Factoring in the Low-Volatility Factor

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Abstract

Low-volatility stocks have historically delivered higher risk-adjusted returns than their high-volatility peers. Despite extensive evidence and widespread adoption in the investment industry, the so-called low-volatility factor is absent from standard asset pricing models. This paradox is attributable to asymmetry in factor legs and real-life investment frictions. A low-volatility factor substantially improves performance of factor models once accounting for these dimensions in various in-sample and out-of-sample exercises, across different low-risk measures and across methodological choices. We advocate integrating the low-volatility factor into asset pricing models, accounting for the asymmetry and frictions.

Keywords: Asset pricing, model selection, low-volatility, low-beta, investment frictions, factor asymmetry
JEL: G11, G12, G15

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1 Introduction

Low-risk stocks, commonly identified by low volatility or beta, systematically provide higher risk-adjusted returns than high-risk stocks (see, e.g., Black, Jensen, and Scholes, 1972; Haugen and Heins, 1975 for early evidence). A large stream of subsequent work confirms the presence of this low-risk effect, henceforth low-volatility effect, across sample periods, markets, and risk measures.[1] The low-volatility effect tends to be one of the strongest and most robust effects across hundreds of anomalies that are part of the factor zoo (Jensen, Kelly, and Pedersen, 2023), with several studies providing evidence for economic channels of the low-volatility effect.[2] Academic interest in low-volatility investing has helped catalyze a sizable industry dedicated to exploiting this anomaly.[3] Moreover, leading equity risk model providers (such as MSCI, Axioma, Factset, Bloomberg and Style Analytics) include low-volatility as a distinct factor.

Despite the strong academic evidence and popularity in the industry, we observe a paradox: the low-volatility factor is not included in well-known asset pricing models.[4] In this

[1]The low-volatility effect holds for different risk measures, such as market beta (Frazzini and Pedersen, 2014), volatility (Blitz, 2007), and idiosyncratic volatility (Ang, Hodrick, Xing, and Zhang, 2009), is present within and across industries and countries (Baker, Bradley, and Taliaferro, 2014), holds among the largest and most liquid U.S. stocks (Auer and Schuhmacher, 2015). It also holds after controlling for other factor premia, is robust over time (e.g., Ang, Hodrick, Xing, and Zhang, 2009; Blitz, 2007; Frazzini and Pedersen, 2014; Blitz, 2020; Baker and Haugen, 2012), is present in corporate bonds (Houweling and Van Zundert, 2017), government bonds, commodities, currencies and equity markets (Frazzini and Pedersen, 2014; Baltussen, Swinkels, and Van Vliet, 2021). Baltussen, van Vliet, and Van Vliet (2023) even confirm the low-volatility effect out-of-sample over the 61 years before 1926, the inception date of the CRSP sample.

[2]These include leverage constraints (Frazzini and Pedersen, 2014), skewness preference (Bali, Cakici, and Whitelaw, 2011), co-skewness risk (Schneider, Wagner, and Zechner, 2020), arbitrage asymmetry (Stambaugh, Yu, and Yuan, 2015), agency issues (Baker, Bradley, and Wurgler, 2011), and institutional investors shifting their demand from high- to low-risk stocks as volatility increases (Barroso, Detzel, and Maio, 2025).

[3]In fact, in the industry the low-volatility effect is one of the most exploited factors, with the largest asset managers and most quantitative managers recognizing and investing based on the low-volatility effect. As of May 2025 assets under management in low volatility investment solutions have surged to over $392 billion, with data from ETFdb showing 110$ billion invested in low-volatility ETFs, while investment fund and mandate data from eVestment reveals assets under management in other strategy vehicles that utilize volatility as factor to equate $283 billion.

[4]Seminal studies to factor models that price the cross-section of stock returns do not consider the low-volatility factor. Factors in these models include size and book-to-market (Fama and French, 1993, 2015; Barillas and Shanken, 2018), profitability and investments (Fama and French, 2015), return on equity and asset growth (Hou and Zhang, 2015; Barillas and Shanken, 2018), equity issuance and post-earnings announcement drift (Daniel, Hirshleifer, and Sun, 2020), or momentum (Barillas and Shanken, 2018).

paper we examine this paradox and find that the low-volatility effect should be factored into factor models.

To better understand this paradox, we start by acknowledging that prior studies typically define the investment opportunity set by building on two important assumptions: (i) the long and short legs that constitute factor mimicking portfolios are considered equal, and (ii) investment frictions such as investability, transaction costs, or shorting costs, are absent. Although commonly unaddressed in the literature, the long and short legs of factor portfolios are composed of distinct stocks with different return drivers or risk exposures (Fama and French, 2018; Blitz, Baltussen, and van Vliet, 2020), limits to arbitrage (Stambaugh, Yu, and Yuan, 2012, 2015), and investment frictions (Blitz, Baltussen, and van Vliet, 2020). Factor returns tend to be lower and correlations higher for short legs of canonical factors (Fama and French, 2018; Blitz, Baltussen, and van Vliet, 2020), short sellers face greater limits to arbitrage than long buyers (Stambaugh, Yu, and Yuan, 2015), creating an "arbitrage asymmetry" between overpriced and underpriced stocks, and shorting stocks involves sizable frictions on investability. Consequently, long and shorts of factors may have different asset pricing implications, creating a potential factor pricing asymmetry.

Further, Detzel, Novy-Marx, and Velikov (2023) (henceforth DNV) highlight serious issues associated with ignoring real-world concerns in financial research, as investment frictions such as transaction costs significantly impact the realized performance of investors. An approach focused on gross returns favors factors that maximize model performance irrespective of their transaction costs potentially leading to misleading inferences. DNV show that accounting for transaction costs fundamentally alters the outcome when evaluating asset pricing models.

We start our analyses by constructing factor returns from well-known asset pricing models using U.S. common stocks from the CRSP database between January 1972 and December 2023. We build upon the standard factor testing framework which we expand by embedding investability considerations, such as the low accessibility of micro-caps and impact of data missingness.[5] Subsequently, we compare factor models by using the maximum Sharpe ratio as the primary criterion to assess a model's asset pricing power, following Barillas and Shanken (2017). If the tangency portfolio of a model's factors is mean-variance efficient, adding more factors does not increase the maximum Sharpe ratio. Hence, asset pricing models can be selected using this criterion. We consider the major factor pricing models examined in the literature: the models of Fama and French (1993, 2015, 2018); Hou and Zhang (2015); Barillas and Shanken (2018); Daniel, Hirshleifer, and Sun (2020).

In our baseline specification, we focus on long-short factors without considering transaction and shorting costs. Our findings are in line with the design of existing factor models: augmenting standard factor models with a low-volatility factor raises the maximum Sharpe ratio across various factor models by a mere 0.57%. Hence, the low-volatility adds no significant residual pricing power in a frictionless framework. Although the low-volatility factor has high average returns it tends to be subsumed by factors like profitability and investment factors, in line with Novy-Marx (2014) and Fama and French (2016). This result may partially explain why the low-volatility factor is excluded from well-known factor models.

Next, we depart from the standard long-short framework and recognize the asymmetry between long and short legs of factors. To account for factor asymmetry, we decompose long–short factor portfolios into their two market-hedged legs. For the long leg, we neutralize its market exposure by taking an offsetting short position in the market factor; for the short leg, we hold the short portfolio and offset its market beta with a long position in the market factor. By construction, the sum of these two hedged legs reproduces the original long–short return. For each factor model, we maintain the same set of factor portfolios as in the baseline analyses, but we allow the long and short legs to enter as separate factors to allow for their

[5] Several studies argue that equity anomalies are influenced by micro-caps (Fama and French, 2008; Hou and Zhang, 2020), which are typically inaccessible for most investors, since they have limited capacity and are expensive to trade. In addition, a commonly ignored aspect in empirical asset pricing research is data missingness. Evaluating factor models requires data availability on all factor metrics per stock or an explicit approach for dealing with missing data. For example, Bryzgalova, Lerner, Lettau, and Pelger (2024) show that missing data is prevalent in Compustat fundamentals data, especially when considering multiple metrics, and importantly, missing data is not random with stock returns depending on missingness.

different asset pricing implications.

This decomposition reveals that hedged long legs on average generate higher average returns than hedged short legs. When comparing "long-plus-short" factor models, where each hedged leg receives its own weight, we observe higher maximum Sharpe ratios relative to the "long-short" approach. Moreover, factor models improve by a substantial 11.9% when including hedged low and high volatility factor legs, with the hedged low-volatility leg commanding an average weight of 26.2%. In spanning regressions, all factor models show significant improvements at the 1% significance level when factoring in low-volatility. This decomposition offers a novel perspective on prior claims that the low-volatility premium is subsumed by profitability and investment factors (Novy-Marx, 2014; Fama and French, 2016). We find that the apparent subsumption arises due to the short side of the low-volatility factor: high-volatility stocks exhibit strong correlations with low profitability and high investment stocks, and thus add no incremental pricing power once those factors enter the model. In contrast, the hedged long side of low-volatility contains distinct economic information and advances the performance of factor models.

In our third specification, we embed transaction and shorting costs in factor composition and returns. Prior studies typically ignore these costs, favoring models with factors constructed to maximize gross returns. We compute factor portfolio returns - considering both legs - net of these costs using the bid-ask spread estimator of Hasbrouck (2009) and shorting fee measures. We show that accounting for costs changes factor performances substantially as different factors are impacted by investment frictions to varying extents, consistent with DNV. To illustrate, the PEAD factor has an average long-short return of 3.45% per annum, but earns -0.87% per annum after accounting for costs. Interestingly, the performance of the low-volatility factor remains robust, especially on the hedged long-side with average gross (3.18%) and net (2.70%) returns per annum, respectively. While the inference across factor models is impacted by cost considerations, the low-volatility factor consistently improves all factor models by an average of 13.0%. In spanning regressions, all factor models show significant improvements when factoring in low-volatility. In addition, short legs - including the high-volatility leg - receive little to no weight in mean-variance efficient portfolios, whereas the low-volatility leg receives 26.4% weight, on average. The best performing models are the augmented Fama-French model (Fama and French, 2015) with cash-based operating profitability and momentum, or the model shown by Barillas and Shanken (2018), both including the low-volatility factor. In other words, accounting for factor asymmetry factors in the low-volatility factor into asset pricing models, especially when accounting for real-life investment frictions.

Finally, we only consider long legs in our analysis to account for the practical frictions associated with implementing the short leg of factor strategies. Beyond shorting costs, investors face additional operational constraints. Some stocks may be difficult or impossible to short due to limited availability or the risk of recall. Moreover, legal barriers can further restrict short-selling, including (partial) bans. Institutional mandates may exclude short selling; many mutual funds and pension funds are prohibited by charter from engaging in short sales for speculative purposes. Given these considerations, we argue that analyzing the long leg of factors separately is useful for understanding factor models in practice. Interestingly, we find that dropping short legs from factor models yields largely similar models and maximum Sharpe ratios as models that consider short legs. In addition, all factor models significantly improve with the addition of the long low-volatility factor, and the best performing models remain the augmented Fama-French model and the model shown by Barillas and Shanken (2018) including the low-volatility factor while having similar factor weights and Sharpe ratios as in previous tests.

So far, our results are centered around the maximum Sharpe ratio criterion, inspired by Barillas and Shanken (2018), estimated in-sample. However, Fama and French (2018) argue that this metric can be upward biased, especially for models with more factors and smaller time dimensions. To address this concern, we complement our in-sample results with out-of-sample tests using ex-ante recursive mean-variance efficient portfolios or a bootstrapped-based model comparison exercise where we compute the probability that a factor model outperforms all other factor models. We find that our results hold up across these tests. The low-volatility factor improves all factor models significantly and is factored into factor models across the majority of simulation runs when accounting for factor asymmetry and real-life investment frictions.

Next, we consider several robustness tests. We examine results over extended samples starting in January 1930 or July 1963 for the factors that can be constructed across those periods. We find that the low-volatility factor provides added value in these deeper samples once accounting for factor asymmetry and investment frictions. Furthermore, we examine seven variations of the low-volatility variable, including market beta (Frazzini and Pedersen, 2014), idiosyncratic volatility (Ang, Hodrick, Xing, and Zhang, 2006), or leveraging the low-volatility legs (as in Frazzini and Pedersen, 2014). We find that factor models generally improve by factoring in the low-volatility factor across specifications after accounting for factor asymmetry and investment frictions.

Lastly, we account for methodological uncertainty in portfolio construction choices. Menkveld (2024) show that degrees of freedom in research design choices can lead to major variation in outcomes, resulting in "non-standard errors". Soebhag, Van Vliet, and Verwijmeren (2024) show that these errors are substantial in factor models and recommend researchers to conduct specification checks. Hence, we construct thousands of different versions of every factor in our sample across a set of twelve factor construction choices. Our findings indicate that the low-volatility factor adds to existing asset pricing models in the large majority of variations considered and is independent of a particular testing choice. Overall, we conclude that the low-volatility factor consistently and robustly improves the performance of existing asset pricing models when accounting for factor pricing asymmetry and investment frictions.

Our findings contribute to several important strands of literature. First, our study adds to the growing body of literature on the comparison and selection of asset pricing models. Barillas and Shanken (2017) argue that with traded factors, the comparison of asset pricing models can be reduced to a comparison of the Sharpe ratios, which we heavily use in our analyses. Fama and French (2018) use out-of-sample Sharpe ratios from bootstrap simulations to compare models. Kan, Wang, and Zheng (2024) show that in-sample estimated Sharpe ratios are overstated and recommends the use of out-of-sample Sharpe ratios. In our study, we conduct both in-sample and out-of-sample model comparison tests. Other related studies compare major factor models in a horse-race (e.g. Ahmed, Bu, and Tsvetanov, 2019, Hou, Mo, Xue, and Zhang, 2019, Detzel, Novy-Marx, and Velikov, 2023, Barillas and Shanken, 2018 and Hanauer, 2020), as do we. Closest to our study is DNV, who also conduct model selection of asset pricing models accounting for transaction costs. We extent on their work by accounting for multiple additional investment frictions: investability, arbitrage asymmetry, transaction costs (including shorting fees), and shorting constraints. Moreover, our focus is to assess to what extent asset pricing models improve when the low-volatility factor is included.

Second, we add to the extensive body of research on low-risk anomalies, which documents the empirical regularity that low volatility or low beta stocks tend to outperform their high-risk counterparts on a risk-adjusted basis, contrary to traditional asset pricing theory. This includes the extensive literature on the betting-against-beta factor (Frazzini and Pedersen, 2014; Novy-Marx and Velikov, 2022), and papers examining drivers of the low-volatility factor (e.g., Barroso, Detzel, and Maio, 2025). We contribute to this literature by showing (i) the importance of factor asymmetry of the low-volatility effect - its main results are driven by the long leg, (ii) the low-volatility effect is robust after accounting for costs and other investment frictions, and, most importantly, (iii) it should be factored into factor models.

The remainder of this paper is organized as follows. We describe the factor data and portfolio construction process in Section 2. Our main empirical results are presented in Section 3. Section 4 presents robustness tests. Finally, Section 5 concludes.

2 Data and Factors

In this section, we outline the data and methodology used in our empirical analysis. In Section 2.1, we describe how the low-volatility factor is constructed. In Section 2.2, we introduce our set of asset pricing models, followed by the data description in Section 2.3. Lastly, in Section 2.4, we provide several test specifications that incrementally account for factor asymmetry and investment frictions. Overall, our primary objective is to assess the model improvement after the inclusion of the low-volatility factor, including accounting for factor asymmetry and investment frictions.

2.1 Constructing the Low-Volatility Factor

We follow the standard approach in constructing long-short factor portfolios: we construct the low-volatility factor portfolio (VOL) using the standard deviation of daily excess returns over the previous 252 trading days. We use a 2×3 sorting procedure that resembles that of Fama and French (2015). At the end of each June, we categorize firms into Small (S) and Big (B) groups based on market equity depending on whether the firm's market equity is below or above the NYSE median size breakpoint. Independently, each firm is also sorted in three volatility groups based on prior 252-day return volatility at the end of month $t-1$. We use 30% and 70% breakpoints based on NYSE firms. The intersection of these two groups yields six portfolios.[6] We compute the value-weighted excess returns for each portfolio. We estimate 36-month rolling market betas by regressing the post-formation realized return of these portfolios onto market excess returns, inspired by Novy-Marx and Velikov (2022). Following Novy-Marx and Velikov (2022) and Barroso, Detzel, and Maio (2025), we define

[6] We obtain the following six portfolios: Small Low Volatility (SLV), Small Medium Volatility (SMV), Small High Volatility (SHV), Big Low Volatility (BLV), Big Medium Volatility (BMV), and Big High Volatility (BHV). Low (High) Volatility is the average of SLV (SHV) and BLV (BHV).

the VOL factor to target market beta-neutrality (in a zero-investment portfolio) as follows:

$$VOL_t = r_{L,t} - r_{H,t} - (\beta_{L,t-1} - \beta_{H,t-1}) * MKT_t, \tag{1}$$

where $\beta_{L,t-1}$ ($\beta_{H,t-1}$) denotes the prior month's market betas of the low- and high-volatility legs, respectively, and MKT_t is the value-weighted excess market return.[7] This procedure avoids the errors-in-variables and illiquidity problems inherent in the procedure of Frazzini and Pedersen (2014), which estimates betas as the holdings-weighted average beta estimates of the stocks they hold and subsequently leverages stock exposures using these betas (see Novy-Marx and Velikov, 2022).[8] In Section 4.2 we consider alternative definitions of the low-volatility factor, including leveraging the low-volatility legs using its portfolio betas, as in Frazzini and Pedersen (2014); Baltussen, van Vliet, and Van Vliet (2023).

2.2 Candidate Factor Models

To conduct model comparison tests including and excluding the VOL factor, we use a wide range of candidate models originating from Fama and French (2015), Hou and Zhang (2015), Fama and French (2018), Barillas and Shanken (2018), and Daniel, Hirshleifer, and Sun (2020). Table 1 provides summary statistics of the factors employed in the various factor models. The table includes the stock-level characteristic used for portfolio construction, the basic construction methodology, and the rebalancing frequency. A more detailed description of each factor is provided in Appendix A. None of the candidate factor models contain the VOL factor. In our study, we construct all factor models following similar procedures as mentioned by the original authors. However, to limit degrees of freedom across factor models and avoid resulting differences in factor performances due to methodological choices,

[7] To avoid excessive hedging, we cap portfolio betas between 0.25 and 2.0, and impose that the maximum beta for low-volatility portfolios is restricted to 1.

[8] In the absence of market exposure hedging using $(\beta_{L,t-1} - \beta_{H,t-1}) * MKT_t$, the VOL factor would yield statistically insignificant average returns (as it runs at a materially negative market beta), despite delivering a significantly positive alpha relative to the market.

we consistently use the 30th and the 70th percentile as breakpoints, following Fama and French (1993).[9]

2.3 Data

To construct the low-volatility and other factors, we obtain return and price data for U.S. equities from the Center for Research in Security Prices (CRSP). Accounting information is retrieved from the Compustat Annual and Quarterly Fundamental Files. Our sample consists of stocks listed on the NYSE, AMEX, and Nasdaq with share codes 10 or 11. We exclude financial firms and firms with negative book-to-market ratios from our sample. The sample period spans January 1972 to December 2023. We start in 1972 as we require quarterly earnings announcement dates (to construct the price earnings announcement drift factor) and quarterly book equity data (to construct the return on equity factor). To construct the volatility factor, we use return data starting in January 1968.

2.4 Model Specifications

In this paper, we critically re-examine the role of the low-volatility factor in popular asset pricing models, accounting for factor asymmetry and investment frictions. To this end, we introduce a range of universe specifications that incrementally add various investment frictions to our empirical analysis. We consider four specifications in total, discussed below.

2.4.1 Long-minus-Short Factors

The standard practice in academic factor portfolios is to construct hypothetical long positions in stocks with attractive characteristics offset with short positions in stocks with unattractive characteristics. Our baseline specification mimics this 'standard' academic setting that often ignores investment frictions.

[9]We refer to Soebhag, Van Vliet, and Verwijmeren (2024) and Section 4 for impact of this choice.

Departing from most academic studies, we impose two additional data filters to account for investability. Several studies argue that equity anomalies are influenced by micro-caps (Fama and French, 2008; Hou and Zhang, 2020), which are typically inaccessible for most investors due to limited capacity and high trading costs. Micro-caps are on average only 3% of the market value of the CRSP universe, but account for about 60% of the total number of stocks (Hou and Zhang, 2020). Micro-caps have been shown to exhibit the highest cross-sectional standard deviations in returns, are more likely to end in the extreme portfolios of anomaly sorts (Fama and French (2008)), and may bias anomaly returns upwards. Due to high transaction costs and illiquidity, anomaly returns in micro-caps are unlikely to be achievable in practice. Nevertheless, micro-caps are typically included in many studies. Hence excluding micro-caps provides a more realistic investable universe. To address this concern, we filter out micro-caps by excluding stocks below the 20th NYSE size percentile in the sorting month.

In addition, a commonly overlooked issue in empirical asset pricing research is data missingness. Evaluating factor models requires data availability on all factor metrics per stock or an explicit approach for dealing with missing data. Stock characteristic data is often incomplete, especially for smaller or less frequently traded firms. Prior studies are often silent about their approach to dealing with missing data. For example, missing data are not random and stock returns depend on missingness (Bryzgalova, Lerner, Lettau, and Pelger, 2024).[10] To address this concern, we require stocks to have non-missing data for at least 9 out of 13 characteristics in the sorting month, and exclude all other stocks from our sample. This filter helps balance the trade-off between coverage and quality, yielding a cleaner and more robust sample for empirical analysis. As such, our factors are constructed using stocks that are accessible for most investors and well-covered in terms of data availability.[11]

[10] Bryzgalova, Lerner, Lettau, and Pelger (2024) show that missing data are very prevalent in Compustat fundamentals data, especially when considering multiple metrics, and importantly, missing data are not random as stock returns depend on missingness.

[11] The results presented in this paper are not materially impacted by these two data filters.

Figure 1 plots the number of stocks over time, with and without the micro-cap and data completeness filters. In the baseline specification, the sample begins with approximately 1,700 stocks, peaks at around 5,100 in 1999, and declines to roughly 3,000 by the end of the sample period. Imposing the data completeness filter ("Data") reduces the sample by approximately 200 stocks per year, on average. The largest reduction occurs when excluding micro-caps, as reflected by the "Micro" line in the figure. Consistent with Fama and French (2008), this filter removes about 60% of the original sample. Applying the data completeness filter on top of the micro-cap exclusion eliminates an additional 50 stocks per year, suggesting that missing data are concentrated in the micro-cap segment.

2.4.2 Long-plus-Short Factors

The "Long-minus-Short" specification assumes that both factor legs contain information that is relevant for investor portfolios and for understanding asset prices. We depart from this assumption by acknowledging that long and short legs are composed of distinct stocks with different return drivers, risk exposures, limits to arbitrage, and investment frictions (e.g., Stambaugh, Yu, and Yuan, 2012, 2015; Blitz, Baltussen, and van Vliet, 2020). Consequently, they may have distinct asset pricing implications, creating a potential factor pricing asymmetry between the long and short legs of factors. Fama and French (2018) and Blitz, Baltussen, and van Vliet (2020) provide first evidence of the impact of this factor asymmetry on factor comparisons, as typically short legs of factors are more correlated.[12] Further, Stambaugh, Yu, and Yuan (2015) document that short sellers face greater limits to arbitrage than long buyers, creating "arbitrage asymmetry" between overpriced and underpriced stocks. In addition, shorting stocks involves additional frictions that affect investability, such as short-selling related risks and costs. Factor models are based on the unique pricing power of a factor and assume frictionless arbitrage. These assumptions may not hold when accounting for real-world frictions.

[12] Related, Daniel and Moskowitz (2016) find that the short leg of the momentum factor tends to be the main driver of momentum crashes.

Hence, we argue that it is important to examine the long and short legs of factor premiums separately. For our second specification, we disentangle factor premiums into long-leg and short-leg premiums. Each long-short factor can be broken down as follows:

$$R_{L-S,t} = R_{L,t} - R_{S,t} = (R_{L,t} - R_{mkt,t}) + (R_{mkt,t} - R_{S,t}) \tag{2}$$

where $R_{L,t}$, $R_{S,t}$ and $R_{mkt,t}$ are the long factor leg, the short factor leg, and the value-weighted market return. We refer to $(R_{L,t} - R_{mkt,t})$ and $(R_{mkt,t} - R_{S,t})$ as the hedged long and short leg, respectively. For the volatility factor, the breakdown is based on equation 1, where the long leg is $(R_{L,t} - \beta_{L,t} R_{mkt,t})$ and the short leg is $(\beta_{S,t} R_{mkt,t} - R_{S,t})$.[13] For each factor model, we maintain the same set of factor portfolios as in the "Long-minus-Short" case, but we allow the long and short legs to enter a factor model separately to account for the gross exposure of each leg. Note that in practice highly liquid and cost-efficient index futures can be used to hedge out the market exposure of each factor leg.

2.4.3 Factors Net of Transaction and Shorting Costs

Accounting for costs is a first-order concern for investors in practice. An approach focused on gross returns favors factors that maximize model performance, irrespective of their transaction costs, potentially leading to misleading inferences as returns do not represent what is realistically achievable by investors. A model's ability to price assets reflects how close its factors come to spanning the efficient frontier. If some combination of the model's factors are ex post mean-variance efficient, then no other asset can be used to improve performance. However, investment frictions, such as transaction and shorting costs, can influence inference on factor models. In fact, strategies that require high transaction costs can improve

[13]Note that we opt for the market as the hedging portfolio. As pointed out by Blitz, Baltussen, and van Vliet (2020), this might result into a size effect distortion: 50% of the long-leg consists of long small stocks, whereas 50% of the short-leg shorts small stocks. To overcome this problem, a neutral hedging portfolio can also be defined as 50% big stocks (B) plus 50% small stocks (S). We find similar results when using this hedging portfolio.

the mean-variance frontier before costs but may fail to do so after costs, as shown by DNV, as arbitrage capital can only be expected to eliminate true abnormal trading opportunities. One could argue that the right factor model should be able to explain returns that fall outside the bounds given by such frictions. We address these concerns in our third specification by accounting for transaction costs and shorting costs in factor portfolios.

We follow the approach of DNV and estimate individual stock-level transaction costs using the method of Hasbrouck (2009). This procedure yields effective spreads that are highly correlated (≥95%) with those from the high-frequency Trade and Quote (TAQ) database and allows for an estimation of effective spreads for public companies in the CRSP database using their daily price series. The procedure entails estimating transaction costs using a Bayesian-Gibbs sampler on the generalized stock price models of Roll (1984).

Investors in practice do not engage in naive portfolio construction, but reduce transaction costs by cost-mitigating techniques that eliminate unrewarded turnover. A cost-mitigation technique should reduce unnecessary turnover while maintaining a high exposure to a given strategy's underlying signal. Novy-Marx and Velikov (2016, 2019) compare commonly used mitigation strategies, and finds that a banding strategy is most effective in reducing transaction costs while maintaining exposure to the underlying factor used to select stocks. As such, we apply a banding strategy with a bandwidth of 10% around the 30th & 70th percentiles when constructing portfolios.[14] We compute portfolio-level effective spreads, for a given leg, as follows:

$$TC_{leg,t} = \sum_{i=1}^{N_t} |W_{i,t} - W_{i,t-1,end}| * c_{i,t} \tag{3}$$

where $c_{i,t}$ denotes the estimated transaction cost for stock i in period t.

[14]For example, when a stock enters the 80th percentile based on a certain characteristic, we buy the stock and hold this until it falls out of the hold range to below the 60th percentile. For market capitalization, for which we use the NYSE median as a breakpoint, we change positions when a stock falls below (above) the 40th (60th) percentile on market capitalization. Note that banding affects both portfolio-level gross returns and transaction costs.

Second, we account for shorting costs on the short leg of factors. Stocks in the short leg typically have substantial shorting fees, and anomalies might even disappear for stocks with low lending fees (Beneish, Lee, and Nichols, 2015; Drechsler and Drechsler, 2014; Muravyev, Pearson, and Pollet, 2025). Muravyev, Pearson, and Pollet (2025) report an average borrow fee of 1.64% per annum, while Drechsler and Drechsler (2014) report an average annual shorting fee of 0.84% per annum, which, however, increases to approximately 1.24% for the bottom three deciles—corresponding to the short legs of long-short strategies over a set of eight anomalies.[15] Since we do not directly observe shorting fees in our sample, we assume a (slightly lower) fixed shorting fee (SF) of 1.00% per annum on the short leg of a factor. Note that using higher shorting fee numbers would decrease the attractiveness of short legs of factors. Moreover, elevated shorting costs for short legs of factors are especially pronounced for anomalies tied to high-volatility stocks, such as those based on beta or idiosyncratic volatility (see Drechsler and Drechsler, 2014; Muravyev, Pearson, and Pollet, 2025), and hence our approach might understate the actual shorting fee for HV stocks and thereby overstate its attractiveness.

Total portfolio costs for the long-short portfolios are equal to the sum of the transaction costs and shorting fee of each leg. Hedged net returns of a long (short) leg of a factor f are then given by[16]:

$$NetRet_{long,t} = Ret_{long,t} - Ret_{mkt,t} - TC_{long,t} \tag{4}$$

$$NetRet_{short,t} = Ret_{mkt,t} - Ret_{short,t} - TC_{short,t} - SF_t \tag{5}$$

[15] Other studies find roughly similar levels of shorting fee. Kolasinski, Reed, and Ringgenberg (2013) find annual costs generally above 100 bps between 2003 and 2007. Porras Prado, Saffi, and Sturgess (2016) find average value-weighted loan fees of 116 bps per year between 2006 and 2010. Cohen, Deither, and Malloy (2007) use proprietary lending data from a large institution and found average loan fees of around 400 bps for small-cap stocks and 40 bps for large-cap stocks in the 1999–2003 period. Beneish, Lee, and Nichols (2015) find a mean loan fee of 33.2 bps for the 10% easiest-to-borrow stocks between 2004 and 2013; the amount quadrupled for the next 10% easiest-to-borrow stocks.

[16] Following DNV we assume that the market portfolio is free of transaction costs.

2.4.4 Net Long-Market Factors

In addition, the standard academic approach studies long-short factor returns under the assumption that both legs contain information that is relevant for portfolios and for understanding asset prices. However, multiple studies argue that short-selling faces several constraints (see, for example, Miller (1977), Shleifer and Vishny (1997), and Chen, Hong, and Stein (2002)), especially for short legs of factors (e.g., Drechsler and Drechsler, 2014; Muravyev, Pearson, and Pollet, 2025).[17]

This implies that mispricing on the short-leg may be considerably harder to correct than on the long-leg. In addition, stocks can be sold short to a limited extent, not at all, or may be recalled unexpectedly (see D'avolio, 2002 and Geczy, Musto, and Reed, 2002). Furthermore, short-selling exhibits additional risks, including unlimited loss potential, short squeezes, counterparty risk, and reputational concerns (Angel and McCabe, 2009). Finally, legal impediments to short selling may exist. For instance, many countries have either a partial or a full ban on short selling. Additionally, some mutual funds in the industry are not allowed to short-sell by charter. As such, building efficient portfolios to capture the short leg of factor premia may bring in additional risks and trading hurdles.

In light of these considerations, we argue that studying the long-only dimension separately reflects an important view of investors who harvest common factor premia. Hence, our fourth specification, "Net Long-Market", considers only the long legs (market-hedged) net of trading costs.

[17] Several studies have found that stocks designated for shorting by a factor strategy (i.e., the short legs) are harder and more expensive to short. Geczy, Musto, and Reed (2002) show that growth stocks, loser stocks, and small-cap stocks are significantly harder to short than other stocks. Beneish, Lee, and Nichols (2015) find that the short returns for several accounting-based anomalies are attributable to special stocks; the short sides of the profitability and investment stocks were present only among hard-to-borrow stocks, and their returns became statistically indistinguishable from zero once shorting costs were accounted for. Drechsler and Drechsler (2014) find more than triple the shorting fees for the short-leg value, momentum, volatility-related, and profitability portfolios, and constraints on shorting execution substantially reduce the profitability on the short side; profits disappear altogether for stocks with low lending fees (see also Muravyev, Pearson, and Pollet, 2025).

3 Main Results

In this section, we present our main empirical results on factor model comparisons. We start with summarizing factor returns across the various specifications in Section 3.1. Next, we focus on in-sample model comparisons with maximal testing power to assess the added value of the VOL factor to a range of asset pricing models in Section 3.2. We use both maximum Sharpe ratio tests (see Barillas and Shanken, 2017; Fama and French, 2018) and test on how VOL expands the efficient frontier across factor models (see Novy-Marx and Velikov, 2016). Lastly, in Section 3.3, we employ out-of-sample techniques of Fama and French (2018) for model comparison across factor models.

3.1 Factor Performances

Table 2 reports the average annualized returns for a broad set of commonly studied factors over the period from January 1972 to December 2023. The table distinguishes between long-short, long leg, and short leg of factors and considers both gross (Panel A) and net (Panel B) returns, with net returns accounting for transaction and shorting costs. In Panel A, the largest long-short average return is observed for the FIN (Financing) factor, yielding 7.18% per annum, followed closely by the UMD (Momentum) factor, at 5.41%. The VOL factor ranks third, with an average long-short return of 5.01% per annum. In contrast, CMA and HML_d deliver relatively modest returns of 1.92% and 2.09% per annum.

Comparing the long and short legs of factor premia, we observe that for most factors, the hedged long returns are higher than the hedged short returns. For example, long momentum stocks earn 3.67% compared to 1.74% for low momentum stocks. Likewise, low volatility stocks earn 3.18% outperformance per annum, whereas high volatility stocks earn 1.83% per annum. Some other factors, such as HML and CMA, even exhibit near-zero average returns on the corresponding short leg. Overall, on a gross basis, the short leg of factors seems to contribute less than the long leg to the total factor return, reflecting an asymmetry in factor performance.

Accounting for transaction and shorting costs substantially attenuates factor returns. Panel B shows that FIN remains the factor with highest long-short return, averaging 3.83% per annum. The VOL factor has the second highest average return of 3.22% per annum. UMD, characterized by high turnover, incurs greater trading costs and delivers a post-cost return of 2.17% per annum. Notably, PEAD significantly deteriorates in performance after accounting for costs, with an average net return of -0.87% per annum. Distinguishing the long and short legs, we find that most short factor legs actually exhibit negative net returns.

These summary statistics suggest that while several factor strategies generate economically significant gross returns, only a subset remains profitable on a net basis, and that most of the net performance stems from the long side of factor returns. Moreover, the low-volatility factor remains robust after accounting for costs, driven by the outperformance of the long leg.

3.2 Model Comparison using In-sample Tests

3.2.1 Maximum Sharpe Ratios

In this section, we evaluate the performance of various asset pricing models with and without the VOL factor. We use the maximum Sharpe ratio (SR) as our primary model evaluation metric. The ability of an asset pricing model to price assets depends on the extent to which its factors span the mean-variance efficient portfolio. If the tangency portfolio of the factors in an asset pricing model is mean-variance efficient, then adding additional factors does not improve the maximum squared Sharpe ratio. Gibbons, Ross, and Shanken (1989) and Barillas and Shanken (2017) show that the asset pricing model with the highest squared Sharpe ratio of the model factors best prices the cross-section of asset returns. The maximum

Sharpe ratio is given by:

$$SR = \sqrt{(max_{\omega} \frac{\omega' \mu}{\sqrt{\omega' \Sigma \omega}})^2} \tag{6}$$

where μ and Σ denote the mean return of factors and the variance-covariance matrix, respectively. Note that for ease of interpretation we take the square-root of the squared Sharpe ratio in what follows, a choice that maintains the ordering of competing asset pricing models.

In the absence of transaction costs, the maximum Sharpe ratio is obtained by computing the mean-variance efficient (MVE) portfolio using all factors within factor model f, which has a closed-form solution. In case of net returns, the optimization problem is subject to the constraint that the weights ω_i are non-negative and sum to one, following DNV.

We compute the maximum Sharpe ratio for each candidate factor model with and without the VOL factor under each specification, and report the results in Table 3. We also report the percentage increase in the maximum Sharpe ratio ($\%\Delta SR$) after the inclusion of the VOL factor relative to the factor model without the VOL factor, and the MVE weight of the VOL factor (ω_{VOL}) or its long and short legs (ω_{LV} and ω_{HV}, respectively) in each asset pricing model.

Long-minus-Short. In Panel A of Table 3, we assess model improvements after VOL inclusion using the standard "Long-minus-Short" specification without any consideration for factor asymmetry or investment costs. Without the VOL factor, we find that the DHS factor model results in the highest maximum Sharpe ratio (1.25), followed by the BS model with a Sharpe of 1.23. Turning to the VOL factor, we find that its added value to factor models is limited. Fama-French factor models marginally improve by between 0.18% and 1.96% when including the volatility factor, with MVE weights ranging from 2.38 and 8.92%. Likewise, the remaining factor models also do not improve much when including the VOL factor. For example, the BS 6-factor model improves marginally (0.24%) and even assigns a negative weight to the VOL factor ($\omega = -2.72\%$). Overall, under the classical "Long-minus-Short" view, the VOL factor is spanned by other factors included in the factor models that we consider, even though VOL ranks third in terms of average returns across all factors considered. Although the low-volatility factor has high average returns, it tends to be subsumed by other factors like profitability and investment factors, in line with the earlier findings of Novy-Marx (2014); Fama and French (2016). This result partly explains why the VOL, even though popular in the industry, has not been included in major asset pricing models to date. However, a limitation of this view is that it does not account for factor asymmetry, nor do the results reflect what is actually achievable in practice by investors.

Long-plus-Short. In Panel B, we examine the "Long-plus-Short" view, where both hedged long and short factors are used simultaneously in the factor models. Under this view, each leg of a factor can have its own weight. We compare the factor models with and without the inclusion of the VOL legs: LV and HV (both market-hedged). First, we observe that the Sharpe ratios of all factor models improve relative to the "Long-minus-Short" models, with the Sharpe ratio increasing on average from 1.11 to 1.28 across all factor models considered. Interestingly, on average 42.77% weight is assigned to short legs of factors, and hence the majority of weight (57.33%) to long legs. Second, we find that the optimal Sharpe ratios across all "Long-plus-Short" models improve with the inclusion of both volatility legs. For example, Fama-French factor models improve maximum Sharpe ratios between 10.50% and 19.25% and assign weights between 25.48% and 30.65% to the hedged low-volatility leg. For non-FF models, the LV factor obtains sizable weights between 15.59% and 35.63%. On average "Long-plus-Short" factor models increase their Sharpe ratios by a sizable 11.90% and assign a weight of 26.23% to low-volatility.

Interestingly, this decomposition offers a novel perspective on prior claims that the low-volatility factor is subsumed by profitability and investment factors (Novy-Marx, 2014; Fama and French, 2016). Once accounting for different asset pricing implications of long and short legs of the low-volatility factor, we find a strong nuance: these prior findings arise due to the short side of the low-volatility factor. High-volatility stocks exhibit strong correlations with, for example, low profitability and high investment stocks, and thus add no incremental pricing power once those factors enter the model. In contrast, the hedged long side of the low-volatility factor contains distinct economic information and advances the performance of factor models.

Net Long-plus-Short. Subsequently, we account for transaction costs on both legs and shorting costs on the short leg in Panel C. Optimal Sharpe ratios of factor models become naturally lower once we account for costs, but they are more realistic in terms of what is achievable to the investor. On average, Sharpe ratios in Panel C drop by 33% compared to those in Panel B. Accounting for costs decreases the attractiveness of most short legs, as the weight to short legs averages to a relatively small 7.10%. Overall, we find that the highest Sharpe ratios are obtained for the augmented Fama-French model (Fama and French, 2018). with cash-based operating profitability and momentum factor, and the model of Barillas and Shanken (2018). Furthermore, non-FF factor models tend to experience a larger drop in Sharpe ratios after accounting for transaction costs. This is because most factors in those models are rebalanced monthly, leading to higher turnover and transaction costs.

Most importantly, we find that the volatility factor, on a net-of-cost basis, improves model performances across all factor models. The Sharpe ratio improvement across factor models averages 13.01%, and ranges between 9.35% and 38.47%. Further, optimal weights to the long leg, LV, averages 26.36% and ranges between 18.96% and 38.47%. In contrast, weight to the short leg of VOL, HV, is zero across all factor models. The best performing models remain the augmented Fama-French model (Fama and French, 2015) with cash-based operating profitability and momentum, or the model shown by Barillas and Shanken (2018), both including the low-volatility factor. In other words, accounting for factor asymmetry and trading costs *factors in* the low-volatility factor into asset pricing models.

Net Long-Market. Finally, we adopt a long leg-only approach, thus only considering long legs, in our analysis to account for the practical frictions associated with implementing the short leg of factor strategies. Panel D presents the results. Excluding the short leg essentially drops half of the number of portfolios in each factor model, but yields largely similar models and maximum Sharpe ratios as the models that also considered short legs. On average, we find that optimal Sharpe ratios of each factor model drop by a small 2%. For some factor models, the optimal Sharpe ratios do not even change, since short legs were assigned 0% in the mean-variance optimal portfolio. For HXZ and DHS, we find a relatively larger drop in the optimal Sharpe ratio compared to Panel C, since the short legs in those models receive considerable non-zero weights. Similar to Panels B and C, we find that the addition of the low-volatility leg results in substantial factor model improvement. On average, we find that factor models improve by 17.03%, and LV receives a considerable weight of 29.07%. The augmented Fama-French factor model and the model of Barillas and Shanken (2018) *including* the low-volatility leg remains the model with the highest optimal Sharpe ratios.

In sum, while traditional asset pricing tests — assuming frictionless markets and unconstrained shorting — find only marginal contributions from a standalone volatility factor, our approach displays a different picture. Once factor asymmetry and trading frictions are accounted for, the low-volatility leg consistently delivers economically and statistically significant Sharpe improvements across factor models. This reconciliation between the academic perspective and the sizable demand in the investment industry for low-risk exposures underscores that the true value of the low-volatility factor emerges when portfolios are constructed using similar approaches and constraints as faced by real-world investors.

3.2.2 Efficient Frontier Expansion

The results from the previous sub-section have shown that VOL improves the performance of an asset pricing model when factor asymmetry and investment frictions are taken into account. In this subsection, we test the extent to which the model including the low-volatility factor ("$M0+VOL$") expands the efficient frontier relative to the model excluding the factor

("$M0$").

To this end, we implement the multi-factor version of the generalized alpha of Novy-Marx and Velikov (2016). We run a spanning regression of the excess returns of the ex-post MVE portfolio constructed from "$M0 + VOL$" on the returns of the MVE portfolio using the factors from model "$M0$". This approach allows us to express both economic significance, in terms of average alphas per year, and statistical significance of adding the low-volatility factor to existing factor models. Table 4 reports the results for each factor model under each specification. We report the annualized alphas (α) and corresponding t-statistics.

Long-minus-Short. In Panel A, under the classical "Long-minus-Short" specification, adding the VOL factor produces virtually no expansion of the efficient frontier. The annualized spanning alphas are small, varying between 0 bps and 13 bps per annum, and none are statistically significant. This finding mirrors our maximum Sharpe results in Table 3, confirming that when factors are combined in a zero-cost, frictionless manner the incremental information in the low-volatility factor is absorbed by the existing factor models.

Long-plus-Short. In Panel B, we consider the "Long-plus-Short" specification, where both legs of a factor can receive their own weight. Overall, we find that the addition of the low-volatility legs significantly expands the efficient frontier of factor models. Spanning alphas range between 0.25% (BS model) and 0.62% (HXZ model), all statistically significant at the 1% level. On average, the alpha of factor models increases by 0.43%.

Net Long-plus-Short. In Panel C, we consider all factor legs while accounting for transaction costs and shorting costs. We again find that the addition of the low-volatility legs significantly expands the efficient frontier of factor models. Spanning alphas are of similar magnitude as in Panel B, with an average of 0.44%, and attain high levels of statistical significance. Thus, accounting for factor asymmetry and investment frictions factors in low-volatility in existing factor models.

Net Long-Market. Lastly, in Panel D, we exclude all short legs of factors. Again, these results confirm earlier results presented in Table 3. The average spanning alpha increases to

0.56% and ranges between 0.35% and 1.08%. Furthermore, spanning alphas are statistically significant for every factor model. Especially for the HXZ and DHS model, we find substantial spanning alphas of 1.03% and 0.95% per annum, respectively. Consistent with our previous results, we find significant gains from adding the low-volatility factor to existing asset pricing models when taking factor asymmetry and real-life considerations into account.

3.3 Model Comparison using Out-of-Sample Tests

The results from the previous section show that the addition of the low-volatility factor substantially improves ex-post maximum Sharpe ratios for a range of factor models, once we depart from the classical “long-minus-short” view. Next, we turn to out-of-sample analyses.

In all our results so far, we have used full-sample estimation to compute maximum Sharpe ratios. One disadvantage of our analysis is that factors may obtain excessive weights in the MVE portfolio when these factors have high average returns relative to expected returns. This will cause MVE-based weights to be overfitted, and consequently may bias estimates of the maximum Sharpe ratio upward. The bias in estimating the maximum Sharpe ratio is especially problematic when comparing non-nested models. The bias becomes more pronounced for smaller samples, as the parameter estimates have more sampling error. To alleviate these concerns, we conduct two out-of-sample exercises. In sub-section 3.3.1, we estimate MVE weights recursively using past data. In sub-section 3.3.2, we conduct a bootstrapped-based simulation.

3.3.1 Recursive Estimation of MVE Portfolios

A simple way to reduce the bias obtained from in-sample (IS) estimation is to estimate MVE weights recursively using information only available at the time of the portfolio formation, i.e., in month $t-1$. Our recursive estimation requires a burn-in period of 10 years of data. As such, our factor returns start in January 1982. We repeat the analysis conducted in Table

3, but now for the recursively estimated MVE portfolios. The results are reported in Table 5.

In Panel A, we consider the "Long-minus-Short" specification. Overall, we find that the Sharpe ratios of factor models are lower than their in-sample Sharpe ratios from Table 3, suggesting that in-sample estimates are potentially upward biased. In line with the in-sample results, though, we find that adding the VOL factor does not result in improvements in the maximum Sharpe ratio of a factor model. In fact, we find that factor models marginally deteriorate out-of-sample (by -0.54%) in terms of out-of-sample Sharpe ratios when including the VOL factor. On average, the weight of the VOL factor is again small (4.86%), with a minimum and maximum of 0.01% and 7.94%, respectively.

In Panel B, we consider the "Long-plus-Short" specification. In line with the in-sample results, we find that maximum out-of-sample Sharpe ratios improve substantially (13.66%, on average) when including VOL. Sharpe ratios improve across all factor models, ranging between 4.88% (DHS) and 24.24% (FF5), and allocations to the low-volatility leg vary between 5.17% and 23.25%.

Accounting for transaction costs and shorting costs, as shown in Panel C, yields even stronger results with Sharpe ratio improvements between 15.58% (FF5$_c$) and 27.32% (HXZ). In Panel D, where we drop the short legs of portfolios, we find an average improvement of 30.15%, ranging between 16.58% (FF5M$_c$) and 74.67% (HXZ) across factor models. Moreover, allocations to the low-volatility leg are again sizable, varying between 16.67% (FF5$_c$) and 48.37% (DHS). Overall, our out-of-sample recursive estimation results align with the in-sample results from the previous section, with low-volatility improving factor models once accounting for factor asymmetry and costs.

3.3.2 Bootstrapped Out-of-Sample Tests

In addition to our recursive estimation, we also conduct bootstrapped simulations. We follow the procedure of Fama and French (2018) and split the 624 months of our sample period

(January 1972 to December 2023), into 312 adjacent pairs: months (1,2), (3,4),$\cdots$, (623, 624). In each bootstrap, we randomly select one month from the pair to be in-sample (IS), the other to be out-of-sample (OS), and then draw 324 random pairs with replacement. For each asset pricing model, we estimate the MVE weights as the weights on each factor that yield the maximum Sharpe ratio over the 324 IS months. Subsequently, we compute the OS portfolio return as the sum of the OS factor returns multiplied by the IS weights. For each factor model, for both IS and OS periods, we compute the percentage of simulation runs for which the factor model has the highest maximum Sharpe ratio across all factor models. We name this percentage the "win probability". In case of ties, there is no "winner" and hence the total win probability does not necessarily sum up to 100%. We perform 10,000 simulation runs

Table 6 presents the bootstrapped IS and OS "win" probabilities. The bottom row reports the total probability of selecting any factor model with VOL as the "winning" model ($Pr(VOL)$). For the "Long-minus-Short" specification, reported in Panel A, we find that DHS + VOL is selected in 53% of in-sample runs, followed by BS + VOL at 46%. However, the IS estimates should be interpreted with caution, because the risk of overfitting is greater in shorter samples (312 months of in-sample data). The OS estimates indicate that the BS + VOL model is the "winning" model in 65% of runs, and that models that include VOL are selected in 74% of cases

In Panel B, where we consider the "Long-plus-Short" specification, we find that the OS Sharpe Ratio of the BS + VOL is greater than all other models in roughly 97% of the bootstrap samples. In Panel C, accounting for transaction costs and shorting fees, we observe that the OS Sharpe ratio of the HXZ + LV model is the highest in 31% of the sample runs, followed by the FF5M$_c$ + LV model (26%). Lastly, when dropping short legs of the factors in Panel D, we find that FF5M$_c$ + LV emerges as the factor model with the highest OS Sharpe ratio in 28% of the time. In 79% of the simulation runs, we find that the winning asset pricing model includes LV. Overall, both our IS and OS bootstrap sample results confirm that including the VOL factor enhances asset pricing model performance when accounting for real-world considerations.

4 Robustness

Results in the previous section have shown that adding the low-volatility factor materially improves the performance of prominent factor models once one accounts for factor asymmetry and trading frictions across in-sample and out-of-sample tests. In this section, we run four additional robustness tests to further corroborate our findings and ensure that the observed improvements are not driven by specific sample periods or model specifications. First, in Section 4.1 we consider pre-1972 evidence of the VOL factor. Second, we consider other definitions or measures of the low-volatility factor, such as betting-against-beta and idiosyncratic volatility, in Section 4.2. Lastly, we perform a large-scale specification check across a broad set of portfolio design choices in Section 4.3.

4.1 Pre-1972 Sample Evidence

Our results so far are based on the sample period from January 1972 to December 2023. In this section, we extend our analysis by utilizing data that reaches back to the start of the CRSP sample. Expanding the sample period helps increase the power of our empirical tests and confirm robustness of the post-1972 sample results. We consider two extended sample periods: as of January 1930 or as of July 1963. As of 1972, we have access to the PEAD factor included in the DHS model, but availability of CMA, RMW or ROE factors starts as of July 1963. Further, as of January 1930 (after a 4-year burn-in period to estimate beta), only the Fama-French three-factor model, momentum, and the CAPM are available.

We summarize the results in Table 7 only for the "Long-minus-Short" and "net Long-Market" specifications, in Panels A and B, respectively. In Panel A, we find that the low-volatility factor significantly improves the CAPM between 1930 and 2023. Adding the VOL

factor increases the maximum Sharpe ratio by 33.92% and yields a spanning alpha of 2.79% per annum (t-statistic = 3.86). We observe comparable results when we consider the Fama-French 3-factor model. However, the VOL factor does not improve the 5-factor model (and its variants) in the sample period 1963-2023, in line with our results presented above. These results change when we account for factor asymmetry and trading costs, as shown in Panel B. In the net long-only specification, we find net annualized alphas that range between 0.38% and 2.01%, all being statistically significant at the 1% level. Overall, this longer sample evidence confirms that the low-volatility factor significantly improves factor model performances.

4.2 Other Low-Risk Definitions

So far, we have computed the low-volatility factor using past 252-day return volatility as a low-risk measure. In this section, we examine alternative definitions and low-risk measures. First, we construct a low-volatility factor based on 3-year monthly return volatility (VOL_{36M}). Increasing the look-back period to three years may yield a more stable volatility estimate, thereby reducing turnover and transaction costs of the low-volatility factor.

Second, our factor construction methodology does not embed any sector structure. Daniel et al. (2020) shows that the standard, unconditional, long-short approach leads to unpriced risks and exposures, and potentially excessive concentration in a few sectors. This, in turn, gives rise to sector-specific risk. Sector-concentrated portfolios have less diversification since stocks in the same sectors are correlated. To address this issue, we construct industry-neutral low-volatility factor portfolios, (VOL_{Ind}), to reduce sector-related concentration risks. Whereas unconditional factors sort stocks into portfolios by the raw return predictors, our industry-neutral factors are constructed using industry-adjusted predictors. Specifically, the industry-adjusted predictor subtracts the cross-sectional industry median of the raw predictor and scales the result by the within-industry range.

Third, we construct a leveraged low-volatility factor ($VOL_{Levered}$). Our approach thus far has focused on low-volatility zero-investment factor portfolios using market beta hedging, as in Novy-Marx and Velikov (2022); Barroso, Detzel, and Maio (2025). Another approach common in the literature is to achieve market neutrality by leveraging up (down) the low-volatility (high-volatility) leg using its market beta to target a market-neutral factor (e.g., Frazzini and Pedersen, 2014; Blitz, Baltussen, and van Vliet, 2020; Baltussen, van Vliet, and Van Vliet, 2023). More specifically, each month t we scale the low (high) volatility portfolio's excess return by its ex-ante market beta from month $t-1$.[18] Asset pricing theory and the maximum Sharpe ratio test build upon unlimited leverage assumptions (e.g., Gibbons, Ross, and Shanken, 1989), implying that the choice of leveraging the low-volatility factor to market-neutrality should not materially impact factor model inference.

Lastly, we construct four alternative low-volatility measures. Instead of using past 252-day return volatility, we use 252-day idiosyncratic return volatility ($IVOL$) (Ang, Hodrick, Xing, and Zhang, 2006), market beta (BAB) (Frazzini and Pedersen, 2014), downside beta ($DBETA$) (Ang, Chen, and Xing, 2006), or downside volatility ($DVOL$) (Post and Van Vliet, 2006). The factors are constructed in the same manner as our main VOL factor, targeting market neutrality following Novy-Marx and Velikov (2022); Barroso, Detzel, and Maio (2025). For the $IVOL$ factor, we sort by the idiosyncratic volatility at the end of the previous month, where idiosyncratic volatility is based on residuals obtained from regressing excess returns on the 3-factor model of Fama and French (1993) using past 252-day returns. For the BAB factor, we sort stocks by their market beta at the end of the previous month, where market beta is computed using past 252-day returns. For the $DBETA$ factor, we sort on the market beta (restricted to negative market days) using past 252-day returns. For the $DVOL$ factor, we sort on the downside volatility (negative stock return days) using past 252-day returns.

[18] We estimate beta using a 36-month rolling window and cap betas between 0.25 and 2.00 to limit the impact of estimation noise. Note that the resulting market beta of each leg equals unity, and market-neutrality of each separate factor leg is achieved by hedging out the excess market return. Further, note that this approach uses borrowing at the risk-free rate to achieve market-neutrality with zero net investment.

As in Section 3.2.2, we compute the multi-factor version of the generalized alpha, where we compute the MVE portfolio excess return series for models $M0$ and $M0 + LowRisk$, where $LowRisk$ is one of the aforementioned low-risk factors. For brevity, we only report spanning alphas and corresponding t-statistics for the "Long-minus-Short" specification and the "Net Long-Market" specification. We report the results in Table 8.

In Panel A, under the standard "Long-minus-Short" specification, adding any of the seven $LowRisk$ factors results in virtually no expansion of the in-sample efficient frontier. This finding is consistent with Table 4. Panel B shows the results for net returns under a long-only perspective. Unlike Panel A, all low-volatility factor measures exhibit economically and statistically significant spanning alphas. Further, the alternative VOL specifications perform comparably, each exhibiting significant alphas across all models. This shows that our main results are robust to using longer look-back periods, addressing sector concentration, applying leverage to equalize market exposure, or using idiosyncratic, beta or downside risk measures. These results further corroborate the notion that the low-volatility factor significantly improves asset pricing models once accounting for factor asymmetry and investment frictions.

4.3 Portfolio Design Choices

So far, we have focused on a specific set of portfolio design choices. In our final robustness test, we consider a broad range of portfolio design choices. Menkveld (2024) show that degrees of freedom in research design choices can lead to major variation in outcomes, resulting in "non-standard errors". Soebhag, Van Vliet, and Verwijmeren (2024) show that non-standard errors are substantial as portfolio design choices materially affect factor returns and model selection exercises. They recommend researchers to conduct specification checks to mitigate concerns that results are driven by selecting a particular set of choices. We also construct thousands of different versions of every factor in our sample across a set of 12 factor construction choices.

Specifically, we alternate between the following decisions: 30-70 breakpoints versus 20-80 (i), NYSE versus full-sample breakpoints (ii), including versus excluding financial firms (iii), applying industry neutralization or not (iv), independent versus dependent sorts (v), monthly rebalancing or yearly (at the end of June) (vi), value-weighting returns or equal-weighting (vii), including or excluding micro-caps (viii), remove or keep stocks with negative book-to-market ratios (ix), remove or keep stocks with prices below 5 dollars in the sorting month (x), include or exclude utility stocks (xi), and banding portfolio returns and transaction costs (xii).[19] Combining these 12 choices results in 4,096 unique specifications, and thereby different versions of factor returns. For each, we compute the returns and run our model selection exercises.

Figure 2 illustrates the distribution of Sharpe ratios for various non-market and non-size equity factors based on the 4,096 distinct factor construction specifications. Each density plot represents the empirical distribution of Sharpe ratios for hedged long-leg portfolios, with transaction costs accounted for. The shape of these distributions offers insight into the robustness and performance consistency of each factor under diverse construction methodologies. Most notably, the low-volatility factor exhibits an empirical Sharpe ratio distribution concentrated at higher Sharpe ratios, suggesting stronger and more consistent risk-adjusted returns across different portfolio specifications. For example, the low-volatility has the second highest mean (median) Sharpe ratio of 0.605 (0.612). The standard deviation of the Sharpe ratio of the low volatility factor is 0.10. In contrast, factors such as high PEAD and ROE show more dispersed distributions.

Lastly, we assess factor model improvements after the inclusion of the low-volatility factor across all choice sets. Table 9 reports the average maximum Sharpe ratio per factor model with and without the low-volatility factor, the alpha improvement of the MVE-efficient portfolio, and the fraction of choice sets for which the alpha is statistically significant at the 5% level. Panel A considers the Long-minus-Short factor models without accounting for

[19] All factors are constructed via 2-by-3 sorting on size and a factor characteristic.

factor asymmetry and costs. Average Sharpe ratios without the low-volatility factor ($\mathrm{SR}_{w/o}$) range from 1.14 (FF5) to 1.56 (BS6) and increase only marginally once the low-volatility factor is included. The fraction of specifications for which the spanning alpha is statistically significant (at the 5% level) does not exceed 35%. These results are consistent with our baseline analysis from Section 3.

In Panel B, we consider the Long-Market factor models net of costs. $\mathrm{SR}_{w/o}$ ranges between 0.82 (DHS) and 1.20 (BS6), and improve substantially to between 0.90 (DHS) and 1.32 (BS6) once we include the low-volatility leg. Spanning alphas range between 0.40% and 0.70% per year, and the fraction of specifications with statistically significant spanning alphas is between 69% and 97%. Thus, adding the low-volatility factor substantially improves the net performance of long-only factor models across thousands of testing variations. Overall, we conclude that the low-volatility factor consistently improves the performance of existing asset pricing models when accounting for factor pricing asymmetry and investment frictions.

5 Conclusion

Although extensively supported by academic research and adopted in the investment management industry, the low-volatility factor remains absent from factor models. We show that this paradox stems from factor asymmetry and investment frictions not sufficiently considered in most academic studies. Incorporating factor asymmetry and investment frictions reveals that the low-volatility factor deserves a central place in asset pricing models. While redundant in a frictionless setting, its long leg carries unique pricing information overlooked when treated symmetrically with its short leg. The low-volatility factor significantly improves all major factor models, increasing Sharpe ratios by 13–17% and receiving 26–29% allocation in efficient portfolios. Robustness checks, including out-of-sample validation, extended time samples, alternative measurements, and thousands of variations in factor construction methods, confirm the consistency of these findings.

Our study shows that the low-volatility factor plays a prominent role in asset pricing models once factor asymmetry and frictions are considered, with broader implications. We illustrate how departures from standard specifications can alter research conclusions. We encourage researchers to be mindful of investment frictions in asset pricing tests. While this paper addresses transaction and shorting costs, other frictions, such as tax efficiency, investment horizon, and slippage, remain largely unexplored. A valuable future avenue for research is to assess these frictions' impact on asset pricing models or develop frameworks that robustly reflect real-world settings, rather than an idealized world.

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Table 1: Candidate Asset Pricing Models

This table lists the non-market factors used by the asset pricing model, indicated by a ✓in the columns headed by the model names. We provide three properties of the factor construction methodology: the sorting characteristic (”Characteristic”), the rebalancing frequency (”Rebalancing”), and the sorting method (”Construction”). In each model, factor returns are defined as the equal-weighted average of the returns on the portfolios with high (or low) values of the primary sorting characteristic minus the equal-weighted average of the portfolios with low (or high) values. SMB returns are given by the simple average of the returns on all portfolios with low size minus the average of the returns on all portfolios with large size in three independent 2x3 sorts of stocks on size and each of the following characteristics: book-to-market, growth in book assets, and operating profitability (or cash profitability). ME returns are given by the simple average of the returns on all portfolios with low size minus those with large size in 2x3x3 sorts on size, growth in book assets, and return on equity. FF5 (FF5M) denote the Fama and French (2015) five-factor model (augmented with UMD). FF5$_c$ and FF5M$_c$ denote versions of the FF5 and FF5M, respectively, that use cash-based operating profitability instead of accruals operating profitability Fama and French (2018). HXZ denotes the Hou and Zhang (2015) four-factor model. BS denotes the Barillas and Shanken (2018) six-factor model. DHS denotes the Daniel, Hirshleifer, and Sun (2020) three-factor behavioural model.

Factor	Characteristic	Rebalancing	Construction	FF5	FF5M	FF5$_c$	FF5M$_c$	HXZ	BS	DHS
SMB	Market capitalization	Annual	2x3	✓	✓	✓	✓		✓	
HML	Book-to-market	Annual	2x3	✓	✓	✓	✓			
CMA	Growth in book assets	Annual	2x3	✓	✓	✓	✓			
RMW	Accruals operating profitability	Annual	2x3	✓	✓					
RMW$_{cp}$	Cash operating profitability	Annual	2x3			✓	✓			
MOM	$R_{t-12,t-2}$	Monthly	2x3		✓		✓		✓	
ME	Market capitalization	Monthly	2x3x3					✓		
IA	Growth in book assets	Monthly	2x3x3					✓	✓	
ROE	Quarterly returns-on-equity	Monthly	2x3x3					✓	✓	
HML$_m$	Book-to-market	Monthly	2x3						✓	
FIN	Net and composite share iss ance	Annual	2x3							✓
PEAD	4-day CAR earnings announcements	Monthly	2x3							✓

Figure 1: Universe Coverage Over Time

This figure shows the annual number of stocks included in the sample under various filtering criteria. "Base" refers to the full sample with no filters applied. "Data" removes stocks with fewer than 9 (out of 13) non-missing sorting characteristics in a given month. "Micro" excludes stocks below the 20th percentile of NYSE market capitalization. "Investability" applies both the Data and Micro filters simultaneously. The sample period spans from January 1968 to December 2023.

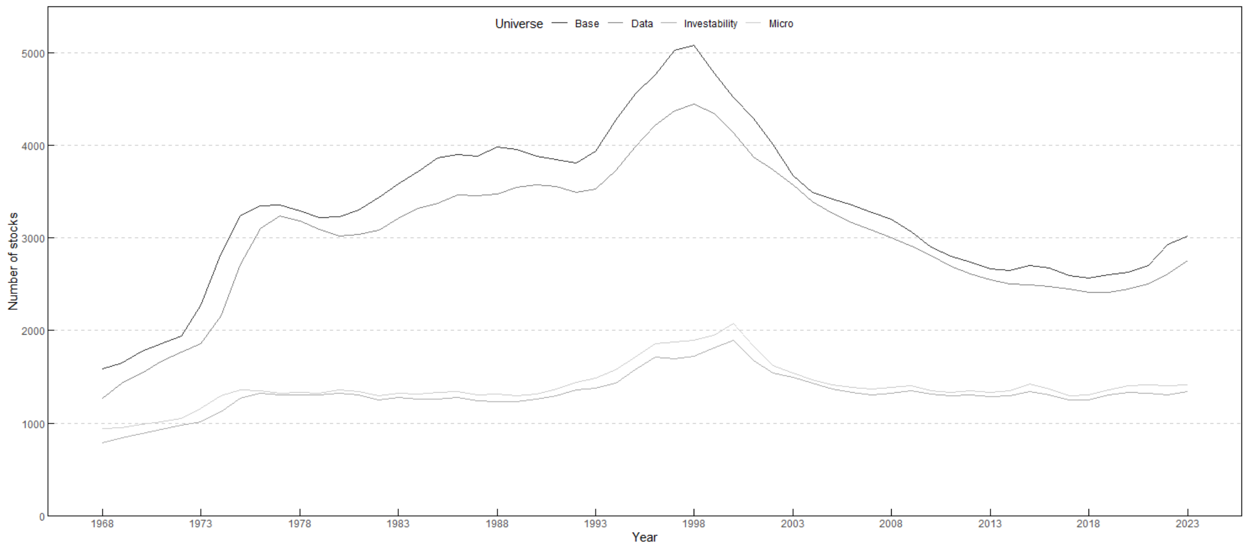

Table 2: Factor Summary Statistics

This table presents average annualized returns of non-market factors using gross (Panel A) and net returns (Panel B). Net returns account for transaction and shorting costs. VOL is the low volatility factor. SMB, HML, RMW and CMA denote the Fama and French (2015) size, value, profitability, and investment factors, respectively. SMB_c, UMD and RMW_c denotes the Fama and French (2018) size, momentum and cash profitability factor, respectively. ME, ROE, and IA denote the Hou and Zhang (2015) size, profitability, and investment factors, respectively. HML(m) denotes the monthly updated value factor of Asness and Frazzini (2013). FIN and PEAD are the financing and post-earning announcement drift factor of Daniel, Hirshleifer, and Sun (2020). "L–S", "L–M", and "M–S" refer to long-minus-short, long-minus-market (i.e., long leg), and market-minus-short (i.e., short leg) returns. The sample period is January 1972 to December 2023.

	Panel A: Gross			Panel B: Net		
	L-S	L-M	M-S	L-S	L-M	M-S
VOL	5.01	3.18	1.83	3.22	2.70	0.52
SMB	2.01	1.95	0.06	0.50	1.68	-1.19
SMB_{cp}	2.06	1.98	0.08	0.49	1.66	-1.18
HML	2.09	1.97	0.12	1.06	2.06	-1.00
HML_d	2.62	2.59	0.03	0.87	2.16	-1.29
RMW	3.07	2.15	0.92	1.19	1.90	-0.71
RMW_{cp}	4.28	2.86	1.42	2.02	2.19	-0.17
CMA	1.92	1.96	-0.03	0.70	1.97	-1.26
UMD	5.41	3.67	1.74	2.17	2.52	-0.35
ME	2.21	2.01	0.19	-0.12	1.22	-1.34
IA	2.80	2.20	0.59	1.07	1.81	-0.74
ROE	4.67	3.20	1.47	2.39	2.52	-0.13
PEAD	3.45	2.44	1.01	-0.87	0.92	-1.79
FIN	7.18	4.57	2.61	3.83	3.73	0.10

Table 3: Ex Post Mean-Variance Efficient Portfolios

This table presents the ex post maximum Sharpe Ratio of each factor model without ($SR_{w/o}$) and with (SR_w) the volatility factor. Panel A considers standard long-short factors ('Long-Short'). Panel B considers long and short legs of factors as separate portfolios in the opportunity set ('Long-Plus-Short'). Panel C is equivalent to Panel B, but accounts for transaction and shorting costs ('Net Long-Plus-Short'). Panel D considers only the long legs of factors while accounting for transaction costs ('Net Long-Market'). $\%\Delta SR$ is the percentage improvement in SR of a factor model when adding the volatility factor. ω_{VOL}, ω_{LV}, ω_{HV} are the weights of the volatility factor, its long leg, and short leg in the ex post mean-variance portfolio of a factor model. For Panel B and C, we add ω_{Short}, or the net allocation to short factor legs in the factor model (including the low-volatility factor). The column 'Avg.' summarizes the average results across all factor models. The sample period is January 1972 to December 2023.

	Panel A: Long-Short							
	FF5	FF5M	FF5$_c$	FF5M$_c$	HXZ	BS	DHS	Avg.
$SR_{w/o}$	0.87	1.03	1.09	1.19	1.08	1.23	1.25	1.11
SR_w	0.89	1.04	1.09	1.19	1.08	1.23	1.25	1.11
$\%\Delta SR$	1.96	0.85	0.39	0.18	0.34	0.03	0.24	0.57
ω_{VOL}	8.92	5.48	3.67	2.38	3.40	-0.98	-2.72	2.88
	Panel B: Long-Plus-Short							
	FF5	FF5M	FF5$_c$	FF5M$_c$	HXZ	BS	DHS	Avg.
$SR_{w/o}$	0.90	1.06	1.10	1.21	1.12	1.38	1.28	1.15
SR_w	1.08	1.22	1.23	1.34	1.25	1.50	1.37	1.28
$\%\Delta SR$	19.25	15.10	11.98	10.50	11.80	8.32	6.35	11.90
ω_{LV}	30.65	29.03	25.48	25.57	35.63	15.59	21.66	26.23
ω_{HV}	-12.01	-10.23	-10.81	-9.63	-18.05	-5.68	-11.96	-11.20
ω_{Short}	52.23	46.79	46.92	41.31	21.89	57.58	32.65	42.77
	Panel C: Net Long-Plus-Short							
	FF5	FF5M	FF5$_c$	FF5M$_c$	HXZ	BS	DHS	Avg.
$SR_{w/o}$	0.68	0.75	0.78	0.83	0.73	0.85	0.75	0.77
SR_w	0.79	0.87	0.85	0.92	0.86	0.94	0.82	0.86
$\%\Delta SR$	15.83	16.66	9.35	11.30	17.53	10.98	9.44	13.01
ω_{LV}	28.65	23.03	22.12	19.71	33.55	18.96	38.47	26.36
ω_{HV}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ω_{Short}	0.00	0.00	10.92	6.69	18.19	0.00	13.88	7.10
	Panel D: Net Long-Market							
	FF5	FF5M	FF5$_c$	FF5M$_c$	HXZ	BS	DHS	Avg.
$SR_{w/o}$	0.68	0.75	0.76	0.81	0.63	0.84	0.69	0.74
SR_w	0.79	0.87	0.84	0.92	0.84	0.94	0.79	0.86
$\%\Delta SR$	15.84	16.83	11.76	13.54	34.07	12.56	14.65	17.03
ω_{LV}	28.65	23.03	21.67	19.54	38.36	18.96	53.30	29.07

Table 4: Spanning Regressions

This table reports the annualized alpha obtained from regressing the ex post mean-variance efficient factor model including the volatility factor against the efficient factor model portfolio without the volatility factor. The row 'Avg.' summarizes the average results across all factor models. Panels are defined as in Table 3. The sample period is January 1972 to December 2023. The t-statistics ($t(\alpha)$) are Newey-West adjusted with 6 lags, and are reported in the second column of each Panel within parentheses. Asterisks indicate significance at the 10% (*), 5% (**) or 1% (***) level.

	Panel A		Panel B		Panel C		Panel D	
	α	t(α)	α	t(α)	α	t(α)	α	t(α)
FF5	0.13	(1.20)	0.50***	(3.90)	0.48***	(2.72)	0.48***	(2.73)
FF5M	0.06	(0.89)	0.49***	(3.92)	0.41***	(3.04)	0.42***	(3.03)
FF5$_c$	0.03	(0.64)	0.40***	(3.44)	0.32**	(2.28)	0.35**	(2.54)
FF5M$_c$	0.02	(0.46)	0.41***	(3.49)	0.32***	(2.67)	0.35***	(2.91)
HXZ	0.03	(0.59)	0.62***	(3.61)	0.70***	(3.16)	1.03***	(3.99)
BS	0.00	(0.19)	0.25***	(3.69)	0.29***	(2.84)	0.35***	(3.00)
DHS	0.03	(0.54)	0.36***	(2.96)	0.56**	(2.29)	0.95***	(2.69)

Table 5: Ex Ante Recursive Mean-Variance Efficient Portfolios

This table presents the maximum Sharpe Ratio of each factor model without ($SR_{w/o}$) and with (SR_w) the volatility factor. Each portfolio of factors, within a given factor model, holds factors in proportion to the optimal weights estimated using returns prior to portfolio formation in a recursive window. The data starts in January 1972, but a minimum of 10 years of data is required to estimate initial weights. Thus, the sample period is January 1982 to December 2023. Panels are defined as in Table 3. $\%\Delta SR$ is the percentage improvement in SR of a factor model when adding the volatility factor. ω_{VOL}^{avg} and ω_{LV}^{avg} are the average weights of the volatility factor or its long leg in the ex post mean-variance portfolio of a factor model. The column 'Avg.' summarizes the average results across all factor models.

	Panel A: Long-Short							
	FF5	FF5M	FF5$_c$	FF5M$_c$	HXZ	BS	DHS	Avg.
$SR_{w/o}$	0.73	0.82	0.92	0.97	0.90	0.90	1.19	0.92
SR_w	0.74	0.82	0.92	0.97	0.89	0.89	1.17	0.91
$\%\Delta SR$	0.78	0.03	-0.53	-0.77	-0.89	-0.96	-1.46	-0.54
ω_{VOL}^{avg}	7.55	7.94	4.37	5.55	6.35	0.01	2.28	4.86
	Panel B: Long-Plus-Short							
	FF5	FF5M	FF5$_c$	FF5M$_c$	HXZ	BS	DHS	Avg.
$SR_{w/o}$	0.68	0.78	0.92	0.99	0.89	1.02	1.10	0.91
SR_w	0.85	0.95	1.02	1.10	1.02	1.10	1.15	1.03
$\%\Delta SR$	24.24	21.82	11.08	11.65	14.65	7.30	4.88	13.66
ω_{LV}^{avg}	14.61	20.15	12.03	18.74	23.25	5.17	11.43	15.06
	Panel C: Net Long-Plus-Short							
	FF5	FF5M	FF5$_c$	FF5M$_c$	HXZ	BS	DHS	Avg.
$SR_{w/o}$	0.54	0.50	0.59	0.58	0.55	0.54	0.54	0.55
SR_w	0.69	0.70	0.69	0.70	0.70	0.65	0.62	0.68
$\%\Delta SR$	27.56	40.80	15.58	22.56	27.32	21.22	14.33	24.20
ω_{LV}^{avg}	24.42	22.68	15.05	16.64	30.04	10.77	27.32	20.99
	Panel D: Net Long-Market							
	FF5	FF5M	FF5$_c$	FF5M$_c$	HXZ	BS	DHS	Avg.
$SR_{w/o}$	0.54	0.53	0.57	0.57	0.43	0.58	0.53	0.54
SR_w	0.68	0.70	0.67	0.70	0.76	0.69	0.64	0.69
$\%\Delta SR$	27.01	30.25	16.58	22.35	74.67	19.26	20.92	30.15
ω_{LV}^{avg}	26.36	23.72	16.67	17.85	35.69	13.98	48.37	26.09

Table 6: Bootstrapped-based Model Comparison

This table reports the percentage of bootstrap simulations in which the model specified by the row header has the highest Sharpe ratio among all models in the run, or "win probability" ("Win"), both in-sample (IS) as well as out-of-sample (OS). Simulations are based on 10.000 runs. The IS and OS simulations split the 624 sample months of our sample period, January 1972 through December 2023, into 312 adjacent pairs. A simulation run draws a random sample with replacement of 312 pairs. In the IS simulation run, we randomly pick a month from each pair in the run. Subsequently, we calculate the IS mean-variance tangency portfolio weights and apply those weights in the unused months of the simulation pairs to compute the corresponding OS estimate of the Sharpe Ratio for the IS tangency portfolio. Panel A considers factor models without the volatility (VOL) factor, while Panel B adds the volatility factor to the opportunity set. The final row of Panel B ($Pr(VOL)$) presents the sum by column of the win probabilities across the models including the volatility factor.

	Long-Short		Long-Plus-Short		Net Long-Plus-Short		Net Long-Market	
	Win_{IS}	Win_{OS}	Win_{IS}	Win_{OS}	Win_{IS}	Win_{OS}	Win_{IS}	Win_{OS}
	Panel A: Factor Models Excluding VOL							
FF5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FF5M	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FF5_c	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
FF5M_c	0.00	0.12	0.00	0.00	0.00	0.03	0.00	0.02
HXZ	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
BS	0.00	0.04	0.00	0.00	0.00	0.04	0.00	0.02
DHS	0.00	0.10	0.00	0.01	0.00	0.11	0.00	0.02
	Panel B: Factor Models Including VOL							
FF5+V	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FF5M+V	0.00	0.00	0.00	0.00	0.10	0.00	0.15	0.01
$\text{FF5}_c + V$	0.00	0.01	0.00	0.00	0.00	0.01	0.00	0.01
$\text{FF5M}_c + V$	0.01	0.05	0.03	0.01	0.08	0.26	0.11	0.28
HXZ+V	0.00	0.01	0.00	0.00	0.00	0.31	0.00	0.24
BS+V	0.46	0.65	0.80	0.97	0.42	0.05	0.47	0.14
DHS+V	0.53	0.01	0.17	0.01	0.19	0.04	0.08	0.11
$Pr(VOL)$	1.00	0.74	1.00	0.99	0.79	0.67	0.81	0.79

Table 7: Model Comparison using Extended Sample Periods

This table reports the maximum Sharpe ratio obtained by the row model excluding ($SR_{w/o}$) and including (SR_w) the volatility factor. $\%\Delta SR$ shows the percentual change in max. Sharpe ratios. ω_{VOL} (ω_{LV}) is the weight of the (low) volatility factor in the ex post mean-variance portfolio of a factor model. α reports the intercepts (annualized) obtained from regressing the ex post mean-variance efficient factor model including the volatility factor against the efficient factor model portfolio without the volatility factor. The t-statistics ($t(\alpha)$) are Newey-West adjusted with 6 lags, and are reported in the last column of each Panel. Asterisks indicate its significance at the 10% (*), 5% (**) or 1% (***) level. The sample period spans January 1930 till December 2023 for all factor models, except for the five-factor models, which start in July 1963 due to data availability.

	Panel A: Long-Short					
	$SR_{w/o}$	SR_w	$\%\Delta SR$	ω_{VOL}	α	$t(\alpha)$
CAPM	0.44	0.59	33.92	54.44	2.79***	3.86
FF3	0.48	0.74	53.67	39.63	2.34***	5.48
FF5	0.85	0.87	2.23	9.45	0.14	1.35
FF5M	1.04	1.05	1.22	6.58	0.09	1.16
$FF5_c$	1.08	1.09	0.55	4.24	0.04	0.82
$FF5M_c$	1.22	1.22	0.39	3.43	0.03	0.75
	Panel B: Net Long-Market					
	$SR_{w/o}$	SR_w	$\%\Delta SR$	ω_{LV}	α	$t(\alpha)$
CAPM	0.44	0.69	58.18	83.45	2.01***	5.12
FF3	0.45	0.70	54.89	82.67	1.92***	5.15
FF5	0.61	0.77	24.57	49.80	0.90***	3.41
FF5M	0.70	0.87	23.80	34.32	0.68***	3.75
$FF5_c$	0.82	0.91	10.83	24.34	0.39***	2.77
$FF5M_c$	0.89	1.00	12.41	21.18	0.38***	3.21

Table 8: Other Low-Risk Measures

This table reports the intercepts (i.e., annualized alpha) obtained from regressing the ex post mean-variance efficient factor model including a low-volatility factor, mentioned in each row and as defined in the text, against the efficient factor model portfolio without this factor. The t-statistics ($t(\alpha)$) are Newey-West adjusted with 6 lags, and are reported within parentheses. Asterisks indicate its significance at the 10% (*), 5% (**) or 1% (***) level. Panel A considers standard long-short factors ('Long-Short'). Panel B considers only the long legs of factors while accounting for transaction costs ('Net Long-Market'). The sample period is January 1972 to December 2023.

	Panel A: Long-Short							Panel B: Net Long-Market						
	FF5	FF5M	FF5$_c$	FF5M$_c$	HXZ	BS	DHS	FF5	FF5M	FF5$_c$	FF5M$_c$	HXZ	BS	DHS
VOL_{252d}	0.13	0.06	0.03	0.02	0.03	0.00	0.03	0.48***	0.42***	0.35**	0.35***	1.03***	0.35***	0.95***
	(1.20)	(0.89)	(0.64)	(0.46)	(0.59)	(0.19)	(0.54)	(2.73)	(3.03)	(2.54)	(2.91)	(3.99)	(3.00)	(2.69)
VOL_{36M}	0.03	0.03	0.00	0.00	0.02	0.01	0.06	0.43***	0.41***	0.33**	0.35***	1.03***	0.35***	0.90***
	(0.58)	(0.58)	(0.01)	(0.10)	(0.45)	(0.34)	(0.82)	(2.66)	(3.11)	(2.53)	(3.01)	(4.06)	(3.07)	(2.66)
VOL_{Ind}	0.20*	0.08	0.04	0.01	0.04	0.00	0.00	0.61***	0.49***	0.39***	0.35***	0.71***	0.37***	0.71**
	(1.82)	(1.20)	(0.84)	(0.49)	(0.79)	(0.05)	(0.26)	(3.35)	(3.50)	(2.99)	(3.18)	(3.84)	(3.53)	(2.48)
$VOL_{levered}$	0.26	0.15	0.08	0.05	0.09	0.00	0.01	0.48***	0.41***	0.33**	0.32***	1.15***	0.32***	1.10***
	(1.60)	(1.30)	(0.97)	(0.82)	(1.05)	(0.08)	(0.38)	(2.59)	(2.82)	(2.35)	(2.64)	(3.81)	(2.78)	(2.58)
$IVOL$	0.12	0.05	0.01	0.00	0.01	0.01	0.03	0.56***	0.49***	0.41***	0.41***	1.12***	0.41***	1.10***
	(1.11)	(0.79)	(0.43)	(0.25)	(0.42)	(0.34)	(0.59)	(3.04)	(3.35)	(2.83)	(3.19)	(4.27)	(3.30)	(3.12)
BAB	0.20	0.08	0.11	0.04	0.07	0.00	0.00	0.25*	0.21*	0.19*	0.17*	0.65***	0.16*	0.43*
	(1.51)	(0.95)	(1.22)	(0.79)	(0.93)	(0.01)	(0.12)	(1.95)	(1.95)	(1.88)	(1.92)	(2.95)	(1.94)	(1.58)
$DBAB$	0.04	0.02	0.03	0.02	0.02	0.00	0.04	0.54***	0.48***	0.43***	0.43***	1.03***	0.40***	0.98***
	(0.67)	(0.53)	(0.59)	(0.50)	(0.43)	(0.11)	(0.74)	(2.84)	(3.10)	(2.82)	(3.08)	(4.07)	(3.11)	(2.62)
$DVOL$	0.04	0.02	0.00	0.00	0.00	0.05	0.04	0.41**	0.39***	0.30**	0.33***	0.98***	0.32***	0.85**
	(0.68)	(0.52)	(0.10)	(0.06)	(0.14)	(0.83)	(0.63)	(2.48)	(2.92)	(2.29)	(2.78)	(3.85)	(2.81)	(2.50)

Figure 2: Sharpe Ratio Distribution across Construction Specifications

This figure shows the distribution of Sharpe ratios across 4,096 factor construction specifications. We consider all factors except for the market and size factors, and show Sharpe ratios for the (hedged) long legs of each factor after taking transaction costs into account. The sample period is January 1972 to December 2023.

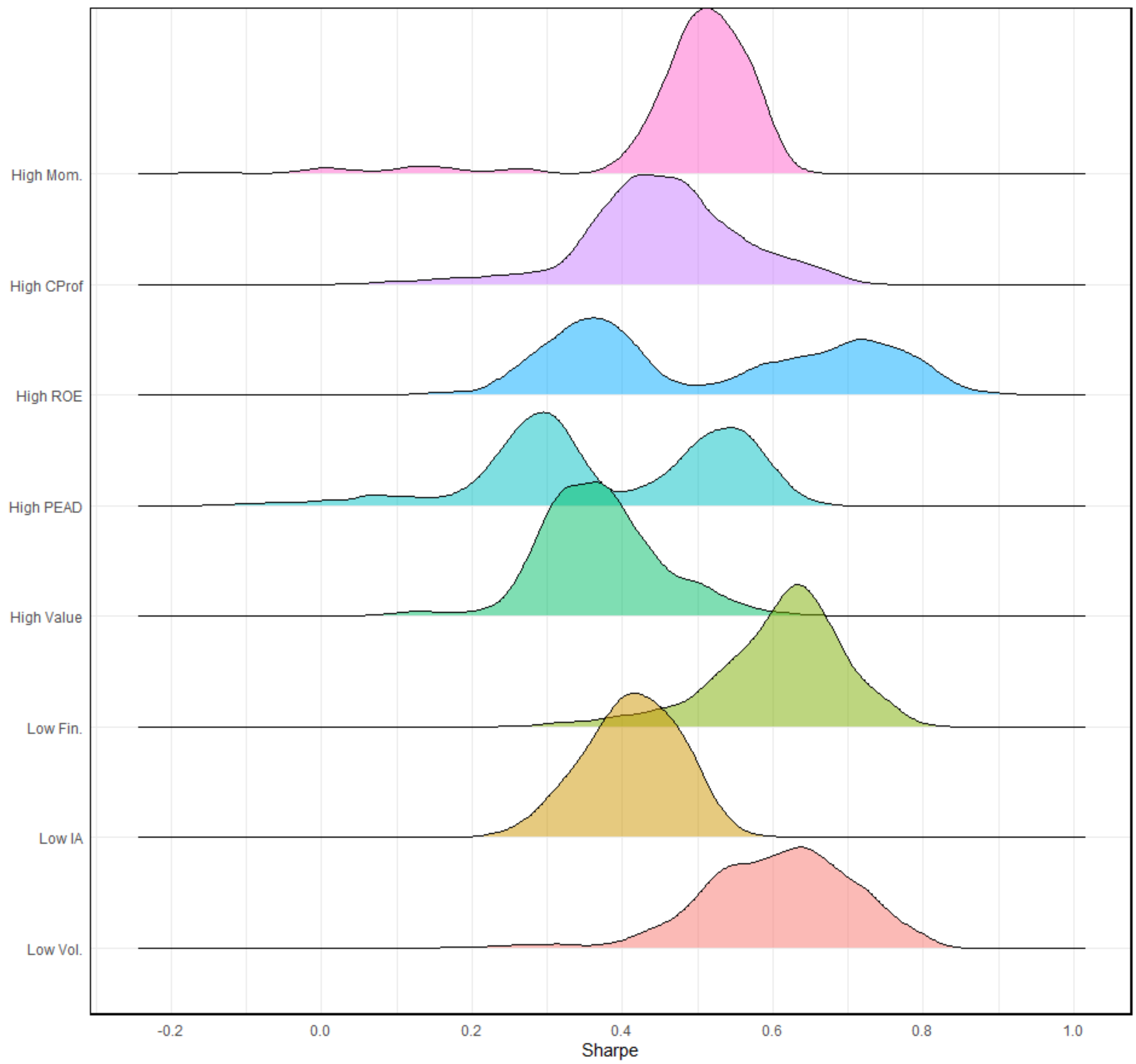

Table 9: Accounting for Methodological Uncertainty

This table reports the presents the average maximum Sharpe Ratio of each factor model without ($SR_{w/o}$) and with (SR_w) the volatility factor across 4,096 factor construction specifications. Furthermore, we report the average intercept (α) obtained from regressing the ex post mean-variance efficient factor model including the volatility factor against the efficient factor model portfolio without the volatility factor, as well as the proportion for which the statistical significance level exceeds a threshold of 5% ($P(t \geq 1.96)$). Panel A considers standard long-short factors ('Long-Short'). Panel B considers only the long legs of factors while accounting for transaction costs ('Net Long-Market'). The sample period is January 1972 to December 2023.

	Panel A: Long-Short				Panel B: Net Long-Market			
	$SR_{w/o}$	SR_w	α	$P(t \geq 1.96)$	$SR_{w/o}$	SR_w	α	$P(t \geq 1.96)$
FF5	1.14	1.17	0.35	0.35	0.83	1.02	0.70	0.97
$FF5_c$	1.26	1.28	0.14	0.12	0.90	1.03	0.52	0.91
FF5M	1.47	1.48	0.07	0.03	1.10	1.26	0.55	0.98
$FF5M_c$	1.54	1.55	0.05	0.03	1.13	1.26	0.43	0.92
HXZ	1.34	1.35	0.09	0.05	0.99	1.14	0.54	0.92
BS	1.56	1.58	0.12	0.13	1.20	1.32	0.40	0.87
DHS	1.49	1.51	0.17	0.16	0.82	0.90	0.65	0.69

Appendices

A Factor Variable Definitions

Variable	Description
Mkt-Rf	Excess market return: The value-weighted portfolio return of all NYSE, Amex, and Nasdaq common stocks with a CRSP share code of 10 or 11 minus the 1-month Treasury bill rate.
SMB	Market Capitalization: prices (CRSP item PRC) times shares outstanding (CRSP item SHROUT)
HML	Book-to-Market: Book equity for June of year t is defined as the total assets for the previous fiscal year-end in calendar year $t-1$, minus liabilities, plus deferred taxes and investment tax credit, minus preferred stock liquidating value if available or redemption value if available, or carrying value. The carrying value is adjusted for net share issuance from the fiscal year-end to the end of December of $t-1$. The book-to-market ratio divides the book equity by the market capitalization. For HML we use book-to-market ratios from previous June. For HML_d we use the previous' month book-to-market ratio.
CMA	Investments: Annual change in total assets (Compustat annual item AT) divided by 1-year-lagged total assets.
RMW	Accrual-based operating profitability: Operating profits is defined as revenues (Compustat annual item REVT) minus cost of goods sold (Compustat annual item COGS) minus selling, general, and administrative expensive (Compustat annual item XSGA) minus interest expense (Compustat annual XINT). We divide operating profits by book equity for the last fiscal year ending in $t-1$.
RMW_c	Cash-based operating profitability: The operating profitability minus accruals for the fiscal year ending in $t-1$. Accruals are the change in accounts receivable from $t-2$ to $t-1$, plus the change in prepaid expenses, minus the change in accounts payable, inventory, deferred revenue, and accrued expenses (Ball, Gerakos, Linnainmaa, and Nikolaev, 2016).
MOM	Momentum: At the beginning of each month t for each stock we calculate the prior 11-month returns from month $t-12$ to $t-2$, skipping month $t-1$.
IA	Investments-to-assets: annual change in total assets (Compustat annual item AT) divided by 1-year-lagged total assets.
ROE	Return-on-Equity: Income before extraordinary items (Compustat quarterly item IBQ) divided by the one-quarter-lagged book equity. We use the quarterly version of the book equity measure as proposed in Davis, Fama, and French (2000).

Variable	Description
FIN	Consists of two sorting variables: Net and Composite Share Issuance. Net Share Issuance of fiscal year $t-1$ equals the natural log of the ratio of split-adjusted shares outstanding of fiscal year $t1$ to split-adjusted shares outstanding of fiscal year $t2$. The split-adjusted shares outstanding is the common share outstanding (CSHO) times the adjustment factor (AJEX).
PEAD	4-day CAR around earnings announcement: The abnormal return is calculated as the four-day cumulative abnormal returns from day $t-2$ and $t+1$ around the latest quarterly earnings announcement date (Compustat quarterly item RDQ). The abnormal normal is used in the months following the quarterly earnings date (RDQ), but within six months from the fiscal quarter end, to exclude stale earnings. In addition, we require, the earnings announcement date to be after the corresponding fiscal quarter end. We require valid daily returns on more than two of the trading days in this window. In addition, we only use earnings that are announced two trading days prior to the end of the month.
VOL	Past 252-day rolling return volatility.
IVOL	Past 252-day rolling idiosyncratic return volatility based on residuals obtained from regressing the excess return on the 3-factor model of Fama and French (1993).
BAB	Past 252-day rolling market beta obtained from regressing the excess return of a stock on the excess market return factor of Fama and French (1993).