ETH zürich   D BSSE   Computational Biology Group

Evolutionary Dynamics

Exercise 8

Prof. Dr. Niko Beerenwinkel
Johannes Gawron
Michael Schneider
Norio Zimmermann

16th November 2023

Problem 1: Weak selection

Consider a population of size N engaged in a two-player evolutionary game. The population consists of two types of individuals, A and B, with frequencies x_A and $x_B = 1 - x_A$, respectively. The fitness of the two types depends on the payoffs from the game and is given by the payoff matrix:
Consider the two-strategy game

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} a & b \\ c & d \end{pmatrix}. \end{array}$$

The average payoffs (fitness) of individuals of type A and B, denoted by f_A and f_B, respectively, are given by:

$$f_A = ax_A + b(1 - x_A) = ax_A + bx_B$$
$$f_B = cx_A + d(1 - x_A) = cx_A + dx_B$$

(a) Write down the replicator equation for the change in the frequency x_A of type A in the population over time. **(1 Point)**

(b) Now assume weak selection, where the fitness is given by $1 + \delta f_i$, where f_i is the payoff for type i, and δ is a small selection strength parameter. Linearize the replicator equation to first order in δ. **(1 Point)**

(c) Determine the condition for evolutionary stability (ESS) of type A. What condition must the payoff parameters a, b, c, d satisfy for A to be evolutionarily stable? How does this depend on the choice of δ? **(2 Points)**

(d) Suppose A and B represent two strategies in the classic Hawk-Dove game, with the following payoff matrix:

	Hawk	Dove
Hawk	$\frac{V-C}{2}$	V
Dove	0	$\frac{V}{2}$

where V is the value of the contested resource and C is the cost of fighting. Analyze the condition for evolutionary stability under weak selection for this game. **(1 Point)**

Problem 2: Strong selection
Consider the two-strategy game

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} a & b \\ c & d \end{pmatrix}. \end{array}$$

(a) In an *infinite* population with replicator dynamics, decide for all games of this type whether strategies A and B are dominant, coexisting, or bi-stable, based on the two variables $\xi = a - c$ and $\zeta = d - b$. **(2 Points)**

Now consider a population of *finite* size N that evolves according to a unstructured Moran process. Suppose the fitness of A and B are given respectively by

$$f_i = \frac{a(i-1) + b(N-i)}{N-1}$$
$$g_i = \frac{ci + d(N-i-1)}{N-1}.$$

Note that this corresponds to limit of strong selection, $w = 1$, as compared to the lecture.
We want to classify the evolutionary stability of A and B as a function of the population size N and the payoff values a, b, c, and d. To this end we analyze the difference in fitness $h_i = f_i - g_i$.

(b) Show that

$$h_i = \xi' \frac{i}{N-1} - \zeta' \frac{N-i}{N-1}$$

with

$$\xi' = \xi - \frac{a-d}{N} \quad \text{and} \quad \zeta' = \zeta + \frac{a-d}{N}.$$

(1 Point)

(c) Show that for $\xi' > 0 > \zeta'$ strategy A is dominant. Derive a criterion for the dominance of B. **(1 Point)**

Problem 3: Hawk-Dove game example
Consider an (infinite) population where individuals can adopt one of two strategies: Hawk (H) or Dove (D). The payoff matrix for interactions between individuals using these strategies is given by:

	Hawk	Dove
Hawk	$\frac{V-C}{2}$	V
Dove	0	$\frac{V}{2}$

where V is the value of the resource $V > 0$ and C is the cost of conflict. Assume $V < C$. Which (if any) of the following statements is true regarding the conditions for an Evolutionarily Stable Strategy (ESS) in this game? Please provide a proper rationale for your answer.
A) Strategy Hawk (H) is an ESS if $\frac{V-C}{2} > 0$ and $V \geq \frac{V}{2}$.
B) Strategy Dove (D) is an ESS if $\frac{V}{2} > 0$ and $0 \geq V$.
C) Strategy Hawk (H) is an ESS if $\frac{V-C}{2} > V$ and $V \geq \frac{V}{2}$.
D) Strategy Dove (D) is an ESS if $\frac{V}{2} > 0$ and $0 \geq \frac{V-C}{2}$.
(1 Point)