Practical - Satellite Control report

Digital Control (Technische Universiteit Delft)

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Digital Control Assignment
Satellite Attitude Control

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Contents

Introduction

A Satellite Attitude Control System is provided which is modeled as a fourth order transfer function. The main objective being accurate pointing of the instruments in presence of vibrations and electrical noise. Figure 0.1 shows the system under consideration and the transfer function describing the system is shown in equation (0.1)

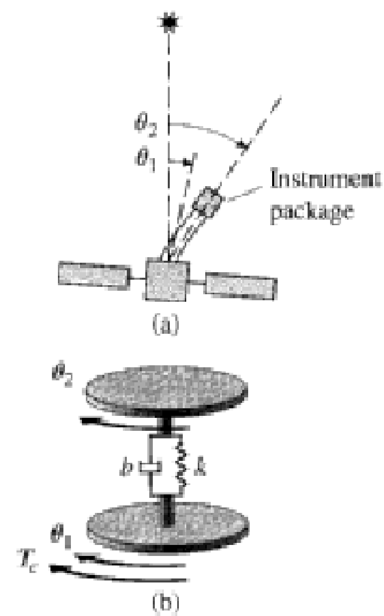

Figure 0.1: The Satellite Attitude Control System

$$G(s) = \frac{0.2s + 1}{s^2(s^2 + 0.2s + 1)} \tag{0.1}$$

The fourth order transfer function is used for design and evaluation of the controller in continuous time and discrete time. In this report, the design of controllers for the given system in continuous time and discrete time are addressed. Classical Control Methods (P, PD, PID etc) and the Modern Control theory techniques (State Feedback, Output Feedback and LQ Control) are used to design the controllers. Simulations are performed using `Matlab` and `Simulink`. This report consists of 13 sections, each section addresses the corresponding question in the assignment.

1 Continuous Time Controller for Set-point Tracking

In this section, the given system is analyzed and a controller is developed for set-point step input tracking utilizing the techniques of Classical Control Engineering. The control objectives for a set-point step are

- Minimal Settling Time
- Maximum Overshoot of 5%
- Zero Steady State Error

1.1 System Anatomy

In order to design an effective controller, the fourth order transfer function describing the system is analyzed. The transfer function is reproduced below

$$G(s) = \frac{0.2s + 1}{s^2(s^2 + 0.2s + 1)} \tag{1.1}$$

The poles and zeros of the transfer function are.

- Poles: 0 of algebraic multiplicity 2, a complex conjugate pair at $-0.1 \pm j0.9950$
- Zeros: -5

Clearly, the system is not stable since there are two poles at the origin. The systems response to the step input was observed and is shown in figure 1.1. As expected the system blows up as $t \to \inf$.

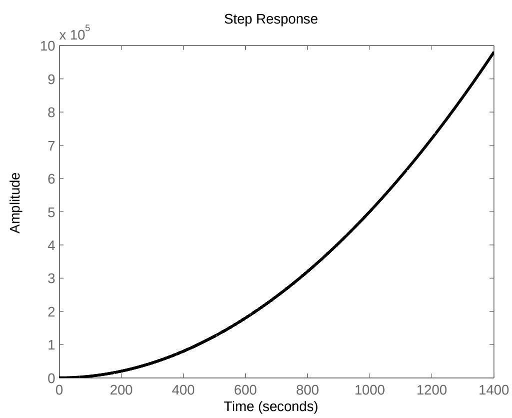

Figure 1.1: Open Loop Response of the System for Step input

The step response was computed analytically and is given as

$$c(t) = -u(t) + \frac{1}{2}t^2 + 1.01e^{-0.1t}\sin(0.99t + 0.47)$$

The above equation contains the term t^2, which is unbounded and hence the system blows up as shown in figure 1.1. The system connected in the negative feedback loop has the poles at $0.4525 \pm j0.8918$ and $-0.5525 \pm j0.8335$. Since a complex conjugate pair has a positive real value, the system is rendered unstable. In the next subsections, the controller design for the set-point tracking for the step input is discussed.

1.2 Controller Design

In this section, the results of different controllers obtained are discussed. **Loop Shaping** technique has been employed to design a controller which makes the closed loop system stable and meets the control objectives. Since the controller is designed in frequency domain, it is natural to first look at the bode plot of the given system. Figure 1.3 shows the bode plot obtained for the plant. The phase margin and the gain margin is negative, which reaffirms the point mentioned in previous section that the system is unstable in closed loop configuration.

Generally, the first step in the design of the controller would be to introduce a Proportional Controller with a gain K_p. However, as already mentioned, we have an unstable plant and from the bode plot it is evident that the phase plot starts at $-180°$ and drops further. We need to have a lead in phase in the frequency range before the gain crossover frequency in order to have a positive phase margin which can be achieved by adding a zero (in addition to the proportional gain) to the controller transfer function. With this idea, a PD Controller was designed followed by modifications to the controller transfer function to achieve the control objectives. The following sub-sections discusses the different controllers used and their results. Matlab's `SISOtool` was used for loop shaping which allows to change the controller parameters and view the resulting frequency plot instantaneously easing the iterative procedure involved. The controller structure is given in figure 1.2

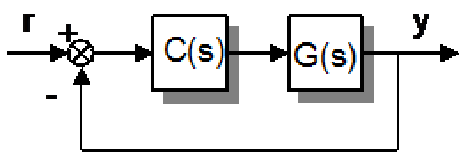

Figure 1.2: Controller Structure for Set-point step tracking

1.2.1 PD Controller

PD Controller when viewed as a filter is a High Pass Filter or a Phase Lead Compensator. The effect is to introduce phase lead in a certain frequency range by addition of a zero. Hence a Phase Lead Compensator was designed which can then be easily shown as a PD Controller. Also the idea of having a phase lead over certain frequency range sounds intuitive and can be visualized easily when using frequency domain methods (Loop Shaping in this case). The transfer function of the Phase Lead Compensator has the form as given in equation (1.2).

$$C(s) = \frac{1}{a}\frac{1 + aTs}{1 + Ts} \text{ with } a > 1 \tag{1.2}$$

From the bode plot of the system in figure 1.3 we can observe that the corner frequency associated with the complex conjugate pair present in the denominator of the plant is $\omega_n = 1$ rad/s and the damping ratio is $\zeta = 0.1$. As a result we have a sudden (due to low damping ratio) drop in phase from $-180°$ towards $-360°$ at 1 rad/s before it picks up and stays at $-270°$ due to the zero present at 5 rad/s.

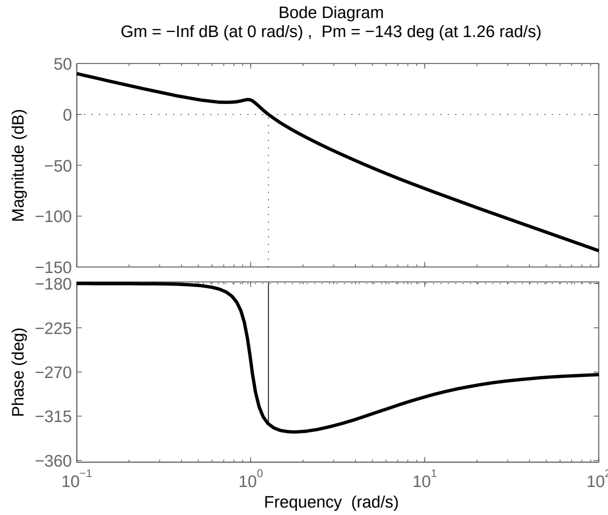

Figure 1.3: Bode Plot of the Plant

- In order to have a positive gain margin, a positive phase was required in the frequency range $0 - \omega_n rad/s$. Here very low frequencies are considered due to the presence of complex pole which decays at the rate of -80 dB/decade at the corner frequency ω_n resulting in negative phase margin. Since the phase shift induced by the zero requires two decades to rise from $-180°$ to $-90°$, the zero of the phase lead compensator is placed very close to the origin which in effect improves the rise time and the settling time. A zero was placed at 0.01 rad/s and a pole at 1 rad/s. The resulting controller transfer function is given below followed by the bode plot of loop transfer function in figure 1.4.

$$C(s) = \frac{1 + 100s}{1 + s} \text{ where } aT = 100 \text{ and } T = 1$$

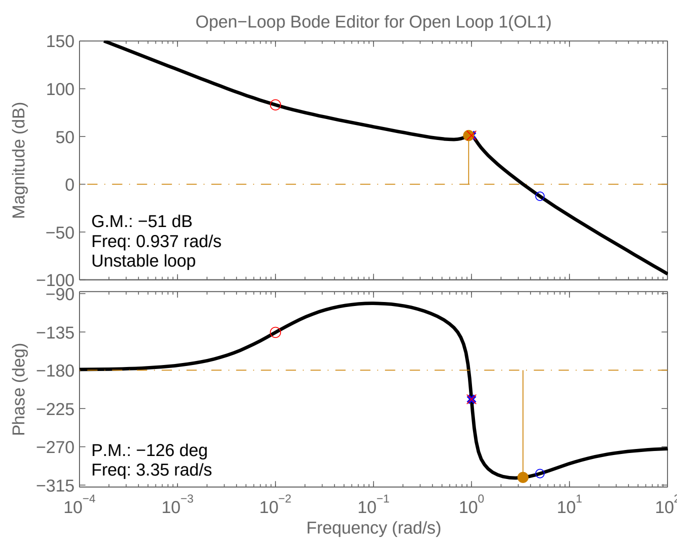

Figure 1.4: Bode Plot of the Plant with Phase Lead Compensator

- The bode plot given in figure 1.4 shows that the phase margin is $-126°$ and gain margin is -51 dB. This implies that we need to reduce the gain of the system to get a positive gain margin. Assuming the requirement of having a gain margin of 15 dB, we need to reduce the gain by $-51 - 15 = -66$ dB which corresponds to the gain of $K = 0.0005$. The bode plot and step response of the system is shown in figure 1.5 and 1.6.

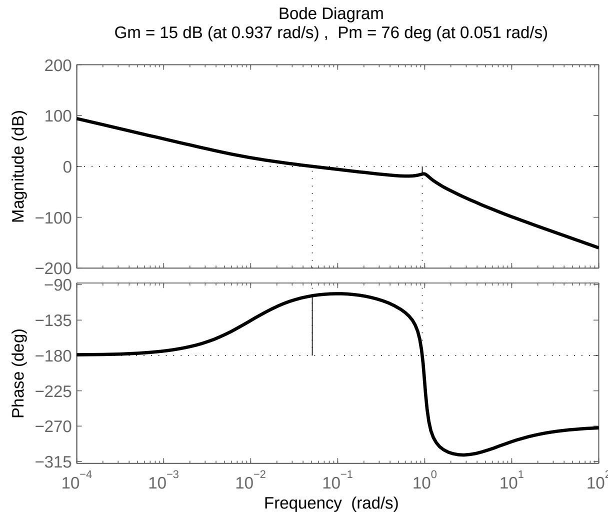

Figure 1.5: Bode Plot of the Loop transfer function with gain K = 0.0005

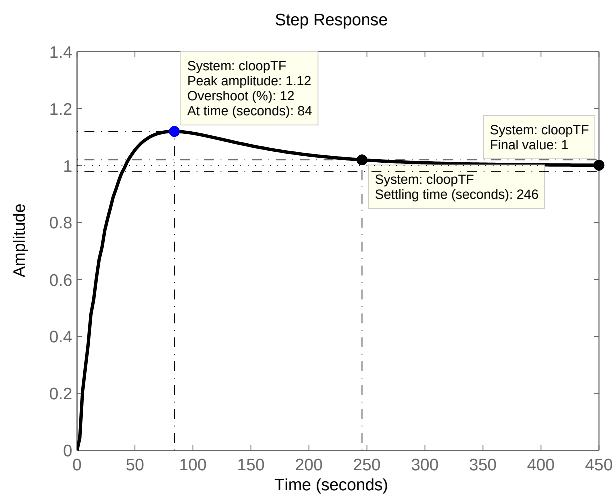

Figure 1.6: Step Response of the Controlled System

- The settling time is 246 seconds with overshoot of 12%. This certainly doesn't satisfy the control requirements except the steady state error which is 0. The above procedure was repeated with parameters of the controller tuned around the value obtained. Since the overshoot is more than 5 %, the zero was shifted much closer to the origin at 0.001 rad/s and the pole was placed at 0.36 rad/s. Having the pole at a higher frequency than 0.36 rad/s would introduce ripples in the step response during the rise time which could be attributed to the fact that the resonant peak at ω_n almost touches the 0 dB line. The gain was recalculated as above to obtain a high positive phase margin to keep the overshoot low. The transfer function of the controller is given below.

$$C(s) = 0.00013\frac{1 + 1000s}{1 + 2.8s} \text{ where } aT = 1000 \text{ and } T = 2.8 \tag{1.3}$$

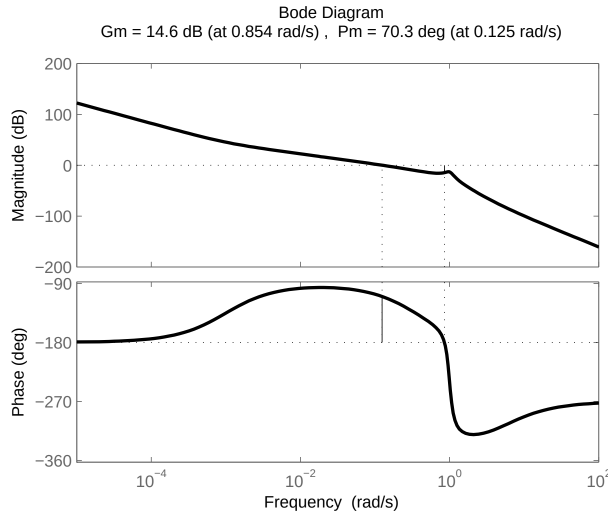

Figure 1.7: Bode Plot of the Loop transfer function with controller given in (1.3)

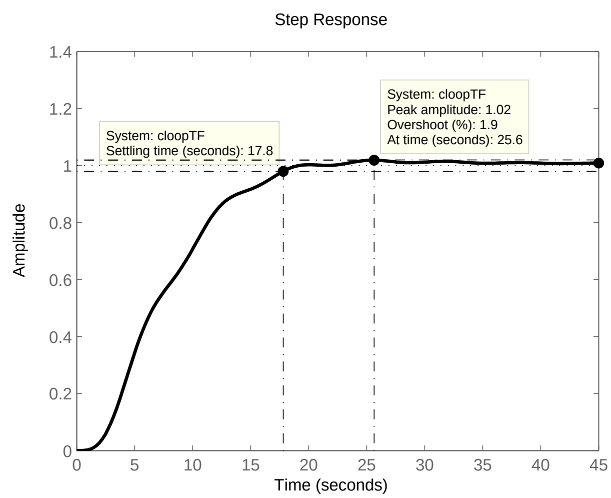

Figure 1.8: Step Response of the System with controller given in (1.3)

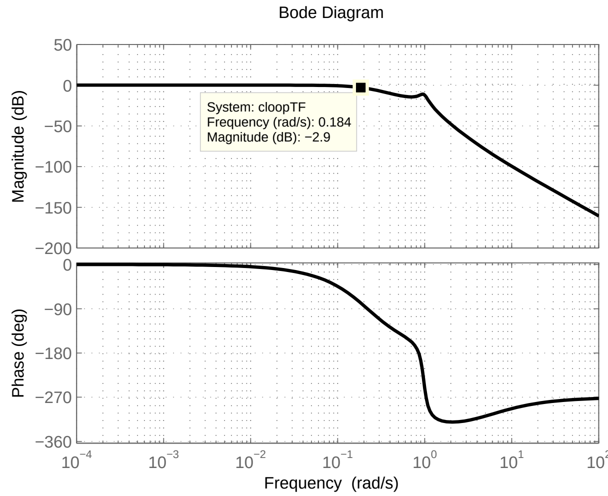

Figure 1.9: Bode Plot of the closed loop transfer function with controller given in (1.3)

The bode plot of the system and the step response of the closed loop system is shown in figure 1.7 and 1.8. The maximum overshoot is 1.9% and the settling time is 17.8 seconds. This is minimum settling time which could be obtained with a Phase Lead Compensator. Figure 1.9 shows the bode plot of the closed loop system from which the bandwidth is found to be 0.189 rad/s. The Phase lead compensator can be written as a PD controller as follows

$$C(s) = K_p + \frac{K_d s}{f_p s + 1} \text{ where } K_p = 0.00013, f_p = 2.8 \text{ and } K_d = 0.1296 \tag{1.4}$$

Steady State Analysis of the PD Controller

The steady state error of the system is given by

$$e(\infty) = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \frac{sR(s)}{1 + \lim\limits_{s\to 0} C(s)G(s)}$$

where $R(s)$ is the Laplace transform for step input which is $\frac{1}{s}$
$C(s)G(s)$ is forward path transfer function
$\lim\limits_{s\to 0} C(s)G(s)$ is the DC gain of the forward path transfer function

For the steady state error to be zero, we need the DC gain of forward path transfer function to be infinity. Due to presence of two integrators in the plant transfer function we have the forward path DC gain to be infinity and hence the steady state error is 0. In order to obtain a much faster response, the PD controller of equation (1.4) was modified which is described in the next section.

1.2.2 Modification to the PD Controller

In the previous section, a PD Controller was designed which met the control objectives (with minimal settling time of 17.8 seconds in the sense, that was the minimum settling time which could be achieved). However, in order to further reduce the settling time, the inverse of the plant was taken as controller in addition to the Phase Lead Compensator already developed. The two poles at origin of the plant was included in the controller to avoid pole zero cancellation of unstable poles.

- The complex pole of the plant was introduced as zero in the controller resulting in cancellation of the complex pole. This was done in order to increase the bandwidth of the system. It was this complex pole, which restricted in achieving a better settling time in the PD Controller design of previous section.

- The zero of the plant was included as the controller pole, this zero along with the zero of Phase Lead compensator (which is very close to origin), provides the positive phase required to get a positive gain margin (which would reduce the maximum overshoot). The controller transfer function is given in equation (1.5)

$$C(s) = 2\frac{(1+14s)(s^2+0.2s+1)}{(1+0.2s)(1+0.0033s)(1+0.01s)} \tag{1.5}$$

- Two additional real poles were placed at the higher frequencies of 100 rad/s and 300 rad/s whose response would decay faster. These poles are placed such that the positive phase induced by the zeros are reduced and the phase plot cuts the -180^o line at a higher frequency to render a positive gain margin and increased bandwidth. It would also ensure that the controller transfer function is proper and realizable.

- The Matlab `SISOtool` was used for loop shaping interactively. The Bandwidth of the System could be increased considerably from 0.189 rad/s to 42 rad/s. The step response of the closed loop system is shown in figure 1.10. Figure 1.11 shows the bode plot of the system with controller given by equation (1.5). The bode plot of the closed loop system is shown in figure 1.12from which bandwidth can be determined.

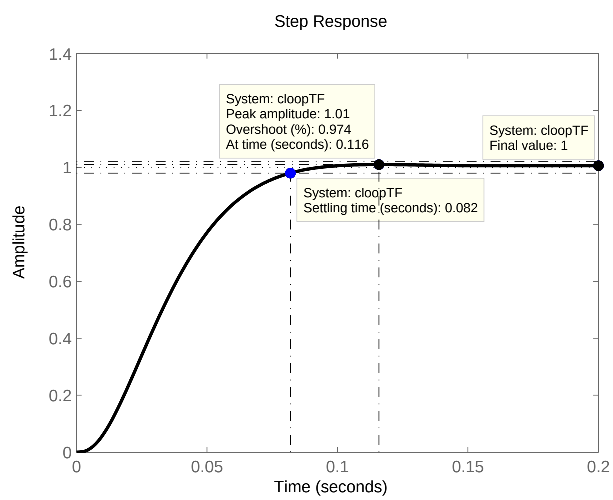

Figure 1.10: Step Response of the System with controller given in (1.5)

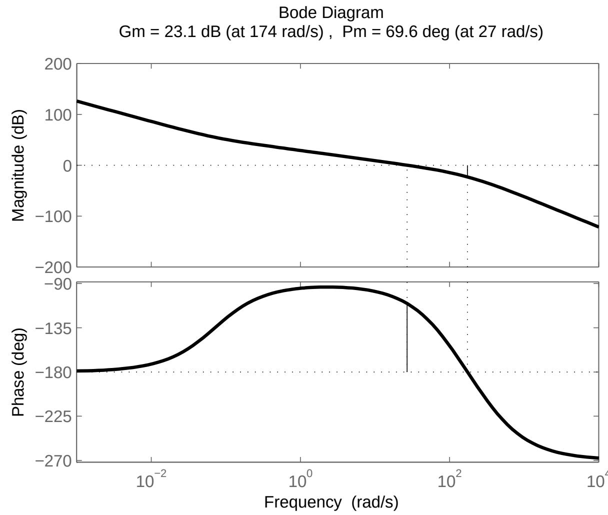

Figure 1.11: Bode Plot of the Loop Transfer function with controller given in (1.5)

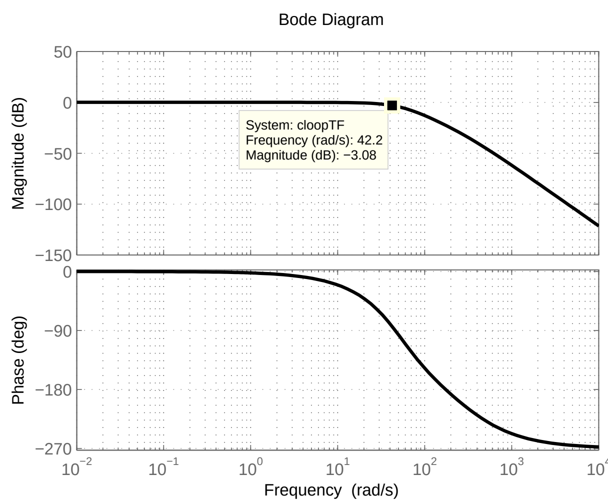

Figure 1.12: Bode Plot of the closed loop System with controller given in (1.5)

From the step response of the system in figure 1.10 we can see that the settling time has drastically reduced to 0.082 seconds with maximum overshoot of 0.974 %. The steady error is zero and the argument regarding steady state error presented for PD Controller design holds true in this case as well. Using this controller the control objectives are met.

2 Disturbance Rejection using Continuous Time Controllers

In this section we design a controller in continuous time when a step disturbance input is applied at the plant input and the reference input is 0. The Controller then happens to be in the feedback path and the structure is shown in figure 2.1.

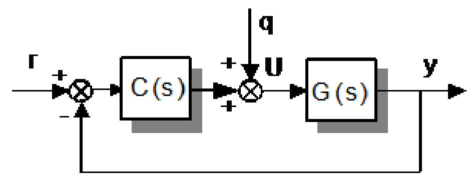

Figure 2.1: Controller Structure for Disturbance Rejection

The closed loop transfer function from the disturbance input to the output is then given as in equation (2.1). In order to reject the disturbance successfully, transfer from disturbance input to the output must be zero. This is possible when the DC gain of loop transfer function G(s)C(s) is infinity

$$G_{cl}(s) = \frac{Y(s)}{Q(s)} = \frac{G(s)}{1 + G(s)C(s)} \tag{2.1}$$

Naturally the first step was to test the controller developed for set-point tracking in section 1 for disturbance rejection. A high loop gain is required to be able to reject the disturbance. Hence the gain of the controller was increased and an integrator was added to the controller to eliminate the offset/steady state error. Also the damping ratio of the complex zero in the controller was increased to 1 to damp the oscillations in the response. Figure 2.2 shows the response of the system for step disturbance input for damping ratio of 1 and figure 2.3 shows the same response for damping ratio of 0.1. From the figure we can see that the settling time is 6.81 seconds and steady state error is zero (for damping ratio 1). The final form of the controller is shown in (2.2)

$$C(s) = \left(40 + \frac{27.68s}{0.2s + 1} + \frac{40}{s}\right) \frac{s^2 + 2s + 1}{(0.01s + 1)(0.0033s + 1)} \tag{2.2}$$

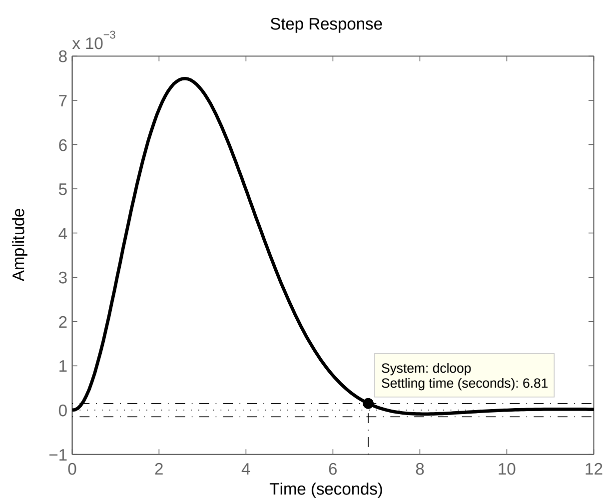

Figure 2.2: Step Response of the system for Disturbance Rejection with damping ratio 1

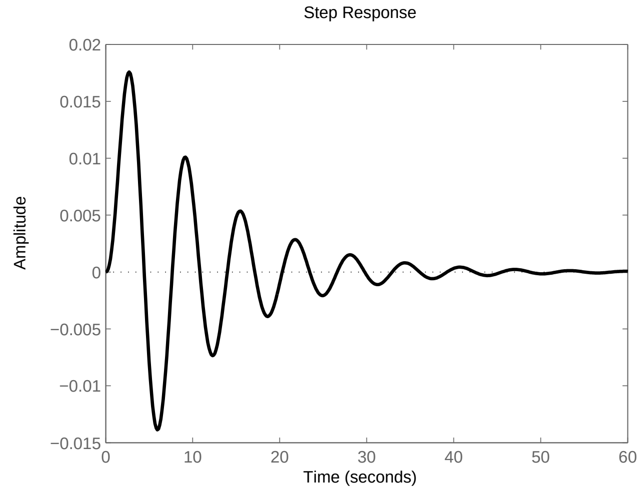

Figure 2.3: Step Response of the system for Disturbance Rejection with damping ratio 0.1

3 State Space Notation in Continuous and Discrete Time

The given plant transfer function (0.1)was converted into the state space model using the Matlab command `ss`. The state space representation is given below. The state space equations are in controllable canonical form.

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

where $A = \begin{bmatrix} -0.2 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, B = $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, C = $\begin{bmatrix} 0 & 0 & 0.2 & 1 \end{bmatrix}$ and D = 0

The state space model obtained above is discretization of the continuous time state space model using zero order hold sampling described in [2]. The sampling period h was chosen to be 10 times the bandwidth of the closed loop system or equivalently 10 samples per rise time. The bandwidth of the closed loop system was found to be 42 rad/s in section 1.2.2. Hence the sampling period $h = 10\,(2\pi/\omega) = 2\pi/420 = 0.01s$ was chosen.
Matlab function `c2d` was used to discretize the system. The sampled discrete time state space model of the system is given as follows.

$$x(kh+h) = \Phi x(kh) + \Gamma u(kh)$$
$$y(kh) = Cx(kh) + Du(kh)$$

with $\Phi = e^{Ah} = \begin{bmatrix} 0.9980 & -0.01 & 0 & 0 \\ 0.01 & 1 & 0 & 0 \\ 0 & 0.01 & 1 & 0 \\ 0 & 0 & 0.01 & 1 \end{bmatrix}$

$\Gamma = \int_0^h e^{As} ds B = \begin{bmatrix} 0.01 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

C = $\begin{bmatrix} 0 & 0 & 0.2 & 1 \end{bmatrix}$ and D = 0

The sampling period of 0.01 selected here is used in rest of the assignment to design controllers in discrete time.

Physical Meaning of the states

The physical meaning of the states can be interpreted as follows, the fourth state is the angular position/attitude of the satellite, 3rd, 2nd and 1st states are the angular velocity, acceleration and jerk associated with the angle respectively.

4 Discretization of Continuous time controllers

The continuous time controllers developed for set-point tracking and disturbance rejection in section 1 and 2 respectively are discretized in this section using a sampling interval of h = 0.01 s. The Matlab command `c2d` was used for discretization of the controller and plant. The resulting discretized model of the plant was unstable because the discretization was done using zero order hold. Hence Tustin approximation was used for discretization which maps the open left half plane of s-plane to a unit circle in z-plane.
The discrete time transfer function of the plant is given in equation (4.1), transfer function of the controller for set-point step tracking is given in equation (4.2) and the transfer function of the controller for disturbance rejection is given in (4.3). All the transfer functions are obtained from their continuous counterparts using the sampling period of 0.01 seconds.

$$G(z) = \frac{2.56 \times 10^{-8} z^4 + 5.245 \times 10^{-8} z^3 + 3.746 \times 10^{-9} z^2 - 4.745 \times 10^{-8} z - 2.435 \times 10^{-8}}{z^4 - 3.998 z^3 + 5.994 z^2 - 3.994 z + 0.998} \tag{4.1}$$

$$C_{sp}(z) = \frac{1.102 \times 10^6 z^3 - 3.302 \times 10^6 z^2 + 3.299 \times 10^6 z - 1.099 \times 10^6}{z^3 - 1.08 z^2 + 0.05397 z + 0.06494} \tag{4.2}$$

$$C_{dr}(z) = \frac{1.422 \times 10^6 z^4 - 5.639 \times 10^6 z^3 + 8.388 \times 10^6 z^2 - 5.545 \times 10^6 z + 1.375 \times 10^6}{z^4 - 2.08 z^3 + 1.134 z^2 + 0.01097 z - 0.06494} \tag{4.3}$$

Figure 4.1 shows the step response of the discretized closed loop system for set-point step input tracking and figure 6.3 shows the response of the discretized closed loop system for disturbance rejection.

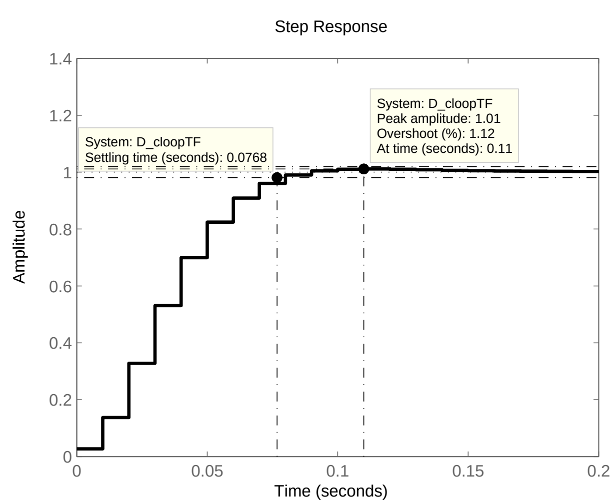

Figure 4.1: Step Response of the system for Set-point step tracking

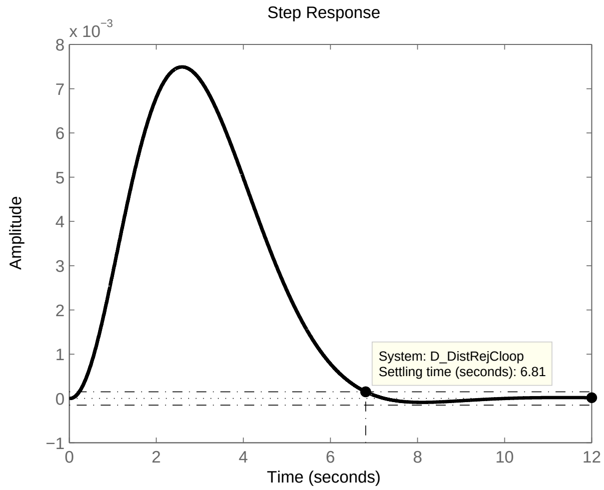

Figure 4.2: Step Response of the system for Disturbance Rejection

5 Discrete State Feedback Controller

In this section, state feedback controller is developed using pole placement technique for the discrete time pulse transfer function of the plant given by equation (4.1). The discrete plant was converted to state space model in controllable canonical form using the Matlab command `tf2ss`. The sampling period used here is 0.01 seconds. The state space model of the plant in controllable canonical form is given in equation below

$$x(kh+h) = \Phi x(kh) + \Gamma u(kh)$$
$$y(kh) = Cx(kh) + Du(kh)$$

with

$$\Phi = \begin{bmatrix} 3.9979 & -5.9938 & 3.9939 & -0.9980 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

C = $10^{-6} \begin{bmatrix} 0.0337 & 0.1045 & -0.0953 & -0.0329 \end{bmatrix}$ and D = 0

Note that the discretized state space model of the plant given in section 4 and model given above are equivalent and can be transformed through a similarity transformation matrix T. The Matlab command `ctrb` was used to get the Controllablity Matrix or the Kalman matrix which was found to be of full rank. Hence the pair (Φ, Γ) is controllable. In the following, different controllers are obtained by placing the desired poles of the closed loop system at different regions within the unit disc in z-plane. Matlab command `place` was used to compute the feedback gain. In each of these controllers, the closed loop transfer function $G_{cl}(z)$ was obtained and its DC gain was calculated using the Matlab command `dcgain`. The inverse of the DC gain of the closed loop pulse transfer function was then used as the feed forward term for set-point tracking controller. The disturbance rejection abilities of the controller is also discussed. Figure

5.1 Desired poles at 0.2, 0.25, 0.3, 0.35

All poles in this case are real and are the faster modes. The gain feedback matrix obtained is K = [2.8979 -5.5463 3.9142 -0.9928]. The step response of the closed loop system is shown in figure 5.1

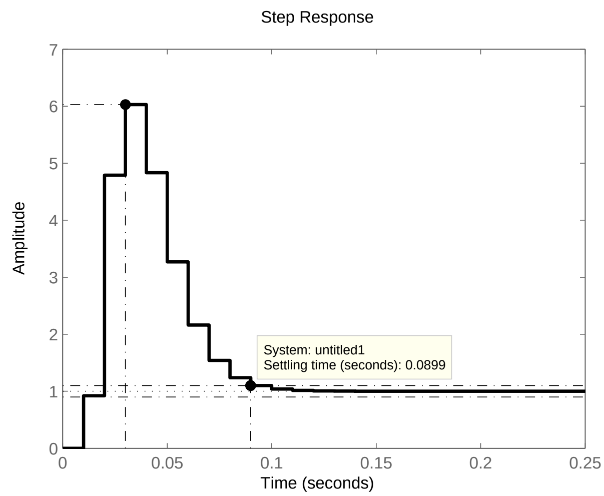

Figure 5.1: Step Response of the system for Set-point step tracking with desired closed loop poles at 0.2, 0.25, 0.3, 0.35

As can be seen from the figure, we have a very high overshoot (peak amplitude of 6 and overshoot of 503 % from the steady state value) and a settling time of 0.089 seconds.

Disturbance Rejection

A unit step was applied at the plant input as load disturbance with the reference input 0. Figure 5.2 shows the response of the system to disturbance step at plant input. It can be observed that the controller is able to reject disturbances of unit step with settling time of approximately 0.11 seconds and reaches the steady state value of 3.7×10^{-8} which is a negligible value and can be considered as zero. However, as the magnitude of the input disturbance increases, the steady state error increases. For example when the magnitude of disturbance was increased to 1000, the steady state error obtained was 3.7×10^{-4}. When the magnitude of load disturbance equals the feed forward term(from reference input, which is 2.73×10^{7}), we have a steady state error of 1.

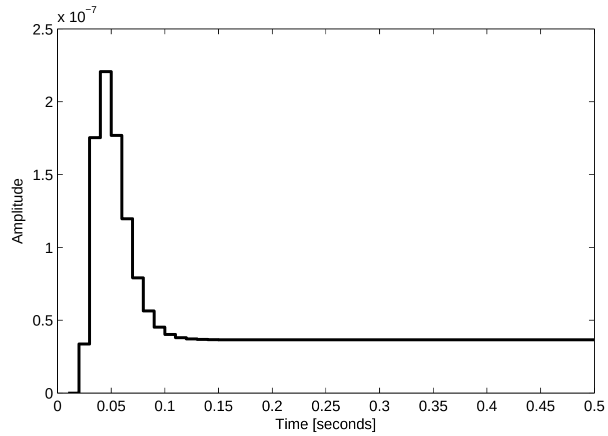

Figure 5.2: Step Response of the system for Disturbance Rejection with desired closed loop poles at 0.2, 0.25, 0.3, 0.35

5.2 Desired poles at 0.5, 0.55, 0.6, 0.65

Having seen a very high overshoot of 503 %, the next step is chose relatively slower poles to reduce the overshoot. The gain matrix obtained is K = [1.6979 -4.0163 3.2407 -0.8908]. Figure 5.3 shows the step response of the system. As expected, the overshoot has reduced to 179 % (which is still high and does not meet the control objectives) and the settling time has increased to 0.2 seconds.

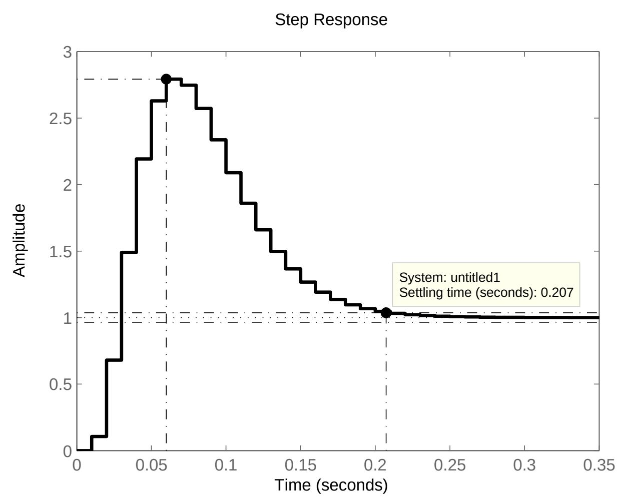

Figure 5.3: Step Response of the system for Setpoint Tracking with desired closed loop poles at 0.5, 0.55, 0.6, 0.65

Figure 5.4 shows the response of system for a unit step disturbance at plant input. The steady state error in this case is again very small (3.18×10^{-7}) but has relatively increased when compared to the previous case.

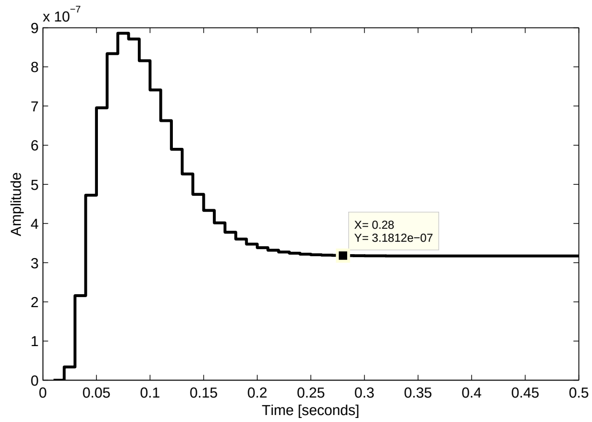

Figure 5.4: Step Response of the system for Disturbance Rejection with desired closed loop poles at 0.5, 0.55, 0.6, 0.65

5.3 Desired poles at 0.8, 0.85,0.9,0.95

The desired closed loop poles were chosen almost near to the unit circle i.e, slower poles were chosen. The gain matrix obtained is K = [0.4979 -1.4063 1.3252 -0.4166]. Figure 5.3 shows the step response of the system to unit step input as reference. We can see that we have zero overshoot and settling time is 0.5 seconds. The steady state error is also zero and this particular choice of controller meets the control objectives.

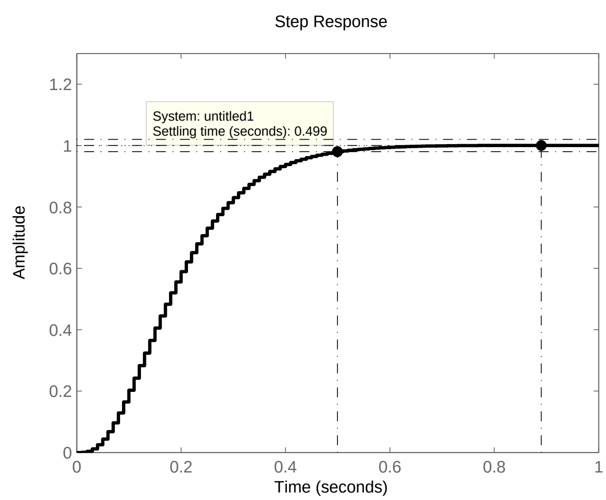

Figure 5.5: Step Response of the system for Setpoint Tracking with desired closed loop poles at 0.8, 0.85, 0.9, 0.95

Figure 5.6 shows the response of system to unit step disturbance at plant input. The steady state error has increased to 6.6×10^{-5} in this case. For unit step disturbance the error could possibly be neglected, however as the magnitude of disturbance increases, we will have a higher steady state error. In all the three cases discussed, the steady state error reaches a magnitude of 1 when the load disturbance is of magnitude of feed forward term.

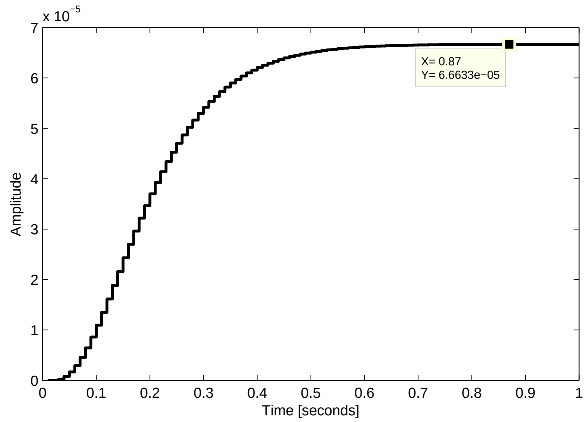

Figure 5.6: Step Response of the system for Disturbance Rejection with desired closed loop poles at 0.8, 0.85, 0.9, 0.95

We have only considered real poles as desired poles until now. Real poles correspond to a damping factor 1, when we consider complex conjugate pole pairs we have damping factor less than 1. Selecting complex poles as desired poles would induce oscillations in the response of the system with frequency equal to the natural frequency associated with the complex pole. This in effect would increase the overshoot and reduce the rise time.

5.4 Desired Poles at 0.2, 0.3, 0.9, 0.95

Figure 5.7 shows the response of the controller for step reference input. Here we have taken two slower poles (0.9 and 0.95) which are the dominant modes of the system and remaining two poles are the faster poles. As result we have a better performance than the previous section. The overshoot is still zero with settling time of 0.35 seconds.

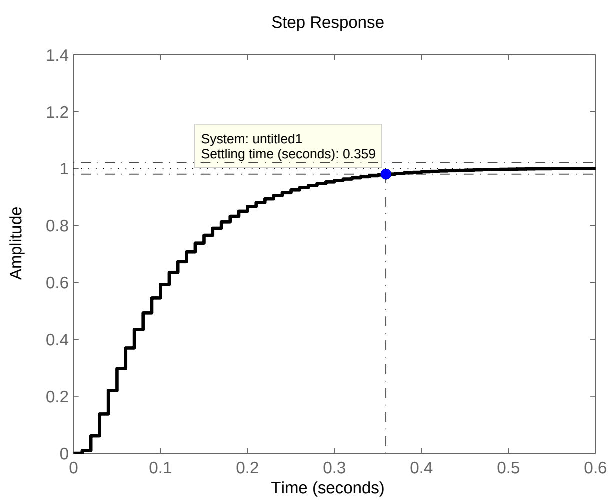

Figure 5.7: Step Response of the system for Setpoint Tracking with desired closed loop poles at 0.2, 0.3, 0.9, 0.95

In this section, we designed the state feedback controller without having any constraints of gain and actuator saturation. As a result we could find multiple closed loop pole locations which would meet the desired control objectives. In this section, two such different pole locations were obtained which met the control objectives. The effect of location of poles on the magnitude of controller output (plant input) would be discussed in further sections when limits on controller outputs are introduced. The Controller corresponding to the desired poles of 0.2, 0.3, 0.9, 0.95 will be used for design and evaluation of Observer and an Output Feedback Controller in the next section.

6 Discrete Time Output Feedback Controller

In the previous section, a state feedback controller was developed with an assumption that the measurement of states were available. In this section, an Output feedback controller will be developed by adding a dynamic observer to the system which would estimate the states of the system which would then be used by the controller (developed in previous section) for feedback. The case of placing slow and fast observer poles are discussed below. In the simulation results presented in this section, the initial states of the system are taken to be [20000, -20000, -5000, 6000] and the initial states of the observer are taken to be [0 0 0 0]. These high discrepancies in the initial estimates of the states are used to evaluate the performance of the observers with different pole locations. The observability matrix of (Φ, C) was found to be of full rank and hence the system is observable.

6.1 Slow Observer Poles: 0.8, 0.85, 0.92, 0.94

The Matlab command `place(`Φ', C'`, des_poles)'` was used to find the static gain matrix of the observer. Figure 6.1 shows the response of the system for a reference step input. As can be seen from the response, the observer output estimate converges to the output of the system. The estimates converge in approximately 0.6 seconds and the step response has a settling time of 0.8 seconds.Comparing the response of this Output Feedback Controller to the response of figure 5.7, we can see that due to the unknown initial estimates of the state (which effects the input, as the controller uses this wrong initial state estimates for computation of control action), we have a high overshoot and increase in settling time from 0.35 seconds to 0.8 seconds. This suggests that there is a need for speedy convergence (and hence faster Observer poles) of the observer state estimates to the true states to obtain a Output Feedback Controller which meets the control requirements.

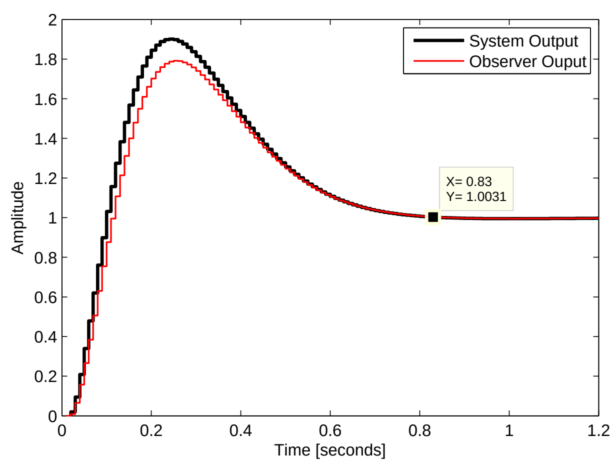

Figure 6.1: Step Response of the system for Setpoint Tracking with desired observer poles at 0.8, 0.85, 0.92, 0.94

6.2 Fast Observer Poles: 0.5, 0.55, 0.6, 0.65

The observer poles placed in this case are faster than the dominant poles of the controller (0.9 and 0.95). The most dominant observer pole 0.65 (-43.07 in continuous time) is 8.4 times faster than the most dominant controller pole 0.95 (-5.13 in continuous time). Hence all other observer poles are at least 8 times faster than dominant controller poles (0.9, 0.95). The response of the system for step input as reference is shown in figure 6.2

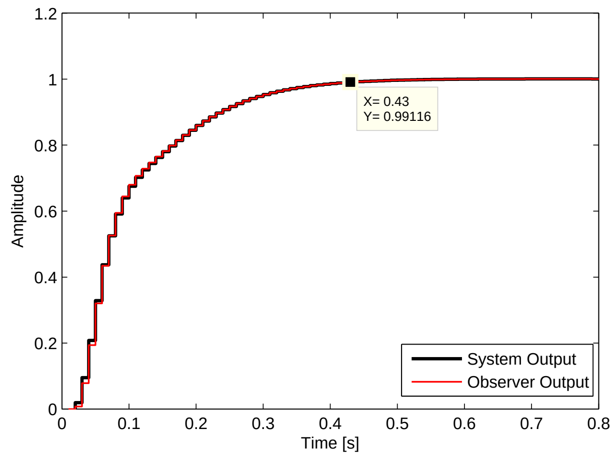

Figure 6.2: Step Response of the system for Setpoint Tracking with desired observer poles at 0.5, 0.55, 0.6, 0.65

From the figure we can observe that the overshoot is zero. The settling time too has reduced to 0.4 seconds and does meet the control objectives. Comparing this response with the one obtained in 5.7 we can see that this output feedback controller exhibits similar performance to that of the state feedback controller. The observer poles can be made much faster, but it would make the system sensitive to measurement noise (which in this case is not considered).

6.3 Plant Output Disturbance Rejection

Figure 6.3 shows the response of the system for unit step disturbance at plant output. As can be seen, observer is quick enough to track the output and the unit step disturbance is completely rejected without any steady state error. The settling time with disturbance is 0.4 seconds. No integrator was needed as the steady state error was zero.

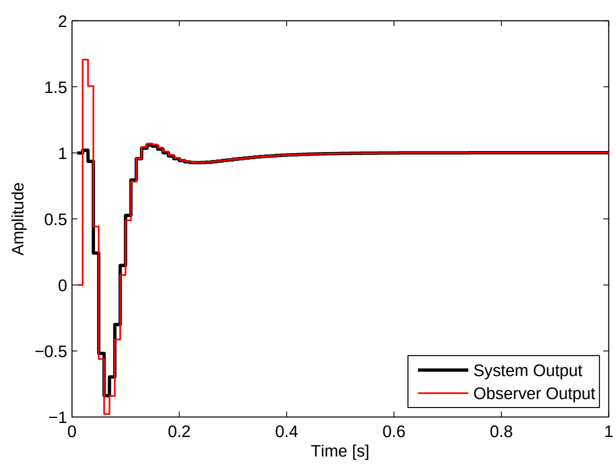

Figure 6.3: Step Response of the system for unit step disturbance at plant output

7 LQ Controller

In this section, a LQ controller is developed with the states of the system estimated by a dynamic Luenberger Observer developed in section 6. In the pole placement method, the desired closed loop poles of the system are specified whose response would meet the required control objectives. An alternate method to develop the linear feedback controller is LQ control. The parameters to be tuned are the weighing matrix Q and R which would place penalties on states and control input respectively to provide a optimal gain matrix K which aims to minimize a quadratic cost function. The Matrix Q is of size $n \times n$ where n is the number of states and R is of size $m \times m$ where m is the number of inputs. In this case we have Q of size 4×4 and R is a scalar (single input). The diagonal entries in Q place weight on each state individually. It is required that Q is positive semi-definite and R is positive definite matrix. We start with both Q and R equal to identity. The LQ Controller gains are calculated using the Matlab command `dlqr`. First we discuss the effect of Matrix Q on response of the system to reference unit step input by holding the R constant. The effects of R are then discussed by holding Q constant.

7.1 Constant R = 1 and varying Q

The weighing matrix R was kept constant (identity) and the 4th state i.e the angular position of the satellite was penalized by varying the 4th diagonal element of Q = 1, 100, 1000. Figure 7.1 shows the output of the controller and figure 7.2 shows the response of the system to the reference unit step input. From the figure we can see that higher we penalize the states, faster the response and higher the magnitude of the controller output. The different elements of the matrix Q penalizes error in each of the states (with respect to the reference value). Since LQ controller finds the optimal gain to minimize a quadratic cost function, placing higher values in Q would result in faster response reducing rise time and increase overshoot.

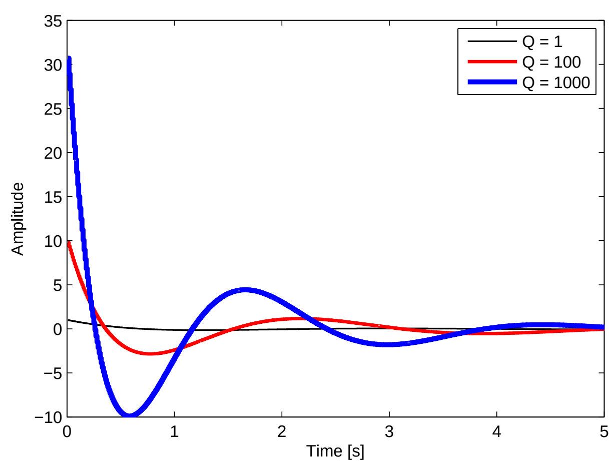

Figure 7.1: Output value of controller for varying Q and R = 1

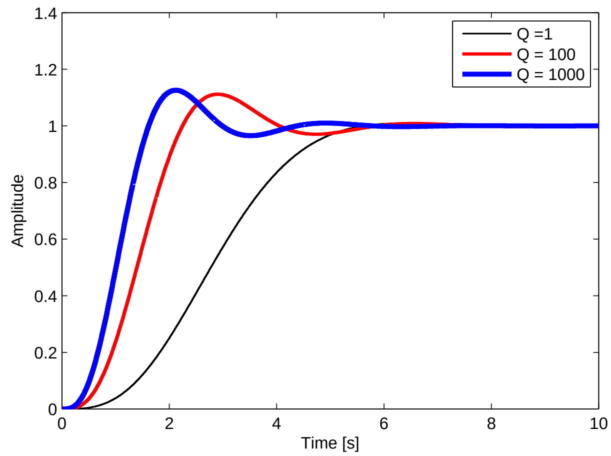

Figure 7.2: Step Response of system for varying Q and R = 1

7.2 Constant Q = diag(1,1,100,1000) and varying R

Now the Matrix Q was made constant and the controller output was penalized by varying R = 0.01, 1, 100. Figure 7.3 shows the controller output and 7.4 shows the step response of the system. Lesser the value of R, less penalty is placed on controller output and hence the magnitude of the control action can be quiet large. As a result, the response of the system is fast. As the value of R is increased, optimal gain matrix K is constructed in such a way that reduces the magnitude of the resulting control action. The response of the system is hence slower . Selecting appropriate weighing matrices Q and R is a trade off between response time of the system and resulting magnitude of the control action.

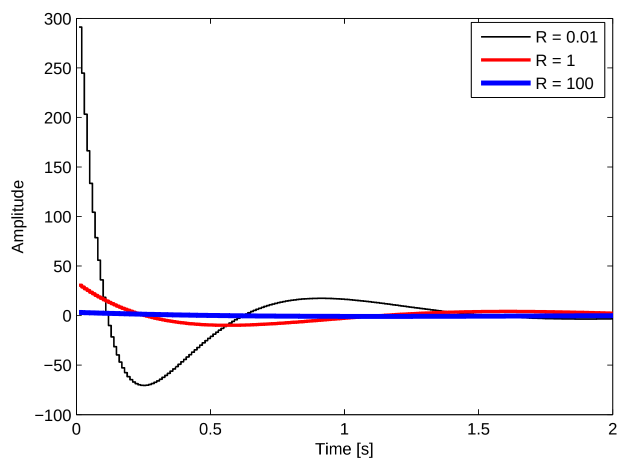

Figure 7.3: Output value of controller for varying R and Q = diag(1,1,100,1000)

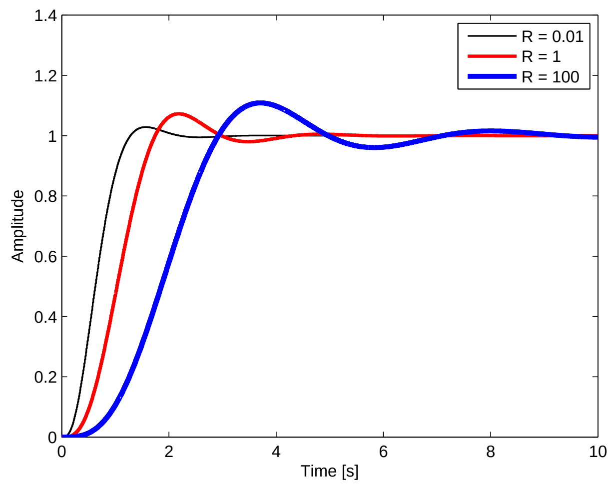

Figure 7.4: Step Response of system for varying R and Q = diag(1,1,100,1000)

7.3 LQ Controller

Now that the influence of Q and R on the response of the system have been discussed, a LQ controller that meets the control objectives is developed by tuning the Q and R matrices. Figure 7.5 shows the response of the system for reference step input. Settling time is 1.79 seconds with a overshoot of 2.86%. The Q matrix used to obtain this response is Q = diag(1,1,100,1000). High Penalty was placed on angular position to obtain a faster response and penalty of 100 was placed on angluar velocity (3rd state) which acts like damper to reduce the overshoot. Low penalty of R = 0.01 was placed on control action to achieve fast response

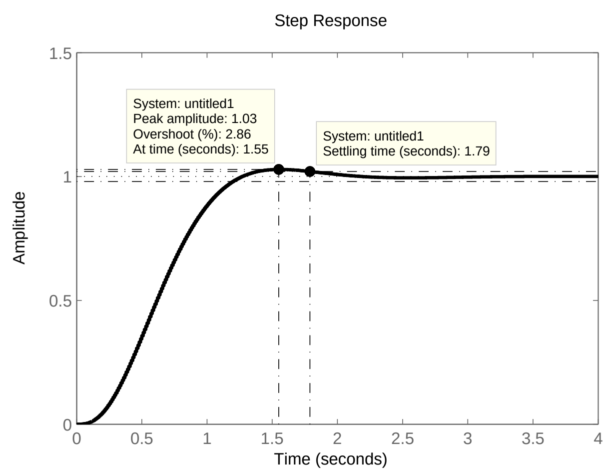

Figure 7.5: Step Response of closed loop system with LQ Controller

Until now, no limitations were placed on the magnitude of controller output and controllers were designed without taking the effects of actuator saturation in to account. In the following sections, the controller outputs of the controllers are examined and effects of actuator saturation are discussed. The effect of sampling period is also discussed.

8 Dealing with Actuator Saturation

In this section we examine the controller output of all the controllers designed in previous sections and impose limitations on the controller outputs. The controllers are then re tuned to avoid saturation in section 9, 10 and 11.

8.1 Control action generated by Discrete controllers

Figure 8.1 and 8.2 shows the output of the controller developed in section 4 for set-point tracking and disturbance rejection respectively.

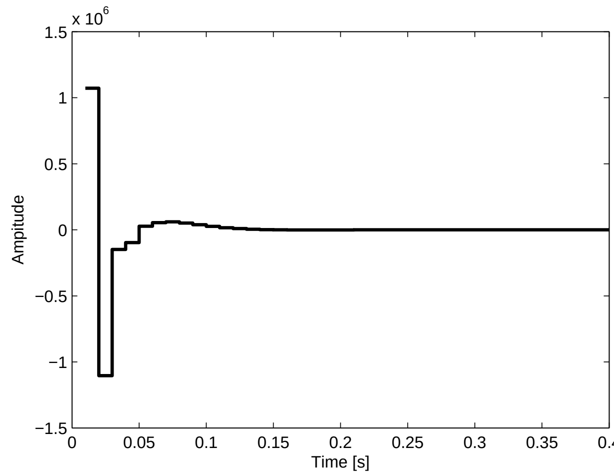

Figure 8.1: Discrete Time Set - point Tracking Controller output

The maximum amplitude of the control signal for tracking of a reference step input is approximately $\pm 1 \times 10^6$ (with settling time of approximately 0.1 seconds), which is a very huge value and can easily saturate the actuators.

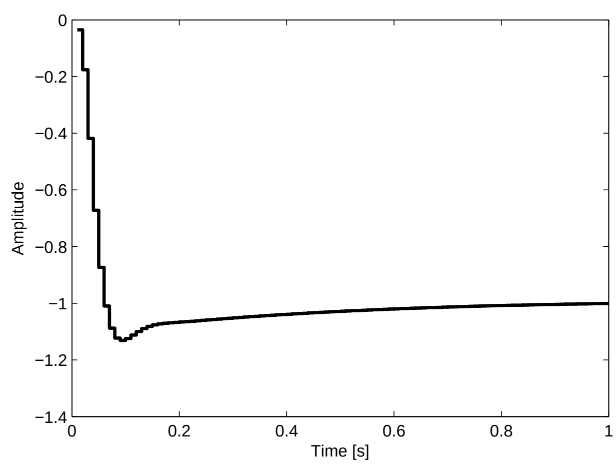

Figure 8.2: Discrete Time Disturbance Rejection Controller output

The maximum amplitude of the controller output for a unit step disturbance at the plant input is around -1.2

8.2 State Feedback Controller output

Figure 8.3 and 8.4 shows the output of the State Feedback Controller for set-point tracking and disturbance rejection. The magnitude of the controller output for set-point tracking in this case is within the limits $\pm 3 \times 10^5$ to achieve a settling time of 0.35 seconds. The output of the controller for disturbance rejection is < -1.8. Having faster poles as the poles of closed loop system to achieve fast response has resulted in high magnitude of control signal.

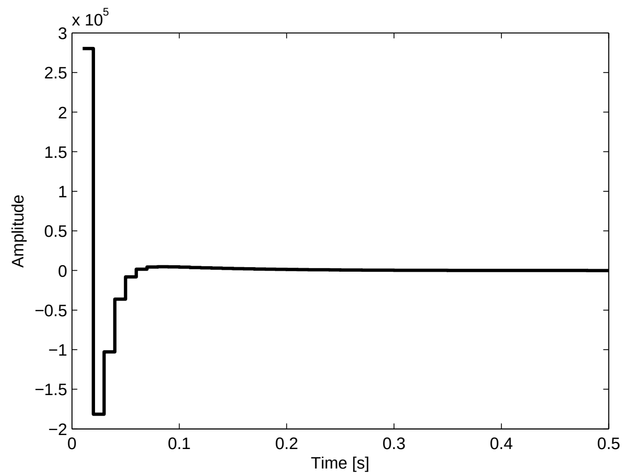

Figure 8.3: State Feedback Set - point Tracking Controller output

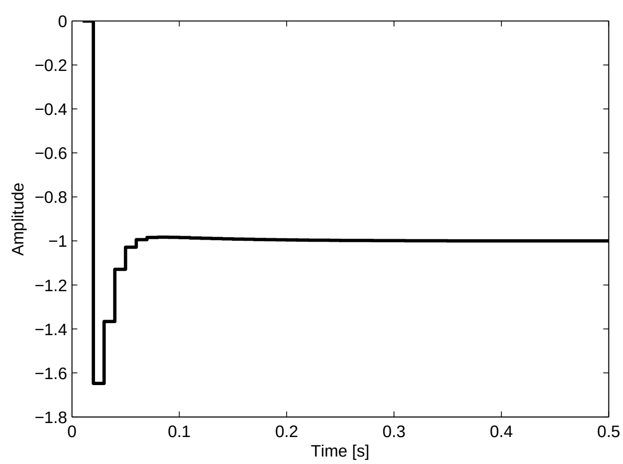

Figure 8.4: State Feedback Disturbance Rejection Controller output

8.3 Output Feedback Controller output

Figure 8.5 and 8.6 shows the controller output for setpoint tracking and disturbance rejection. They too have magnitude similar to the state feedback controller (since the controller poles have been placed at same locations as in state feedback controller). Magnitude of the control signal is higher in this case for disturbance rejection.

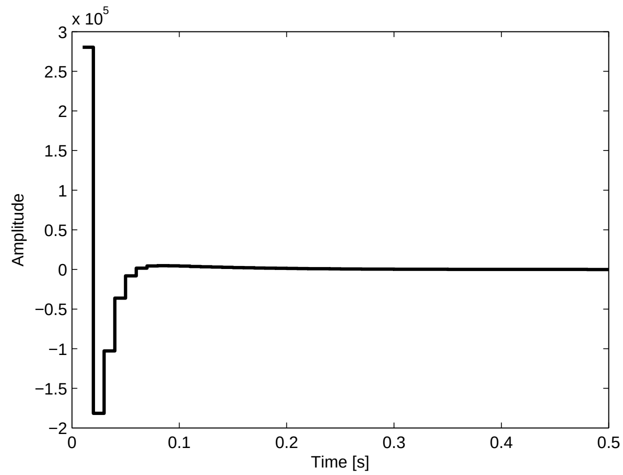

Figure 8.5: Output Feedback Set - point Tracking Controller output

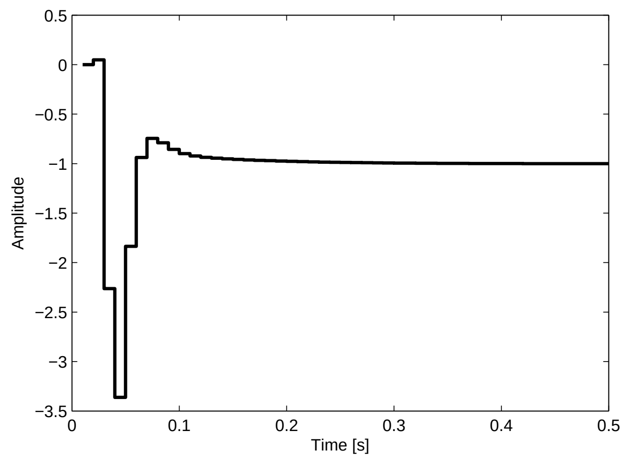

Figure 8.6: Output Feedback Disturbance Rejection Controller output

8.4 LQ Controller Output

Figure 8.7 and 8.8 shows the control signals generated for setpoint tracking and disturbance rejection. Here the magnitude of the control signal for setpoint tracking is way lower than the previous controllers and within limits ± 300. However the settling time in this case was 1.79 seconds. This is due to the nature of the LQ controller which optimizes the performance by considering the trade-off between the magnitude of control signal and deviation in the states through R and Q matrices respectively.

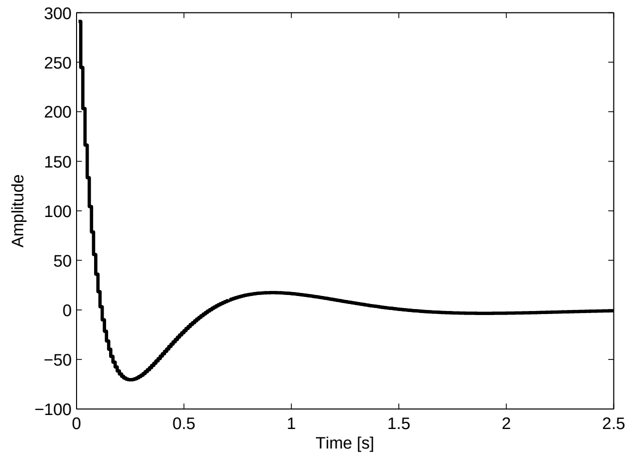

Figure 8.7: LQ Controller Set - point Tracking output

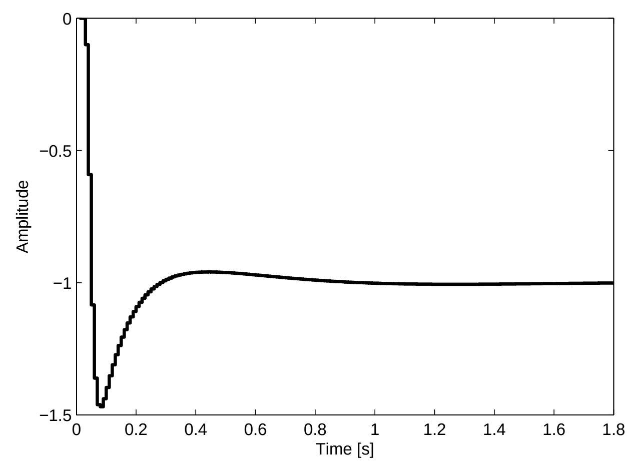

Figure 8.8: LQ Controller Disturbance Rejection output

Clearly , in all the controllers designed, we are trying to achieve a faster response resulting in larger magnitudes of control action. We did not take into account the limitations on controller output to avoid actuator saturation. By limiting the controller output to a value of 0.2, we need to re tune the controllers to avoid actuator saturation and still be able to achieve the control objective. Here the trade off between the magnitude of the control signal and the response time needs to be made among all the controller designs. Section 9 discusses the controllers designed in section 4 with actuator saturation. Section 10 revisits the Output feedback controller and in section 11 we tune the parameters Q and R to obtain a LQ controller which avoids actuator saturation.

9 Discrete Time Controllers revisited

In section 8, it was seen that the magnitude of control signal was quite large for tracking of reference step input. Hence the controller need to be changed to avoid saturation of actuator. With the limits on the controller output of ± 0.2 placed using a saturation block in simulink, the controller developed in section 4 (equation (4.2)) was tested for unit step reference input. Figure 9.1 shows the response of the system with a saturation block in place. It can be seen that the system tracks the reference input with settling time of 64 seconds after initial oscillatory behaviour during the rise time. It is to be noted that the controller continues to produce signals of magnitude beyond 0.2. The controller output here is only limited by a saturation block in simulink.

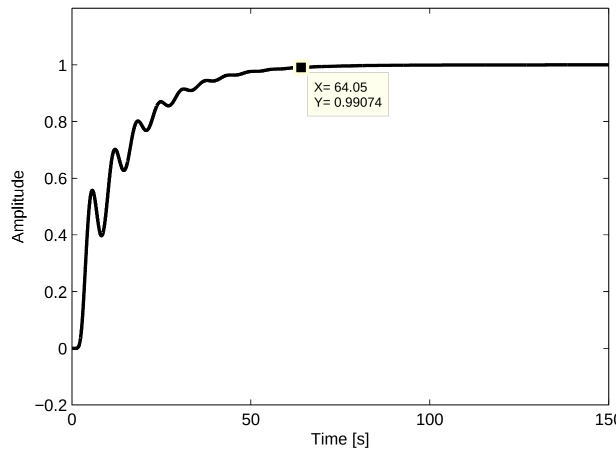

Figure 9.1: Step Response of Controller with saturation

In order to limit the output of the controller to 0.2, we use the PD Controller designed in Section 1 (equation (1.3)). The PD Controller is discretized using matlab command `c2d` with a sampling period of h = 0.01. Figure 9.2 shows the step response of the system for PD Controller. It can be seen that the settling time is 19 secs and from figure 9.3 it can be seen that the controller output is within limits to avoid saturation of actuator. Here in order to reduce the magnitude of control signal the response time was traded off.

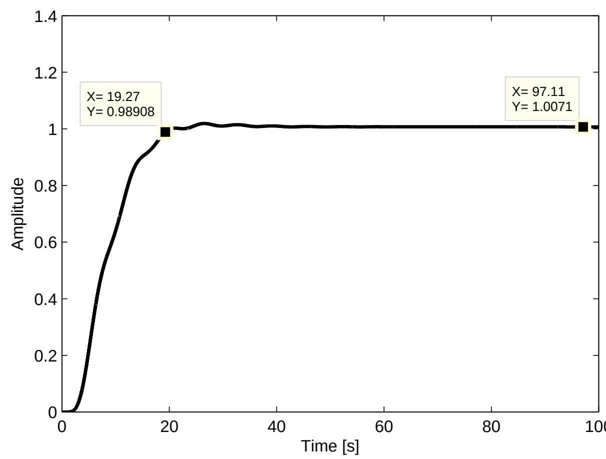

Figure 9.2: Step Response of Controller with saturation for PD Controller

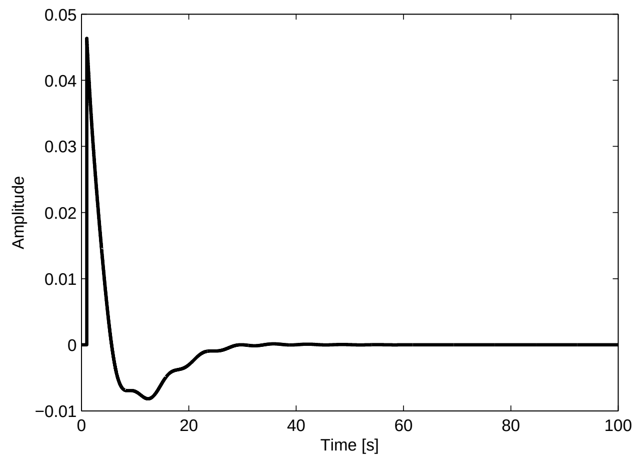

Figure 9.3: PD Controller output for reference step input

Regarding the disturbance rejection, the magnitude of the disturbance was scaled down to 0.1. The controller developed in section 4 for disturbance rejection (equation (4.3)) was tested and the controller output was already within the specified limits. Figure 9.4 shows the response of system for disturbance input and figure 9.5 shows the control signal generated by the controller. It can be seen that the controller output is within the prescribed limits

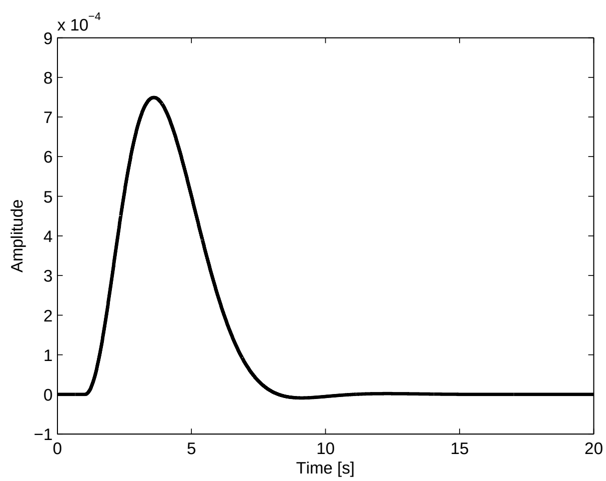

Figure 9.4: Step Response of Controller with saturation for Disturbance rejection of magnitude 0.1

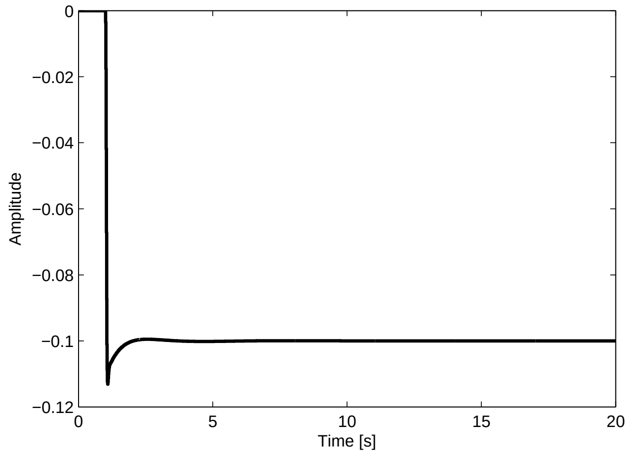

Figure 9.5: Controller output for disturbance rejection

10 Output Feedback Controller revisited

In section 8 it was seen that the output of the controller was beyond the limits of 0.2. In order to avoid the saturation of actuator, the slow poles were chosen as the desired closed loop poles. Earlier the desired closed loop poles were [0.2, 0.3, 0.9, 0.95]. The desired closed loop poles were chosen as [0.992, 0.993, 0.994, 0.995]. The numbers upto three decimal places were considered because the sampling interval of 0.01 was too small and higher decimal places become important when dealing with small sampling instants. Figure 10.1 shows the response of the system for tracking reference step input and figure 10.2 shows the plot of control action generated by the controller. It can be seen that the response time (settling time of approximately 14 - 16 seconds) has been compromised to keep the control signal within limits.

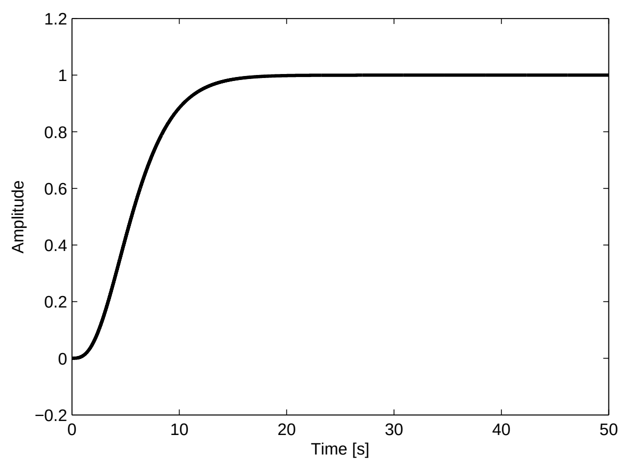

Figure 10.1: Step Response of Output Feedback Controller

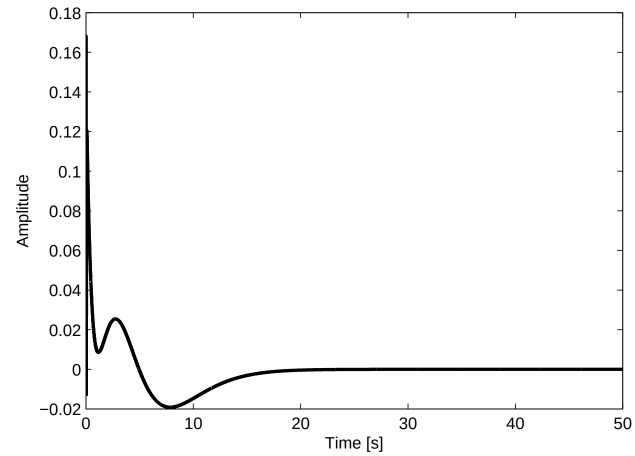

Figure 10.2: Control Signal generated by Output Feedback Controller for reference tracking

The same controller was tested for disturbance rejection with a step input of magnitude of 0.01 at plant input. Figure 10.3 shows the response of system for disturbance at plant input. It can be clearly seen that the disturbance is amplified and an offset is caused in the response. The control signal is however within the specified limits as can be seen from figure 10.4. The elimination of this offset is discussed in section 12.

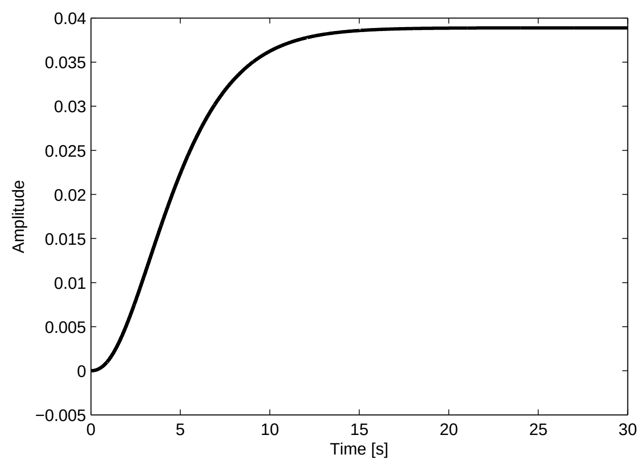

Figure 10.3: Response of system for disturbance rejection using Output Feedback Controller

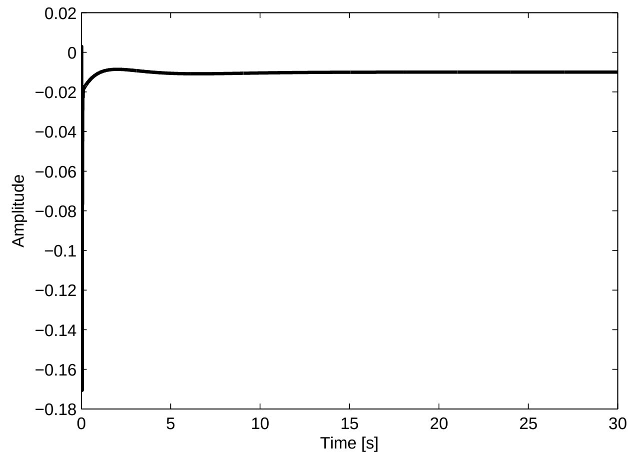

Figure 10.4: Control Signal generated by Output Feedback Controller for disturbance rejection

11 LQ Controller revisited

The LQ controller developed in section 7 have been tuned to avoid saturation of actuator. The penalty on the input was increased to limit the controller output. The response of the LQ Controller for reference step input is shown in figure 11.1. The weighing matrices chosen are R = 500 (high penalty on control action) and Q = diag(1, 1, 300, 20). High penalty on 3rd state (angular velocity) was placed to add damping to the response. The settling time obtained was 16.5 seconds. Figure 11.2 shows that the control signal generated is within the limits 0.2.

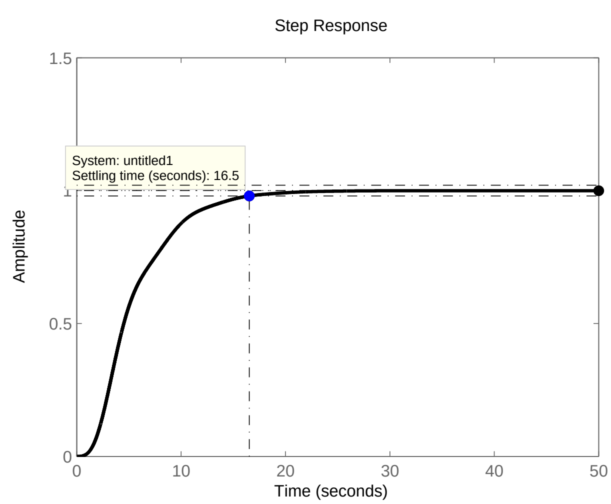

Figure 11.1: Step Response of LQ Controller

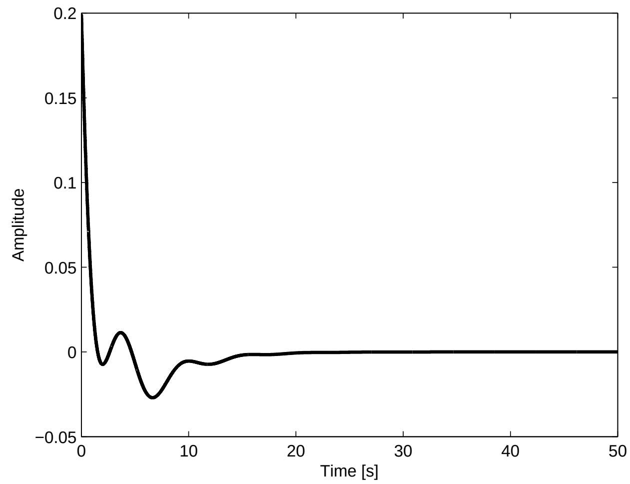

Figure 11.2: Control Signal generated by LQ Controller for reference tracking

Similar to section 10, the LQ controller was evaluated for step disturbance input at plant input of magnitude 0.01. Figure 11.3 shows the response of the system for disturbance input. In this case, we do have a offset induced by the disturbance. Section 12 provides a solution to get rid of the offset/steady state error. Figure 11.4 shows the generated control signal.

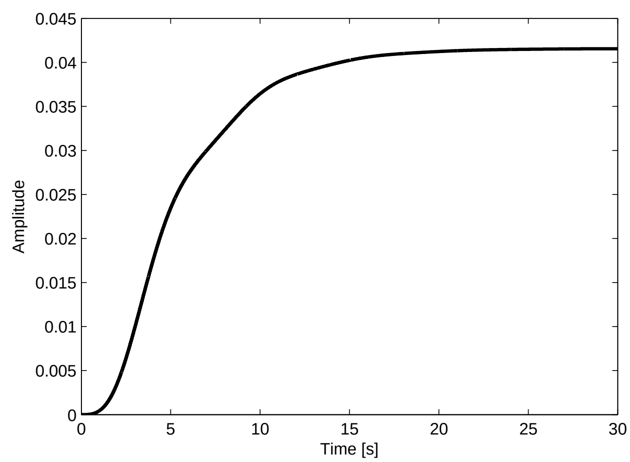

Figure 11.3: Response of system for disturbance rejection using LQ Controller

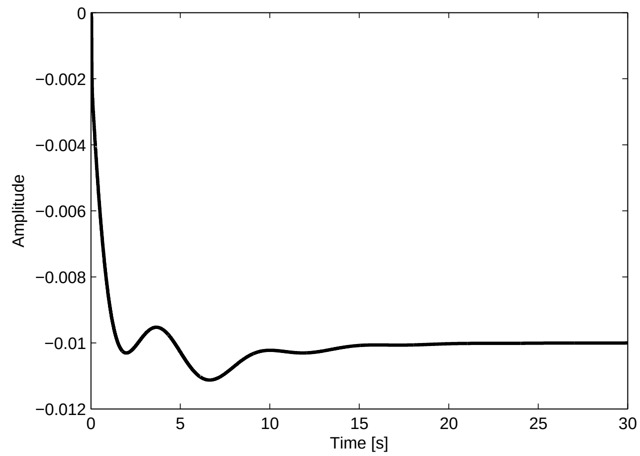

Figure 11.4: Control Signal generated by LQ Controller for disturbance rejection

12 Dealing with steady state errors

In section 10 and 11, it was seen that the steady state error/offset becomes significant for a step disturbance at plant input when the controller output is limited to 0.2. To completely remove this offset an integrator was added which integrates the error between the output and the reference and provides it as control input to the plant in addition to existing state/output feedback or LQ Controller.
Figure 12.1 shows the block diagram generated in simulink which shows the addition of integrator. This scheme was used in State/Output Feedback controllers and LQ controllers to remove the steady state error during disturbance rejection.

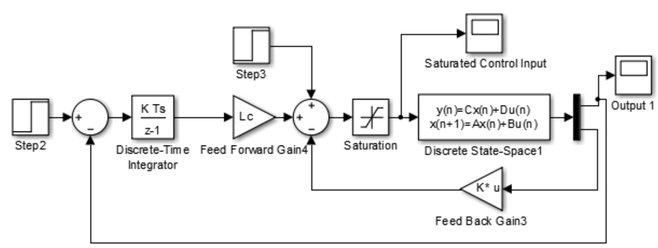

Figure 12.1: Simulink model of the controller with integrator

12.1 Elimination of Steady State error for Output Feedback Controller

Figure 12.2 shows the response of the system for step disturbance input of magnitude 0.01 acting at plant input. It can be seen that the steady state error is completely removed and the output dies down to in 40 seconds. Figure 12.3 shows that the control signal generated is still within the limits. Same results were obtained for State Feedback Controller as well.

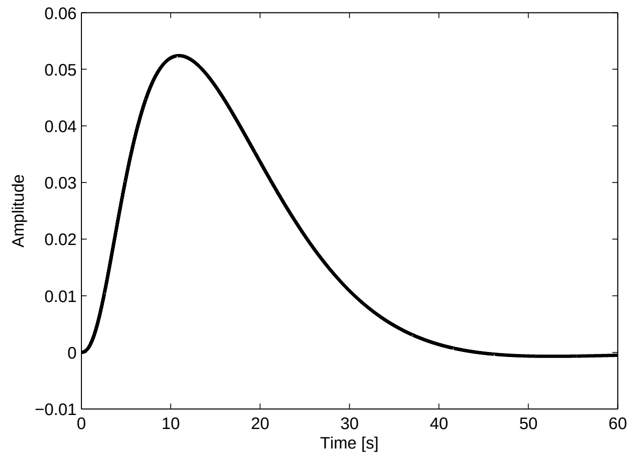

Figure 12.2: Output of system for step disturbance input for output feedback controller with integrator

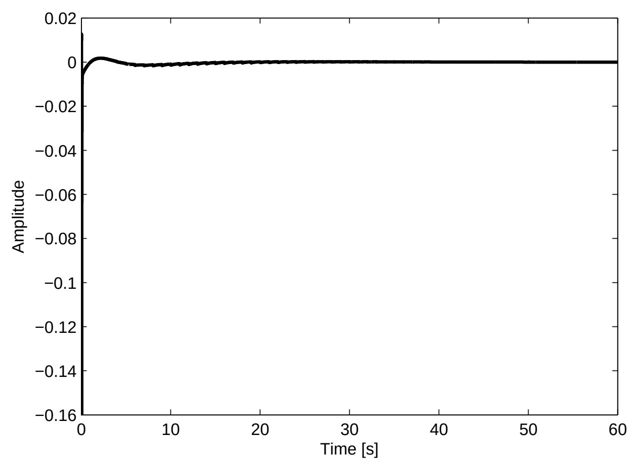

Figure 12.3: Control Signal generated by output feedback controller with integrator

12.2 Elimination of Steady State Error for LQ Controller

Similar to previous section, figure 12.4 shows that the addition of integrator is able to remove the steady state error in LQ Controller while the control signal generated is within the limits (figure 12.5).

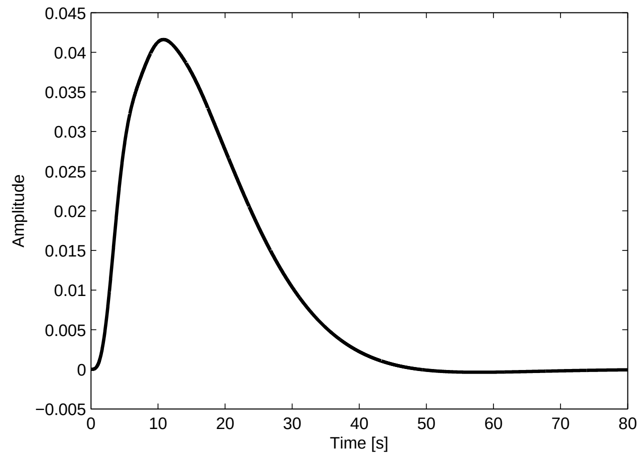

Figure 12.4: Output of system for step disturbance input for LQ controller with integrator

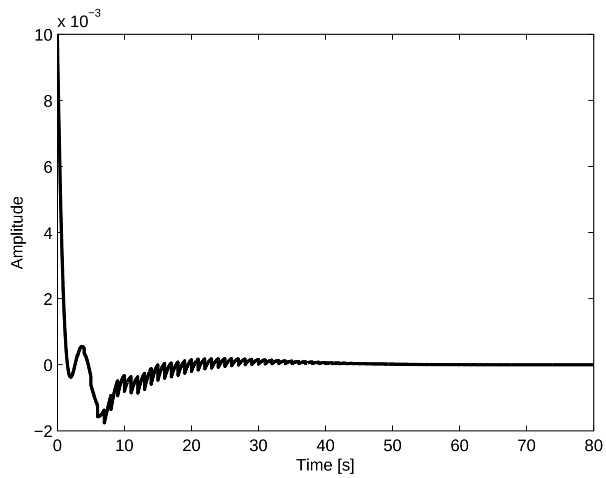

Figure 12.5: Control Signal generated by LQ controller with integrator

13 Handling Delays

A computational delay of one sampling interval was introduced in the simulink models to see the effects of delay on the performance of the controllers developed. In this case since sampling period of 0.01 seconds was very small, the delay made no significant effect on the performance of the controllers. In order to understand the effects of delay, the sampling period was increased to 0.1 seconds and the output feedback controller developed in section 10 was simulated. Figure 13.1 shows the response of the system when the sampling time was increased to 0.1 seconds. It can be seen that the system exhibits a very different performance than expected. There is a very large overshoot before the controller settles to the reference value. The settling time to has increased to around 150 seconds.

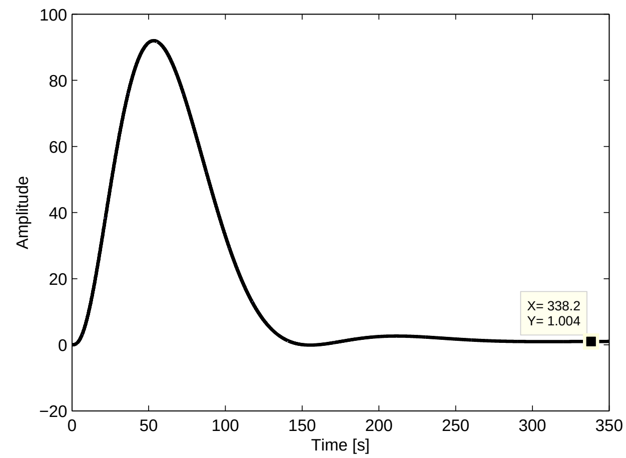

Figure 13.1: Output of system for step reference input for output feedback controller

In the following the Output Feedback Controller was re-tuned to achieve the required control performance by augmenting the state space model with the delayed input. The augmented state space model is given as follows

$$\begin{bmatrix} x(kh+h) \\ u(kh) \end{bmatrix} = \begin{bmatrix} \Phi & \Gamma_1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x(kh) \\ u(kh-h) \end{bmatrix} + \begin{bmatrix} \Gamma_0 \\ I \end{bmatrix} u(kh) \tag{13.1}$$

where $\Phi = e^{Ah}$, $\Gamma_0 = B$ and $\Gamma_1 = \int_0^h e^{As} ds B$

With the above augmented state space model and the desired pole locations of the system changed to [0.95, 0.93, 0.9, 0.85, 0.8], the response of the system obtained for step reference input is shown in figure 13.2. Figure 13.3 shows the control signal generated. It can be seen that the control signal too is within the limits of ± 0.2

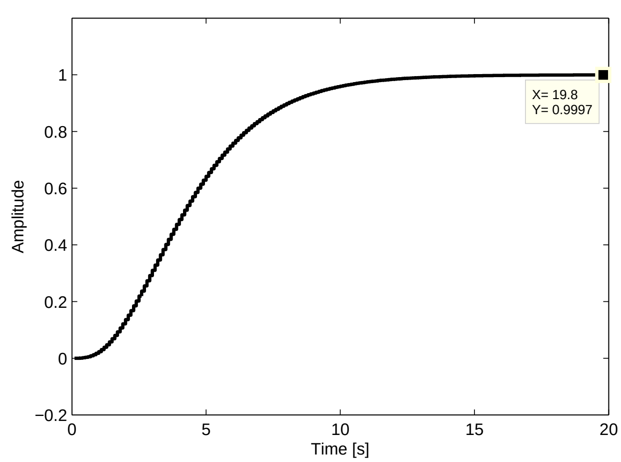

Figure 13.2: Response of the output feedback controller

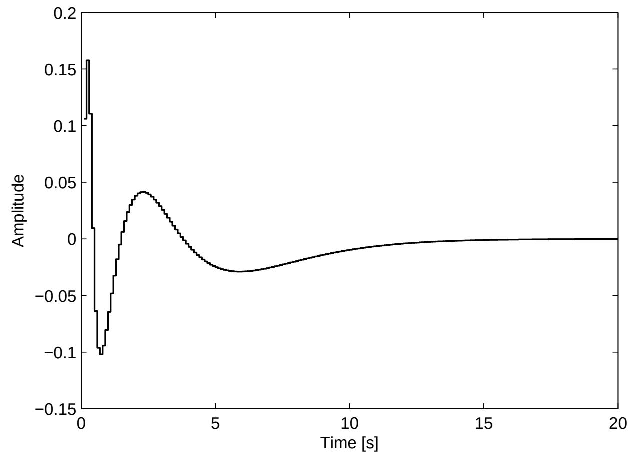

Figure 13.3: Control Signal generated by the output feedback controller

Now the Output feedback controller with integrator developed in section 12 was tested with the augmented state space model with a step disturbance of magnitude 0.01 acting on the plant input. Figure 13.4 shows the response of the system, we can see that the controller is able to reject the disturbances and the control signal generated (figure 13.5) is also within the specified limits.

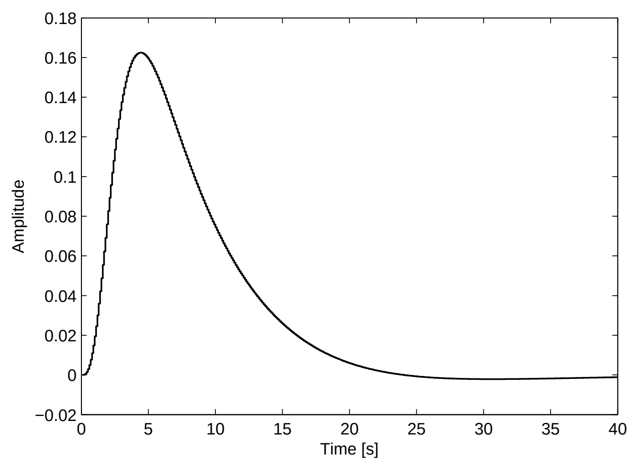

Figure 13.4: Response of the output feedback controller for disturbance rejection

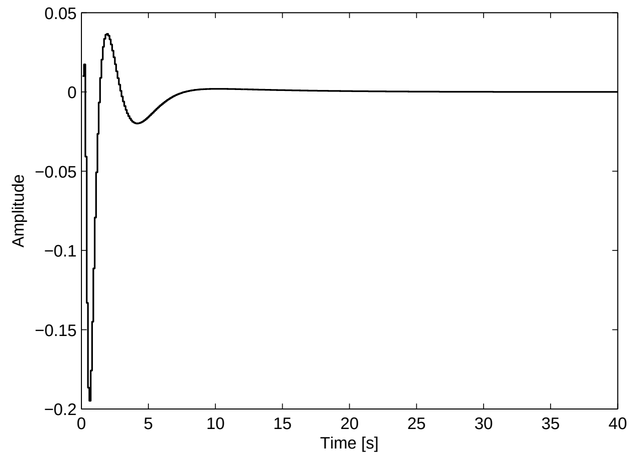

Figure 13.5: Control Signal generated by the output feedback controller for disturbance rejection

Conclusion

In this assignment, a satellite attitude control system was developed using tools from Continuous and Discrete Time Control system theory. The system was modeled as a fourth order transfer function which was used for design and evaluation of controllers. Tools from classical control theory (P, PD, PID, Phase Lead etc) and from modern control theory (Pole Placement, Observers, LQ Control) were used to develop controllers. The controllers were first developed using classical control theory in continuous time in section 1. It was found that a PD Controller could achieve a minimal settling time of 17.8 seconds and lesser settling time could be achieved only by cancellation of the complex pole of the plant. Such a controller was also developed and a minimal settling time of 0.08 seconds could be achieved. These controllers were also tested for the disturbance rejection abilities in section 2 and an integrator was introduced in the controller to remove the steady state error caused by step disturbance at plant input.
The given transfer function was converted in state space in continuous time in section 3. This model was then converted to discrete model using zoh sampling of the continuous time state space model with a sampling period of 0.01 seconds. The controllers of section 1 and 2 were converted into discrete time using tustin approximation and it was seen that the zoh approximation could make a stable continuous system unstable in discrete time. In section 5, a state feedback controller was developed followed by adding a dynamic observer in section 6. The effect of location of closed loop poles on response of the system and rate of convergence of observer estimates to the true states were studied. In section 7, LQ controller was developed and effects of Q and R Matrices on response of system was studied. While the pole placement technique allows us to explicitly specify the desired closed loop poles, the magnitudes of control signals can be quiet large. This is where LQ Controller scores, in that we can obtain a balance between speed of response and magnitude of the generated control signals. However, tuning Q and R matrices can be quite tedious when compared to selecting the closed loop poles in pole placement technique
Until now, the controllers were developed to achieve faster response ignoring the magnitude of control signal generated in the process. Limitations were posed on the output of the controllers and all the controllers were retuned. It was seen that all the controllers developed previously generated control signals of very large magnitude. In section 9, PD controller of section 1 was revisited and it was found that the PD controller achieved the desired performance and control signal generated was within the limits ± 0.2. In Section 10 and 11, the desired closed loop poles selected were the slow poles so that the controller output magnitude can be limited to specified limits. In such cases, it was seen that the steady state error can be significant during the disturbance rejection and hence an integrator was introduced to drive the steady state error to zero in section 12. While trying

to avoid saturation of the actuators, speed of response was compromised and settling time of all the controllers were around 15 - 20 seconds. In section 13, effects of delay was assessed. Due to small sampling period of 0.01 seconds, the delay made no significant difference in the response of the system for reference step input and step disturbance at plant input. However, when the sampling period was increased from 0.01 seconds to 0.1 seconds, we witnessed a very high overshoot and larger settling time of around 150 seconds. To incorporate the delay, the state space model was augmented to accommodate the delayed input signal and the output feedback controller was retuned to obtain a desired performance with settling time of around 12 - 14 seconds.