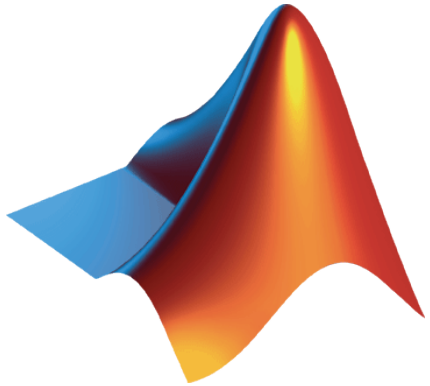
Mathematical Modeling with MATLAB



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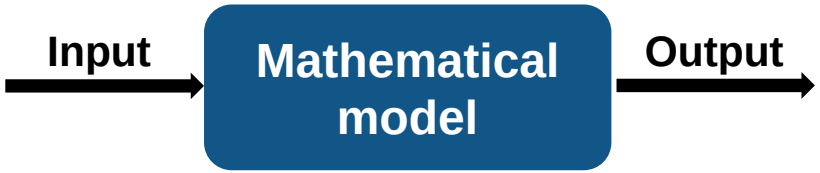
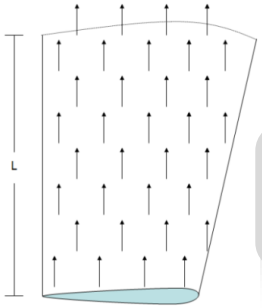
Agenda

- MATLAB for Mathematical Modeling
- Mathematical Modeling Examples
  - Parametric modeling
  - Black box modeling
  - First principles modeling
- Summary

We use math to describe our world

- Mathematical modeling is the use of a mathematical language to describe a system or process.



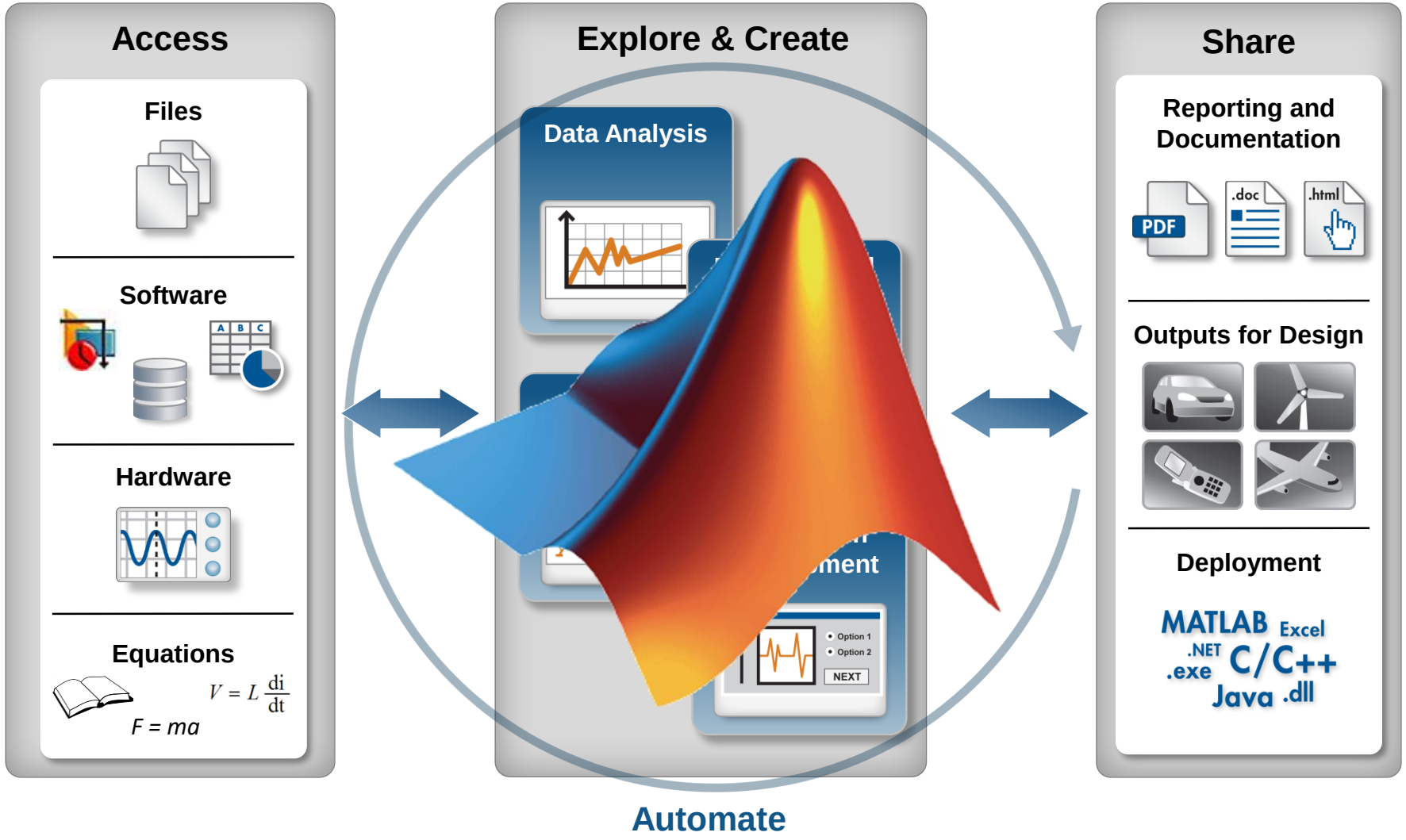


$$q_l = \frac{2W_{to}n\sqrt{L^2 - x^2}}{L^2\pi}$$

Lift on aircraft wing

- Uses of Mathematical Modeling
  - **Forecast system behavior**
    Predict and gain insight into system behavior for various “what-if” scenarios
  - **Optimize system behavior**
    Identify parameters that optimize system performance
  - **Design control systems**
    Develop model to represent plant during control system design

Technical Computing Workflow



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Focusing on Three Types of Modeling

- Parametric Modeling
  - Uses a known model that maps the independent variables to the dependent variable with a set of constant unknowns
- Black-box Modeling
  - Uses an automatically created model that “learns” to map the independent variables to the dependent variable
- First Principles Modeling
  - Uses a model derived directly from the laws of physics without making assumptions such as empirical or fitted parameters

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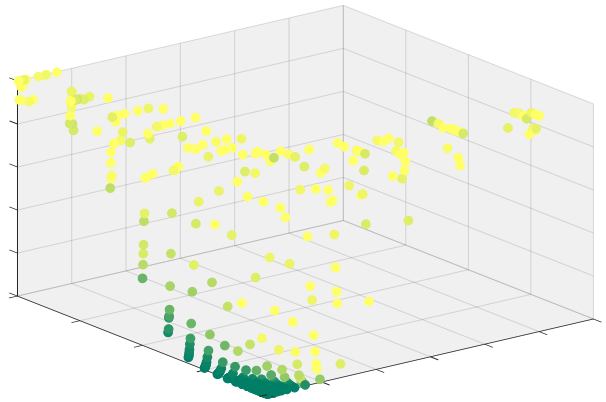
Parametric Modeling

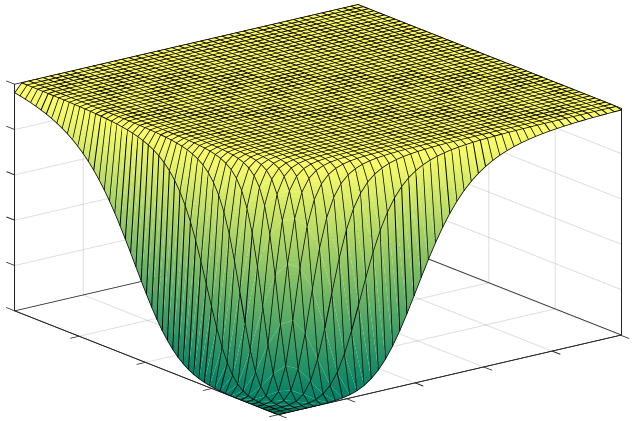
- Used when a relationship between predictors and response is known
- Build the model by estimating constant unknown parameters
- Common methods
  - Linear regression (`fitlm`)
  - Curve and surface fitting (`fit`)



Input

Model

$$R = \frac{\left(\frac{C_O}{IC50_O} + \frac{C_S}{IC50_S} + \alpha \frac{C_O}{IC50_O}\frac{C_S}{IC50_S}\right)^n}{1 + \left(\frac{C_O}{IC50_O} + \frac{C_S}{IC50_S} + \alpha \frac{C_O}{IC50_O}\frac{C_S}{IC50_S}\right)^n}$$



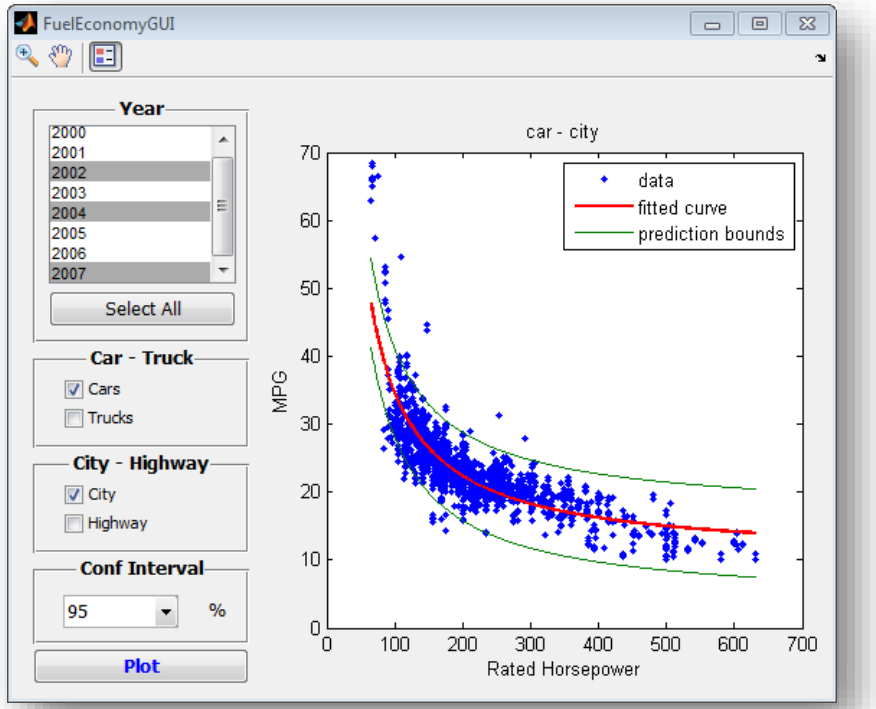
Output

Demo: Fuel Economy Analysis

Parametric modeling in MATLAB

Products Used
- MATLAB
- Statistics and Machine Learning Toolbox
- Parallel Computing Toolbox

- **Objective**:
  - Study the relationships between fuel economy, horsepower, and type of vehicle
- **Inputs**
  - Historical fuel economy data for cars manufactured between 2000 and 2012 from Excel
- **Approach:**
  - Interactively visualize and explore trends
  - Fit a curve to the data
  - Document results



$$MPG = b_1 + b_2/RatedHP$$

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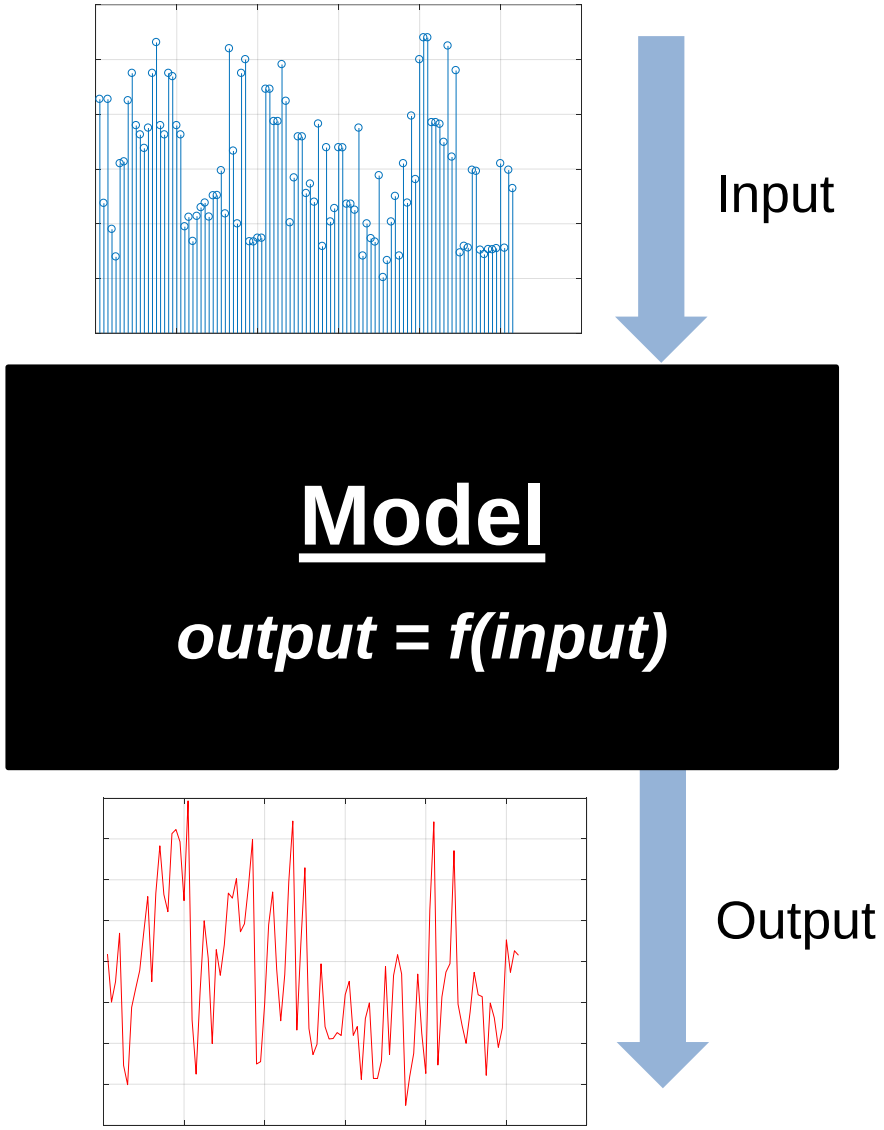
Black-Box Modeling

- Used when relationship between predictors and response is unknown
- Primary interest is in fitting the data rather than the structure of the model
- Model structures
  - Neural networks (`network`)
  - Support vector machines (`fitrsvm`)
  - Gaussian processes (`fitrgp`)
  - Decision trees (`fitrtree`)

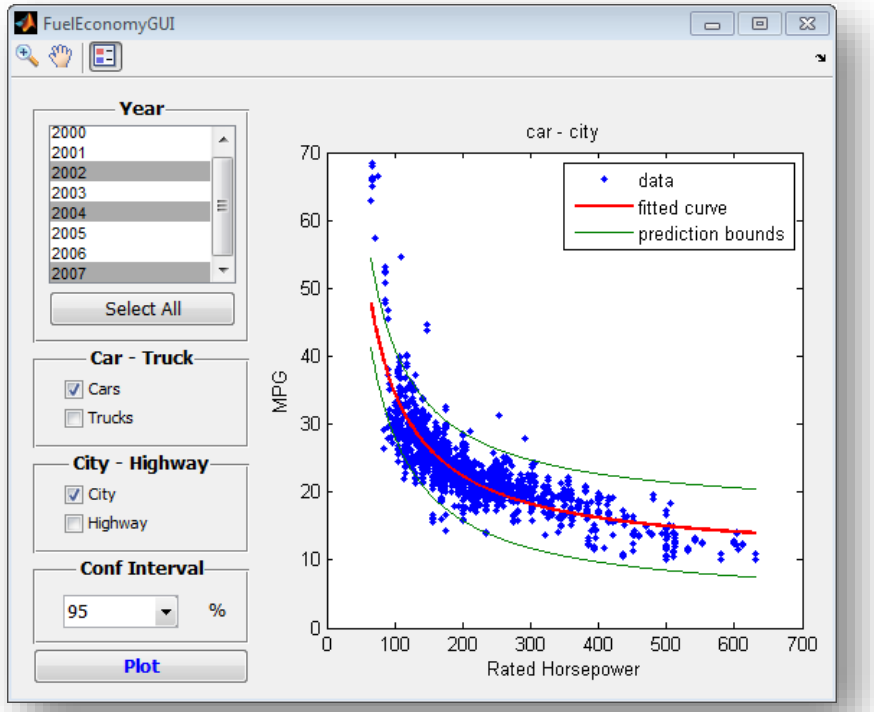


Demo: Fuel Economy Analysis

Black-box modeling in MATLAB

Products Used
- MATLAB
- Statistics and Machine Learning Toolbox
- Parallel Computing Toolbox

- **Objective**:
  - Study the relationships between fuel economy, horsepower, and type of vehicle
- **Inputs**
  - Historical fuel economy data for cars manufactured between 2000 and 2012 from Excel
- **Approach:**
  - Interactively visualize and explore trends
  - Train a regression model
  - Document results



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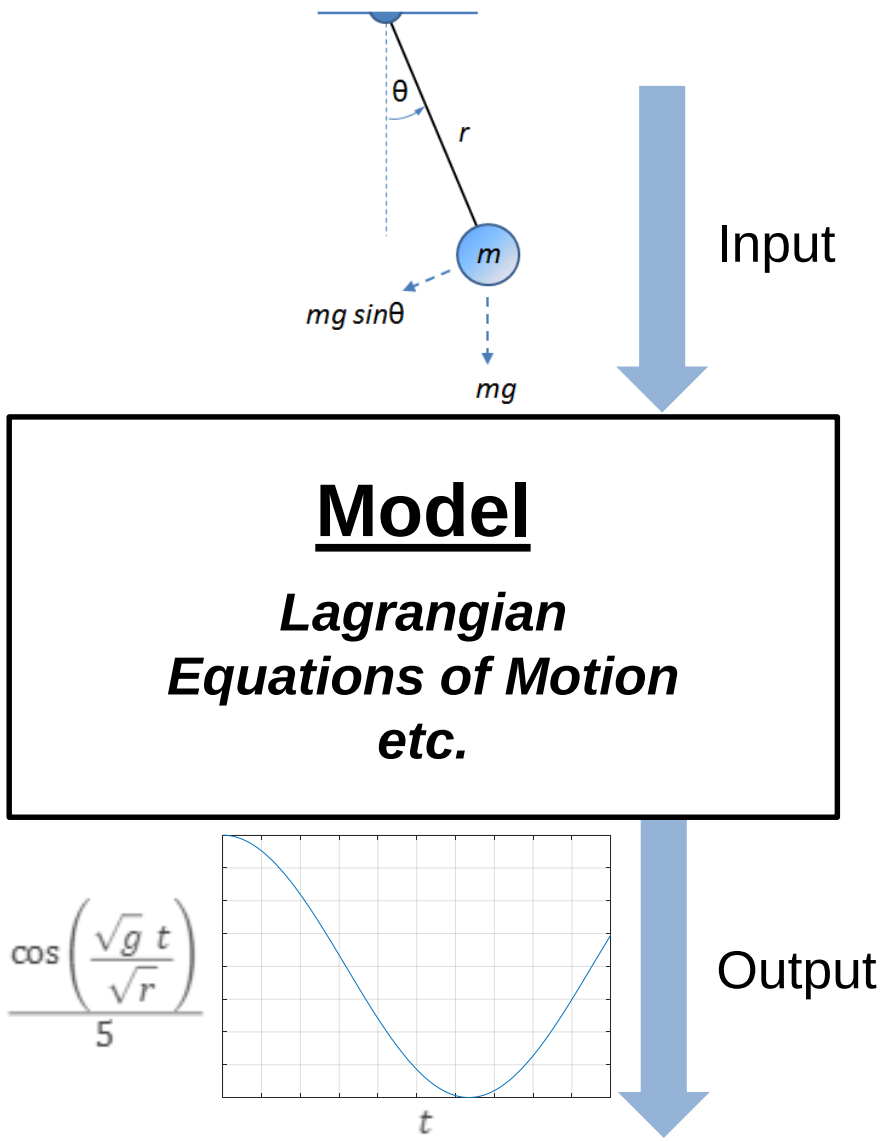
First Principles Modeling

- Used when relationship between predictors and response can be defined with analytical equations
- Build the model directly from first principles
- Solve differential equations
  - Symbolic solution (`dsolve`)
  - Numerical integration (`ode45, ode23s`)
  - Constrained minimization (`fmincon`)



Demo: Optimizing Gantry Crane Motion

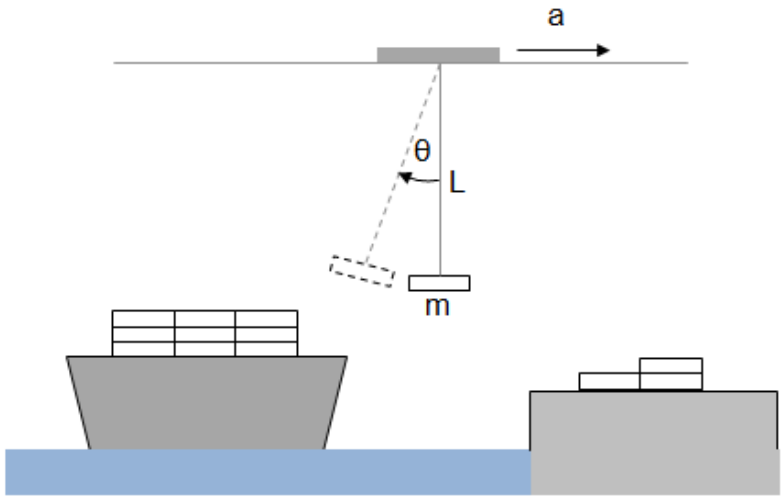
First principles modeling in MATLAB

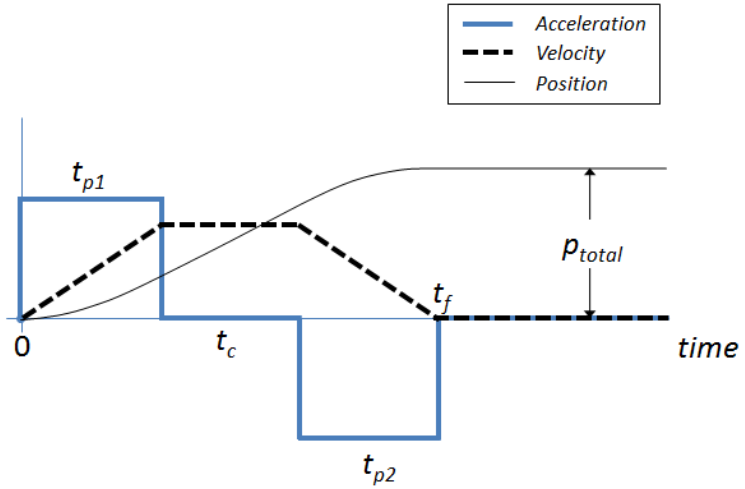
Products Used
- MATLAB
- Symbolic Math Toolbox
- Optimization Toolbox
- Parallel Computing Toolbox

- **Objective:** Determine acceleration profile that minimizes payload swing

- **Inputs:**
  - Free body diagram and first principles

- **Approach:**
  - Derive equations of motion
  - Determine ideal acceleration profile
    - ODE solver
    - Constrained minimization
  - Find all possible solutions





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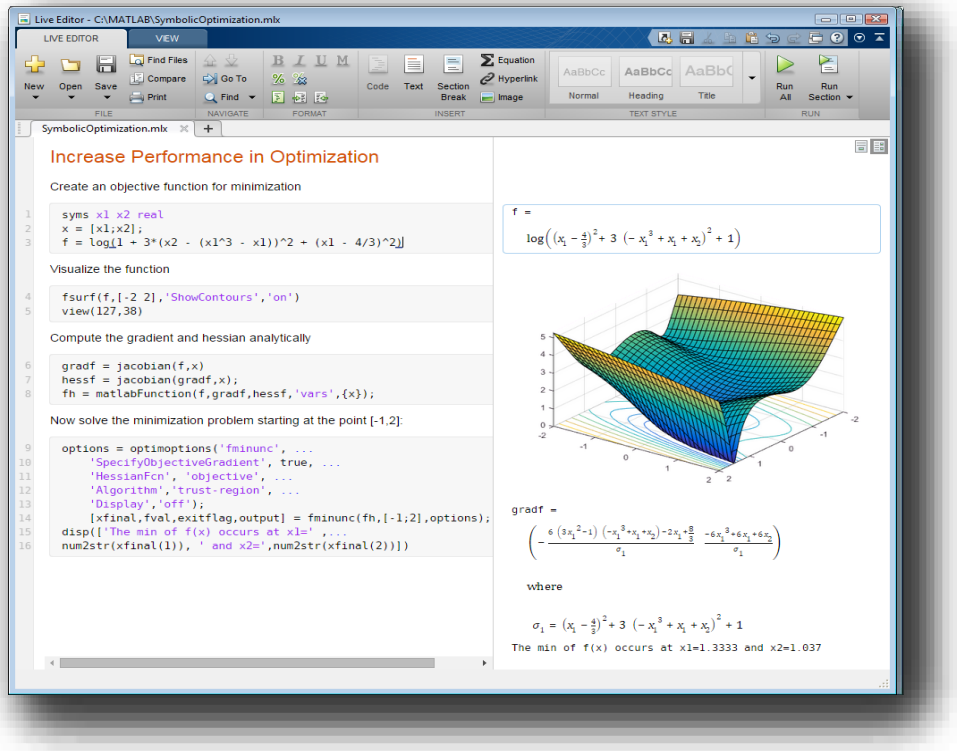
Why MATLAB for Mathematical Modeling

- Explore and integrate different modeling approaches in different domains
  - Numeric: approximate solution in vector form
  - Symbolic: exact solution in form of analytical expression

Mathematical Models for…

- **Engineering**
- **Controls**
- **Science**
- **Finance**
- **Optimization**



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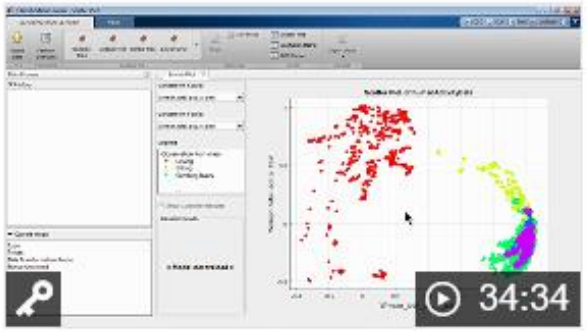
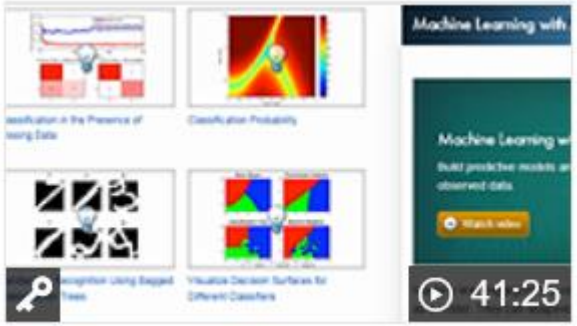
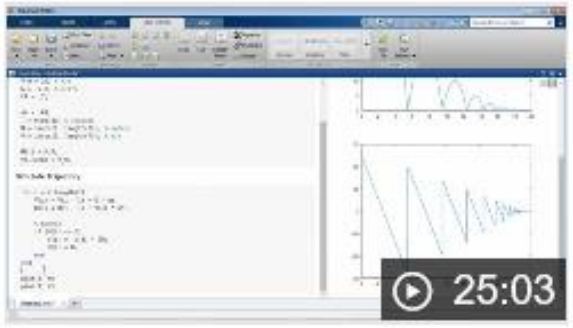
Machine Learning Made Easy



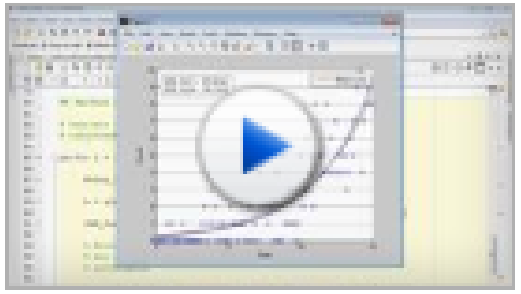
Machine Learning with MATLAB



Introducing the MATLAB Live Editor



Fitting with MATLAB: Statistics, Optimization, and Curve Fitting



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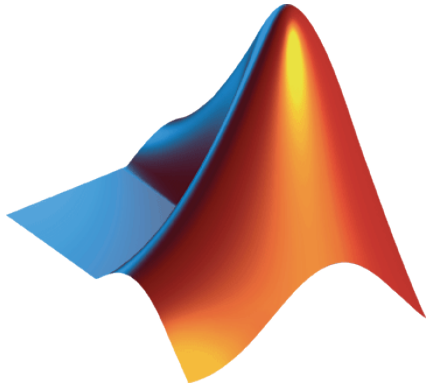

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