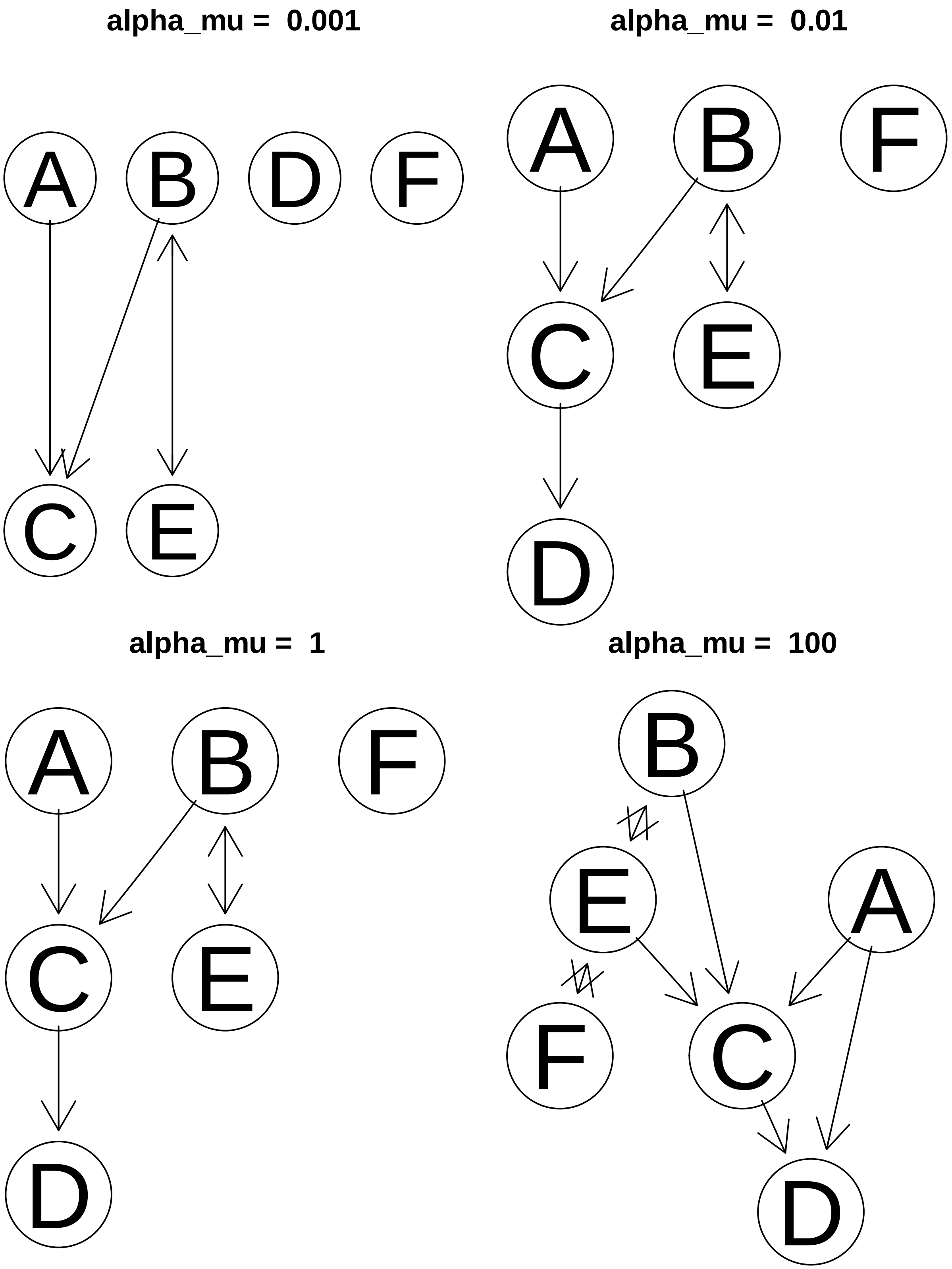




Problem 27: Running the partition MCMC algorithm

```r
par(mfrow=c(1,2))

ams <- c(1e-3, 1e-2, 1, 1e2, 1e3)
for (am in ams) {
  score <- scoreparameters(scoretype = "bge", mvn.dag, bgepar = list(am=am, aw=NULL))
  maxBN <- learnBN(score, algorithm = "orderIter")
  plot(graphAM(as.matrix(maxBN$CPDAG), edgemode = "directed"), main = paste("alpha_mu = ", am))
}
```

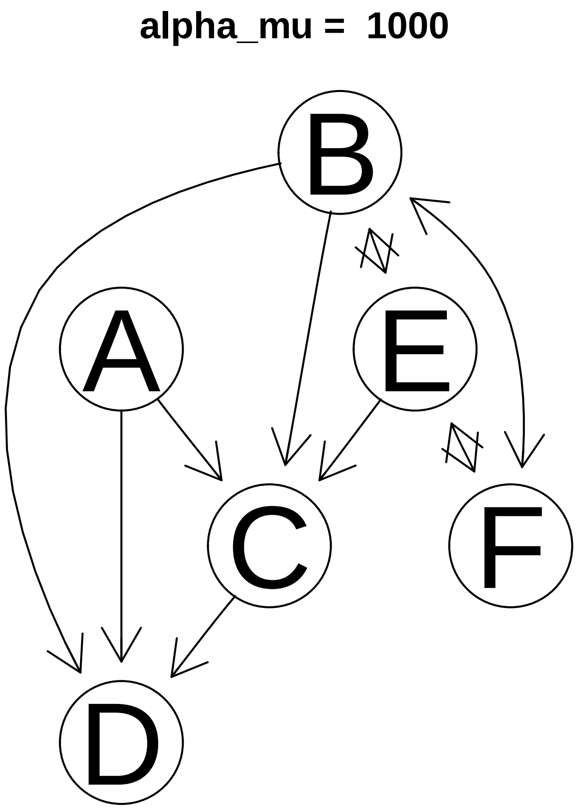


alpha_mu = 0.001

alpha_mu = 0.01

alpha_mu = 1

alpha_mu = 100

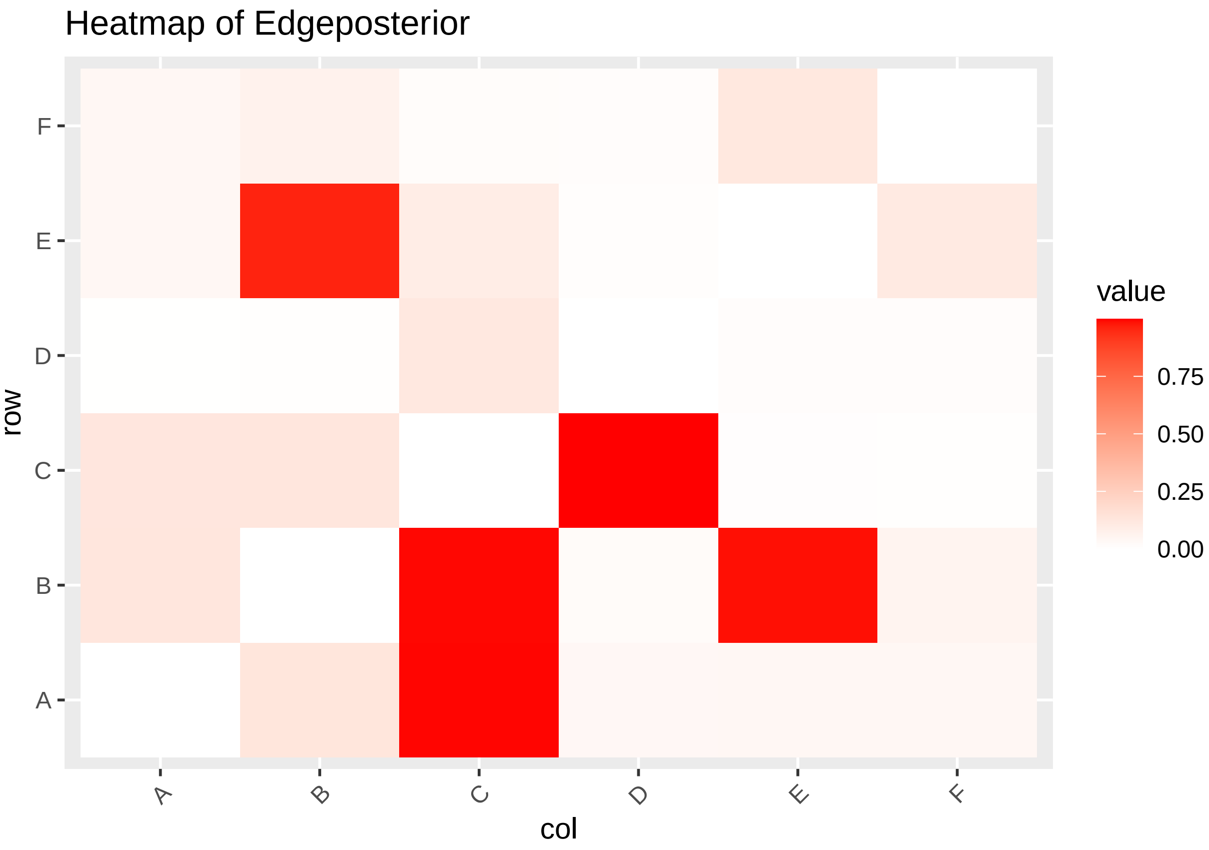


We see that as α_μ increases, the graph become denser and denser. The colliders are preserved for every learned graph though.

```
score <- scoreparameters(scoretype = "bge", mvn.dag, bgepar = list(am=1, aw=NULL))
maxBN <- learnBN(score, algorithm = "orderIter")
partition_sample <- sampleBN(score, algorithm = "partition", startspace = maxBN$endspace)
edgeposterior <- edgep(partition_sample, pdag = TRUE)
```

```
# Reshape data to long format
edgeposterior_df <- melt(as.matrix(edgeposterior), varnames = c("row", "col"))

# Create heatmap of edgeposterior
ggplot(edgeposterior_df, aes(x = col, y = row, fill = value)) +
  geom_tile() +
  scale_fill_gradient(low = "white", high = "red") +
  ggtitle("Heatmap of Edgeposterior") +
  scale_x_discrete(limits = c("A", "B", "C", "D", "E", "F")) +
  scale_y_discrete(limits = c("A", "B", "C", "D", "E", "F")) +
  theme(axis.text.x = element_text(angle = 45, hjust = 1))
```


Heatmap of Edgeposterior
row
F
E
D
C
B
A
col
A
B
C
D
E
F
value
0.75
0.50
0.25
0.00